$\mathcal{O}(a^2)$ Corrected Renormalization Constants $N_f = 4$

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OUTLINE

Operators

$$
\begin{aligned}
\mathcal{O}^{\mathbf{S}} &= \hat{1} \\
\mathcal{O}^{\mathbf{P}} &= \gamma^5 \\
\mathcal{O}^{\mathbf{V}} &= \gamma^\mu \\
\mathcal{O}^{\mathbf{A}} &= \gamma^5\,\gamma^\mu \\
\mathcal{O}^{\mathbf{T}} &= \gamma^5\,\sigma^{\mu\,\nu} \\
\mathcal{O}^{\mathbf{DV}} &= \gamma^{\{\mu}\,\overleftrightarrow{D}\,^{\nu\}} \\
\mathcal{O}^{\mathbf{DA}} &= \gamma^5\,\gamma^{\{\mu}\,\overleftrightarrow{D}\,^{\nu\}}
\end{aligned}
$$

- No mixing with lower dimension operators
- No disconnected diagrams
- symmetrization and subtraction of the traces

Momentum Source Method

- ★ Dirac equation solved with momentum source
- ★ $\sharp$ of inversion depends on the $\sharp$ of momenta considered
- ★ Application of any operator
- ★ High statistical accuracy is achieved

Statistics

	β	k_{cr}	μ_0	$L^3 \times T$
3p	1.95	0.160826	0.0180	$24^3 \times 48$
3m	1.95	0.161229	0.0180	$24^3 \times 48$
2p	1.95	0.160826	0.0085	$24^3 \times 48$
2m	1.95	0.161229	0.0085	$24^3 \times 48$
8p	1.95	0.160524	0.0020	$24^3 \times 48$
8m	1.95	0.161585	0.0020	$24^3 \times 48$
5p	2.10	0.155949	0.0078	$32^3 \times 64$
5m	2.10	0.156291	0.0078	$32^3 \times 64$
4m	2.10	0.156250	0.0064	$32^3 \times 64$
3p	2.10	0.156017	0.0046	$32^3 \times 64$
2ap	2.10	0.166042	0.0030	$32^3 \times 64$
2am	2.10	0.166157	0.0030	$32^3 \times 64$

$\beta = 1.95\,(\mu = 0.0180,\ 0.0085)$	$\beta = 1.95\,(\mu = 0.002)$	$\beta = 2.10$
(n_t,2,2,2), $n_t : 1-7$	(n_t,2,2,2), $n_t : 10-12$	(n_t,2,2,2), $n_t : 3-12$
(n_t,3,3,3), $n_t : 1-10$	(n_t,3,3,3), $n_t : 5-10$	(n_t,3,3,3), $n_t : 1-13$
(3,3,3,2)	(3,3,3,2), $n_t : 8-10$	(n_t,4,4,4), $n_t : 4-15$
(3,3,3,4)	(3,3,3,4)	(n_t,5,5,5), $n_t : 2-11$
(3,3,4,4)	(3,3,4,4)	(3,3,3,2),(3,3,2,2)
		(3,3,3,4),(3,3,4,4)

$\sim$ **10-50 configurations per momentum**

Renormalization Conditions (RI$'$-MOM scheme):

$$Z_q = \frac{1}{12} \mathrm{Tr}[(S^L(p))^{-1} \, S^{(0)}(p)]\Big|_{p^2=\mu^2}$$

$$Z_q^{-1} \, Z_{\mathcal{O}}^{\mu\nu} \, \frac{1}{12} \mathrm{Tr}[\Gamma^L_{\mu\nu}(p) \, \Gamma^{(0)-1}{}_{\mu\nu}(p)]\Big|_{p^2=\mu^2} \quad = \quad 1$$

$$S^{(0)}(p) \quad = \quad \frac{-i \sum_\rho \gamma_\rho \sin(p_\rho)}{\sum_\rho \sin(p_\rho)^2}$$

$$\Gamma^{(0)}_{\mu\nu}(p) \quad = \quad -i \, \tilde{\Gamma}_{\{\mu} \, \sin(p_{\nu\}})$$

$$\frac{1}{L^2} \ll \Lambda^2_{\mathrm{QCD}} \ll \bar{\mu}^2 \ll \frac{1}{a^2}$$

- **Reliable perturbation theory (Wilson coefficients)**
- **Small $\mathcal{O}(a)$ lattice effects**

In practice: $0.5 \leq (a\, p)^2 \leq 4.5$

Example of Perturbative Results:

- **Iwasaki gluons ,** $c_{\rm SW} = 0$
- **Landau gauge**
- $m = 0$**,** $\mu_0 = 0$

$$
\begin{aligned}
\frac{1}{4}\mathrm{Tr}\left[(S^L(p))^{-1}\, S^{(0)}_{\rm tree}(p)\right]\Big|_{p_\rho=\mu_\rho} &= 1+\tilde{g}^2\Big\{-8.11657 \\
&+ a^2\Big[\mu^2\,(0.620227-0.0748167\ln(a^2\,\mu^2)) \\
&+ \frac{\sum_\rho \mu_\rho^4}{\mu^2}(1.85344-0.963033\ln(a^2\,\mu^2))\Big]\Big\} \\
&+ \mathcal{O}(a^4\,g^2, g^4)
\end{aligned}
$$

- **Ambiguity on the choice of the momentum direction**

- **(3,3,3,3):** $(a\,p)^2 = 2.06045,\ (a\,p)^4 = 1.18557$
- **(8,2,2,2):** $(a\,p)^2 = 2.06045,\ (a\,p)^4 = 1.75809$

Criterion for choosing the momenta

arXiv:1004.1115

$$\text{local}: \frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} \leq 0.28 \qquad \text{twist}-2: \frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} \leq 0.4$$

Minimizes the $\sum_\rho p_\rho^4$ contributions of the perturbative expressions

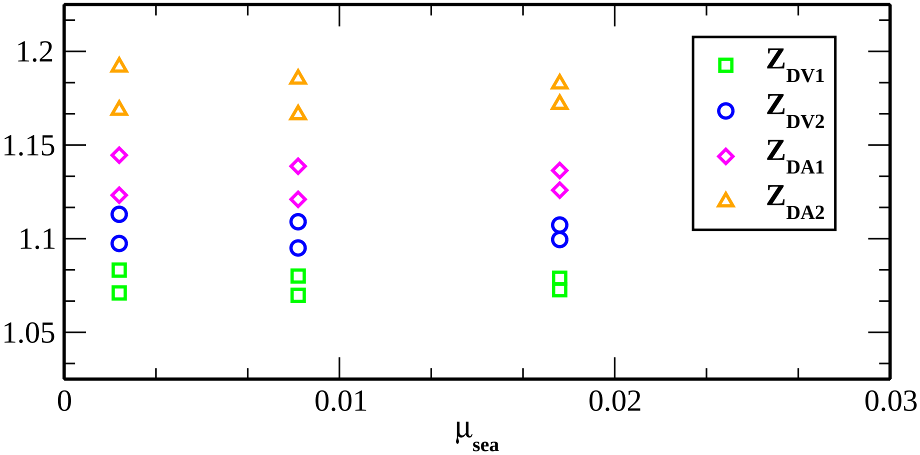
$$Z_{\mathrm{DV1}}^{\mathrm{impr}} = Z_{\mathrm{DV1}}$$
$$-a^2\tilde{g}^2\Bigg[c_1\, p2 + c_2\, p_{\nu_1}^2 + \frac{c_3\, p_{\nu_1}^4}{p2} + \frac{c_4\, p_{\nu_1}^4}{8p_{\nu_1}^2 + p2} + \frac{c_5\, p_{\nu_1}^6}{(8p_{\nu_1}^2 + p2)^2} + \left(c_6\, p2 + c_7\, p_{\nu_1}^2 + \frac{c_8\, p_{\nu_1}^4}{(8p_{\nu_1}^2 + p2)}\right)\ln[a^2 p2]$$
$$+p4\left(c_9\frac{1}{p2} + c_{10}\frac{p_{\nu_1}^2}{p2^2} + c_{11}\frac{1}{8p_{\nu_1}^2 + p2} + c_{12}\frac{p_{\nu_1}^2}{(8p_{\nu_1}^2 + p2)^2}\right.$$
$$\left. + \left(c_{13}\frac{1}{p2} + c_{14}\frac{1}{(8p_{\nu_1}^2 + p2)}\right)\ln[a^2 p2]\right)\Bigg]$$

No momentum criterion was applied here.

Mass dependence

$\beta = 1.95$

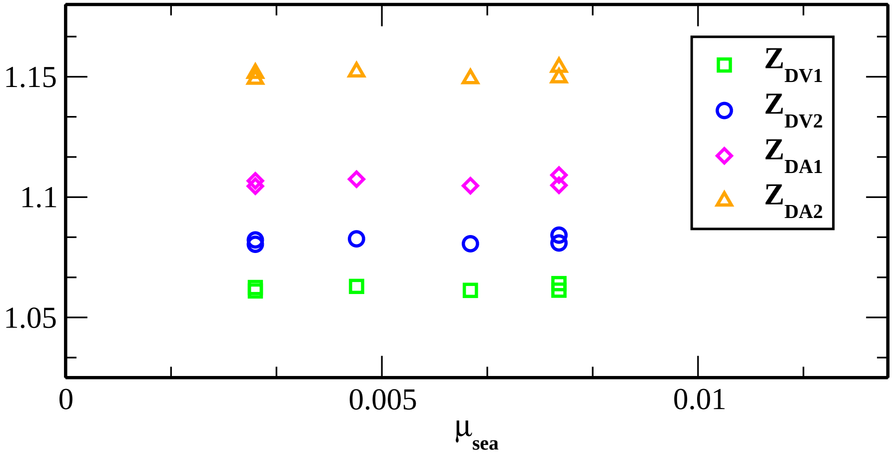
$24^3 \times 48$



Mass dependence

$\beta = 2.10$

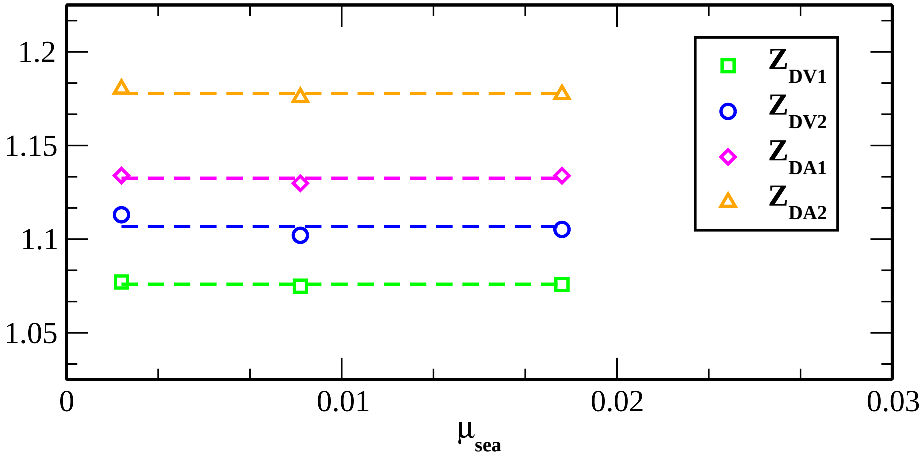
$32^3 \times 64$



Theta average: Mass dependence

$\beta = 1.95$

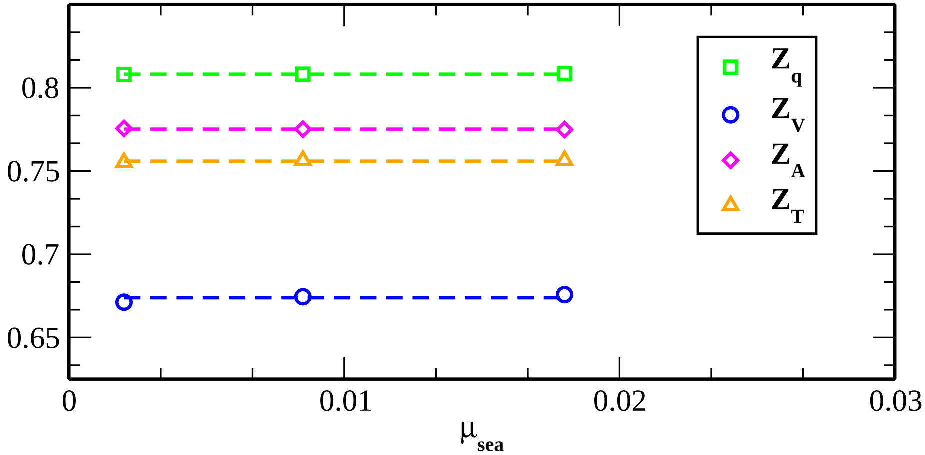
$24^3 \times 48$



Theta average: Mass dependence

$\beta = 1.95$

$24^3 \times 48$



Volume effects

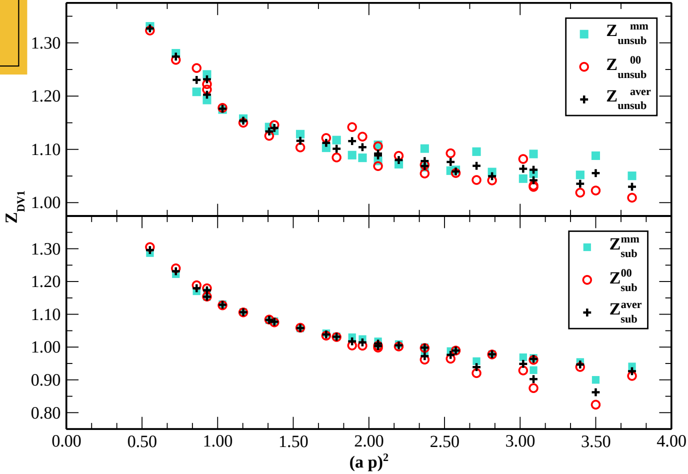
Not studied yet

We don’t expect significant dependence on volume

Averaging over different directions

$\beta = 1.95$

$24^3 \times 48$



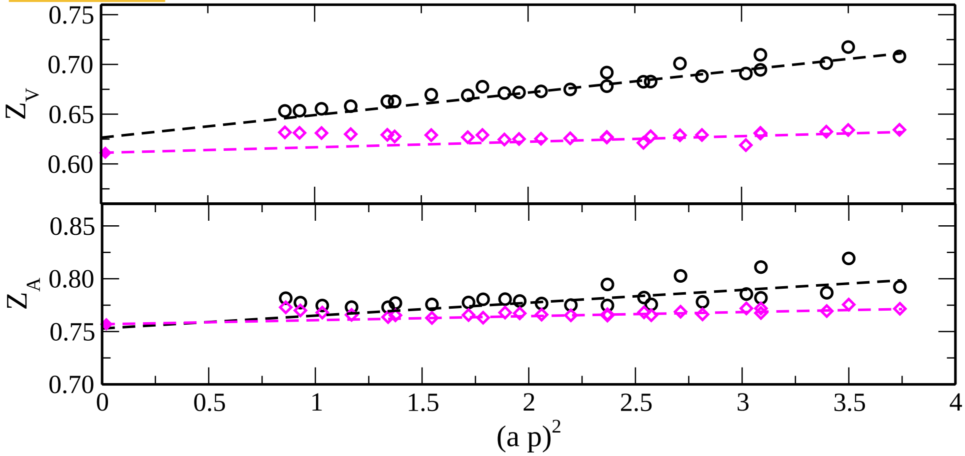
$$Z^{aver}_{DV1} = \left(Z^{00}_{DV1} + \sum_{i=1}^{3} Z^{ii}_{DV1}/3 \right)/2$$

Conversion to $\overline{\text{MS}}$: $Z^{\overline{\text{MS}}}_{DV1} = C^{aver}\, Z^{aver}_{DV1}$

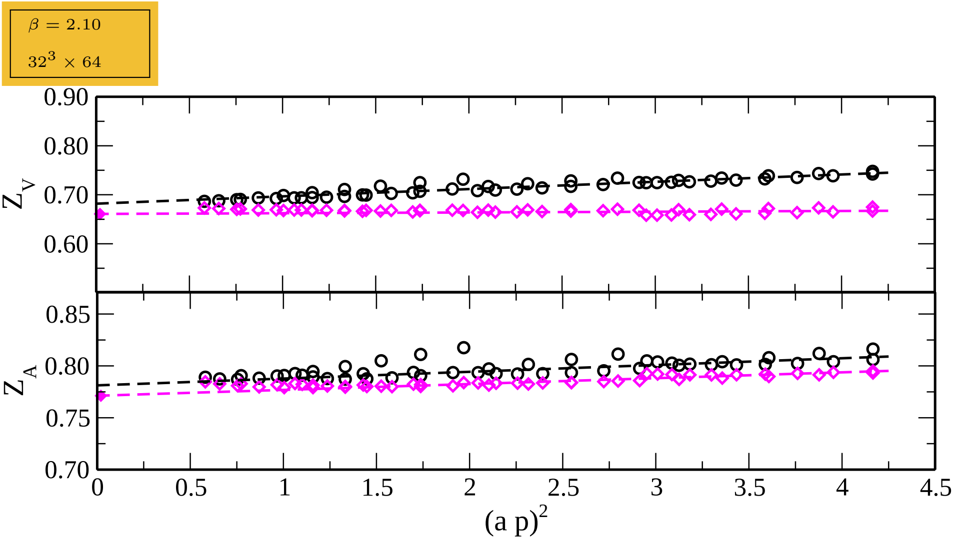
Z_V, Z_A

$\beta = 1.95$

$24^3 \times 48$



Z_V, Z_A

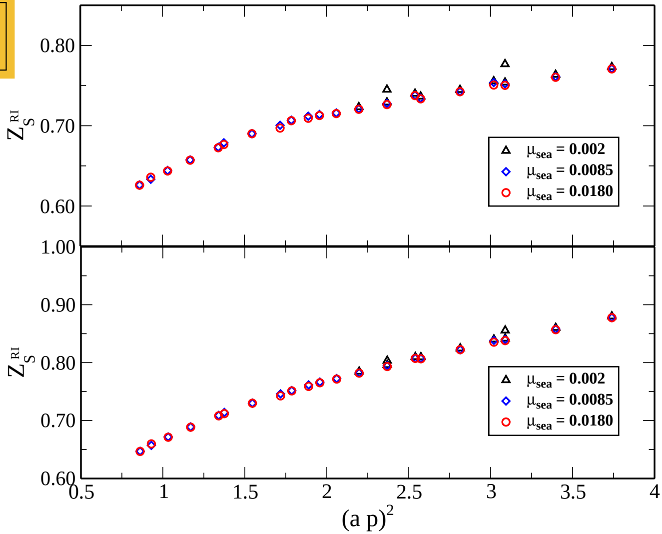


β = 2.10
32³ × 64
0.90
0.80
0.70
0.60
Z_V
0.85
0.80
0.75
0.70
Z_A
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
(a p)²

Pion pole for Z_S

$\beta = 1.95$

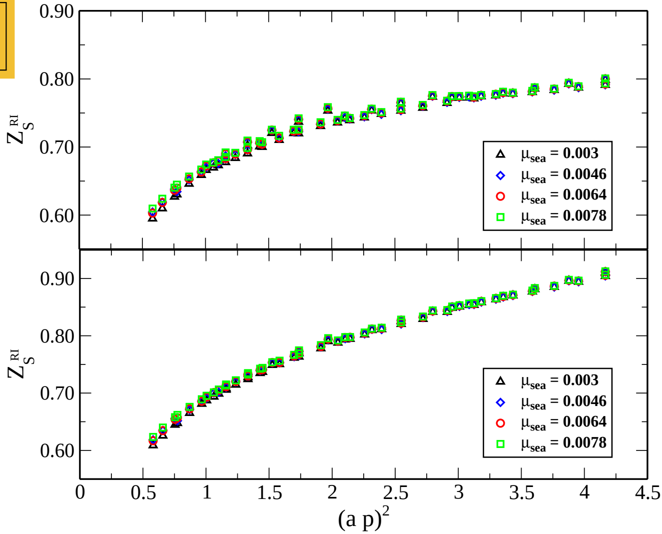
$24^3 \times 48$



Pion pole for Z_S

$\beta = 2.10$

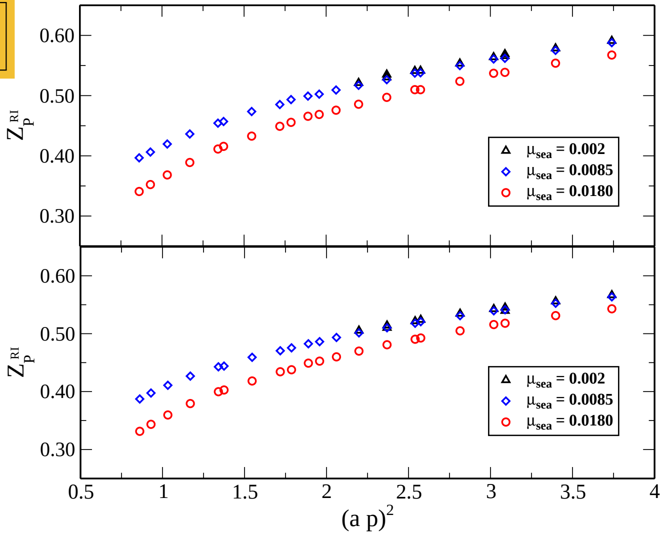
$32^3 \times 64$



Pion pole for Z_P

$\beta = 1.95$

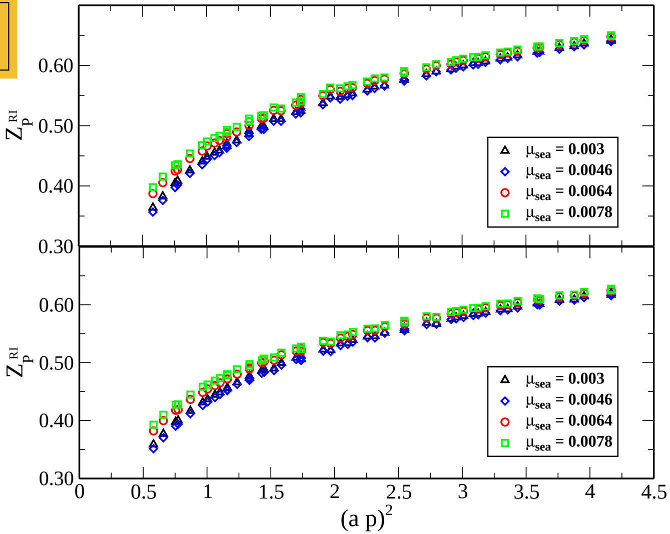
$24^3 \times 48$



Pion pole for Z_P

$\beta = 2.10$

$32^3 \times 64$



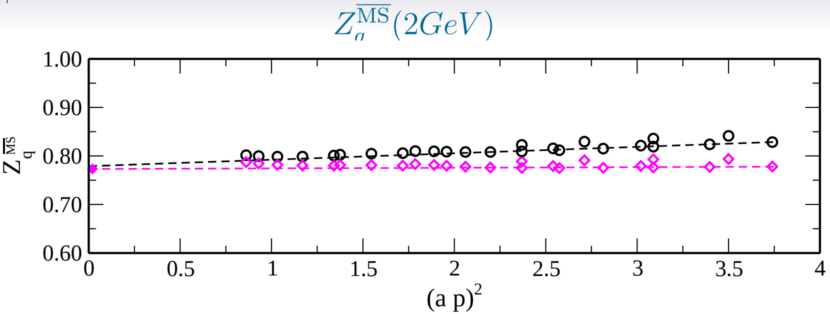
Conversion to $\overline{\mathrm{MS}}$

ultra-local operators: 2-loop formulae

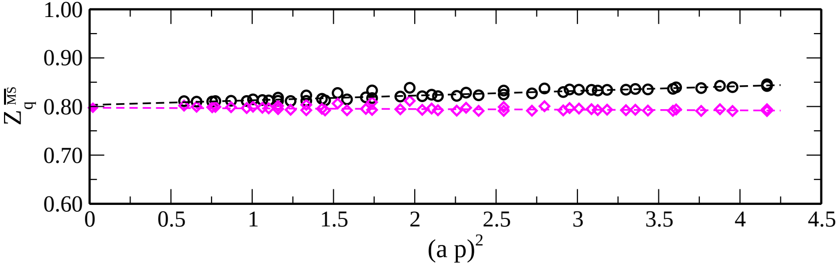
one-derivative operators: 3-loop formulae

★ **bare coupling**

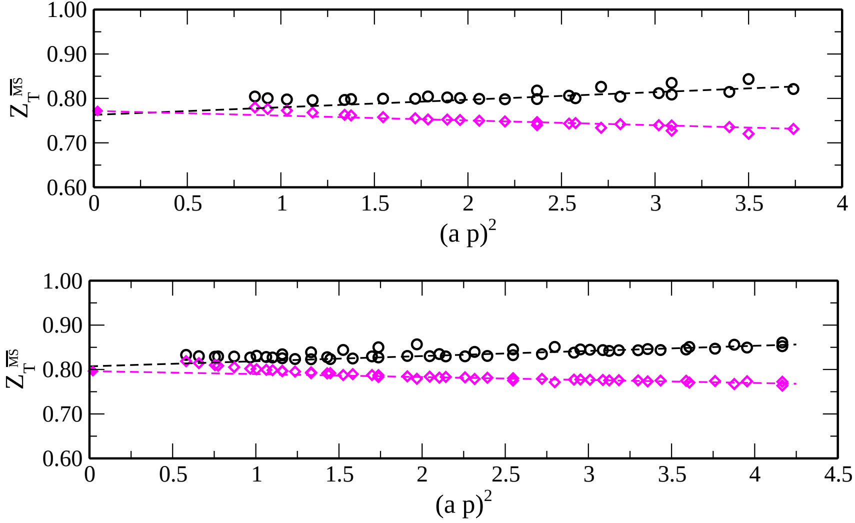
$\beta = 1.95$



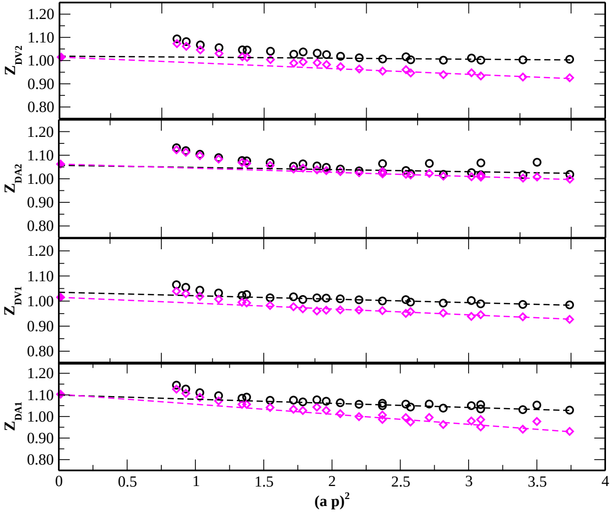
$\beta = 2.10$


1.00
0.90
0.80
0.70
0.60
Z q MS
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
(a p)2

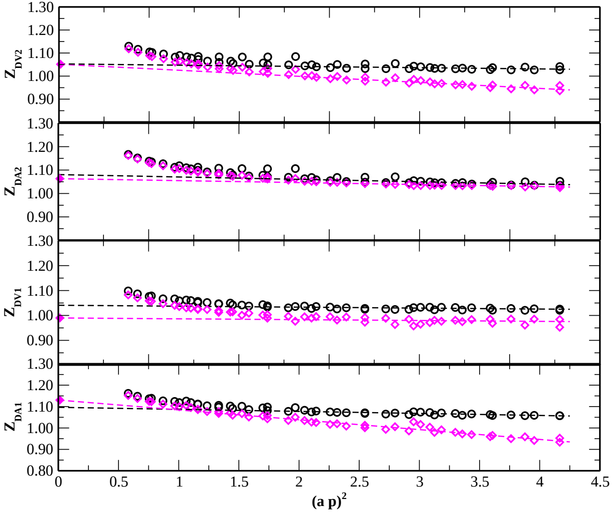
$Z_T^{\overline{\rm MS}}(2GeV)$



$Z^{\overline{\rm MS}}_{\rm DV}(2GeV)$, $Z^{\overline{\rm MS}}_{\rm DA}(2GeV)$



$Z_{\mathrm{DV}}^{\overline{\mathrm{MS}}}(2GeV)$, $Z_{\mathrm{DA}}^{\overline{\mathrm{MS}}}(2GeV)$



Fitting ($\overline{\mathrm{MS}}$ at 2GeV)

Very Preliminary Results

β	Z_V	Z_A	Z_T	Z_q
1.95	0.611(9)	0.757(3)	0.773(2)	0.7719(9)
2.10	0.661(5)	0.771(2)	0.796(3)	0.798(3)

β	Z_{DV1}	Z_{DV2}	Z_{DA1}	Z_{DA2}
1.95	1.015(6)	1.015(9)	1.103(9)	1.062(3)
2.10	0.990(9)	1.052(9)	1.13(1)	1.063(3)

Systematic errors: Not computed yet

Summary

- $\mathcal{O}(a^2)$ **subtraction are crucial**
- **Mass dependence is insignificant after θ-averaging**

Future Work

- **Extend momentum range**
- **Explore volume dependence**
- **Pion pole subtraction for Z_S and Z_P**
- **Apply criterion for momentum selection (each operator separately)**
- **Systematic and statistical errors**

THANK YOU