The Strong CP problem and Planck scale physics

A short story

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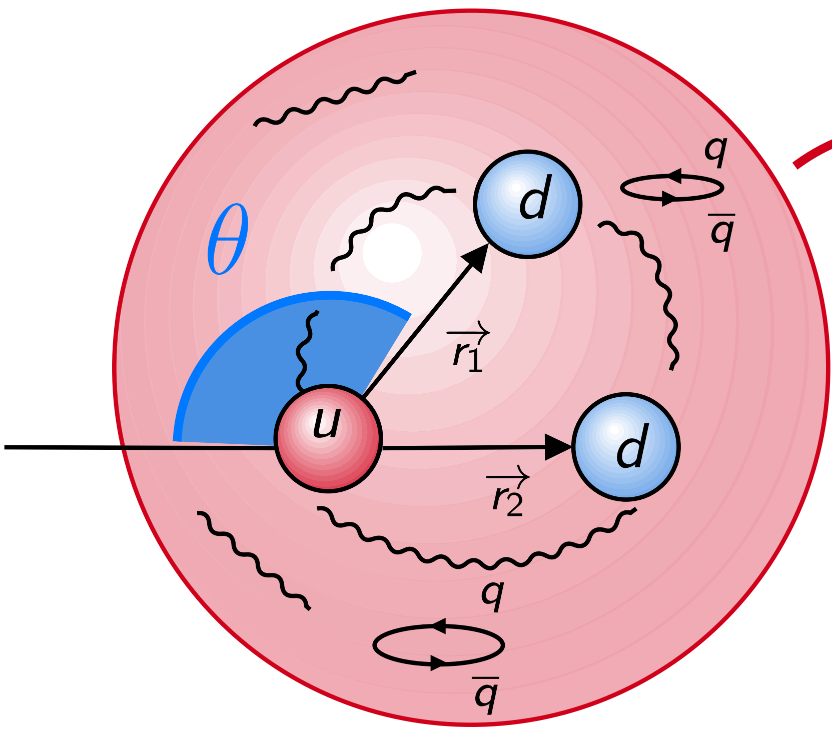
PhD seminars

(Maximal) plan of the talk

- Neutron eDM theoretical estimate,
- Neutron eDM measurement & the Strong CP problem,
- Possible solutions to the Strong CP problem (symmetries, relaxation mechanism,..),
- The Peccei-Quinn axion,
- The Axion Quality Problem.

Neutron eDM theoretical estimate

At its heart, the Strong CP problem is a question of why the neutron electric dipole moment (eDM) is so small.
At the cartoon level, we can depict a neutron - a (udd) QCD bound-state - as follows



electric dipole moment

$$\vec{d_n} = \sum_i Q_i \vec{r_i} \simeq 10^{-13} cm\ e\ \sqrt{1-\cos\theta}$$

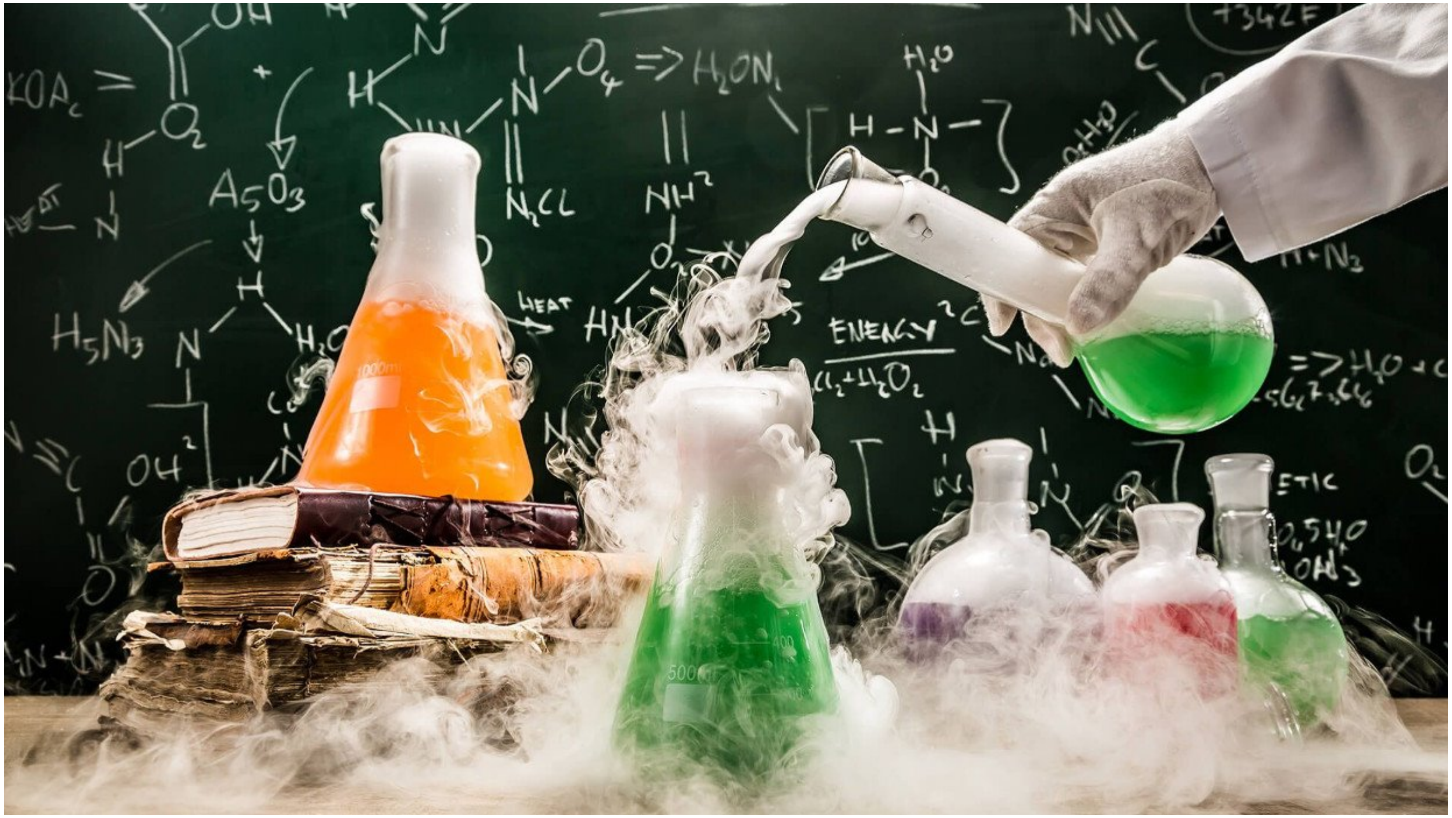
classical neutron radius

$$Expectations: \sqrt{1-\cos\theta} = \mathcal{O}(1)$$

$$\vec{d_n} \simeq \mathcal{O}\left(10^{-13}\right) cm\ e$$

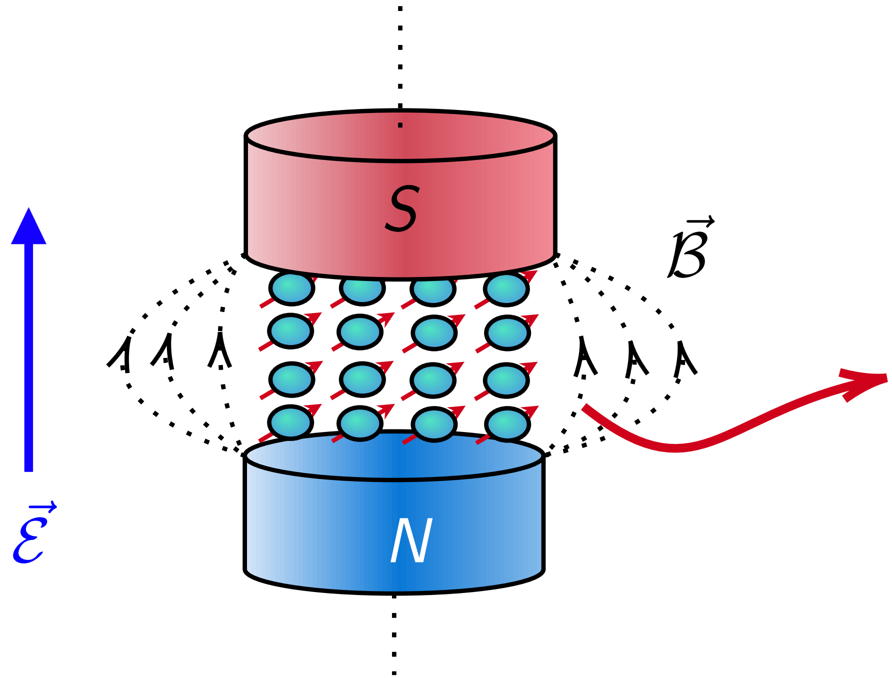
$$Q^{down}_{em} = -1/3$$
$$Q^{up}_{em} = 2/3 \Rightarrow (udd) \textit{ is neutral } \checkmark$$

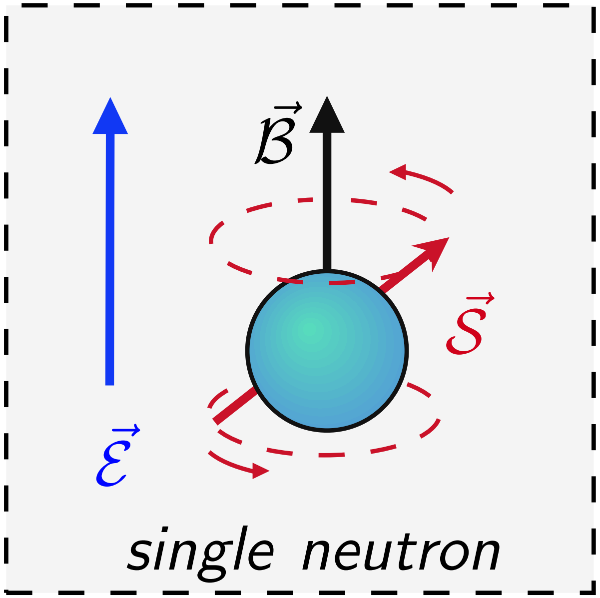
What is the actual value of the neutron eDM?



Many experiments have attempted to measure the neutron eDM and the simplest conceptual way to do so is via a **precession experiment**.

Consider a sample of neutrons, and apply external magnetic and electric fields, respectively $\vec{\mathcal{B}}$, $\vec{\mathcal{E}}$, with $\vec{\mathcal{B}} \parallel \vec{\mathcal{E}}$



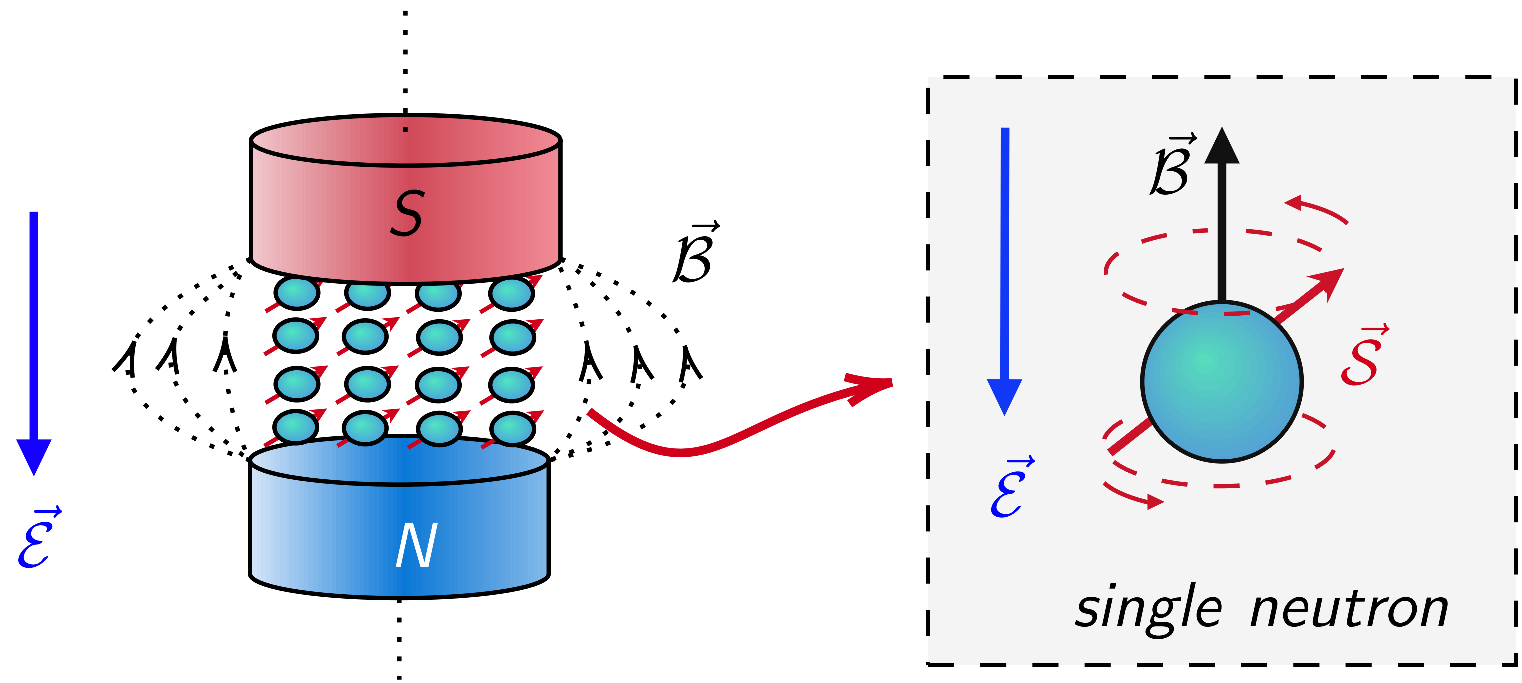


Planck constant

$$hf_+ = |\mu_n \vec{\mathcal{B}} + d_n \vec{\mathcal{E}}|$$

Larmor frequency

We consider the same sample, but the external fields $\vec{\mathcal{B}}$ and $\vec{\mathcal{E}}$ we apply are anti-parallel, i.e. $\vec{\mathcal{B}} \parallel -\vec{\mathcal{E}}$



Planck constant

$$hf_- = |\mu_n \vec{\mathcal{B}} - d_n \vec{\mathcal{E}}|$$

Larmor frequency

Summarizing, we measure the frequencies

$$hf_+ = \left|\mu_n \vec{\mathcal{B}} + d_n \vec{\mathcal{E}}\right| ,$$
$$hf_- = \left|\mu_n \vec{\mathcal{B}} - d_n \vec{\mathcal{E}}\right| ,$$

from which we deduce that

$$|d_n| = \frac{2h\Delta f}{\mathcal{E}} \leq 10^{-26} \mathrm{e\,cm}$$

The Strong CP problem

Theory

$$d_n \simeq \mathcal{O}\left(10^{-13}\right)\, e\ cm$$

Experiment

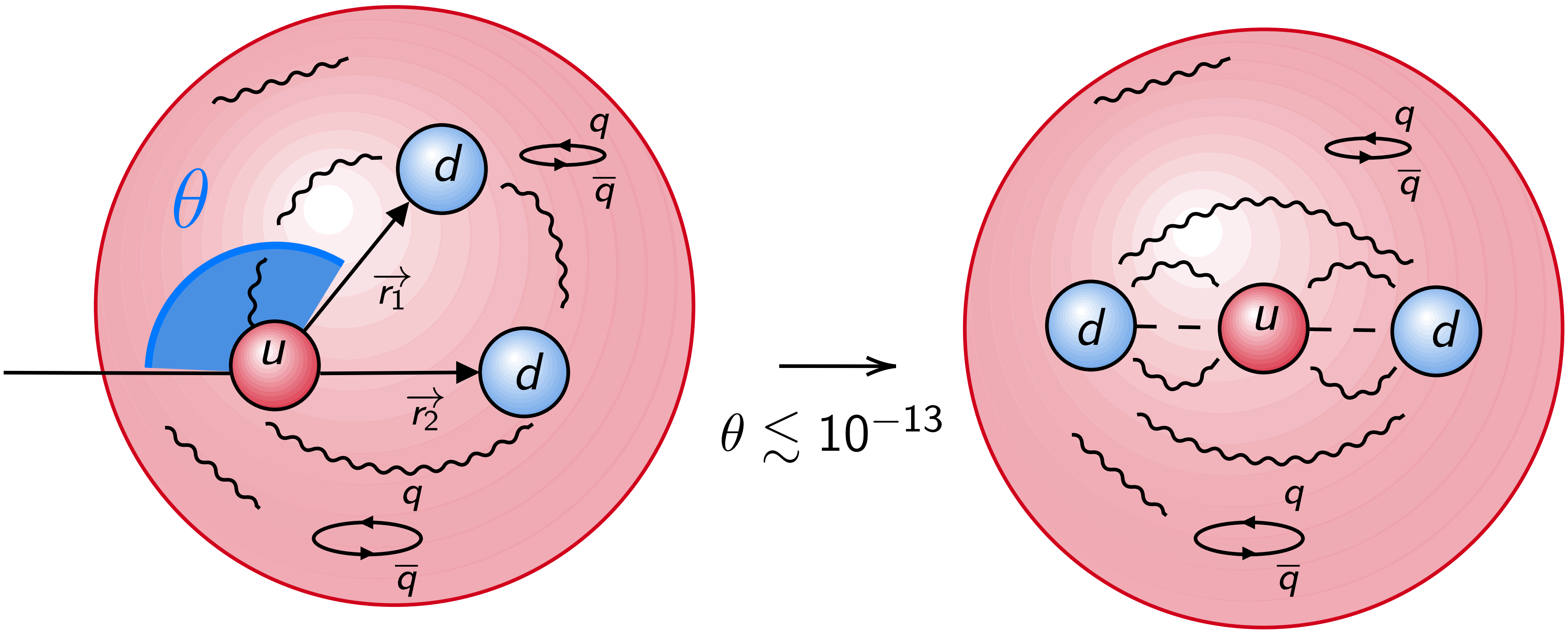
$$d_n \lesssim \mathcal{O}\left(10^{-26}\right)\, e\ cm$$

$$d_n \simeq 10^{-13}\sqrt{1-\cos\theta}\ e\ cm \Rightarrow \theta \lesssim 10^{-13}$$

Why is θ so small?

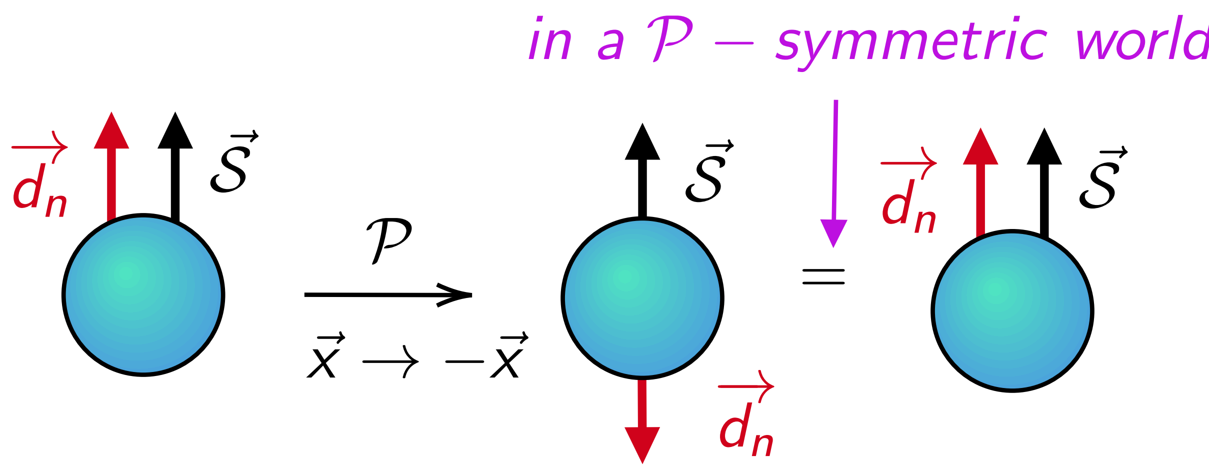
The Strong CP problem and its (possible) classical solutions

Phrased in another way, the Strong CP problem is simply the statement that we should have drawn all of the quarks on the same line



There are three solutions to the Strong CP problem that can be described at the classical level:

- requires Parity ($\mathcal{P}$) to be a good symmetry of nature, indeed



$$\Rightarrow \overrightarrow{d_n} = -\overrightarrow{d_n} \Leftrightarrow \overrightarrow{d_n} = 0!$$

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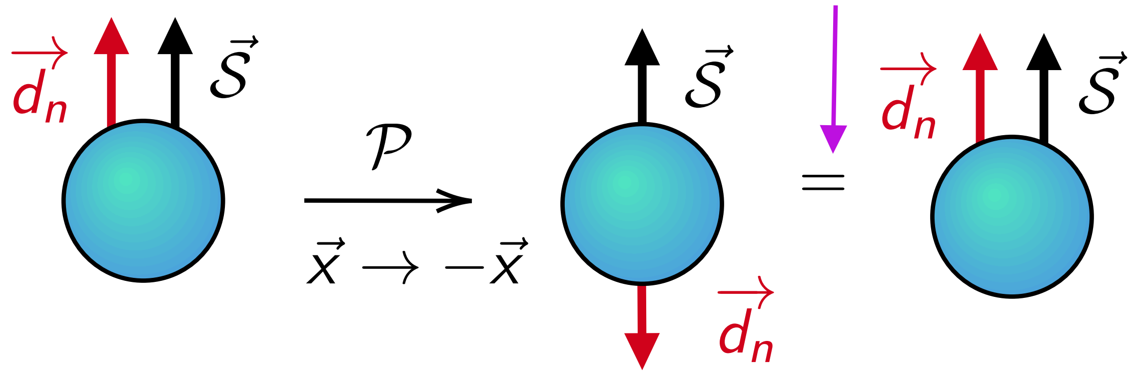
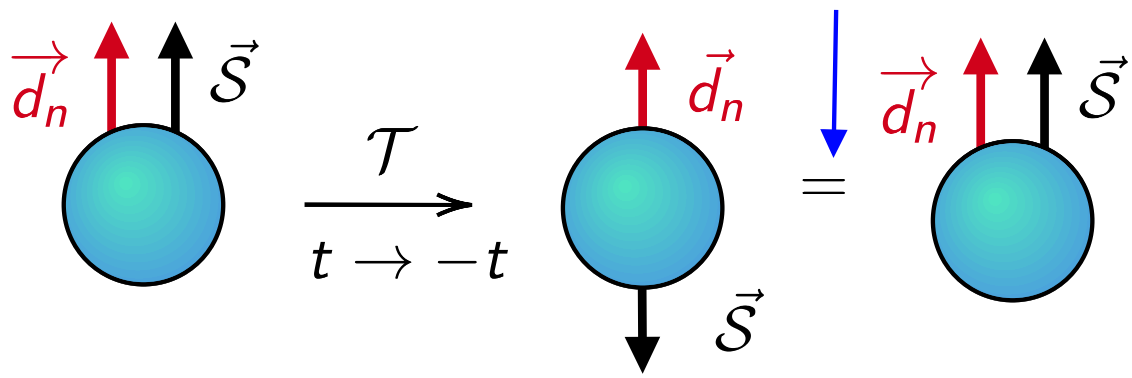
in a $\mathcal{P}$ − symmetric world



$$\Rightarrow \vec{d_n} = -\vec{d_n} \Leftrightarrow \vec{d_n} = 0!$$

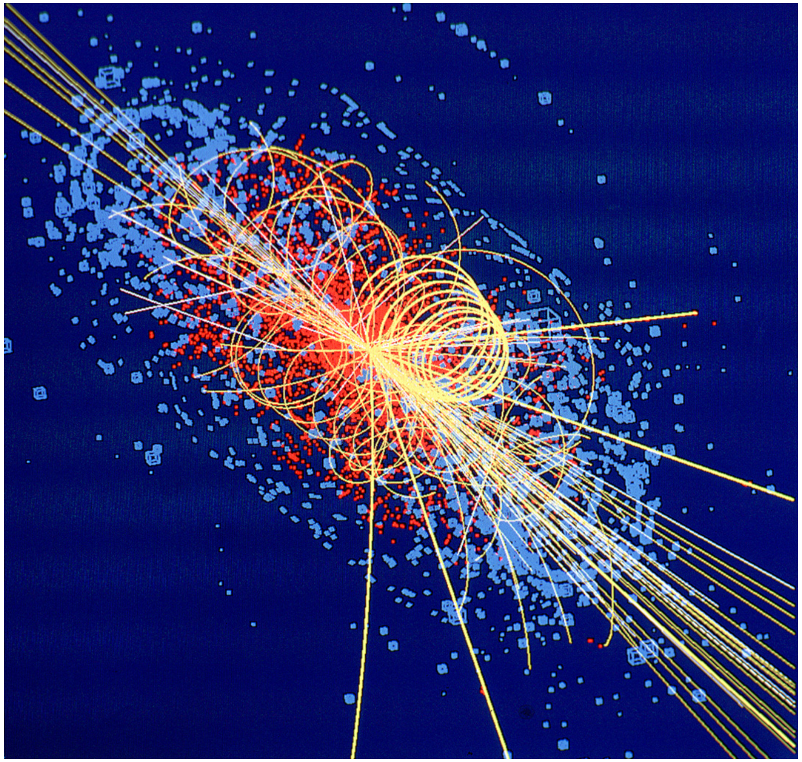
- requires Charge conjugation + Parity ($\mathcal{CP}$), i.e. *time reversal* ($\mathcal{T}$), to be a good symmetry of nature, indeed

in a $\mathcal{T}$ − symmetric world



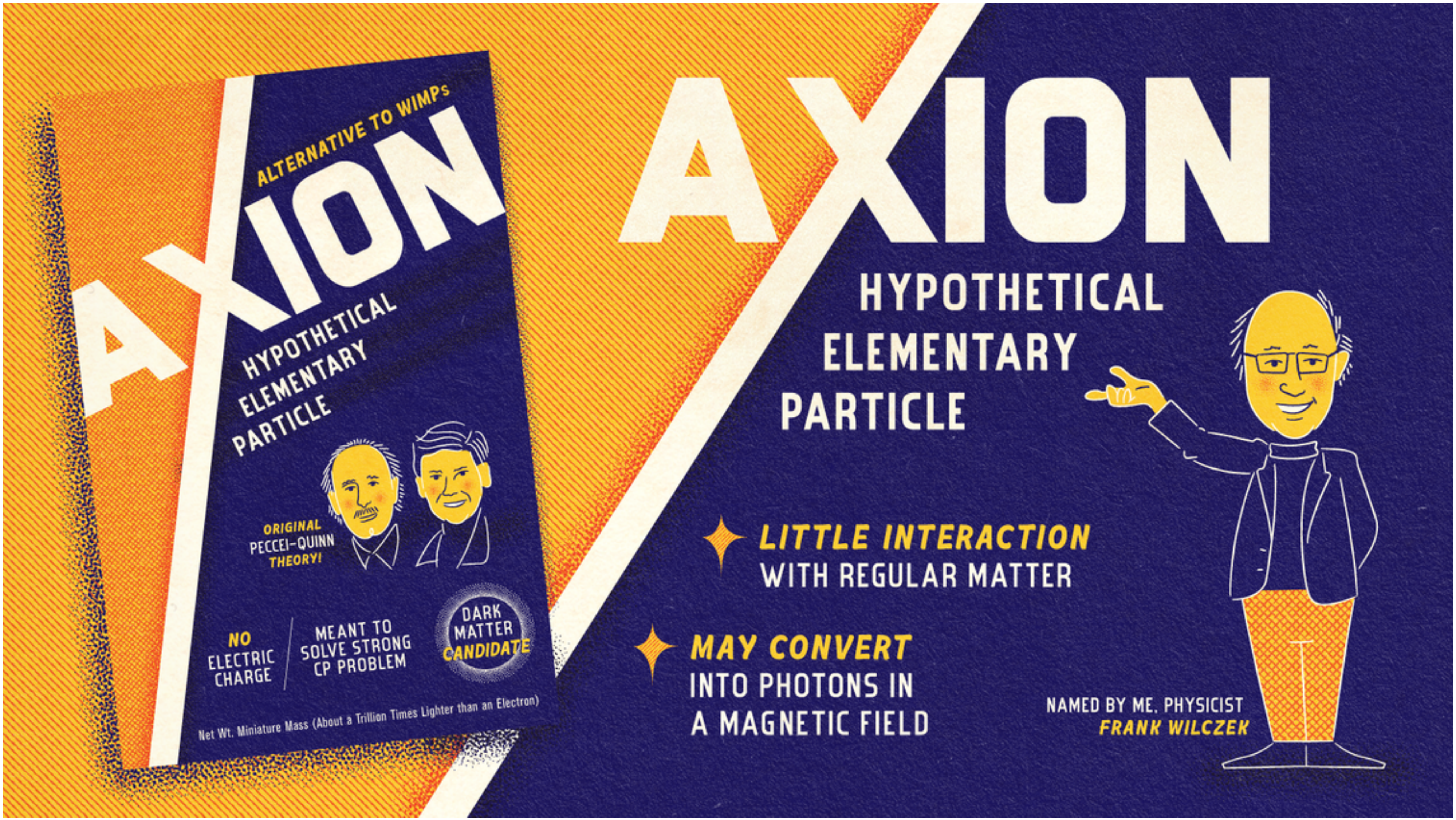
$$\Rightarrow \vec{d_n} = -\vec{d_n} \Leftrightarrow \vec{d_n} = 0!$$

However, neither Parity nor $C\mathcal{P}$ are good symmetries of the universe:



- the W bosons couple only to right-handed particles,
- $C\mathcal{P}$ is violated by the Cabibbo angle.

The Peccei-Quinn Axion solution



The Peccei-Quinn Axion solution

The strong CP problem concerns the *smallness* of θ. Up to now we have tried, unsuccesfully, to solve this puzzle via symmetry arguments. Another way to deal to try to solve this puzzle is via a **relaxation mechanism**. The so-called **Peccei-Quinn mechanism**.

The Peccei-Quinn mechanism

The last solution at the classical level is the axion solution.

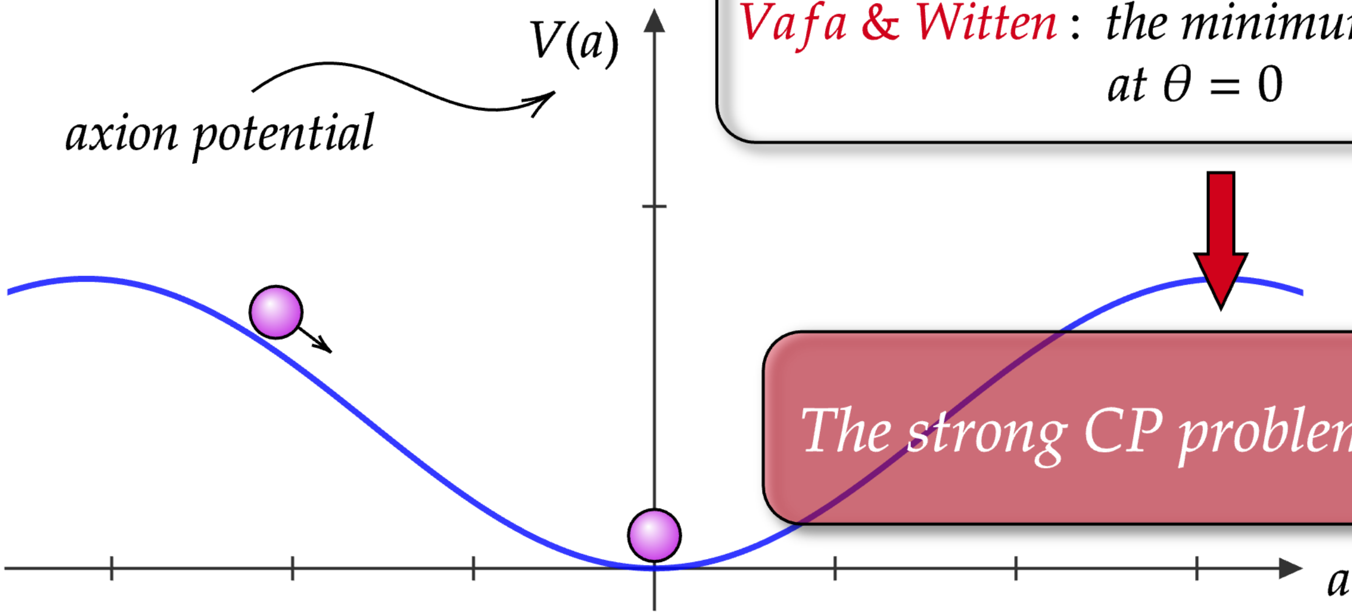
The Peccei-Quinn mechanism

The last solution at the classical level is the axion solution.
Idea: the angle θ is dynamical and can change, relaxing to its *minimal value*:

$\theta \to \frac{a(x)}{f_a}$

a(x) is a dynamical field, the so-called Peccei-Quinn axion

$f_a \sim 10^{12}\,GeV$ *is the axion decay constant*



Vafa & Witten: the minimum will always be at $\theta = 0$

The strong CP problem is SOLVED!

Considerations on the vacuum structure of Strong Interactions

The vacuum structure of QCD (Yang-Mills theories) is highly non-trivial: each vacuum is labelled by a topological number, called the *winding number* n

$$n = \frac{g^2}{32\pi^2}\int d^4x \tilde{F}^a_{\mu\nu}F^{\mu\nu,a}, \quad \tilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^a_{\rho\sigma}$$

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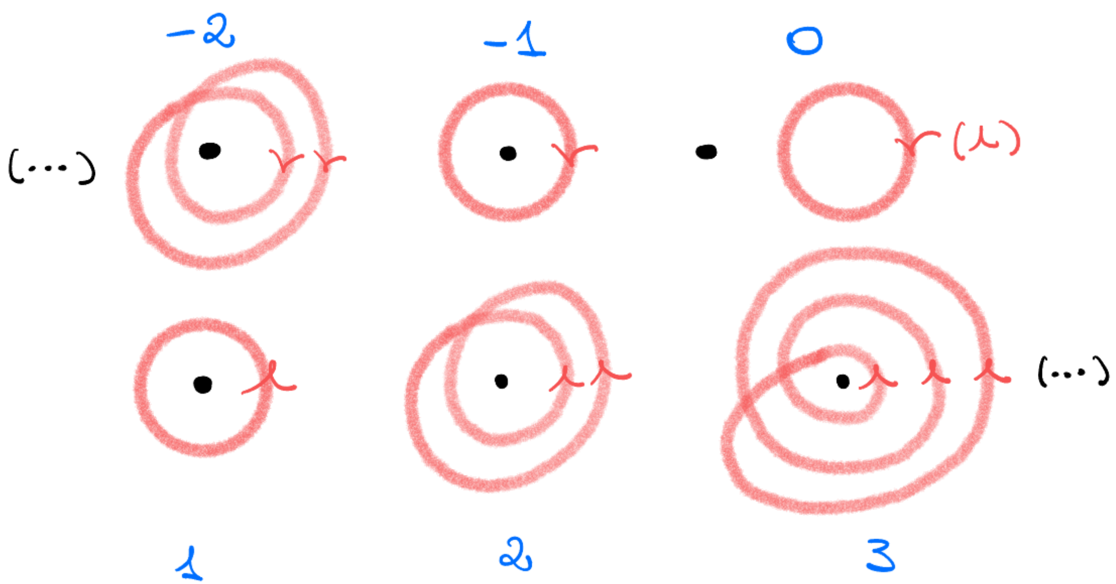
The winding number is simply the number of times a circle wraps around another circle:

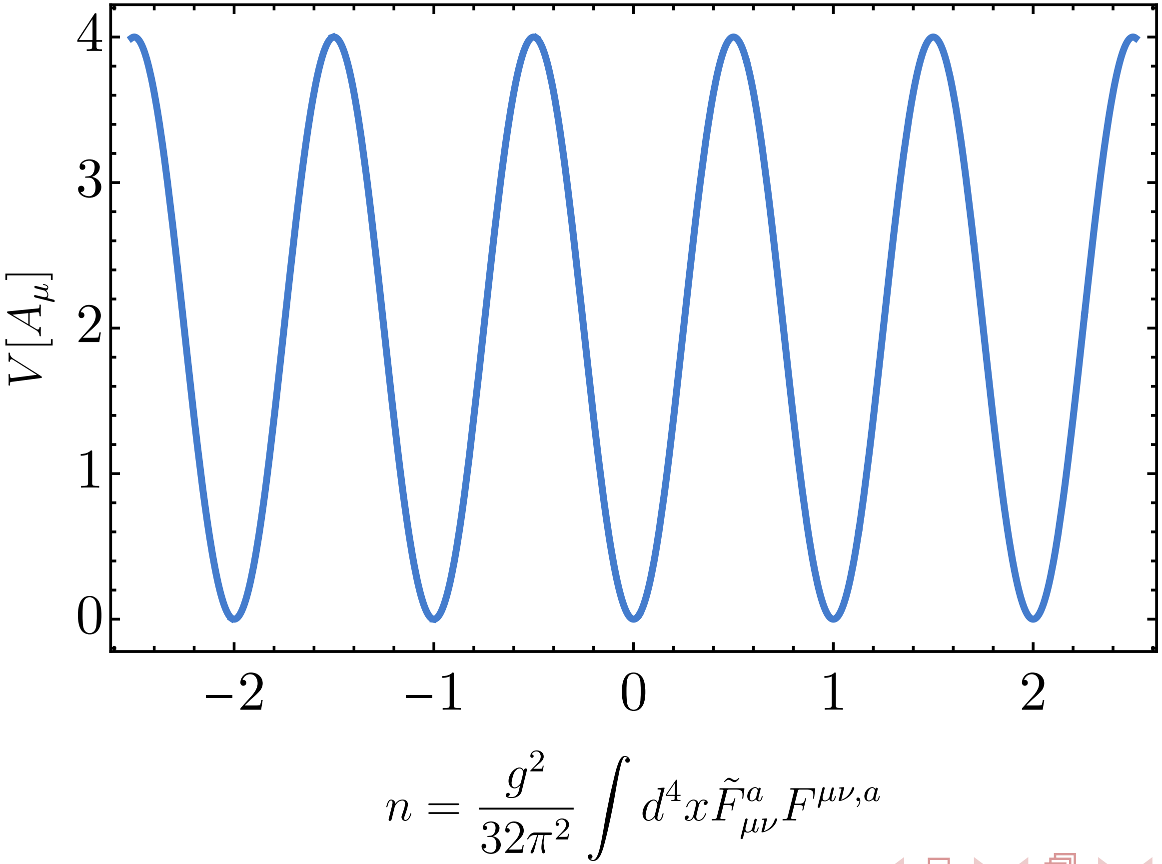


Considerations on the vacuum structure of Strong Interactions

Therefore,

$$n = \frac{g^2}{32\pi^2}\int d^4x \tilde{F}^a_{\mu\nu}F^{\mu\nu,a} = integer\ ,$$

and we have the gauge "potential"



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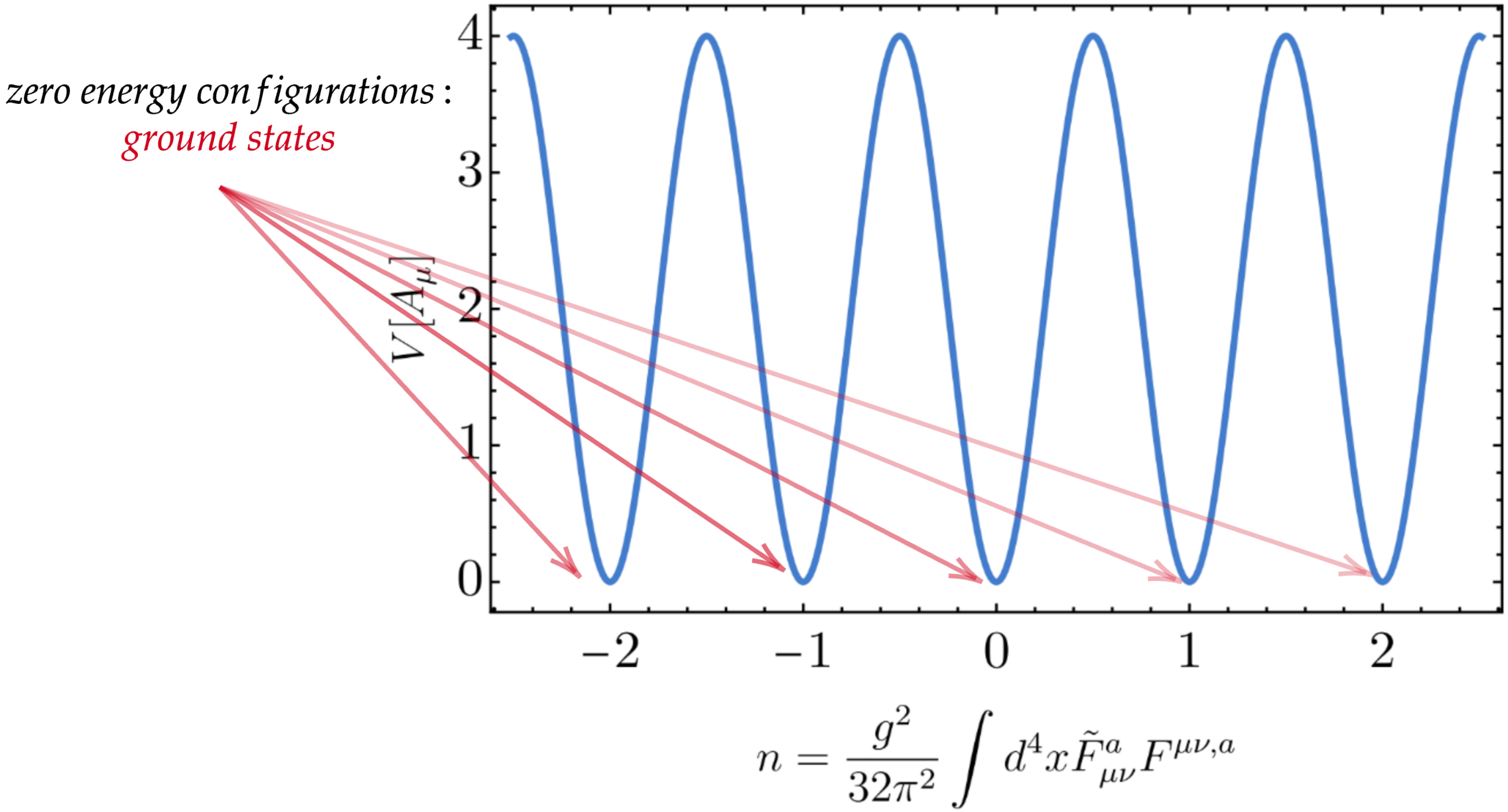
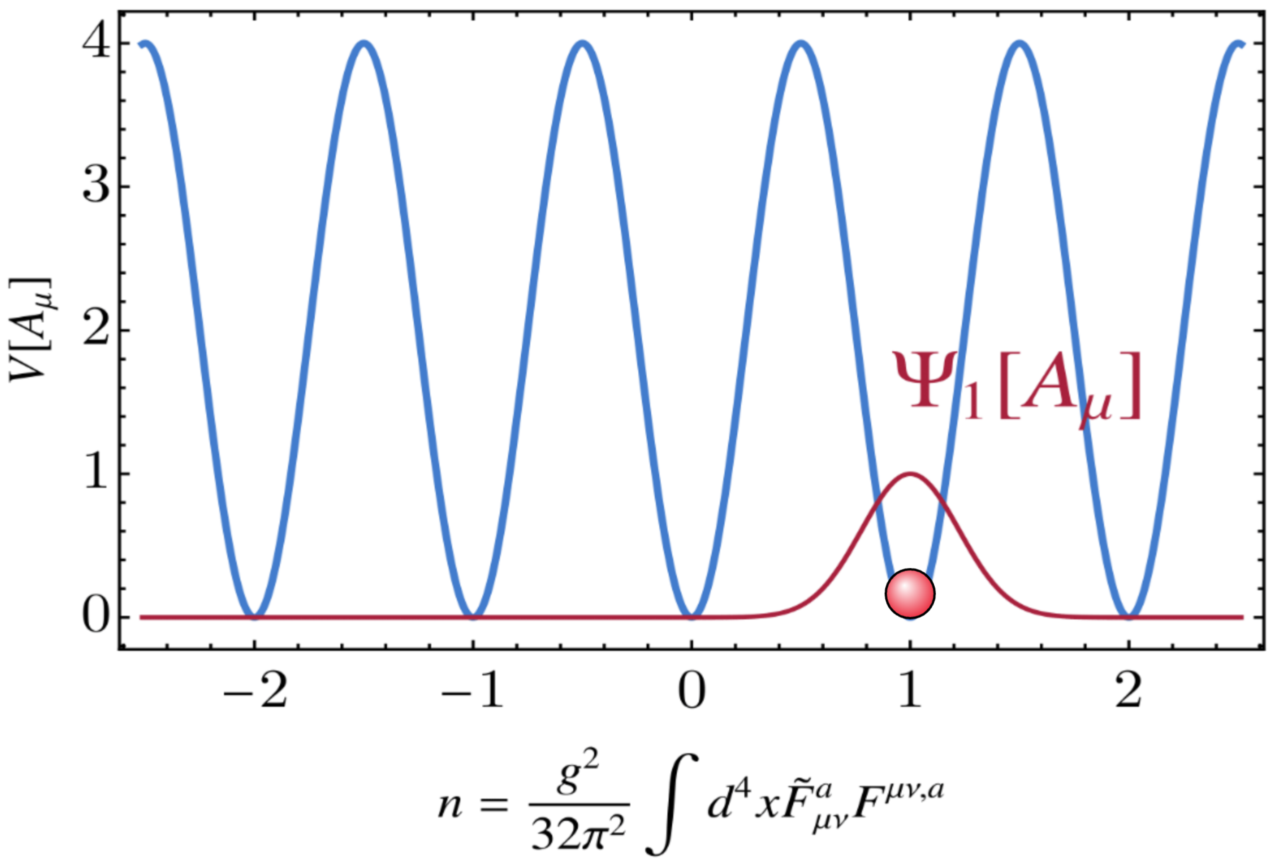
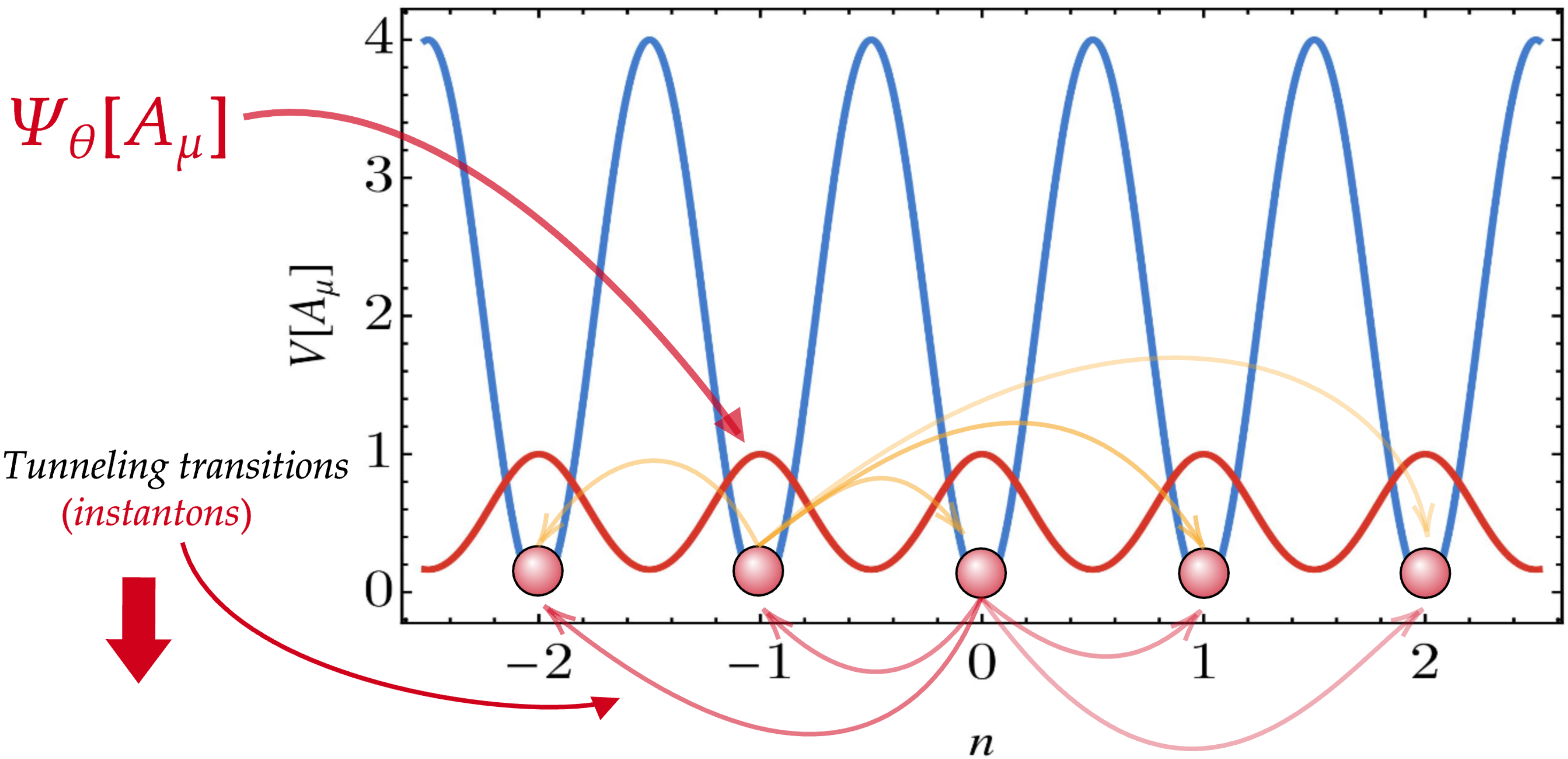


The θ term

Without tunneling events (*instanton* configurations), the wave functional of the sistem would look like





- *The wave – functional will be like the wave – function of a Bloch electron*
- *These tunneling phenomena produce a term in the Lagrangian :*

$$\mathcal{L}_\theta = \frac{g^2\theta}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\widetilde{F}_{\mu\nu,a}$$

this is the so – called θ – term : CP violating → *AXION to wash out θ*

The Axion Quality Problem

The Peccei-Quinn axion solution relies on a $U(1)_{PQ}$ global symmetry. Essentially, the axion is a pseudo Nambu-Goldstone boson associated with the breaking of this $U(1)_{PQ}$ global symmetry, which happens at the scale f_a.

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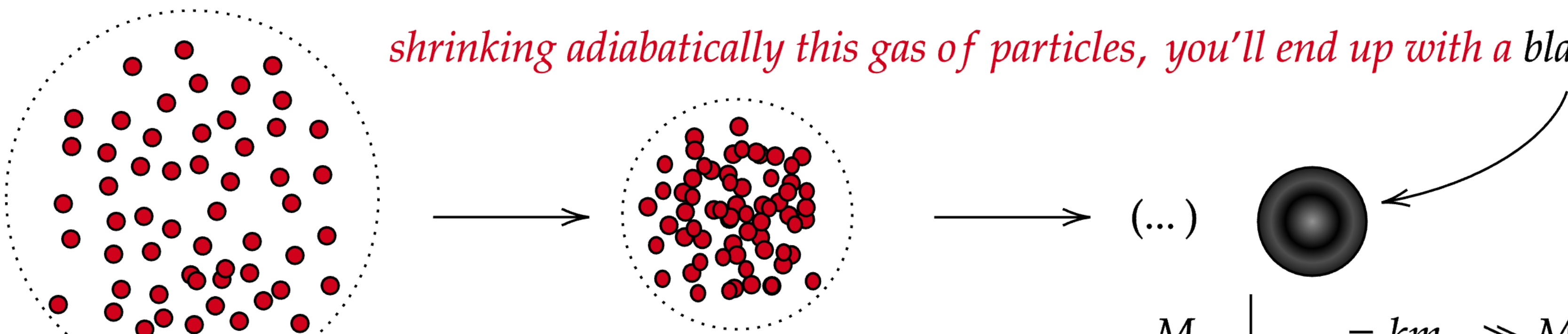
Below this symmetry breaking scale, the Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{\alpha_S}{8\pi^2}\left(\frac{a(x)}{f_a} + \theta\right) G^a_{\mu\nu}\tilde{G}_{\mu\nu,a} + \frac{\partial_\mu a(x)}{2f_a}\sum_f \overline{\psi}_f\gamma_5\gamma_\mu\psi_f + \dots$$

Because of this shift symmetry,
$a(x) \to a(x) + const$
the axion couples derivatively.

The Axion Quality Problem

shrinking adiabatically this gas of particles, you'll end up with a black hole



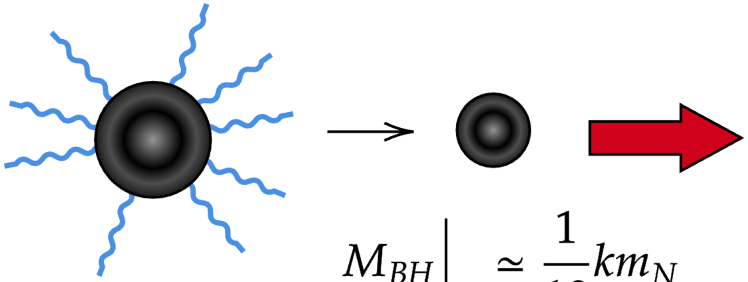
$$M_{BH}\Big|_{initial} = k m_N \gg M_{Planck}$$

$k,\ m_N > 0,\ Q_i = 1$
interacting

Black holes emit photons with black – body themperature

$$T_{BH}\Big|_{initial} \simeq \frac{M^2_{Planck}}{M_{BH}}$$

Our black hole will lose some of its mass via Hawking radiation



$$M_{BH}\Big|_{t_1} \simeq \frac{1}{10} k m_N$$

It is kinematically impossible to emit k particles. This means that :

Quantum gravity breaks global symmetries

The Axion Quality Problem

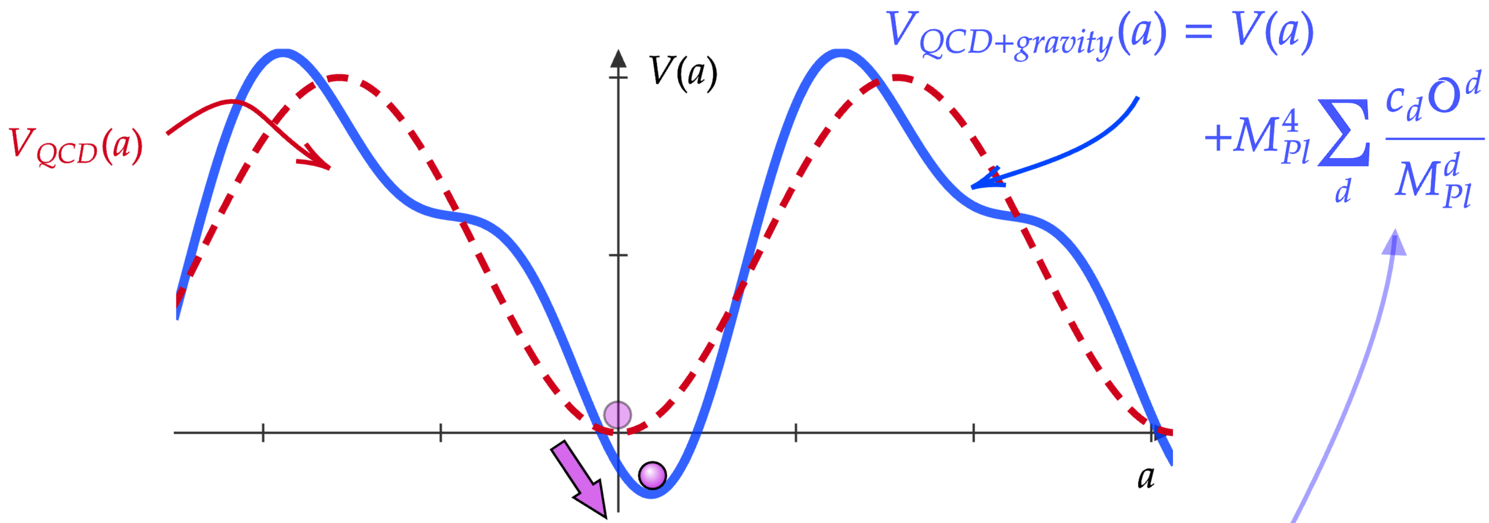
$$\overbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxx}}^{shift\ symmetric:\ a(x) \to a(x) + const}$$

$$\mathcal{L} \quad = \quad \frac{1}{2}(\partial_\mu a)^2 + \frac{\alpha_s}{8\pi^2}\left(\frac{a(x)}{f_a} + \theta\right) G^a_{\mu\nu}\widetilde{G}_{\mu\nu,a} + \frac{\partial_\mu a}{2 f_a}\sum_f \overline{\psi}_f \gamma_5 \gamma^\mu \psi_f$$

$$+\sum_{d>4} \frac{c_d}{M^{d-4}_{Planck}} \mathcal{O}^d(a(x))$$

$\mathcal{L}$ is no more shift symmetric ⇒ *Gravity induces an axion potential!*

The Axion Quality Problem



In presence of gravity, the axion potential is no more minimized at $a \simeq \theta = 0$!

Even if $M_{Pl} \simeq 10^{18}$ *GeV, you must have*

$$|\theta| \lesssim 10^{-13}$$

"high − quality" $U(1)_{PQ}$ symmetry :

$$c_1 = c_2 = c_3 = \ldots = c_{12} = 0$$

Problems/Opportunities

- The axion quality problem is a conjecture based on String Theory arguments and black hole evaporation.
- Is it possible to give a **precise** and **controllable** estimate on how gravitational interactions may affect the axion solution?
- Gravitational instantons (Eguchi-Hanson & K3 instantons).
- **Is quality problem really a problem?**

Thank You four your attention!