Emergence of inflaton potential from asymptotically safe gravity

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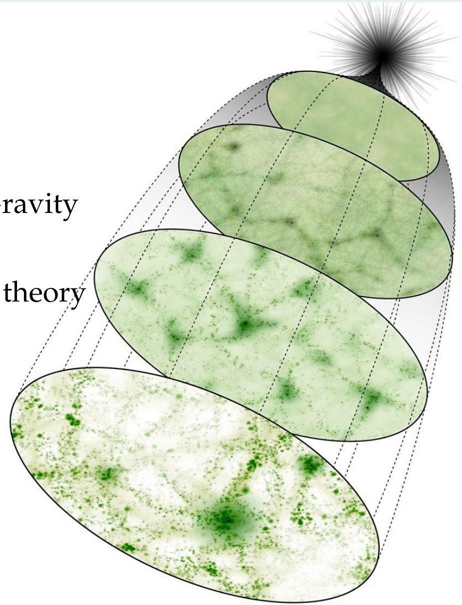
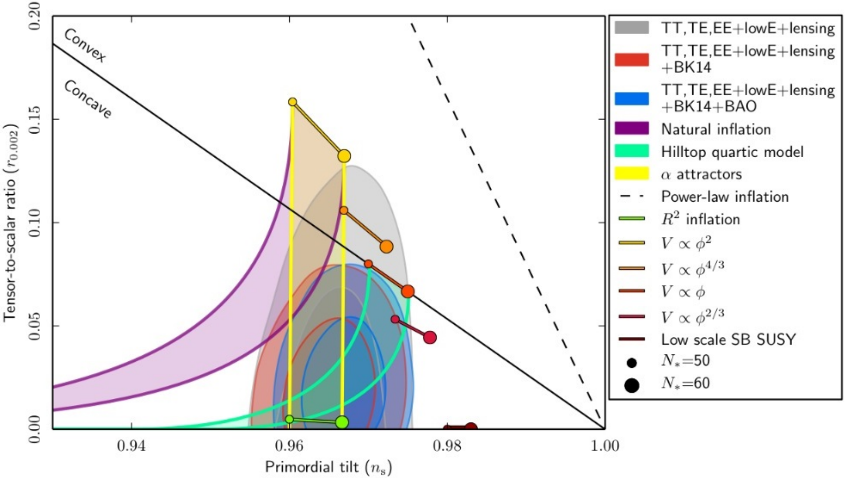
Presentation Overview

1. Cosmological Inflation - Some open questions
2. Asymptotic Safety Hypothesis for Quantum Gravity
3. Renormalization Group Flow of Scalar-Tensor theory
4. Emergence of Inflation from the UV
5. Conclusions



Cosmological Inflation - Some open questions



[1]

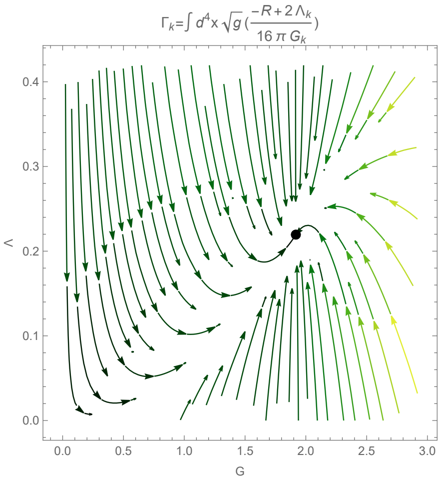
- Where do these models come from? Can we build one from fundamental physics?
- Can we predict the initial conditions of the inflaton field?

$\Downarrow$

Using Asymptotic Safety, we can!

[1] Credit image: Planck 2018 results. X. Constraints on inflation. arXiv:1807.06211

Asymptotic Safety



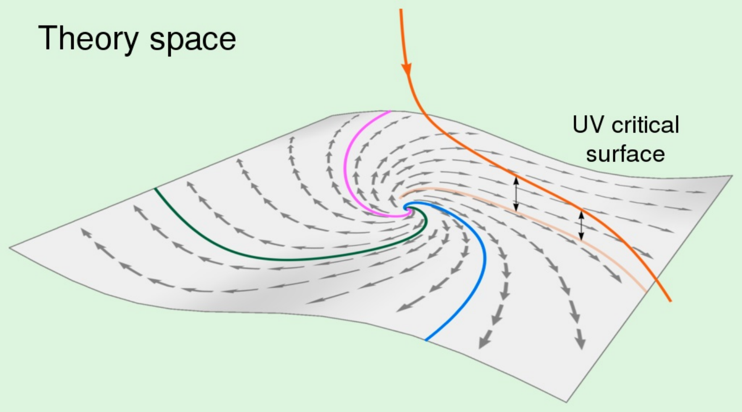
Wetterich RG flow Equation

$$k \frac{d\Gamma_k}{dk} = \frac{1}{2} \mathbf{Tr}\left[\frac{k \frac{dR(k)}{dk}}{\left(\frac{\delta^2 \Gamma_k}{\delta \phi^2} + R(k) \right)} \right]$$

1. Ansatz Wilsonian effective action:
   $\Gamma_k = \sum_{n=1}^{N} \tilde{g}_k^n \, \mathcal{O}_n \qquad 0 \leq k < \infty$
2. Get beta functions of the couplings:
   $k \frac{\partial g_n}{\partial k} = \beta^n$
3. Search for UV-fixed point:
   $\beta|_* = 0 \, , \, g_* \neq 0$

FRGE: Christof Wetterich. arXiv:1710.05815

Predictive Power of Asymptotically Safe QFTs



UV-Fixed point with finite amount of attractive directions

$\Downarrow$

Finite amount of free parameters

=

Finite amount of measurements needed

$\Downarrow$

Predictive power!

What can we do with Asymptotically Safe QFTs?

We can make predictions coming from the UV-fixed point!

$$\Gamma(k=\infty) = \textit{UV fixed point action}$$
$$\Downarrow$$
$$\Gamma(k=0) = \textit{Full effective action}$$

$\rightarrow$ Study the phenomenology of the emergent $\Gamma(k=0)$

4D Scalar-Tensor EFT: RG Flow

① Scalar-Tensor Model: Unknown $f(k,\phi^2)$ and $v(k,\phi^2)$ $\qquad (\phi^2 \equiv G_k \varphi^2)$

$$\Gamma(k) = \int d^4x\sqrt{g}\Big(\frac{-R}{16\pi G_k} + \frac{2\lambda_k}{(16\pi G_k)^2} - \frac{f(k,\phi^2)}{16\pi G_k}R + \frac{v(k,\phi^2)}{(16\pi G_k)^2} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\Big)$$

$\Downarrow$

② **RG flow** of f and v given by **Non-linear PDEs of second order**

$\Downarrow$

③ **Shift-symmetric** ($\phi \to \phi + c$), **UV-fixed point**

$$\lim_{k\to\infty}\{G_k,\lambda_k\} = \Big\{\frac{48\pi}{41}, \frac{6912\pi^2}{1681}\Big\} \qquad \lim_{k\to\infty} f(k,\phi^2) = \lim_{k\to\infty} v(k,\phi^2) = 0$$

4D Scalar-Tensor EFT: RG Flow

Scalar-Tensor Model: Unknown $f(k,\phi^2)$ and $v(k,\phi^2)$ $\quad (\phi^2 \equiv G_k \varphi^2)$

$$\Gamma(k) = \int d^4x \sqrt{g}\left(\frac{-R}{16\pi G_k} + \frac{2\lambda_k}{(16\pi G_k)^2} - \frac{f(k,\phi^2)}{16\pi G_k} R + \frac{v(k,\phi^2)}{(16\pi G_k)^2} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi \right)$$

$$\Downarrow$$

④ The RG flow has **4 degrees of freedom**, but 2 are fixed by observations!

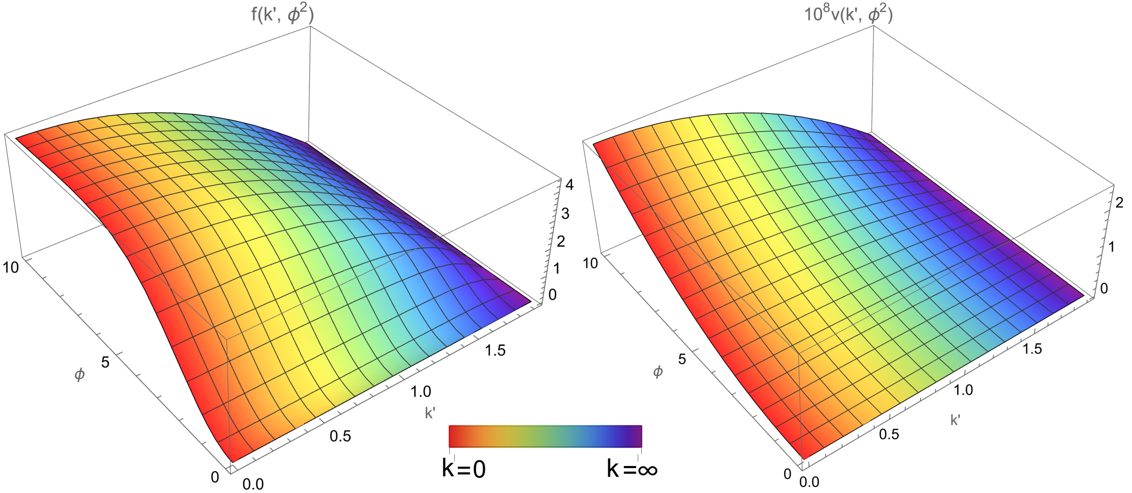
$\tilde{G}_0 \simeq 6.71 \times 10^{-39} GeV^{-2}$, $\lambda_0 \simeq 10^{-120}$,

$$m_0^f \equiv \frac{\partial^2 f}{\partial \phi^2}\Big|_{\phi=0,k=0} \qquad m_0^v \equiv \frac{\partial^2 v}{\partial \phi^2}\Big|_{\phi=0,k=0}$$

Only 2 free parameters: m_0^f, m_0^v

4D Scalar-Tensor EFT: Numerical Solutions

$m_0^f = 0.42 \qquad m_0^v = 2.75 \times 10^{-10}$



Emergence of Inflation from the UV

By means of a conformal transformation ($g \to g_E$), one goes to the Einstein frame

$$\Gamma_k = \int d^4x \sqrt{g_E}\left(-\frac{R_E}{16\pi G_k} + \frac{V_{eff}(k,\phi(\sigma))}{(16\pi G_k)^2} + \frac{\partial_\mu \sigma \partial^\mu \sigma}{2\,G_k}\right)$$

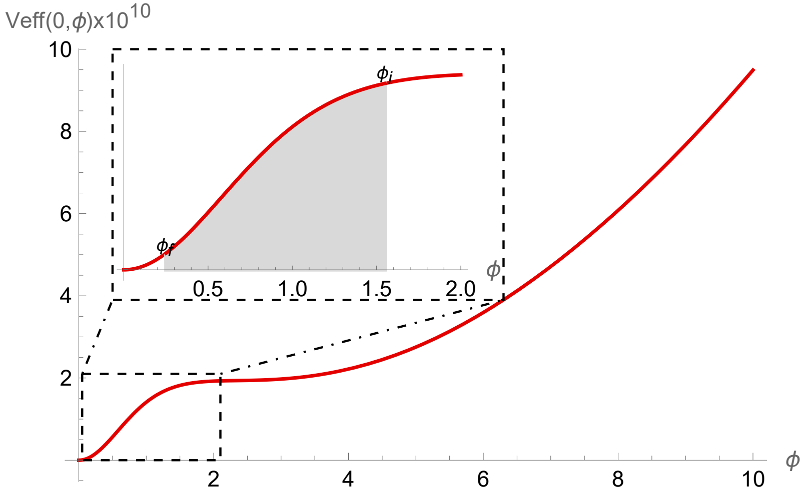
Where the effective potential turns out to be

$$\boxed{V_{eff}(k,\phi(\sigma)) \doteq \frac{2\lambda_k + v(k,\phi^2(\sigma))}{(1+f(k,\phi^2(\sigma)))^2}}$$

with the UV and IR limits

$$\lim_{k\to\infty} V_{eff}(k,\phi(\sigma)) = 2\lambda_* \quad \Big| \quad \lim_{k\to 0} V_{eff}(k,\phi(\sigma)) = \frac{2\lambda_0 + v(0,\phi^2(\sigma))}{(1+f(0,\phi^2(\sigma)))^2}$$

The inflaton potential emerged from Quantum Gravity!



Slow-Roll Inflation

$\phi_i = 1.55$

$\phi_f = 0.25$

$n_s \simeq 0.965$

$r \simeq 0.005$

$N_{ef} \simeq 66$

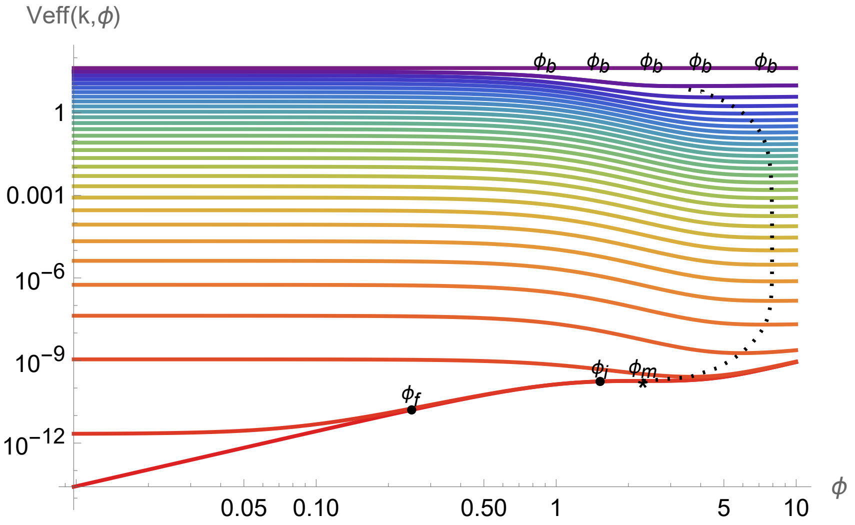
$A_s \simeq 2.06 * 10^{-9}$

Emergence of Inflation from the UV

Universe starts near UV-fixed point with arbitrary initial value of ϕ (shift-symmetry)

$\Downarrow$

Adiabatical evolution gives initial condition of inflaton!



Take home messages

We studied the Renormalization Group Flow of scalar-tensor theories and found a UV-Fixed point (Asymptotic Safety).

$\Downarrow$

We connected the UV with the IR by solving the RG flow equations and obtained non trivial emergent potentials.

$\Downarrow$

The emergent potentials could give rise to an inflationary period fitting current observations.

$\Downarrow$

Following RG flow from the UV to the IR we explained initial conditions inflaton.

☺ Thank you for your attention ☺

Appendix 0: RG Flow Equations

We studied these kind of scalar tensor theories

$$\boxed{\Gamma(k) = \int d^4x\sqrt{g}\left(-F(k,\varphi)R + V(k,\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)}$$

$$\Downarrow$$

Non-linear, second order RG flow equations [2]

$$k\,V^{(1,0)}(k,\varphi) = \varphi V^{(0,1)}(k,\varphi) - 4V(k,\varphi) + \frac{1}{16\pi^2} + \frac{3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)}{32\pi^2\left(3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)\left(V^{(0,2)}(k,\varphi)+1\right)\right)}$$

$$k\,F^{(1,0)}(k,\varphi) = \varphi F^{(0,1)}(k,\varphi) - 2F(k,\varphi) + \frac{37}{384\pi^2} + \frac{F(k,\varphi)\left(\left(3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)\right)\left(-3F^{(0,2)}(k,\varphi) + 3V^{(0,2)}(k,\varphi) + 1\right) + 2F(k,\varphi)V^{(0,2)}(k,\varphi)^2\right)}{96\pi^2\left(3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)\left(V^{(0,2)}(k,\varphi)+1\right)\right)^2}$$

[2] Flow Equations: Roberto Percacci, Gian Paolo Vacca. arXiv:1501.00888

Appendix 1: Conformal Transformation

The conformal transformation to go from the Jordan frame to the Einstein frame is

$$g^E_{\mu\nu} \equiv (1 + f(k, \phi^2))g_{\mu\nu}$$

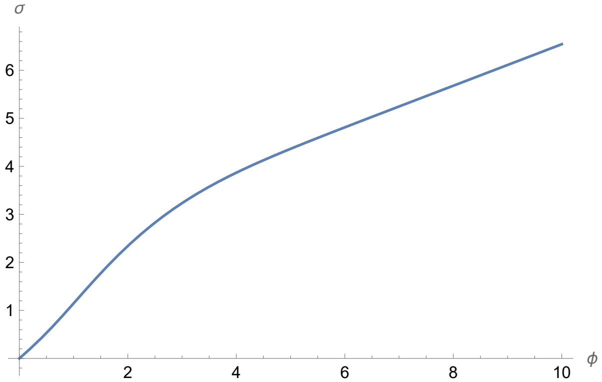
$$\Downarrow$$

For slow-roll inflation this gives the relations at $k = 0$

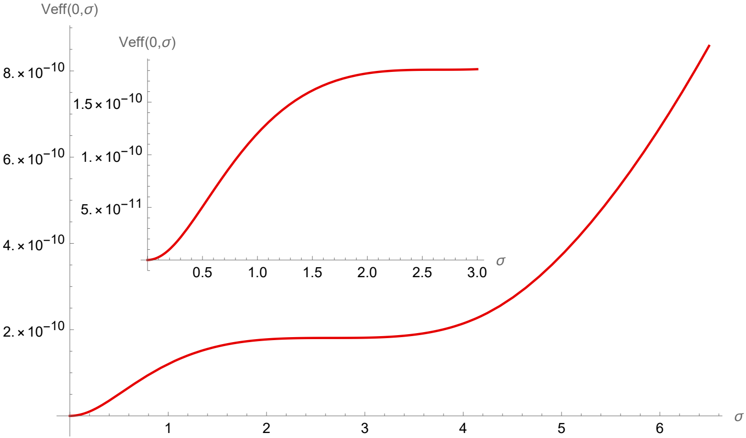
$$\phi_i = 1.55 \rightarrow \sigma_i \simeq 1.8$$

$$\phi_f = 0.25 \rightarrow \sigma_f \simeq 0.26$$

$$\left(\frac{\partial\sigma}{\partial\phi}\right)^2 = \frac{1}{(1 + f(k, \phi^2))} + \frac{3\,(f'(k, \phi^2))^2}{(1 + f(k, \phi^2))^2}$$



Appendix 1: Conformal Transformation



One can of course, compute the potential as a function of the new field σ. The overall shape is similar to the shape as a function of ϕ.

$$\sigma_i \simeq 1.8$$
$$\sigma_f \simeq 0.26$$
$$n_s \simeq 0.965$$
$$r \simeq 0.005$$
$$N_{ef} \simeq 66$$
$$A_s \simeq 2.06 * 10^{-9}$$

Appendix 2: Slow-roll observables

$$\left(\frac{\partial \sigma}{\partial \phi}\right)^2 = \frac{1}{(1 + f(k, \phi^2))} + \frac{3\,(f'(k, \phi^2))^2}{(1 + f(k, \phi^2))^2}$$

Einstein Frame	Jordan Frame
$\epsilon = \frac{1}{8\pi}\frac{1}{2}\left(\frac{V'_{eff}(\sigma)}{V_{eff}(\sigma)}\right)^2 \quad \eta = \frac{1}{8\pi}\left(\frac{V_{eff}(\sigma)''}{V_{eff}(\sigma)}\right)$	$\epsilon = \frac{1}{8\pi}\frac{1}{2}\left(\frac{V'_{eff}(\phi)}{V_{eff}(\phi)}\right)^2\left(\frac{\partial \sigma}{\partial \phi}\right)^{-2}$
$n_s = 1 - 6\,\epsilon + 2\,\eta \quad r = 16\,\epsilon$	$\eta = \frac{1}{8\pi}\left(\left(\frac{V_{eff}(\phi)''}{V_{eff}(\phi)}\right) - \left(\frac{V'_{eff}(\phi)}{V_{eff}(\phi)}\right)\frac{\frac{\partial^2 \sigma}{\partial \phi^2}}{\frac{\partial \sigma}{\partial \phi}}\right)\left(\frac{\partial \sigma}{\partial \phi}\right)^{-2}$
$A_s = \frac{1}{4}\frac{V_{eff}}{24\pi^2 \epsilon} \quad N_{ef} = 8\pi \int_{\sigma_i}^{\sigma_f} \frac{d\,\sigma}{\sqrt{2\,\epsilon}}$	$n_s = 1 - 6\,\epsilon + 2\,\eta \quad r = 16\,\epsilon$
	$A_s = \frac{1}{4}\frac{V_{eff}}{24\pi^2 \epsilon} \quad N_{ef} = 8\pi \int_{\phi_i}^{\phi_f} \frac{d\,\phi}{\sqrt{2\,\epsilon}}\frac{\partial \sigma}{\partial \phi}$

Appendix 3: Dimensionless variables

The determination of the RG flow and the existence of fixed points is usually done for dimensionless variables. In our case, we start with dimenisonful ($\tilde{}$)

$$\Gamma(k) = \int d^4\tilde{x}\sqrt{g}(-\tilde{F}(k,\tilde{\varphi})\tilde{R} + \tilde{V}(k,\tilde{\varphi}) + \frac{1}{2}\partial_\mu\tilde{\varphi}\partial^\mu\tilde{\varphi})$$

and end up with

$$\Gamma(k) = \int d^4x\sqrt{g}(-F(k,\varphi)R + V(k,\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi)\,,$$

where $F \equiv \frac{\tilde{F}}{k^2}$, $V \equiv \frac{\tilde{V}}{k^4}$, $\varphi \equiv \frac{\tilde{\varphi}}{k}$, $R \equiv \frac{\tilde{R}}{k^2}$ and $x \equiv \tilde{x}\,k$.
More in detail, if one has individual couplings instead of functionals, like the newton coupling, one uses $G_k \equiv \tilde{G}_k\,k^2$.
Notice that since $\lim_{k\to\infty} G_k = G_*$ then $\lim_{k\to\infty} \tilde{G}_k = 0$.

Appendix 4: Fixing Constant Dimensionless field on the flow

One is usually used to writing the flow equations in terms of the dimensionless variables φ defined as $\tilde{\varphi} = \frac{\varphi}{k^{d(d-1)/2}}$, and keeping them constant as functions of k. For this, one usually defines, for example $\tilde{F}(k, \tilde{\varphi}) = k^2 F(t, \varphi)$. In this work we used other variables defined as $\psi = \phi^2 = G\varphi^2$, and we kept them constant as functions of k. This amounts to the transformations

$$F(t, G\varphi^2) \doteq f(t, \psi)$$

$$F^{(0,n)}(t, \varphi) \to G^n f^{(0,n)}(t, \psi)$$

$$\frac{\partial F^{(0,n)}(t, \varphi)}{\partial t} \to G^n \left(\frac{n\beta_G}{G} f^{(0,n)}(t, \psi) + f^{(1,n)}(t, \psi) + \psi \frac{n\beta_G}{G} f^{(0,n+1)}(t, \psi) \right)$$

Appendix 5: Final Equations S_{UV}

The change of variables realized to compactify the domain is $k' \equiv G(k)$. Furthermore, we also used $\psi \equiv \phi^2$. This amounts to replacing $f(k,\phi^2) \to f(G,\psi)$ and $v(k,\phi^2) \to v(G,\psi)$, and the respective derivatives $kf^{(1,0)}(k,\phi^2) \to f^{(1,0)}(G,\psi)\beta_G$ and $k\,v^{(1,0)}(k,\phi^2) \to v^{(1,0)}(G,\psi)\beta_G$, and the obvious chain rule for ψ. This allows us to solve the equations in the domain $G \in [0, G_*]$ instead of $k \in [0,\infty]$. The resulting equations are

$$
\begin{aligned}
0 = {} & \frac{256\pi G^7 \left(4\pi(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right) - v^{(0,1)}(G,0)\left(3\psi f^{(0,1)}(G,\psi)^2 + 4\pi(f(G,\psi)+1)\right)\right)}{\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)} \\
& - \frac{\left(24576\pi^3G^7f^{(0,1)}(G,0) - G\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)\right)\left(2v(G,\psi)-\psi v^{(0,1)}(G,\psi)\right)}{24\pi\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
& - \frac{G^2\left(\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)-24576\pi^3G^6f^{(0,1)}(G,0)\right)v^{(1,0)}(G,\psi)}{48\pi\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
& + G\left(-2\psi v^{(0,1)}(G,\psi)+4v(G,\psi)\right)
\end{aligned}
$$

Appendix 5: Final Equations S_{UV}

$$
\begin{aligned}
0 = &-\frac{2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)-49152\pi^3G^6f^{(0,1)}(G,0)}{2G^2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2}\\
&-\frac{\psi\left(24576\pi^3G^6f^{(0,1)}(G,0)-\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)\right)f^{(0,1)}(G,\psi)}{G^2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2}\\
&-\frac{\left(\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)-24576\pi^3G^6f^{(0,1)}(G,0)\right)f(G,\psi)}{G^2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2}\\
&-\frac{\left(24576\pi^3G^6f^{(0,1)}(G,0)-\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)\right)f^{(1,0)}(G,\psi)}{2G\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2}\\
&+\frac{8(f(G,\psi)+1)\left(256\pi^2G^4\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)\left(-3f^{(0,1)}(G,\psi)-6\psi f^{(0,2)}(G,\psi)+8\pi\right)\right)}{\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)^2}\\
&+\frac{8(f(G,\psi)+1)\left(48\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)}{\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)^2}+\frac{48\pi\psi(f^{(0,1)}-f(G,\psi)-1)(G,\psi)}{G^2}\\
&+\frac{8(f(G,\psi)+1)\left((f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)^2\right)}{\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)^2}+37
\end{aligned}
$$

Appendix 6: Allowed boundary conditions

One can expand the flow equations near $\phi = 0$, by assuming $f(G,\phi^2) = f_1(G)\phi^2$ and $v(G,\phi^2) = v_1(G)\phi^2$ and expanding up to order ϕ^2. Furthermore, if one assumes $f_1(G), v_1(G) << 1$, one can expand to the lowest non-trivial order in f_1 and v_1. The resulting equations are

$$82G\, v_1(G) + \left(48\pi - 41G^2\right) v_1'(G) = 0$$

$$12G f_1(G)^2 + \left(96\pi^2 - 82\pi G^2\right) f_1'(G) = 0$$

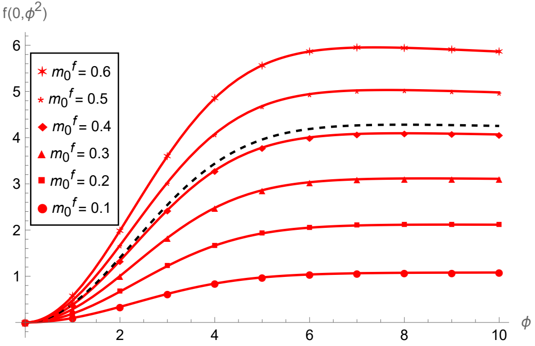
and the solutions are (m_0^v, m_0^f are free parameters)

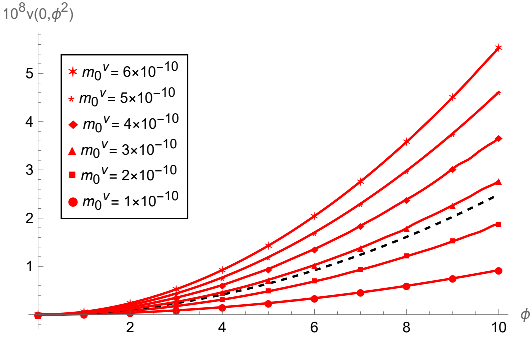
$$v^{(0,1)}(G,0) = v_1(G) = \frac{41\left(\frac{48\pi}{41} - G^2\right) m_0^v}{48\pi} \qquad \lim_{G\to 0} v_1(G) = m_0^v \qquad \lim_{G\to G_*} v_1(G) = 0$$

$$f^{(0,1)}(G,0) = f_1(G) = \frac{m_0^f}{1 - \frac{3}{41\pi}\, m_0^f \log\left(\frac{48\pi - 41G^2}{48\pi}\right)} \qquad \lim_{G\to 0} f_1(G) = m_0^f \qquad \lim_{G\to G_*} f_1(G) = 0$$

Appendix 7: Different choices of free parameters m_0^v and m_0^f

Different choices for the parameters m_0^v, m_0^f lead to different results of the emergent effective potentials. This is somewhat similar to the swampland program in string theory, where only some parameters are compatible with observations.





Appendix 8: Scaling of interactions and k limits

The construction of the S_{UV} is subject to the knowledge of the limits of the interactions as functions of the scale k. In the case of a negative mass dimension coupling $\tilde{G} = \frac{G}{k^n}$ (like Newton's constant)

$$\lim_{k->0} \tilde{G} = \tilde{G}_0 = \lim_{k->0} \frac{G}{k^n} \quad \rightarrow \quad \lim_{k->0} G = 0$$

In this case, one can use the coupling G to turn all the other relevant interactions (u^α) into negative mass dimension, and all the irrelevant interactions (v^ν) into dimensionless. In this case, the S_{UV} will be functions $v(u)$, with domain $[0, u_*]$, where $v(u_*) = v_*$ in the UV, and $v(0) = v$ is the effective coupling in the IR. An example of how to make a coupling dimensionless, can be the cosmological constant

$$\frac{2\tilde{\Lambda}}{16\pi\tilde{G}} \rightarrow \frac{2\lambda}{(16\pi\tilde{G})^2} \quad (\lambda \equiv 16\pi\tilde{G}\tilde{\Lambda})$$

Appendix 8: Scaling of interactions and k limits

The same thing happens with the fields. For example, a scalar field $\tilde{\varphi} = \frac{\varphi}{k^{d(d-1)/2}}$. Since we want to work with variables that finite in the limit of $k \to 0$, to be able to do numerical calculations, we can work at constant $\psi \doteq \tilde{G}\tilde{\varphi}^2 = G\varphi^2$ where

$$\lim_{k->0} \tilde{G}\tilde{\varphi}^2 = \lim_{k->0} G\varphi^2 = \tilde{G}_0\tilde{\varphi}^2 = \psi < \infty$$

also

$$\lim_{k->\infty} \tilde{G}\tilde{\varphi}^2 = \lim_{k->\infty} G\varphi^2 = G_*\varphi^2 = \psi < \infty$$

working with these type of field variables allows one to map the UV and the IR without needing to to infinite values of the field, as one would have to do when working with constant φ defined as $\tilde{\varphi} = \frac{\varphi}{k^{d(d-1)/2}}$. This is because the only well defined variable between those 2 in the IR is $\tilde{\varphi}$, and for finite $\tilde{\varphi}$, one must study $\varphi \to \infty$.