Overlap Valence Quarks on a Twisted Mass Sea – an update

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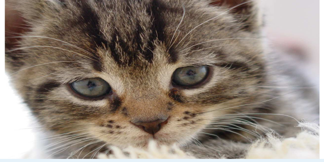
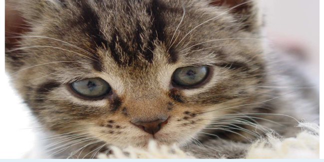


Presentation outlook

1. Introduction and motivation
2. Pion decay constant continuum limit scaling test – an update
3. Overlap valence quarks on ETMC $N_f = 2+1+1$ configurations – short feasibility study
4. Mixed action χPT – extraction of LECs



Setup

We analyze a mixed action setup of:

- MTM sea quarks,
- overlap valence quarks.

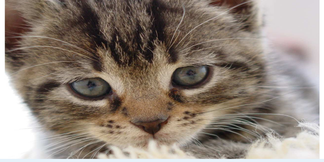
We work with:

- small volume $L \approx 1.3$ fm,
- pion masses $m_\pi \approx 300$ and $m_\pi \approx 450$ MeV.

Main motivation for an update of the continuum limit scaling test of f_π is the availability of a new small-volume ensemble:

$$\beta = 4.35,\ L/a = 32,\ a \approx 0.04 \text{ fm},\ a\mu = 0.00175,$$

matched to other ensembles that we have been using so far.



Overlap fermions

The overlap Dirac operator was given by H. Neuberger in 1997:

$$D_{overlap} = \frac{1}{a}\left(1 - \frac{am}{2}\right)\left(1 - A(A^\dagger A)^{-1/2}\right) + m,$$
$$A = 1 - aD_{Wilson},$$
$$D_{Wilson} = \frac{1}{2}\left(\gamma_\mu(\nabla^*_\mu + \nabla_\mu) - ar\nabla^*_\mu\nabla_\mu\right).$$

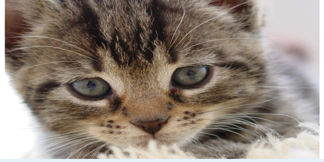
At zero quark mass, this operator obeys the Ginsparg-Wilson relation:

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D.$$

In 1998 Lüscher found that the Ginsparg-Wilson relation leads to a non-standard realization of lattice chiral symmetry. The action is invariant under:

$$\psi \to e^{i\theta\gamma_5\left(1-\frac{aD}{2}\right)}\psi,$$
$$\bar\psi \to \bar\psi e^{i\theta\gamma_5\left(1-\frac{aD}{2}\right)}.$$

Chiral symmetry protects from additive mass renormalization and $O(a)$ artefacts.



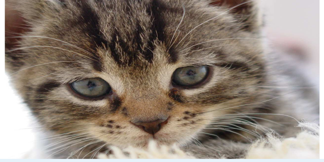
Mixed action approach

- The mixed action approach has potential difficulties, originating from the fact that the fermionic determinant comes from an action which is different than the one of the observables and the spectra of D_{TM} and $D_{overlap}$ are different.
- We have many different competing effects: standard FSE, topological FSE, discretization effects (standard ones, isospin violation, zero modes, ...).
- However, the continuum limit of this approach should be the same as of the unitary approach – continuum limit scaling test should check universality.
- One needs a matching of quark masses – the matching condition can be (for fixed lattice spacing and volume):

  ⋆ $m_\pi^{VV} = m_\pi^{SS}$ or

  ⋆ $m_{q,ren}^{valence} = m_{q,ren}^{sea}$.

  At the matching point, other observables should be matched up to $\mathcal{O}(a^2)$ lattice artefacts.

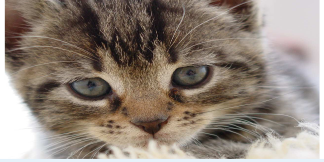


Setup of the scaling test

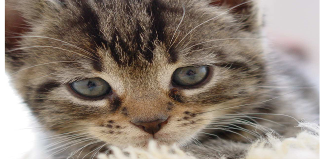
We would like to test the scaling behaviour towards the continuum limit of overlap fermions in fixed volume.

Table 1: The summary of ensembles that we have used.

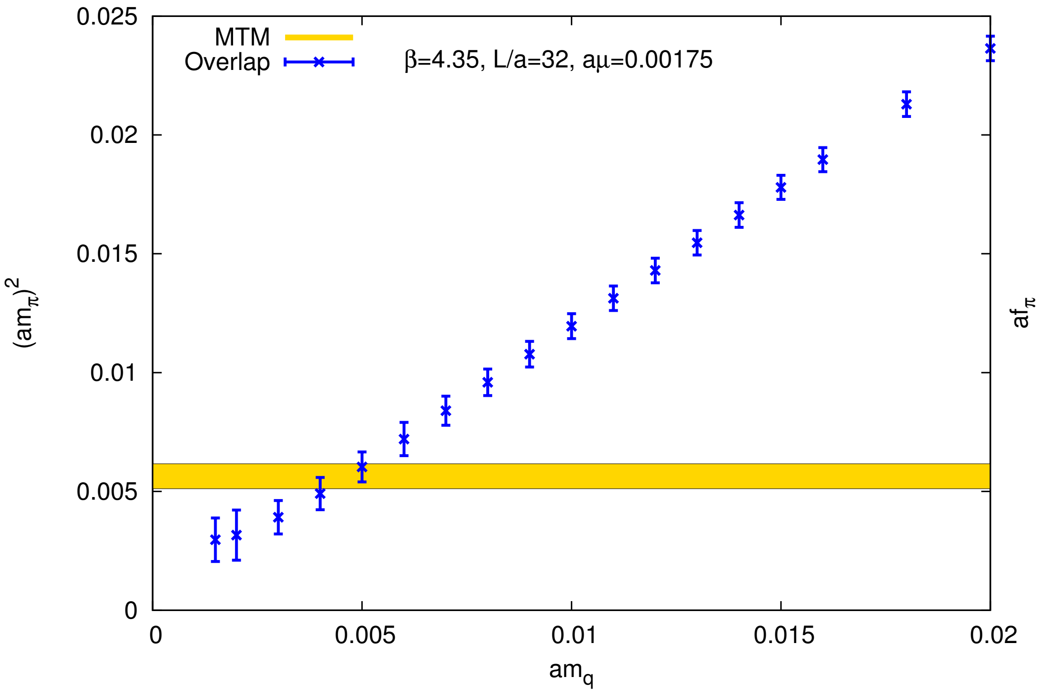
β	L/a	a [fm]	L [fm]	$a\mu$	m_π [MeV]	$r_0 m_\pi$	L/r_0
3.9	16	0.079	1.3	0.004	300	0.84	3.05
4.05	20	0.063	1.3	0.003	300	0.80	3.03
4.2	24	0.051	1.3	0.002	300	0.82	2.88
4.35	32	0.042	1.3	0.00175	300	0.74	3.26
3.9	16	0.079	1.3	0.0074	450	1.03	3.05
4.05	20	0.063	1.3	0.006	450	1.00	3.03
4.2	24	0.051	1.3	0.005	450	1.04	2.88
3.9	20	0.079	1.6	0.004	300	0.73	3.81
3.9	24	0.079	1.9	0.004	300	0.71	4.57

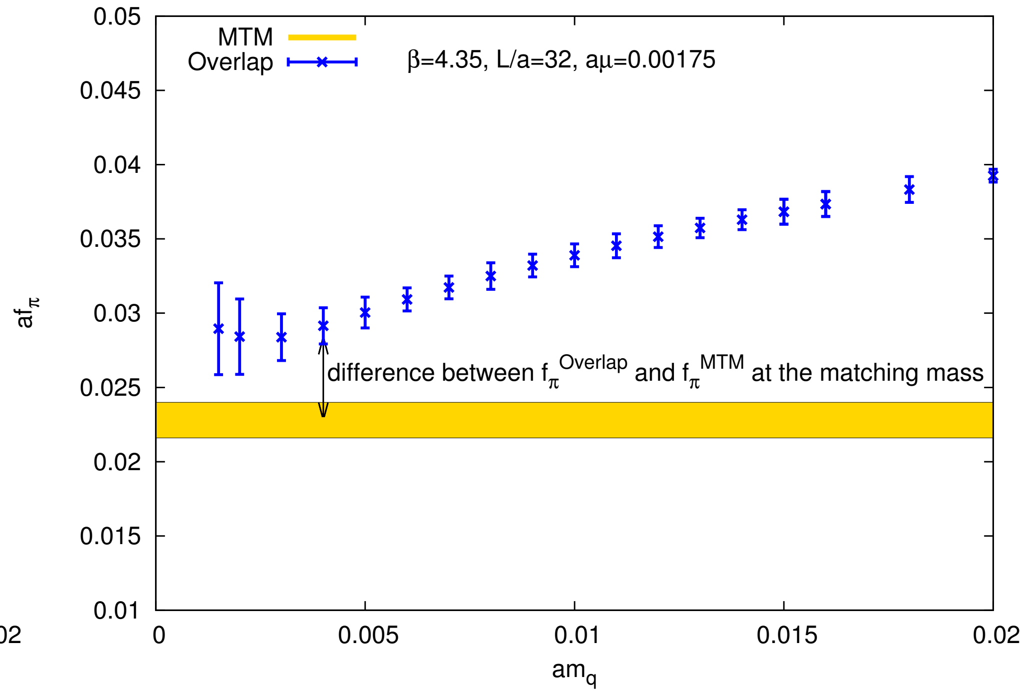


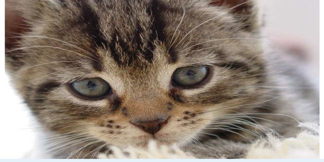
Scaling test



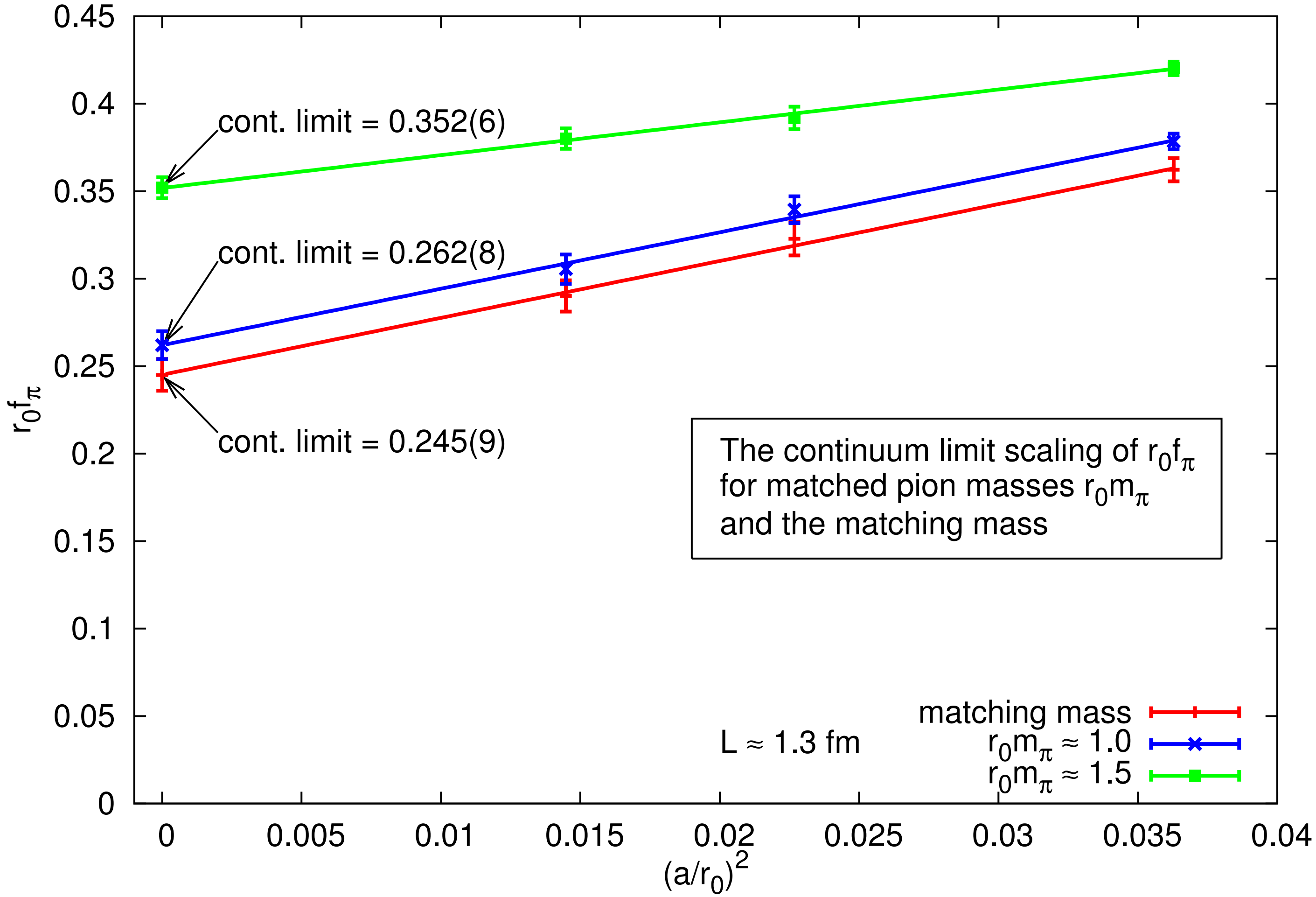
Matching the pion mass



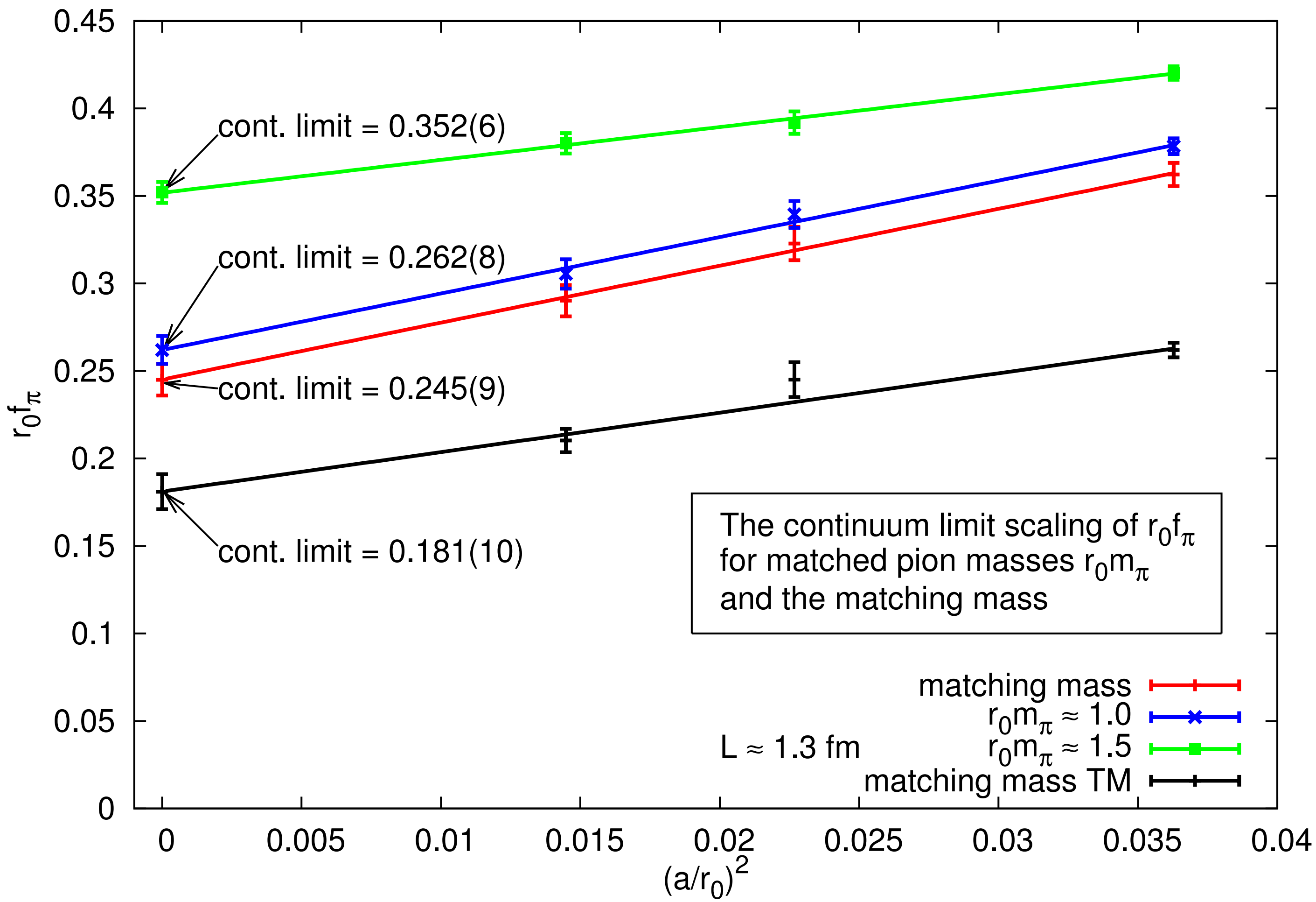




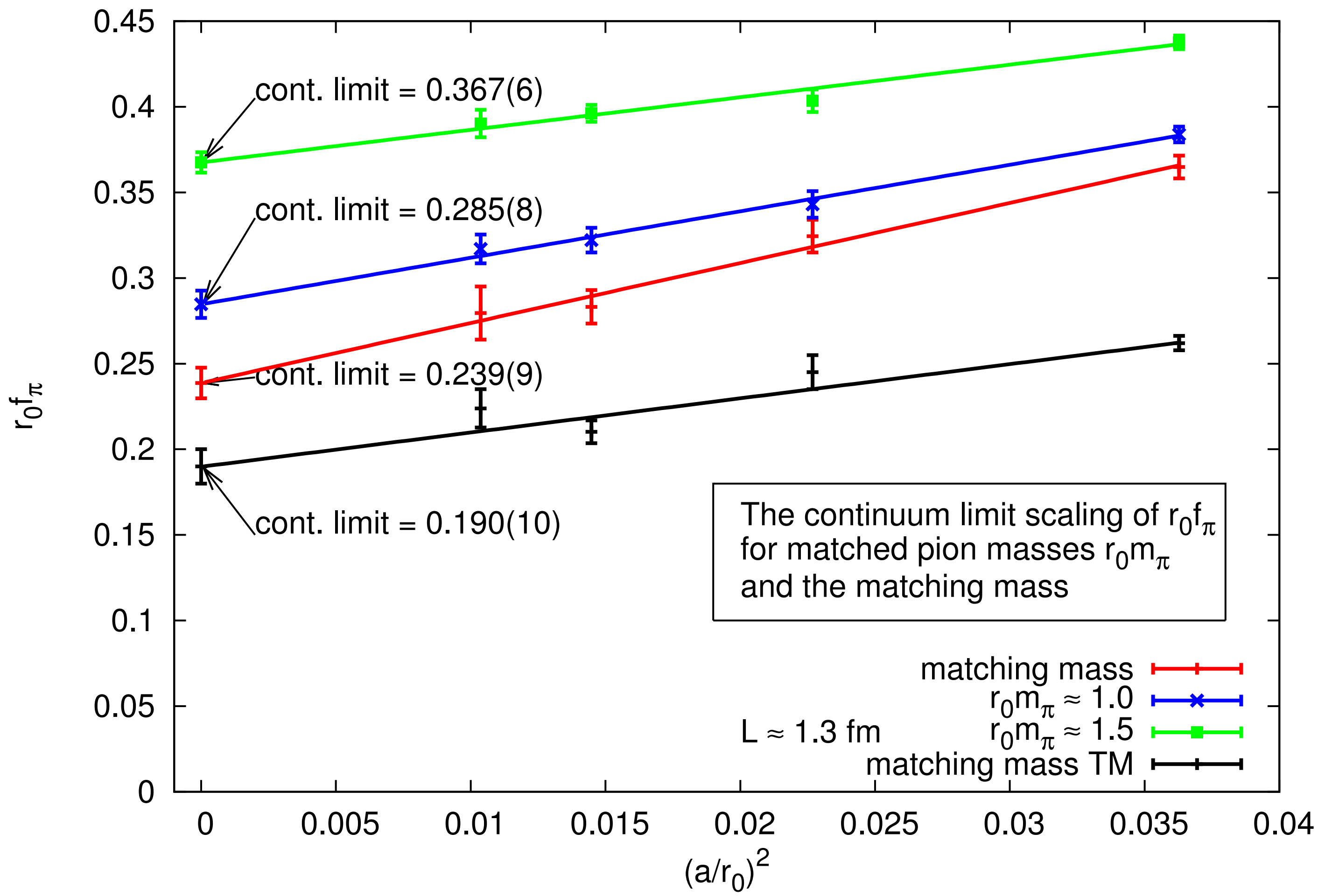
Scaling test – reminder

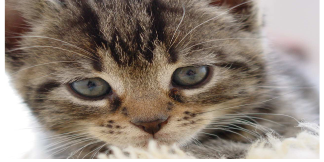


Scaling test – reminder



Scaling test – new with $\beta = 4.35$





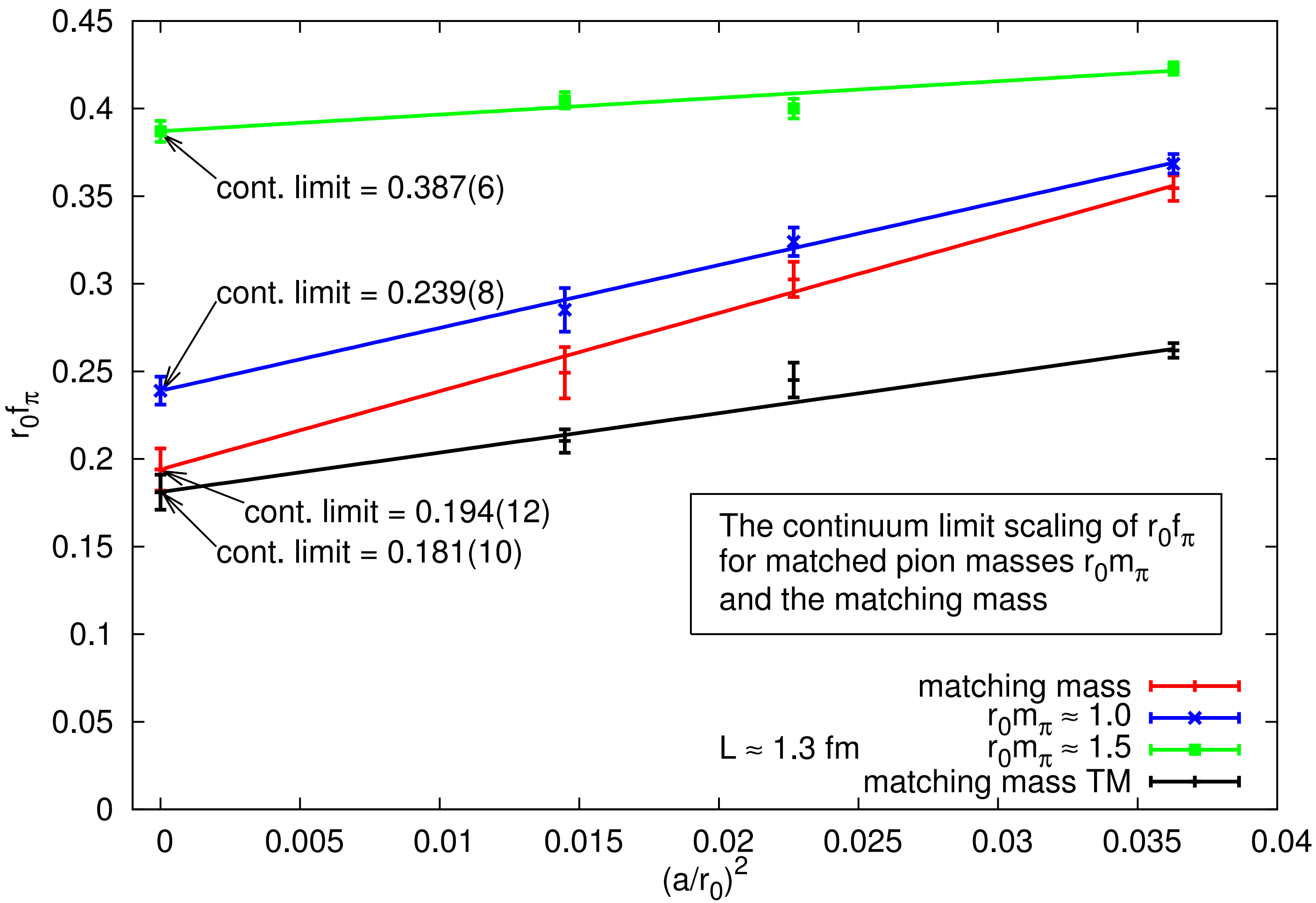
Pion decay constant mismatch & zero modes of the overlap Dirac operator

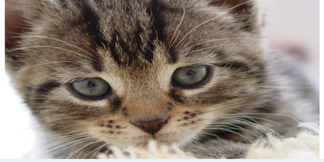
- At the matching point, we should have:

$$f_\pi^{Overlap} = f_\pi^{MTM} + O(a^2)$$

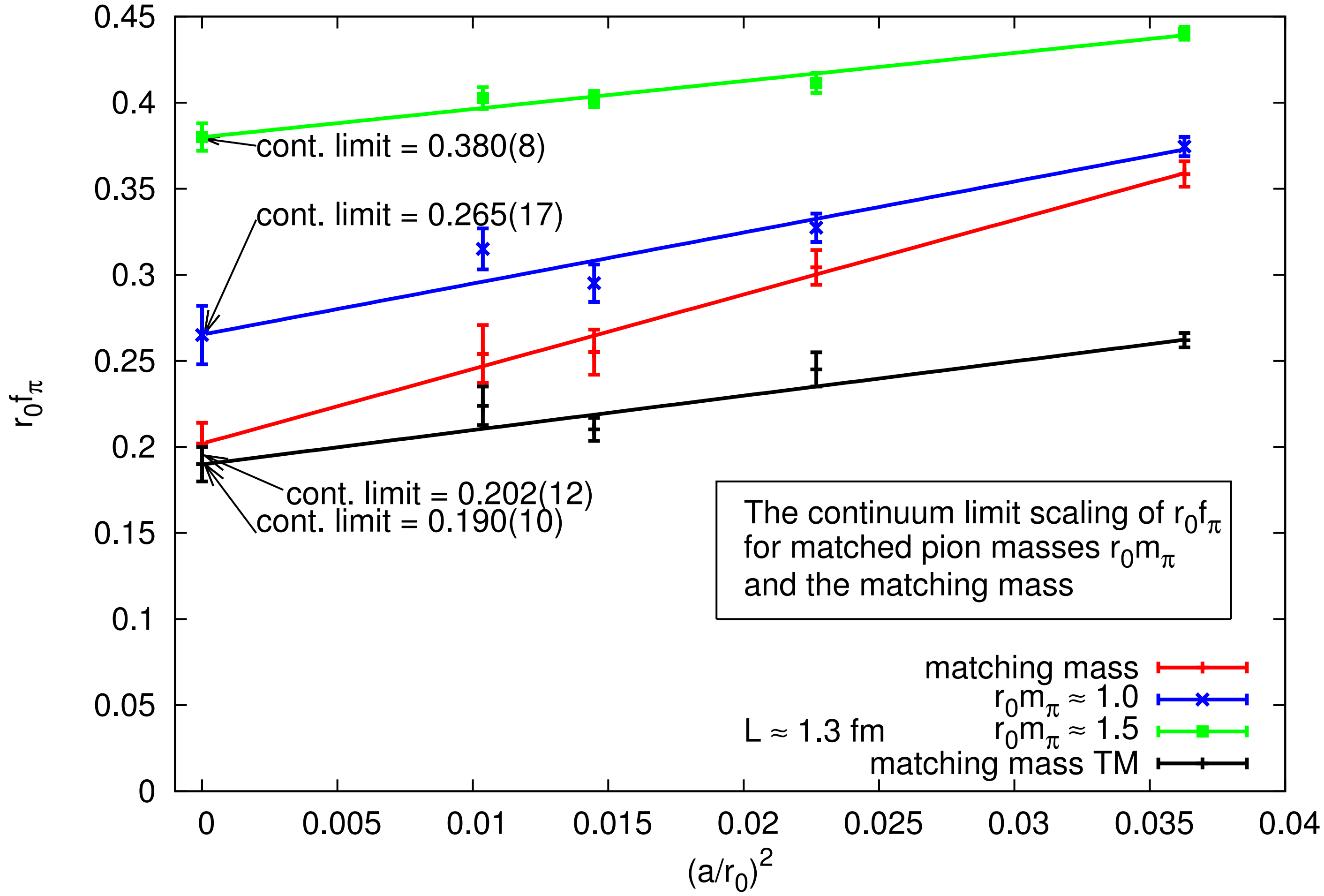
  However, we observe quite large discrepancies between $f_\pi^{Overlap}$ and f_π^{MTM} that go away very slowly when we approach the continuum limit.
- The overlap Dirac operator admits chiral zero modes at any value of the lattice spacing.
- The MTM Dirac operator needs sufficiently small lattice spacing to develop chiral zero modes (by far smaller than the values currently reached).
- Hence, in our mixed action setup the contribution of the zero modes of the overlap operator is not suppressed by the fermionic determinant.
- This can give large artefacts in some correlation functions, such as PP and SS.
- The leading contribution is proportional to $1/m^2$ and also it should vanish in the infinite volume limit, but can be very important in small volume.
- The zero modes contribute equally to the PP and SS correlators. Thus, their contribution vanishes in the difference $C_{PP-SS}(t) = C_{PP}(t) - C_{SS}(t)$.

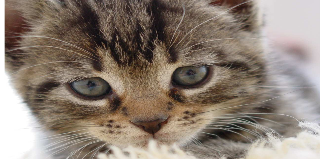
Scaling test – PP-SS





Scaling test – PP-SS – new with $\beta = 4.35$

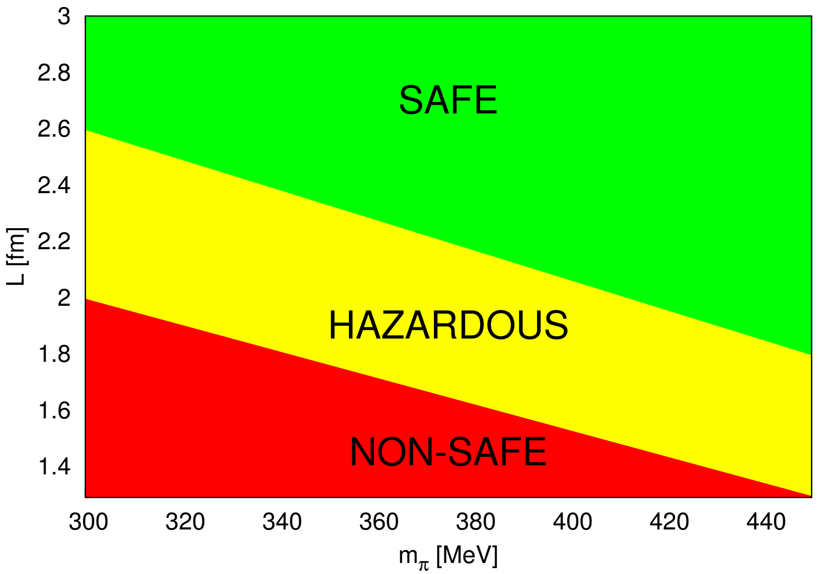




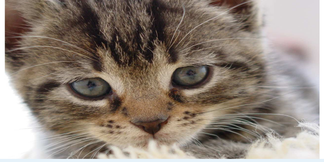
CONGO FLAG!!!

We seem to be safe against the zero modes when:

- The volume is large enough – at $\beta = 3.9$ (and $m_\pi \approx 300$ MeV) we need something of the order of 2.6 fm, i.e. $32^3 \times 64$!
- The sea quark mass is large enough – at $m_\pi \approx 450$ MeV we need something of the order of 1.8 fm, i.e. $24^3 \times 48$.



SAFE $\iff m_\pi L > 4$, HAZARDOUS $\iff m_\pi L \in [3, 4]$

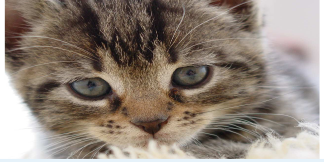


Overlap valence quarks on $N_f = 2+1+1$ ETMC configurations – short feasibility study

- Overlap fermions are approx. 2 orders of magnitude more expensive than MTM fermions
- Timings for $N_f = 2$ propagator computation (8 inversions per conf) and index computation:

β	L/a	L [fm]	propagator [CPUh]	index [CPUh]
3.9	16	1.3	200	50
4.05	20	1.3	500	125
4.2	24	1.3	1000	250
4.35	32	1.3	4000	1000
3.9	20	1.6	1200	300
3.9	24	1.9	5000	–
3.9	32	2.6	30000	–

CPU hours on HLRB-II SGI Altix 4700 in Munich



Overlap valence quarks on $N_f = 2+1+1$ ETMC configurations – short feasibility study

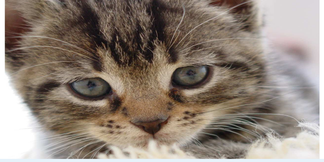
Examples of $N_f = 2+1+1$ ensembles that should be safe against the zero modes problem:

β	L/a	$a\mu$	$m_\pi L$	200 inversions [CPUh]
1.95	24	0.0085	4.7	0.7 M
1.95	32	0.0075	5.8	4 M
1.95	32	0.0055	5.0	5 M
1.95	32	0.0035	4.0	6 M
2.10	48	0.0030	4.7	30 M
2.10	48	0.0020	3.9	30 M

Things to do before actual computation:

1. Find the matching mass.
2. Check that one is safe against the zero modes!

Mixed action χPT LECs

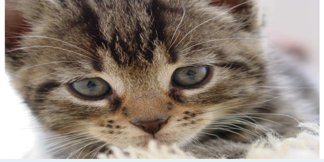


Light pseudoscalar meson masses

The expressions for the light pseudoscalar meson masses at LO and at maximal twist read:

$$
\begin{aligned}
M_{\pm}^2 &= 2B_0 m_s\,, \\
M_0^2 &= 2B_0 m_s - \hat{a}^2 \frac{16}{f^2}\left(2W_6' + W_8'\right), \\
M_{\mathrm{VV}}^2 &= 2B_0 m_v\,, \\
M_{\mathrm{VS}}^2 &= B_0(m_v + m_s) + \hat{a}^2 \frac{2}{f^2} W_M - \hat{a}^2 \frac{4}{f^2} W_8'\,,
\end{aligned}
$$

where $\hat{a} = 2W_0 a$, $M_{\pm,0}$ are the charged and neutral SS meson masses, the convention for f is such that $f_\pi = 92.4\,\mathrm{MeV}$.



Mixed meson masses

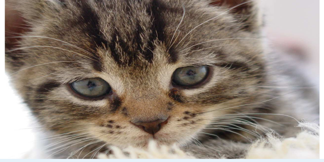
From the mixed meson mass $M_{\rm VS}$ it is possible to determine the LECs $W_M - 2W_8'$ in the following way:

$$\left(r_0^6 W_0^2\right)\left(W_M - 2W_8'\right) = r_0^2 \left(M_{VS}^2 - M_{VV}^2\right) \frac{(r_0 \hat{f})^2}{16} \left(\frac{r_0}{a}\right)^2$$

where $\hat{f} = \sqrt{2} f$ was used in order to match the more frequent lattice convention for the decay constant. This formula assumes that we work at the matching point $M_{SS} = M_{VV}$. If we allow other masses as well, we have:

$$\left(r_0^6 W_0^2\right)\left(W_M - 2W_8'\right) = r_0^2 \left(M_{VS}^2 - \frac{M_{VV}^2 + M_{SS}^2}{2}\right) \frac{(r_0 \hat{f})^2}{16} \left(\frac{r_0}{a}\right)^2,$$

where M_{SS} is the charged pseudoscalar meson mass.



Mixed meson masses 2

Alternative parametrization:

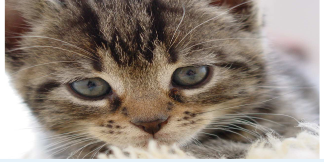
$$M_{VS}^2 = B_0(m_v + m_s) + a^2 C_{Mix}.$$

Thus:

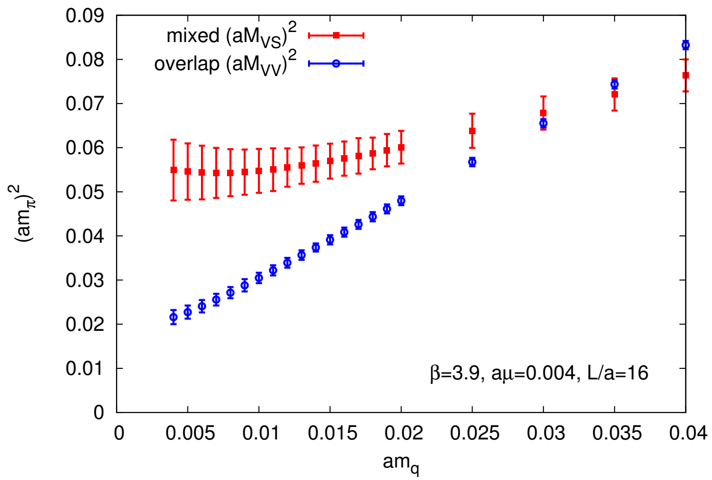
$$a^4 C_{Mix} = (aM_{VS})^2 - \frac{(aM_{VV})^2 + (aM_{SS})^2}{2}.$$

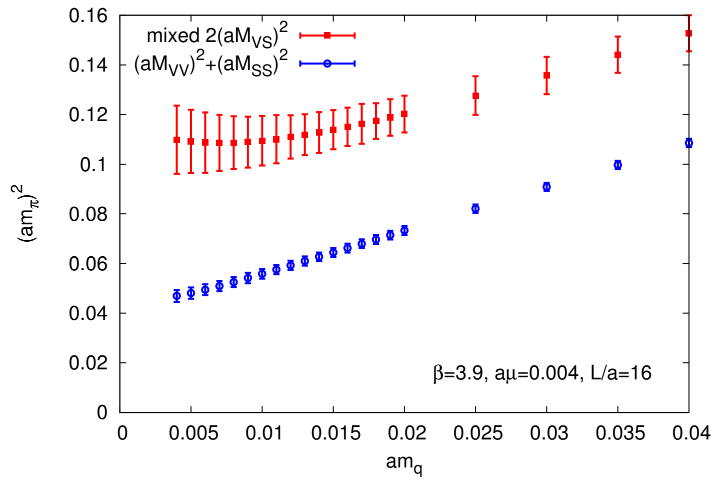
In terms of $\left(r_0^6 W_0^2\right)\left(W_M - 2W_8'\right)$:

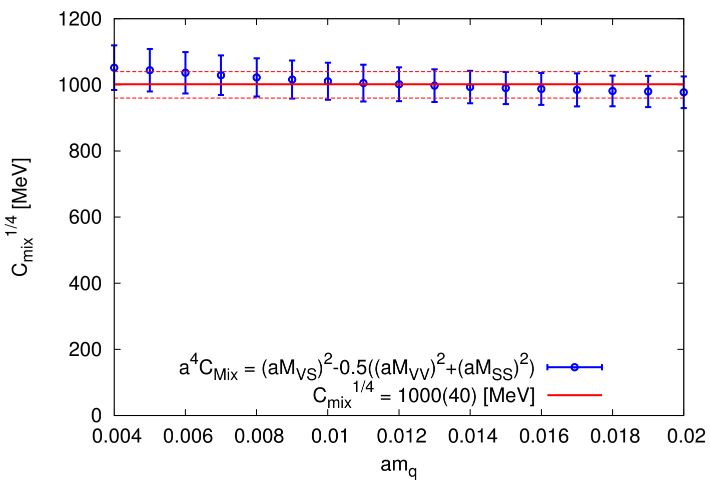
$$a^4 C_{Mix} = \frac{16\left[\left(r_0^6 W_0^2\right)\left(W_M - 2W_8'\right)\right]}{(r_0/a)^6 (a\hat{f})^2}.$$

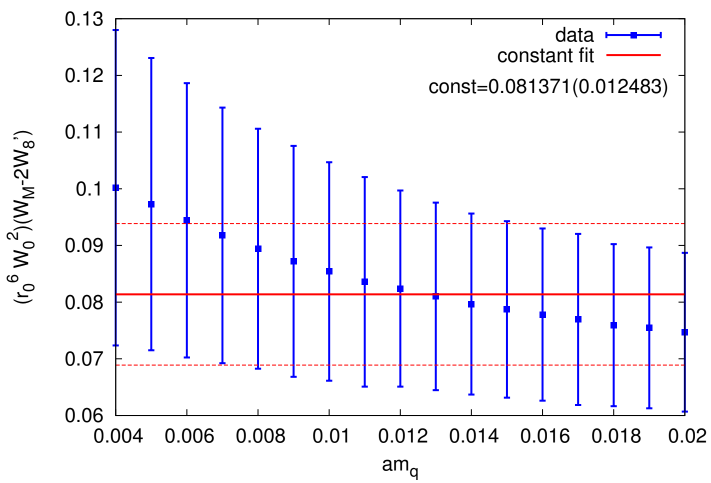


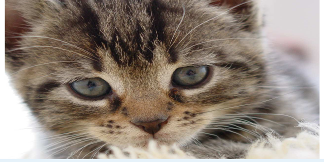
$\beta = 3.9,\ L/a = 16,\ a\mu = 0.004$



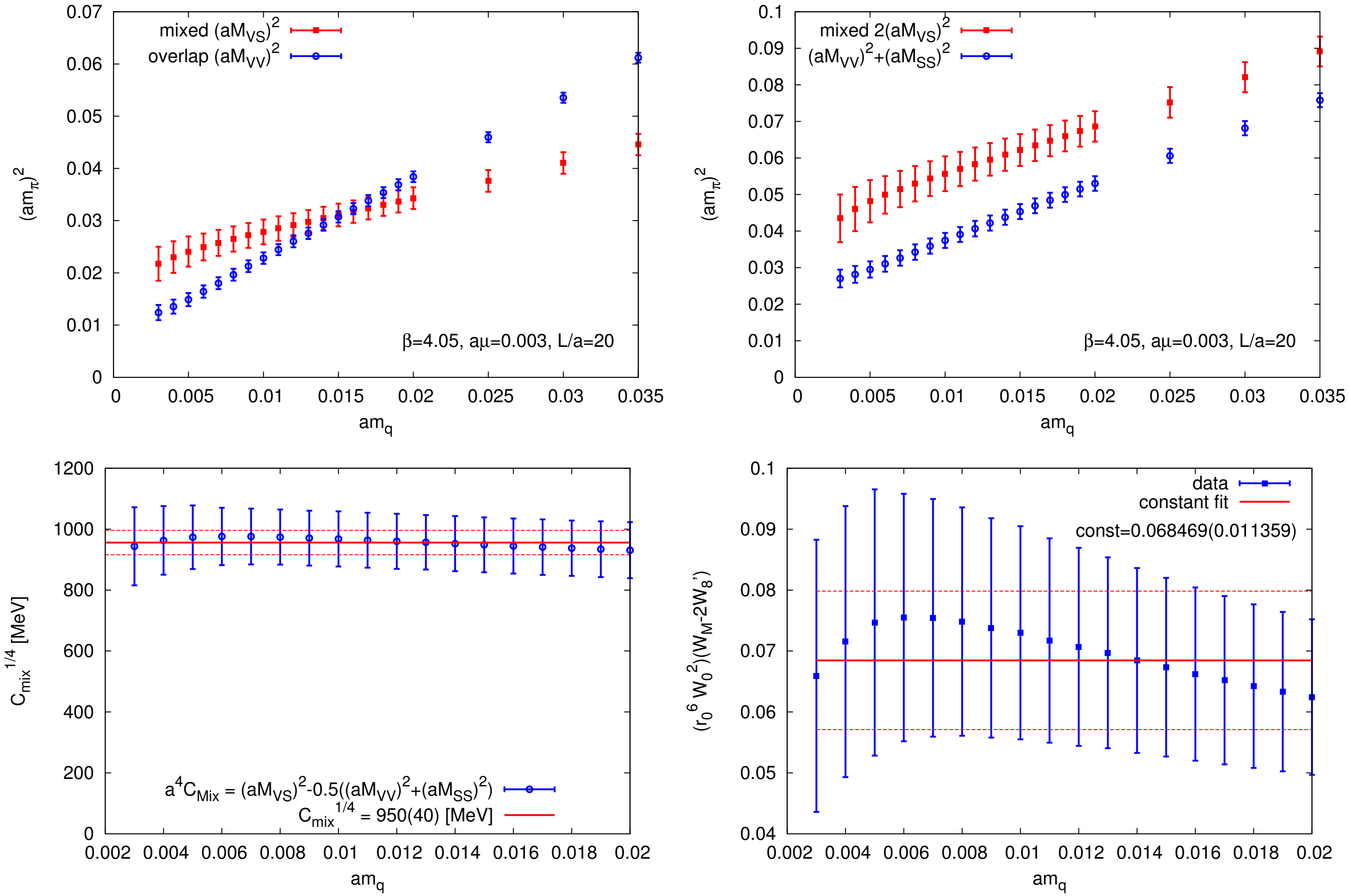


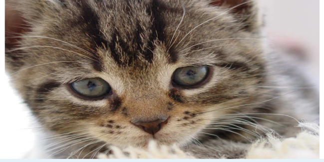




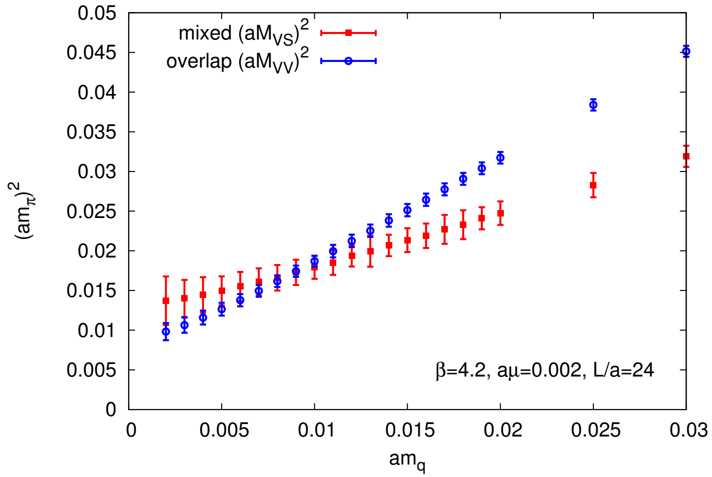


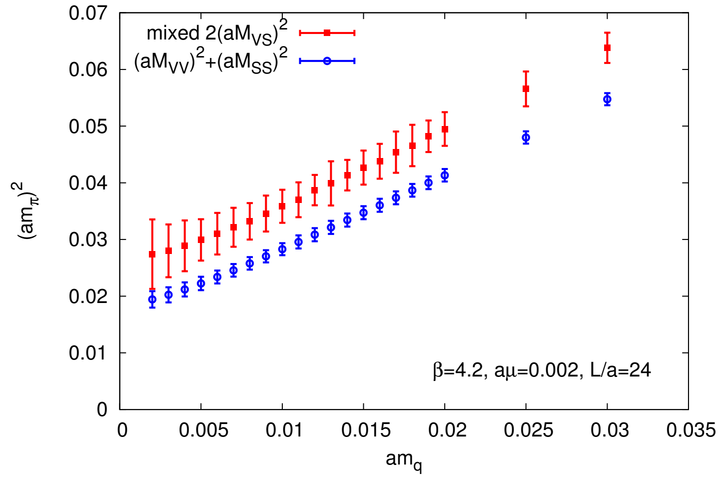
$\beta = 4.05,\ L/a = 20,\ a\mu = 0.003$

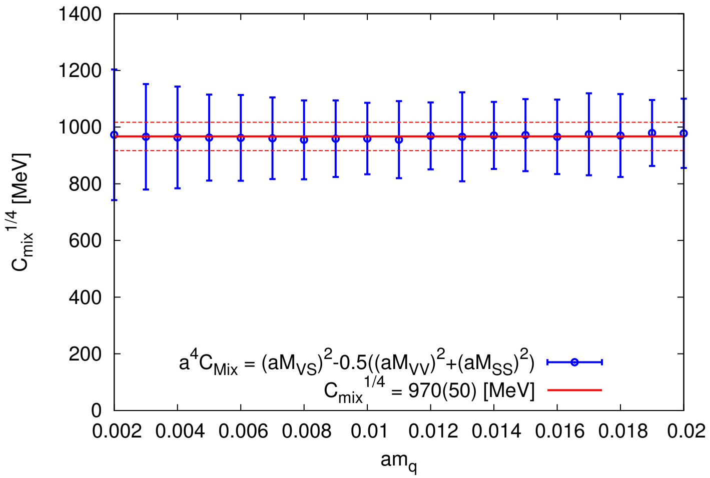


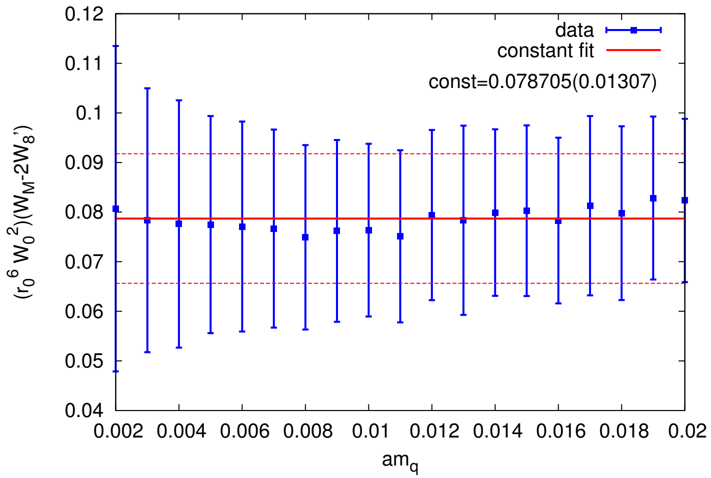


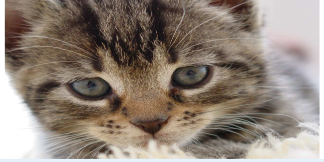
$\beta = 4.2,\ L/a = 24,\ a\mu = 0.002$



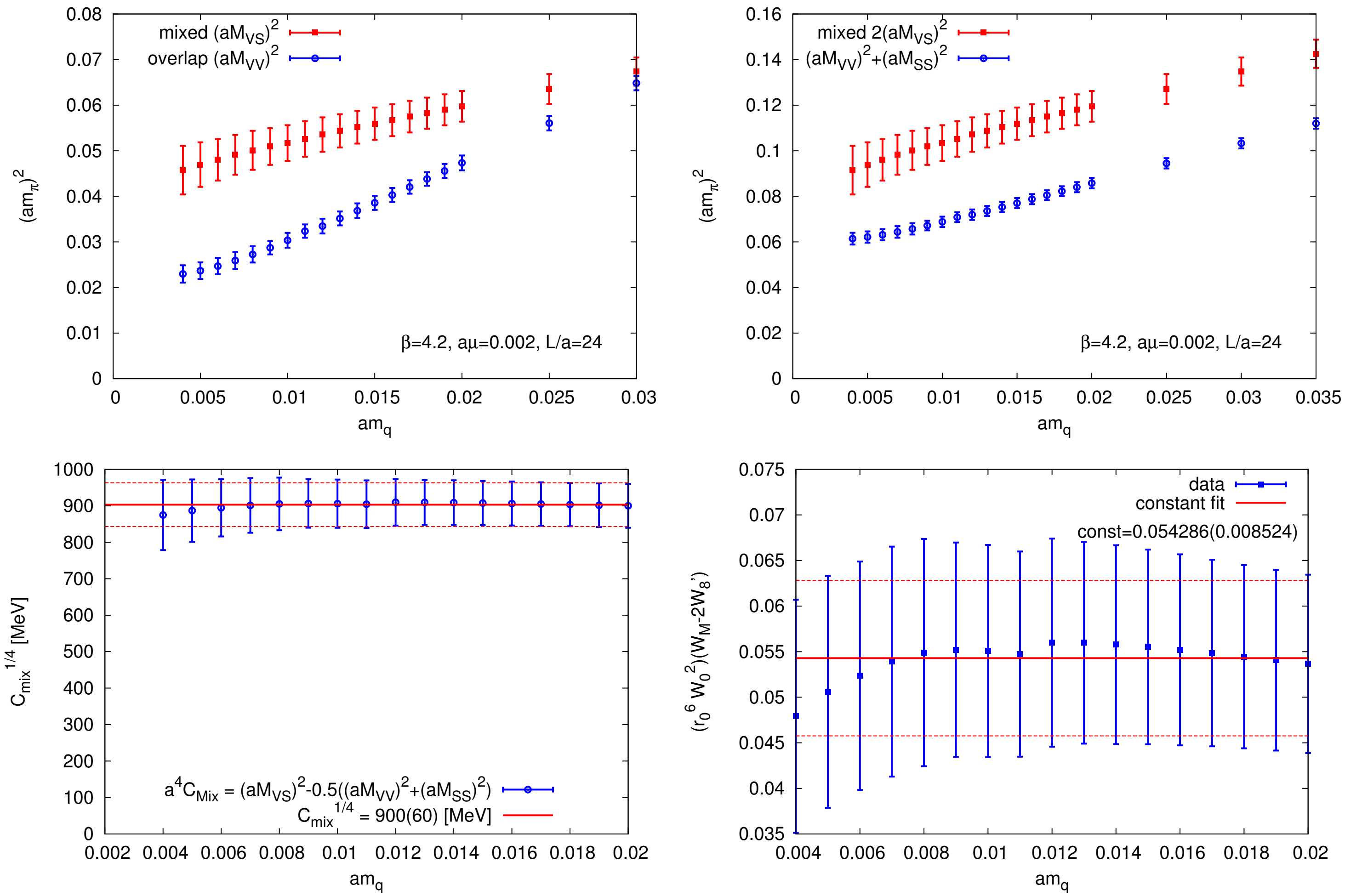


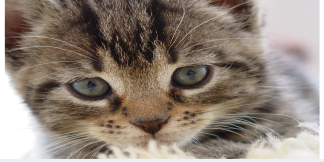






$\beta = 3.9,\ L/a = 16,\ a\mu = 0.0074$



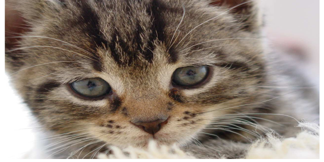


$W_M - 2W_8'$ – summary

β	matching mass	$\left(r_0^6 W_0^2\right)\left(W_M - 2W_8'\right)$		$C_{Mix}^{1/4}$
	am_v	matching point	constant fit	[MeV]
3.9	0.007	0.092(23)	0.081(12)	1000(40)
4.05	0.005	0.075(22)	0.068(11)	950(40)
4.2	0.002	0.081(33)	0.079(13)	970(50)
3.9h	0.015	0.056(11)	0.054(9)	900(60)

$C_{Mix}^{1/4}$ for other mixed actions:

- overlap on clover – 872 MeV
- domain-wall on staggered – 708, 664 MeV
- overlap on domain-wall – -427 MeV



Unitarity violations in the scalar correlator

The flavour non-singlet scalar correlator is given by:

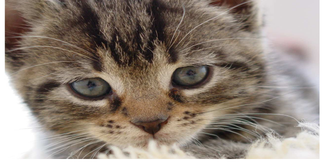
$$C_{sca}^{VV}(t) = -2\left(M_{\pm}^2 - M_{VV}^2 + \hat{a}^2\,\frac{16}{f^2}\,W_8'\right)\cdot B_{DP}(T,L,t,M_{VV},M_{VV}) \\ -2B_{SP}(T,L,t,M_{VV},M_{VV}) + 2B_{SP}(T,L,t,M_{VS},M_{\mathrm{VS}})\,.$$

In the limit $T \to \infty$, and at large t, the bubble contributions can be simplified to:

$$C_{sca}^{VV}(t) \;\to\; \frac{B_0^2}{2L^3}\left[\frac{e^{-2M_{VS}t}}{M_{VS}^2} - \frac{e^{-2M_{VV}t}}{M_{VV}^2}\frac{1}{M_{VV}^2}\left(M_{VV}^2 + \hat{a}^2\,\frac{8}{f^2}\,W_8'(1+M_{VV}\,t)\right)\right].$$

Assuming that the $\mathcal{O}(a^2)$ effects in the mixed meson mass are small, a Taylor expansion of the contribution from M_{VS} leads to the following expression:

$$C_{sca}^{VV}(t) \;\to\; \frac{B_0^2}{2L^3}\,\frac{e^{-2M_{VV}t}}{M_{VV}^4}\left[-\hat{a}^2\,\frac{2}{f^2}(W_M + 2W_8')(1+M_{VV}\,t)\right].$$



Unitarity violations in the scalar correlator

Fitting ansatz (fitting parameters are α and M_{VV}):

$$C_{sca}^{VV}(t) = -\alpha \left[(1 + M_{VV} t) e^{-2M_{VV} t} + (1 + M_{VV}(T-t)) e^{-2M_{VV}(T-t)} \right],$$

where (having eliminated B_0 using $M_{VV}^2 = 2B_0 m_v$):

$$\alpha = \frac{2a^2}{L^3 m_v^2 \hat{f}^2} (W_0)^2 (W_M + 2W_8').$$

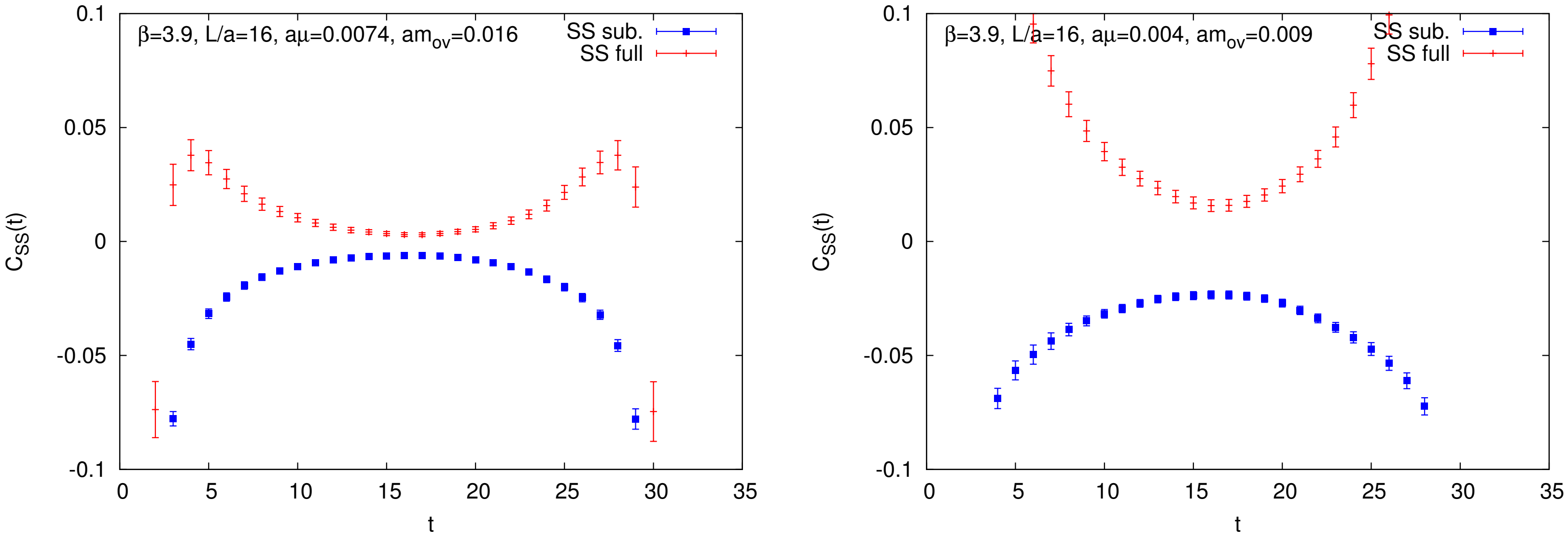
We want the combination $r_0^6 W_0^2 (W_M + 2W_8')$, which in terms of the fitting parameter α (in lattice units: $a^3\alpha$) is given by:

$$r_0^6 W_0^2 (W_M + 2W_8') = \frac{1}{2} \left(\frac{r_0}{a} \right)^4 \left(\frac{L}{a} \right)^3 (a m_v)^2 (r_0 \hat{f})^2 (a^3 \alpha).$$

Explicit subtraction of zero modes

The comparison of the SS correlator computed from full propagators (SS full) and with explicitly subtracted zero modes (SS sub.).
The effect of zero modes is more pronounced for smaller quark mass.

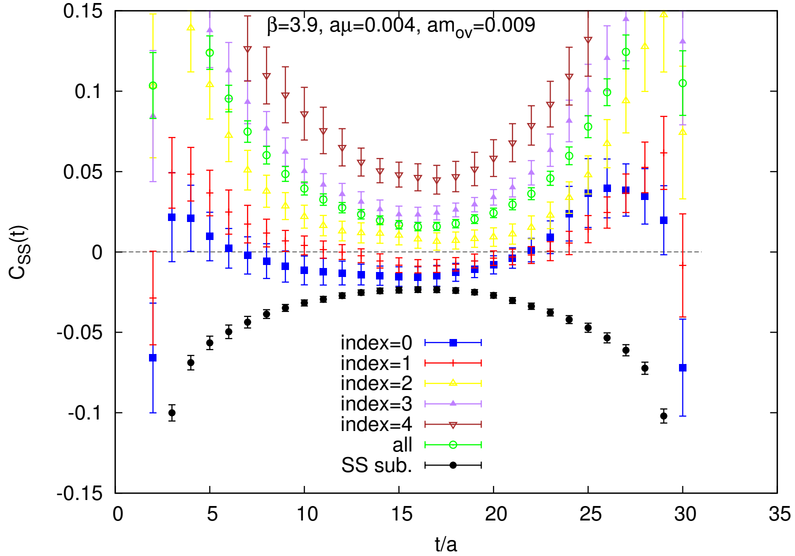
$\beta = 3.9,\ L/a = 16$
Left: $am_s = 0.0074,\ am_v = 0.016$
Right: $am_s = 0.004,\ am_v = 0.009$.

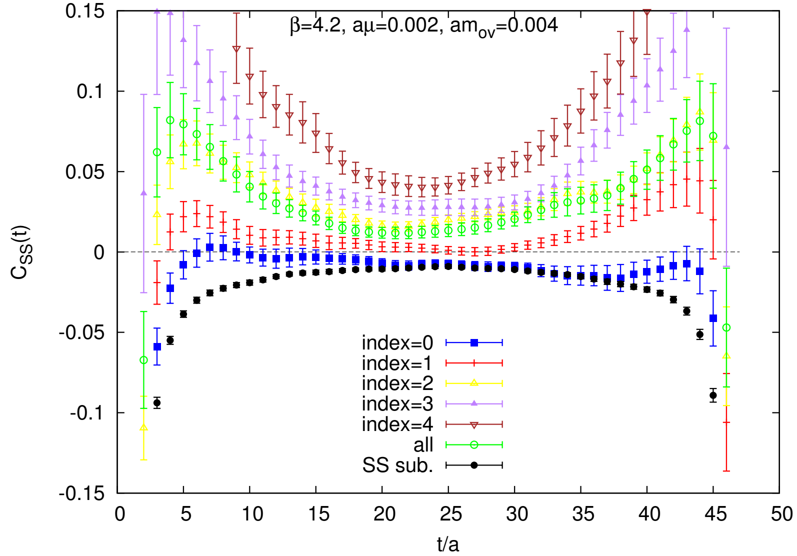


Dependence of the SS correlator on topology

The SS correlator from full propagators (index 0,1,2,3,4 and all configurations) and from propagators with explicitly subtracted zero modes (SS sub.).

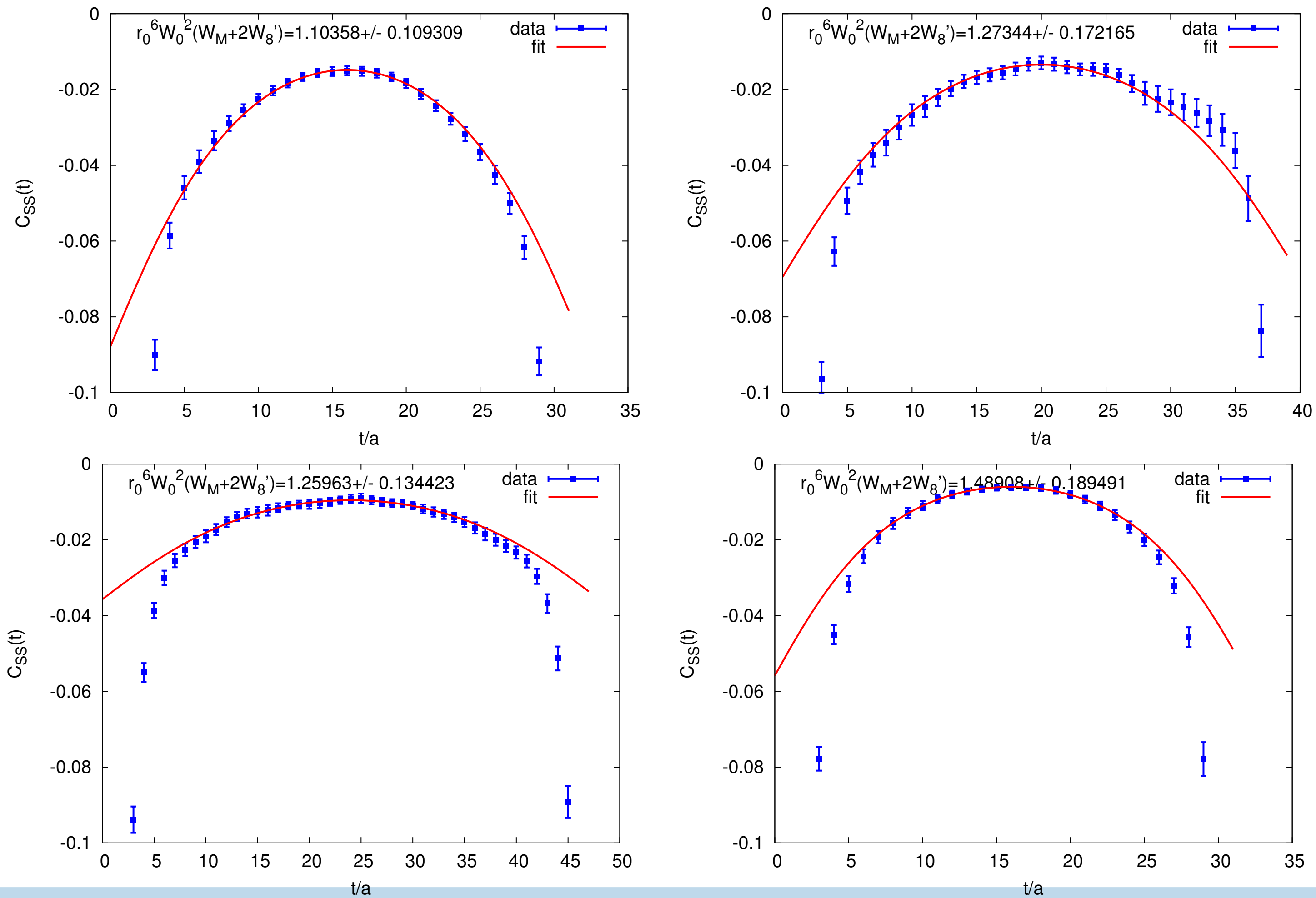
Left: $\beta = 3.9$, $L/a = 16$, $am_s = 0.004$, $am_v = 0.009$
Right: $\beta = 4.2$, $L/a = 24$, $am_s = 0.002$, $am_v = 0.004$.

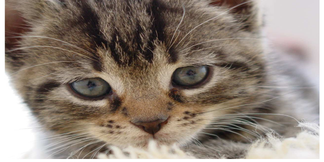




Extraction of $W_M + 2W_8'$

Upper left: $\beta = 3.9$, $a\mu = 0.004$. Upper right: $\beta = 4.05$, $a\mu = 0.003$. Lower left: $\beta = 4.2$, $a\mu = 0.002$. Lower right: $\beta = 3.9$, $a\mu = 0.0074$.





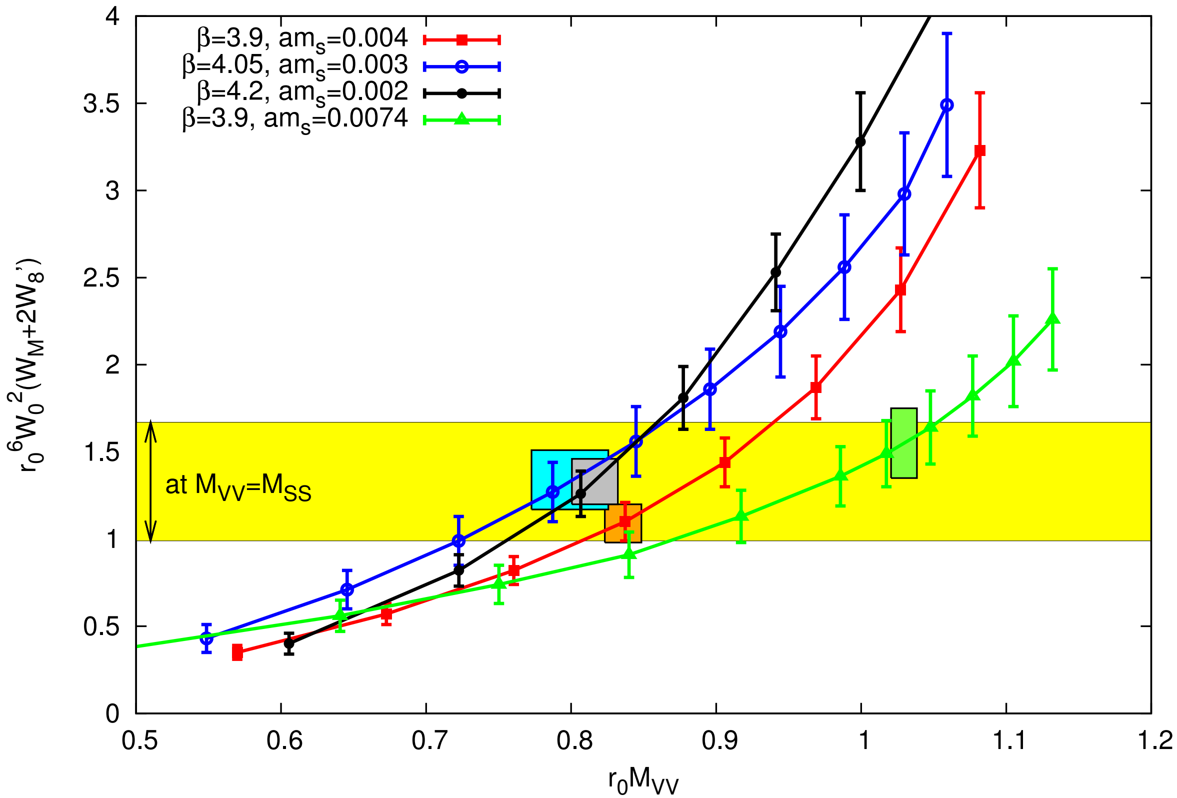
Extraction of $W_M + 2W_8'$

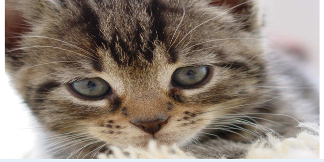
β	$\left(r_0^6 W_0^2\right)\left(W_M + 2W_8'\right)$							
	$am_v^{PPsub.}$	–	am_v^{PP-SS}	–	am_v^{PP}	–	am_v	–
3.9	0.011	1.10(11)	0.011	1.10(11)	0.007	0.57(6)	0.009	0.82(8)
4.05	0.007	1.56(20)	0.006	1.27(17)	0.005	0.99(14)	0.006	0.99(14)
4.2	0.005	1.81(18)	0.004	1.26(13)	0.002	0.40(6)	0.003	0.82(9)
3.9h	0.018	1.82(23)	0.016	1.49(19)	0.015	1.36(17)	0.017	1.64(21)

The results are compatible from all ensembles at the PP-SS matching mass.

Extraction of $W_M + 2W_8'$

The comparison of the pion mass dependence of $(r_0^6 W_0^2)(W_M + 2W_8')$ for all 4 ensembles. The rectangles are centered at the matching mass for each ensemble (their width = error on M_{SS}) and at an appropriate curve (their height = error on $(r_0^6 W_0^2)(W_M + 2W_8')$).





Alternative extraction of W_8'

Alternatively, one can estimate the LEC W_8' directly from:

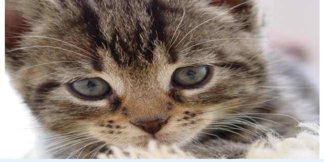
$$\begin{aligned} C_{sca}^{VV}(t) &= 2\left(M_{\pm}^2 - M_{VV}^2 + \hat{a}^2\frac{16}{f^2}W_8'\right)B_{DP}(T,L,t,M_{VV},M_{VV}) + \\ &\quad -2B_{SP}(T,L,t,M_{VV},M_{VV}) + 2B_{SP}(T,L,t,M_{VS},M_{VS}), \end{aligned}$$

where B_{SP} and B_{DP} are the single-pole and double-pole bubble functions. The fitting ansatz is:

$$\begin{aligned} C_{sca}^{VV}(t) &= 2\left(M_{\pm}^2 - M_{VV}^2 + \xi\right)B_{DP}(T,L,t,M_{VV},M_{VV}) + \\ &\quad -2B_{SP}(T,L,t,M_{VV},M_{VV}) + 2B_{SP}(T,L,t,M_{VS},M_{VS}), \end{aligned}$$

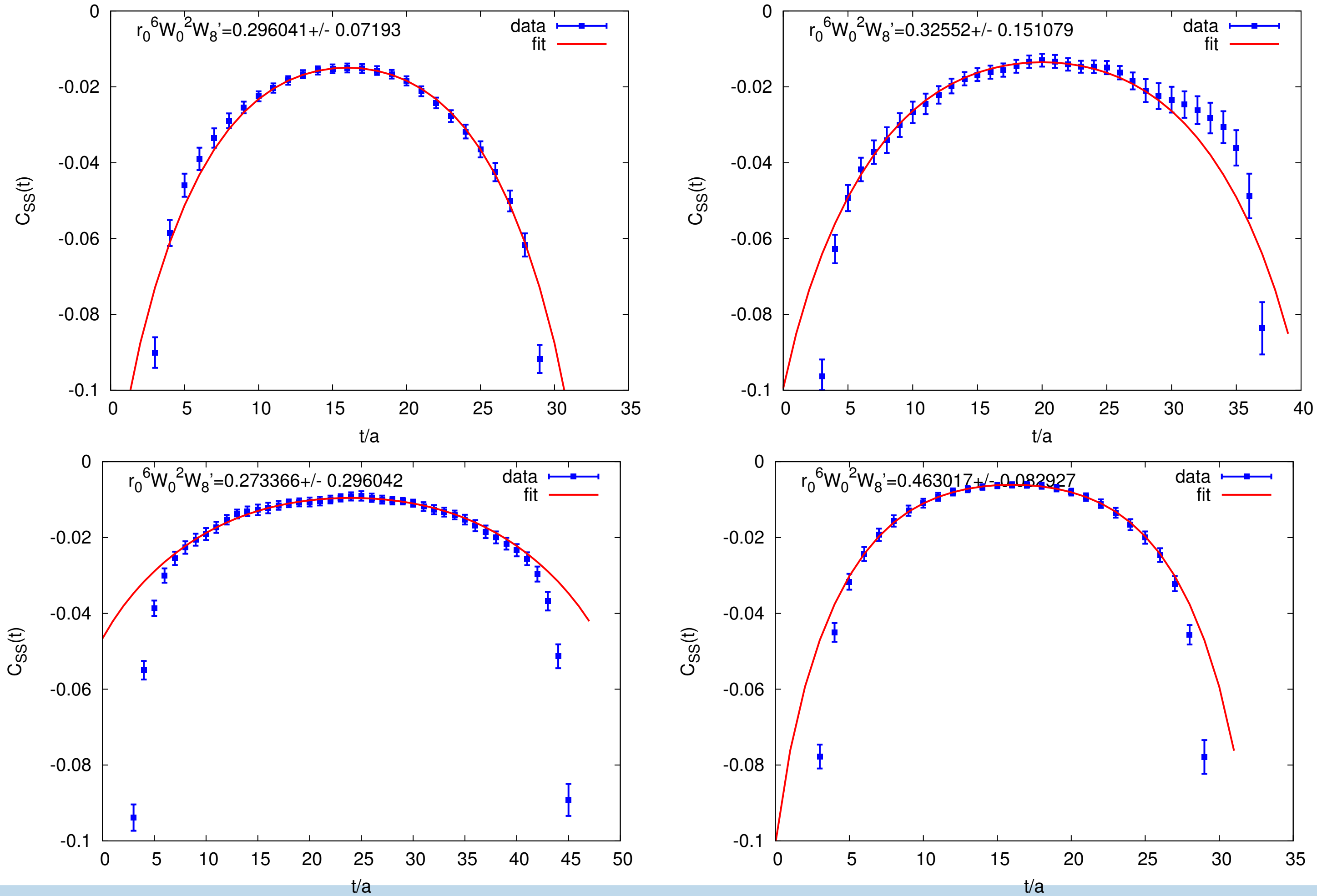
where the fitting parameters are ξ and M_{VV}. Moreover, the bubble functions contain the LEC B_0, which can be eliminated by setting $B_0 = M_{VV}^2/2m_v$.
The LEC $r_0^6 W_0^2 W_8'$ is thus related to ξ:

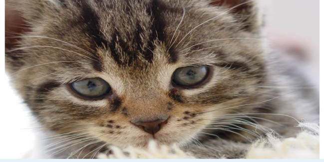
$$r_0^6 W_0^2 W_8' = \frac{1}{128}(a^2\xi)(r_0\hat{f})^2\left(\frac{r_0}{a}\right)^4.$$



Alternative extraction of W_8'

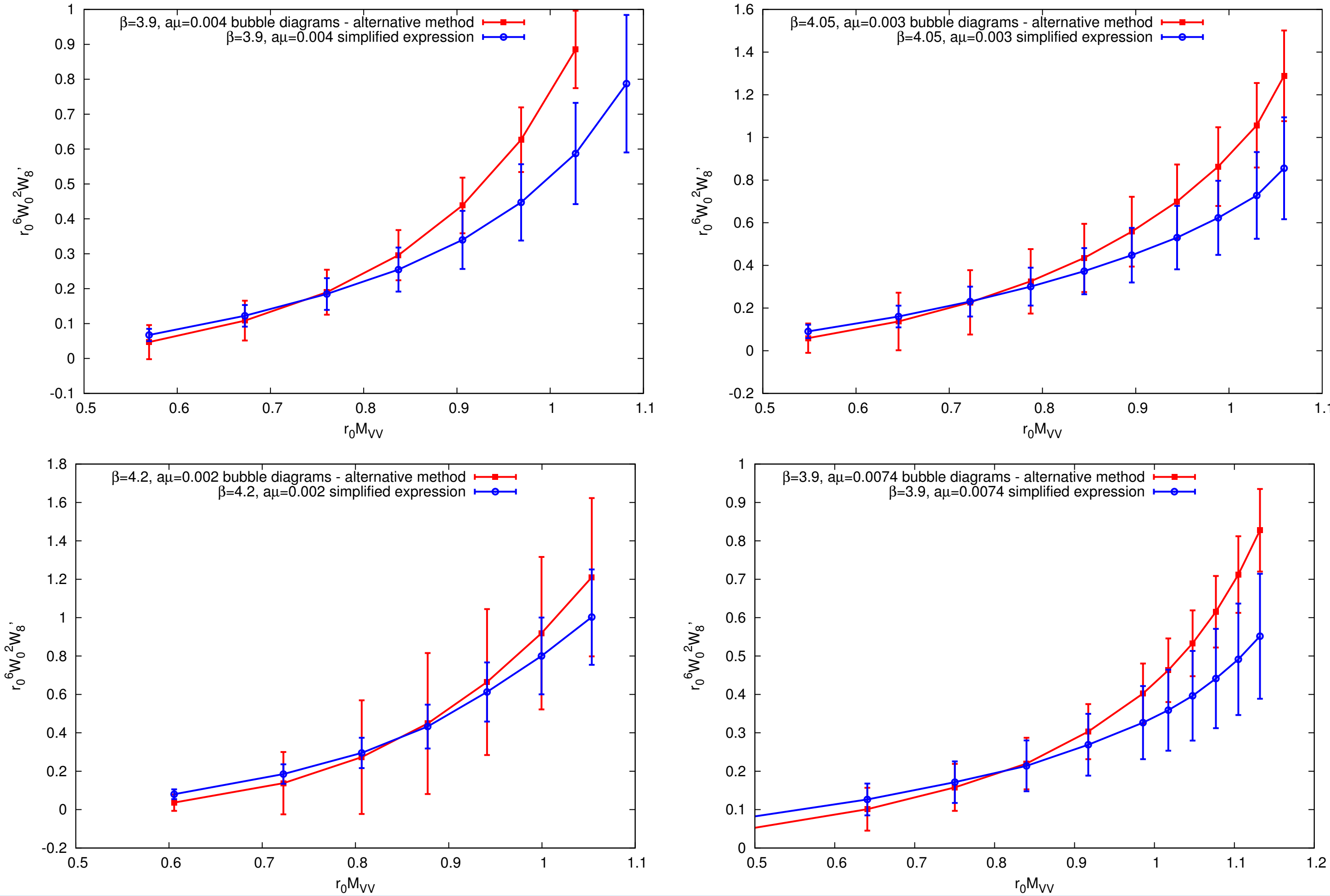
Upper left: $\beta = 3.9$, $a\mu = 0.004$. Upper right: $\beta = 4.05$, $a\mu = 0.003$.
Lower left: $\beta = 4.2$, $a\mu = 0.002$. Lower right: $\beta = 3.9$, $a\mu = 0.0074$.



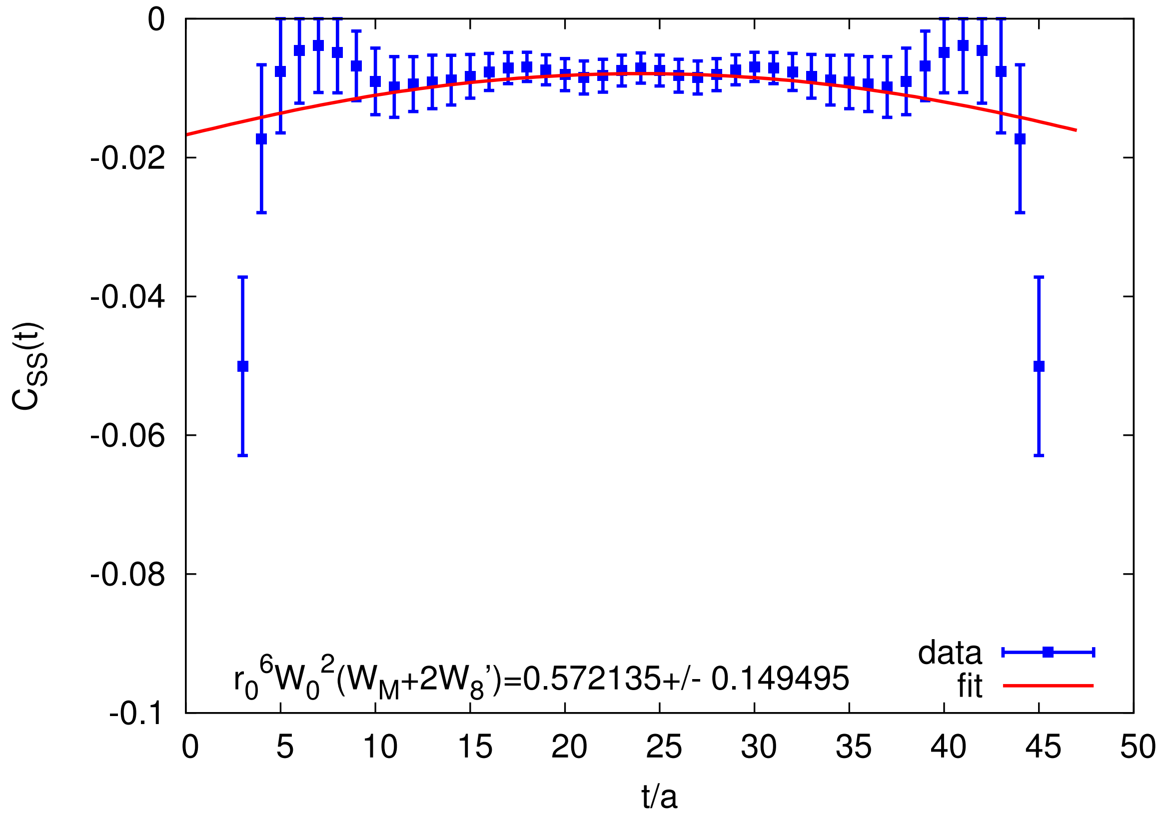


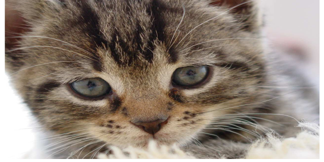
Alternative extraction of W_8'

Upper left: $\beta = 3.9$, $a\mu = 0.004$. Upper right: $\beta = 4.05$, $a\mu = 0.003$. Lower left: $\beta = 4.2$, $a\mu = 0.002$. Lower right: $\beta = 3.9$, $a\mu = 0.0074$.



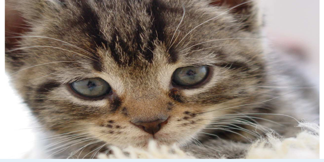
The determination of $(r_0^6 W_0^2)(W_M + 2W_8')$ for $\beta = 4.2$, $L/a = 24$, $a\mu = 0.002$, $am_v = 0.004$ (PP-SS matching mass)





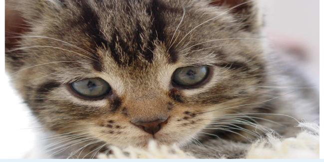
Summary – LECs

LEC	$\beta = 3.9$	$\beta = 4.05$	$\beta = 4.2$	$\beta = 3.9h$	all
$\left(r_0^6 W_0^2\right)(W_M - 2W_8')$	0.081(12)	0.068(11)	0.079(13)	0.054(9)	0.068(15)
$\left(r_0^6 W_0^2\right)(W_M + 2W_8')$	1.10(11)	1.27(17)	1.26(13)	1.49(19)	1.33(34)
$\left(r_0^6 W_0^2\right)(W_8' + 2W_6')$	0.0026(7)	0.0027(11)	–	–	0.0026(11)
$\left(r_0^6 W_0^2\right) W_M$	0.59(6)	0.67(9)	0.67(7)	0.77(10)	0.70(18)
$\left(r_0^6 W_0^2\right) W_8'$	0.255(31)	0.301(45)	0.295(36)	0.359(50)	0.31(9)
$\left(r_0^6 W_0^2\right) W_6'$	-0.126(16)	-0.149(23)	–	–	-0.14(5)
C_{Mix} [MeV]	1000(40)	950(40)	970(50)	900(60)	960(60)
$r_0^4 c_2$	2.5(6)	2.6(1.0)	–	–	2.5(1.0)



Conclusions

- The new small-volume ensemble at $\beta = 4.35$ confirms our conclusions about the role of the zero modes in the mixed action setup of overlap valence and MTM sea quarks.
- We have estimated the parameters for which one seems to be safe against the effects of the zero modes:
  - ⋆ $L \approx 2.6$ fm at $m_\pi \approx 300$ MeV,
  - ⋆ $L \approx 1.8$ fm at $m_\pi \approx 450$ MeV.
- We have extracted the value of the mixed action χPT LECs W_M and W_8' and their combination $C_{Mix} = 960(60)$ MeV.



Thank you for your attention!