Pseudoscalar flavor-singlet mesons from 2+1+1 tmLQCD

C. Michael, K. Ottnad, C. Urbach, F. Zimmermann

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Bonn-Cologne Graduate School of Physics and Astronomy

Overview

- Want to calculate properties of η, η'-mesons using 2+1+1 dynamic quark flavors
- Determine m_η, $m_{\eta'}$ (and m_{η_c} ?)
- Want to study quark mass dependence
- Extract the corresponding flavor contents of the states
- Check if the c-quark gives any contribution
- Study mixing angles
- Topological quantities (Q_{top}, χ_{top}) are also accessible via disc loops
- Additionally calculated m_{π^0} for each run; results are on the wiki

We work in the 2+1+1 unitary setup:

$$\mathcal{S}_{F,l}[U,\chi_l,\bar{\chi}_l] = a^4 \sum_x \bar{\chi}_l \left(D_W + m_0 + i\mu_l \gamma_5 \tau^3\right) \chi_l$$

$$\mathcal{S}_{F,h}[U,\chi_h,\bar{\chi}_h] = a^4 \sum_x \bar{\chi}_h \left(D_W + m_0 + i\mu_\sigma \gamma_5 \tau^1 + \mu_\delta \tau^3\right) \chi_h \ .$$

- strange and charm quark masses are given by

$$m_{c,s} = \mu_\sigma \pm \frac{Z_P}{Z_S} \mu_\delta$$

- Heavy sector is NOT flavor-diagonal → two additional progagators G_{cs}^{xy} G_{sc}^{xy}
- No (simple) „γ_5-trick“, instead

$$\begin{pmatrix} G_{cc}^{xy} & G_{sc}^{xy} \\ G_{cs}^{xy} & G_{ss}^{xy} \end{pmatrix} = \begin{pmatrix} \gamma_5 \left(G_{cc}^{yx}\right)^\dagger \gamma_5 & -\gamma_5 \left(G_{cs}^{yx}\right)^\dagger \gamma_5 \\ -\gamma_5 \left(G_{sc}^{yx}\right)^\dagger \gamma_5 & \gamma_5 \left(G_{ss}^{yx}\right)^\dagger \gamma_5 \end{pmatrix}$$

⇒ Heavy sector requires a much larger number of contractions for correlation functions

Interpolating operators for η, η'

In the physical basis 2 γ-combinations ($i\gamma_5$, $i\gamma_0\gamma_5$) available; consider only $i\gamma_5$:

$$\eta_l^{phys} = \frac{1}{\sqrt{2}}\bar{\psi}_l i\gamma_5 \psi_l \quad \eta_{c,s}^{phys} = \bar{\psi}_h \left(\frac{1\pm\tau^3}{2} i\gamma_5\right)\psi_h = \begin{cases} \bar{c} i\gamma_5 c \\ \bar{s} i\gamma_5 s \end{cases} .$$

At maximal twist this reads in the twisted basis:

$$\eta_l^{tm} = \frac{1}{\sqrt{2}}\bar{\chi}_l\left(-\tau^3\right)\chi_l \quad \eta_{c,s}^{tm} = \frac{1}{2}\bar{\chi}_h\left(-\tau^1 \pm i\gamma_5\tau^3\right)\chi_h .$$

$\Rightarrow$ heavy operators are a sum of scalars and pseudoscalars!

Considering renormalization we have

$$\eta_{c,renormalized}^{tm} = Z_P\left(\bar{\chi}_c i\gamma_5\chi_c - \bar{\chi}_s i\gamma_5\chi_s\right) - Z_S\left(\bar{\chi}_s\chi_c + \bar{\chi}_c\chi_s\right)$$
$$\eta_{s,renormalized}^{tm} = Z_P\left(\bar{\chi}_s i\gamma_5\chi_s - \bar{\chi}_c i\gamma_5\chi_c\right) - Z_S\left(\bar{\chi}_s\chi_c + \bar{\chi}_c\chi_s\right) .$$

$\rightarrow$ Need $\frac{Z_P}{Z_S}$; how can we avoid this when calculating masses?

Correlation function matrix for η, η'

Choose different set of „heavy“ operators

$$\eta_{S,P} = \eta_c^{tm} \pm \eta_s^{tm} = \begin{cases} \frac{1}{\sqrt{2}}\left(\bar{\chi}_c\chi_s + \bar{\chi}_s\chi_c\right) \\ \frac{1}{\sqrt{2}}\left(\bar{\chi}_c i\gamma_5\chi_c - \bar{\chi}_s i\gamma_5\chi_s\right) \end{cases} .$$

$\Rightarrow$ This corresponds to an additional rotation of the basis.

In the twisted basis we have to calculate this correlation matrix:

$$\mathcal{C}^\eta(t) = \begin{pmatrix} \eta_l(t)\eta_l(0) & \eta_l(t)\eta_S(0) & \eta_l(t)\eta_P(0) \\ \eta_S(t)\eta_l(0) & \eta_S(t)\eta_S(0) & \eta_S(t)\eta_P(0) \\ \eta_P(t)\eta_l(0) & \eta_P(t)\eta_S(0) & \eta_P(t)\eta_P(0) \end{pmatrix} .$$

- in our basis the vectors corresponding to pure $SU(4)$ symmetric states are

$$e_{ll} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e_{ss} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}, \quad e_{cc} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} .$$

- η and η' are not pure states but will be mixtures
- Eigenvectors of $\mathcal{C}^\eta(t)$ are a linear combination of these basis vectors
- Eigenvalues allow to extract masses for η and η'

Generalized eigenvalue problem

Use of N operators allows to extract N excited states:

$$C^{\eta}_{ij}(t) \simeq \sum_{n=0}^{N-1} \phi_i^{(n)} \exp\left(-E_n t\right) \left(\phi_j^{(n)}\right)^* , \qquad \phi_i^{(n)} = \langle 0 | \eta_i | n \rangle \ .$$

For $\eta_i = \eta_i^\dagger$, $C^{\eta}_{ij} = \left(C^{\eta}_{ij}\right)^\dagger$ one has to solve a generalized eigenvalue problem:

$$\mathcal{C}^{\eta}(t)\phi^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0)\, C^{\eta}(t_0)\phi^{(n)}(t,t_0) \ ,$$

where $\phi^{(n)}$ is the eigenvector corresponding to n-th state.

Masses are either obtained via (multi-state) cosh-fit from eigenvalues

$$\lambda^{(n)}(t,t_0) = \exp\left(-m^{(n)}t\right) - \exp\left(-m^{(n)}(T-t)\right) \ ,$$

or from

$$\frac{\lambda^{(n)}(t,t_0)}{\lambda^{(n)}(t+1,t_0)} = \frac{\exp\left(-m^{(n)}t\right) - \exp\left(-m^{(n)}(T-t)\right)}{\exp\left(-m^{(n)}(t+1)\right) - \exp\left(-m^{(n)}(T-(t+1))\right)} \ .$$

Flavor contents and mixing angle

Flavor contents of the states are given by

$$c_l^{(k)} = \frac{1}{N^{(n)}}\phi_0^{(n)} \; , \quad c_s^{(n)} = \frac{1}{N^{(n)}}\left(-\frac{Z_S}{Z_P}\phi_1^{(n)} + \phi_2^{(n)}\right) \; , \quad c_c^{(n)} = \frac{1}{N^{(n)}}\left(\cdot\frac{Z_S}{Z_P}\phi_1^{(n)} - \phi_2^{(n)}\right) \; ,$$

where

$$N^{(n)} = \sqrt{\left(\phi_0^{(n)}\right)^2 + \left(\frac{Z_S}{Z_P}\phi_1^{(n)}\right)^2 + \left(\phi_2^{(n)}\right)^2} \; .$$

- This information allows to define a mixing matrix for physical and pure flavor singlet states
- Neglecting charm contribution the mixing matrix with one angle reads

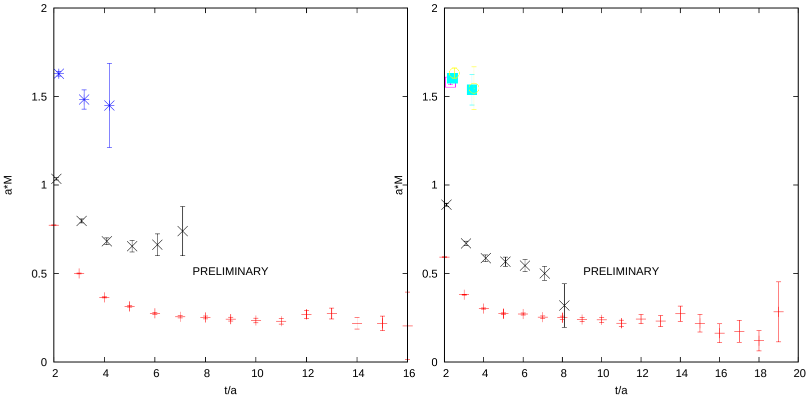
$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} |ll\rangle \\ |ss\rangle \end{pmatrix}$$

Setup

We used the following setup:

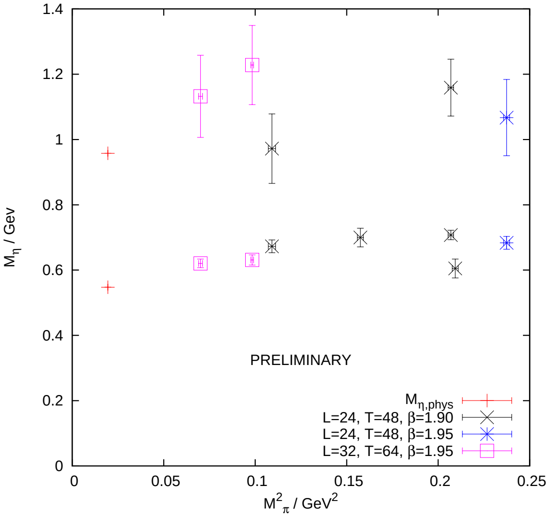
- For the moment we use 7 ensembles: B25.32, B35.32, B85.24, A40.24, A60.24, A80.24, A80.24s
- Also want to use the remaining B- and all D-ensembles (and as many of the Axx.32 ensembles as possible)
- Two different lattice sizes, i.e. $48x24^3$ and $64x32^3$
- Two different lattice spacings corresponding to $\beta = 1.90$ and $\beta = 1.95$
- Three different volumes
- Charged pion masses range from $\approx 230\,\mathrm{MeV}$ to $\approx 460\,\mathrm{MeV}$

Masses from GEVP



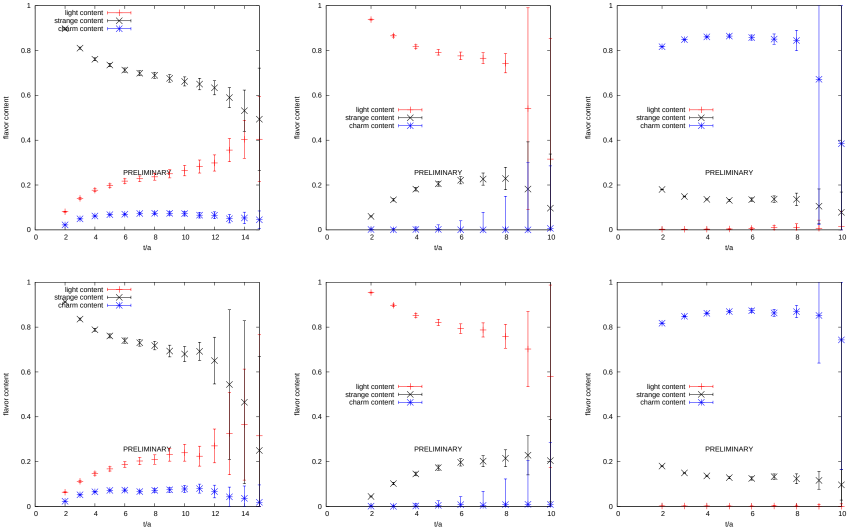
Masses from GEVP for 3x3 and 6x6 correlation function matrix (B35.32 ensemble)

Masses for η, η'



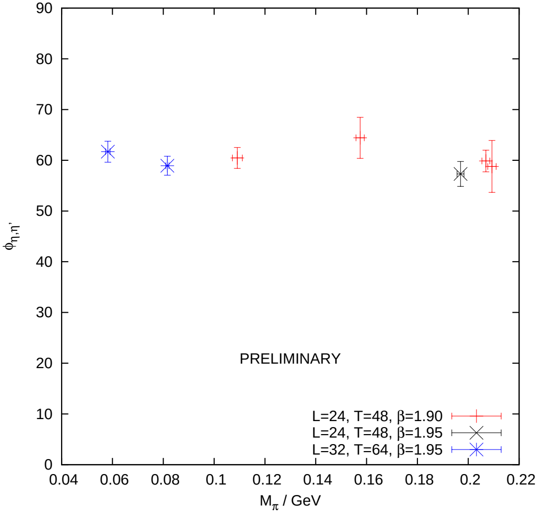
- $m_\eta \approx 0.606 \ldots 0.777\,\mathrm{GeV}$ comes out larger than the phys. value $m_{\eta,phys} = 0.548\,\mathrm{GeV}$, as expected
- The quark mass dependence points in the right direction
- The η appears to be sensitive to the strange quark mass
- No large dependence on lattice spacing / volume
- η'-signal still noisy; plateau fit fails in most cases

Flavor contents



Flavor contents for η, η' and η_c (?) from B35.32 (top) and A80.24 (bottom) ensemble

$SU(3)_f$-mixing angle



Mixing angle between physical and pure $SU(3)_f$-symmetric states

Topological Quantities

Can also use disc loops to calculated topological quantities; via fermionic definition of topological charge Q_{top} (in phys. basis):

$$Q_{top} = \int d^4x \rho(x) \ , \qquad \rho(x) = \frac{1}{N_f} \sum_{f=1}^{N_f} m_f \bar{\psi}_f \gamma_5 \psi_f$$

For the topological susceptibility we have

$$\chi_{top} = \frac{\langle Q_{top}^2 \rangle}{V}$$

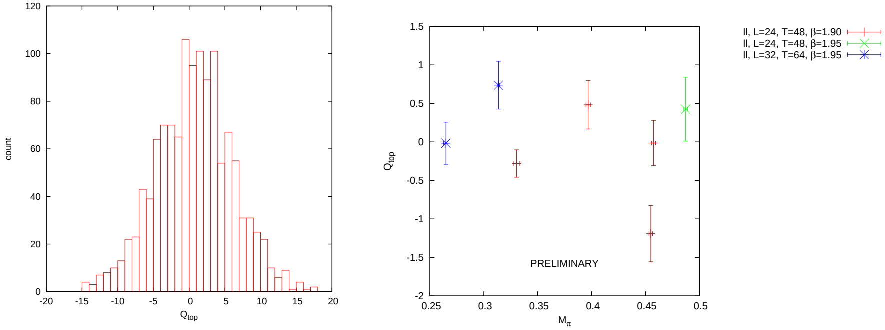
From $SU(2)$-χPT one expects

$$\chi_{top,SU(2)} = \frac{m_l \Sigma_{SU(2)}}{2} + \mathcal{O}\left(m_l^2\right)$$

where $\Sigma_{SU(2)} = \langle \bar{u}u + \bar{d}d \rangle$ is the $SU(2)$-chiral condensate. For $N_f = 4$:

$$\chi_{top} = \frac{\Sigma}{\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s} + \frac{1}{m_c}} + \mathcal{O}\left(m^2\right) \quad (?)$$

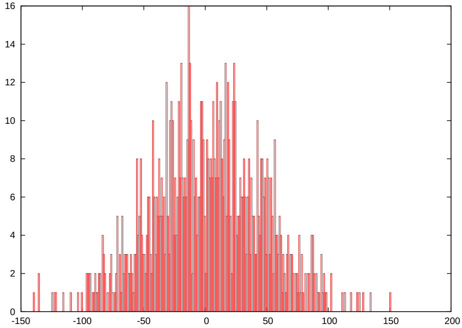
Topological Charge (1)

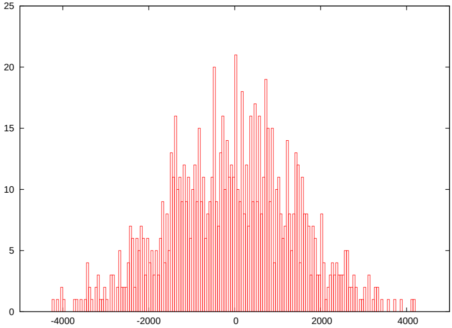


$Q_{top,l}$-histogram (binwidth=1) for B35.32 (left) and $Q_{top,l}$ for all ensembles (right)

- $Q_{top,f}$ is Gaussian-distributed around zero, as expected
- the width ($\sim \chi_{top}$) increases with larger quark mass

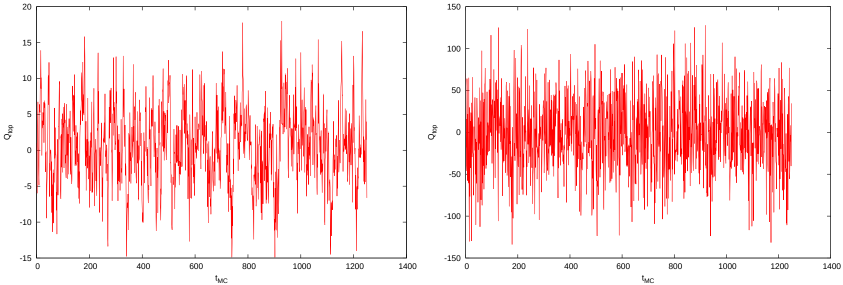
Topological Charge (2)





$Q_{top,s}$-histogram (binwidth=1) (left) and $Q_{top,c}$-histogram (binwidth=50) for B35.32 (right)

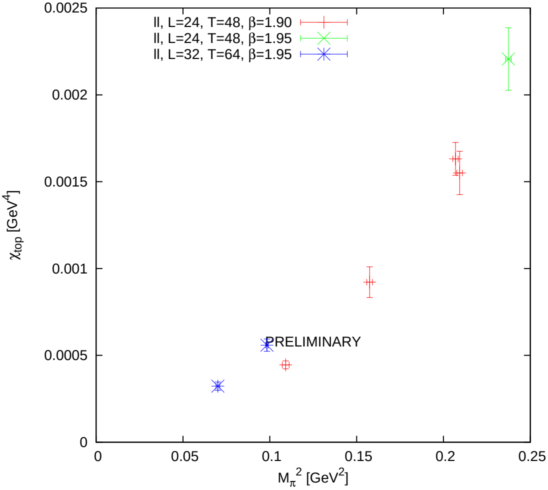
MC History and autocorrelation



Monte-Carlo Histories for light (left) and strange (right) for B35.32

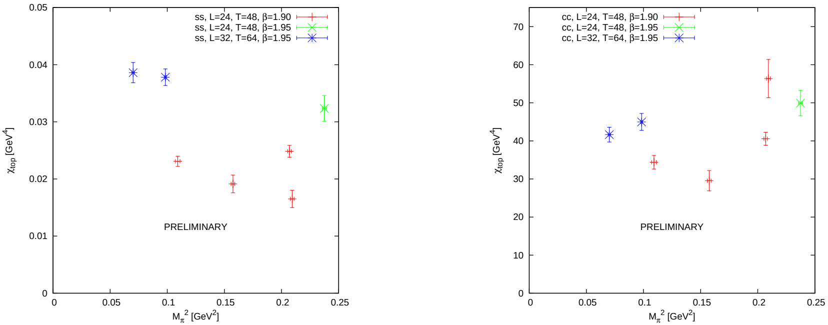
Label	B25.32	B35.32	B85.24	A40.24	A60.24	A80.24	A80.24s
ΔN	2	2	5	2	10	2	10
$\tau_{int,l}$	3.03(69)	3.21(66)	1.34(28)	0.97(24)	0.72(15)	2.83(58)	0.505(89)
$\tau_{int,s}$	0.542(60)	0.582(56)	0.526(62)	0.61(13)	0.467(60)	0.524(42)	0.475(60)
$\tau_{int,c}$	1.65(30)	1.61(26)	0.92(16)	0.428(82)	0.49(11)	1.73(29)	0.400(73)

Topological susceptibility (1)



- $\chi_{top,l}$ shows clear dependence on $M_\pi^2 \sim m_l$
- assuming linear behavior of $\chi_{top,l}(m_l)$ seems not compatible with $\chi_{top,l}(0) = 0$

Topological susceptibility (2)

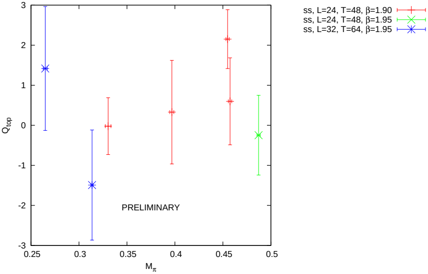


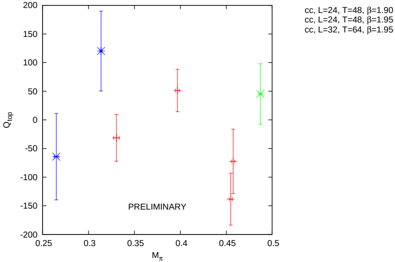
$\chi_{top,s}$ (left), $\chi_{top,c}$ (right) as function of M_π^2

- (almost) no dependence on $M_\pi^2 \sim m_l$
- χ_{top} dominated by charm contribution; plot looks basically the same

Summary and Outlook

- Results for m_η look reasonable, statistical errors are already small
- Need to increase operator basis to obtain better signal for η'
- Can also extract flavor contents and (in principle) give estimates for mixing angles
- Still increasing statistics
- Calculations for more ensembles, larger volumes
- Alternative ansatz: mixed action setup
- Also calculate top. quantities from gauge field definition $\sim G_{\mu\nu}\tilde{G}^{\mu\nu} \rightarrow$ Falk
- Compare results for different definitions for Q_{top}, χ_{top}





$Q_{top,s}$ and $Q_{top,c}$ for all ensembles