Kaon Oscillations from tmLQCD using $N_f = 2+1+1$ dynamical sea quarks

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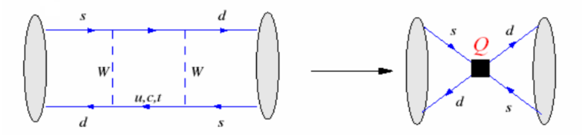
CP Violation in $\overline{K}^0 - K^0$ Mixing



Kaon oscillations are possible through a weak interaction of second order.

In the SM the effective operator responsible of the transition takes contribution only from

$$Q_1 = \frac{1}{4}[\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d]$$

$$\langle \overline{K}^0 | \mathcal{H}_{eff}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} \underbrace{\left[\sum_{l,m=u,c,t} C_1^{(l,m)}(\mu) V_{ls}^* V_{ld} V_{ms}^* V_{md} \right]}_{\text{perturbative}} \underbrace{\langle \overline{K}^0 | \hat{Q}_1(\mu) | K^0 \rangle}_{\text{non perturbartive}}$$

$$\langle \overline{K}^0 | \hat{Q}_1(\mu) | K^0 \rangle = \underbrace{\langle \overline{K}^0 | \hat{Q}_1 | K^0 \rangle_{VIA}}_{8/3 f_K^2 M_K^2} \hat{B}_1(\mu)$$

CP Violation in $\overline{K}^0 - K^0$ Mixing

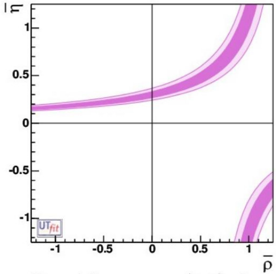
ϵ_K is a measurement of the indirect CP violation

$$K_L \sim \overbrace{(K^0 - \overline{K}^0)}^{CP=-1} + \epsilon_K \overbrace{(K^0 + \overline{K}^0)}^{CP=+1}$$
$$\hookrightarrow \pi\pi \qquad\qquad \hookrightarrow \pi\pi$$

$$\epsilon_K = \frac{G_F^2 M_W^2 f_K^2 M_K^2}{6\sqrt{2}\Delta M_K} \hat{B}_1(\mu) \mathrm{Im}\left[\sum_{l,m=u,c,t} C_1^{(l,m)}(\mu) V_{ls}^* V_{ld} V_{ms}^* V_{md}\right]$$

Since ϵ_K can be measured experimentally and $\hat{B}_1(\mu)$ extracted from the lattice simulations one can constrain the vertex of the unitary triangle

$$\bar{\eta}(1 - \bar{\rho}) = \text{constant}$$



Bound from ε_K on $(\bar{\rho}, \bar{\eta})$ plane.

Lattice action

Bare Operators:

- Iwasaki Gauge Action
- Wilson Twisted Mass Action at maximal twist with $N_f = 2+1+1$ dynamical sea quarks & Osterwalder-Seiler valence quark action

$$S_l^{sea,Mtm}(x) = \sum_x \{\overline{\chi}_l(x)\left[D_W[U] + m_{0,l} + i\mu_l\gamma_5\tau_3\right]\chi_l(x)\}$$

$$S_h^{sea,Mtm}(x) = \sum_x \overline{\chi}_h(x)\left[D_W[U] + m_{0,h} + i\mu_\sigma\gamma_5\tau_1 + \mu_\delta\tau_3\right]\chi_h(x)$$

$$S^{val,OS} = \sum_x \sum_{f=d,d',s,s'} \overline{q}_f(x)\left[D_W[U] + m_{0,f} + i\mu_f\gamma_5 r_f\right]q_f(x)$$

Chiral improved Wilson fermions

$r_s = r_s' = r_d = -r_d'$

- $\mathcal{O}(a)$ improvement
- Continuum like renormalization pattern

R.Frezzotti and G.C. Rossi. JHEP, 10:070, 2004

Renormalization Constants:

- Dedicated runs with Nf=4 degenerate sea quarks

Simulation for the bare parameter

	μ_{sea}	M_{PS}^{ll}(MeV)	$L^3 \times T$	#confs
	0.0030	~280	$32^3 \times 64$	144
β=1.90	0.0040	~320	$32^3 \times 64$	96
($a \sim 0.087$ fm)	0.0040	~320	$24^3 \times 48$	150+26=176
$\mu_\sigma = 0.15$ $\mu_\delta = 0.19$	0.0050	~360	$32^3 \times 64$	144
$\mu_{val} = \mu_{sea}$,0.0151,0.0185,0.0225	0.0060	~400	$24^3 \times 48$	144
	0.0080	~450	$24^3 \times 48$	150+58=208
	0.0100	~500	$24^3 \times 48$	150+58=208
	0.0025	~270	$32^3 \times 64$	144
β=1.95	0.0035	~320	$32^3 \times 64$	144
($a \sim 0.077$ fm)	0.0055	~400	$32^3 \times 64$	144
$\mu_\sigma = 0.135$ $\mu_\delta = 0.17$	0.0075	~460	$32^3 \times 64$	80
$\mu_{val} = \mu_{sea}$,0.0141,0.0180,0.0219	0.0085	~490	$24^3 \times 48$	150+94=244
β=2.10	0.0015	~220	$48^3 \times 96$	96
($a \sim 0.062$ fm)	0.0020	~250	$48^3 \times 96$	96
$\mu_\sigma = 0.12$ $\mu_\delta = 0.1385$	0.0030	~310	$48^3 \times 96$	96

Simulation details for the RC

- Dedicated runs with $N_f = 4$ degenerate quarks
- Working out of maximal Twist: average over p & m ensembles to achieve $\mathcal{O}(a)$-improvement in the RCs (arXiv:1101.1877)

$\beta = 1.95$	$a^{-4}(L^3 \times T) = 24^3 \times 48$		
$a\mu_{\rm sea}$	ensemble	$a\mu_{\rm val}$	$N_{\rm meas}$
0.0085	B4[1p**]	0.0085 0.0150 0.0203 0.0252 0.0298	304
0.0085	B4[1m**]	0.0085 0.0150 0.0203 0.0252 0.0298	304
0.0085	B6[2p]	0.0085 0.0150 0.0203 0.0252 0.0298	352
0.0085	B7[2m]	0.0085 0.0150 0.0203 0.0252 0.0298	352
0.0180	B[3p]	0.0060, 0.0085 0.0120 0.0150 0.0180 0.0203 0.0252 0.0298	352
0.0180	B[3m]	0.0060, 0.0085 0.0120 0.0150 0.1080 0.0203 0.0252 0.0298	352
0.0085	B[4p]	0.0060, 0.0085 0.0120 0.0150 0.0180 0.0203 0.0252 0.0298	224
0.0085	B[4m]	0.0060, 0.0085 0.0120 0.0150 0.1080 0.0203 0.0252 0.0298	224
0.0085	B[7m]	0.0085 0.0150 0.0203 0.0252 0.0298	400
0.0085	B[7p]	0.0085 0.0150 0.0203 0.0252 0.0298	400
0.0020	B[8m]	0.0085 0.0150 0.0203 0.0252 0.0298	400
0.0020	B[8p]	0.0085 0.0150 0.0203 0.0252 0.0298	400

Lattice estimator for the bare B-parameter

B-parameters mesure the deviation of the VIA value of the matrix elements which contribute to kaon oscillation from its physical value

$$\langle \overline{K^0}|Q_i|K^0\rangle = B_i\langle \overline{K^0}|Q_i|K^0\rangle_{VIA} = B_i F_i f_K^2 M_K^2$$

$$F_1 = \frac{8}{3}$$
$$F_i = C_i \left[\frac{M_K}{m_s+m_d}\right]^2 \quad i=2,\ldots,5 \quad C_i = \{-5/3, 1/3, 2, 2/3\}$$

$$Q_1 = \tfrac{1}{4}[\bar{s}^a\gamma^\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1-\gamma_5)d^b]$$
$$Q_2 = \tfrac{1}{4}[\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b]$$
$$Q_3 = \tfrac{1}{4}[\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1-\gamma_5)d^a]$$
$$Q_4 = \tfrac{1}{4}[\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b]$$
$$Q_5 = \tfrac{1}{4}[\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1+\gamma_5)d^a]$$

Since (in the CoM system)

$$\langle \overline{K^0}|A_0|0\rangle\langle 0|A_0|K^0\rangle = f_K^2 M_K^2$$

The bare B_1 estimator can be obtained from the ratio

$$B_1 = \frac{\langle \overline{K^0}|Q_1|K^0\rangle}{F_1\langle \overline{K^0}|A_0|0\rangle\langle 0|A_0|K^0\rangle} = \frac{\langle P^{34}|Q_1|P^{21}\rangle}{F_1\langle P^{34}|A_0^{43}|0\rangle\langle 0|A_0^{12}|P^{21}\rangle}$$

$$|K^0\rangle = \bar{d}\gamma_5 s \equiv |P^{21}\rangle = \bar{q}_2\gamma_5 q_1 \qquad |\overline{K^0}\rangle = \bar{d}\gamma_5 s \equiv |P^{43}\rangle = \bar{q}_4\gamma_5 q_3$$

Lattice estimator for the bare B-parameter

We define two "K-meson walls" at y_0 and $y_0 + T/2$

$$W^{21}(y_0) = \sum_{\vec{y}} \bar{q}_2(\vec{y}, y_0)\gamma_5 q_1(\vec{y}, y_0)$$

$$W^{43}(y_0 + T/2) = \sum_{\vec{y}} \bar{q}_4(\vec{y}, y_0 + T/2)\gamma_5 q_3(\vec{y}, y_0 + T/2)$$

Introducing the 4-fermion operators at $x = (\vec{x}, x_0)$ we construct the 2- and 3-point correlation functions

$$C_3(x_0) = \sum_{\vec{x}} \langle W^{43}(y_0 + T/2) Q_i(x) W^{21}(y_0)\rangle$$

$$C_2(x_0) = \sum_{\vec{x}} \langle A_0^{12}(x)) W^{21}(y_0)\rangle$$

$$C_2'(x_0) = \sum_{\vec{x}} \langle W^{43}(y_0 + T/2) A_0^{34}(x)\rangle$$

The B_1 parameter can be estimated from the ratio

$$R_1(x_0) = \frac{C_3(x_0)}{C_2(x_0)C_2'(x_0)} \xrightarrow[y_0 << x_0 << y_0 + T/2]{} \frac{\langle P^{34}|Q_1|P^{21}\rangle}{\langle P^{34}|A_0^{43}|0\rangle\langle 0|A_0^{12}|P^{21}\rangle} = F_1 B_1$$

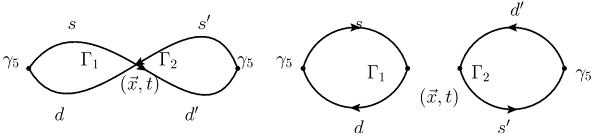
Lattice estimator for the bare B-parameter

Each one of the five B-parameters comes from a particular

$$C_3(x_0,\vec{x}) = \sum_{\vec{x}\vec{x}_L\vec{x}_R} \langle (\bar{d}'\gamma_5 s')(y_0+T/2,\vec{x}_R) 2 Q^{\Delta S=2}(x_0,\vec{x})(\bar{d}\gamma_5 s)(y_0,\vec{x}_L)\rangle$$

$$Q^{\Delta S=2} = (\bar{s}\Gamma d)(\bar{s}'\Gamma d') + (\bar{s}\Gamma d')(\bar{s}'\Gamma d)$$

$$C_3(x_0,\vec{x}) = 2 \sum_{\vec{x}\vec{x}_L\vec{x}_R} \mathrm{Tr}[S'_d(y_0+T/2,x)\Gamma S_s(x,y_0)\gamma_5 S_d(y_0,x)\Gamma S_S(x,y_0+T/2)\gamma_5]$$
$$-\mathrm{Tr}[S'_d(y_0+T/2,x)\Gamma S_s(x,y_0+T/2)\gamma_5]\cdot\mathrm{Tr}[S_d(x,y_0)\gamma_5 S_S(y_0,x)\Gamma]$$



Analysis Steps

1. Compute the bare B-parameters $B_i(\beta; \mu_{sea}, \mu_{val})$ by performing a fit of the correlation functions
2. Extrapolations to the physical quark masses
   - Chiral extrapolation in the $\mu_l \to \mu_{u/d}$
   - Interpolation in $\mu_h \to \mu_s$
3. Renormalization

Standard model contribution

We focus on B_1

Computation of the RCs is still in progress

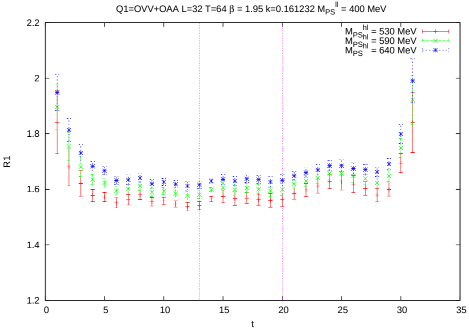
Only $\beta = 1.95$ is analyzed
No continuum limit extrapolation has been done yet

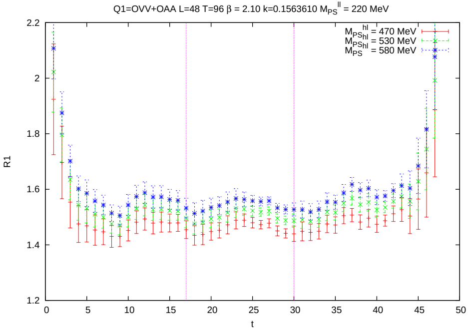
Ratio Fits

$B_1(\beta; \mu_{sea}, \mu_{val})$ have been computed by performing a constant fit of the ratio

$$R_1(t) = \frac{C_1^{(3)}(t)}{C_{AP}^{(2)}(t)C_{AP}^{(2)}(t)}$$

over the range where this function reaches a plateau.



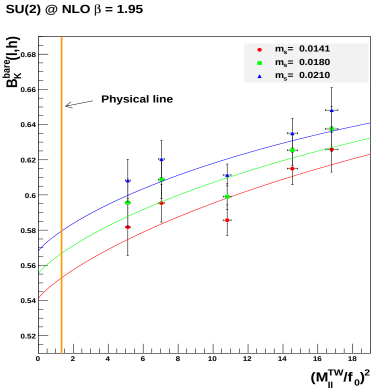


Physical quark mass Extrapolation (β =1.95)

- For each value of M_{ll} (i.e m_l) we obtain the value of B_K^{bare} at three values of M_{hh}(i.e m_s).
- Three chiral fits for each strange value are performed based on

SU2 @NLO: $B_K^{bare}(M_{ll}, M_{hh}) = B_K^{'}(M_{hh})\left[1 + b^{'}(M_{hh})\frac{(M_{ll})^2}{f_0^2} - \frac{(M_{ll})^2}{32\pi^2 f_0^2}\log\frac{(M_{ll})^2}{16\pi^2 f_0^2}\right]$

or a **polynomial fit:** $B_K^{bare}(M_{ll}, M_{hh}) = B_K^{'}(M_{hh})\left[1 + b^{'}(M_{hh})(M_{ll})^2\right]$



SU(2) @ NLO

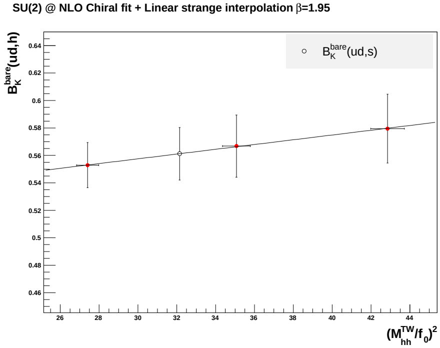
am_h	$\chi^2/d.o.f$	$B_K^{bare}(M_\pi, M_{hh})$
0.0141	2.38	0.552(16)
0.0180	2.62	0.57(2)
0.0219	2.60	0.58(3)

Polynomial fit

am_h	$\chi^2/d.o.f$	$B_K^{bare}(M_\pi, M_{hh})$
0.0141	2.04	0.56(2)
0.0180	2.21	0.58(2)
0.0219	2.14	0.59(2)

Physical quark mass Extrapolation ($\beta = 1.95$)

- Interpolation in $(M_{hh})^2$ to the physical kaon mass : $(M_{hh})^2 = 2M_K^2 - M_\pi^2$



	$B_K^{bare}(m_u, m_s, \beta = 1.95)$
SU(2)@NLO	0.561(19)
Polynomial Fit	0.572(21)

4-Fermion Renormalization Constants

In the formalism of OPE

$$\langle \overline{K}^0 | \mathcal{O} | K^0 \rangle = c_w(\mu) \langle \overline{K}^0 | \mathcal{O}(\mu) | K^0 \rangle = c_w(\mu) Z_{\mathcal{O}}(a\mu) \langle \overline{K}^0 | \mathcal{O}(a) | K^0 \rangle$$

$\mathcal{O}^{\Gamma_1 \Gamma_2}$ can mix with any dimension-six operator provided they have the same quantum numbers.

Mixing

The Wilson term induces explicit chiral symmetry breaking. That implies mixing with operators of the same dimensionality but with the wrong naive chirality.

OS fermions

The four-fermion operators are renormalizable as in the continuum once the Wilson parameters appearing in the action of the OS fermions of the valence satisfy

$r_s = r_{s'} = r_d = -r'_d$

R.Frezzotti and G.C. Rossi. JHEP, 10:070, 2004

4-Fermion Mixing

Classification in terms of their discrete simmetries:

- Parity

$$
\begin{array}{llll}
PC & & PV & \\
Q_1^{\pm} \equiv & O^{\pm}_{[VV+AA]} & \mathcal{Q}_1^{\pm} \equiv & O^{\pm}_{[VA+AV]} \\
Q_2^{\pm} \equiv & O^{\pm}_{[VV-AA]} & \mathcal{Q}_2^{\pm} \equiv & O^{\pm}_{[VA-AV]} \\
Q_3^{\pm} \equiv & O^{\pm}_{[SS-PP]} & \mathcal{Q}_3^{\pm} \equiv & -O^{\pm}_{[SP-PS]} \\
Q_4^{\pm} \equiv & O^{\pm}_{[SS+PP]} & \mathcal{Q}_4^{\pm} \equiv & O^{\pm}_{[SP+PS]} \\
Q_5^{\pm} \equiv & O^{\pm}_{[TT]} & \mathcal{Q}_5^{\pm} \equiv & O^{\pm}_{[T\tilde{T}]}
\end{array}
$$

$$
O^{\pm} = \tfrac{1}{2}[O \pm O^F] \qquad O_{\Gamma_1\Gamma_2} = (\bar{\psi}_1\Gamma_1\psi_2)(\bar{\psi}_3\Gamma_2\psi_4)
$$

- Charge Conjugation + Flavour exchange symmetries

$$
\hat{Q}^{\pm} = Z_{\chi}^{\pm}[I + \Lambda^{\pm}]Q^{\pm} \qquad \hat{\mathcal{Q}} = \mathcal{Z}^{\pm}\mathcal{Q}
$$

$$
Z_{\chi}^{\pm} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{44} & Z_{55} \end{pmatrix}^{\pm} \qquad \mathcal{Z}_{\chi}^{\pm} = \begin{pmatrix} \mathcal{Z}_{11} & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Z}_{22} & \mathcal{Z}_{23} & 0 & 0 \\ 0 & \mathcal{Z}_{32} & \mathcal{Z}_{33} & 0 & 0 \\ 0 & 0 & 0 & \mathcal{Z}_{44} & \mathcal{Z}_{45} \\ 0 & 0 & 0 & \mathcal{Z}_{44} & \mathcal{Z}_{55} \end{pmatrix}^{\pm}
$$

$$
\Lambda^{\pm} = \begin{pmatrix} 0 & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\ \Lambda_{11} & 0 & 0 & \Lambda_{24} & \Lambda_{25} \\ \Lambda_{31} & 0 & 0 & \Lambda_{34} & \Lambda_{35} \\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & 0 & 0 \\ \Lambda_{51} & \Lambda_{52} & \Lambda_{53} & 0 & 0 \end{pmatrix}^{\pm} \xrightarrow{\text{Chiral limit}} 0
$$

RI-MOM method

Doing in the lattice what we do in the continuum

RI/MOM consists in imposing renormalization conditions non-perturbatively directly on quark and gluon Green functions, in a fixed gauge, with given off-shell external states

RI/MOM 4-fermion RC

$$Z^{\pm} = Z_{\psi}^{-2}[D^{\pm T}]^{-1}$$

$$\Lambda^{\pm}_{\Gamma_1\Gamma_2}(p) = \underbrace{S^{-1}S^{-1}}_{\Lambda^{\pm}}\overbrace{G_{\Gamma_1\Gamma_2}}^{\text{amputed GF}}S^{-1}S^{-1}$$

$$\underbrace{D^{\pm}}_{\text{dynamical matrix}} = \underbrace{P^{\pm}}_{\text{tree-level projector}}$$

$$P^{\pm}\Lambda^{\pm(0)} = I$$

Steps for the computation of the RCs

B_K Renormalization in the SM

Note that:

$$Q_1^+ \equiv O^+_{[VV+AA]} = [\bar{\psi}_{d_1}\gamma_\mu(1-\gamma_5)\psi_{s_1}][\bar{\psi}_{d_2}\gamma_\mu(1-\gamma_5)\psi_{s_2}]$$

$(Z_{11})^+_{PC}$ is the only 4-fermion operator relevant for the computation of $\hat{B}_K$

θ-av

$$D_{ij}(p, M_{val}, M_{sea}) = 0.5\{D_{ij}(p, M_{val}, M_{sea}, p) + D_{ij}(p, M_{val}, M_{sea}, m)\}$$
$$D_{ij}(p, 0, M_{sea}) = 0.5\{D_{ij}(p, 0, M_{sea}, e=p) + D_{ij}(p, 0, M_{sea}, e=m)\}$$
$$Z_{ij}(p, 0, M_{sea}) = 0.5\{Z_{ij}(p, 0, M_{sea}, e=p) + Z_{ij}(p, 0, M_{sea}, e=m)\}$$
$$Z_{ij}(p, 0, 0) = 0.5\{Z_{ij}(p, 0, 0, e=p) + Z_{ij}(p, 0, 0, e=m)\}$$

1. Computation of $D_{ij}(p, M_{val}, \text{ensemble})$: $D^{\pm} = P^{\pm}\Lambda^{\pm}$
2. Valence Chiral limit with substraction of the Goldstone Pole $\rightarrow D_{ij}(p, M_{val}=0, \text{ensemble})$ and θ-average
3. Computation of $Z_{ij}(p, M_{val}=0, M_{sea})$ with removal of $O(a^2g^2)$ cuttof effects : $Z^{\pm} = Z_{\psi}^{-2}[D^{\pm T}]^{-1}$
4. Sea Chiral Limit$\rightarrow$ $Z_{ij}(p, M_{val} = M_{sea} = 0)$
5. Evolution: RI'-MOM$\rightarrow$RGI and p^2 fit

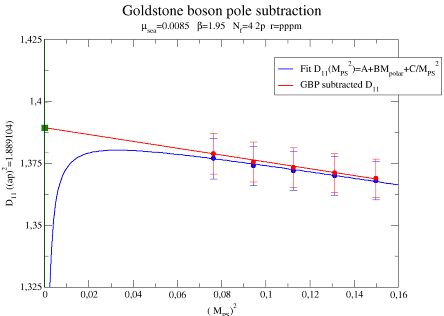
Dynamical Matrix Valence Chiral Limit

$$D_{ij}(p, M_{val}, e) = Z_{ij}(p^2, e) + A_{ij}(p^2, e) M_{POL,val} + \overbrace{\frac{B_{ij}(p^2, e)}{M_{POL,val}}}^{\text{simple pole}} + \overbrace{\frac{C_{ij}(p^2, e)}{M^2_{POL,val}}}^{\text{double pole}}$$

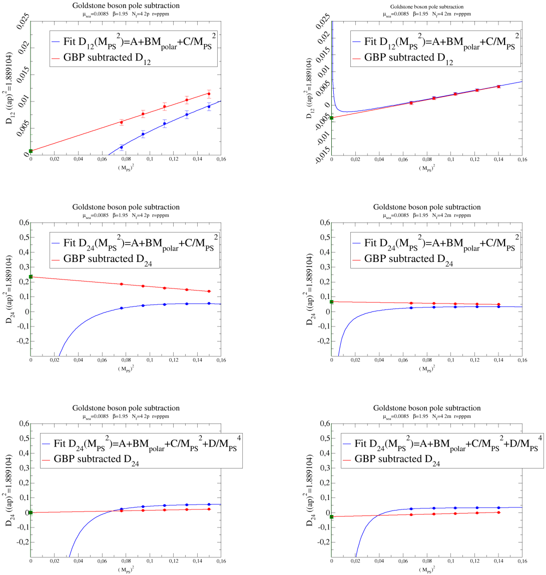
$$D_{ij}(p, M_{val}, e) = Z_{ij}(p^2, e) + A_{ij}(p^2, e) M^2_{PS,val} + \frac{B_{ij}(p^2, e)}{M^2_{PS,val}} + \frac{C_{ij}(p^2, e)}{M^4_{PS,val}}$$

$$D_{ij}(p, M_{val}, e) = Z_{ij}(p^2, e) + A_{ij}(p^2, e) M_{POL,val} + \frac{B_{ij}(p^2, e)}{M^2_{PS,val}} + \frac{C_{ij}(p^2, e)}{M^4_{PS,val}}$$

with $\begin{cases} C_{ij} = 0 \\ C_{ij} \neq 0 \\ \begin{cases} C_{0j} = 0 \\ C_{ij} \neq 0 \text{ if } i \neq 0 \end{cases} \end{cases}$



Absence of Wrong Chirality Mixing in the Dynamical Matrix



$$\begin{pmatrix} D_{11} & 0 & 0 & 0 & 0 \\ 0 & D_{22} & D_{23} & 0 & 0 \\ 0 & D_{32} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & D_{44} & D_{45} \\ 0 & 0 & 0 & D_{54} & D_{55} \end{pmatrix}$$

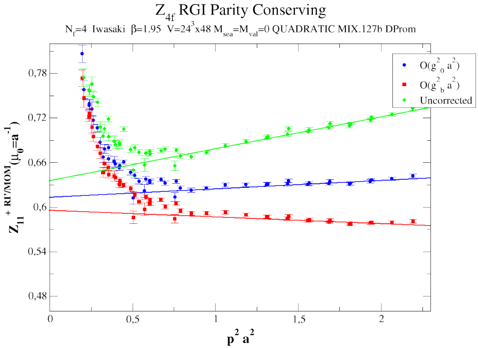
- Goldstone boson pole contributions are more significant for ensembles p
- Double pole substraction is necessary for D_{ij} with $i \neq 0$

Z computation & $\mathcal{O}(a^2g^2)$ substraction

- θ-average: $D_{ij}(p,0,M_{sea},\theta\text{-av}) = 0.5\{D_{ij}(p,0,M_{sea},p) + D_{ij}(p,0,M_{sea},m)\}$
- Removal of $\mathcal{O}(a^2g^2)$ discretization errors

$$D_{11}(p)^{\text{pert}} = 1 + \frac{g^2}{16\pi^2}\left[b_{11}^{(1)} + b_{11}^{(2)}\log(a^2p^2)\right] + \frac{g^2}{16\pi^2}a^2\left[p^2\left(c_{11}^{(1)} + c_{11}^{(2)}\log(a^2p^2)\right) + c_{11}^{(3)}\frac{\sum_\rho p_\rho^4}{p^2}\right] + \mathcal{O}(a^4g^2, g^4)$$

- Computation of Z_ψ : $Z_\psi^{WI} = -i/4\mathrm{Tr}[\not{p}S(p)^{-1}/p^2]|_{p^2=-\mu^2}$
- Computation of $Z(p, M_{val} = 0, M_{sea}, \theta - av) = Z_\psi^{-2}[D^T]^{-1}$

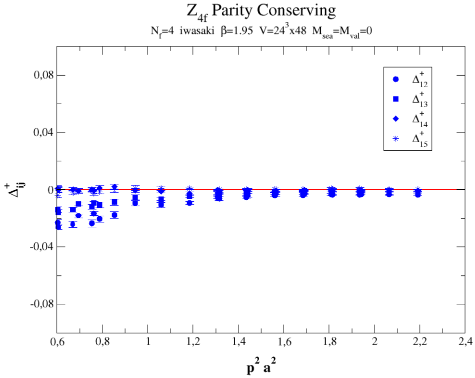


Absence of wrong chirality mixings

Wrong chirality mixings can affect the renormalization of $[Q_1]^+_{PC}$

$$[\hat{Q}_1]^+_{PC} = [Z_{11}]^+_{PC} \left[[Q_1]^+_{PC} + \sum_{j=2}^{5} \Delta_{1j} Q_j \right]$$

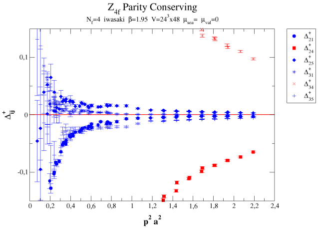
Using OS valence fermions with $r_s = r_{s'} = r_d = -r_{d'}$: $\Delta_{ij} \xrightarrow{\text{chiral limit}} 0$

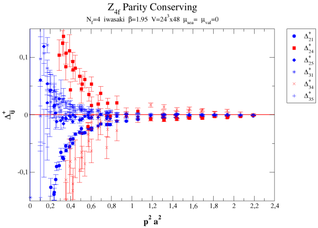


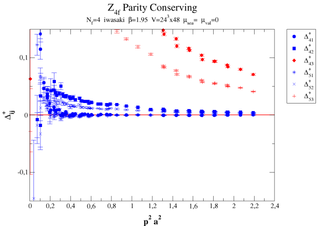
Absence of Wrong Chirality Mixing

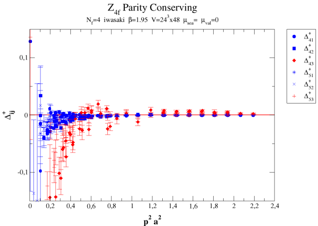
$$\Lambda^{\pm} = \begin{pmatrix} 0 & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\ \Lambda_{11} & 0 & 0 & \Lambda_{24} & \Lambda_{25} \\ \Lambda_{31} & 0 & 0 & \Lambda_{34} & \Lambda_{35} \\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & 0 & 0 \\ \Lambda_{51} & \Lambda_{52} & \Lambda_{53} & 0 & 0 \end{pmatrix}^{\pm} \xrightarrow{\text{Chiral limit}} 0$$

$\Delta_{24}, \Delta_{34}, \Delta_{43}$ and Δ_{53} $\xrightarrow{\text{Chiral limit}}$ 0 only if we substract a double goldstone pole





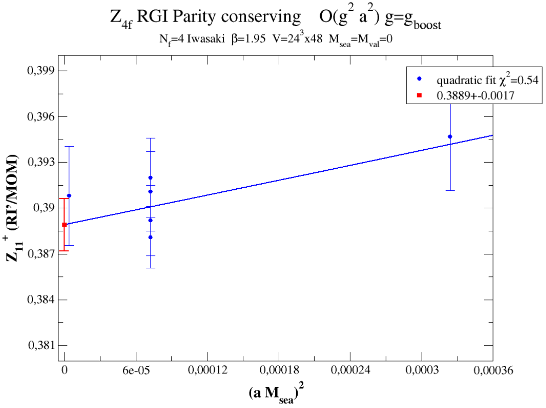




RCs Sea Chiral Limit

$$Z_{ij}(p^2, \mu_{val} = 0, \mu_{sea}) = A(p) + B(p) M_{POL,sea}$$
$$Z_{ij}(p^2, \mu_{val} = 0, \mu_{sea}) = A(p) + B(p)(M_{POL,sea} + C(p)\cos(2\theta))$$
$$Z_{ij}(p^2, \mu_{val} = 0, \mu_{sea}) = A(p) + B(p) M^2_{POL,sea}$$
$$Z_{ij}(p^2, \mu_{val} = 0, \mu_{sea}) = A(p) + B(p)(M^2_{POL,sea} + C(p)\cos(2\theta))$$

Since at this point we have performed the θ-av : $M_{POL,sea} = \mu_{sea}$



p^2-fit & RI'/MOM →RGI

- $Z^{RI}(\mu_{val}=0,\mu_{sea}=0,p^2) \xrightarrow[\text{running}]{NLO} Z^{RI}(\mu_{val}=0,\mu_{sea}=0,\mu_0^2=a^{-2}) \equiv Z^{RI}(\mu_0^2)$

M1: p^2 linear fit method $(ap)^2 \subset [1.5, 2]$
M2: p^2-window method $(ap)^2 \subset [1.8, 2]$

	M1	M2
$\left[(Z_{11})^+_{PC}\right]^{RI'/MOM}(\mu_0=a^{-1})$	0.597(06)	0.580(02)

$$Z_{B_K} = \frac{(Z_{11})^+_{PC}}{Z_A Z_V}$$

	M1	M2
Z_A	0.738(05)(01)	0.713(02)(01)
Z_V	0.610(02)(01)	0.614(02)(01)

where the $Z2f$ have been computed over the same ensembles as $Z4f$ with

- Valence chiral fit $Z_O(p, M_{sea}, M_{val}) = z_0 + z_1 M^2_{PS,val}$
- Sea chiral fit $Z_O(p, M_{sea}, 0) = z_0 + z_1 (M^2_{PS,sea})^2$

sistematic error comes from $M^2_{PS} \to M_{POL}$

$\hat{B}_1^{RI'/MOM}(\mu_0 = a^{-1})$

$$\hat{B}_K = \frac{(Z_{11})^+_{PC}}{Z_A Z_V} B_K$$

	M1	M2
SU(2) @ NLO	**0.744(04)(11)**	0.743(07)(04)
Polynomial Fit	0.758(04)(11)	0.757(07)(05)

FINAL RESULT

$$B_K^{RI'/MOM}(\mu_0 = a^{-1}, a = 0.087\,\text{fm}) = 0.744(04)(17)$$

- 0.004: statistical error from boostrap analysis
- 0.017: sistematic error resulting from
  - 0.014 from the extrapolation of the bare parameter to the chiral point: SU(2) @ NLO or Polynomial fit
  - 0.010 from the renormalization
    - 0.001 due to the spread between M1 and M2
    - 0.007 from the variation of the p^2 interval
    - 0.003 from varying the choice of the valence chiral extrapolation variable: M_{PS}^2 or M_{POL}
    - 0.004 due to the election of simple or double goldstone pole
    - 0.005 from the difference between quadratic or linear extrapolation in M_{sea}
    - error from performing the θ-av in $D(p, M_{sea}, M_{val} = 0)$ or in $Z(p, M_{sea}, M_{val} = 0)$ <0.001

Conclusions

FINAL RESULT

$$B_K^{RI'/MOM}(\mu_0 = a^{-1}, a = 0.087\,\text{fm}) = 0.744(04)(16)$$

RESULT N_f=2 (CONTINUUM LIMIT EXTRAPOLATED)

$$B_K^{RGI} = 0.729(30)$$

- We need to understand theoretically the goldstone pole contribution out of maximal twist
  - Some Δ_{ij} need double pole substraction in order to $\xrightarrow{\text{Chiral limit}} 0$
  - Contributions are larger for p ensembles
- Combination of R_i $i = 2, 3, 4, 5$ with Z_{ij} in order to compute the supersymmetric contributions
- B_K bare parameters have been analysed at $\beta = 1.90$ & $\beta = 2.10$ $\rightarrow$We need 4-Fermion RCs for these βs

BACKUP SLIDES

Stochastic Propagators

Consists in computing quark propagators by inverting the lattice Dirac operator for the relevant OS valence quark flavours on random sources of the form:

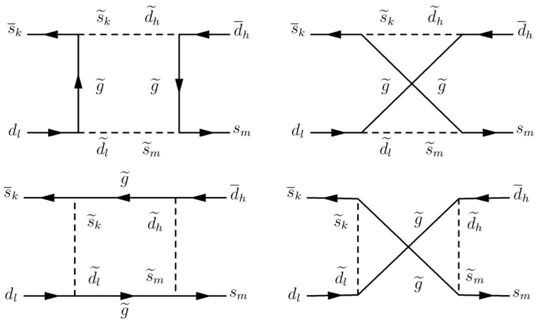
$$\delta_{\alpha\gamma}\delta_{x_o,z_0}\delta_{\vec{x},\vec{z}}\eta^{(z_0)}(k,\vec{z}) \quad ,\gamma = 1,2,3,4, \quad z_0 = y_0, y_0 + T/2$$

with free indices
α: spin
x_0: time
i: colour
$\vec{x}$: space
The Z_2-valued η vectors carry only colour and three-space indices (colour and space "dilution") and are normalized according to:

$$\langle \eta^{(z_0)}(k,\vec{z})\eta^{(z_0)}(k',\vec{z}')\rangle = \delta_{k,k'}\delta_{\vec{z},\vec{z}'}$$

Supersymmetric Contributions

Supersymmetric box diagrams contributing to $K^0 - \overline{K}^0$ oscillations at LO with $h, k, l, m = L, R$



$$\mathcal{H}_{eff}^{\Delta S=2} = \frac{G_F^2 M_W^2}{16\pi^2} \left\{ \sum_{i=1}^{5} C_i(\mu)\hat{Q}_i(\mu) + \sum_{j=1}^{3} \widetilde{C}_j(\mu)\hat{\widetilde{Q}}_j(\mu) \right\}$$

$\widetilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by exchanging the sign in both helicities.

From Lattice to Supersymmetric basis

- Each B-parameter comes directly from an operator of the supersymmetric basis
- The lattice basis is easier to implement

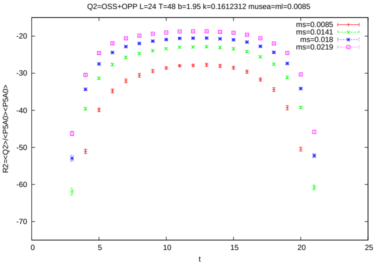
Supersymmetric Basis	Lattice Basis
$Q_1 = \frac{1}{4}[\bar{s}^a\gamma^\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1-\gamma_5)d^b]$	$O^{SS} = \frac{1}{4}(\bar{s}d)(\bar{s}d)$
$Q_2 = \frac{1}{4}[\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b]$	$O^{VV} = \frac{1}{4}(\bar{s}\gamma_\mu d)(\bar{s}\gamma^\mu d)$
$Q_3 = \frac{1}{4}[\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1-\gamma_5)d^a]$	$O^{TT} = \frac{1}{4}(\bar{s}\sigma^{\mu\nu}d)(\bar{s}\sigma_{\mu\nu}d)$
$Q_4 = \frac{1}{4}[\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b]$	$O^{AA} = \frac{1}{4}(\bar{s}\gamma^\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma^5 d)$
$Q_5 = \frac{1}{4}[\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1+\gamma_5)d^a]$	$O^{PP} = \frac{1}{4}(\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d)$

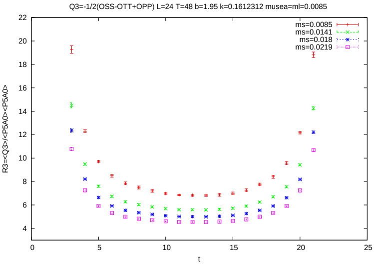
Both basis are related by a Fierz Matrix

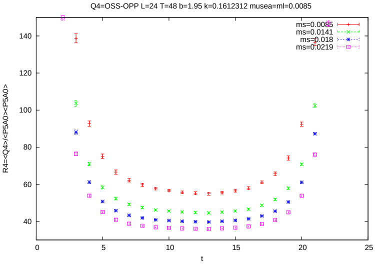
$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} O^{VV} + O^{AA} \\ O^{VV} - O^{AA} \\ O^{SS} - O^{PP} \\ O^{SS} + O^{PP} \\ O^{TT} \end{pmatrix} \rightarrow$$

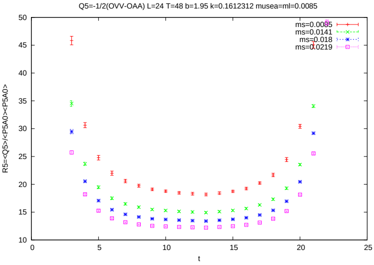
Only parity even operators are relevant

Supersymmetric ratios

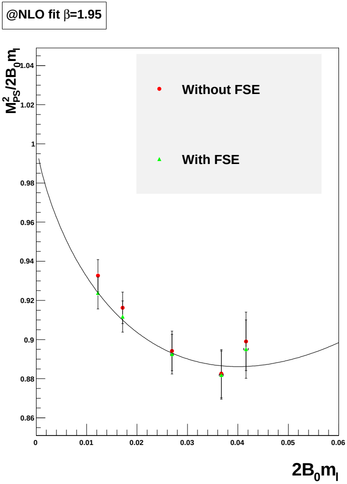


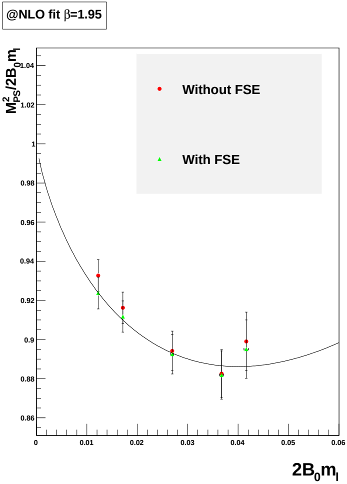




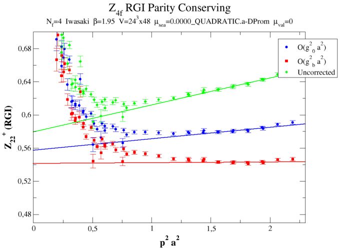


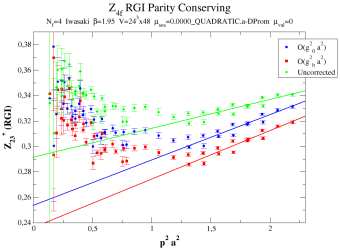
Light Physics

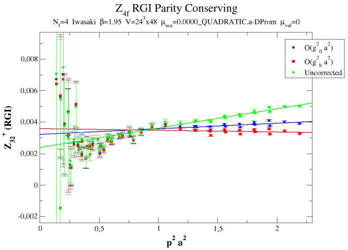


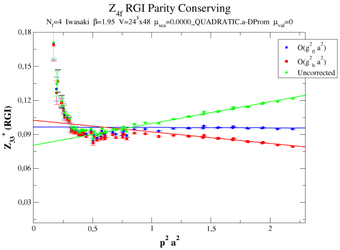


4-Fermion RCs









4-Fermion RCs