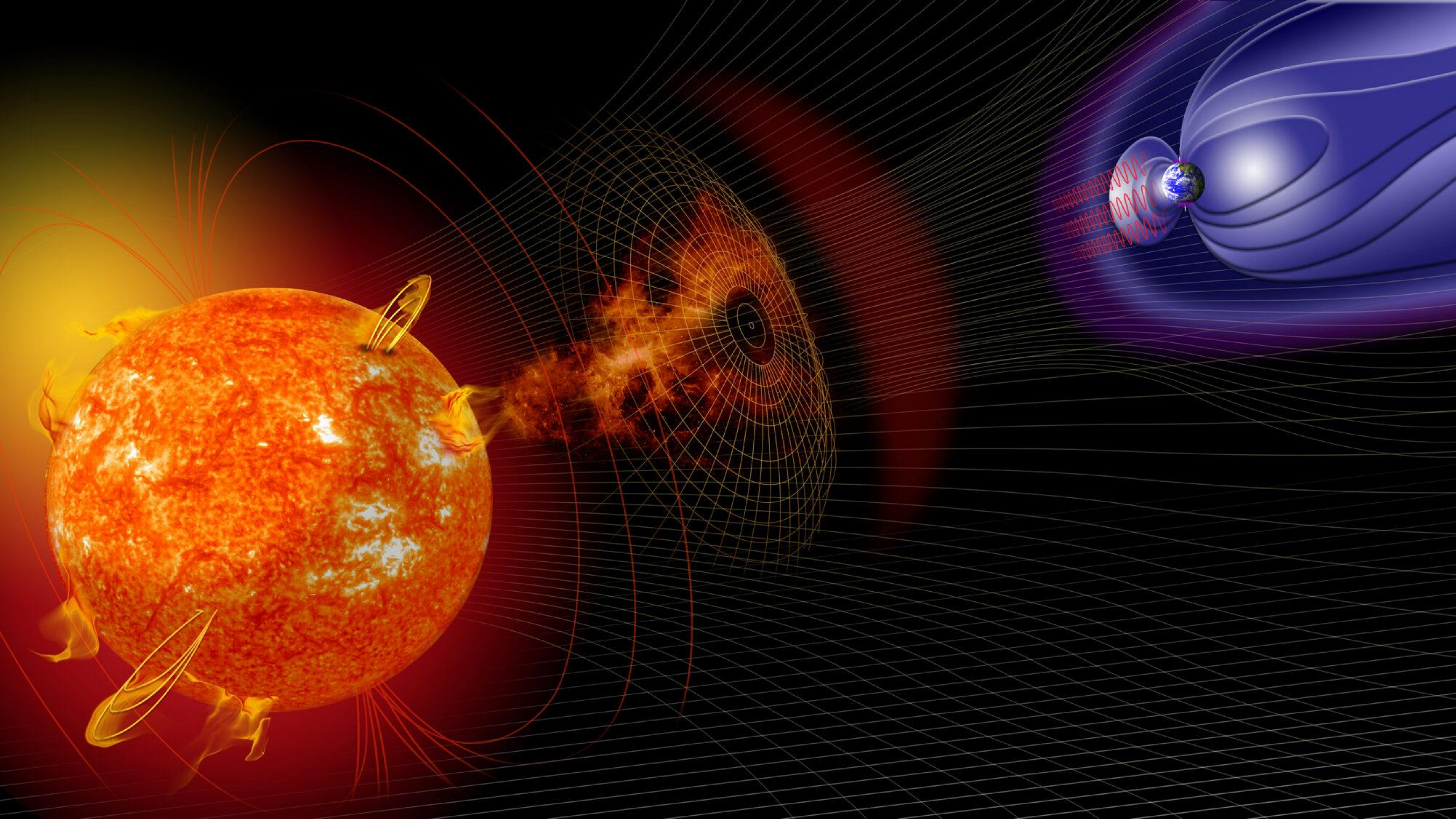
Status 2024 of CR Backtracing in the Magnetosphere

Davide Grandi & Davide Rozza

On behalf of
INFN Milano Bicocca group

AMS Analysis Meeting April 2024



Trapped Particles & Trajectories

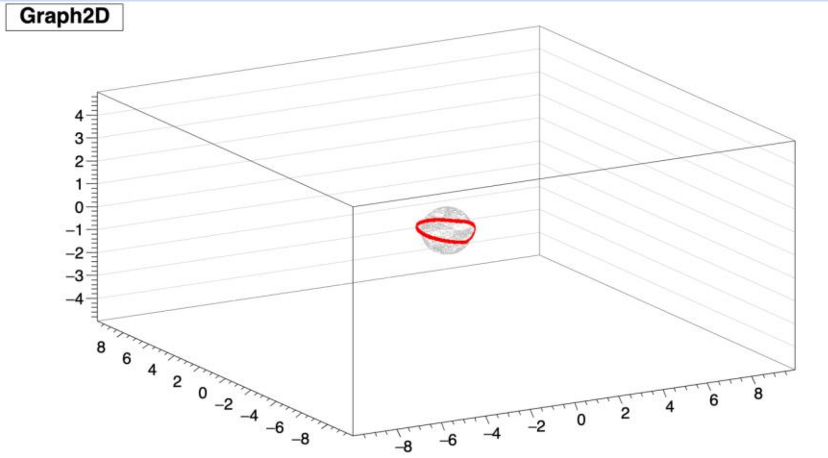
Ions from A. Oliva

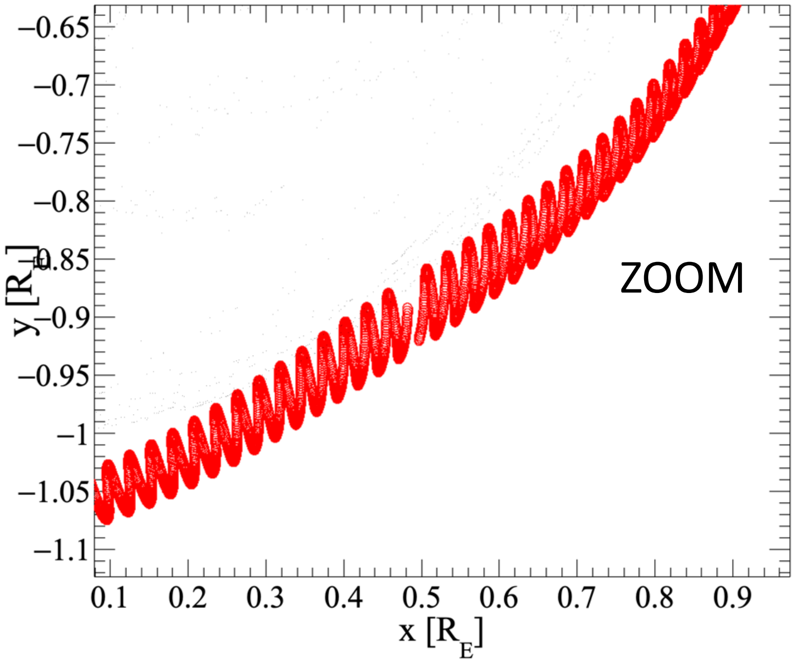
Backtracing and analysis performed by D. Grandi and D. Rozza

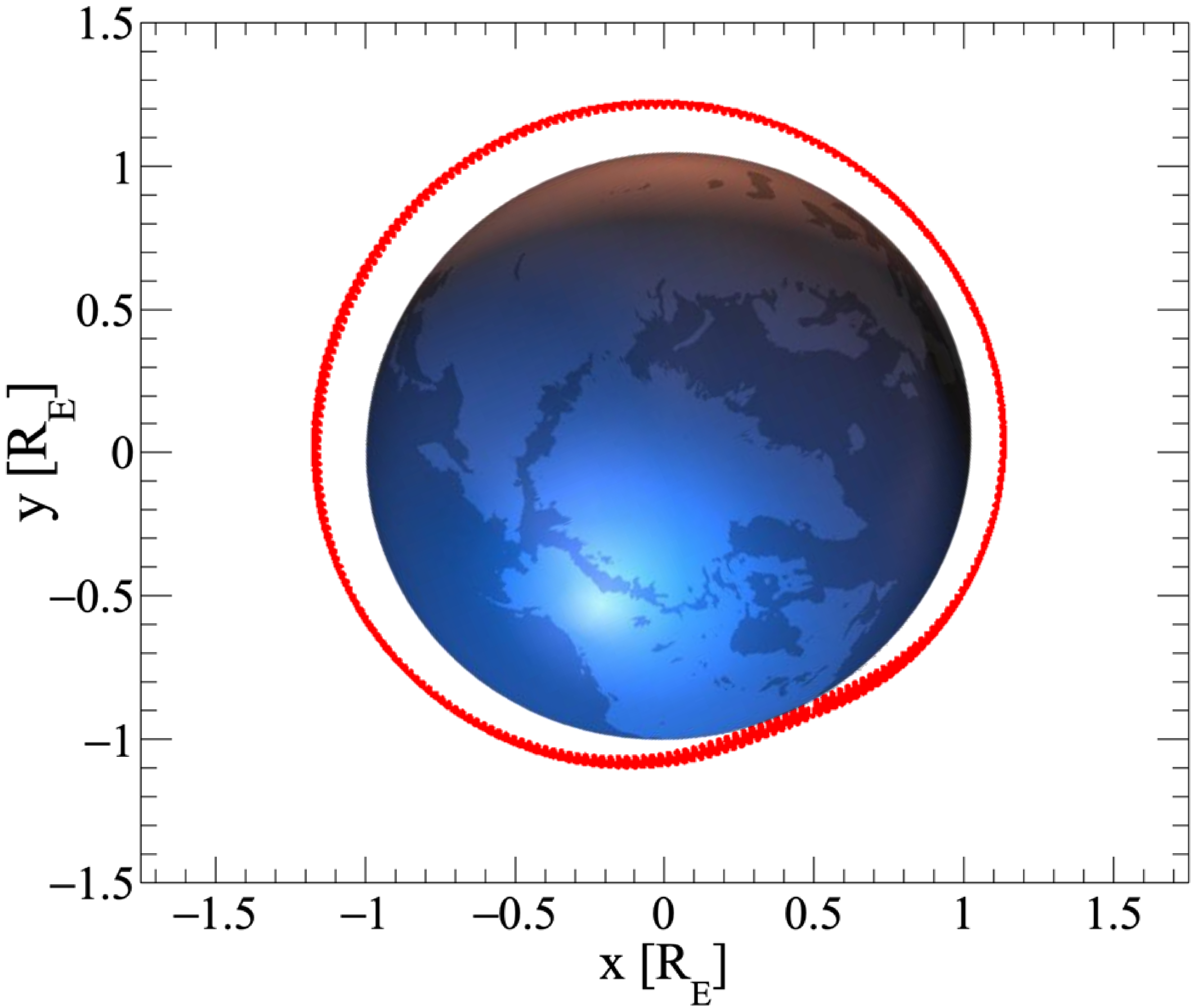
Tailored code for trapped particles:

- High precision (10^{-4} rad angle between 2 consecutive velocities)
- Inner boundary decreased to 20 km (typical altitude of CR air showers starting point)
- Increase time limit (up to 10 min of “real” trajectory time)
- Particles main request to be considered as trapped: a WHOLE drift shell (> 360° in longitude)

Lithium







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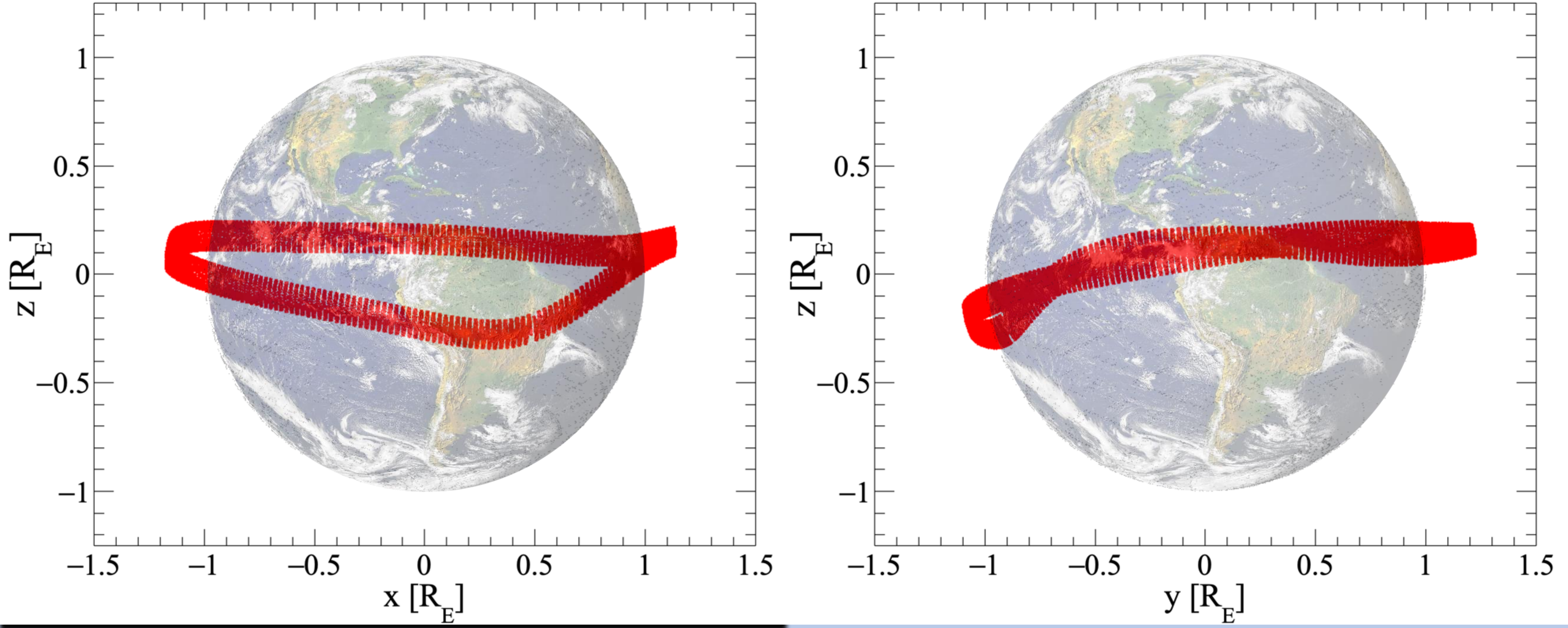
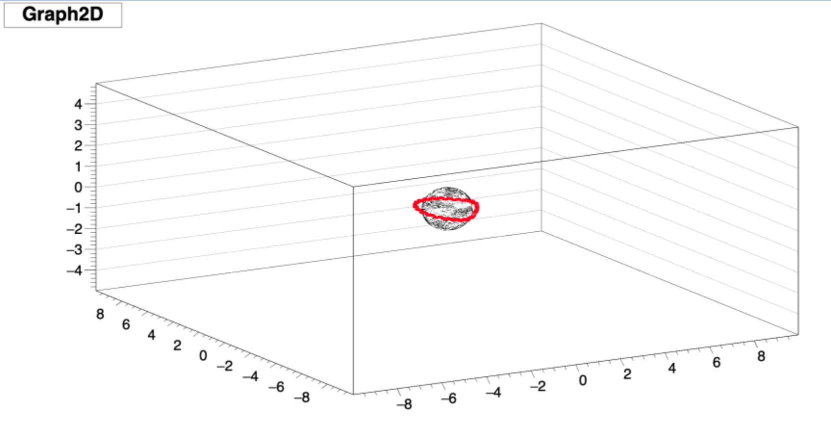


Carbon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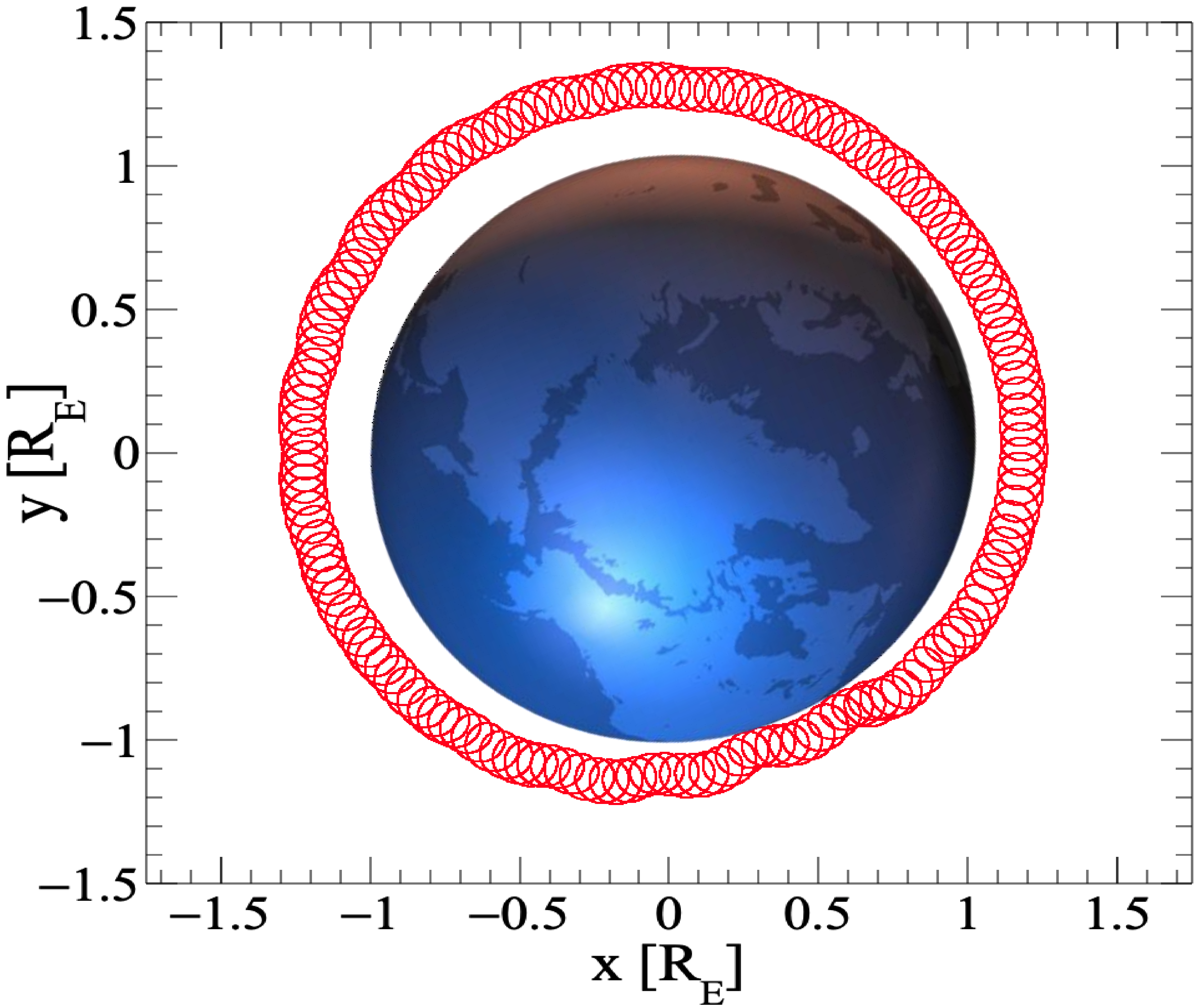
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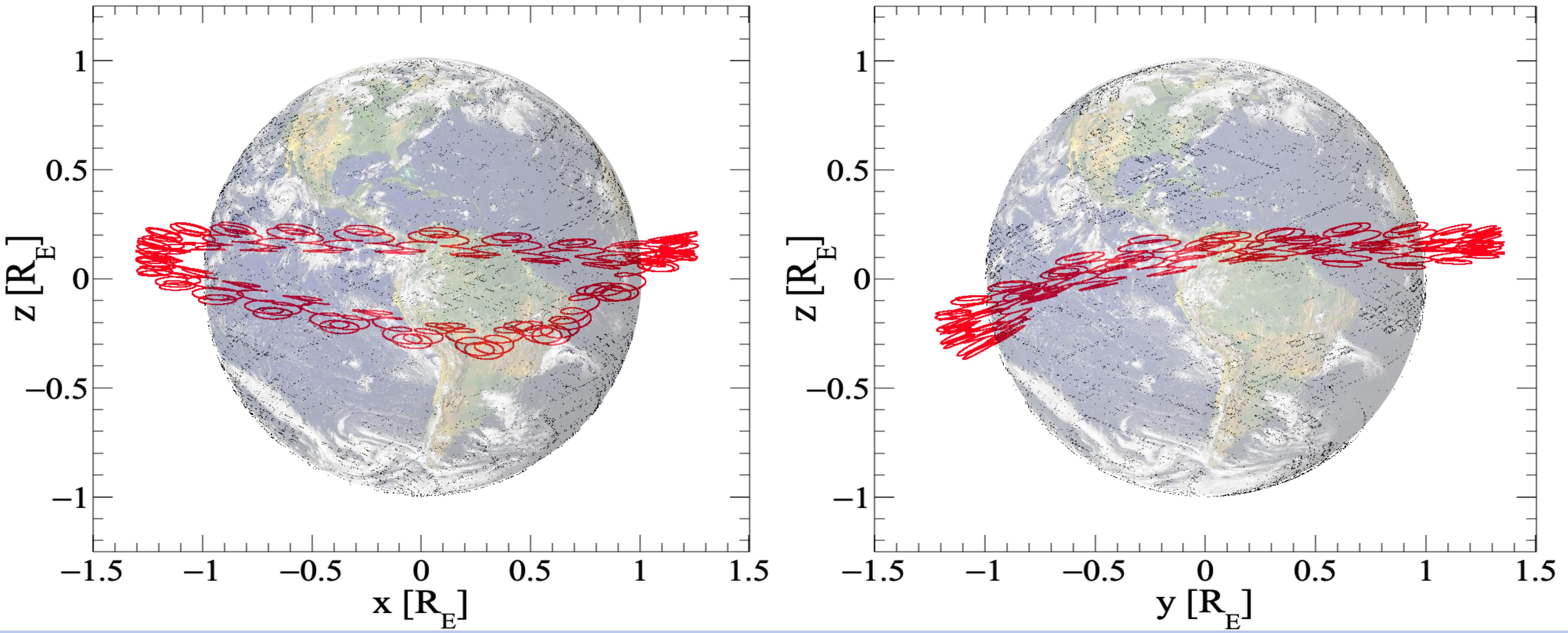
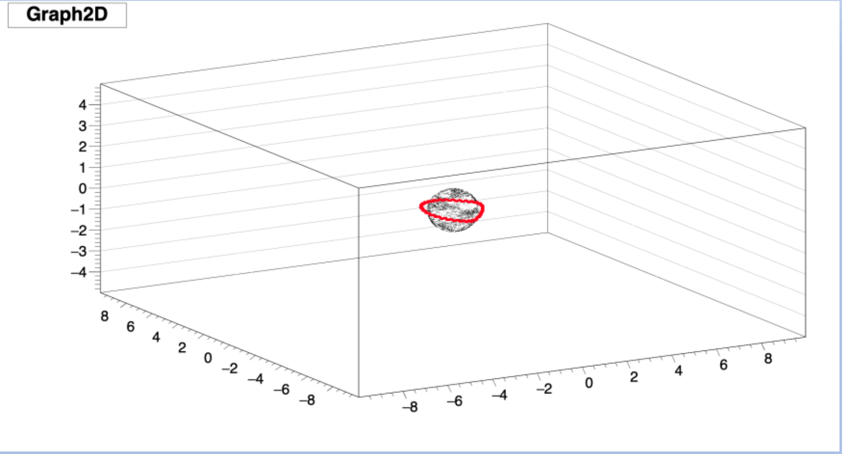
Carbon



Berillium


Graph2D
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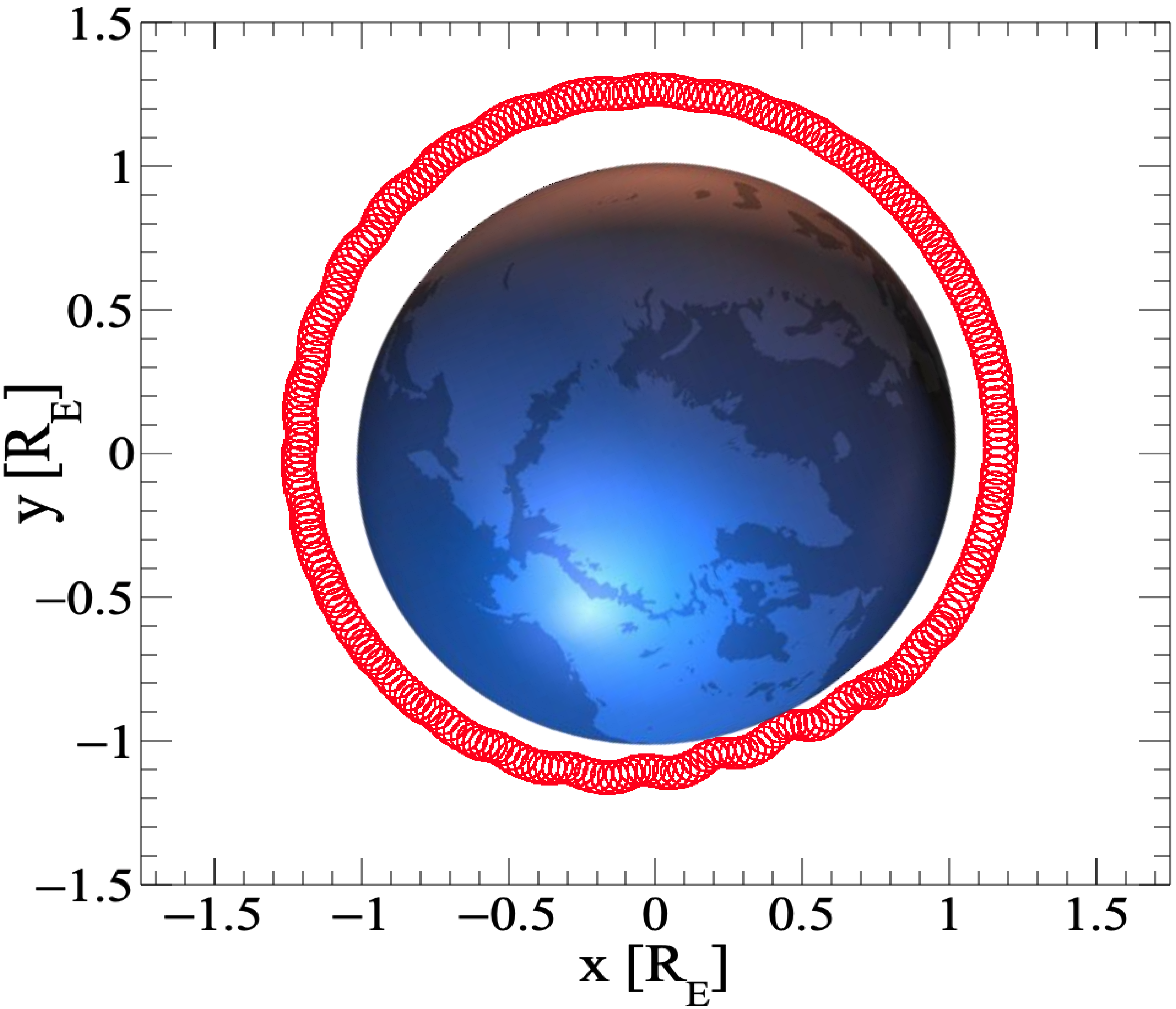
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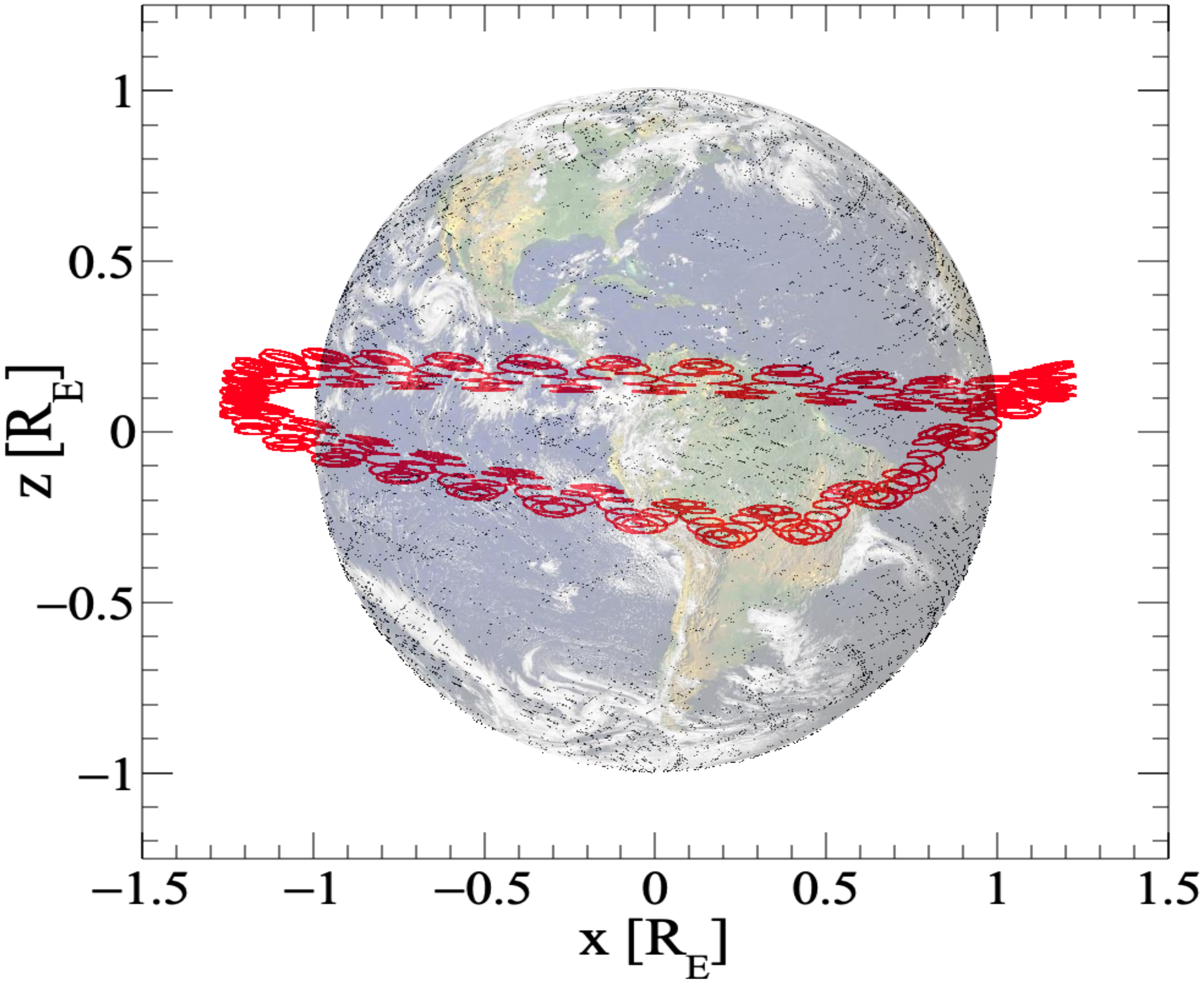
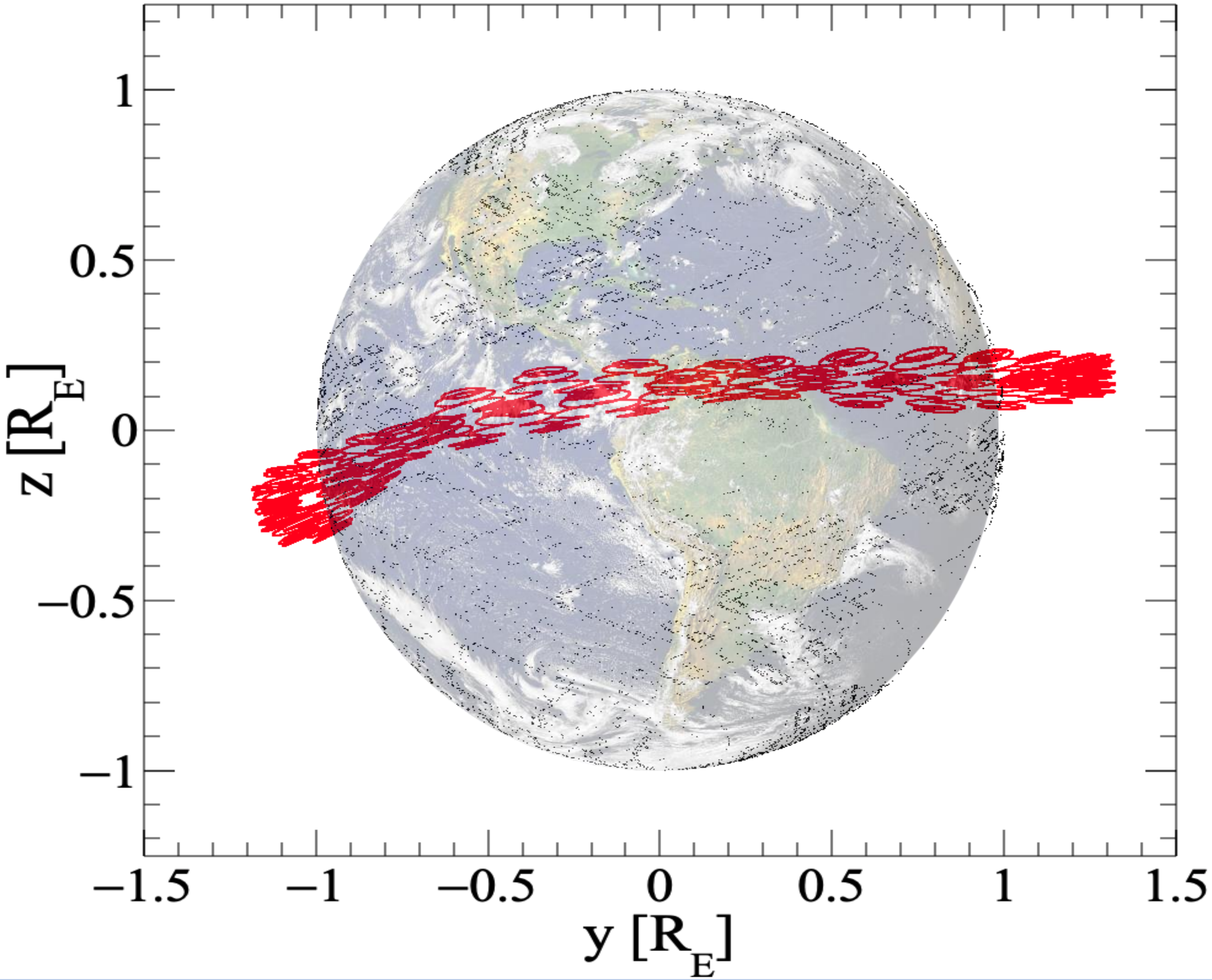
1.5

Berillium





Preliminary results

Trapped Ions

- From both Inner and Outer radiation belt?
- Different detection positions
- Different lifetime and trajectories
- From both samples (upgoing and downgoing particles)
- Needed: Equatorial Pitch Angle and L-Shell analysis

Trapped Particles & Pitch Angle

Pitch Angle calculation from Dipole Approximation (IGRF)

For this calculation we need

- The Local Pitch Angle
  - the angle between the particle direction and the Magnetic field
- This can be obtained by particle rigidity, charge and mass (to obtain velocity value) and particle arrival direction
- The unix time of the detection (to evaluate the magnetic field)
- After we can evaluate the correspondent equatorial pitch angle (and this can be done for all particles but it “**makes sense”** only for trapped once that can be considered to have three adiabatic invariants)

Pitch Angle – Adiabatic Invariant

$$sin^2(\theta)/B \;=\; sin^2(\theta_0)/B_{eq} =$$

B_{eq} = equatorial Magnetic Field

$$sin^2(\theta_M)/B_M = \; sin^2(\theta_{eq})/B_{eq} = 1/B_M$$

B_M = Mirror Point Magnetic Field

Pitch Angle – Adiabatic Invariant

$$B = B_0 \, (R_e/r)^3 \sqrt{1 + 3\, sin^2(\lambda_m)}$$

$$B/B_0 = (R_e/r)^3 \sqrt{1 + 3\, sin^2(\lambda_m)}$$

$$B_0 = 0.3\ Gauss\ (Earth\ Surface)$$

Pitch Angle – Adiabatic Invariant

$$B_{loc} = B_0\,(\mathrm{R_e}/\mathrm{r_{loc}})^3\sqrt{1+3\,sin^2(\lambda_{mloc})}$$

$$B_{eq} = B_0\,(\mathrm{R_e}/\mathrm{r_{eq}})^3\sqrt{1+3\,sin^2(\lambda_{meq})}$$

$$r_{loc} = r_{eq}\,cos^2(\lambda_m)$$

In the same Field Line

Pitch Angle – Adiabatic Invariant

$$\boldsymbol{sin}^2(\theta_{loc})\boldsymbol{B}_{eq}/\boldsymbol{B}_{loc} = \boldsymbol{sin}^2(\theta_{eq})$$

$$\boldsymbol{B}_{eq}/\boldsymbol{B}_{loc} = \frac{(\mathbf{Re/req})^3\sqrt{\mathbf{1+3}\ \boldsymbol{sin}^2(\lambda_{meq})}}{(\mathbf{R_e/r_{loc}})^3\sqrt{\mathbf{1+3}\ \boldsymbol{sin}^2(\lambda_{mloc})}}$$

Pitch Angle – Adiabatic Invariant

$$\boldsymbol{sin}(\lambda_{\boldsymbol{eq}}) = \boldsymbol{0}$$

$$\boldsymbol{B_{eq}/B_{loc}} = \frac{\boldsymbol{(R_e/r_{eq})^3}}{\boldsymbol{(R_e/r_{loc})^3}\sqrt{\boldsymbol{1 + 3\,sin^2}(\lambda_{\boldsymbol{mloc}})}}$$

Pitch Angle – Adiabatic Invariant

$$\boldsymbol{sin(\lambda_{eq}) = 0}$$

$$\boldsymbol{B_{eq}/B_{loc} = \frac{(r_{loc})^3}{(r_{eq})^3\sqrt{1 + 3\,sin^2(\lambda_{mloc})}}}$$

$$\boldsymbol{cos^2(\lambda_{mloc}) = \frac{r_{loc}}{r_{eq}}}$$

Pitch Angle – Adiabatic Invariant

$$\frac{sin^2(\theta_{loc})B_{eq}}{B_{loc}} = sin^2(\theta_{eq}) =$$

$$\frac{(cos^2(\lambda_{mloc}))^3 * sin^2(\theta)_{loc}}{\sqrt{1+3\ sin^2(\lambda_{mloc})}} = sin^2(\theta_{eq})$$

Trapped Particles & L-Shell Calculation

L-Shell calculation from Dipole Approximation (IGRF)

Calculation:

- Using IGRF we can evaluate
  - The geomagnetic latitude
  - Distance from the dipole center
- These parameters can be used to evaluate the Mc-Ilwain parameter (L-Shell)
- Following slides with formulas we have used

Magnetic Field Line – L Shell

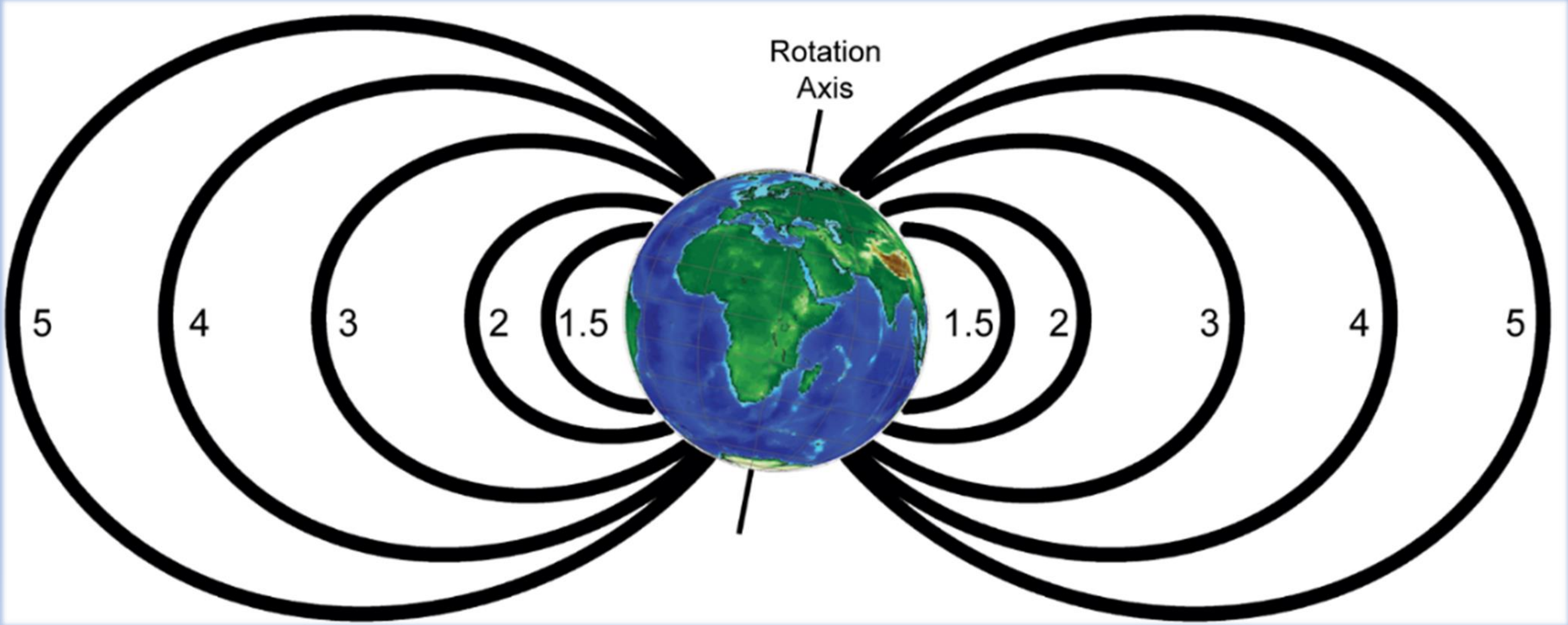
Field Line → $r = r_0\, cos^2(\lambda_m)$

$$r_0 = r_{eq}$$

L - shell → $r = L\, cos^2(\lambda_m)$

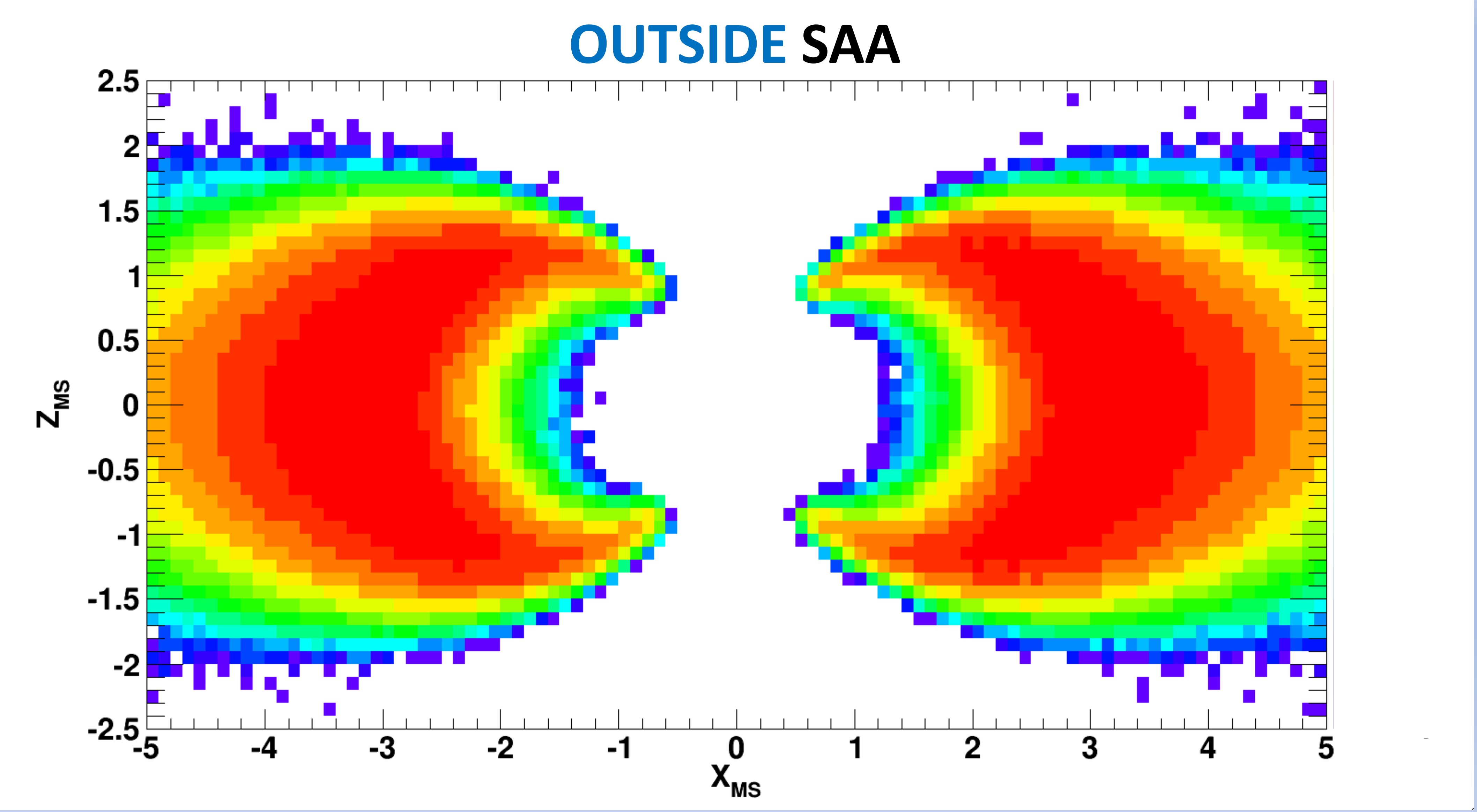
r is the position with respect to the geomagnetic frame (shifted and tilted as from IGRF 13 parameters)

L - shell → $r = L\cos^2(\lambda_m)$

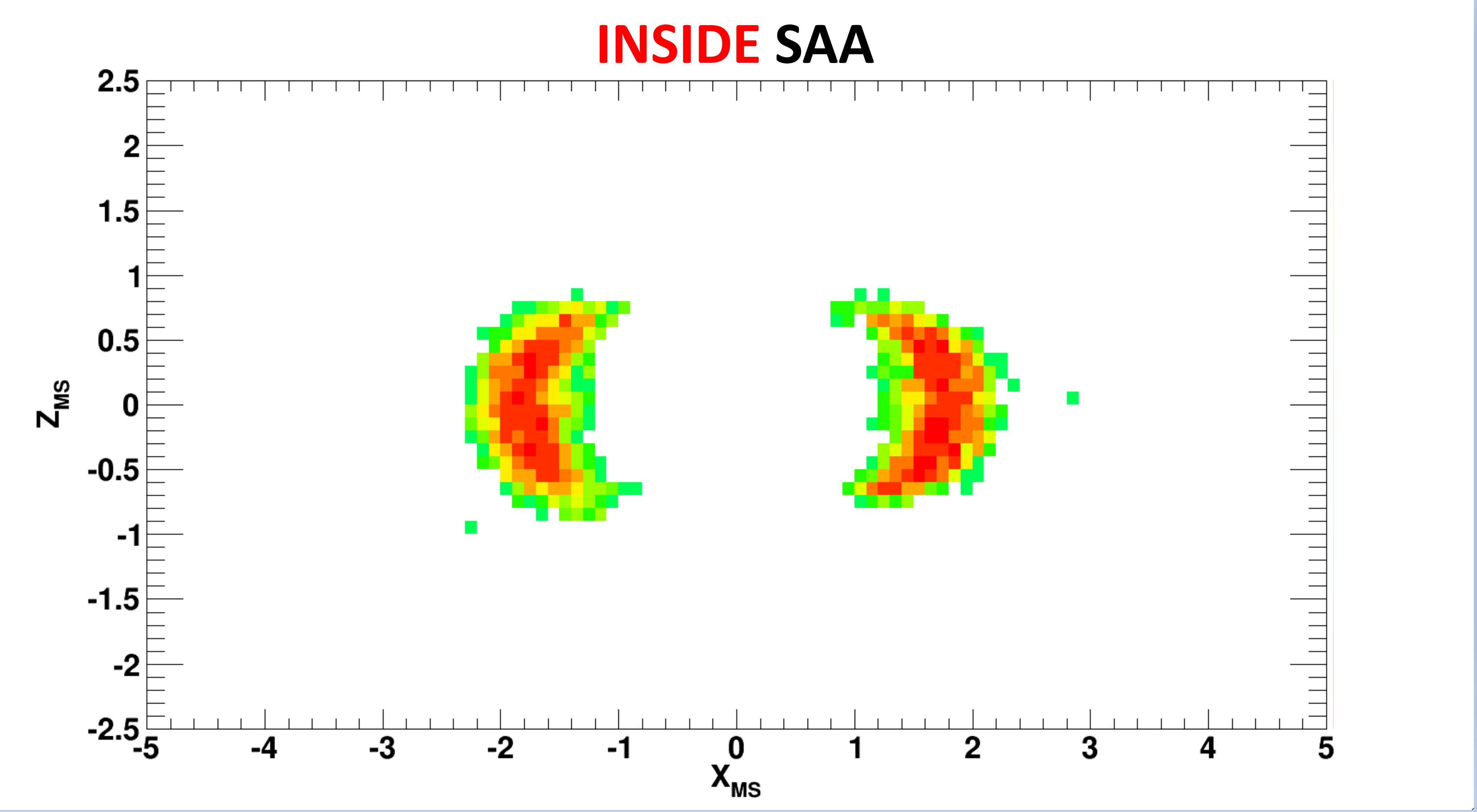


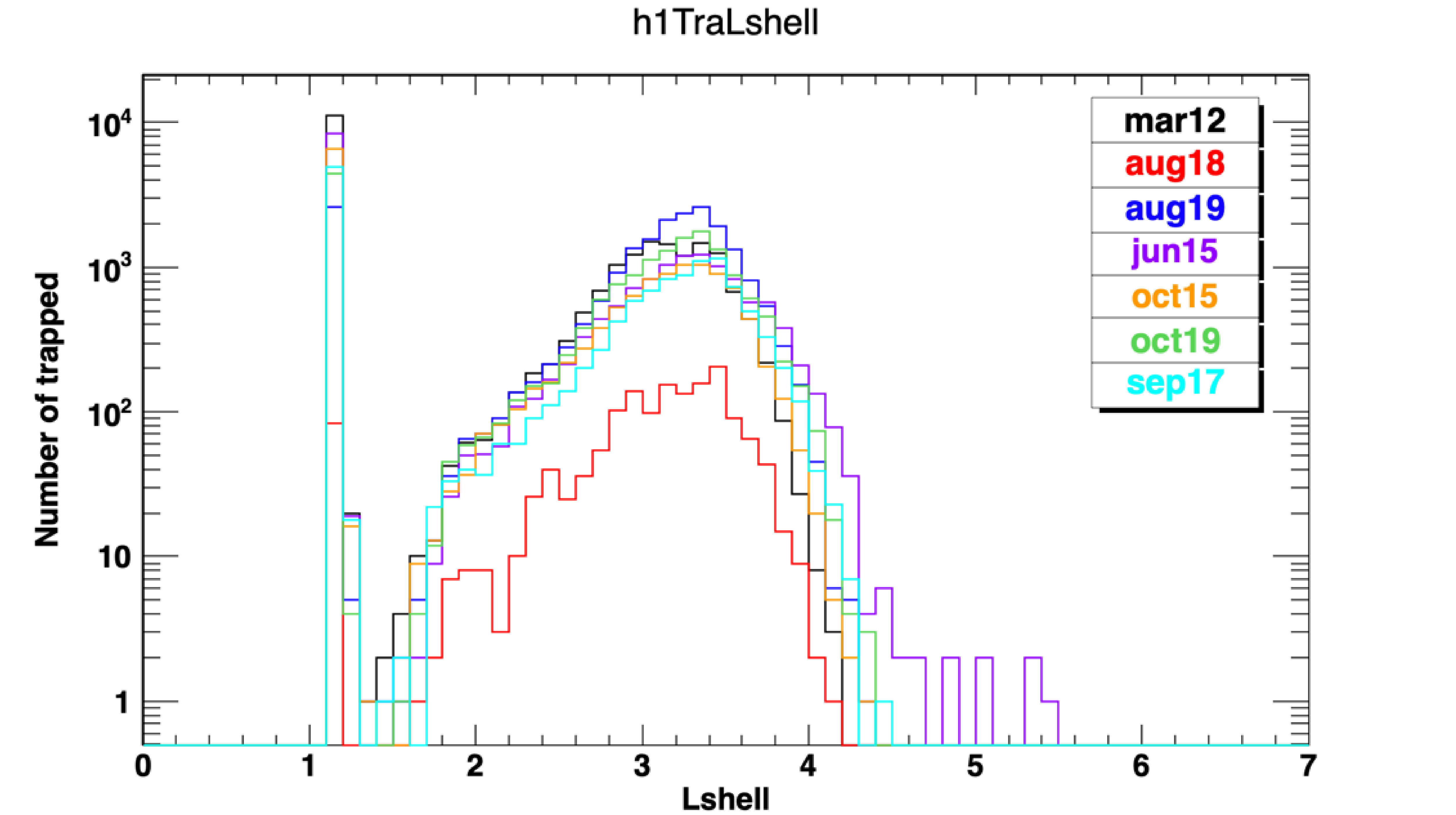
Trapped Protons

Trapped CR: Final position after 10 seconds of backtracing



Trapped CR: Final position after 10 seconds of backtracing

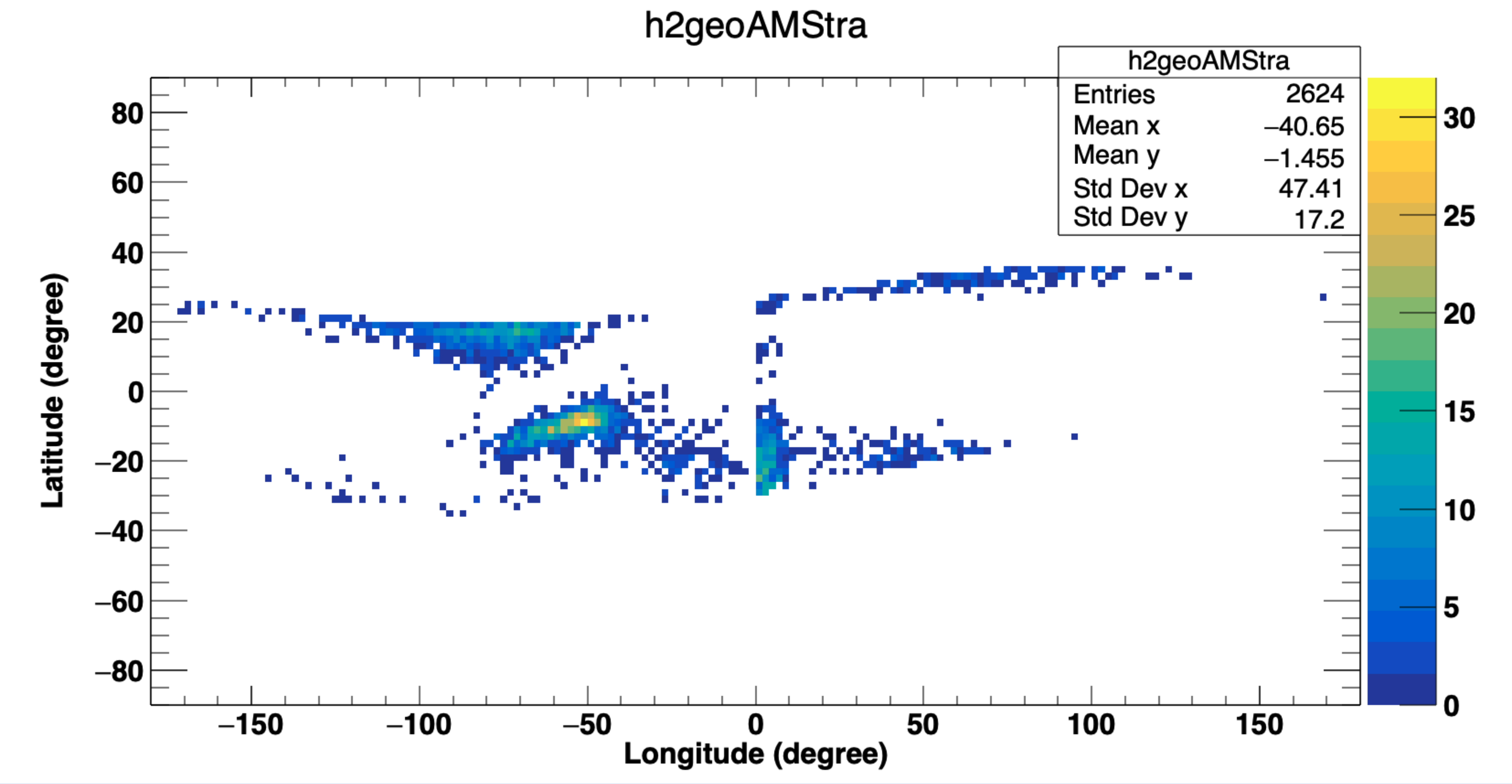




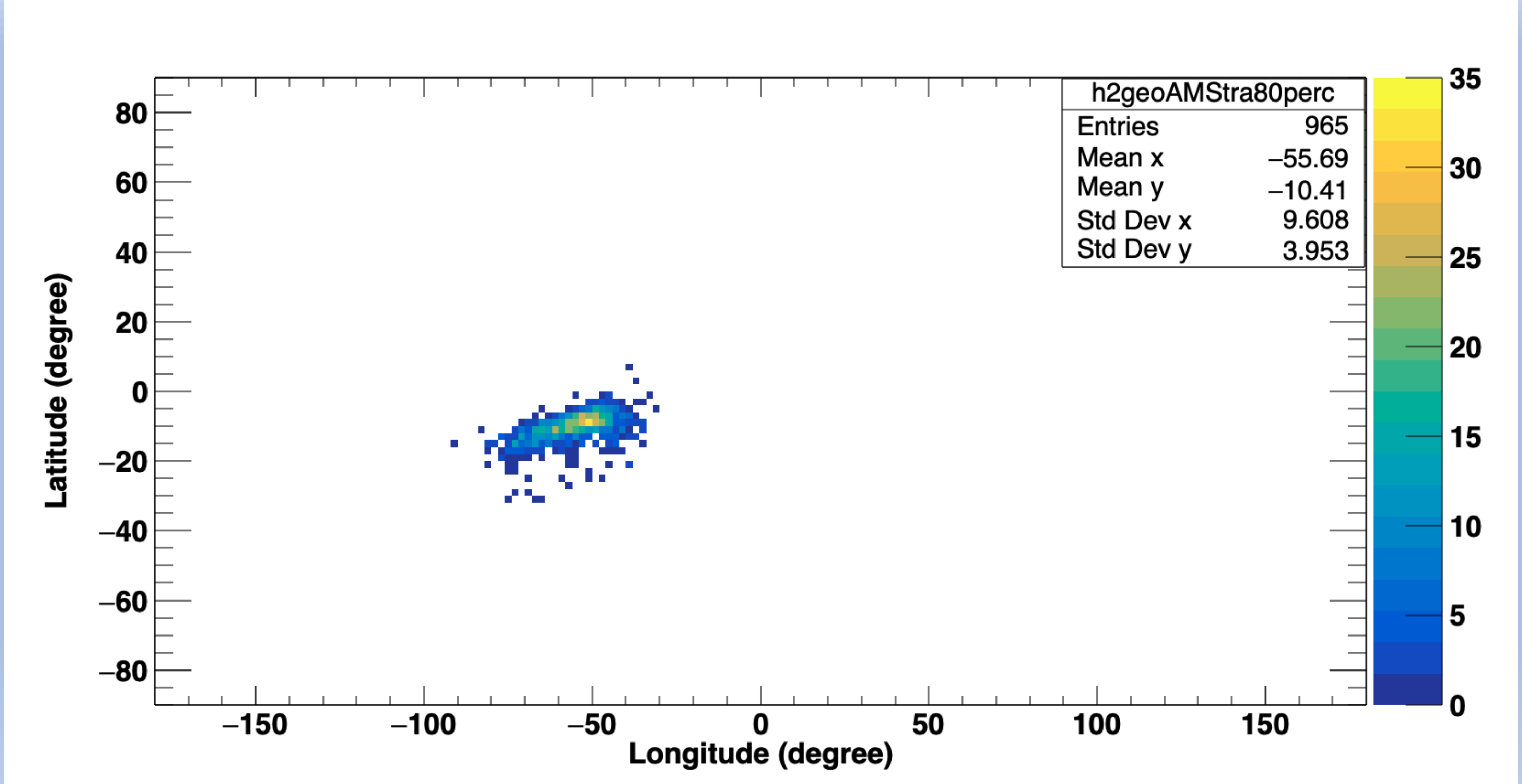
h1TraLshell
mar12
aug18
aug19
jun15
oct15
oct19
sep17
Number of trapped
Lshell
10^4
10^3
10^2
10
1
0
1
2
3
4
5
6
7

Trapped Ions?

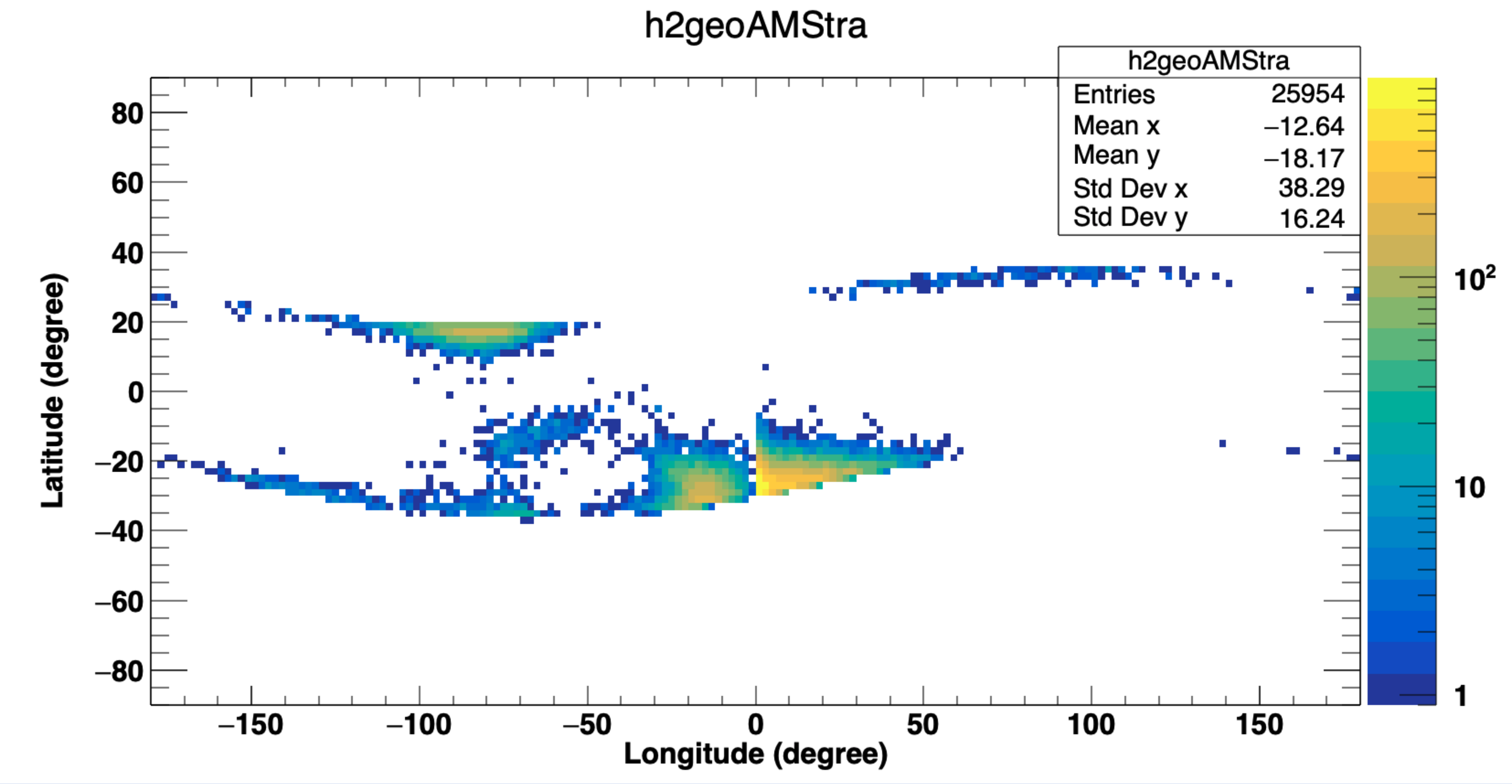
Upgoing ions from GeoMagSphere BkT



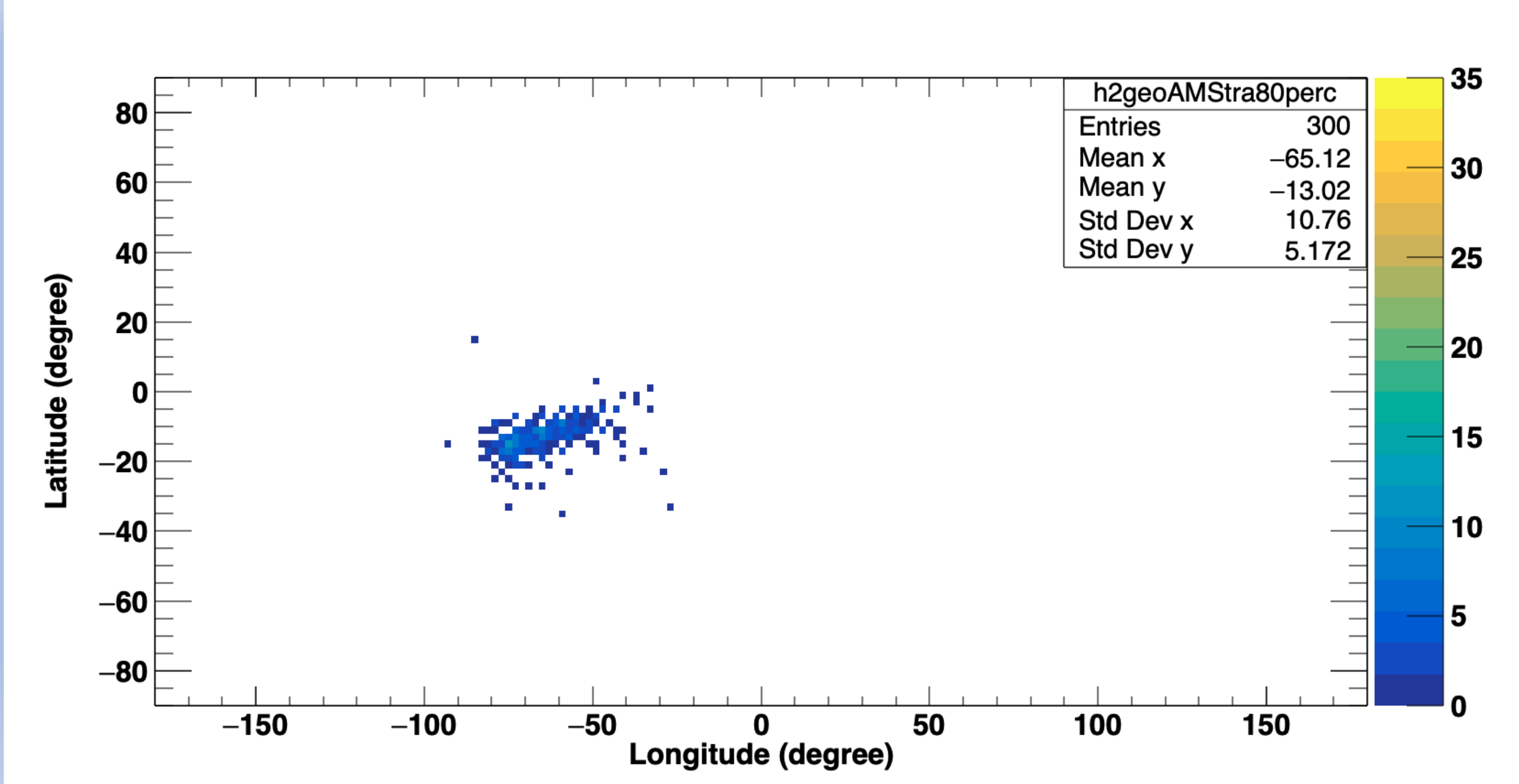
Upgoing ions from GeoMagSphere BkT requiring 8/10 particles **generated** are trapped



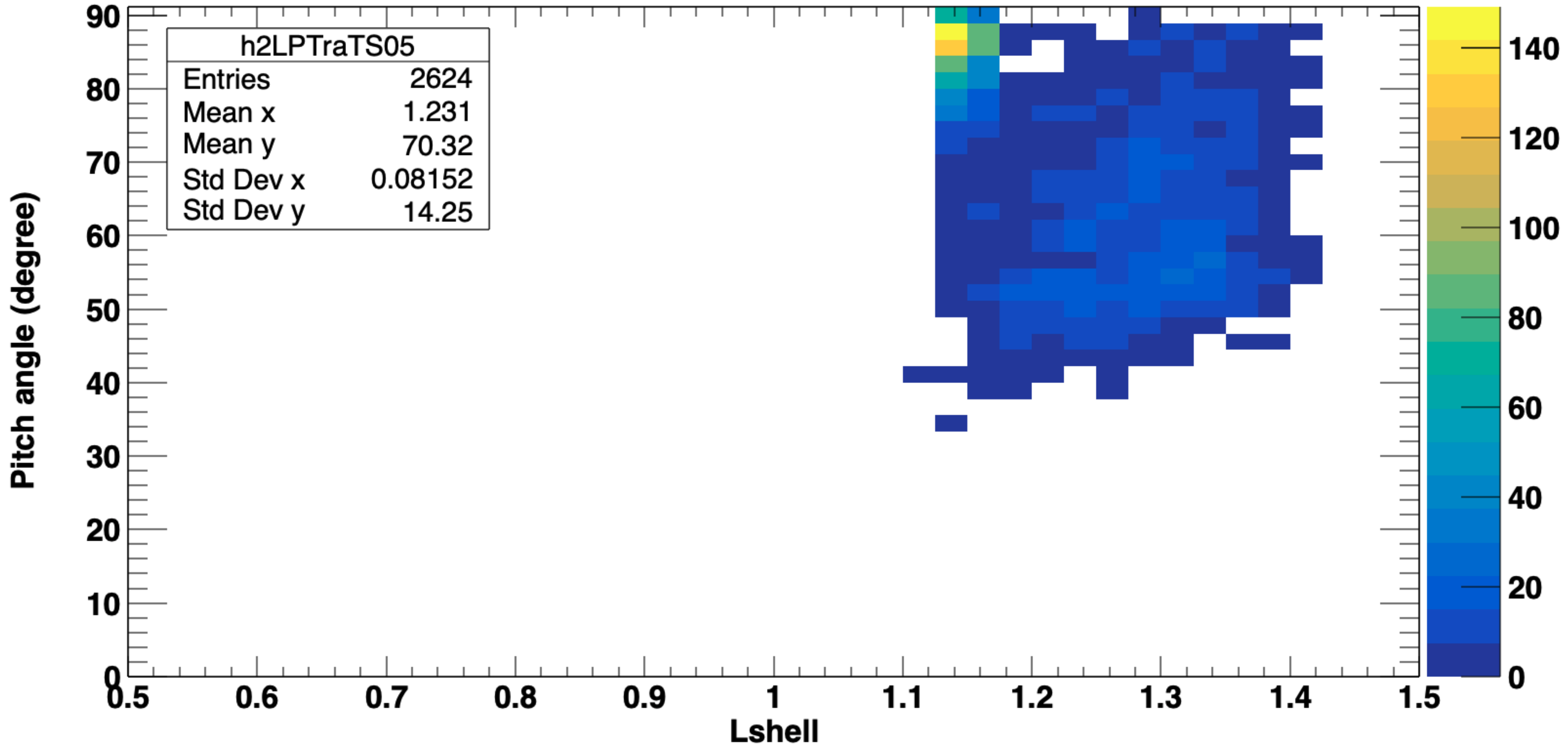
Downgoing ions from GeoMagSphere BkT



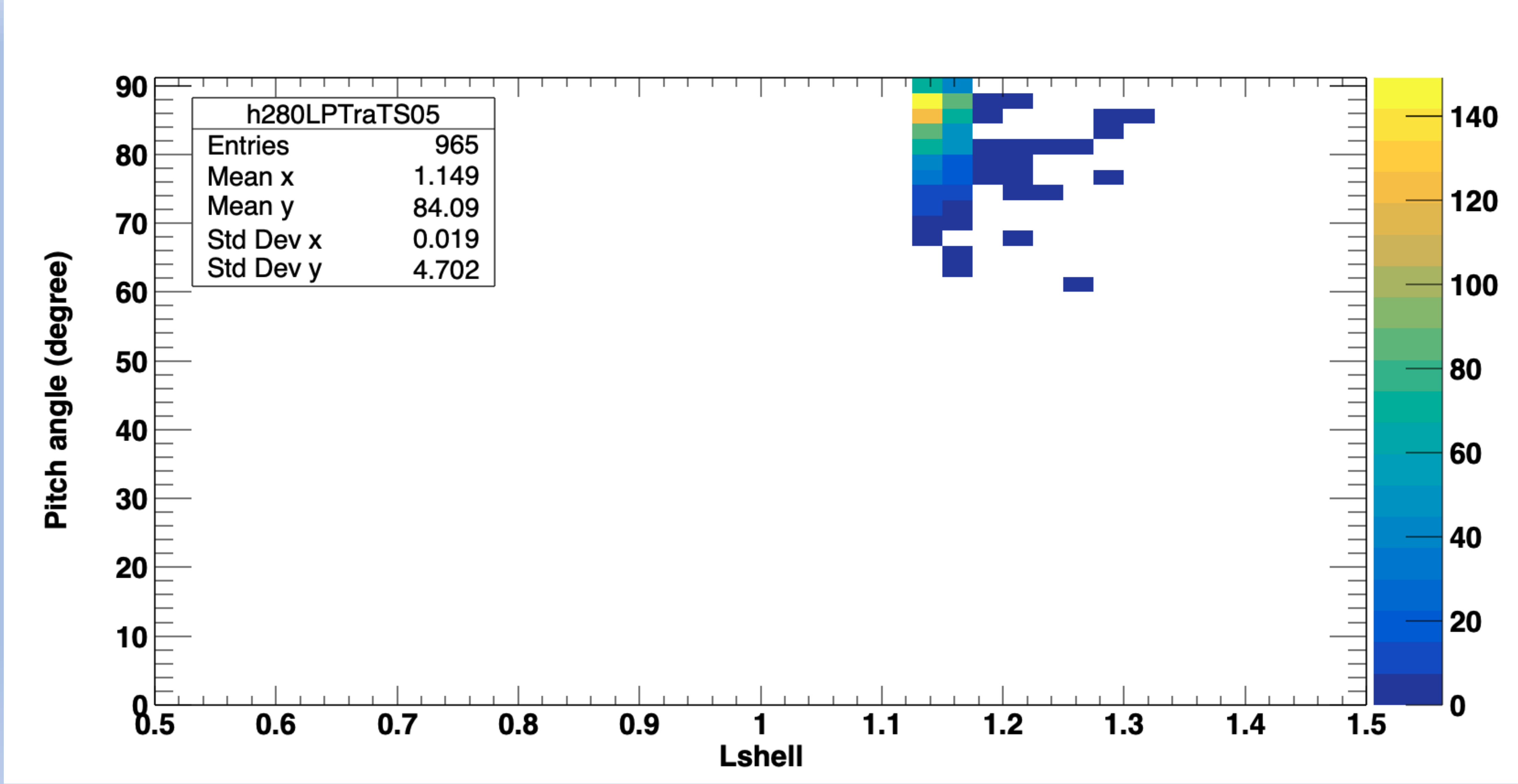
Downgoing ions from GeoMagSphere BkT requiring 8/10 particles **generated** are trapped



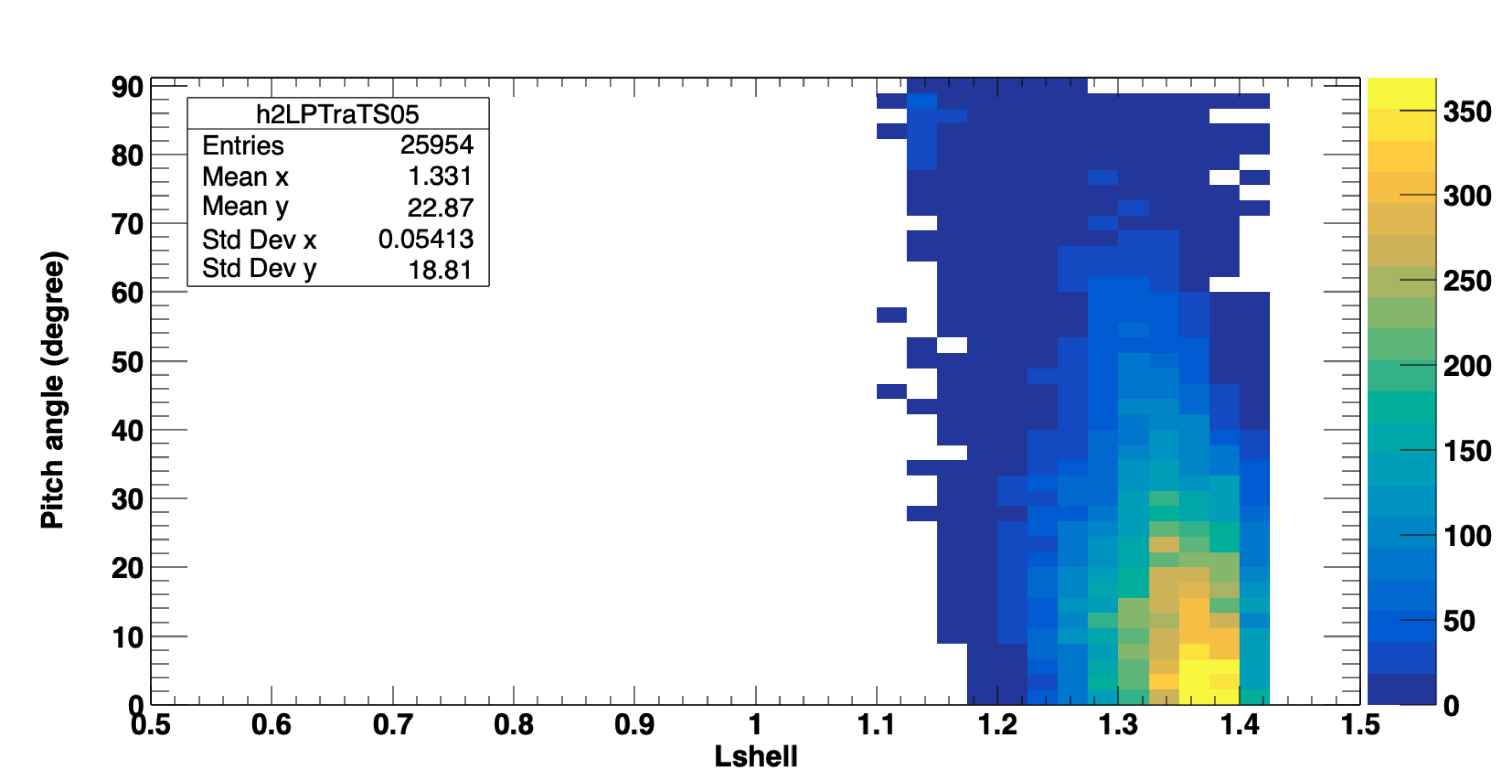
Upgoing ions Pitch angle vs Lshell



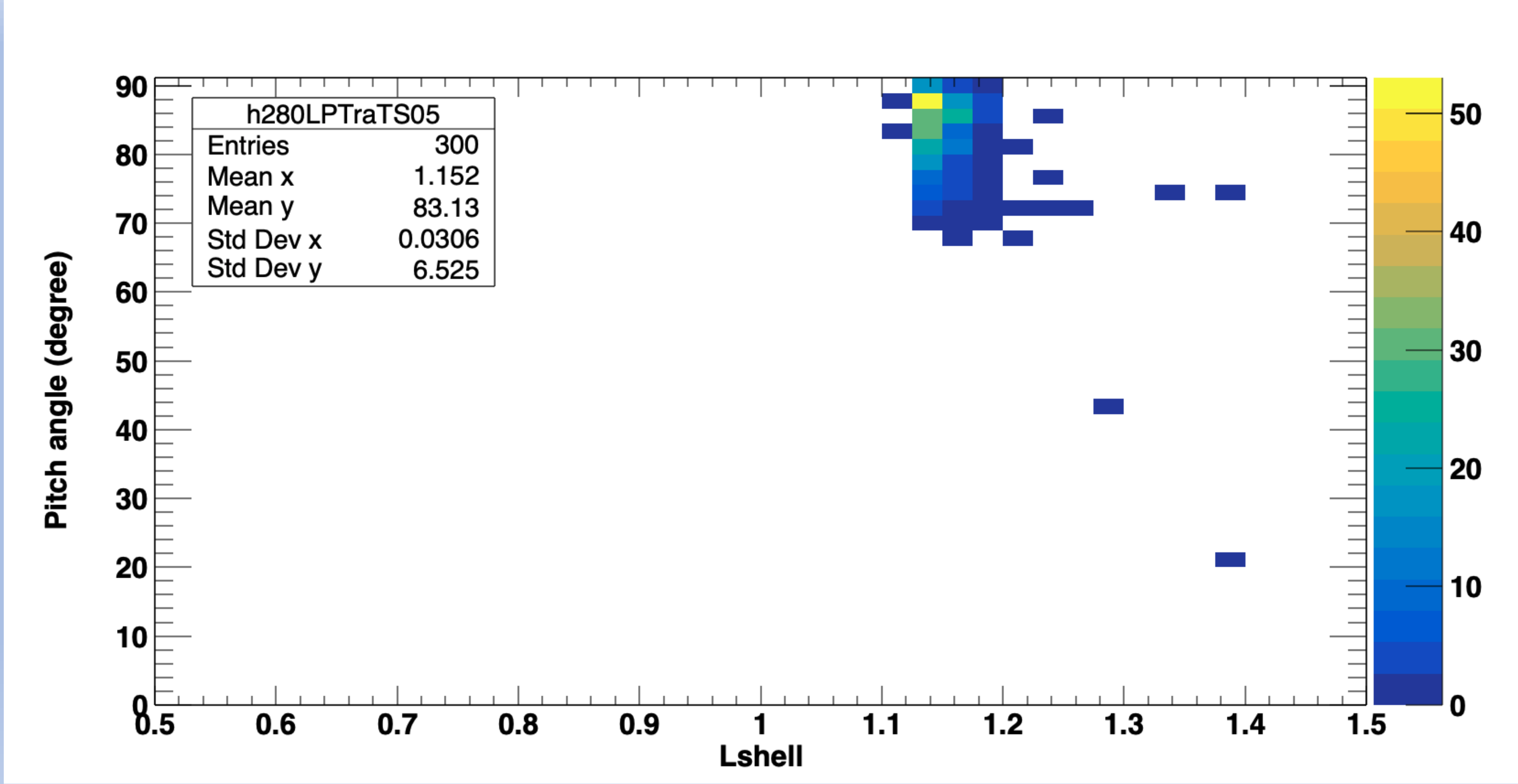
Upgoing ions Pitch angle vs Lshell requiring 8/10 particles **generated** are trapped



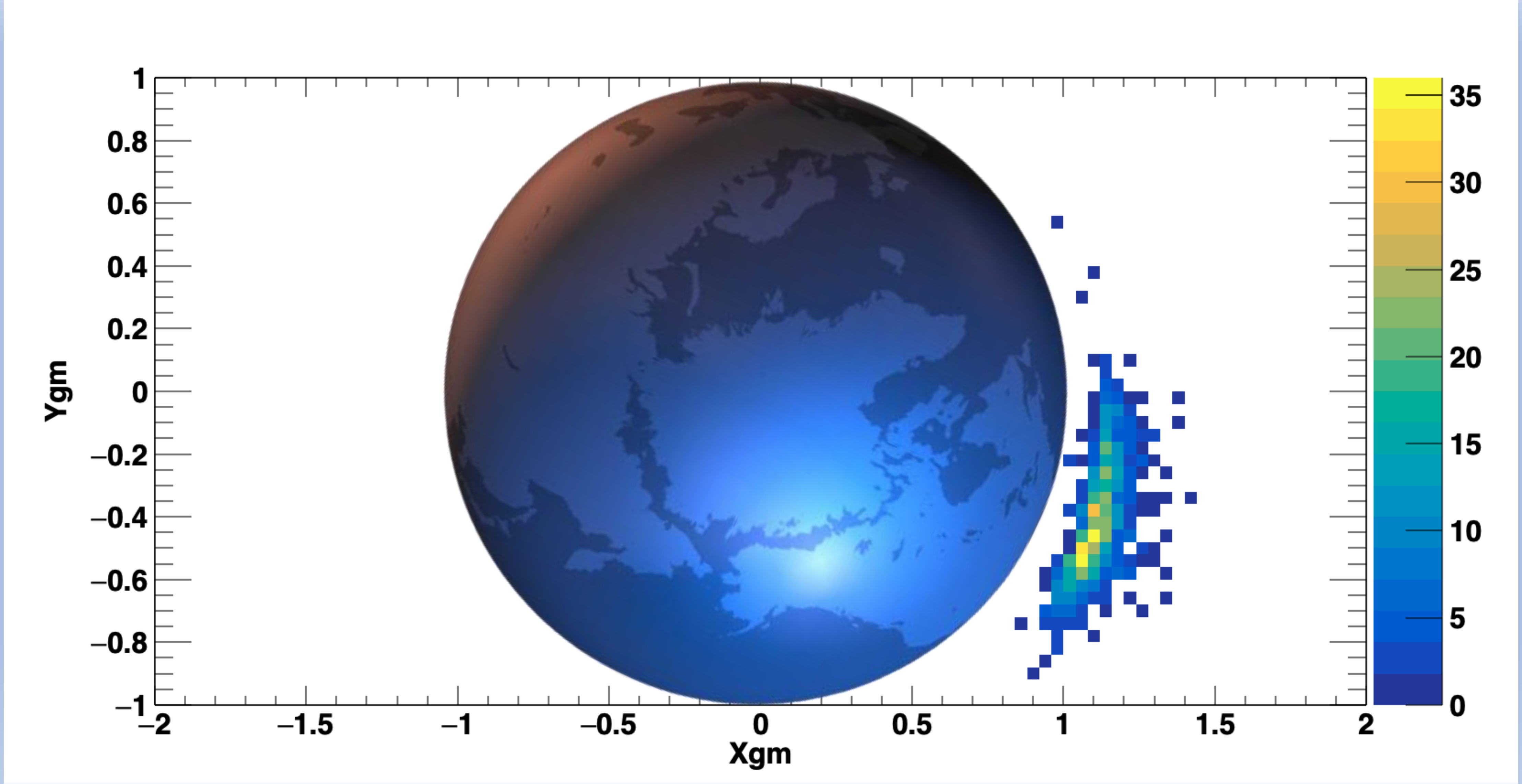
Downgoing ions Pitch angle vs Lshell



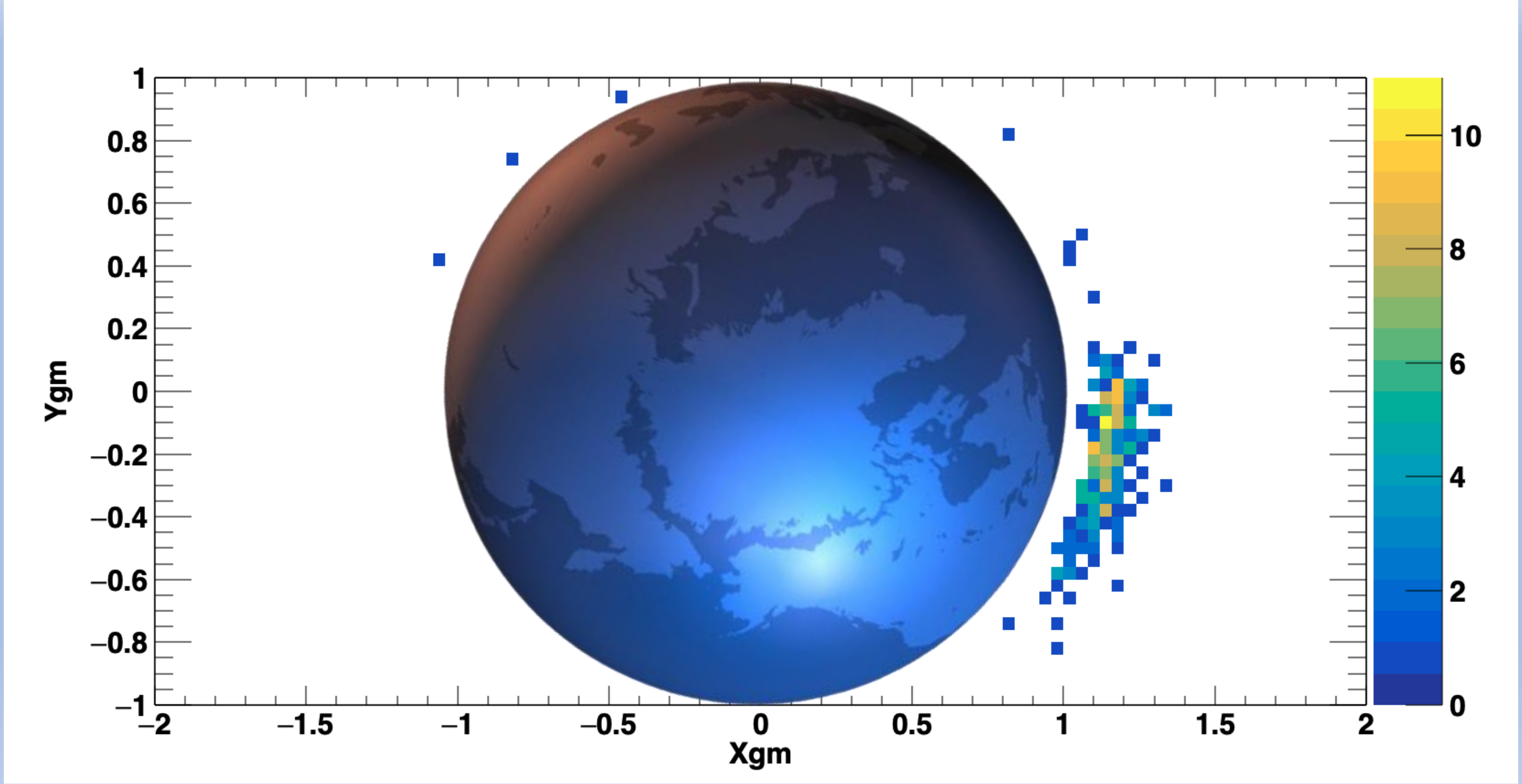
Downgoing ions Pitch angle vs Lshell requiring 8/10 particles **generated** are trapped



Upgoing ions Final position after Backtracing requiring 8/10 particles **generated** are trapped

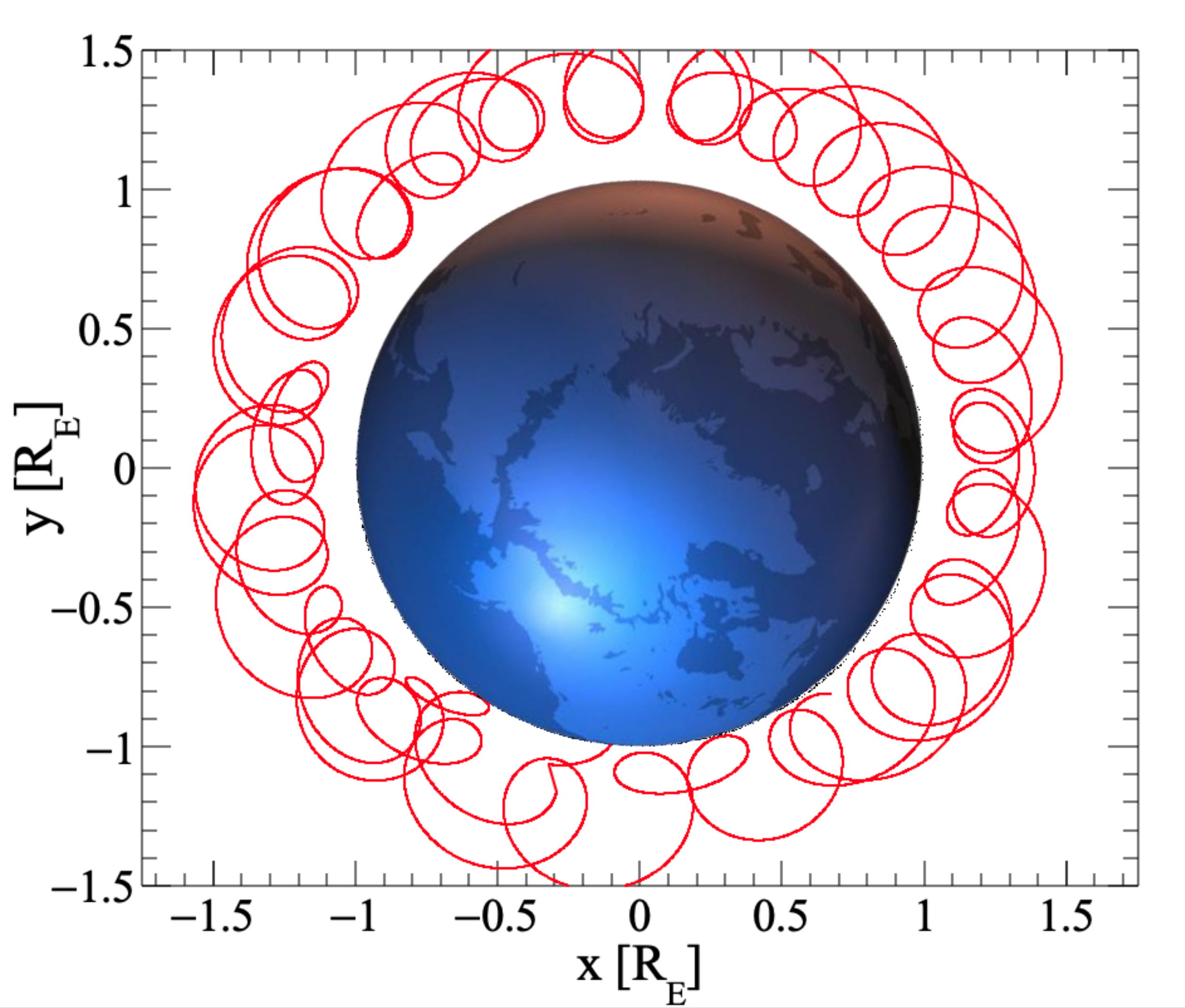


Downgoing ions Final position after Backtracing requiring 8/10 particles **generated** are trapped

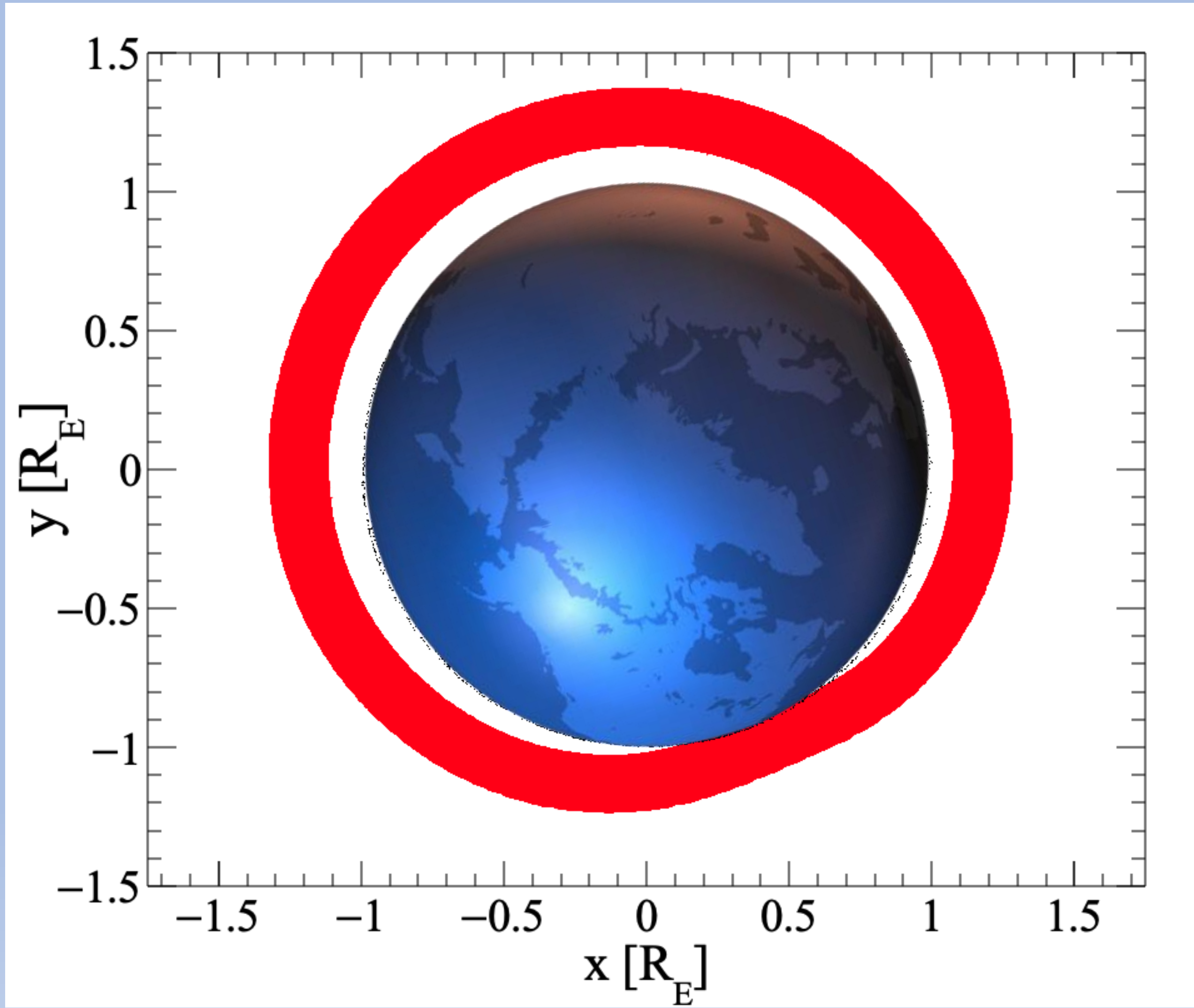


Upgoing ions

Boron (5.65 GV)

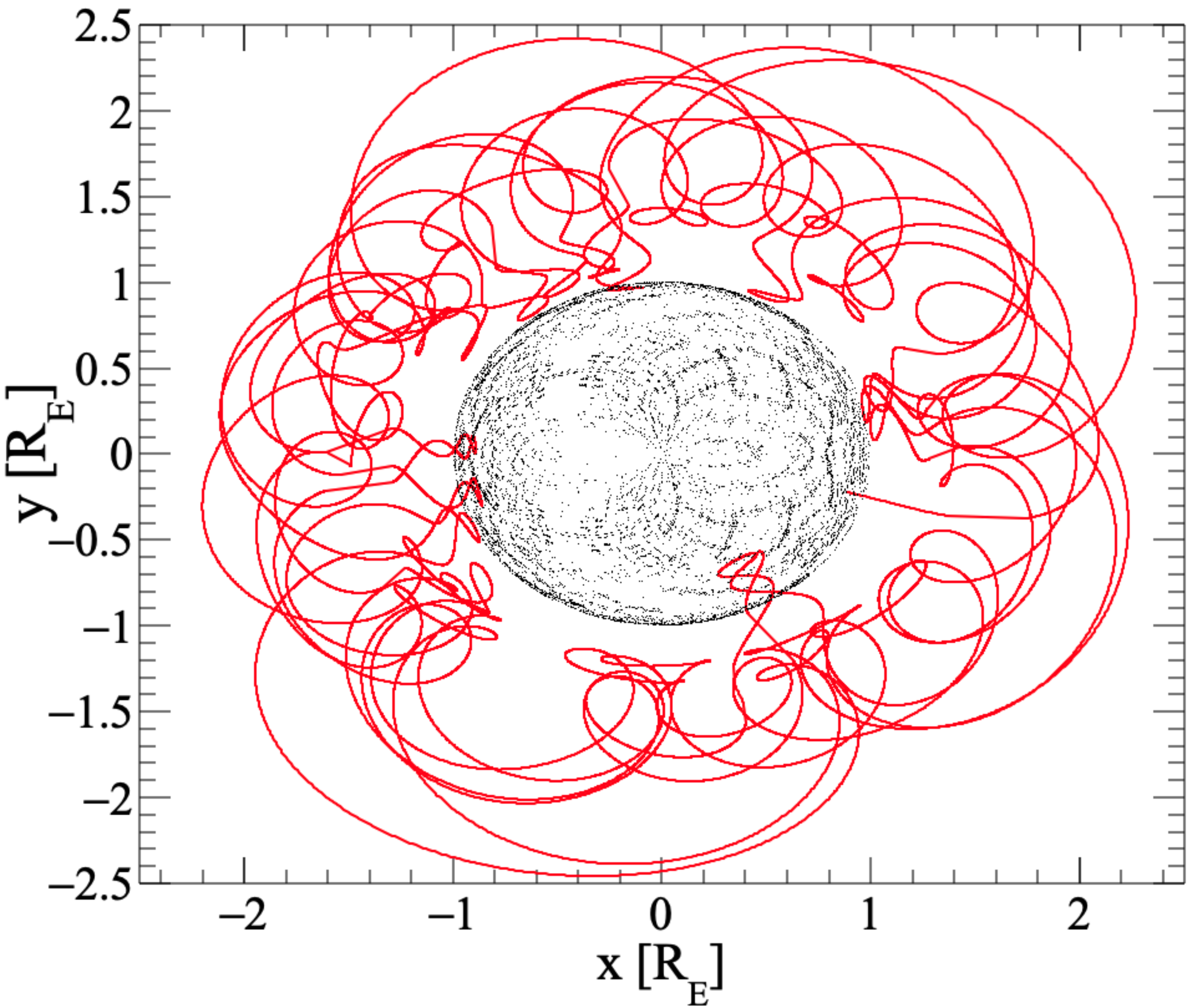


Carbon (3.02 GV)

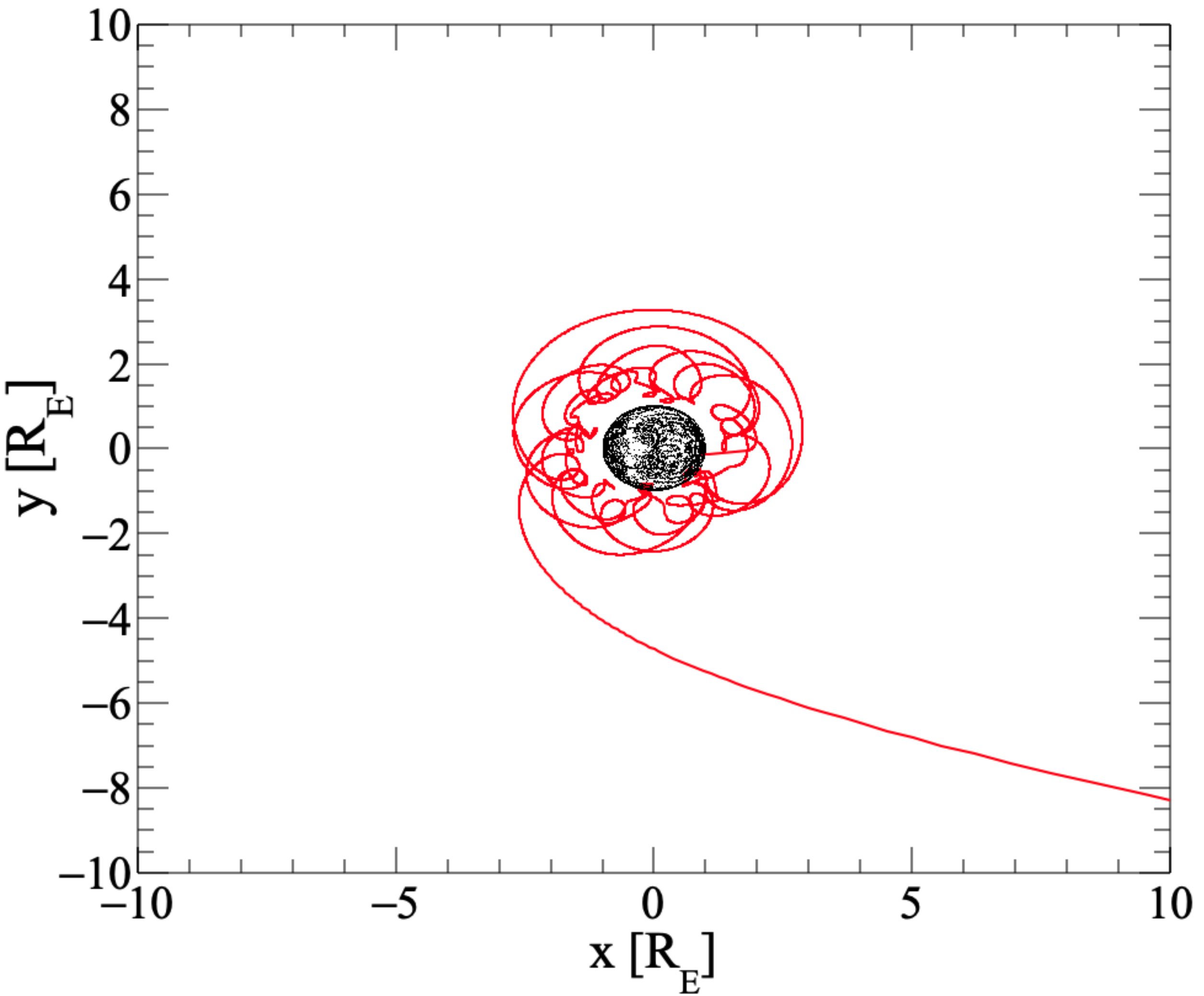


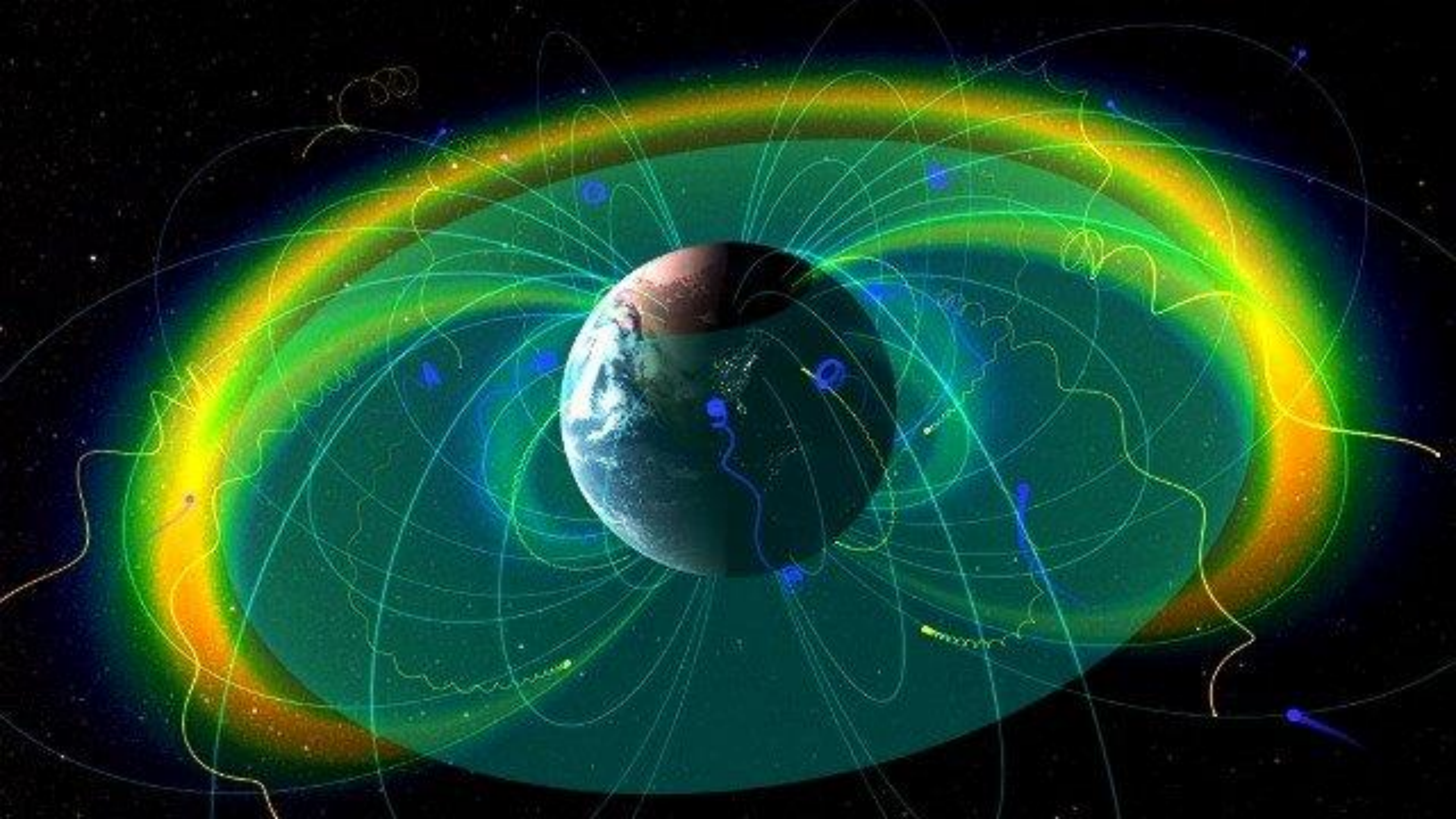
Downgoing Particles

Secondary?

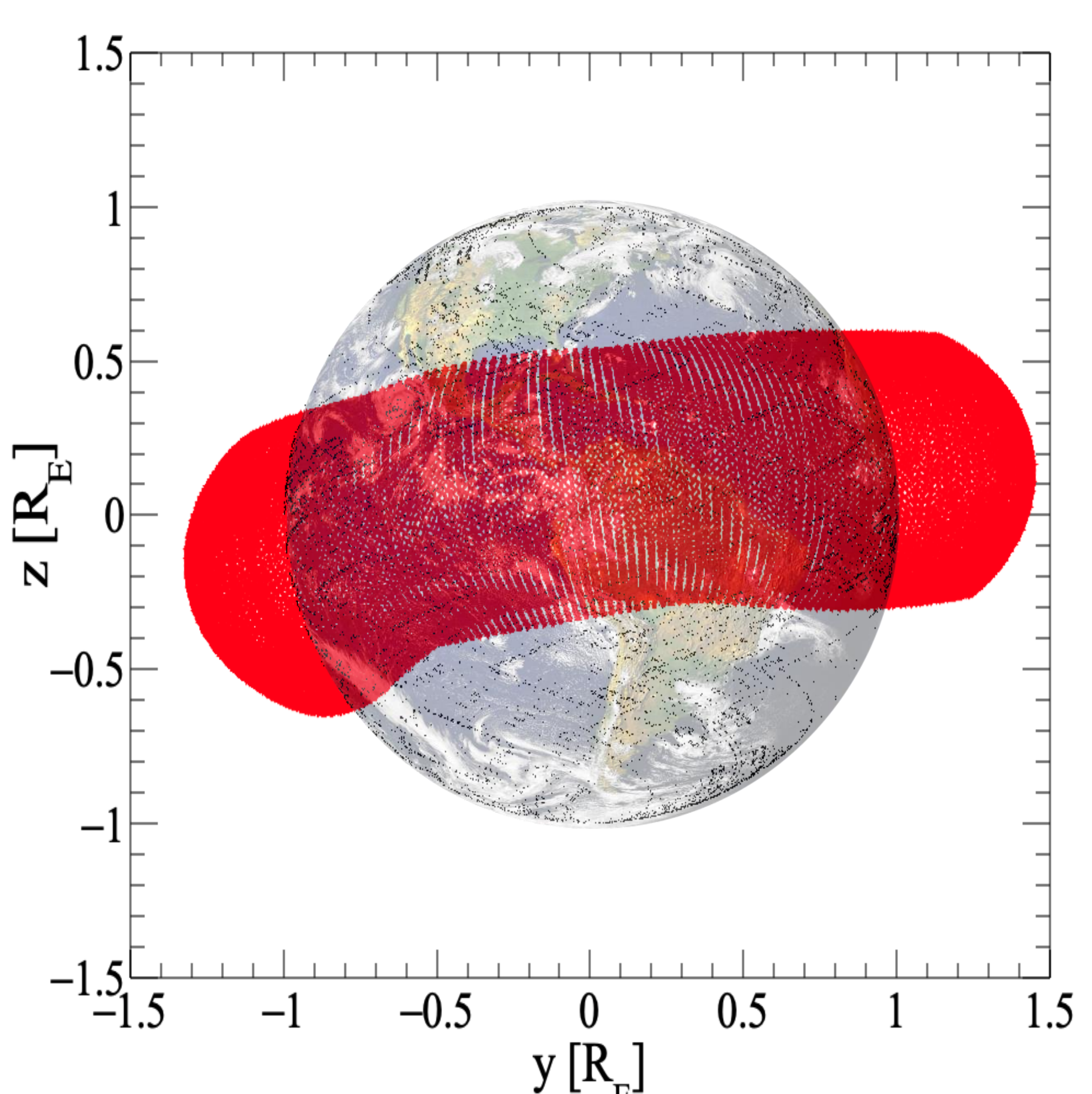


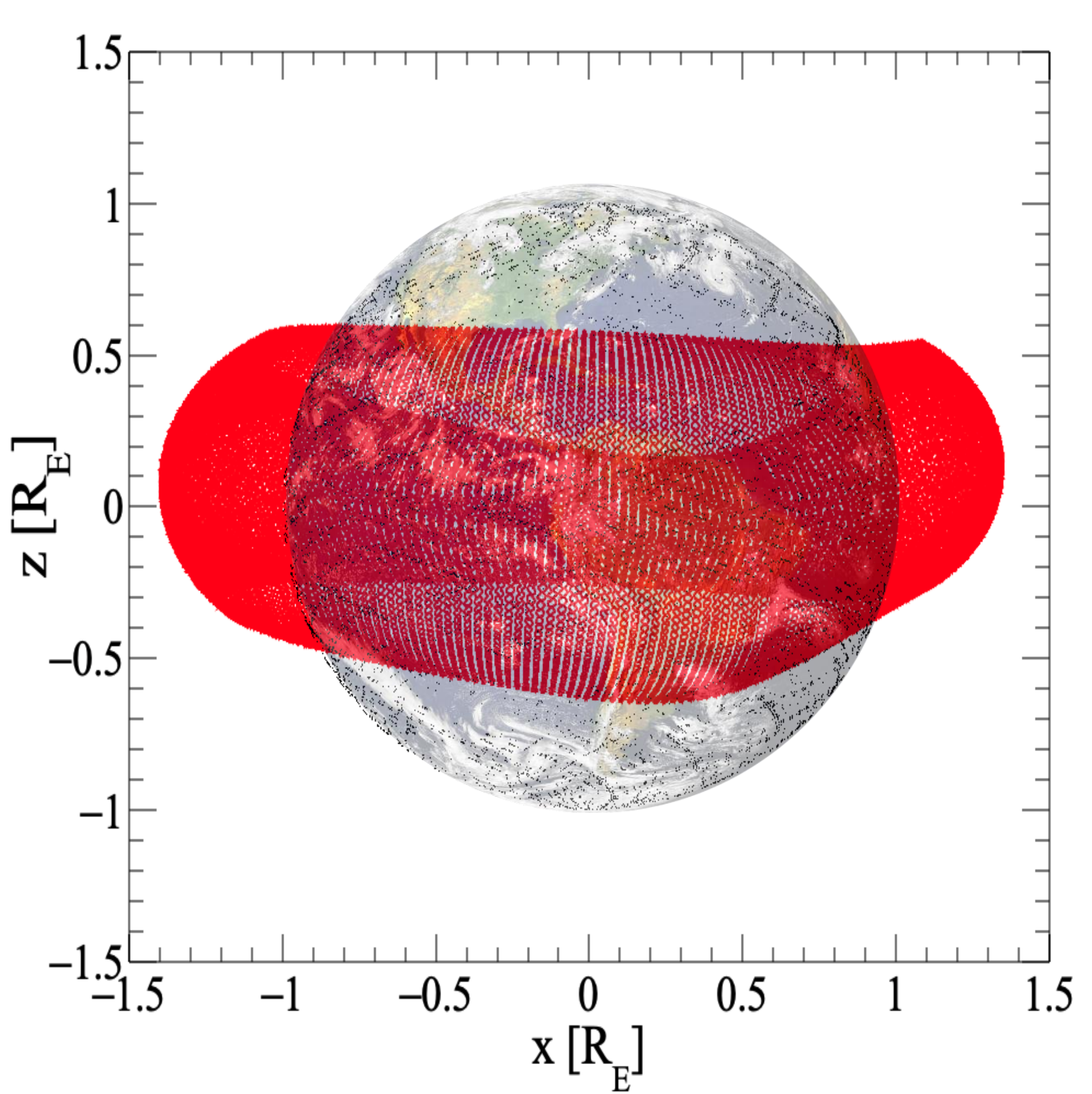
Primary?



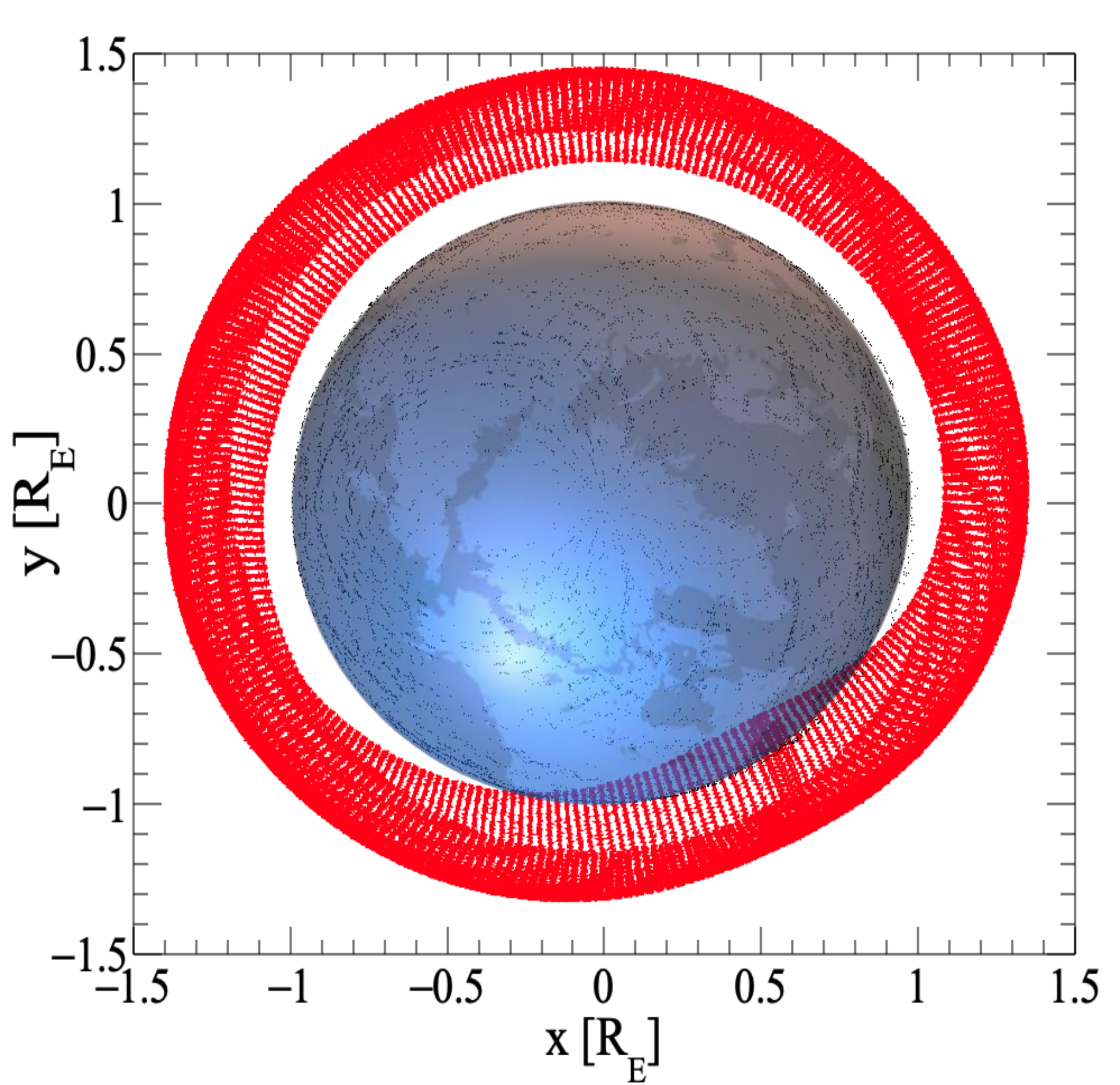


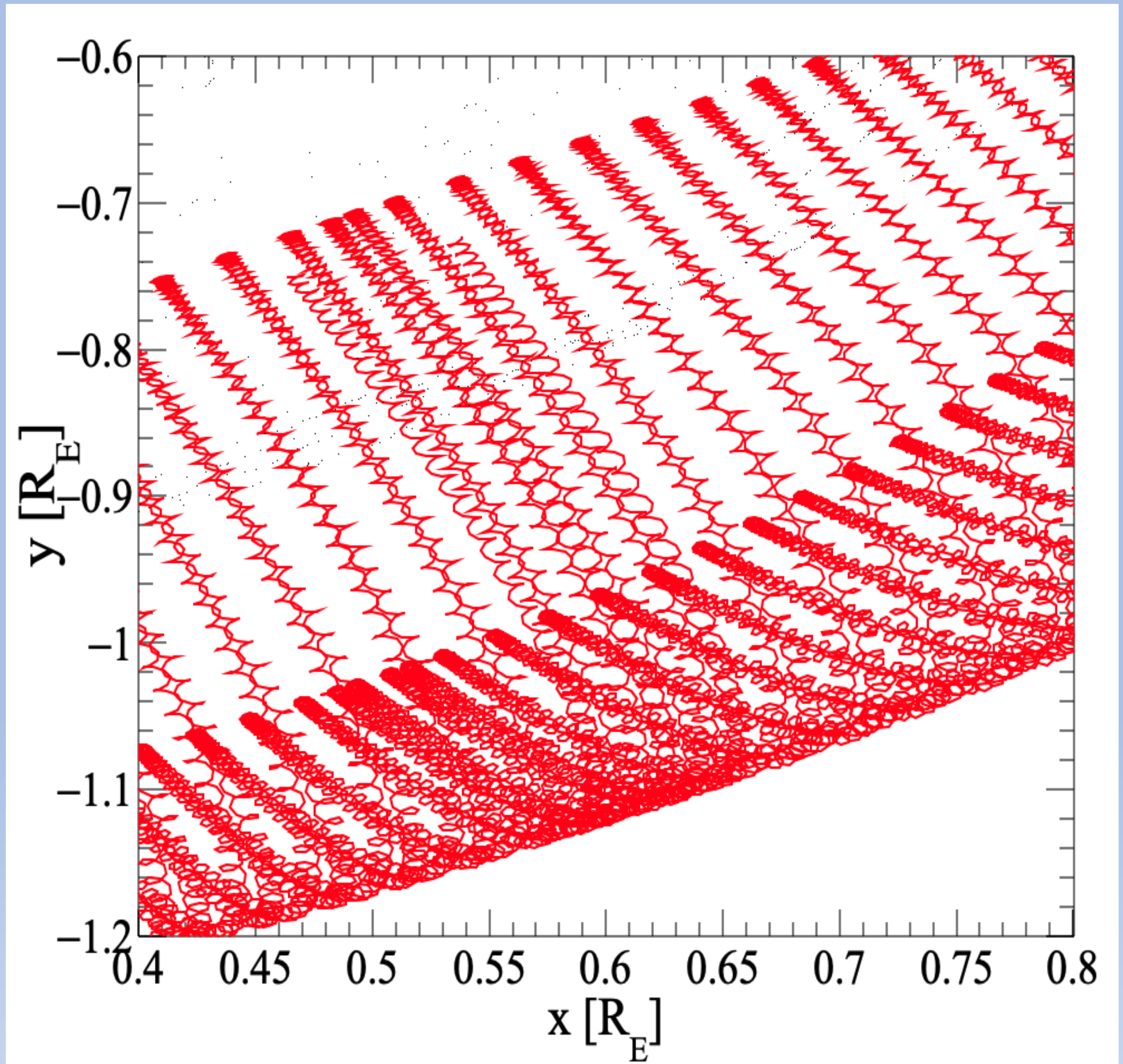
Trapped Lithium (0.16 GV)





Trapped Lithium





Website implementation

- **New analysis parameters has been added as a online calculator to our main website www.geomagsphere.org**
- **Through some dedicated webpages the user can obtain the**
  - **The L-Shell can be evaluated for all kind of particles**
  - **Equatorial Pitch Angle as for now can be calculated for positive particles (protons and ions)**
- **N.B. The needed parameters are specified in each calculation webpage**

Website L-Shell Calculation

L-Shell Calculation

- **GeoMagSphere Web Calculators**

GeoMagSphere data sets and results can be freely downloaded or copied. However, the user should make the appropriate acknowledgment or citation, e.g., see Citations or Bibliography pages. The data generated by using the Calculator may be used by GeoMagSphere developers for statistical purpose.

- **L-Shell**

This page enables users to calculate the L-Shell (or McIlwain) parameter for any particle and for any position. The L-Shell parameter is calculated in the shifted tilted dipole approximation, with the simple formula you can find here, using the updated IGRF-13 last parameters (https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html).

To perform the L-Shell Calculation please follow this link:

- L-Shell Calculator

Input Parameters

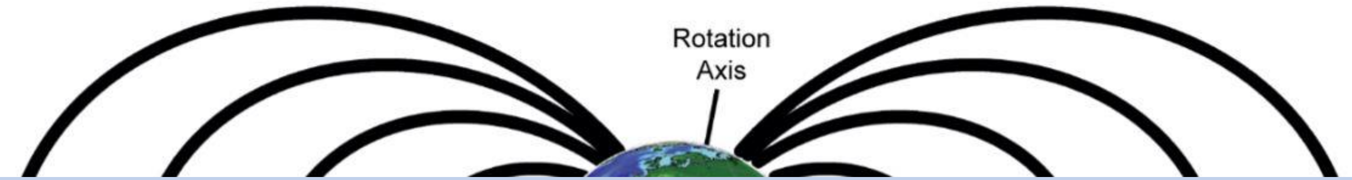
Input values are

- Geographical latitude (from 88 to -88 deg.)
- Geographic longitude (from 0 to 360 deg.)
- Altitude from the Earth's surface (in km)
- Time (year,month,day,hour,min,sec)

Output Parameters

Ouputs are:

- L-Shell parameter (in unit of Earth radii)



Website Equatorial Pitch Angle Calculation

Equatorial Pitch Angle

Trapped Particles can be studied through adiabatic invariants (see Adiabatic Invariants). In particular any kind of detailed study should deal with the McIlwain's L-parameter (see L-Shell Calculation) and the picth angle distribution. This last is related to the magnhetic flux that, under the specific trapping mechanism, can be considered contstant.

Pitch Angle

The direction of particle's velocity with respect to the magnetic field line is called pitch angle α. This one can be calculated in every position, but when the magnetic flux is constant, it is related to the equatorial pithc angle, so the angle that the particle velocity (in its circling, bouncing and drifting motion) has with respct toi the magnetic field line at the Earth geomagnetic equator. More this value is close to 90° more the mirror point are close one another and the drift shell is not reaching the geomagnetic poles, as can be seen in Eq. 1.

$$\frac{\sin^2 \alpha_{loc}}{B_{loc}} = \frac{\sin^2 \alpha_{eq}}{B_{eq}} = \frac{1}{B_M} \tag{1}$$

Because $\alpha_M = 90^\circ$ so $\sin^2 \alpha_M = 1$. In addition to this relation we know that the magnetic field line, in the (shifted and tilted) dipole approximation follow the equation 2:

$$r = r_{eq} \cos^2 \lambda_m \tag{2}$$

where r is the position of the field line with respect to the dip{ole center and r_{eq} is the distance of the field line from the dipole center at the equator. This equation cal be also written as

$$r = L \cdot \cos^2 \lambda_m \tag{3}$$

where L in Eq. 3 is the McIlwain parameter. In the shifted-tilted dipole approximation of the Earth magnetic field, the local value of the magnetic field B can be obtained as a function of the Earth surface value B_s from the formula:

Website Equatorial Pitch Angle Calculation

$$B = B_s \cdot (\frac{Re}{r})^3 \cdot \sqrt{1 + 3 \cdot \sin^2 \lambda_m} \quad (4)$$

where R_e is the Eart

- **GeoMagSphere Web Calculators**

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h radii, r is the distance from the dipole center and λ_m is the magnetic latitude (see Geomagnetic Coordinates). Using this Eq. 4 we can obtain:

$$B_{loc/eq} = B_s \cdot (\frac{Re}{r_{loc/eq}})^3 \cdot \sqrt{1 + 3 \cdot \sin^2 \lambda_{mloc/meq}} \quad (5)$$

And in addition to the fact that $\sin \lambda_{meq} = 0$ and using Eq. 2 with $r = r_{loc}$we can have:

$$\frac{\sin^2 \alpha_{loc} \cdot B_{eq}}{B_{loc}} = \sin^2 \alpha_{eq} \quad (6)$$

and

$$\frac{B_{eq}}{B_{loc}} = \frac{(\frac{Re}{r_{eq}})^3}{(\frac{Re}{r_{loc}})^3 \cdot \sqrt{1 + 3 \cdot \sin^2 \lambda_{mloc}}} = \frac{(r_{loc})^3}{(r_{eq})^3 \cdot \sqrt{1 + 3 \cdot \sin^2 \lambda_{mloc}}} \quad (7)$$

but

Website Equatorial Pitch Angle Calculation

$$\frac{r_{loc}}{r_{eq}} = \cos^2 \lambda_{mloc} \quad (8)$$

so finally we can obtain:

$$\frac{\sin^2 \alpha_{loc} \cdot B_{eq}}{B_{loc}} = \sin^2 \alpha_{eq} = \frac{(\cos^2 \lambda_{mloc})^3 \cdot \sin^2 \alpha_{loc}}{\sqrt{1 + 3 \cdot \sin^2 \lambda_{mloc}}} \quad (9)$$

And this can tells us that once we are able to evaluate the pitch angle at the detection point r_{loc} plus the magnetic latitude (in the shifted tilted dipole approximation, using IGRF-13) we can estimate the equatorial pitch angle in case of trapped particles (and conservation of the magnetic flux).

- **GeoMagSphere Web Calculators**

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- **Pitch Angle Calculation**

This page enables users to calculate the Equatoria Pitch Angle for any trapped particle and for any position, using the previous formulas.

To perform the L-Shell Calculation please follow this link:

- Equatorial Pitch Angle Calculator

Input Parameters

Input values are:

- Particle (in mass number so 1 for Protons, 2 for Helium nuclei, etc., up to 26 for iron)
- Particle Rigidity value in GV
- Geographical latitude (from 88 to -88 deg.)
- Geographic longitude (from 0 to 360 deg.)
- Altitude from the Earth's surface (in km)
- Time (year,month,day,hour,min,sec)

Output Parameters

Output data are:

- Equatorial Picth Angle

Future plans

- Trapped Protons & Helium
  - Sample inside the SAA & outside the SAA (polar)
  - to be used as a “comparison”
- Trapped Ions
  - Reduce the request of sigma in “generated” particle (for 8/10)
  - Extend in time the Ions to be backtraced