Open questions in particle physics

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Limitations of the Standard model

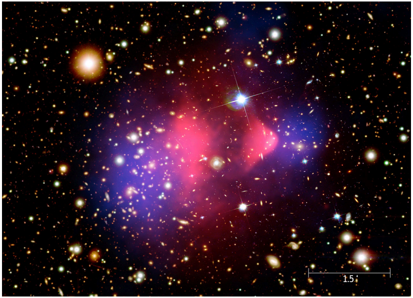
Limitations of the Standard model

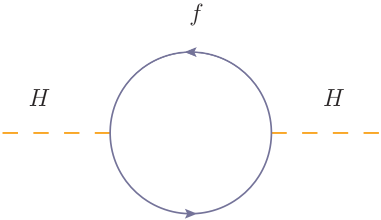
- Missing theoretical elements: the SM does not contain a quantum theory of gravity
- Unable to explain observed phenomena: the SM can not explain dark matter/dark energy
- Theoretical issues: the hierarchy problem

However: the SM explains very well the phenomena to which we have access with our experiments.

Question: up to which energy the SM can be valid?

- $\Lambda < M_{PL}$ – otherwise quantum gravity effects are important
- stability of the Higgs potential
- Hierarchy problem: Higgs mass unstable under quantum corrections $\delta M_H^2 \simeq \Lambda^2$





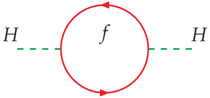
The hierarchy problem

Loop corrections to the propagator

The mass of a particle is defined as the pole of the propagator

- Free propagator: 

$\frac{-i}{p^2 - M_H^2}$



- Corrected propagator: $\frac{-i}{p^2 - M_H^2 + \Sigma_H}$

Let's take a look at the structure of the quantum corrections

$$\Sigma_H^f \sim N_f \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{k^2 - m_f^2)^2} \right)$$

for large momenta $\Lambda \to \infty$

$$\Sigma_H^f \sim N_f \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{d^4k}{k}}_{\sim \ln \Lambda} \right)$$

Sensitivity to the UV scale and fine tuning

For $\Lambda = M_{pl}$ (i.e. NP is at the Planck scale)

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{Pl}^2 \qquad \to \qquad \delta M_H^2 \approx 10^{30} M_H^2$$

- No additional symmetry for $M_H = 0$
- No protection against large loop corrections

The hierarchy problem is the sensitivity of the Higgs to the UV scale, in a theory where the is a new UV scale very high above the weak scale It is also called the fine-tuning problem because one needs to cancel this very large corrections up a very large precision in order to have a weak-scale Higgs mass

Example: the grand unification scale (i.e. the scale at which gauge couplings unify), in a Grand Unified Theory: $\delta M_H^2 \approx M_{\mathrm{GUT}}^2$

There is another fine tuning problem in elementary physics, namely the cosmological constant problem

Solution to the hierarchy problem

The hierarchy problem has been for many years one of the driving efforts of the theory community (but not the only one) Many different solutions has been devised

- Supersymmetry – add an additional symmetry that “protects” the Higgs mass from these corrections
- Composite models – the Higgs is a composite object (as the hadrons are in QCD), and not far above the EW scale one could probe its constituents
- Cosmological solution – the relaxion mechanism and its derivation (the Higgs potential evolves as a function of the time scale of the Universe, being the vev another field with its own evolution)
- Other solutions (Neutral naturalness etc.)

Supersymmetry

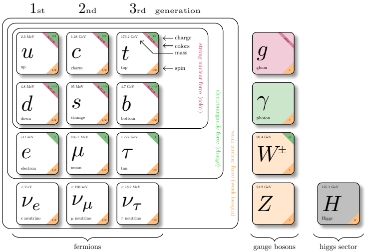
Supersymmetry

Supersymmetry is a symmetry that links fermions and bosons, schematically we have

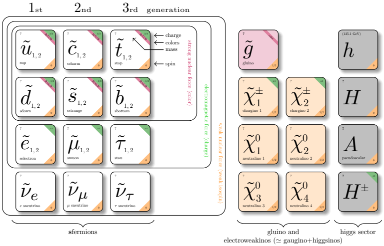
$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

where Q is the supersymmetry generator. At the pratical level, that means that for each SM state there is a SUSY partner

The Standard Model of particle physics

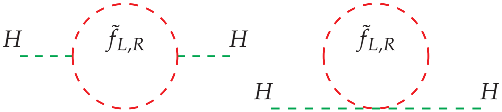


Supersymmetric particles



The hierarchy problem in Supersymmetry

The scalar superpartners to a (heavy) fermion contributes as well to the Higgs mass correction



$$\Sigma_H^{\tilde{f}} = N_{\tilde{f}} \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{ terms without quadratic divergeces}$$

From which we have

$$\Lambda \to \infty \qquad \Longrightarrow \qquad \Sigma_H^{\tilde{f}} \approx N_{\tilde{f}} \lambda_{\tilde{f}}^2 \Lambda^2$$

We see that the quadratic divergences diverge if

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda_{\tilde{f}}^2 = \lambda_f^2$$

while the complete correction vanishes if furthermore we have

$$m_{\tilde{f}} = m_f$$

(Soft) SUSY breaking

However, we are interested in the case where SUSY is not exact, i.e. the mass of the selectron (if it exists) is clearly not the same as the mass of the electron

$$m_{\tilde{f}}^2 = m_f^2 + \Delta, \qquad \lambda_{\tilde{f}}^2 = \lambda_f^2$$

results in

$$\Sigma_H^{f+\tilde{f}} \sim N_f \lambda_f^2 \Delta^2 + \ldots$$

- We see that if the split is sufficiently small, the corrections are still at an acceptable level.
- This is realized if the SUSY mass scale is $M_{\mathrm{SUSY}} \lesssim 1$ TeV

SUSY breaking

How does this work? First we recall that supersymmetry relates bosonic and fermionic states

$$Q|\text{fermion}\rangle > = |\text{boson}\rangle$$
$$Q|\text{boson}\rangle > = |\text{fermion}\rangle$$

In a simplified way we can write for instance write

$$Q|\text{top } t, \text{fermion}\rangle > = |\text{scalar top } \tilde{t}, \text{boson}\rangle$$
$$Q|\text{gluon } g, \text{vector}\rangle > = |\text{gluino } \tilde{g}, \text{fermion}\rangle$$

$\rightarrow$ We double the degrees of freedom of the SM
Unbroken SUSY: all the states belong to the same multiplet $\rightarrow$ they have the same mass

The breaking of SUSY is achieved by hadding SUSY-breaking terms to the Lagrangian

In this we can make the SUSY states heavy and satisfy the experimental observations that they are not at the same mass scale as their "SM partners".

SUSY multiplets

- The SUSY multiplets that we need are of two kinds, chiral and vector
- The chiral multiplets contain spin-0 and spin-1/2 states
- The vector multiplets contain spin-1/2 and spin-1 states

Therefore, if we want to build the minimal supersymmetric extension of the SM (i.e. we simply makes the SM Lagrangian supersymmetric) we have

- SM spin 0 bosons
  spin-0 state $\rightarrow$ (spin-0, spin-$\frac{1}{2}$) chiral multiplet ($LH_{\chi}SF$)
- SM spin-$\frac{1}{2}$ fermions
  spin-$\frac{1}{2}$ state $\rightarrow$ (spin-0, spin-$\frac{1}{2}$) chiral multiplet ($LH_{\chi}SF$)
- SM spin 1 bosons
  spin-1 state $\rightarrow$ (spin-$\frac{1}{2}$, spin-1) vector multiplet

How to achieve soft SUSY breaking?

- We have understood that SUSY must be broken
- The only satisfactory way to do that is via spontaneous symmetry breaking

Soft SUSY breaking terms do not alter dimensionless couplings (i.e. the dimension of the coupling constants of soft SUSY breaking terms is one or more). If that is not the case, one re-introduces the hierarchy problem. Indeed, in this way the cancellation of the quadratic divergences still persists.

Unfortunately we do not know how SUSY is broken (otherwise we would now the mass of the SUSY partners). There are different soft SUSY breaking schemes that yield at low scale a Lagrangian which is supersymmetric aside from the so-called “soft SUSY breaking terms”.

Soft SUSY breaking terms

There are different kinds of SUSY breaking terms [L. Girardello, M. Grisaru '82]

- scalar mass terms: $m^2_{\phi_i}|\phi_i|^2$
- trilinear scalar interactions: $T_{ijk}\phi_i\phi_j\phi_k + h.c.$
- gaugino mass terms: $\frac{1}{2}m\bar{\lambda}\lambda$
- bilinear terms: $B_{ij}\phi_i\phi_j + h.c.$
- linear terms: $C_i\phi_i$

Note that all the couplings are dimensionfull, and that there are no additional mass terms for the chiral fermions.

SUSY breaking scenarios

Two classes of models:

- Unconstrained models

No assumption is made on the SUSY breaking mechanism; one writes the most general low-energy effective Lagrangian with soft SUSY breaking terms.

In the most general case we have 105 new parameters with respect to the SM: new masses, phases and mixing angles.

- Constrained models

One assumes a specific SUSY breaking scenarios. This in turn yields a specific prediction for the structure of the Lagrangian at the low scale. In this case one has specific patterns for the soft SUSY breaking terms. In principle the breaking mechanism could be pinpointed experimentally, once the low-energy SUSY parameters are determined experimentally.

SUSY breaking scenarios

The core idea of SUSY breaking is that there is a "Hidden sector" where SUSY is broken, and this is "transferred"to the "visible sector", i.e. the MSSM. Schematically, we have

"Hidden sector"	$\rightarrow$	"Visible sector"
SUSY breaking MSSM		MSSM

Examples of SUSY breaking mechanisms are

- "gravity-mediated" – CMSSM/mSUGRA, where the mediating interaction is the gravitational one
- "gauge-mediated" – GMSB, for which mediating interactions are EW or QCD interactions
- "anomaly-mediated" – AMSB, in the case of which the breaking happens on a different brane in a higher-dimensional theory

The Minimal Supersymmetric Standard Model (MSSM)

We want to build the minimal SUSY extension of the SM (i.e. keeping the field numbers at minimum)

- The SM matter fields have different quantum numbers that SM gauge bosons

$\rightarrow$ the fields have to be included in different supermultiplets
$\rightarrow$ no SM fermion is a gaugino

Moreover, in the MSSM the Higgs is not to be the scalar superpartners of the neutrinos (gauge numbers ok, but it does not work due to the Yukawa pattern; one needs modifications, see e.g. [Riva, Biggio, Pomarol '12].

We are agnostic on how SUSY breaking is achieved $\rightarrow$ parametrization of all the possible soft SUSY-breaking terms $\rightarrow$ the most general case has 105 new parameters (mass terms, mixing angles, phases)

Building the MSSM: fermions and sfermions

SM fermions are part of left-handed chiral supermultiplets, together with their partners, the sfermions (right handed (s)fermions are included via their conjugate). We have, for each generation

- $LH_\chi SF$ Q: quark, squark – SU(2) doublets
- $LH_\chi SF$ U: quark, squark, up-type, SU(2) singlets
- $LH_\chi SF$ D: quark, squark, down-type, SU(2) singlets

And for the leptons

- $LH_\chi SF$ L: lepton, slepton – SU(2) doublets
- $LH_\chi SF$ E: (charged) lepton, slepton, SU(2) singlets

We see that we need 5 $LH_\chi SF$ to describe a single SM generation.

Building the MSSM: gauginos and the Higgs sector

Gauge bosons and gauginos are embedded in a so-called vector supermultiplet. We have

- gluons g and gluinos $\tilde{g}$
- W bosons $W^{1,2,3}$ and the winos $\tilde{w}^{1,2,3}$
- B boson B_0 and the bino $\tilde{B}^0$

For the Higgs boson the situation is slightly more complicated. As we will see in the next slides, we need two Higgs doublets. Those two Higgs doublets (scalars) are part of two different chiral supermultiplets. Their fermionic partners are called Higgsinos.

The Higgs sector of the MSSM

If you remember, in the SM, to give mass to both LH and RH fermions we used the same Higgs filed, writing the Yukawa terms as

$$\mathcal{L}_{SM,\mathrm{Yukawa}} = \underbrace{m_d \bar{Q}_L H d_R}_{d-quarkmass} + \underbrace{m_u \bar{Q}_L \tilde{H} u_R}_{u-quarkmass}$$

where

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L , \quad \tilde{H} = i\sigma_2 H^* \quad \text{so that} \quad H \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \tilde{H} \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

However, supersymmetry forbids the $\bar{Q}_L H^*$. The superpotential is a holomorphic function of chiral supermultiplets, i.e. it should depend only on ϕ_i (and not on ϕ_i^*). Moreover, we have seen that soft SUSY-breaking masses are allowed for chiral fermions.
$\rightarrow$ We need two Higgs doublets, H_d and H_u to give mass (separately) to down- and up-type fermions

- Moreover, two doublets are required so that the fermion partners (the Higgsinos) do not break the cancellation of the anomalies

The fields of the MSSM

Chiral supermultiplets

Name	Symbol	spin 0	spin 1/2	$(SU(3)_C, SU(2)_L, U(1)_Y)$
squarks,quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
(×3 families)	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons,leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
(×3 families)	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgses, Higgsinos	H_u	(H_u^+, H_u^0)	$(\widetilde{H}_u^+, \widetilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
	H_d	(H_d^0, H_d^-)	$(\widetilde{H}_d^0, \widetilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Gauge supermultiplets

Name	spin 1/2	spin 1	$(SU(3)_C, SU(2)_L, U(1)_Y)$
gluino,gluon	$\tilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W}^\pm \quad \widetilde{W}^0$	$W^\pm \quad W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

R-parity

On the top of this theoretical structure, we add another discrete Z_2 symmetry called R-parity (not to be confused with R-symmetry).
We assign the R-parity to a field with the assignment rule

$$P_R = (-1)^{3B+L+2S}$$

where B is the baryon number, L is the lepton number and S is the spin of the field.
Note that

- all SM fields, and the Higgs bosons, have even R-parity $P_R = +1$
- all superpartners have odd R-parity $P_R = -1$

There are two very important implications for phenomenology from this

- SUSY particles can be produced/appear only in pairs
- The lightest SUSY particle (LSP) is stable

Soft SUSY-breaking terms

The most general soft SUSY-breaking Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{\rm soft} = & -\frac{1}{2}\left(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + M_3\tilde{g}\tilde{g}\right) + h.c. \\
& - m^2_{H_u} H_u^\dagger H_u^\dagger H^u - m^2_{H_d} H_d^\dagger H_d - (B_\mu H_u H_d + h.c.) \\
& - \left(\tilde{u}_R T_u \tilde{Q} H_u - \tilde{d}_R T_d \tilde{Q} H_d - \tilde{e} T_e \tilde{L} H_d\right) + h.c. \\
& - \tilde{Q}^\dagger m^2_Q \tilde{Q} - \tilde{L}^\dagger m^2_L \tilde{L} - \tilde{u}_R m^2_u \tilde{u} - \tilde{d}_R m^2_d \tilde{d} - \tilde{e}_R m^2_e \tilde{e}
\end{aligned}
$$

where m^2_i and T_i are 3×3 matrices in family space.

Source of the many new parameters that depend on the SUSY breaking mechanism and that we do not know.

The effect of EWSB on the SUSY and Higgs sector

- We will discuss more in details the Higgs sector in the next lecture. For now, the important point is that both Higgs doublets acquire a non-zero vacuum expectation value, v_u and v_d (with the parameter $\tan\beta = v_u/v_d$.
- The Higgs spectrum is larger than in the SM, with five physical states: h^0 and H^0, neutral and CP-even; A, neutral and CP-odd; $H^\pm$ is a charged Higgs.

- The breaking of EW symmetry also causes the mixing of left- and right-handed sfermions with each other, and of the higgsinos with the EW gauginos.

$$\tilde{f}_L\,,\,\tilde{f}_R \to \tilde{f}_1\,\tilde{f}_2$$
$$\tilde{W}^\pm\,,\;\tilde{H}^\pm_{u,d} \to \tilde{\chi}^\pm_{1,2} \qquad \text{(charginos)}$$
$$\tilde{B}_0\,,\tilde{W}^0\,,\;\tilde{H}^0_{u,d} \to \tilde{\chi}^0_{1,2,3,4} \qquad \text{(neutralinos)}$$

Sfermion mixing

Taking as an example the stop and sbottom mass matrices (with $X_t = A_t - \mu/\tan\beta$, $X_b = A_b - \mu\tan\beta$) we have

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + D_{\tilde{t}_1} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + D_{\tilde{t}_2} \end{pmatrix} \overbrace{\longrightarrow}^{\text{diagonalization}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + D_{\tilde{b}_1} & m_b X_b \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + D_{\tilde{b}_2} \end{pmatrix} \overbrace{\longrightarrow}^{\text{diagonalization}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

where the D represents the so-called D-terms, and $M^2_{\tilde{t}_L} = M^2_{\tilde{b}_L} = M^2_{\tilde{Q}^3}$ due to gauge invariance. In other terms, the mass eigenstates are obtained by rotation of an angle $\theta_{\tilde{t}}$

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{t}} & -\sin\theta_{\tilde{t}} \\ \sin\theta_{\tilde{t}} & \cos\theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

Charginos

As you remember we have that

$$\tilde{W}^{\pm}\,,\ \tilde{H}^{\pm}_{u,d} \to \tilde{\chi}^{\pm}_{1,2}$$

Diagonalization of the mass matrix

$$X = \begin{pmatrix} M_2 & \sqrt{2}\sin\beta M_W \\ \sqrt{2}\cos\beta M_W & \mu \end{pmatrix}$$

that we can diagonalize using two unitary matrices U and V

$$M_{\tilde{\chi}^{\pm}} = V^{*} X^{\mathsf{T}} U^{\dagger} = \begin{pmatrix} m_{\tilde{\chi}^{\pm}_1} & 0 \\ 0 & m_{\tilde{\chi}^{\pm}_2} \end{pmatrix}$$

to obtain the chargino mass eigenstates $\tilde{\chi}^{\pm}_{1,2}$.

Neutralinos

As you remember we have

$$\tilde{B}_0\,,\tilde{W}^0\,,\;\tilde{H}^0_{u,d} \to \tilde{\chi}^0_{1,2,3,4}$$

To obtain the mass eigenstates we diagonalize the neutralino mass matrices

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W \cos\beta & M_Z s_W \sin\beta \\ 0 & M_W & M_Z c_W \cos\beta & -M_Z c_W \sin\beta \\ -M_Z s_W \cos\beta & M_Z c_W \cos\beta & 0 & -\mu \\ M_Z s_W \sin\beta & -M_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

obtaining

$$M_{\tilde{\chi}^0} = N^* Y N^\dagger = \mathrm{diag}\left(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4}\right)$$

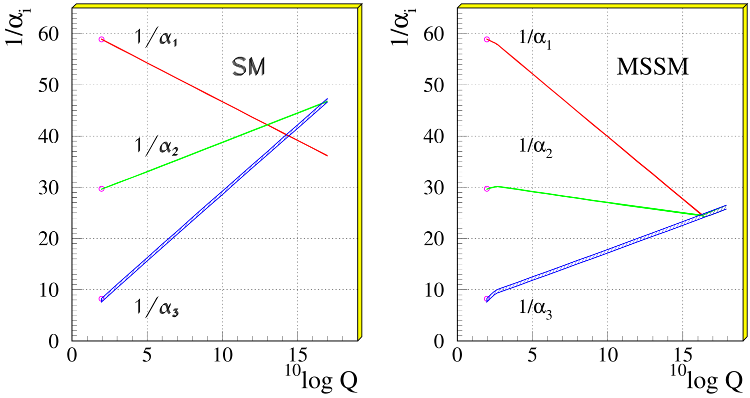
Note that

- The chargino mass matrix depends on M_2, μ and $\tan\beta$
- The neutralino mass matrix depends on M_1, M_2, μ and $\tan\beta$
- The neutralino and chargino spectra are connected $\to$ important element for collider phenomenology

Features of SUSY theories

Gauge coupling unification

- What happens if we evolve the gauge couplings to high scales? Theoretical arguments would like to see the unification of the forces, but this does not happen in the SM
- However, this comes for free in the MSSM



Radiative electroweak symmetry breaking

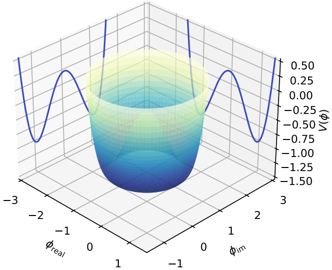
If you recall, in the SM we have that

- Higgs field, SU(2) scalar doublet: $\mathbf{\Phi} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
- Higgs potential

$$V(\mathbf{\Phi}) = \mu^2 |\mathbf{\Phi}^\dagger \mathbf{\Phi}| + \lambda |\mathbf{\Phi}^\dagger \mathbf{\Phi}|^2$$

  with $\lambda > 0$ (vacuum stability)
- To have EWSB one imposes $\mu^2 < 0$
- Minimum of the potential for

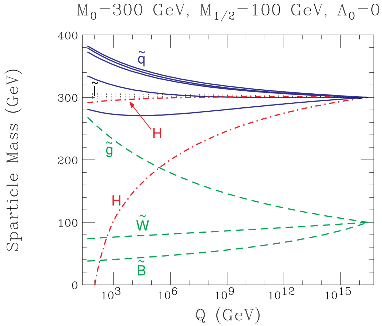
$$|\langle \mathbf{\Phi}_0 \rangle| = \sqrt{\frac{-mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$$



Radiative electroweak symmetry breaking

In the MSSM the sign of the equivalent of μ comes for free if

- One assumes SUSY breaking at the GUT scale
- One assumes universal input parameters at the GUT scale (as in the CMSSM)
- One run the parameters down to the EW scale



This works only if

- $M_T = 150 \ldots 200$ GeV (satisfied experimentally, $M_T \sim 173$ GeV)
- $M_{\text{SUSY}} = 1$ TeV

R-parity

The most general gauge-invariant and renormalizable superpotential with chiral superfields in the MSSM is

$$\mathcal{V} = \mathcal{V}_{MSSM} + \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j E_k + \lambda'^{ijk} L_i Q_j D_k + \mu'^{i} L_i H_u}_{\text{violates lepton number}} + \underbrace{\frac{1}{2}\lambda''^{ijj} U_i D_j D_k}_{\text{violates baryon number}}$$

- If both lepton and baryon numbers are violated $\rightarrow$ rapid proton decay
- Experimentally, the proton is very long lived $\tau_p > 10^{34}$ seconds

Imposing R-parity ($P_R = -1^{3B+L+2S}$), under which all the SM states and the Higgses have R-parity 1, and the SUSY partners have R-parity -1, do now allow us to write these operators $\rightarrow$ good motivation to impose R-parity

R-parity and dark matter

- Due to R-parity, the lightest supersymmetric particle (LSP) is stable
- If the LSP is neutral, it could be a good DM candidate
- Indeed, if the LSP is the lightest neutralino $\tilde{\chi}_1^0$, we can satisfy the constraint on the measured relic density

The consequence of this on the collider phenomenology is that

- The decay chains of sparticles produced (in pair) at the LHC contains two $\tilde{\chi}_1^0$
- Large MET in the detector – typical SUSY signature (caveat: long lived charged state)

Relation between SUSY parameters and other constraints

SUSY imposes specific relations between couplings, for instance we have that

gauge boson-fermion coupling = gaugino-fermion-sfermion coupling

for all the gauge groups. Moreover

- There is an upper bound on the mass of the lightest CP-even Higgs boson (and a prediction of its values in terms of the other parameters of the Lagrangian)
- There is a relation between the mass of charginos and neutralinos
- There is a relation between the mass of the sfermions, for instance

$$m^2_{\tilde{e}_L} = m^2_{\tilde{n}u_L} - M^2_W \cos 2\beta$$

All these relations receive loop corrections, which in turns imply that they depend on the whole set of soft SUSY-breaking parameters and on EWSB. After a discovery, they provide an experimental test to verify the structure of the theory.

Constrained models

SUSY breaking scenarios

Important point: the soft SUSY-breaking mechanism influences the phenomenology at the low scale

"Hidden sector" SUSY breaking MSSM	$\rightarrow$	"Visible sector" MSSM

Examples of SUSY breaking mechanisms are

- "gravity-mediated" – CMSSM/mSUGRA, where the mediating interaction is the gravitational one
- "gauge-mediated" – GMSB, for which mediating interactions are EW or QCD interactions
- "anomaly-mediated" – AMSB, in the case of which the breaking happens on a different brane in a higher-dimensional theory

Note that all these constrained models are variants of the MSSM

Gravity mediated SUSY breaking

A quantum theory of supergravity would include a graviton (spin-2) and a gravitino (spin-3/2). However, a quantum theory of spin-2 and spin-3/2 fields is not renormalizable. That implies that

- This QFT could not be extended at arbitrarily higher energies $\rightarrow$ interpreterable only as a EFT
- Best candidate for the UV theory: string theory

Since this is an effective theory, it means that it contains higher-dimensional operators suppressed by powers of M_{pl}.

SUSY breaking in the hidden sector

- supergravity Lagrangian contains non-renormalizable terms that communicate between the hidden and the visible sectors that are suppressed by $\sim \frac{1}{M_{pl}^n}$

Gravity mediated SUSY breaking

Dimensional analysis

- SUSY breaking in the hidden sector is caused by a vev $\langle F \rangle$ (with $[\langle F \rangle] = \text{mass}^2$)
- In the limit $\langle F \rangle \to 0$, we want that $m_{\text{soft}} \to 0$; the same for $M_{pl} \to \infty$ (no gravitational interaction)

From this we deduce that

$$m_{\text{soft}} \simeq \frac{\langle F \rangle}{M_{pl}}$$

Since we would like to have $m_{\text{soft}} \lesssim 1$ TeV (to avoid the hierarchy problem) $\to$ $\sqrt{\langle F \rangle} \simeq 10^{1}1$ GeV is the scale of SUSY breaking in the hidden sector.
Gravitino phenomenology:

- In general, we have $m_{\text{gravitino}} = m_{3/2} \simeq \frac{\langle F \rangle}{M_{pl}}$
- From which we have $m_{3/2} \simeq m_{\text{soft}}$
- Gravitino not important for collider phenomenology

Supergravity Lagrangian

The non-renormalizable terms in the supergravity Langrangian are

$$\mathcal{L}_{NR} = -\frac{1}{M_{Pl}} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + h.c. - \frac{1}{M_{pl}^2} F_X F_X^* k^i_j \phi_i \phi^{*j}$$
$$- \frac{1}{M_{Pl}} F_X \left(\frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j \right) + h.c.$$

where

- F_X is the auxiliary field of the chiral supermultiplet X in the hidden sector
- ϕ_i , λ^a are the scalar and gaugino fields of the MSSM

If $\sqrt{\langle F \rangle} \sim 10^{10}$-$10^{11}$ GeV $\rightarrow$ soft SUSY-breaking terms of the MSSM with $m_{\mathrm{soft}} \simeq 10^2$-$10^3$ GeV

- Assuming a "minimal" form for the supergravity Lagrangian $\rightarrow$ soft SUSY-breaking terms should obey "universality" and "proportionality"

The soft SUSY-breaking Lagrangian

With these assumptions, at low energy we obtain exactly the same soft SUSY-breaking Lagrangian that we have discussed before

$$
\begin{aligned}
\mathcal{L}_{\rm soft} = & -\frac{1}{2}\left(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + M_3\tilde{g}\tilde{g}\right) + h.c. \\
& - m^2_{H_u} H^\dagger_u H^\dagger_u H^u - m^2_{H_d} H^\dagger_d H_d - (B_\mu H_u H_d + h.c.) \\
& - \left(\tilde{u}_R T_u \tilde{Q} H_u - \tilde{d}_R T_d \tilde{Q} H_d - \tilde{e} T_e \tilde{L} H_d\right) + h.c. \\
& - \tilde{Q}^\dagger m^2_Q \tilde{Q} - \tilde{L}^\dagger m^2_L \tilde{L} - \tilde{u}_R m^2_u \tilde{u} - \tilde{d}_R m^2_d \tilde{d} - \tilde{e}_R m^2_e \tilde{e}
\end{aligned}
$$

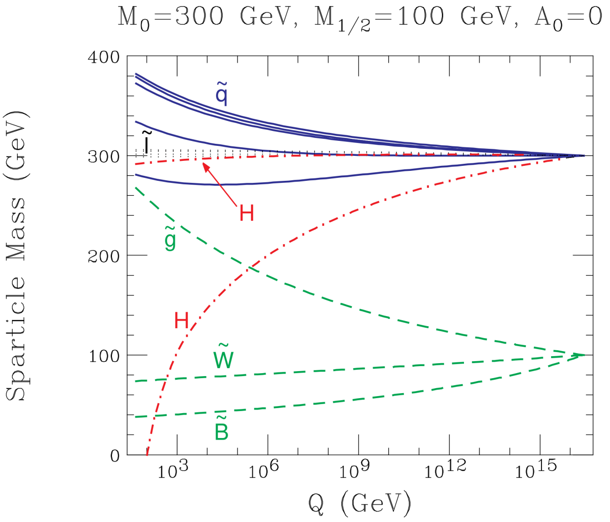
with the matching condition at the GUT scale given by

$$
\begin{aligned}
M_1 = M_2 = M_3 = m_{1/2} \\
m^2_{H_u} = m^2_{H_d} = m^2_Q = m^2_L = m^2_u = m^2_d = m^2_e = m^2_0 \\
T_u = T_d = T_e = T_0
\end{aligned}
$$

Moreover, we have two other free parameters B_μ and μ (the latter from the superpotential)

Running at the low scale

Using the RGE of the MSSM, we can run these parameters at the low scale, at which we will determine the physical spectrum of the theory.



μ, B_μ and EWSB

We have five parameters left (forgetting about possible complex phases)

$$m_0\,, m_{1/2}\,, A_0\,, B_\mu\,, \mu$$

However, we observe that from electroweak symmetry breaking we have

$$|\mu|^2 + m_{H_d}^2 = B_\mu \tan\beta - \frac{M_Z^2}{2}\cos 2\beta$$
$$|\mu|^2 + m_{H_u}^2 = B_\mu \cot\beta - \frac{M_Z^2}{2}\cos 2\beta$$

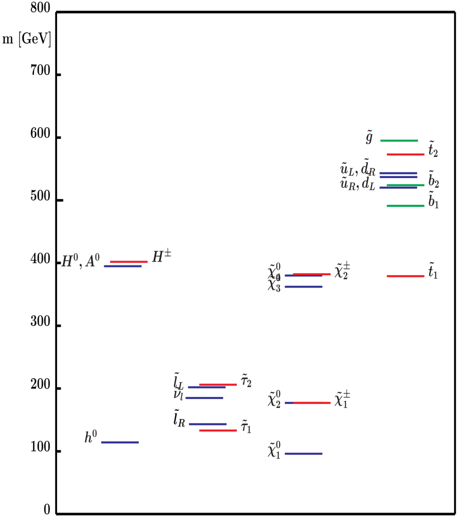
we can exchange $|\mu|$ and B_μ for $\tan\beta$ and the sign of μ.

We find therefore that this scenario is characterized by the following parameters

$$m_0\,, m_{1/2}\,, A_0, \tan\beta\,, \mathrm{sign}(\mu)$$

This is usually called the CMSSM (constrained MSSM) or 'mSUGRA' (minimal supergravity)

Example of a CMSSM point



Non universal Higgs mass models

It is possible to relax some of the assumptions that we have incorporated in the CMSSM NUHM1 – Non-universal Higgs Mass model 1

- We relax the assumption on the unification of the soft susy breaking mass parameters of the sfermions and the Higgs

$$m_0^2 \neq M_H^2 \left(= M_{H_u}^2 = M_{H_d}^2\right) \tag{1}$$

Effectively that means that we have either M_A or μ as free parameters at the EW scale, besides the other CMSSM parameters (one parameter more).

NUHM2 – Non-universal Higgs Mass model 2

- We furthermore relax the assumptions of the soft SUSY-breaking mass terms of the Higgs fields H_u and H_d

$$m_0^2 \neq M_{H_u}^2 \neq M_{H_d}^2 \tag{2}$$

Effectively that means that we have M_A *and* μ as free parameters at the EW scale, besides the other CMSSM parameters (two parameters more).

Minimal gauge mediate SUSY breaking: mGMSB

- New chiral supermultiplets, the "messengers", couple to SUSY breaking in the hidden sector
- They also couple indirectly to the MSSM fields via gauge interactions

$\hookrightarrow$ mediation of SUSY breaking via EW and QCD gauge interactions
$\hookrightarrow$ SUSY breaking is approximately flavor diagonal SUSY breaking is already in the messenger spectrum
$\hookrightarrow$ soft SUSY-breaking mass terms arise from loop diagrams with messenger particles, with vertexes of gauge-interactation strenght

$$m_{\mathrm{soft}} \simeq \frac{\alpha_i}{4\pi} \frac{\langle F \rangle}{M_{mess}} , \quad M_{mess} = \sqrt{\langle F \rangle}$$

Requiring $m_{soft} \lesssim 1$ TeV $\Rightarrow \sqrt{\langle F \rangle} \simeq 10^4$-$10^5$ GeV

- The SUSY breaking scale is much lower than in SUGRA

Generation of the mass terms in mGMSB

- **Gravitino**

The gravitino mass is M_{pl} suppressed, so that we have

$$m_{3/2} \simeq \frac{\langle F \rangle}{M_{pl}} \simeq 10^{-9}\ \mathrm{GeV}$$

The gravitino is always the LSP in mGMSB

- **Gaugino mass terms**

They are generated at one-loop order, $m_\lambda \simeq \frac{\alpha_i}{4\pi}$

- **Scalar mass terms**

They are generated at one-loop order, $m_\phi^2 \simeq \left(\frac{\alpha_i}{4\pi}\right)^2$

The fact that generation of the soft SUSY-breaking terms proceeds via the gauge interactions induces a hierarchy between the strongly and weakly interacting particles due $\sim \alpha_3/\alpha_2/\alpha_1$

The mGMSB scenario

The input parameters are

$$M_{\mathrm{mess}}\,, N_{\mathrm{mess}}, \Lambda, \tan\beta, \mathrm{sign}(\mu)$$

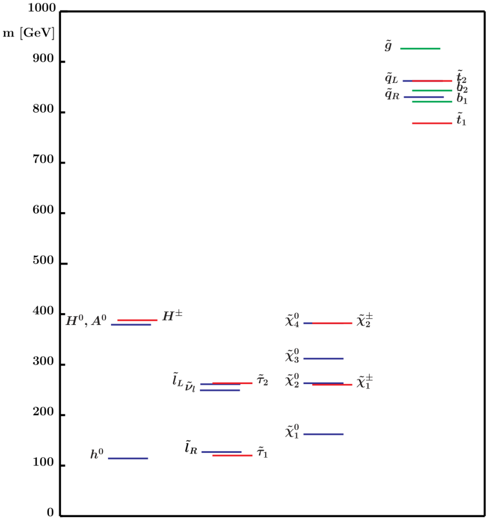
where

- M_{mess} is the messenger mass scale
- N_{mess} is the number of messenger multiplets
- $\Lambda = \frac{<\langle F \rangle>}{M_{\mathrm{mess}}}$ is the universal soft SUSY-breaking mass scale induced in the low-energy sector

Phenomenological features

- LSP is always the gravitino
- the next-to-lightest SUSY particle (NLSP) is either the $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$
- They can be long-lived $\Rightarrow$ can decay outside the detector, or metastable charged track

Example mGMSB point



Minimal anomaly mediated SUSY breaking: mAMSB

Two branes connected via "the bulk".
$\langle F \rangle$ enters via RGEs (anomaly)

$$
\begin{aligned}
m^2_{\tilde{f}_k} &\sim \frac{|\langle F \rangle|^2}{(16\pi^2)^2} g_k^4 + m_0^2 \\
&\sim \frac{m^2_{3/2}}{(16\pi^2)^2} g_k^4 + m_0^2 \\
M_i &\sim \frac{\langle F \rangle}{16\pi^2} g_i^2 \sim \frac{m_{3/2}}{16\pi^2}
\end{aligned}
$$

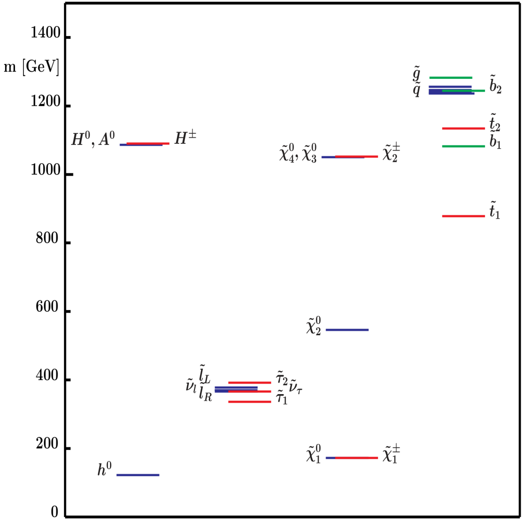
The input parameters for the mAMSB scenario are

$$m_{3/2}\,, m_0\,, \tan\beta\,, \mathrm{sign}(\mu)$$

where

- $m_{3/2} = \frac{\langle F \rangle}{M_{Pl}}$ is the overall scale of SUSY particle masses
- m_0 phenomenological parameter (universal scalar mass term) to avoid the slepton masses becoming negative

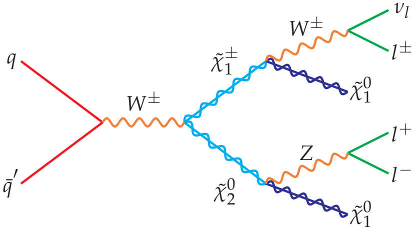
Example of mAMSB point



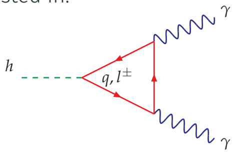
Testing SUSY at the collider

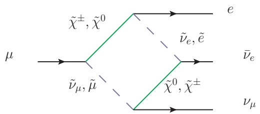
As for any BSM model, there are two ways to probe the existence of supersymmetry

- **Direct searches**
  One probe the existence of a new state by testing its production at a high energy experiment (i.e. at collider)



- **Indirect searches**
  One measure very precisely an observable that features only SM states as the external leg, and compare the prediction in the SM vs the one in the BSM model we are interested in.

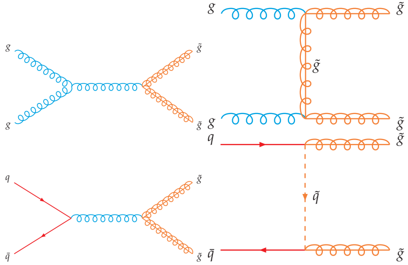


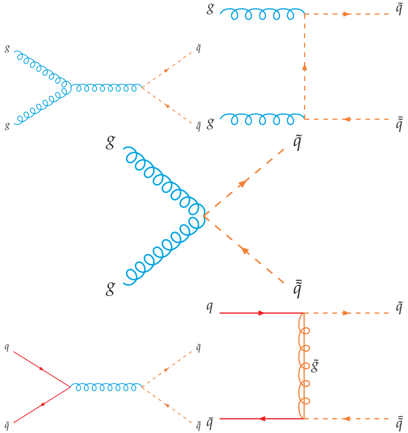


Both approach are important, and they should provide a consistent picture.

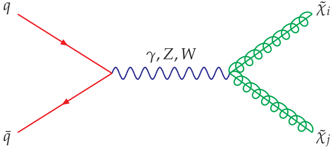
Colored sparticle production

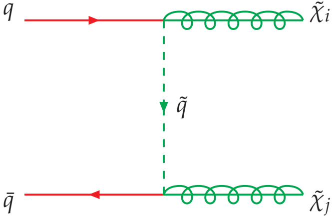
Colored sparticles are heavily produced at the LHC ⇒ the strongest limits from the searches are on these states ⇒ we need precise predictions for these cross sections.

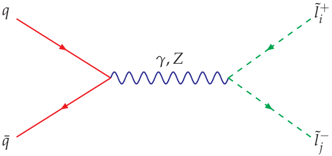




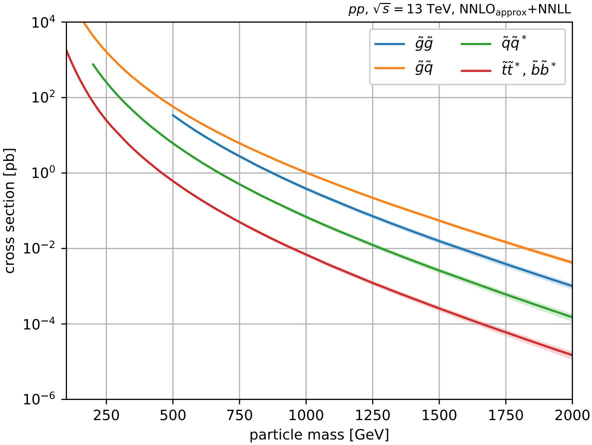
Cross sections for these sparticles are smaller at the LHC $\Rightarrow$ weaker limits from the searches.







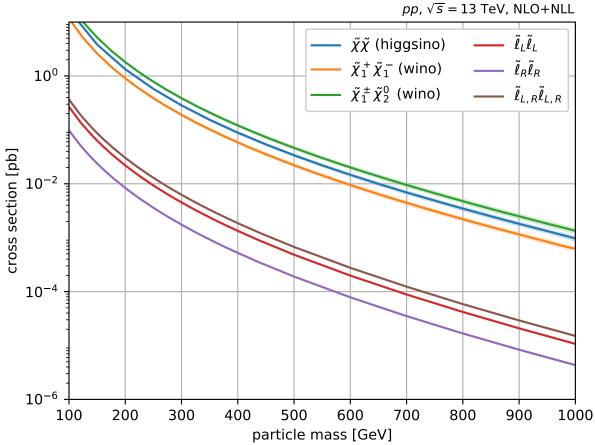
Cross sections: strongly interacting states



[LHC SUSY WG]

- Note: cross sections for the simplified topologies used by the experiments (e.g. gluino cross-sections is with squarks decoupled)

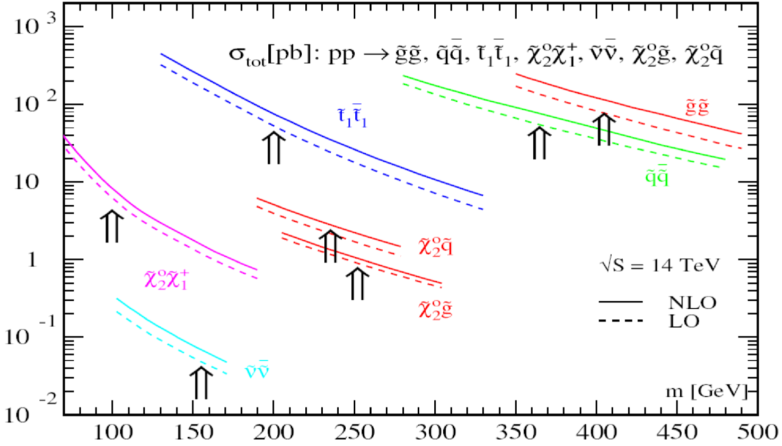
Cross sections



[LHC SUSY WG]

- Note: cross sections for the simplified topologies used by the experiments (e.g. $\tilde{l}_L\tilde{l}_L$, $\tilde{l}_R\tilde{l}_R$ etc.)

Impact of QCD corrections



[Prospino collaboration]

K-factors important for a proper interpretation of the data

Long decay chains

Depending on the spectrum, the production of a SUSY state at the LHC can result in long decay chain/complicated final state.
Another possibility is that the NLSP is long-lived $\rightarrow$ long charged tracks

Note that the production of uncolored particles via cascade decays often dominates over the direct production of the same states $\rightarrow$ it needs to be taken into account

Example of cascade decays

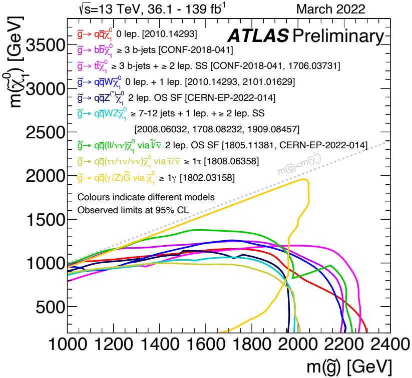
- Different patterns according to the SUSY breaking mechanism
- Many different final states

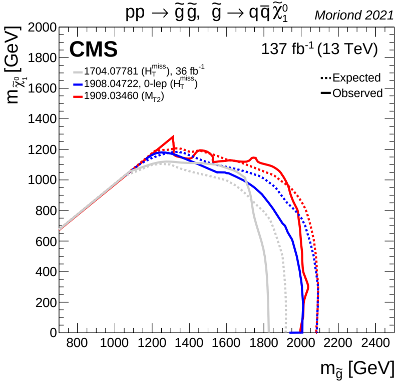
Signature	Motivating Model(s)	Comments
1 Jet + 0 Lepton + MET 70/nb	• Large Extra Dim (ExoGraviton) • strong qG production, G propagate in extra Dim • Planck Scale is MD in 4+δ dim • Normal Gravity >> R • SUSY • qg→ISR + 2 Neutralino or squark + Neutralino	• Not primary discovery channel for SUGRA, GMSB, AMSB... but helps in characterization • Possible leading discovery for neutralino NLSP with nearly degenerate gluino
NEW 2,3,4 [b]-Jet + 0 Lepton + MET 310/nb for b-jets 35/pb	• squark/gluino production • squark→q+LSP, gluino→q+squark+LSP	• Possible leading squark/gluino discovery channel • Must manage QCD bkg
NEW 2,3,4 [b]-Jet + 1 Lepton + MET 310/nb for b-jets 35/pb	• squark/gluino production with cascades which include electroweak (or partner) decays • high tan β leads to more τ's	• Lepton requirement suppresses QCD • τ's partially covered by e/μ
2 lepton + MET 70/nb	• Same sign: gluino cascade can have either sign lepton... squark/gluino prod can produce same sign. • Opposite sign: squark/gluino decay dediated by Z (or partner) • Same flavor: 2 leptons from same sparticle cascade must be same flavor	• Reduced SM backgrounds for same sign • Opposite Sign-Flavor Subtraction
3 lepton + MET	• SUSY events ending in Chargino/neutralino pair decays • Weak Chargino/Neutralino production • Exotic sources	• Low SM bkgs
2 photon + MET 3.1/pb	• GMSB models with gravitino LSP and neutralino or stau NLSP • UED- each KK partons cascade to LKP which decays to graviton + γ	• No SUSY limit (not sensitive at the time)

[Farbin '11]

Gluino searches

Gluinos are probably the most constrained SUSY states (however, its mass is not so relevant for the Higgs sector)





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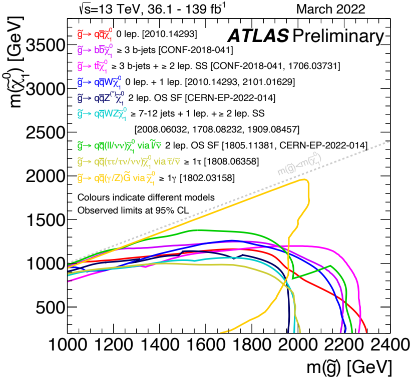

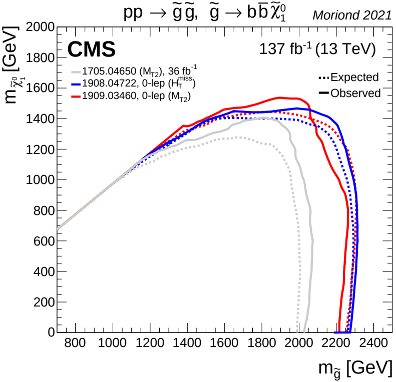

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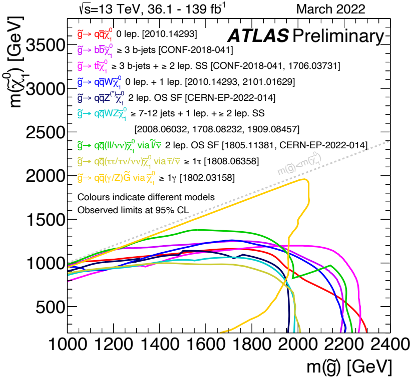

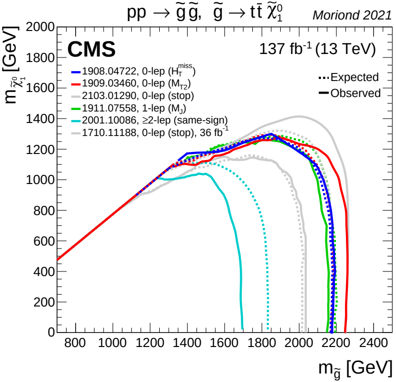

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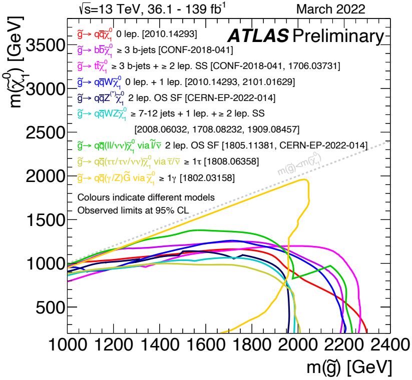
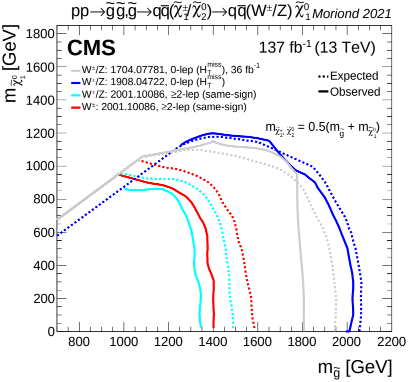
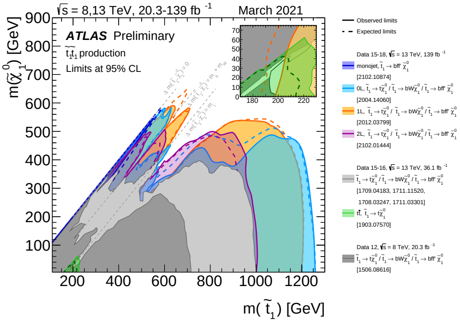
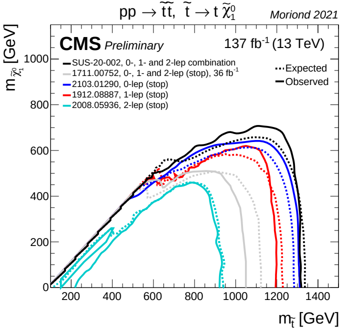




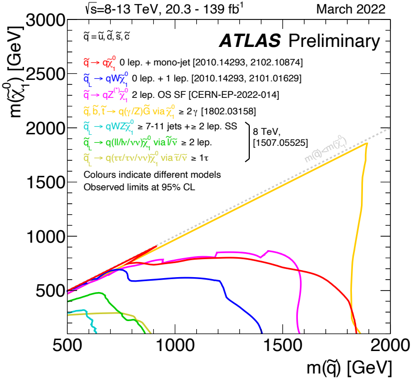
Stop searches

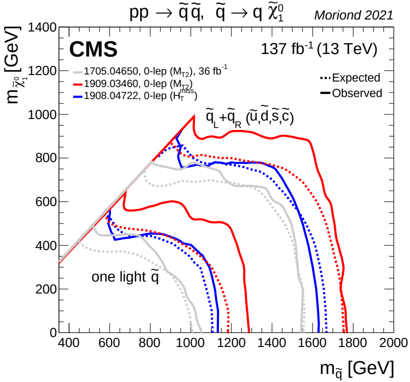
As we have seen, stop searches are important because the stop mass scale correlate directly with the Higgs mass prediction in the MSSM





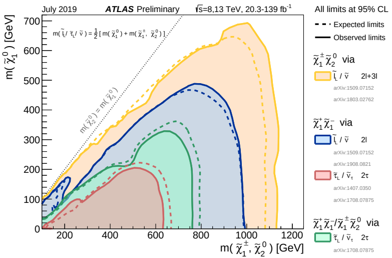
Searches for the first two generations. Note the assumptions on the degeneracy of the masses.

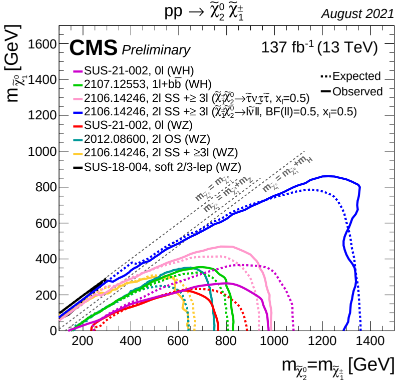




Electroweakino searches

- Simplified models depend on the final state and on the interdiate state in the decay chain





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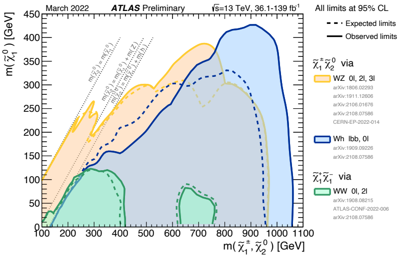
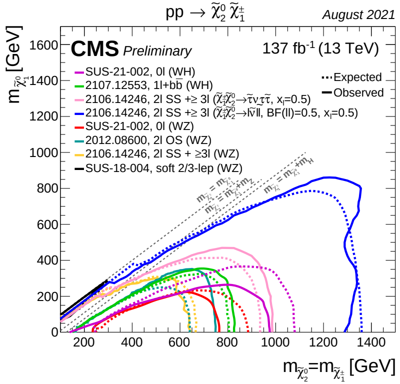
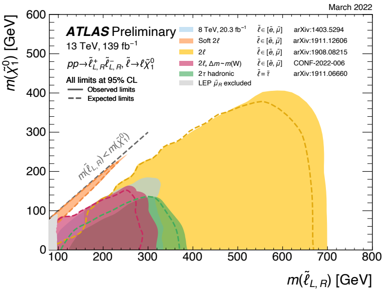
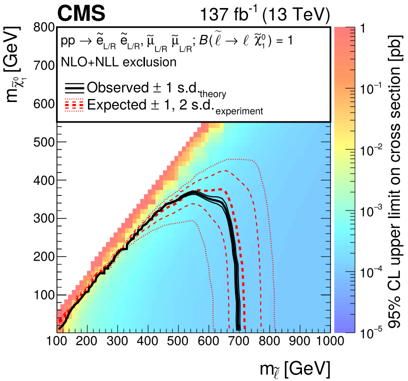




Slepton searches

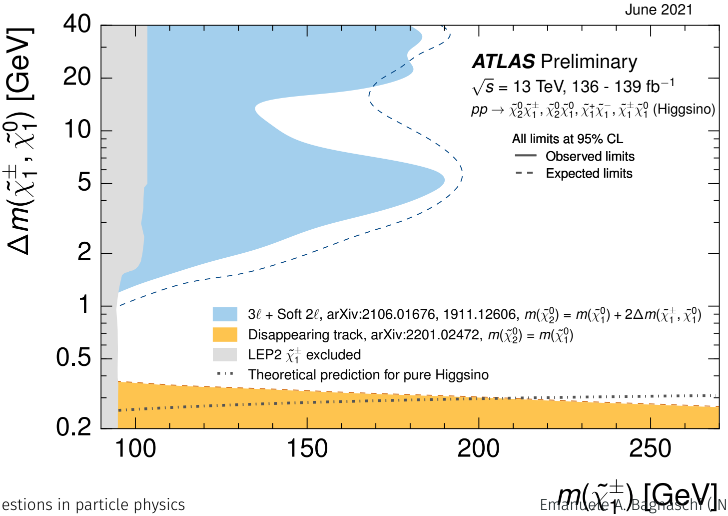




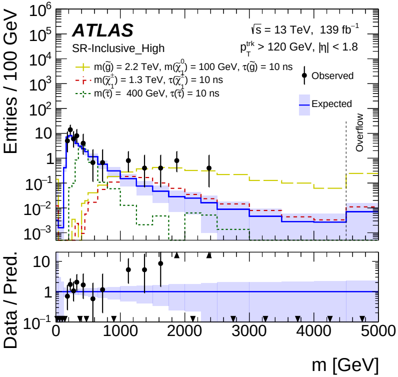
Compressed spectra

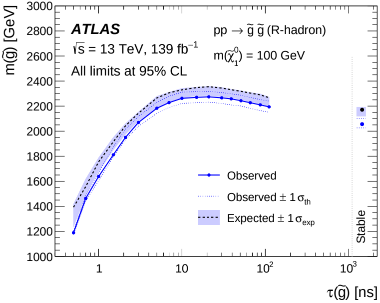
Compressed spectra are region of parameter space where the mass difference between the LSP and the NLSP is small.

They can lead also to long lived particles (more later).



- Recent ATLAS search that use the pixel detector to look for LLP (there are techniques as well)





Long lived particle signatures

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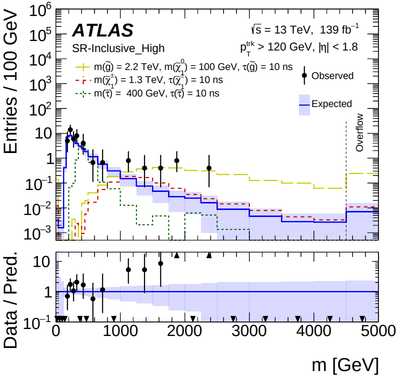

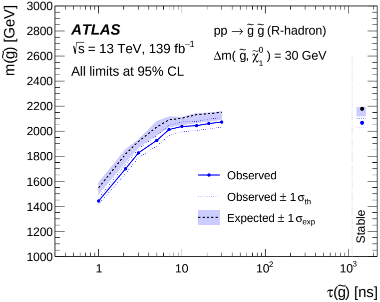

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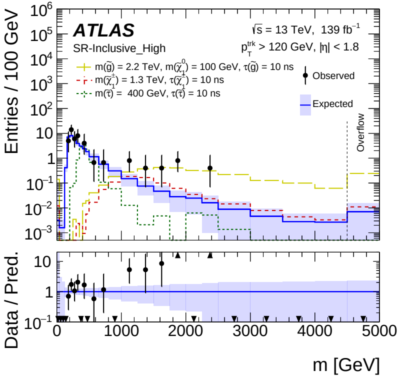

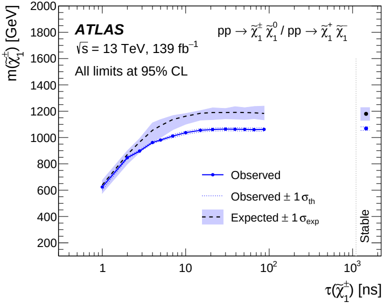

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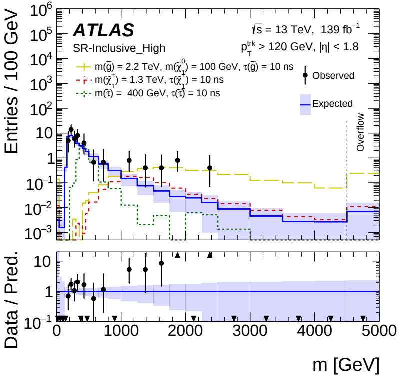
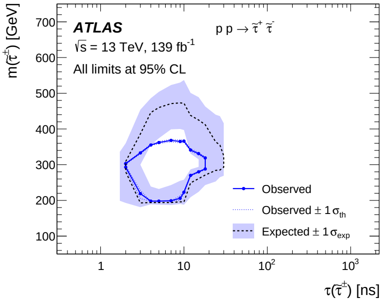
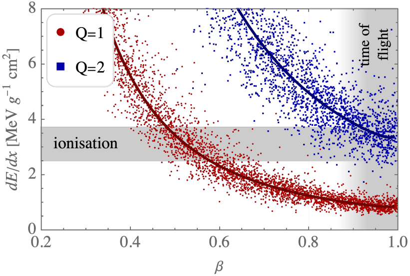
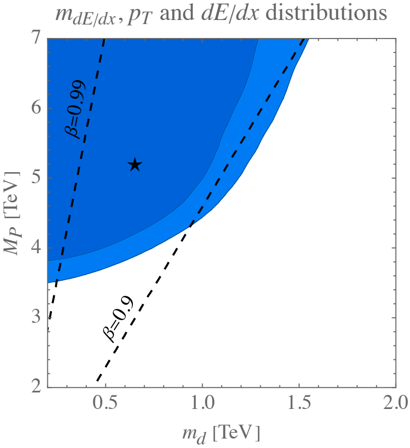
- Recent ATLAS search that use the pixel detector to look for LLP (there are techniques as well)





Solution by MCCullogh et al.

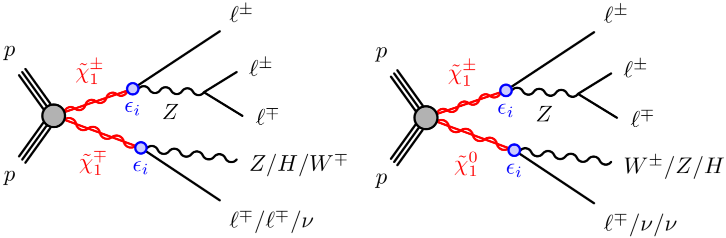




- Inconsistency resolved by assuming is instead coming from a $Q = 2$ LLP which is the decay product of a heavier resonance

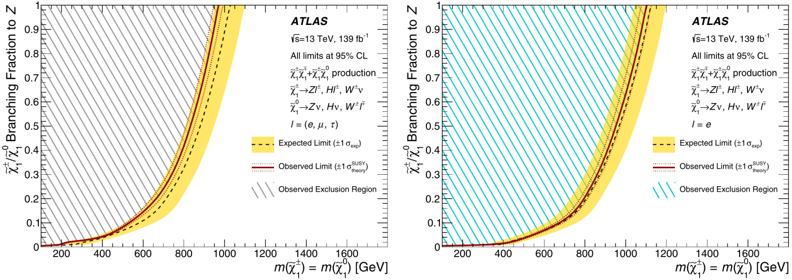
[McCullogh et al. '22]

R-parity violation



[ATLAS '20]

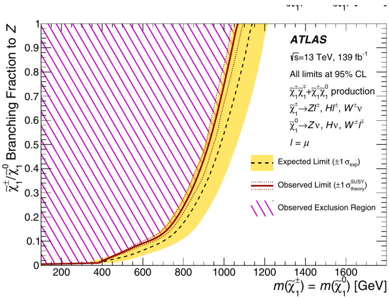
- Assume the existence of a R-parity violating coupling

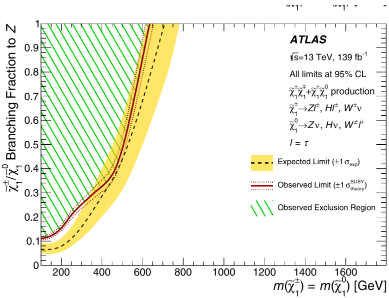


[ATLAS '20]

- Results for various lepton flavor combinations

R-parity violation

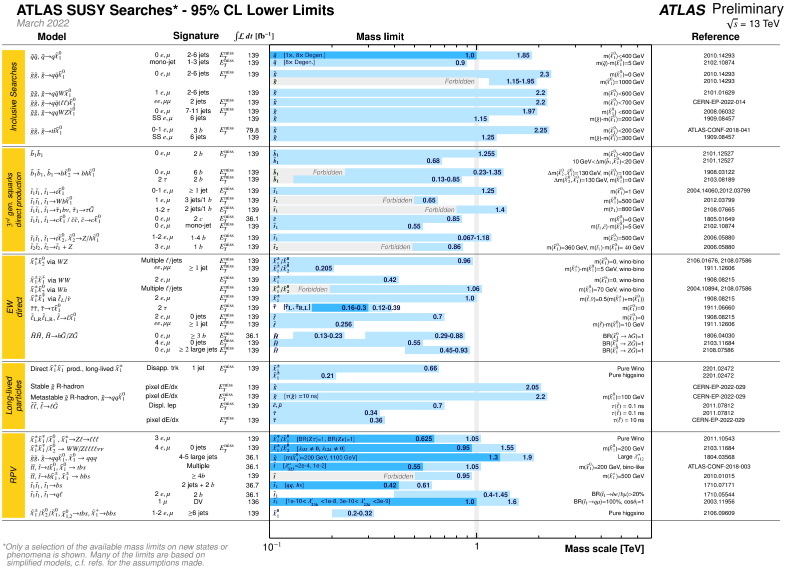




[ATLAS '20]

- Results for various lepton flavor combinations

Overview of simplied model limits



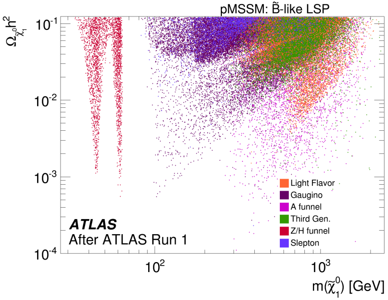
The role of the simplified models

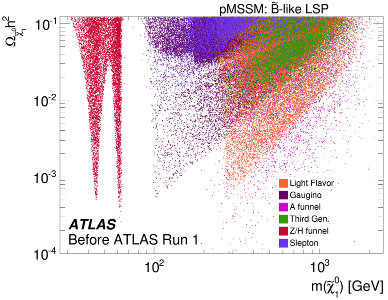
- Simplified models do not represent the real reach of the experiment in excluding a given model (in our case the MSSM)
- They are a way to represent the progress of an experimental in probing a specific experimental signature

To really see the progress in the exclusion of the model, i.e. of the MSSM, one needs to have either

- A complete analysis of the “full-model” points by the experimental collaborations $\rightarrow$ thiis is usually done for very specific models and only at the end of the runs, since it is heavy resource hungry
- Perform the “reinterpretation” (recasting) of the experimental analysis to “map it” to a given model – this is what usually theorists do to study their own preferred model/physics (we will see this in detail in the last lecture)

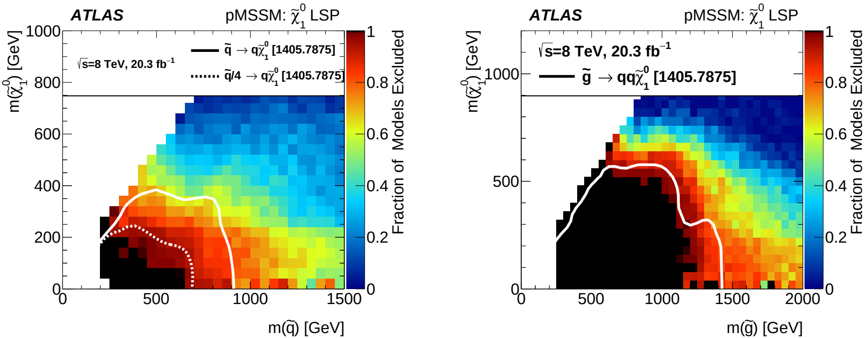
ATLAS pMSSM study





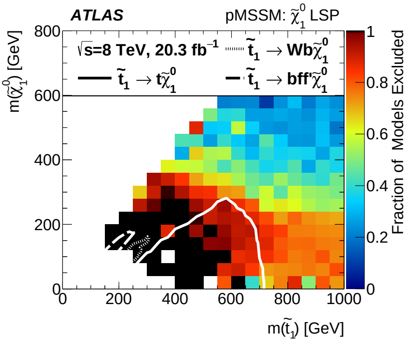
- Comprehensive Run-1 study from ATLAS on pMSSM [1508.06608]

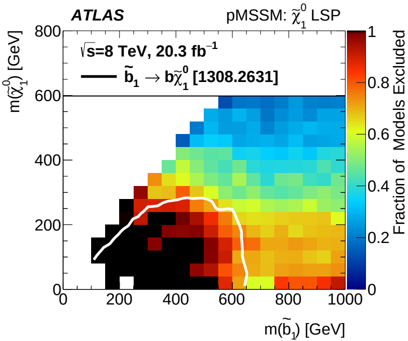
Simplified models vs pMSSM



- Note that the simplified model limits excludes more than what it is in the complete model

Simplified models vs pMSSM





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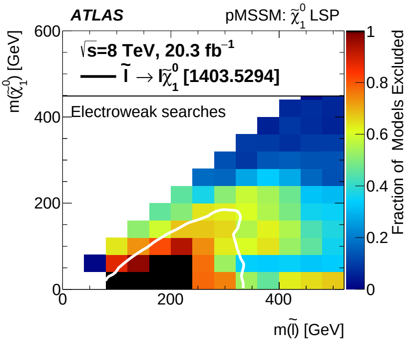

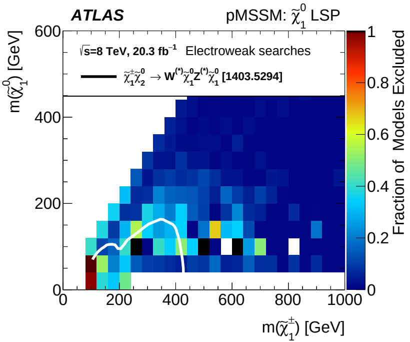

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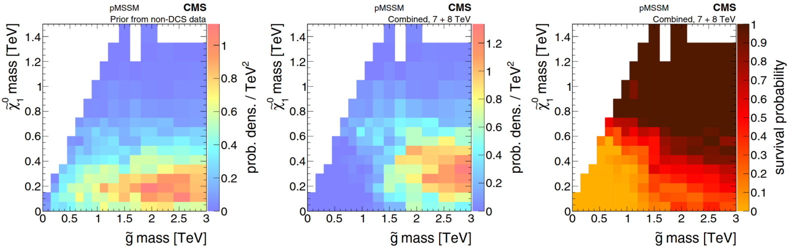
CMS pMSSM study



- Similar study by CMS for Run-1 [1606.03577]

Estabilishing SUSY experimentally

In the case we had a BSM signal at the LHC we would need, before claiming the discovery of the MSSM, to prove

- that the quantum numbers of the would-be superpartners are the same as the corresponding SM state
- that the spin of the would-be superpartner differs by half-unit of spin
- coupling structure
- mass relation between the states
- ...

$\Rightarrow$ to carry out these measurements the precision of the LHC is not enough, especially if the sparticles are heavish and not so heavily produced at the LHC

SUSY at the ILC

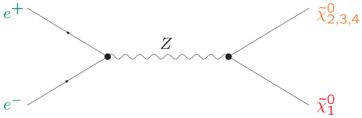
In the case of the ILC (or any other high-energy linear e^+e^- collider) we have:

- much cleaner experimental environment
- threshold scan for the production of the sparticles
- $\Rightarrow$ we can determine with precision properties
- limitation: kinematich reach
- very good prospects for uncolored states
- ...

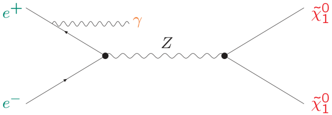
Production processes

Two main final state signatures

- Production with a heavier particle

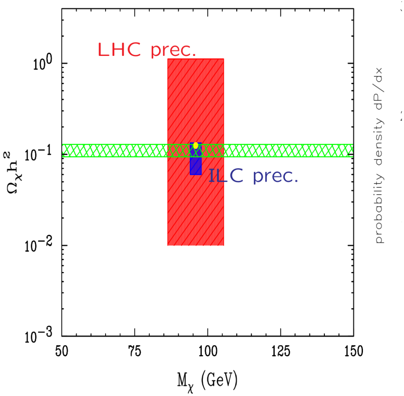


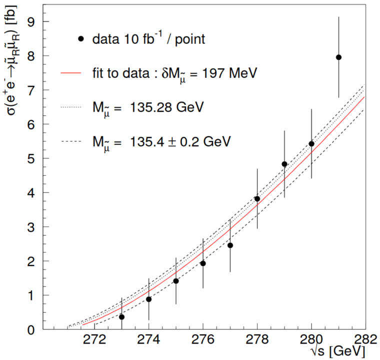
- Production in association with a photon



ILC reach

Two main final state signatures





- Astro vs collider DM
- Smuon mass measurement

Testing the MSSM via EWPOs

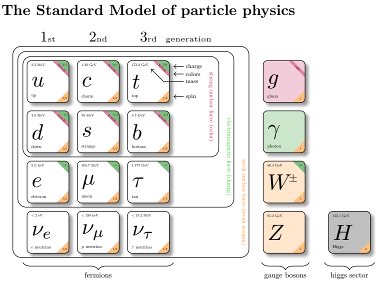
- The idea is to look at (pseudo)-observables involving SM states measured very precisely and to compare the predictions for this quantities in the SM vs a given BSM models
- Deviations are due to the different radiative corrections in the two cases (impact of quantum fluctuations)
- Very high accuracy required $\rightarrow$ many loop calculations

M_W, $\sin^2\theta_{\text{eff}}$, M_h, $(g-2)_\mu$, $\ldots$

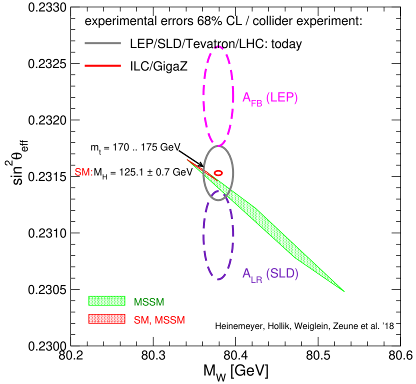
- We have seen the importance of M_W (and in a minor form $\sin^2\theta_{\text{eff}}$) for the SM EW fit
- We have discussed M_h in the previous lecture

An example: the W mass and $\sin^2\theta_{\text{eff}}$ in the MSSM

- Example with two EWPOs: M_W mass and $\sin^2\theta_{\text{eff}}$
- In the MSSM, by using just these two observables, we can probe many possible combinations of MSSM parameters



Experimental measurements



[Heinemeyer et al. '18]

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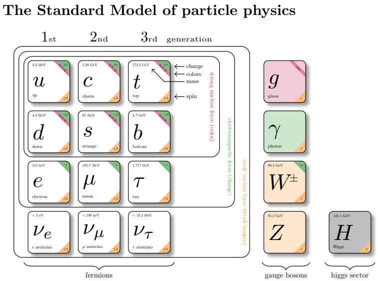

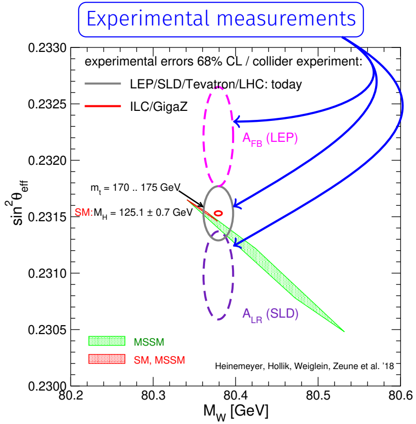

[Heinemeyer et al. '18]

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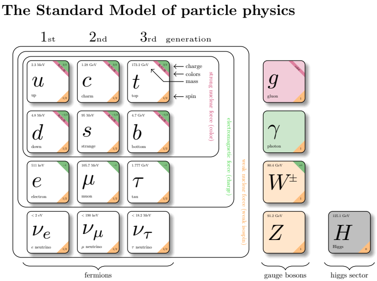

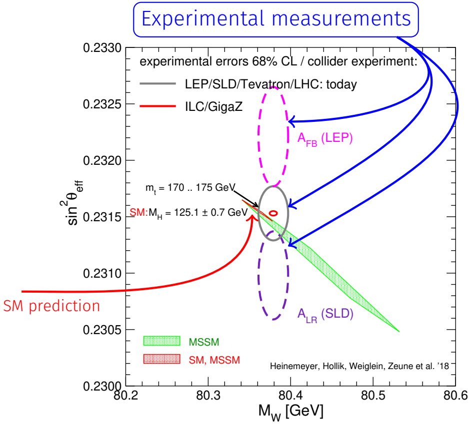

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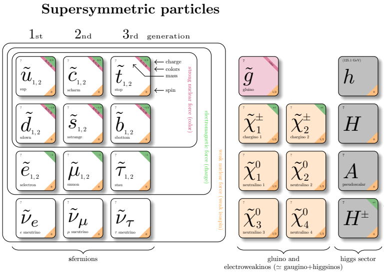

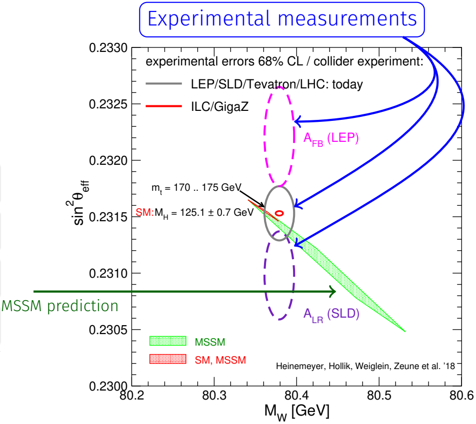

[Heinemeyer et al. '18]

Backup slides

Symmetries, groups and their algebra

symmetry – a group of transformation that leaves the Lagrangian invariant

- The generators of the symmetry group satisfies certain relations that are called their algebra

Examples

- Angular rotations: $\Phi \to \Phi e^{i\theta^a L_a}$
  theory is invariant under rotation
  generators: L_a, **algebra** $[L_a, L_b] = i\epsilon_{abc}L^c$
  quantum numbers: (max. spin)^2, spin $[l(l+1), m = +l \ldots - l]$
- Poincaré symmetry
  space-time symmetries : Lorentz transformations $\Lambda^{\mu\nu}$ (rotations + boosts) and translations P^ρ
  generators: T_a, **algebra** $[T_a, T_b] = if_{abc}T^c$
  quantum numbers: mass, spin
- Internal symmetry groups (e.g. $SU(3) \times SU(2) \times U(1)$) (gauge) symmetry; used to describe the fundamental interactions (QCD, EW forces)
  generators: T_a, **algebra** $[T_a, T_b] = if_{abc}T^c$
  quantum numbers: color, weak isospin, hypercharge (for the SM)

The Lorentz group

Representation of the Lorentz group are labelled by two 'spins', (j_1, j_2) where $j_1, j_2 = 0, \frac{1}{2}, 1 \ldots$. Basic repersentations $M_\alpha{}^\beta$ act on

- $\left(\frac{1}{2}, 0\right)$ – left-handed 2-component Weyl spinor, ψ_α
- $\left(0, \frac{1}{2}\right)$ – right-handed 2-component Weyl spinor, $\bar{\psi}^{\dot{\alpha}}$

The two component Weyl spinors ψ_α (LH) and $\bar{\psi}^{\dot{\alpha}}$ (RH) transform under Lorentz transformation as

$$\psi'_\alpha = M_\alpha{}^\beta \psi_\beta \qquad \bar{\psi}'_{\dot{\alpha}} = (M^*)_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}}$$

$$\psi'^\alpha = \left(M^{-1}\right)^\alpha_\beta \psi^\beta \qquad \bar{\psi}'^{\dot{\alpha}} = \left((M^*)^{-1}\right)_{\dot{\beta}}{}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}}$$

where $M = e^{i\frac{\sigma}{2}(\vec{\theta} - i\vec{\phi})}$ with $\vec{\theta}$ and $\vec{\phi}$ being respectively the three rotation angles, and the boost parameters Summing up, we have

- spinors with undotted indices (first two components of a Dirac spinor), transform under the $\left(\frac{1}{2}, 0\right)$ representation of the Lorentz group
- spinors with dotted indices (last two componenents of a Dirac spinor), transform under the $\left(0, \frac{1}{2}\right)$ representation of the Lorentz group

Spacetime and internal symmetries

The SM is described by

- internal symmetries (the gauge groups of the interactions): T_a
- space-time symmetries (the Poincaré group): $\Lambda^{\mu\nu}$, P^ρ

Note that the internal symmetries are trivial extensions of the Poincaré group

$$[\Lambda^{\mu\nu}, T^a] = 0 \qquad [P^\rho, T^a] = 0$$

$\hookrightarrow$ symmetry is the direct product $(\text{Poincaré group}) \otimes (\text{internal symmetry group})$
Particle states are characterized by the maximal set of community observables:

$$|\underbrace{m, s; \vec{p}, s_3;}_{\textit{spacetime}} \quad \underbrace{Q, I, I_3, Y, \ldots}_{\textit{internal}}\rangle$$

quantum numbers

The Coleman-Mandula theorem

Coleman-Mandula theorem [Coleman, Mandula '67]
Any Lie-group containing both the Poincaré group P and an internal symmetry group $\tilde{G}$ must be the direct product $P \otimes \tilde{G}$

That is, exactly as we have in the SM, one has separately spacetime and quantum numbers

$$| \underbrace{m, s; \vec{p}, s_3;}_{\textit{spacetime}} \quad \underbrace{Q, I, I_3, Y, \ldots}_{\textit{internal}} \rangle$$

quantum numbers

Extensions where a new group $\tilde{G}$ with generators Q^α such that

$$[\Lambda^{\mu\nu}, Q^\alpha \neq 0 \,, \qquad [P^\rho, Q^\alpha] \neq 0$$

are not allowed This implies that no irreducible multiplets can contain particles with different mass or different spin
$\hookrightarrow$ a new symmetry has to predict new particles with the same mass and spin of the SM states
$\hookrightarrow$ this is exluded experimentally

How to extend the symmetry groups of the SM

Coleman-Mandula theorem [Coleman, Mandula '67]

Any Lie-group containing both the Poincaré group P and an internal symmetry group $\tilde{G}$ must be the direct product $P \otimes \tilde{G}$

That is, exactly as we have in the SM, one has separately spacetime and quantum numbers

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$\hookrightarrow$ this is exluded experimentally

The Haag–Łopuszański–Sohnius theorem

First SUSY developments: [Gol'fand, Likhtman '71], [Volkov, Akulov, '72], [Wess, Zumino '73]

$\hookrightarrow$ [Haag, Łopuszański, Sohnius '75] generalized the Coleman-Mandula theorem, showing that another symmetry (mixing spacetime and internal symmetries) was on the other hand possible: supersymmetry

The caveat in the Coleman-Mandula theorem is that it assumes that the generators of the Lie group satisfies commutator relations. However, it is possible to evade the theorem if the generator are fermion spin-1/2 and satisfies instead anti-commutation relations.

$$[\ldots] \to \{\ldots\}$$

In this case, particle with different spins in one multiplet are possible.

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \,, Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Q changes the spin by $\frac{1}{2}$ unit

$\mathcal{N} = 1$ supersymmetry

In the simplest case one has only one fermion genetor Q_α (and its conjugate $\bar{Q}_{\dot{\beta}}$).
The generator (SUSY) algebra is

$$[Q_\alpha, P_\mu] = \left[\bar{Q}_{\dot{\beta}}, P_\mu\right] = 0$$

$$[Q_\alpha, M^{\mu\nu}] = i\,(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$$

$$\{Q_\alpha, Q_\beta\} = \left\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\right\} = 0$$

$$\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu$$

Note that Energy = H = $P_0 \Rightarrow [Q_\alpha, P_0] = 0 \Rightarrow$ the energy is a conserved charge
Superysymmetry is the only possible extension of the Poincaré group in $D = 4$

The harmonic oscillator

The harmonic oscillator (we take $\bar{h} = c = w = \ldots = 1$) Space-momentum commutation relation: $[q, p] = i$.
The construction and destruction operators are

$$a = \frac{1}{\sqrt{2}} (q + ip) \ , a^\dagger = \frac{1}{\sqrt{2}} (q - ip)$$

They satisfy the commutation relation $[a, a^\dagger] = 1$.
We define the number operator $N_b := a^\dagger a$. Its eigenstates $|n\rangle >$ are such that

$$N_b|n\rangle = a^\dagger a|n\rangle = \sqrt{n} a^\dagger |n - 1\rangle = n|n\rangle$$

The hamiltonian of the harmonic oscillator can be written in terms of the nunber operator

$$H_B = \frac{1}{2} \left(p^2 + q^2\right) = N_B + \frac{1}{2} \qquad \Rightarrow \qquad H_B|n\rangle = \left(n + \frac{1}{2}\right) |n\rangle$$

The fermionic harmonic oscillator

Next we consider a two state system analogous to $|\vec{S}^2, S_z\rangle$ for spin $\frac{1}{2}$.

- We define the two states: $|+\rangle := |\frac{1}{2}, +\frac{1}{2}\rangle$ and $|-\rangle := |\frac{1}{2}, -\frac{1}{2}\rangle$
- The spin operators S_x, S_y, S_z satisfy the closed Lie algebra $[S_i, S_y] = i\epsilon_{ijk} S_K$.
- We define the following operators

$$S_{\pm} = S_x \pm i S_y \,, \qquad d^+ := S_+ \,, d := S_-$$

The matrix representations of the states are

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ,|-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

while for the operators we have

$$S_x = \frac{1}{2}\sigma_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , S_y = \frac{1}{2}\sigma_y = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ i & 0 \end{pmatrix} , S_z = \frac{1}{2}\sigma_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Note that $(d^+)^2 = d^2 = 0$ and $\left[d^+, d\right] = 2S_z$, i.e. the commutator $\left[d^+, d\right]$ leaves the algebra of d and d^+.

The fermionic harmonic oscillator

On the other hand, if we look at the anticommutators we have

$$\left\{d^+, d\right\} = 1\,, \{d, d\} = \left\{d^+, d^+\right\} = 0$$

↪ under anti-commutation the algebra is closed.
Then analogously to the bosonic case, we define a number operator $N_F := d^+d$.
The Hamiltonian is then given by $H_F = S_Z = N_F - \frac{1}{2}$. We also find that

$$\begin{aligned}
d^+|-\rangle &= \ldots = |+\rangle\,, d^+|+\rangle = \ldots = 0 \\
d|-\rangle &= \ldots = 0\,, d|+\rangle = \ldots = |-\rangle \\
N_F|+\rangle &= \ldots = |+\rangle\,, H_F|+\rangle = \frac{1}{2}|+\rangle \qquad \text{(fermion)} \\
N_F|-\rangle &= d^+d|- >= 0 \qquad \text{(vacuum)}
\end{aligned}$$

Coupling the two system together

The two-system hamiltonian is given by

$$H := H_B + H_F = N_B + N_F = a^+a + d^+d$$

with eingestates

$$|n\,,+\rangle := |n\rangle \otimes |+\rangle\,, \qquad |n\,,-\rangle := |n\rangle \otimes |-\rangle$$

and we have

$$H|n\,,+\rangle = (a^+a + d^+d)(|n\rangle \otimes |+\rangle) = (n+1)|n\,,+\rangle H|n\,,-\rangle = (n+0)|n\,,-\rangle$$

The lowest energy state of the spectrum is $|0\,,-\rangle$, with $E = 0$, not degenerate.
All the other states are two-fold degenerates

$$\begin{aligned} E = 0 &: |0\,,-\rangle \\ E = 1 &: |1\,,-\rangle\,, |0\,,+\rangle \qquad \text{(multiplet)} \\ E = 2 &: |2\,,-\rangle\,, |1\,,+\rangle \\ &[\ldots] \\ E = n &: |n\,,-\rangle\,, |(n-1)\,,+\rangle \end{aligned}$$

The “SUSY” operator Q

Is there any operator that acts within one multiplet (i.e. that trasnforms one into the other state, leaving the energy unchanged)? That is

$$Q|n\,,+\rangle \to |n+1\,,-\rangle\,, Q^{+}|n+1\,,-\rangle \to |n\,,+\rangle$$

$\Rightarrow Q = c \times a^{+}d$ and $Q^{+} = c^{*} \times ad^{+}$, where c is the normalization factor $c = c^{*} = \frac{1}{\sqrt{2}}$. We have that

- Q, Q^{+} leave the energy unchanged $\Rightarrow [H, Q] = [H, Q^{+}] = 0$
- $Q|vac\rangle = \ldots = 0\,, Q^{+}|vac\rangle = \ldots = 0$
- $[N_F, Q] = \ldots = -Q\,, [N_F, Q^{+} = \ldots = +Q^{+}$
- $[N_B, Q] = \ldots = +Q\,, [N_B, Q^{+} = \ldots = -Q^{+}$
- $\{Q, Q^{+}\} = \ldots = \frac{1}{2}H$, $\{Q, Q\} = 2Q^2 \sim d^2 = 0$, $\{Q^{+}, Q^{+}\} = 2(Q^{+})^2 \sim (d^{+})^2 = 0$

Moreover, the energy expectation value of the Hamiltonian is

$$\begin{aligned}
&\langle n\,,\pm|H|n\,,\pm\rangle \sim \langle n\,,\pm|\{Q, Q^{+}\}|n\,,\pm\rangle \\
&= (\langle n\,,\pm|Q)\left(Q^{+}|n\,,\pm\right) + \left(n\,,\pm|Q^{+}\right)(Q|n\,,\pm\rangle) \\
&= (\ldots) + (\ldots)^{+} \Rightarrow \text{positive definite}
\end{aligned}$$

SUSY algebra

We have therefore demonstrated that Q and Q^+ satisfies the following relations

$$\begin{aligned} \{Q\,, Q^+\} &= \frac{1}{2}H \\ \{Q\,, Q\} = \{Q^+\,, Q^+\} &= 0 \\ [H\,, Q] = [H\,, Q^+] &= 0 \end{aligned}$$

We observe the general structure of commutators and anti-commutators

$$\{F\,, F\} = B\,, \quad [B, B] = B\,, \quad [B, F] = F$$

$\Rightarrow$ Super-Lie/graded Lie algebra

Can SUSY be an exact symmetry?

Let's consider a state $|f\rangle$ with mass m

- Applying the SUSY generator we find a bosonic state $|b\rangle = Q_\alpha|f\rangle$

Remember that $P^2|f\rangle = m^2|f\rangle$. We have for the bosonic state

$$P^2|b\rangle = P^2 Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2 Q_\alpha|f\rangle = m^2|b\rangle$$

That is, the mass of $|b\rangle$ is the same as the fermionic state $|f\rangle$.

In other words, all the states in a given supermultiplet have the same mass.
$\hookrightarrow$ This is clearly experimentally excluded.
$\hookrightarrow$ SUSY must be broken.

Positivity of the Hamiltonian

We recall that the anti-commutation generation of the SUSY generators are

$$\{Q_\alpha\,,\bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu$$

We have

$$\{Q_\alpha\,,\bar{Q}_{\dot\beta}\}\bar\sigma_\nu^{\dot\beta\alpha} = 2\,\underbrace{\sigma^\mu_{\alpha\dot\beta}\bar\sigma_\nu^{\dot\beta\alpha}}_{2g^\mu{}_\nu}P_\mu = 4P_\nu$$

We take now $\nu = 0$

$$H = P_0 = \frac{1}{4}\left\{Q_\alpha\,,\bar{Q}_{\dot\beta}\right\}\bar\sigma_0^{\dot\beta\alpha} = \frac{1}{4}\left(\{Q_1\,,Q_1^\dagger\} + \{Q_2\,,Q_2^\dagger\}\right)$$

where $\bar{Q}_{\dot\alpha} = (Q_\alpha)^\dagger$. However we have also that

$$\{Q_i\,,Q_i^\dagger\} = Q_iQ_i^\dagger + Q_i^\dagger Q_i$$

is clearly hermitian $\rightarrow$ eigenvalues ≥ 0. That shows that

- For any state $|\alpha\rangle$ we have $\langle\alpha|H|\alpha\rangle \geq 0$
- There are no negative eigenvalues, the spectrum of H is bounded from below and ≥ 0

Positivity of the Hamiltonian

We denote our vacuum state as $|0\rangle$

- If the vacuum state is symmetric, i.e. $Q|0\rangle = 0$, $Q^\dagger|0\rangle = 0$ for all Q
- $\Rightarrow$ vacuum state has zero energy, $\langle 0|H|0\rangle = E_{\mathrm{vac}} = 0$

However, if we have spontaneous symmetry breaking, the vacuum is not invariant anymore

- if (global) SUSY is spontaneously broken, i.e. $Q_\alpha|0\rangle \neq 0$
- $\Rightarrow$ then $\langle 0|H|0\rangle = E_{\mathrm{vac}} > 0$
- $\Rightarrow$ Non-vanishing vacuum energy

Multiplet spin structure

We recall that we have that

$$\left\{\bar{Q}_{\dot{\alpha}}\,,\bar{Q}_{\dot{\beta}}\right\}=0$$

from which we see that $\bar{Q}^2_{\dot{\alpha}}=0$ (and analogously $Q^2_{\alpha}=0$). Now, let's consider a massless SUSY multiplet. We start with the state of lowest helicity λ_0

- With an application of $\bar{Q}_{\dot{\alpha}}\rightarrow$ we have one additional state with helicity $\lambda_0+\frac{1}{2}$
- However, from what we have shown above, further applications of $\bar{Q}_{\dot{\alpha}}$ will yield 0! No further states.

In other words, we have shown that a given supermultiplet contains at most one fermionic and one bosonic state (for $\mathcal{N}=1$ SUSY).
if we have N SUSY generators, then 2^{N-1} bosonic and 2^{N-1} fermionic states)
In any case, always the same number of bosonic and fermionic states.

Most relevant supermultiplets

- chiral supermultiplet

The chiral supermultiplet contains a Weyl fermion (spin-1/2) + a complex scalar (spin-0)

- vector supermultiplet

The vector supermultiplet contains a massless vector (spin-1) + a Weyl fermion (spin-1/2)

- graviton supermultiplet

The graviton supermultiplet contains a massless spin-2 (graviton) + a spin-3/2 particle (gravitino)

Superfields and superspace

Recall that

- Translation: generator P_μ, parameter x^μ
- SUSY: generators Q_α, $\bar{Q}_{\dot{\alpha}}$, parameters θ,$\bar{\theta}$

$\hookrightarrow$ θ and $\bar{\theta}$ are Grassmann variables, i.e. anticommuting c-numbers.

A superspace is an extension of the 4-dim. space-time by the coordinates θ^α and $\bar{\theta}^{\dot{\alpha}}$.

Point in superspace	$X = \left(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}\right)$
Superfield	$\phi\left(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}\right)$

Grassmann variables

- Grassmann variables without spinor indexes

$$\{\theta, \theta\} = 0 \qquad \theta\theta = 0$$

- Grassmann variables with spinor indexes

$$\theta\theta \equiv \theta^\alpha \theta_\alpha = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta$$
$$\Rightarrow \theta\theta \neq 0$$

However, if we would Taylor-expand a superfunction $\phi(\theta)$ in terms of the Grassmann variables, the term $\theta^\alpha\theta^\beta\theta^\gamma$ $(\alpha\,,\beta\,,\gamma = 1, 2)$ would be zero.
$\hookrightarrow$ Taylor expansion ends after the second order, i.e. $\phi(\theta) = a + \psi\theta + f\theta\theta$.

- Properties under integration

Similarly it follows that $\int d\theta = 0$ and $\int d\theta\theta = 1$. It follows that

$$\int d^2\theta\phi(\theta) = \int d^2\theta(a + \psi\theta + f\theta\theta) = f \qquad \text{with} \qquad d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^\alpha d\theta^\beta$$

SUSY transformations

Group element of finite SUSY transformation:

$$S(y,\xi,\bar{\xi}) = expi\left(\xi Q + \bar{\xi}\bar{Q} - y^\mu P_\mu\right)$$

in analogy to group elements for Lie groups. Note that $\xi, \bar{\xi}$ are independent of y^μ – global SUSY transformation

- Superfield transformation

We waant to compute $S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta})$. We have

$$S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta}) = \phi\left(x^\mu + y^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}\xi + \theta, \bar{x} + \bar{\theta}\right)$$

Representations of generators are obtained from the infinitesima transformation of the superfield

$$P_\mu = i\partial_\mu\,, \quad Q_\alpha = -i\partial_\alpha + (\sigma^\mu\bar{\theta})_\alpha\partial_\mu\,, \quad \bar{Q}_{\dot{\alpha}} = i\partial_{\dot{\alpha}} - (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

with $\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}\,, \bar{\partial}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}$. From this, we have that the SUSY covariant derivatives are

$$D_\alpha = -i\partial_\alpha - (\sigma^\mu\bar{\theta})_\alpha\partial_\mu\,, \quad \bar{D}_{\dot{\alpha}} = i\bar{\partial}_{\dot{\alpha}} + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

General superfield in component form

Rememembering that we can have at most $\theta\theta$ and $\bar{\theta}\bar{\theta}$ ($\theta\theta\theta = \bar{\theta}\bar{\theta}\bar{\theta} = 0$), the structure of the most general superfield has to be:

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta F(x) + \theta\bar{\theta}H(x) + \theta\sigma^{\mu}\bar{\theta}A_{\mu}(x) \\ + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\xi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x)$$

Components:

- ϕ, F, H, D – scalar fields
- A_{μ} – vector field
- $\psi, \bar{\chi}, \bar{\lambda}, \xi$ – Weyl-spinor fields

$\Rightarrow$ too many components in 4-dimension for irreducible representations of SUSY with spin ≤ 1 (chiral or vector multiplets)
$\Rightarrow$ representation is reducible

Irreducible superfields

Irreducible superfields are obtained by imposing conditions on the most general superfields, namely that it has to be invariant under SUSY transformation

- $\bar{D}_{\dot{\alpha}}\mathbf{\Phi} = 0$: left-handed chiral superfield ($LH_\chi SF$)
- $D_\alpha \mathbf{\Phi} = 0$: right-handed chiral superfield ($RH_\chi SF$)
- $\mathbf{\Phi} = \mathbf{\Phi}^\dagger$: vector superfield

The chiral superfields represent left- or right-handede components of a Weyl fermion plus its scalar partner simplified $LH_\chi SF$ in components:

$$\phi(x, \theta) = \phi(x) + \sqrt{2}\theta\psi(x) - (\theta\theta)F(x) \tag{3}$$

ϕ , F: scalar fields, ψ: Weyl spinor field

Simplified left-handed chiral superfield

$$\phi_L(x,\theta) = \phi(x) + \sqrt{2}\theta\psi(x) - (\theta\theta)F(x)$$

mass dimensions: $[\phi] = 1$, $[\psi] = \frac{3}{2}$, $[F] = 2$.

Recall that: $\theta\theta = \epsilon^{\alpha\beta}\theta_\alpha\theta_\beta$, $\theta^\alpha\theta_\alpha = -\theta_1\theta_2 + \theta_2\theta_1$.
The infinitesimal SUSY transformation are: $\theta^\alpha = \theta^\alpha + \epsilon^\alpha$, $x^\mu = x^\mu + 2i\theta\sigma^\mu\bar{\theta}$.

$$\delta\phi_L = \left(\epsilon\frac{\partial}{\partial\theta} + \bar{\epsilon}\frac{\partial}{\partial\bar{\theta}} + 2i\theta\sigma^\mu\bar{\epsilon}\partial_\mu\right)\phi_L$$

Now we replace ϕ_L with its expression in terms of components

$$\delta\phi_L = 2i\theta\sigma^\mu\bar{\epsilon}\partial_\mu\phi + \sqrt{2}\epsilon^\alpha\psi_\alpha\sqrt{2}2i\theta\sigma^\mu\bar{\epsilon}\partial_\mu\theta^\alpha\partial_\alpha + 2\epsilon^\alpha\theta_\alpha F + \mathcal{O}(\theta^3)$$

where we have used

$$\epsilon^\alpha\frac{\partial}{\partial\theta^\alpha}\theta^\beta\epsilon_{\beta\gamma}\theta^\gamma = \ldots = 2\epsilon^\alpha\theta_\alpha$$

Simplified left-handed chiral superfield

Using

$$\theta^\beta (\sigma^\mu)_{\beta\dot\beta}\bar\epsilon^{\dot\beta}\theta^\alpha = \ldots = -\frac{1}{2}\theta\theta(\sigma^\mu)^\alpha{}_{\dot\beta}\bar\epsilon^{\dot\beta}$$

we can rewrite ϕ_L as

$$\delta\phi_L = 2i\theta\sigma^\mu\bar\epsilon\partial_\mu\phi + \sqrt{2}\epsilon^\alpha\psi_\alpha + \sqrt{2}i(\theta\theta)(\sigma^\mu)^\alpha{}_{\dot\beta}\bar\epsilon^{\dot\beta}\partial_\mu\psi_\alpha + 2\epsilon^\alpha\theta_\alpha F + \mathcal{O}(\theta^3)$$

The SUSY transformation of a $LH_\chi SF$ yields a $LH_\chi SF$

$$\delta\phi_L \overset{!}{=} \delta\phi + \sqrt{2}\theta^\alpha\delta\psi_\alpha + (\theta\theta)\delta F$$

From which

$$\begin{aligned}
&\theta^0 : \delta\phi = \sqrt{2}\epsilon\psi && \text{boson } \to \text{fermion}\\
&\theta^1 : \delta\psi_\alpha = \sqrt{2}\epsilon_\alpha F + i\sqrt{2}(\sigma^\mu)_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha}\partial\phi && \text{fermion } \to \text{boson}\\
&\theta^2 : \delta F = -i\sqrt{2}\partial\left((\sigma^\mu)^\alpha{}_{\dot\beta}\bar\epsilon^{\dot\beta}\psi_\alpha\right) && \text{total derivative}
\end{aligned}$$

Analogous for $RH_\chi SF$

Vector superfield

The components of a vector superfields are

$$
\begin{aligned}
V(x,\theta,\bar\theta) = {} & c(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta v_\mu(x) \\
& + \frac{i}{2}(\theta\theta)(M(x)+iN(x)) - \frac{i}{2}(M(x)-iN(x)) \\
& + i(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_m u\bar\chi(x)\right) \\
& + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right)
\end{aligned}
$$

The number of components can be reduced via a SUSY gauge transformation, the choice of the Wess-Zumino gauge.

$$
\tilde{(}x) = c(x) = M(x) = N(x) \equiv 0 \tag{4}
$$

With this choice, the vector supermultiplet becomes

$$
V(x,\theta,\bar\theta) = \ldots + i(\theta\theta)(\bar\theta)\bar\lambda(x) - i(\bar\theta\bar\theta)\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x) + \ldots
$$

$$
\delta D = -\xi\sigma^\mu\partial_\mu\bar\lambda(x) - \partial_\mu\lambda(x)\sigma^\mu\bar\xi \qquad \mathrm{D} \rightarrow \text{ total derivative}
$$

Supersymmetric Lagrangian

Aim: to build an actiona that is invariant under SUSY transformations

$$\delta \int d^4x\mathcal{L}(x) = 0$$

Satisfied if $\mathcal{L} \to \mathcal{L} + \text{total derivative.}$

F and D terms (the terms with the highst powers of θ and $\bar{\theta}$) of a chiral and vector supermultiplet transform into a total derivative under SUSY transformations $\Rightarrow$

F-terms ($LH_\chi SF$, $RH_\chi SF$) and D-terms (vector SF) to construct an invariant action

$$S = \int d^4 \left(\int d^2\theta\mathcal{L}_F + \int d^2\theta d^2\bar{\theta}\mathcal{L}_D \right)$$

If $\mathbf{\Phi}$ is a $LH_\chi SF$, then also $\mathbf{\Phi}^n$ is a $LH_\chi SF$ (since $\bar{D}_{\dot{\alpha}}\mathbf{\Phi}^n = 0$ if $\bar{D}_{\dot{\alpha}}\mathbf{\Phi} = 0$).
$\Rightarrow$ products of chiral superfields are chiral superfields, products of vector superfields are vector superfields.

Supersymmetric Lagrangian

- **F-term Lagrangian**

$$\mathcal{L}_F = \int d^2\theta \sum_{ijk} \left(a_i \mathbf{\Phi}_i + \frac{1}{2} m_{ij} \mathbf{\Phi}_i \mathbf{\Phi}_j + \frac{1}{3} \lambda_{ijk} \mathbf{\Phi}_i \mathbf{\Phi}_j \mathbf{\Phi}_k \right) + h.c.$$

Terms of higher order in $\mathbf{\Phi}_i$ lead to a non-renormalizable Lagrangian $\hookrightarrow$ F-term Lagrangian contains mass terms, scalar-fermion interactions but no kinetic terms

- **D-term Lagrangian**

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta} V \qquad (5)$$

$\hookrightarrow$ D-term Lagrangian contains kinetic terms for the vector fields

The Wess-Zumino Lagrangian

Construction of the Lagrangian from chiral superfields Φ_i: Φ_i , $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$. Note that the combination $\Phi_i^\dagger\Phi_i$ is a vector superfield ($(\Phi_i^\dagger\Phi_i)^\dagger = \Phi_i^\dagger\Phi_i$).

$$[\Phi_i^\dagger\Phi_i]_{\theta\theta\bar{\theta}\bar{\theta}} = F^\dagger F + (\partial_\mu\phi^*)(\partial^\mu\phi) + \frac{i}{2}\left(\psi\sigma^\mu\partial_\mu\bar{\psi} - \partial_\mu\psi\sigma^\mu\bar{\psi}\right) + \partial_\mu(\ldots)$$

The auxiliar field F can be eliminated via the equations of motions:

$$\text{abelian } F = m\phi^* + g(\phi^*)^2$$
$$\text{non}-\text{abelian } D^G = \ldots\sum_a g_G\left(\phi_i^\dagger(T_G)^a\phi_i\right)$$

From which we have

$$\mathcal{L}_D = FF^* + \frac{1}{2}\sum_G D^G(D^G)^\dagger + \ldots$$

The Wess-Zumino Lagrangian

Combinining everything together

$$\begin{aligned}\mathcal{L}_D = &\frac{i}{2}\left(\psi_i\sigma^\mu\partial_\mu\bar{\psi}_i - (\partial_m u\psi_i)\sigma^\mu\bar{\psi}_i\right) - \frac{1}{2}m_{ij}(\psi_i\psi_j + \bar{\psi}_i\bar{\psi}_j)\\ &+ (\partial_\mu phi_i^*)(\partial^\mu\phi_i) - \sum_i |a_i + \frac{1}{2}m_{ij}\phi_j + \frac{1}{3}\lambda_{ijk}\phi_j\phi_k|^2\\ &- \lambda_{ijk}\phi_i\psi_j\psi_k - \lambda_{ijk}^\dagger\phi_i^2\bar{\psi}_j\bar{\psi}_k\end{aligned}$$

This the Lagrangian for ϕ_i complex scalar fields, and Weyl spinor fields, with the same mass m_{ij}. Note that the relation between the couplings imposed by SUSY.

The superpotential

$\mathcal{L}$ can be rewritten as a kinetic part plus a contribution from the so-called superpotential ν:

$$\nu(\phi_i) = a_i\phi_i + \frac{1}{2}m_{ij}\phi_i\phi_j + \frac{1}{3}\lambda_{ijk}\phi_i\phi_j\phi_k$$

We can write the $\mathcal{L}$ as

$$\mathcal{L} = \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar{\psi}_i - (\partial_\mu\psi_i)\sigma^\mu\bar{\psi}_i) + (\partial_\mu\phi_i^*)(\partial^\mu phi_i) - \sum_i \left|\frac{\partial\nu}{\partial\phi_i}\right|^2 - \frac{1}{2}\frac{\partial^2\nu}{\partial\phi_i\partial phi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\nu^*}{\partial\phi_i^*\partial phi_j^*}\psi_i^*\psi_j^*$$

The superpotential ν determines all interactions and mass terms. The Wess-Zumino model corresponds to the case $a_i = 0$.