Chiral condensate and topological susceptibility with $N_\mathrm{f} = 2 + 1 + 1$ dynamical flavors of maximally twisted mass fermions

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Chiral Condensate and Banks-Casher Relation

- In the continum:

$$\frac{\Sigma}{\pi} = \lim_{\lambda\to 0} \lim_{m\to 0} \lim_{V\to\infty} \rho(\lambda, m)$$

$$\rho(\lambda, m) = \frac{1}{V} \sum_{k=1}^{\infty} \langle \delta(\lambda - \lambda_k) \rangle \,, \qquad \Sigma = - \lim_{m\to 0} \lim_{V\to\infty} \langle \bar{u}u \rangle$$

- mode number $\nu \rightsquigarrow$ average number of eigenmodes of $D_m^\dagger D_m$ with $\lambda \leq M^2$

$$\nu(M, m) = V \int_{-\Lambda}^{\Lambda} d\lambda \rho(\lambda, m), \qquad \Lambda = \sqrt{M^2 - m^2}$$

$\nu(M, m) = \nu_R(M_R, m_R)$ $\rightsquigarrow$ renormalization-group invariant (Giusti & Lüscher)

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$$\Sigma_R \propto \frac{\partial}{\partial M_R} \nu_R$$

for non-vanishing mass and finite volume

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Mode number and Spectral Projectors

- Spectral Projector $\mathbb{P}_M$ to compute $\nu(M, m)$ (Giusti & Lüscher, 2008)

$$\nu(M, m_q) = \langle \mathrm{Tr}\{\mathbb{P}_M\} \rangle$$

- Approximation of $\mathbb{P}_M$:

$$\mathbb{P}_M \approx h(\mathbb{X})^4, \qquad \mathbb{X} = 1 - \frac{2M_\star^2}{D_m^\dagger D_m + M_\star^2}, \qquad M_\star \approx M$$

⇝ $h(x)$ is an approximation to the step function $\theta(-x)$ in the interval $[-1, 1]$.

$$h(x) = \frac{1}{2}\{1 - xP(x^2)\}$$

where $P(x)$ is the polynomial which minimizes

$$\delta = \max_{\epsilon \le y \le 1} ||1 - \sqrt{y}P(y)||$$

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$$\nu(M, m_q) = \langle \mathcal{O}_N \rangle, \quad \mathcal{O}_N = \frac{1}{N}\sum_{k=1}^{N}(\eta_k, \mathbb{P}_M \eta_k)$$

η_k sources generated randomly.

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Is related to distribution of topological charge

$$\chi_{top} = \frac{\langle Q_{top}^2 \rangle}{V} = \int d^4x \langle q(x) q(0) \rangle$$

- Topological Charge $\rightsquigarrow$ property of the gauge fields:

$$Q_{top} = a^4 \sum_{n \in \Lambda} q(n), \qquad q(n) = \frac{1}{2a^3} tr\left[\gamma_5 D\right]$$

- ⋆ Index Theorem: $Q_{top} = n_- - n_+$
  - $\rightsquigarrow$ relates topological charge and number of zero modes of the Dirac operator.
  - $\rightsquigarrow$ We have to simulate chiral fermions, it gets very expensive for large volumes.
- ⋆ Other representation of the topological susceptibility making used of chiral fermions: (Giusti, Rossi & Testa, 2004)
- ⋆ Using density chains the topological susceptibility can be defined in a universally way. (Lüscher, 2004)

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- Using Ginsparg-Wilson fermions we can establish the following

$$\mathrm{Tr}\{\gamma_5 f(D)\} = f(0) Q_{top}$$ $\rightsquigarrow$ analogous to the index theorem

- On example of density chain

$$a^{4r} \sum_{x1\ldots x_r} \langle S_{r1}(x_1)P_{12}(x_2)\ldots P_{r-1r}(x_r)\rangle_F = -\mathrm{Tr}\left\{\gamma 5(D_{m_1}^{-1})\ldots(D_{m_r}^{-1})\right\}$$

for simplicity we omit here terms $(1-\frac{1}{2}aD)$.

- Topological charge:

$$Q_{top} = -m_1\ldots m_r \times a^{4r} \sum_{x_1\ldots x_r} \langle P_{r1}(x_1)S_{12}(x_2)\ldots S_{r-rr}(x_r)\rangle_F \qquad Z_P = Z_S$$

- Topological susceptibility:

$$\chi_{top} = m_1\ldots m_s \times a^{4s-4} \sum_{x_1\ldots x_{s-1}} \langle P_{r1}(x_1)S_{12}(x_2)\ldots S_{r-1r}(x_r)\times P_{sr+1}(x_{r+1})S_{r+1r+2}(x_{r+2})\ldots S_{s-1s}(x_0)\rangle$$

- Topological susceptibility in the continuum:

$$\chi_{top} = m_1\ldots m_5 \int d^4x_1 d^4x_4 \langle P_{31}(x_1)S_{12}(x_2)S_{23}(x_3)P_{54}(x_4)S_{45}(0)\rangle$$

these correlation functions don't have any short-distance singularity

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Topological Susceptibility III

(Giusti & Lüscher, 2008)

- replace the inverse powers of $D_m^\dagger D_m + \mu^2$ for a function of $D_m^\dagger D_m \rightsquigarrow$ Spectral Projectors

$$\chi_{top} = \frac{Z_S^2}{Z_P^2} \frac{1}{V} \langle \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\} \, \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\} \rangle$$

- twisted spectral sums as density chain observables:

$$\sigma_k(\mu, m_q) = \left\langle \mathrm{Tr}\left\{ \gamma_5 (D_m^\dagger D_m + \mu^2)^{-k} \right\} \right\rangle$$

- We can write $\frac{Z_S^2}{Z_P^2}$ as a function of spectral sum since $\sigma_{k,l,R} = \sigma_{k+l,R}$

$$\frac{Z_S^2}{Z_P^2} = \frac{\sigma_{k+l}}{\sigma_{k,l}} = \frac{\langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(x_6)\rangle}{\langle S_{12}(x_1)P_{23}(x_2)S_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(x_6)\rangle} \rightsquigarrow \quad \frac{Z_S^2}{Z_P^2} = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\gamma_5 \mathbb{P}_M \gamma_5 \mathbb{P}_M\}\rangle}$$

- Finally we get

$$\chi_{top} = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\gamma 5 \mathbb{P}_M \gamma 5 \mathbb{P}_M\}\rangle} \frac{\langle \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\} \, \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\}\rangle}{V}$$

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$$\chi_{top} = \frac{Z_S^2}{Z_P^2}\frac{1}{V}\langle \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\}\, \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\}\rangle$$

- twisted spectral sums as density chain observables:

$$\sigma_k(\mu, m_q) = \left\langle \mathrm{Tr}\left\{\gamma_5 (D_m^\dagger D_m + \mu^2)^{-k}\right\}\right\rangle$$

- We can write $\frac{Z_S^2}{Z_P^2}$ as a function of spectral sum since $\sigma_{k,l,R} = \sigma_{k+l,R}$

$$\frac{Z_S^2}{Z_P^2} = \frac{\sigma_{k+l}}{\sigma_{k,l}} = \frac{\langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(x_6)\rangle}{\langle S_{12}(x_1)P_{23}(x_2)S_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(x_6)\rangle} \rightsquigarrow \qquad \frac{Z_S^2}{Z_P^2} = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\gamma_5 \mathbb{P}_M \gamma_5 \mathbb{P}_M\}\rangle}$$

- Finally we get

$$\chi_{top} = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\gamma 5 \mathbb{P}_M \gamma 5 \mathbb{P}_M\}\rangle}\frac{\langle \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\}\, \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\}\rangle}{V}$$

Topological Susceptibility III

(Giusti & Lüscher, 2008)

- replace the inverse powers of $D_m^\dagger D_m + \mu^2$ for a function of $D_m^\dagger D_m \rightsquigarrow$ Spectral Projectors

$$\chi_{top} = \frac{Z_S^2}{Z_P^2}\frac{1}{V}\langle \mathrm{Tr}\{\gamma 5 \mathbb{P}_M\}\,\mathrm{Tr}\{\gamma 5 \mathbb{P}_M\}\rangle$$

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Topological Susceptibility IV

(Lüscher & Palombi, 2010)

- We can make the calculation more efficient:

if now we define $\mathbb{R}_M = h(\mathbb{X})$ which denotes the approximation and not the projector

$$\chi_{top} = \frac{\left\langle \mathrm{Tr}\{\mathbb{R}_M^4\}\right\rangle}{V} \frac{\left\langle \mathrm{Tr}\{\gamma_5 \mathbb{R}_M^2\}\mathrm{Tr}\{\gamma_5 \mathbb{R}_M^2\}\right\rangle}{\left\langle \mathrm{Tr}\{\gamma_5 \mathbb{R}_M^2 \gamma_2 \mathbb{R}_M^2\}\right\rangle} = \frac{\langle \mathcal{A} \rangle}{V} \frac{\left(\left\langle \mathcal{C}^2 \right\rangle - \frac{\langle \mathcal{B} \rangle}{N}\right)}{\langle \mathcal{B} \rangle}$$

$\rightsquigarrow$ What we actually compute; Stochastic observables:

$$\mathcal{A} = \frac{1}{N}\sum_{k=1}^{N}(\mathbb{R}_M^2 \eta_k, \mathbb{R}_M^2 \eta_k),$$

$$\mathcal{B} = \frac{1}{N}\sum_{k=1}^{N}(\mathbb{R}_M \gamma_5 \mathbb{R}_M \eta_k, \mathbb{R}_M \gamma_5 \mathbb{R}_M \eta_k)$$

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$$\mathcal{C} = \frac{1}{N}\sum_{k=1}^{N}(\mathbb{R}_M\eta_k, \gamma_5\mathbb{R}_M\eta_k)$$

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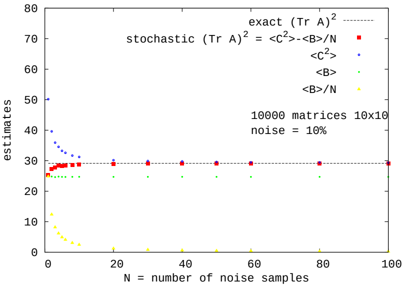
Choice of Stochastic Sources

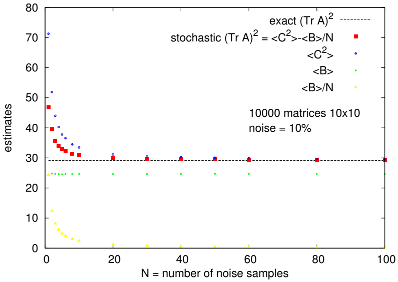
$$\nu(M, m_q) = \langle \mathcal{O}_N \rangle \,, \quad \mathcal{O}_N = \frac{1}{N} \sum_{k=1}^{N} (\eta_k, \mathbb{P}_M \eta_k)$$

η_k sources generated randomly.

Z2 noise:
$p(1+i) = p(1-i) = p(-1+i) = p(-1-i) = 1/4$

gaussian noise:
η real, $p(\eta) = e^{-\eta^2/2}/\sqrt{2\pi}$

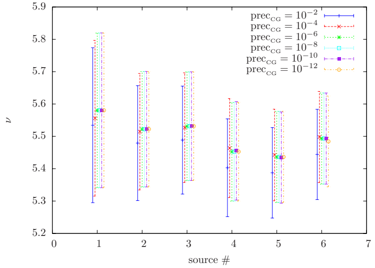




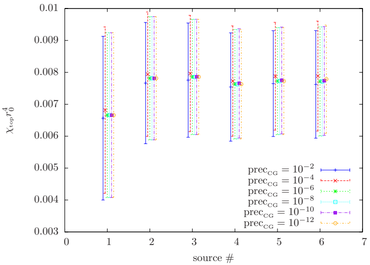
Solver Precision

- $(D^{\dagger}_{tm,\mu} D_{tm,\mu} + M^2_{\star})\psi = \eta$ better conditioned than $D_m\psi = \eta$
- $16^3 \times 32$ lattice with $\beta = 3.9$ for $N_f = 2$ using 100 configs.

⋆ Mode Number ν



⋆ Topological Susceptibility $\chi_{\rm top}$



- Conservatively set precision to $prec_{CG} = 10^{-6}$

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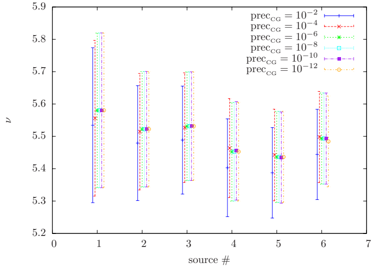

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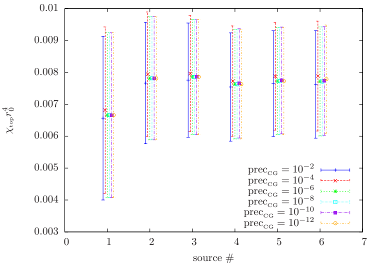

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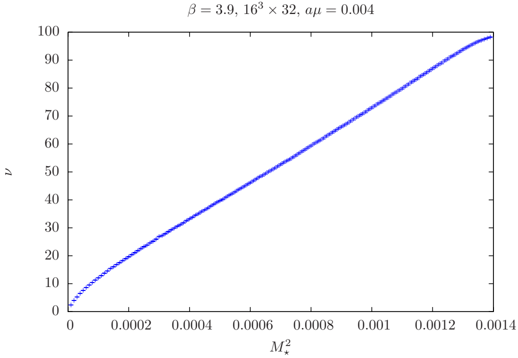
M* for chiral condensate

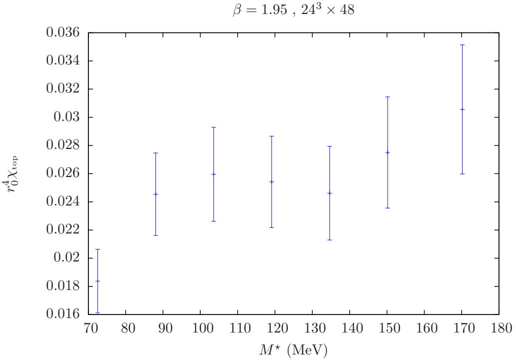
We want to compute the mode number in the linear region.



avoid values $\approx m_q$ and $>> m_q$

M* for topological susceptibility

Optimal value of $M_\star$ to compute $\chi_{\rm top}$



value set to $M_\star \approx 100$ MeV

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Configurations setup

- Wilson Twisted Mass Action at maximal twist
- Iwasaki Gauge Action

- $N_{\rm f} = 2 + 1 + 1$ dynamical fermions
- light valence quark mass $m_{val} = m_{sea}$
- $\beta = 1.95$, $a = 0.078$ fm
- $32^3 \times 64$
  - ⋆ $a\mu_{sea} = 0.0025, 0.0035, 0.0055, 0.0075$
  - ⋆ $270 \text{ MeV} \le M_\pi \le 500 \text{ MeV}$
- $24^3 \times 48$
  - $a\mu_{sea} = 0.0085$

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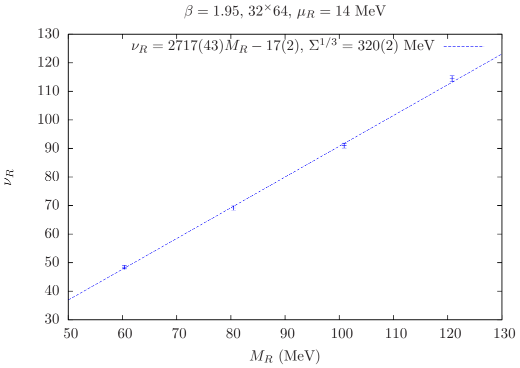
Extracting Σ_R from ν_R

$$\Sigma_R = \frac{\pi}{2V}\sqrt{1-\left(\frac{m_R}{M_R}\right)^2}\frac{\partial}{\partial M_R}\nu_R$$

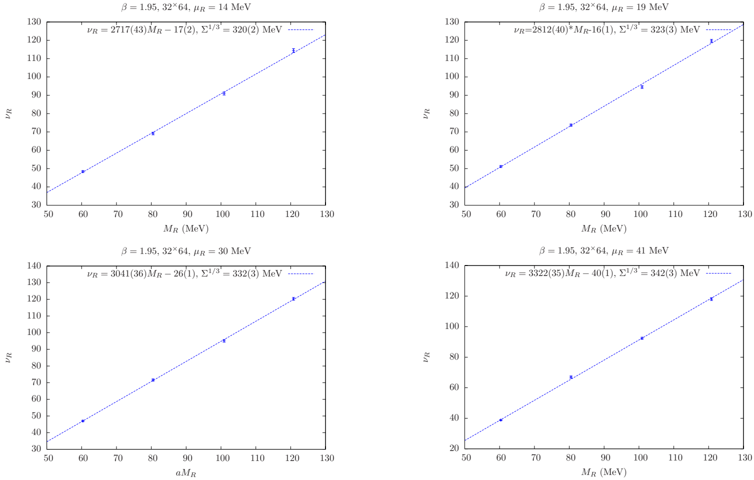
$\nu(M,m) = \nu_R(M_R, m_R)$ $\rightsquigarrow$ renormalization-group invariant (Giusti & Lüscher, 2008)

- We extract the term $\frac{\partial}{\partial M_R}\nu_R$ through the slope of a linear fit.



Extracting Σ_R from ν_R

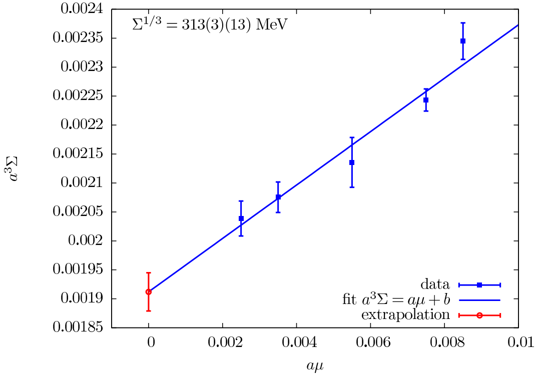
- Thus we get a value of the Σ_R for each value of the m_q



$M_R = 95\ MeV$, $\quad Z_P[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 0.462(6)$ preliminary value!

Chiral Limit of Σ

Performing the chiral extrapolation $\rightsquigarrow$ $\Sigma^{1/3} = 304(3)(13)$ MeV



m_q	$a\Sigma_{m_q}$
13.6	2.0(3)e-03
19.1	2.1(3)e-03
30.0	2.3(3)e-03
41.0	2.5(3)e-03
46.4	2.7(4)e-03

⋆ Comparison to $\Sigma = \frac{-B_0 f_0^2}{2}$ (ETMC, 2010) (1004.5284)

Result obtained through chiral fits results is $\Sigma^{1/3} = 276(2)(11)$ MeV

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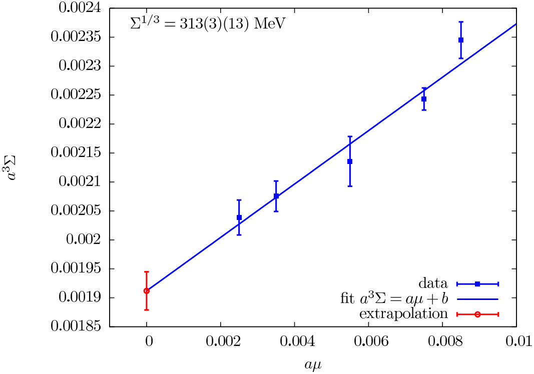

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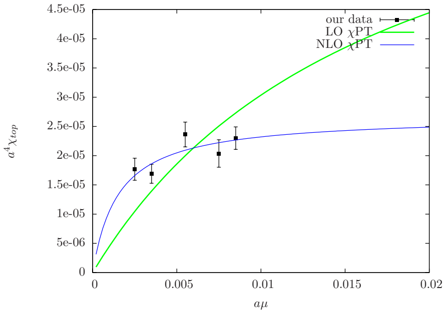
Chiral Behavior Our first go



LO χPT:

$$\chi_{top} = \Sigma \left[\frac{2}{m_u} + \frac{1}{m_s} + \frac{1}{m_c} \right]^{-1}$$

NLO χPT: (Mao & Chiu, 2009)

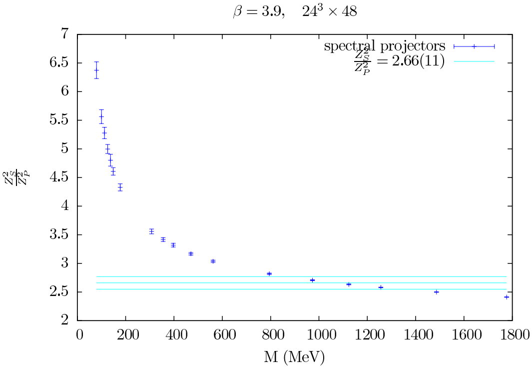
$$\chi_{top} = \Sigma \left[\frac{2}{m_u} \left(1 - \frac{3}{2F_\pi^2} \frac{M_\pi^2}{16\pi^2} \ln \frac{M_\pi^2}{\mu_{sub}^2} - m_u K \right) + \frac{1}{m_s} + \frac{1}{m_c} \right]^{-1}$$

$$, K = 2K_6 + 2K_7 + K_8, \qquad M_\pi^2 = \frac{2m_u \Sigma}{F^2}$$

$\rightarrow$ leads to $\Sigma^{1/3} \approx 500\text{GeV}$

Problem with Z_P/Z_S

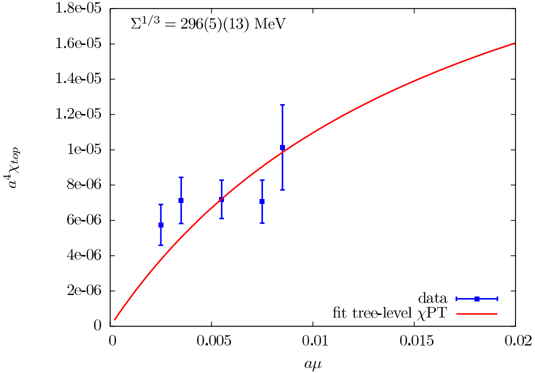
Assumed: Z_P/Z_S independent from M^*



$\rightarrow$ see clear dependence

$\Rightarrow$ use directly computed Z_P/Z_S

Topological susceptibility



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Witten-Veneziano Formula

$$\frac{f_\pi^2}{2N_f}(m_\eta^2 + m_{\eta'}^2 - 2m_K^2) = \chi_\infty$$

- first attempt to understand mass of η'
- is obtained through Ward identities.
- Our goal compute the right hand side χ_∞

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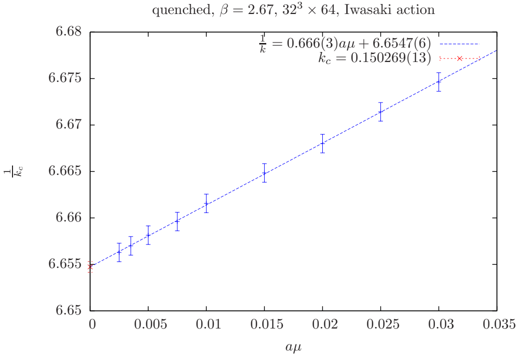
Strategy to Compute χ_{top} quenched

- matching $r_0/a = 5.71(4)$ of dynamical simulation.
- Iwasaki action.
- Compute the k_c for several values of $a\mu$ and extrapolate to the chiral limit.



- Compute the value of $a\mu$ which matches the value of m_π of the dynamical simulation. In our case around $m_\pi = 390$MeV

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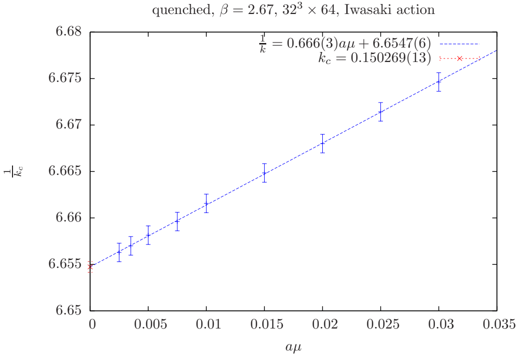

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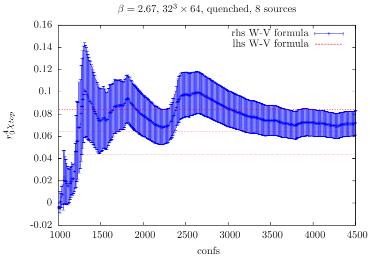
Autocorrelation problems in the quenched case

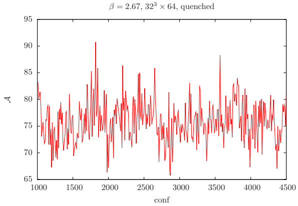
$$\chi_{top} = \frac{\langle \mathcal{A} \rangle \left(\left\langle \mathcal{C}^2 \right\rangle - \frac{\langle \mathcal{B} \rangle}{N} \right)}{V \langle \mathcal{B} \rangle}$$

⇝ acumulated mean of χ_∞:

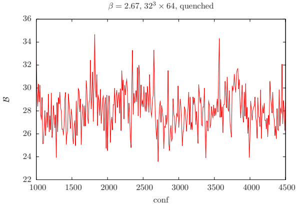


Autocorrelation

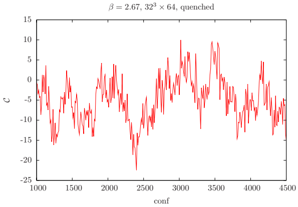
$$\mathcal{A} = \frac{1}{N}\sum_{k=1}^{N}(\mathbb{R}_M^2\eta_k, \mathbb{R}_M^2\eta_k),$$



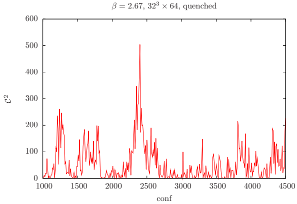
$$\mathcal{B} = \frac{1}{N}\sum_{k=1}^{N}(\mathbb{R}_M\gamma_5\mathbb{R}_M\eta_k, \mathbb{R}_M\gamma_5\mathbb{R}_M\eta_k)$$



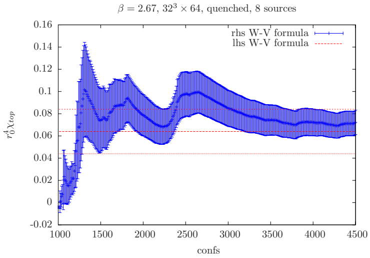
$$\mathcal{C} = \frac{1}{N}\sum_{k=1}^{N}(\mathbb{R}_M\eta_k, \gamma_5\mathbb{R}_M\eta_k)$$

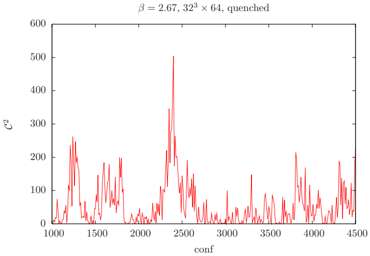


$$\mathcal{C}^2$$



Autocorrelation

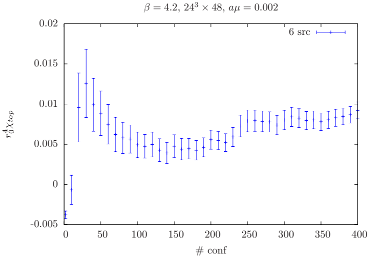




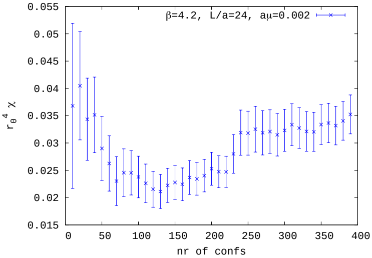
Topological Barriers

$\beta = 4.2, \quad 24^3 \times 48$ small volume

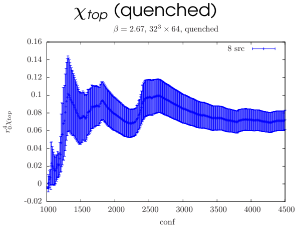
- Spectral Projectors:

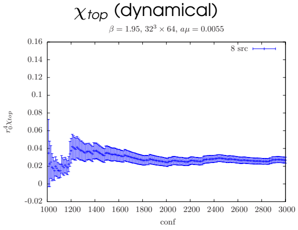


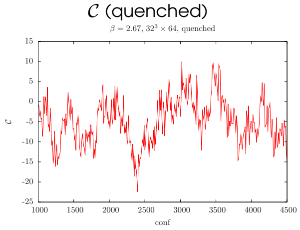
- Index:

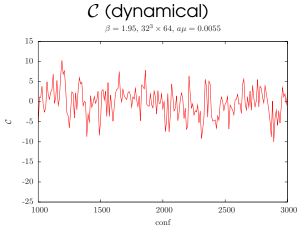


Quenched vs Dynamical









Witten-Veneziano Formula

Preliminary results!

$$\frac{f_\pi^2}{2N_f}(m_\eta^2 + m_{\eta'}^2 - 2m_K^2) = \chi_\infty$$

am_η	$am_{\eta'}$	am_K	af_π	$r_0^4\chi_\infty$
0.230(10)	0.384(24)	0.2280(4)	0.0656(2)	0.073(12)

$$(r_0/a)^4\left(\frac{(af_\pi)^2}{2N_f}((am_\eta)^2 + (am_{\eta'})^2 - 2(am_K)^2)\right) = 0.066(20)$$

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Conclusions and outlook

- We have applied the spectral projector method using $N_f = 2 + 1 + 1$ twisted mass ensembles generated by ETMC.

- Σ for 5 values of $a\mu$ performing the chiral extrapolation.
  Find consistency between methods. Using $Z_P = 0.462(6)$

  ⋆ $\Sigma^{1/3}|_{sp} = 304(3)(13)$ MeV ⋆ $\Sigma^{1/3}|_{\chi} = 296(5)(13)$ MeV ⋆ $\Sigma^{1/3}|_{ch} = 276(2)(11)$ MeV

- We have looked at the Witten-Veneziano formula which is roughly fulfilled.
  - ⋆ We need to increase statistics to overcome problem of autocorrelation.

- Spectral projector good indicator for autocorrelations

- Check lattice spacing effects in quenched and dynamical case ($\beta = 2.1$, $a \approx 0.06$ fm).

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Thank you!

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