A Novel Method for Nucleon Matrix Element Calculations

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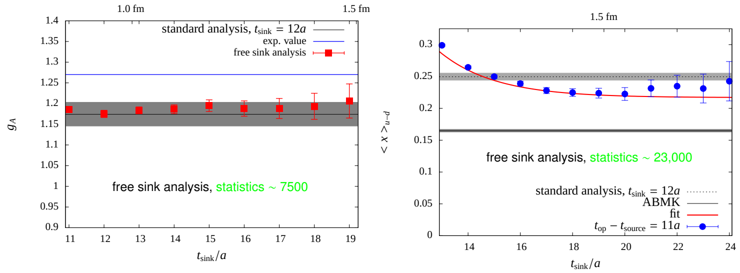
My talk at Lattice'11 in a nutshell

- ∃ some excited state contribution (not necessarily for all matrix elements)

Source-sink separation dependence of g_A and $\langle x \rangle_{u-d}$

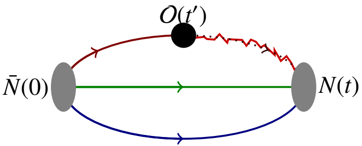


- hint from ratios of matrix elements (e.g. $\frac{\langle \mathbf{x} \rangle_{u-d}}{\langle \mathbf{x} \rangle_{u+d-2s}}$):
  Renormalization might be an issue as well
- GEVP most likely not useful (or not feasible) for elimination of excited state contribution

What to do now?

do not want to blindly continue standard program

idea: explore possibility of evaluating the 3-point functions stochastically



contraction pattern (spin structure suppressed):

$$C_3(t = t_f - t_i, t') \sim \sum_{\vec{x}_f, \vec{x}'} S(x_f, x_i) S(x_f, x_i) S(x_f, x') \mathscr{O}(x') S(x', x_i)$$

$$S(x_f, x') \rightarrow (\eta(x_f)^\dagger \phi(x'))^\dagger$$

$$\Rightarrow C_3(t = t_f - t_i, t') \sim \sum_{\vec{x}_f} [S(x_f, x_i) S(x_f, x_i) \eta(x_f)] \sum_{\vec{x}'} \left[\phi(x')^\dagger \mathscr{O}(x') S(x', x_i)\right]$$

Why would we do that?

Pros:

- dispose of sequential inversion
- Dirac structure of the nucleon for free (“free projector”)
- free sink momentum
- stochastic propagators can be reused
- (stochastic timeslice sources)
  new source-sink separation only requires new forward inversions
  (*i.e.* move source relative to sink)
- also works with the free sink method

Cons:

- stochastic noise may be substantial
- volume dependence?

- idea basically by Dru (whithin ETMC), but has as been employed in semileptonic form factor calculations
  (Ewans, Bali, Collins; arXiv:1008.3293v1 [hep-lat])
  - charm propagator estimated stochastically
  - stochastic method competitive and sometimes superior to sequential method
  - stochastic sources with support on one timeslice only
- QCDSF is exploring the use of stochastic calculation of nucleon matrix elements as well
  (hint from A. Sternbeck in his Lattice'11 talk)

A few words on sink momentum

consider generic matrix element

$$\langle N(p', s')|\mathcal{O}|N(p, s)\rangle \sim f(q^2)[\cdots] + \cdots \quad (1)$$

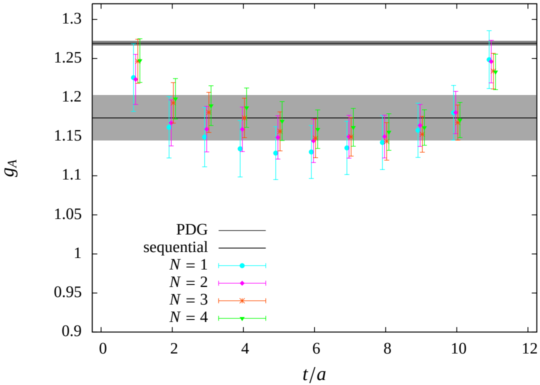
- **sequential method**: vary q^2 via source momentum p (*i.e.* through momentum projection at the operator insertion), new sequential inversions needed for each new sink momentum p'
- **stochastic method**: can in principle take any combination of source and sink momentum p, p' that gives the same q^2 to obtain $f(q^2)$
  - significant gain in statistics
  - but: noise typically increases with momentum

Test setup

- Ensemble B55.32 ($N_f = 2 + 1 + 1$, $L = 2.5$ fm, $a = 0.078$ fm, $m_\pi = 380$ MeV)
- spin-colour diluted timeslice sources (stochastic inversion is as expensive as for point source)
- $\gtrsim 500$ gauge field configurations
- check simple observables first ($q^2 = 0$): g_A, $< x >_{u-d}$
- in progress: non-zero momentum transfer, $q^2 \neq 0$

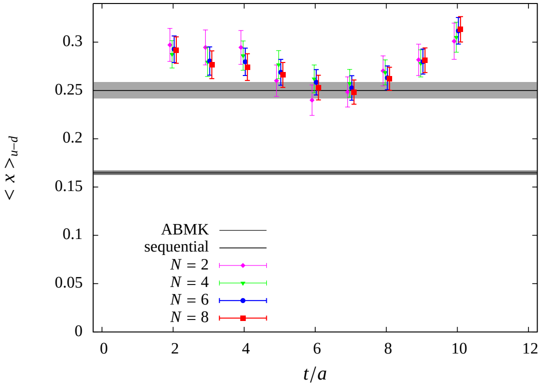
g_A: competitiveness of the stochastic method

- nota bene: effort $N = 1 \leftrightarrow$ sequential method with one projector
- stochastic method: correlators for 3 operators, p and n
  $\Rightarrow$ effectively boost statistics by factor ~ 6



$< x >_{u-d}$: competitiveness of the stochastic method

- stochastic method: correlators for p and n
  $\Rightarrow$ effectively boost statistics by factor ~ 2



Summary and Discussion

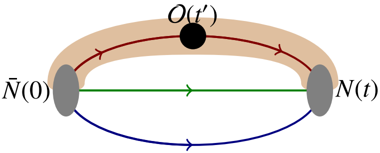
- tested stochastic method for nucleon matrix elements on simple observables
  $\Rightarrow \mathcal{O}(10)$ spin-color diluted sources needed to reach statistical accuracy of sequential method
- on the other hand more flexibility due to free projector and sink momentum
- dilution?
- lower inverter precision?
- volume dependence of signal-to-noise ratio?
- combination with free sink method for better control of excited states

Backup Slides

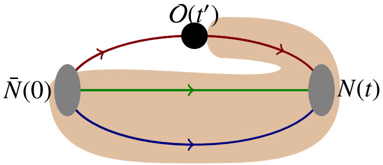
Free Sink Method

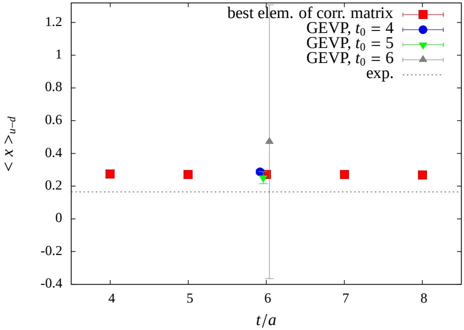
- want to study dependence of matrix elements on source-sink separation t
- free sink method: do not fix t, but insertion timeslice t' and operator $\mathcal{O}$
- drawback: new inversions needed for each operator in contrast to in the standard method

free sink method



standard method
(sequential method through the sink)





- 3×3 correlation matrix using different (Gaussian) smearing and 500 measurements statistics at source-sink separation $12a$ (0.95 fm)
- $\Rightarrow$ no improvement (yet), i.e. no effect seen!
- may need larger matrix, or different interpolating field for nucleon (very noisy)
- work in progress: free sink method using two different local interpolating fields (does not seem feasible . . .)

Local interpolating fields for nucleon (here proton):

$$\chi_\alpha^{(1)} = \varepsilon^{abc} u_\alpha^a \left(d^{b^T} \mathscr{C} \gamma_5 u^c \right)$$

$$\chi_\alpha^{(2)} = \varepsilon^{abc} \left(u^a \gamma_5 \right)_\alpha \left(d^{b^T} \mathscr{C} u^c \right) \qquad \mathscr{C} = i\gamma_2\gamma_0$$

GEVP analysis of 2-point