Preliminary Studies on Black Holes

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Resumo

The investigation of black holes (BHs) and their properties is a subject exerting considerable attraction among those interested in the universe and its astrophysical content. This context motivated the studies conducted by the authors on black hole physics. Their methodology included a bibliographic review of papers and books addressing the theme pedagogically. We start with a historical introduction that highlights the importance of these fascinating objects, we proceed to establish the physical concepts underlying BHs, we qualitatively explore some topics in Einstein's theories of Special Relativity and General Relativity. In particular, we write down the Minkowski metric and Schwarzschild metric, and begin the journey of characterizing them quantitatively(e.g. through understanding their causal structure). The Schwarzschild metric is fundamental to understanding the so-called Schwarzschild BH, a spherical and static object that models the outer region of a massive star that has extinguished its nuclear fuel and collapsed under its own gravity (or curvature). We will present a summary of these preliminary Scientific Initiation studies in our poster.

I. Einstein's Special Theory of Relativity

Albert Einstein's 1905 Special Theory of Relativity explains the concept of how velocity affects mass, space, and time, including a path where the speed of light defines the relationship between energy and matter. Consequently, we understand how Hermann Minkowski, in 1907, introduced a notion regarding the concepts of space and time into this theory, with the following equation:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 \quad (1)$$

This equation, known as the Minkowski line element, which presents the spacetime distance between two events, brings a possible classification related to the sign, which can be greater than, less than, or equal to zero. These possibilities are as follows:

• If the events s_1 and s_2 are separated by a null line element, $ds^2 = 0$, understood as $ds \equiv s_1 - s_2$, the equation becomes:

$$c^2dt^2 = dx^2 + dy^2 + dz^2 \quad (2)$$

Note that (2) leads to the speed of light in a vacuum: $c = \frac{d\vec{r}}{dt}$ with $\vec{\boldsymbol{r}} = \boldsymbol{x\hat{\imath}} + \boldsymbol{y\hat{\jmath}} + \boldsymbol{z\hat{k}}$, the position vector.

• If the events s_1 and s_2 are separated by $ds^2 < 0$, it is understood that the temporal variation present in equation (1) becomes greater compared to the spatial part, being called time-like and is represented by the inequality:

$$c^2dt^2 > dx^2 + dy^2 + dz^2 \quad (3)$$

which can be reformulated and rewritten as follows:

$$c^2 > \left(\frac{d\vec{\boldsymbol{r}}}{dt}\right)^2 = v^2 \quad (4)$$

where $\vec{r}=(x,y,z)$ is the particle's position vector. The separation between events allows for the displacement of massive particles, predicted in Special Relativity with speed $v<c$. Therefore, massive particles follow the so-called time-like trajectories.

• Finally, if the events s_1 and s_2 are separated by $ds^2 > 0$, such an interval is understood as space-like, which occurs when the spatial sector is greater than the temporal sector in equation (1). This leads to the inversion of the inequality in equation (4), which would describe objects in motion with speed $v>c$. This does not allow for any causal scenario, as the object would need a propagation speed greater than the speed of light to link the events, which is prohibited by the postulates of Special Relativity.

As a result of these equations, we understand the intervals in a schematic diagram made by Minkowski, titled the Spacetime Diagram, where there are light cones, as in figure 1:

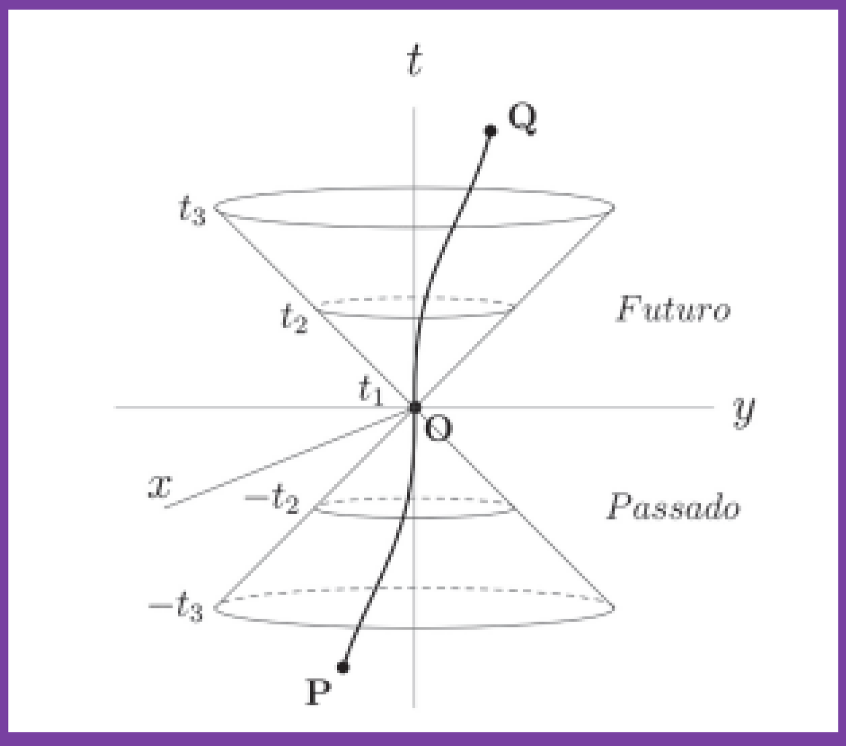

Figure 1: Minkowski Light Cone with Origin at O.
Source: Adapted from [Felipe and Oliveira, 2024].

II. General Theory of Relativity

Newtonian mechanics describes gravitation in terms of forces. Einstein proposes a shift in perspective with his General Theory of Relativity, which describes gravitation as the curvature of space and time. For this, it requires the use of tensors, an object designed to extend the concept of scalars, vectors, and matrices. Einstein's field equations are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (5)$$

where $R_{\mu\nu}$ is the Ricci tensor or Ricci curvature tensor, R is the curvature scalar (the trace of the Ricci tensor), and these quantities depend on the second-order derivatives of the metric tensor $g_{\mu\nu}$. On the right side of (5), $T_{\mu\nu}$ appears, which is the energy-momentum tensor of matter and serves as the source of gravitation (or curvature). In our work, we will not learn to solve (5) directly, but we will study the properties of the solutions of (5) called black holes. Even so, the message of equation (5) is clear: matter ($T_{\mu\nu}$) tells space how to curve ($g_{\mu\nu}$); then, curved space tells matter how to move.

III. Schwarzschild Black Holes

When analyzing a vacuum region around a massive, spherically symmetric, and static body, we find the Schwarzschild solution in Einstein's equations. This solution is described by the following line element:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{1}{\left(1 - \frac{2M}{r}\right)}dr^2 + r^2d\Omega^2 \quad (6),$$

which, within the studies of the Schwarzschild solution, leads us to the equation (7):

$$r_s = 2M = \frac{2Gm}{c^2}. \quad (7),$$

defining the Schwarzschild radius (r_s), which is crucial for understanding the formation of the event horizon of a black hole, the boundary beyond which nothing can escape, not even light.

At the event horizon, at $r = r_s = 2M$, spatial and temporal coordinates switch roles (signs). This means that $ds^2 > 0$ becomes a time-like interval and $ds^2 < 0$ becomes a space-like interval, different from what happens when $r > 2M$. Furthermore, the future light cone is tilted inward, indicating that the observer's future is directed into the black hole, as represented in figure (2). This transition between interval types is fundamental for understanding the structure and behavior of black holes.

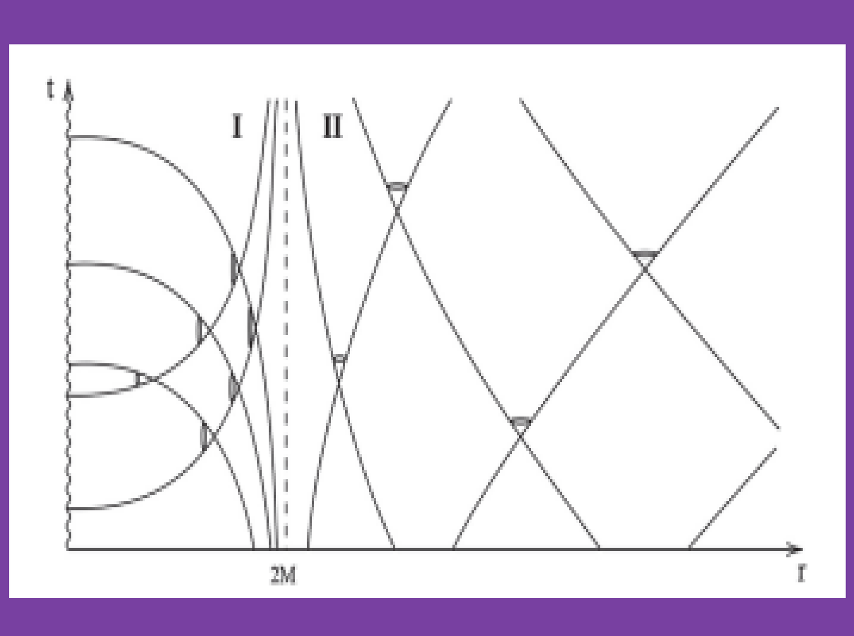

Figure 2: Light Cones in Schwarzschild Spacetime as Seen by Observers at an Infinite Distance from the Event Horizon (r=$2M$).
Source: Adapted from [Felipe and Oliveira, 2024].

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