Superradiant scattering in Bopp-Podolsky wormholes

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Introduction

Superradiance is a physical phenomenon involving the amplification of incident electromagnetic radiation through the extraction of energy from a system. This work analyzes the conditions for the occurrence of superradiance in electrically charged, spherical, and static wormholes, using Podolsky's generalized electromagnetic theory. Given a metric representing the wormhole and its electric field, minimal coupling is applied in the Klein-Gordon equation to determine the necessary conditions for superradiance in the solution of the radial equation. By using Maxwell's electromagnetic theory as a limiting case for generalized electrodynamics, we aim to estimate the parameter arising from the non-minimal coupling in Podolsky's theory. We estimate the reflection coefficient of an electromagnetic wave scattered by a wormhole in Bopp-Podolsky electrodynamics. Preliminary results are presented based on the approximation $\Delta_{00} \approx \Delta_{11}$ in the Bopp-Podolsky wormhole metric, justified by the low relative percentage error between the terms Δ_{00} and Δ_{11} of the metric components.

Wormhole (perturbative) solution

Let us take Einstein's equation below, whose energy-momentum tensor is given according to Podolsky electrodynamics [1]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi\left(T^M_{\mu\nu} + T^a_{\mu\nu} + T^b_{\mu\nu}\right), \tag{1}$$

where:

$$T^M_{\mu\nu} = \frac{1}{4\pi}\left[F_{\mu\sigma}F^{\sigma}{}_{\nu} + g_{\mu\nu}\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\right], \tag{2}$$

$$T^a_{\mu\nu} = \frac{a^2}{4\pi}\left[g_{\mu\nu}F_{\beta}{}^{\gamma}\nabla_{\gamma}K^{\beta} + \frac{g_{\mu\nu}}{2}K^{\beta}K_{\beta} + 2F_{(\mu}{}^{\alpha}\nabla_{\nu)}K_{\alpha} - 2F_{(\mu}{}^{\alpha}\nabla_{\alpha}K_{\nu)} - K_{\mu}K_{\nu}\right], \tag{3}$$

$$T^b_{\mu\nu} = \frac{b^2}{2\pi}\left[-\frac{1}{4}g_{\mu\nu}\nabla^{\beta}F^{\alpha\gamma}\nabla_{\beta}F_{\alpha\gamma} + F^{\gamma}_{(\mu}\nabla^{\beta}\nabla_{\beta}F_{\nu)\gamma} + F_{\gamma(\mu}\nabla_{\beta}\nabla_{\nu)}F^{\beta\gamma} - \nabla_{\beta}\left(F_{\gamma}{}^{\beta}\nabla_{(\mu}F_{\nu)}{}^{\gamma}\right)\right], \tag{4}$$

are terms of the energy-momentum tensor such that $T^M_{\mu\nu}$ is the term related to Maxwell's electrodynamics, while the terms $T^a_{\mu\nu}, T^b_{\mu\nu}$ come from Podolsky electrodynamics, where the latter is due to a non-minimal coupling. Together with:

$$\nabla_{\mu}\left[F^{\mu\nu} - \left(a^2 + 2b^2\right)H^{\mu\nu} + 2b^2S^{\mu\nu}\right] = 0, \tag{5}$$

$$H^{\mu\nu} \equiv \nabla^{\mu}K^{\nu} - \nabla^{\nu}K^{\mu}, \tag{6}$$

$$S^{\mu\nu} \equiv F^{\mu\sigma}R_{\sigma}{}^{\nu} - F^{\nu\sigma}R_{\sigma}{}^{\mu} + 2R^{\mu}{}_{\sigma}{}^{\nu}{}_{\beta}F^{\beta\sigma}, \tag{7}$$

they provide the energy-momentum tensor. Looking for a metric $g_{\mu\nu}$ that is spherically symmetric and static, space-time must be of the following form:

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2d\Omega^2 \tag{8}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $A(r)$ and $B(r)$ are functions to be determined. Due to the spherically symmetric mass distribution, the electric field has only a radial component and the only non-zero term of the electromagnetic tensor is:

$$E(r) = \frac{Q}{r^2}\left(1 - \frac{8Mb^2}{r^3} + \frac{11b^2Q^2}{r^4}\right), \tag{9}$$

where Q is the electrical charge. The metric-tensor is:

$$ds^2 = -\frac{\Delta_{00}}{r^2}dt^2 + \frac{r^2}{\Delta_{11}}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{10}$$

where

$$\frac{\Delta_{00}}{r^2} = \frac{1}{r^2}\left(r^2 - 2M + Q^2 + \frac{2b^2Q^2}{r^2} - \frac{6Mb^2Q^2}{r^3} + \frac{18b^2Q^4}{5r^4}\right), \tag{11}$$

$$\frac{\Delta_{11}}{r^2} = \frac{1}{r^2}\left(r^2 - 2M + Q^2 - \frac{4b^2Q^2}{r^2} + \frac{6Mb^2Q^2}{r^3} - \frac{12b^2Q^4}{5r^4}\right). \tag{12}$$

It is important to highlight that the solution (10) was obtained through perturbative methods by [2], where it was interpreted as a wormhole due to the fact that $A(r) \neq B(r)^{-1}$.

Superradiance condition

To analyze the possibility of superradiant scattering by monochromatic waves, we use the massless Klein-Gordon equation to couple the electromagnetic wave to the field [3]. The Klein-Gordon equation is solved through separation of variables and numerical methods. Due to spherical symmetry, only the radial equation is needed to investigate superradiance:

$$\frac{d}{dr}\left(\Delta\frac{d}{dr}\right)R(r) + \sqrt{\frac{\Delta_{00}}{\Delta_{11}}}\left(r^2\frac{K_P^2}{\Delta_{00}} - \lambda\right)R(r) = 0, \tag{13}$$

where

$$K_P^2 = \left(\omega r - qQ\left(1 - \frac{2Mb^2}{r^3} + \frac{11b^2Q^2}{5r^4}\right)\right)^2, \quad \Delta = \sqrt{\Delta_{11}\Delta_{00}}. \tag{14}$$

Defining $f = rR$ and changing coordinates $dr^*/dr = r^2/\Delta$, we can evaluate the potential term of the equation in two limits: far away from the wormhole and close to the throat. Through this analysis, the solution to equation (13) must have the following structure:

$$f(r_*) = \begin{cases} Z_{tr}e^{-i\left(\omega - \frac{qQ}{r_+}\left(1 - \frac{2Mb^2}{r_+^3} + \frac{11b^2Q^2}{5r_+^4}\right)\right)r_*}, & r_* \to r_+ \\ Z_{in}e^{-i\omega r_*} + Z_{out}e^{i\omega r_*}, & r_* \to \infty \end{cases} \tag{15}$$

We interpret the plus and minus signs in the above functions as outgoing and ingoing waves, respectively. From the definition of the reflection and transmission coefficients, we find the following relationship:

$$|\mathcal{R}|^2 = 1 - \left(\omega - \frac{qQ}{r_+}\left(1 - \frac{2Mb^2}{r_+^3} + \frac{11b^2Q^2}{5r_+^4}\right)\right)|\mathcal{T}|^2. \tag{16}$$

Hence, the superradiance condition for a static, spherical wormhole described by equation (9) is:

$$0 < \omega < \frac{qQ}{r_+}\left(1 - \frac{2Mb^2}{r_+^3} + \frac{11b^2Q^2}{5r_+^4}\right), \tag{17}$$

Preliminary numerical results

The energy-momentum tensor of the equation (1) has two additional terms to Maxwell's, accompanied by the parameters a and b of Podolsky's theory. The first, proposed by Podolsky as a correction to Maxwell's theory, and the second, originating from a non-minimal coupling when considering curved space-time. Both parameters a and b are expected to be small perturbations, so that $a, b \ll 1$.

The reflection coefficient determines the amplitude of the reflected wave in relation to the incident wave. Therefore, we must know the wave equation, solution of the equation (13). The terms Δ, Δ_{00} and Δ_{11} present in this equation come from the wormhole metric (10), obtained in a perturbative way.

When considering the parameter b a small but non-zero disturbance, we make the approximation $\Delta_{00} \approx \Delta_{11}$. In fact, when we take $b = 0$, we obtain the Reissner-Nordström solution, that is, $A(r) = B^{-1}(r)$. As a consequence, making $\Delta \approx \Delta_{11}$ in the equation (13), we can write it as:

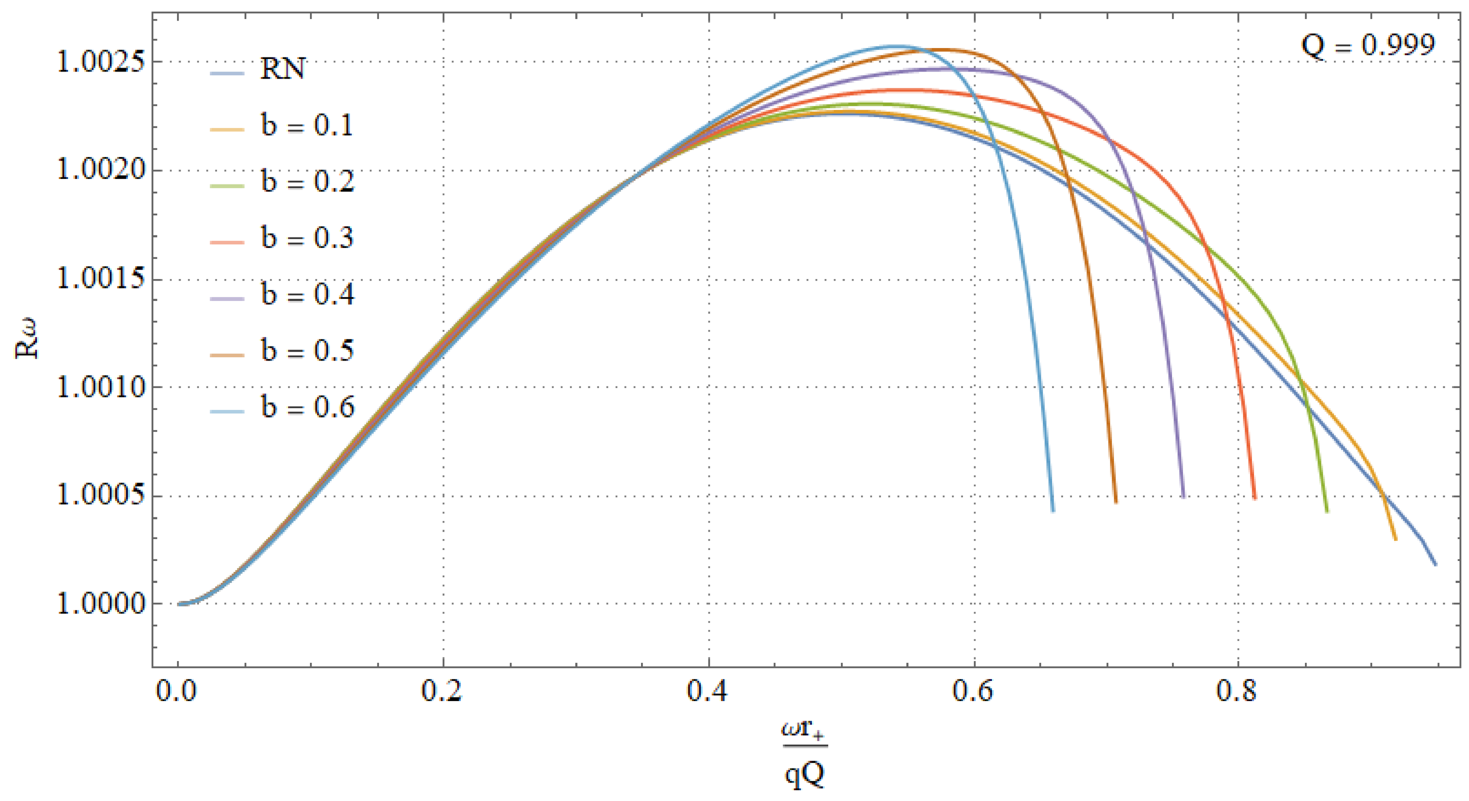
$$\frac{d}{dr}\left(\Delta\frac{d}{dr}\right)R(r) + \left(r^2\frac{K_P^2}{\Delta} - \lambda\right)R(r) = 0, \tag{18}$$

with

$$\Delta = r^2 - 2M + Q^2 - \frac{4b^2Q^2}{r^2} + \frac{6Mb^2Q^2}{r^3} - \frac{12b^2Q^4}{5r^4}, \tag{19}$$

and for $b = 0$, we have the limiting case of a Reissner-Nordström black hole. We interpret the metric (19) as a spherical and static black hole in Podolsky electrodynamics.

The following graph relates the reflection coefficient for different values of the parameter b. To do this, we are considering $b \ll 1$, so we are making the approximation $\Delta_{00} \approx \Delta_{11}$ in the radial equation (13).



Influence of different values of parameter b on the reflection coefficient.

In the graph above, we kept the parameters $M = 1$, $q = 1$ and $Q = 0.999$, and investigated the impact of the non-minimum coupling term on the reflection coefficient, comparing it with the Reissner-Nordström case ($b = 0$). It is observed that as b approaches zero, the reflection coefficient tends to equal that of the Reissner-Nordström case, as expected. However, for larger values of b, even when $b < 1$, the reflection coefficient curve shows significant changes. This shows that the parameter b exerts a substantial influence on the superradiance, modifying the expected behavior of the system.

Final remarks

This work presented the condition for superradiance to occur in Podolsky electrodynamics. Through a perturbative solution of the field equations, a metric was obtained that describes a wormhole in Podolsky electrodynamics, where the parameter b arises from non-minimal coupling. By considering the parameter b sufficiently small, we made the approximation $\Delta_{00} \approx \Delta_{11}$ in the metric, interpreting this result as a perturbation of the Reissner-Nordström black hole. With this result, we graphically analyzed the magnitude of the reflection coefficient for some values of b, comparing it with the Reissner-Nordström limiting case when b tends to zero. Future proposals involve estimating the limiting values of the parameter b through the superradiance condition and determining the reflection coefficient for a more general situation.

References

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