Nucleon matrix elements and disconnected diagrams

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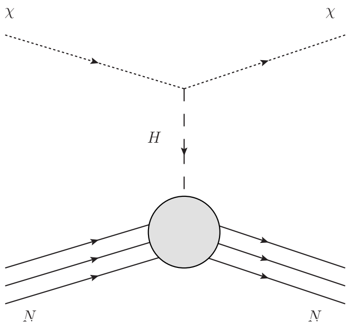
Outline

Strange content of the Nucleon : Motivations

Motivations

- Experimental direct detection of dark matter put bounds on the WIMP-Nucleon cross section
- Results are interpreted using various models (including SUSY) : systematic uncertainty due to $\langle N(p)|\bar{q}q|N(p)\rangle$
  $\longrightarrow$ non-perturbative computation is required.



- sigma term :
  $$\sigma_{\pi N} \equiv m_l \langle N(p)|\bar{u}u + \bar{d}d|N(p)\rangle$$
- dimensionless ratio :
  $$y_N \equiv \frac{2\langle N(p)|\bar{s}s|N(p)\rangle}{\langle N(p)|\bar{u}u+\bar{d}d|N(p)\rangle}$$
- dimensionless coupling :
  $$f_{Tq} = \frac{m_q}{m_N}\langle N|\bar{q}q|N\rangle$$

Lattice setup (I)

$N_f = 2 + 1 + 1$ dynamical simulations

- $N_f = 2 + 1 + 1$ configurations generated by ETMC
- Fixed lattice spacing $a = 0.078$ fm
- $32^3 \times 64$ lattices : $L = 2.5$ fm
- $m_{\mathrm{PS}} L > 3.5$
- 3 pion masses $[300, 450]$ MeV

Mixed action setup

- Mixed action setup : introduce a doublet of twisted mass fermions χ_q with a mass μ_q
- μ_q can be tuned to reproduce the K, D meson masses in the unitary setup
- Multimass solver inversions from the light to the charm quark mass

Lattice setup (II)

- Rotation to the physical basis :

$$\overline{\chi}_q \gamma_5 \tau^3 \chi_q \longrightarrow \bar{q}q + \bar{q}'q' = \begin{cases} \bar{u}u + \bar{d}d & \text{if } q = u, d \\ \bar{s}s + \bar{s}'s' & \text{if } q = s \\ \bar{c}c + \bar{c}'c' & \text{if } q = c \end{cases}$$

- Three-points function definition (disconnected part only):

$$R(t_s, t_{\rm op}) = \frac{\langle C_{\rm 2pt}(t_s) D(t_{\rm op})\rangle - \langle C_{\rm 2pts}(t_s)\rangle \langle D(t_{\rm op})\rangle}{C_{\rm 2pts}(t_s)}$$

- Connected pieces : fixed source-sink separation $t_s/a = 12$

Renormalization

- Discretization that break chiral symmetry suffer from mixing under renormalization of the light and strange content
- Twisted mass property in the mixed action setup : cancellation of chiral violating effects between "loops" of valence quarks regularized with opposite values of r.

$$(\bar{s}s)_R = \frac{Z_P}{2}[(\bar{s}s)_{r=+1} + (\bar{s}'s')_{r=-1}]\,.$$

(R. Frezzotti, G. Herdoiza, G. Rossi)

- The quantity :

$$f_{T_s} = \frac{1}{2}\frac{\mu_s}{m_N}\langle N(p)|\bar{s}s + \bar{s}'s'|N(p)\rangle$$

is thus renormalization free.

Twisted mass fermions and disconnected diagrams

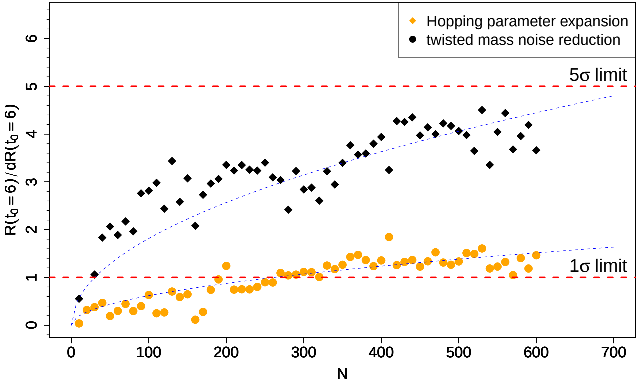
- Twisted mass Wilson Dirac operator : $D_{\pm}[U] = D_{\mathrm{W}}[U] + am_0 \pm ia\mu_q\gamma_5$
- Noise reduction technique specific for twisted mass fermions :
    - Exact relation : (C. Michael, C. Urbach, 2007)

$$\frac{1}{D_-} - \frac{1}{D_+} = 2ia\mu_q \frac{1}{D_-}\gamma_5\frac{1}{D_+}$$

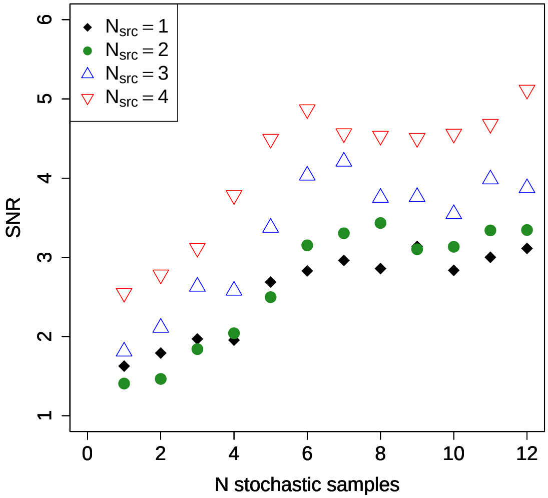
- r.h.s can be evaluated stochastically using "one-end-trick"
- Useful for correlators containing the insertion of the operator $\overline{\chi}(x)\Gamma\tau^3\chi(x)$ (twisted basis)

Performances

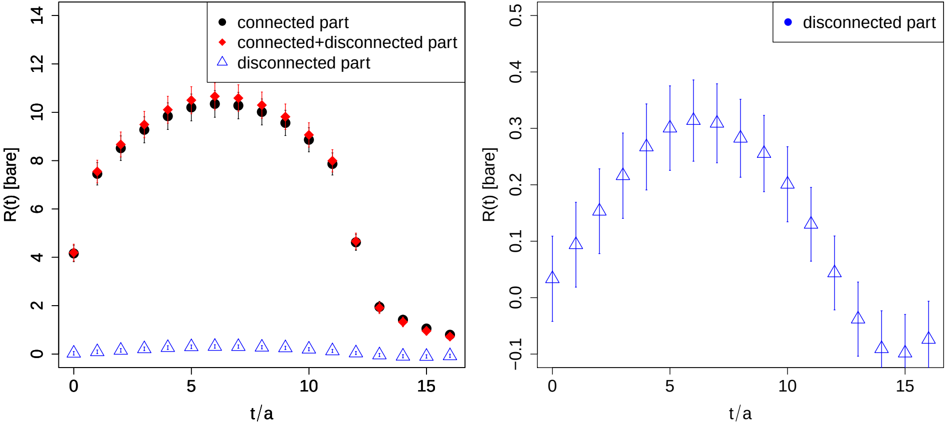
- Signal-over-noise ratio of a typical three point function vs $N_{\rm conf}$
- In comparison with hopping parameter expansion noise reduction save a factor $\sim$ 10 in statistics.



Performances

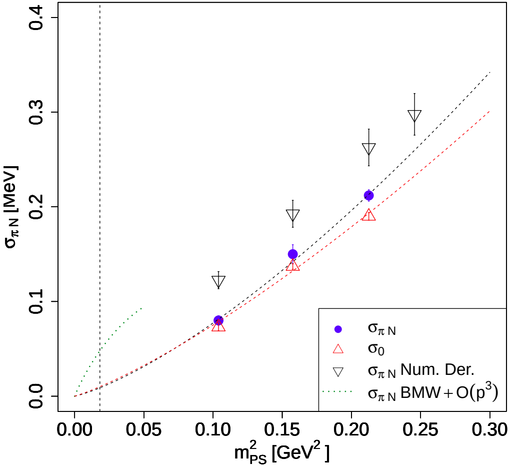


Result : light sector



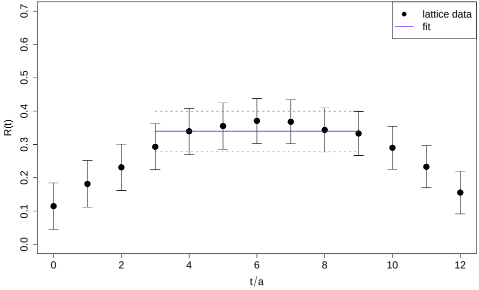
- $m_{\mathrm{PS}} = 380$ MeV
- statistics : $\sim$ 845 gauge configurations
- 12 volume sources (no dilution), 4 sets of two point
- Disconnected piece is different from 0 at 4σ

chiral behaviour : $\sigma_{\pi N}$

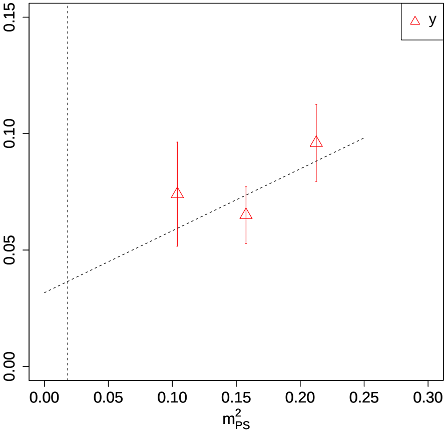


- $\sigma_{\pi N} = m_q \frac{\partial m_N}{\partial m_q} = m_q \langle N|\bar{u}u + \bar{d}d|N\rangle$
- $\sigma_0 = m_q \langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$

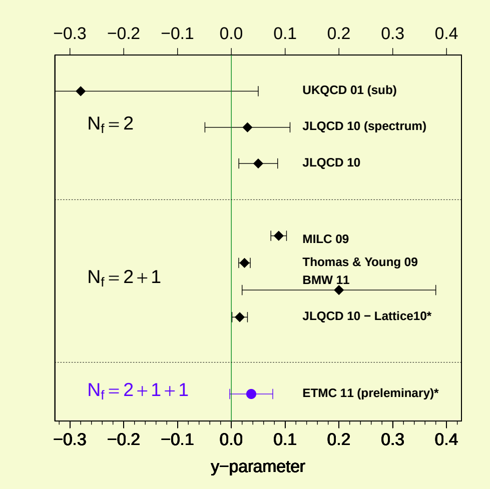
Result : strange sector



chiral behaviour : y_N

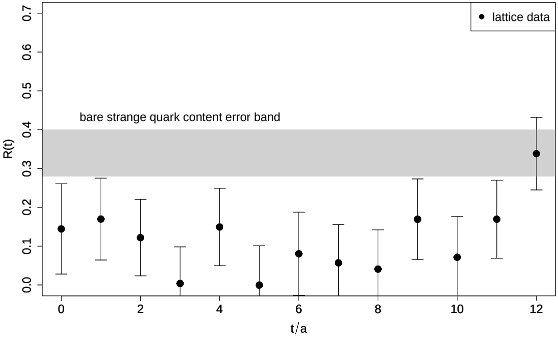


Preleminary results



- Systematic errors not estimated

Result : charm sector



- $\langle N|\bar{c}c|N\rangle < \langle N|\bar{s}s|N\rangle$ but not really interesting
- we want to compare f_{Ts} and f_{Tc}

Motivations

Consider

$$\langle p,s|\bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\lambda^8\psi|p,s\rangle\Big|_{\mu^2} = 2\langle x\rangle_{u+d-2s,\mu^2}p^{\{\mu}p^{\nu\}}, \quad \text{with} \quad \psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

where $\lambda_8 = \mathrm{diag}(1,1,-2)$.

- $\langle x\rangle_{u+d-2s,\mu^2}$ has never been estimated on the lattice
- known experimentaly
- Cancelation of renormalizations factor in $\frac{\langle x\rangle_{u-d,\mu^2}}{\langle x\rangle_{u+d-2s,\mu^2}}$
- Problem : (quark)-disconnected diagrams

Techniques

$$J^8 = \overline{\psi}\Gamma\lambda_8\psi = \frac{1}{\sqrt{3}}\left(\bar{u}\Gamma u + \bar{d}\Gamma d - 2\bar{s}\Gamma s\right) \tag{1}$$

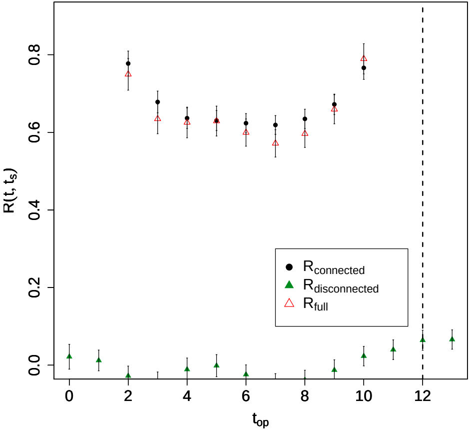
in the twisted basis :

$$\tilde{J}^8 = \overline{\chi}\Gamma\chi - \bar{\chi}_s\Gamma\chi_s \tag{2}$$

$$D_{q,\pm} - D_{q',\pm} = \pm 2i\kappa\gamma_5 a\left(\mu_q - \mu_{q'}\right) \tag{3}$$

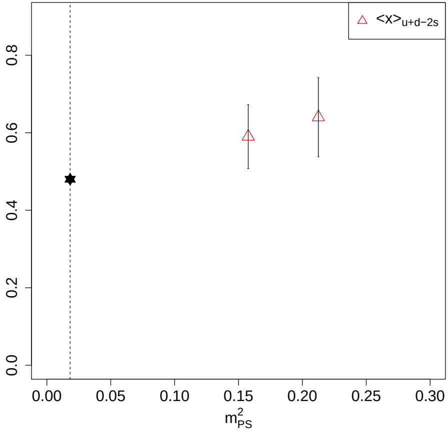
$$\frac{1}{D_{q',\pm}} - \frac{1}{D_{q,\pm}} = \frac{1}{D_{q,\pm}}\left(D_{q,\pm} - D_{q',\pm}\right)\frac{1}{D_{q',\pm}} = \pm 2i\kappa a\left(\mu_q - \mu_{q'}\right)\frac{1}{D_{q,\pm}}\gamma_5\frac{1}{D_{q',\pm}}, \tag{4}$$

Results



$t_s = 12a$ fixed

Chiral behaviour : $< x >_{u+d-2s,\mu^2}$



(Phenomenological value from S. Alekhin)

Conclusion

- Direct computation of the nucleon sigma terms
- Crucial use of the twisted mass noise reduction
- Mixing problems are avoided in our setup
- Disconnected contribution $4-5\sigma$ away from zero in the light and strange sector
- First computation of $< x >_{u+d-2s,\mu^2}$
- Use a generalization of the twisted mass noise reduction
- disconnected contribution compatible with 0, but with relatively small error

Chiral condensate