Neutral Kaon Mixing Beyond the SM from N_f=2 tmQCD

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Petros Dimopoulos

on behalf of

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Motivation

- FCNC and CP violation
  - → provide valuable precise information for the UT
  - → furnish various useful constraints on estimates for degrees of freedom and possible scales in New Physics models/scenarios

- Using BSM *ΔS*=2 box diagrams → Estimates for the upper bounds of off-diagonal elements of the squark mass matrix.
  [Gabbiani & Masiero NPB 1989]
  [Gabbiani *et al.* NPB 1996]
  [Ciuchini *et al.* JHEP 1998]

- Considering BSM *ΔF*=2 ops. in a UT fit analysis → Lower bounds on NP scale for various BSM scenarios with/out MFV.
  [Utfit; Bona *et al*. JHEP 2008]

Exploit the margin between experimental values and SM predictions and get possible constraints for unknown degrees of freedom assumed in the (OPE) Wilson parameters.

⇒ ***<u>Necessary step</u>: calculate Matrix Elements on the lattice.***

Most General Effective Hamiltonian for **ΔS = 2** processes:

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} C_i O_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{O}_i$$

$$\begin{aligned}
O_1 &= [\bar{s}^\alpha \gamma_\mu (1-\gamma_5) d^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) d^\beta] \\
O_2 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1-\gamma_5) d^\beta] \\
O_3 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1-\gamma_5) d^\alpha] \\
O_4 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1+\gamma_5) d^\beta] \\
O_5 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha]
\end{aligned}$$

$$(1-\gamma_5) \leftrightarrow (1+\gamma_5)$$

$$\begin{aligned}
\tilde{O}_1 &= [\bar{s}^\alpha \gamma_\mu (1+\gamma_5) d^\alpha][\bar{s}^\beta \gamma_\mu (1+\gamma_5) d^\beta] \\
\tilde{O}_2 &= [\bar{s}^\alpha (1+\gamma_5) d^\alpha][\bar{s}^\beta (1+\gamma_5) d^\beta] \\
\tilde{O}_3 &= [\bar{s}^\alpha (1+\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha]
\end{aligned}$$

$$< \bar{K}|O_i|K >_{p-even} = < \bar{K}|\tilde{O}_i|K >_{p-even}$$

SM: $$< \bar{K}|O_1|K >_{p-even} \implies B_K$$

Parity-even Op.

$$
\begin{aligned}
Q^{VV} &= (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) \\
Q^{AA} &= (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d) \\
Q^{PP} &= (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \\
Q^{SS} &= (\bar{s}d)(\bar{s}d) \\
Q^{TT} &= (\bar{s}\sigma_{\mu\nu} d)(\bar{s}\sigma_{\mu\nu} d)
\end{aligned}
$$

→

Lattice Basis

$$
\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix} = \begin{pmatrix} Q^{VV} + O^{AA} \\ Q^{VV} - O^{AA} \\ Q^{SS} - O^{PP} \\ Q^{SS} + O^{PP} \\ Q^{TT} \end{pmatrix}
$$

Use Fierz transformation:

$$
\begin{aligned}
O_1 &= Q_1 \\
O_2 &= Q_4 \\
O_3 &= -\tfrac{1}{2}(Q_4 - Q_5) \\
O_4 &= Q_3 \\
O_5 &= -\tfrac{1}{2} Q_2
\end{aligned}
$$

$$
\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C_1(\mu) \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle \left(1 + \sum_{i=2,\ldots,5} \frac{C_i(\mu)}{C_1(\mu)} \frac{\bar{K}^0 | O_i(\mu) | K^0 \rangle}{\bar{K}^0 | O_1(\mu) | K^0 \rangle} \right)
$$

Bag parameters

$$\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \xi_1 \, B_1(\mu) \, m_K^2 f_K^2$$

$$\langle \bar{K}^0 | O_i(\mu) | K^0 \rangle = \xi_i \, B_i(\mu) \left[\frac{m_K^2 f_K}{m_s(\mu) + m_d(\mu)} \right]^2 \quad \text{for } i = 2, \dots, 5$$

with $\xi_i = (8/3, \, -5/3, \, 1/3, \, 2, \, 2/3)$

- $B_1 \equiv B_K$
- $$\frac{\langle \bar{K}^0 | O_i(\mu) | K^0 \rangle}{\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle} = \frac{\xi_i B_i(\mu)}{\xi_1 B_1(\mu)} \frac{m_K^2}{(\mu_s(\mu) + \mu_d(\mu))^2}$$

Extra systematic error due to q-masses… Not small! ($\delta\mu_q$ ~7%)

Direct calculation of the ME ratio

$$\frac{\bar{K}^0|O_i(\mu)|K^0\rangle}{\bar{K}^0|O_1(\mu)|K^0\rangle}$$

In the chiral limit:

- $< \bar{K}|O_1|K > \longrightarrow 0$
- $< \bar{K}|O_{i\,(\neq 1)}|K >$ *finite*

Construct the ME ratio taking care of the chiral properties

(See below)

Mixed Action setup

[Frezzotti & Rossi JHEP 2004]
[Constantinou *et al*. PRD 2010]

- Mtm sea quarks ($N_f = 2$)

$$S_{\rm Mtm} = a^4 \sum_x \bar{\psi}(x) \Big\{ \tfrac{1}{2} \sum_\mu \gamma_\mu (\nabla_\mu + \nabla^*_\mu) - i\gamma_5 \tau^3 \Big[M_{\rm cr} - \frac{a}{2} \sum_\mu \nabla^*_\mu \nabla_\mu \Big] + \mu_{sea} \Big\} \psi(x)$$

- Osterwalder-Seiler valence quarks (Max. twisted)

$$S_{\rm OS} = a^4 \sum_{x,f} \bar{q}_f(x) \Big\{ \tfrac{1}{2} \sum_\mu \gamma_\mu (\nabla_\mu + \nabla^*_\mu) - i\gamma_5 r_f \Big[M_{\rm cr} - \tfrac{a}{2} \sum_\mu \nabla^*_\mu \nabla_\mu \Big] + \mu_f \Big\} q_f(x)$$

q_f : single quark flavour $f = 1, \ldots 4$ → *two replicas for d and s quarks*

$$\left.\begin{array}{l} \mu_1 = \mu_3 \equiv \mu_{``s"} \\ \mu_2 = \mu_4 \equiv \mu_\ell \end{array}\right\} \quad r_1 = r_2 = r_3 = -r_4$$

"OS" meson ($r_1 = r_2$) **"tm" meson** ($r_3 = -r_4$)

→ $Q_{\Gamma\Gamma} = \frac{1}{2} \{ [\bar{q}_1 \Gamma q_2][\bar{q}_3 \Gamma q_4] + (q_2 \leftrightarrow q_4) \}$

- Automatic O(a) improvement **&** Continuum-like ren. pattern for 4f Operators
  (**No** unwanted mixings due chiral (Wilson) SB up to $O(a^2)$)
- Uniratity Violations up to $O(a^2)$

Simulation details

$\beta = 3.80$, $a^{-1} \sim 0.10$ fm			
$a\mu_\ell = a\mu_{sea}$	$a^{-4}(L^3 \times T)$	$a\mu_{“s”}$	N_{stat}
0.0080	$24^3 \times 48$	0.0165, 0.0200, 0.0250	170
0.0110	“	“	180
$\beta = 3.90$, $a^{-1} \sim 0.09$ fm			
0.0040	$24^3 \times 48$	0.0150, 0.0220, 0.0270	400
0.0064	“	“	200
0.0085	“	“	200
0.0100	“	“	160
0.0030	$32^3 \times 64$	“	300
0.0040	“	“	160
$\beta = 4.05$, $a^{-1} \sim 0.07$ fm			
0.0030	$32^3 \times 64$	0.0120, 0.0150, 0.0180	190
0.0060	“	“	150
0.0080	“	“	220

(Correlator functions at β=4.20 have been computed; RCs not ready yet.
β=4.20 *not* included in this analysis.)

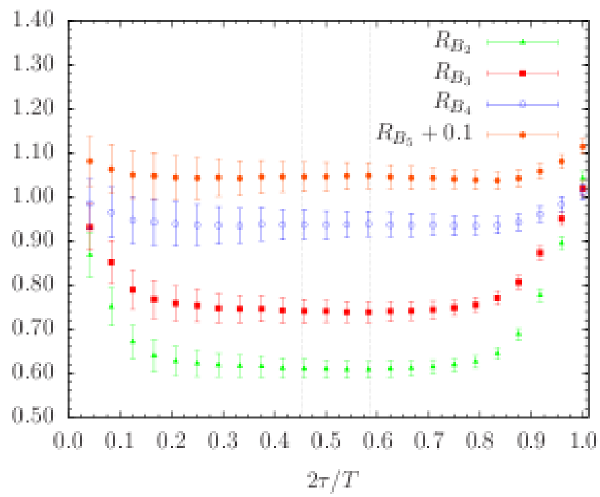
$$R_{B_i}(x_0) \;=\; \frac{C_i(x_0)}{C_{PP}(x_0)\; C'_{PP}(x_0)}, \quad i = 2, \ldots, 5$$

$$C_i(x_0) = \left(\frac{a}{L}\right)^3 \sum_{\vec{x}} \langle \mathcal{P}^{43}(y_0 + \tfrac{T}{2})\, O_i(x)\, \mathcal{P}^{21}(y_0)\rangle \qquad i = 1, \ldots, 5$$

$$C_{PP}(x_0) = \left(\frac{a}{L}\right)^3 \sum_{\vec{x}} \langle P^{12}(x)\, \mathcal{P}^{21}(y_0)\rangle\,,$$

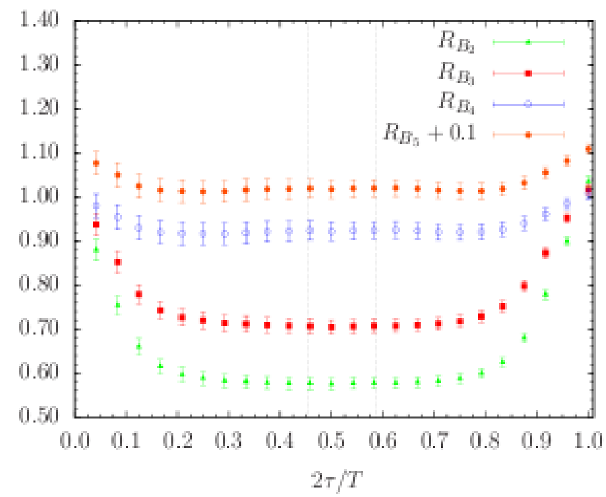
$$C'_{PP}(x_0) = \left(\frac{a}{L}\right)^3 \sum_{\vec{x}} \langle \mathcal{P}^{43}(y_0 + \tfrac{T}{2})\, P^{34}(x)\rangle\,.$$

β=3.80



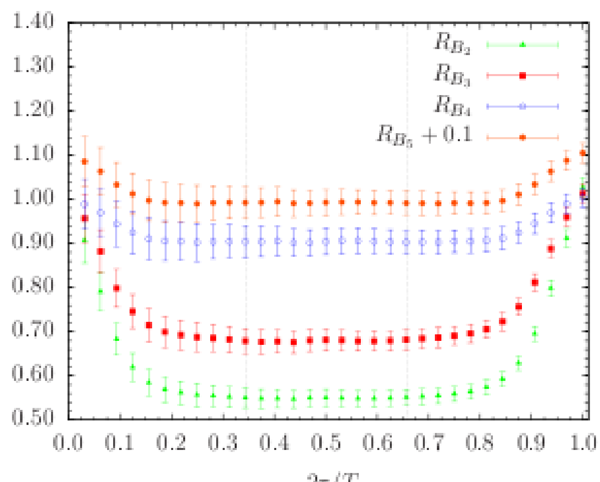
(a)

β=3.90



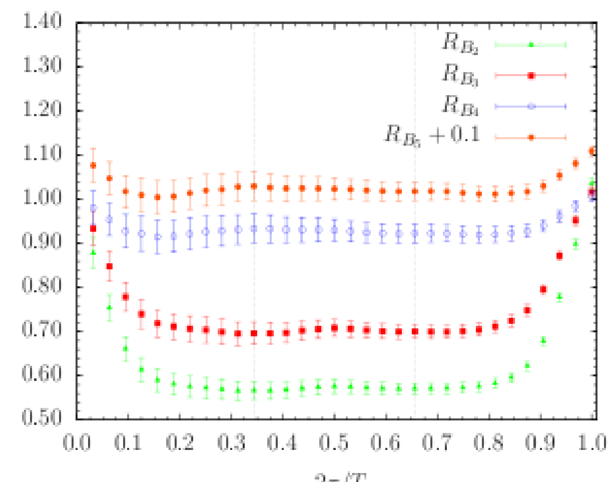
(b)

β=4.05



(c)

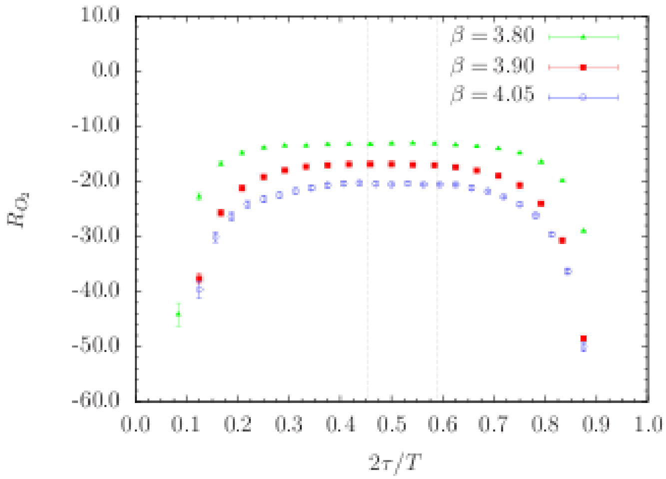
β=3.90



(d)

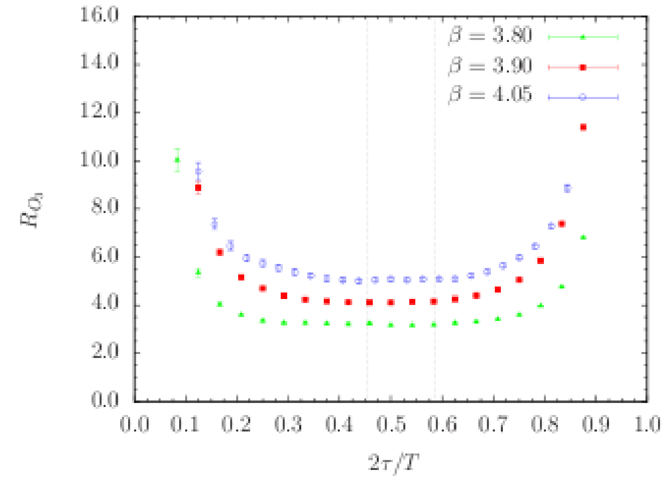
$$R_{O_i}(x_0) = \frac{C_i(x_0)}{C_1(x_0)} \xrightarrow{y_0 \ll x_0 \ll y_0+T/2} \frac{\langle \bar{K}^0|O_i|K^0\rangle}{\langle \bar{K}^0|O_1|K^0\rangle}, \quad i = 2, \ldots, 5$$

R_{O2}



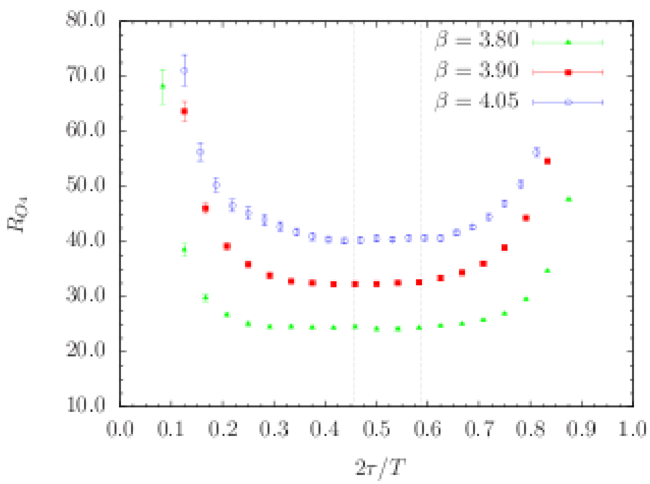
(a)

R_{O3}



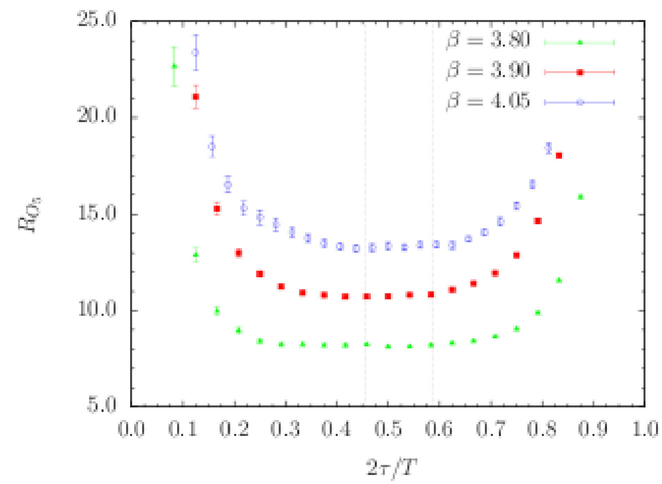
(b)

R_{O4}



(c)

R_{O5}



(d)

Renormalisation

$$\begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \\ O_5 \end{pmatrix}_{\rm R} = \begin{pmatrix} Z^s_{11} & 0 & 0 & 0 & 0 \\ 0 & Z^s_{22} & Z^s_{23} & 0 & 0 \\ 0 & Z^s_{32} & Z^s_{33} & 0 & 0 \\ 0 & 0 & 0 & Z^s_{44} & Z^s_{45} \\ 0 & 0 & 0 & Z^s_{54} & Z^s_{55} \end{pmatrix} \begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \\ O_5 \end{pmatrix}_{\rm b}$$

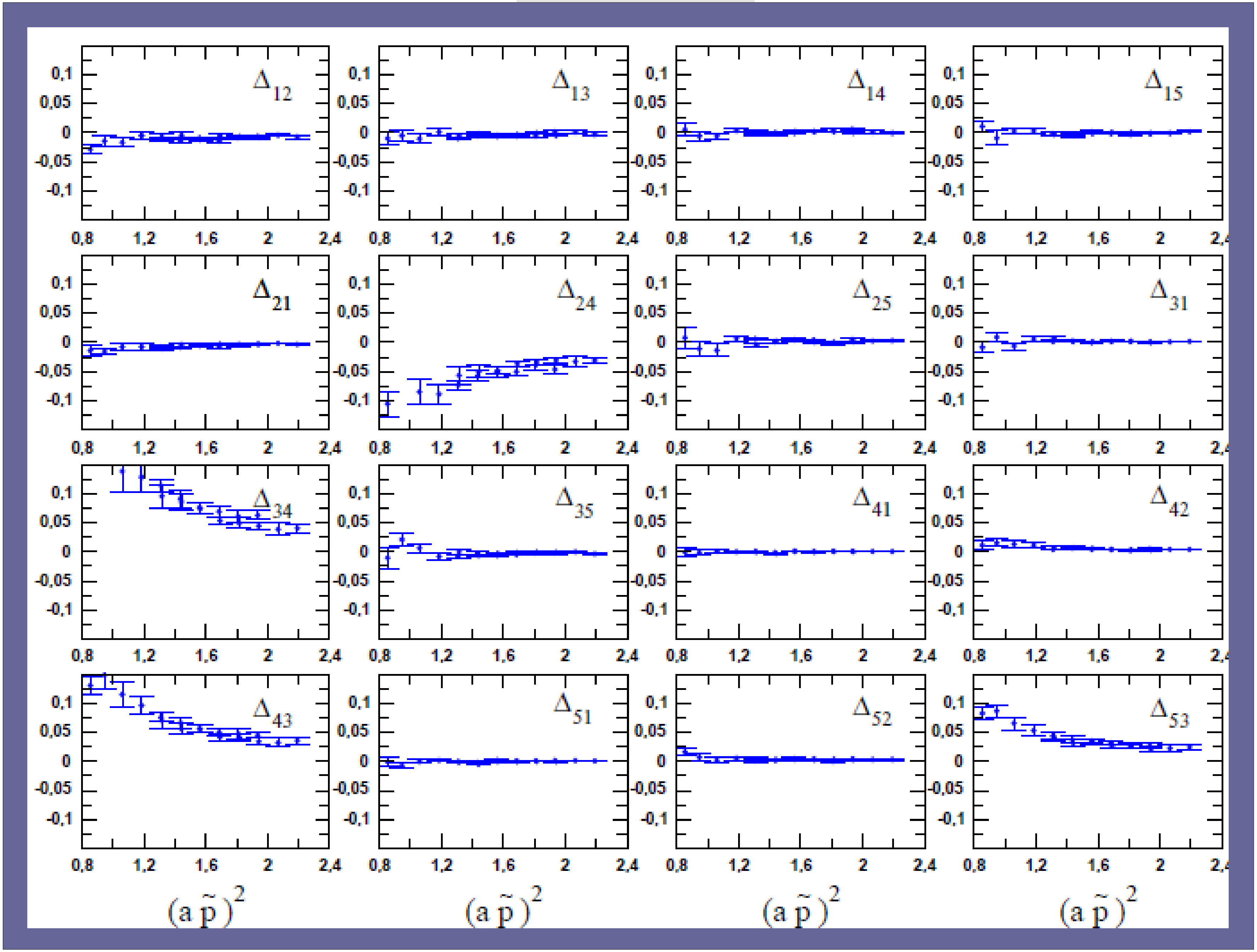
$\Delta_{ij}^{s} \sim O(a^2)$

$$\hat{R}_{B_i}(x_0) = \frac{Z_{ij}}{Z_S\, Z_P}\, R_{B_j}(x_0), \quad i,\, j = 2, \ldots, 5$$

$$\hat{R}_i(x_0) = \left(\frac{f_K}{m_K}\right)^2_{\rm expt.} \left[\frac{M^{12} M^{34}}{F^{12} F^{34}}\, \frac{Z_{ij}}{Z_{11}}\, R_{O_j}(x_0)\right]_{\rm Lat.}, \quad i,\, j = 2, \ldots, 5$$

Δ_{ij} – off diagonal-block RCs

β = 3.80



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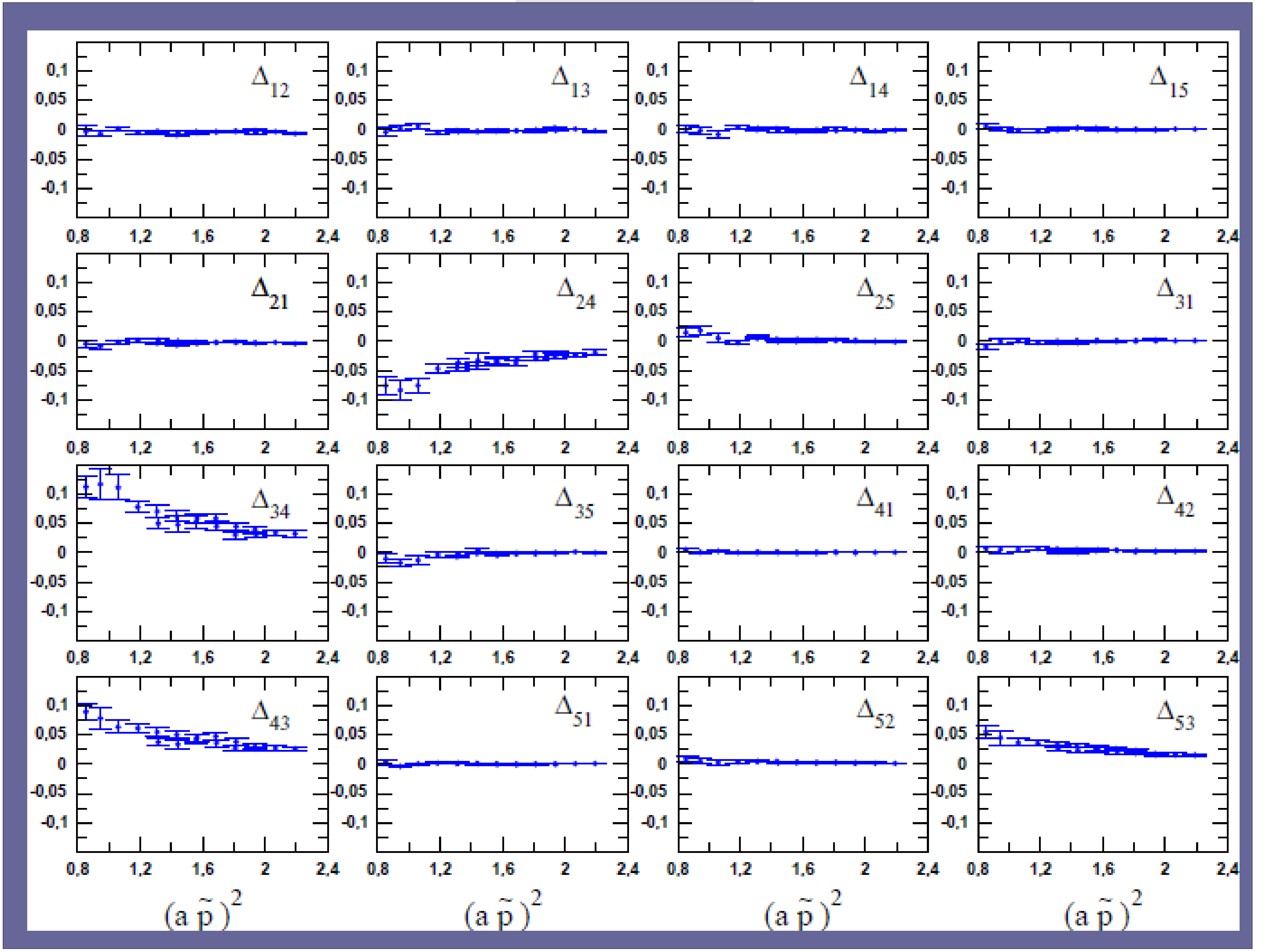

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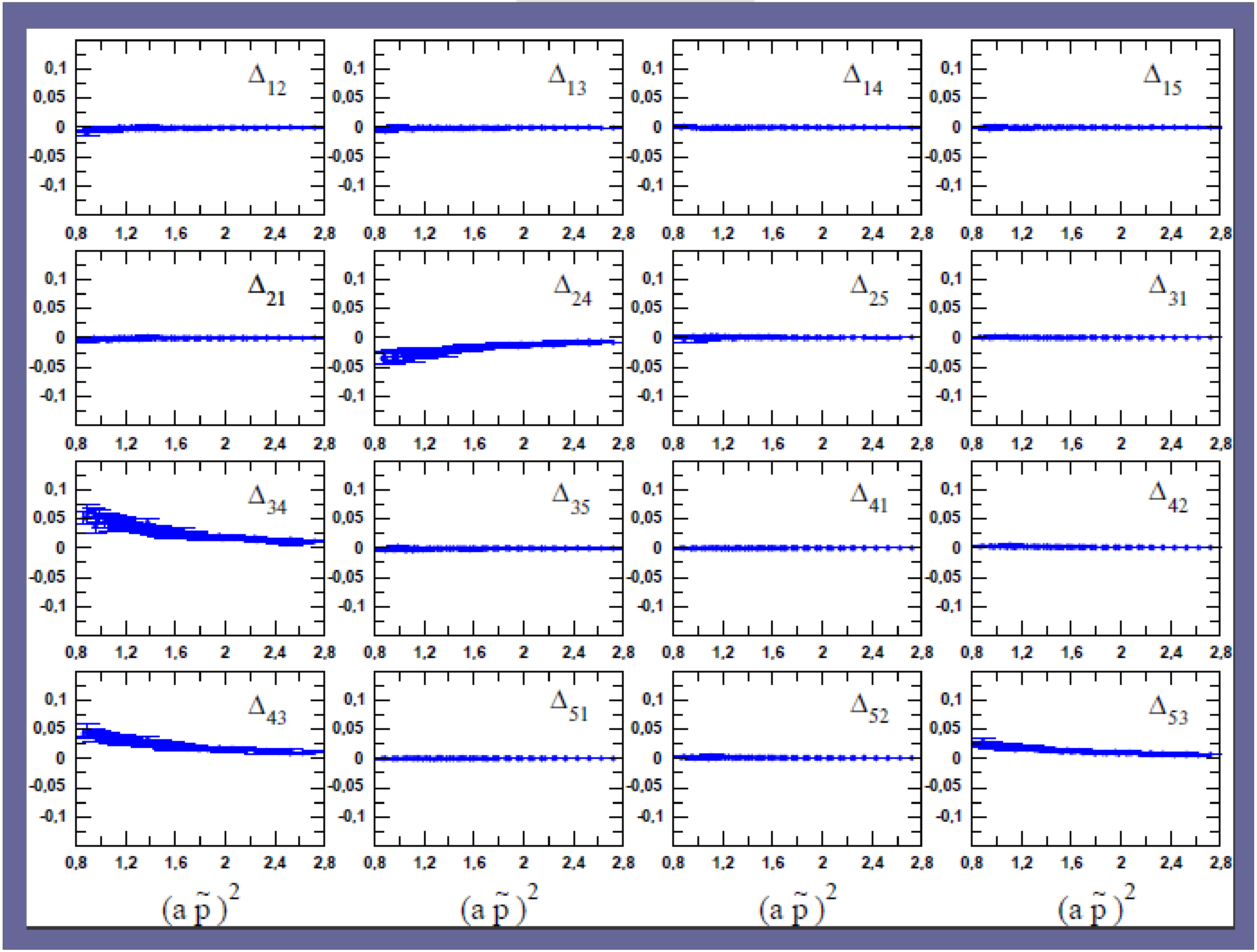
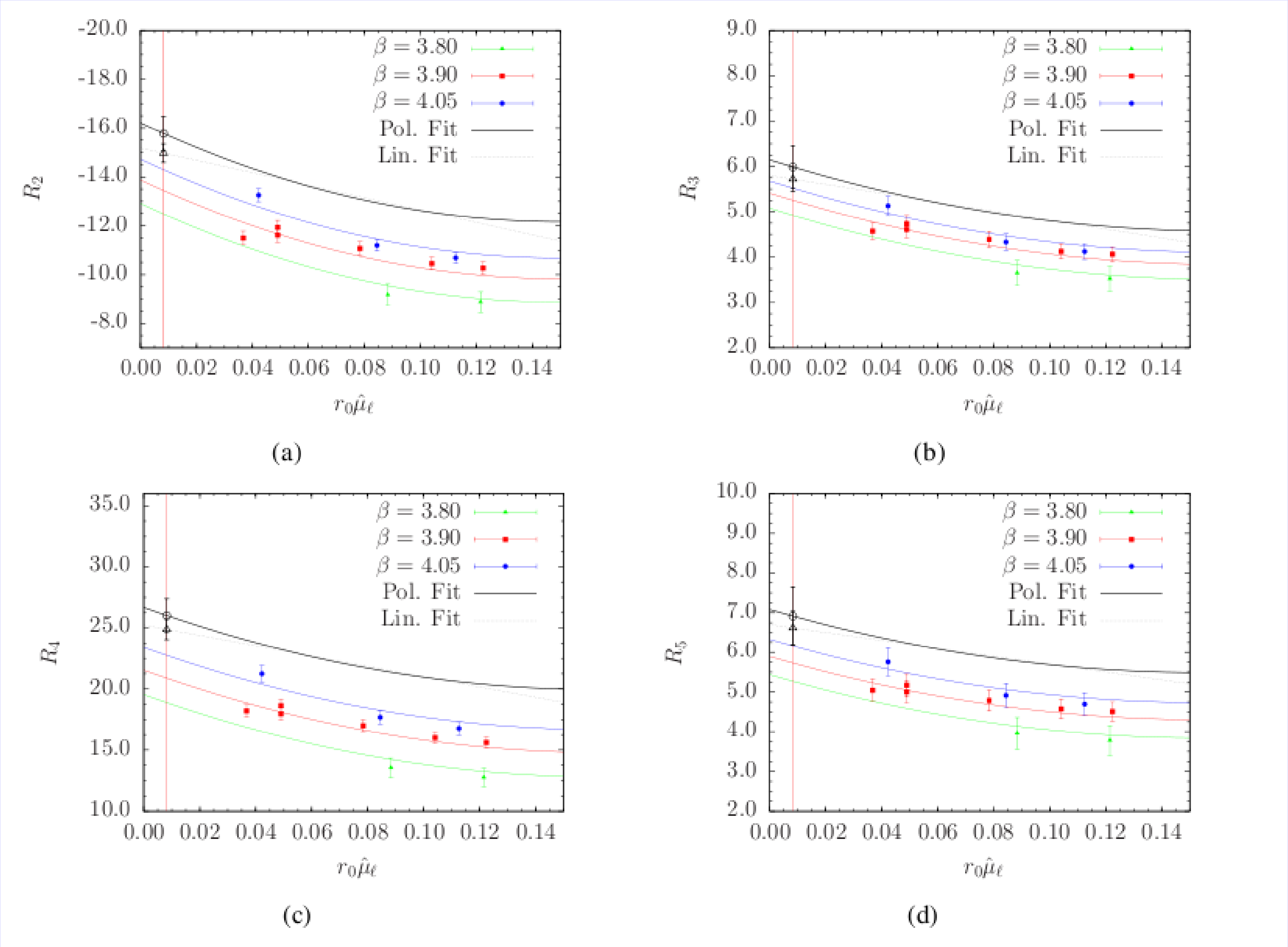


Fit Ansatz $$Y = \sum_{n=0}^{2} A_Y^{(n)}(r_0\hat{\mu}_s)\,[r_0\,\hat{\mu}_\ell]^n + D_Y(r_0\hat{\mu}_s)\left[\frac{a}{r_0}\right]^2$$



- **Results from Linear and Polynomial Fit are compatible within 1 σ.**

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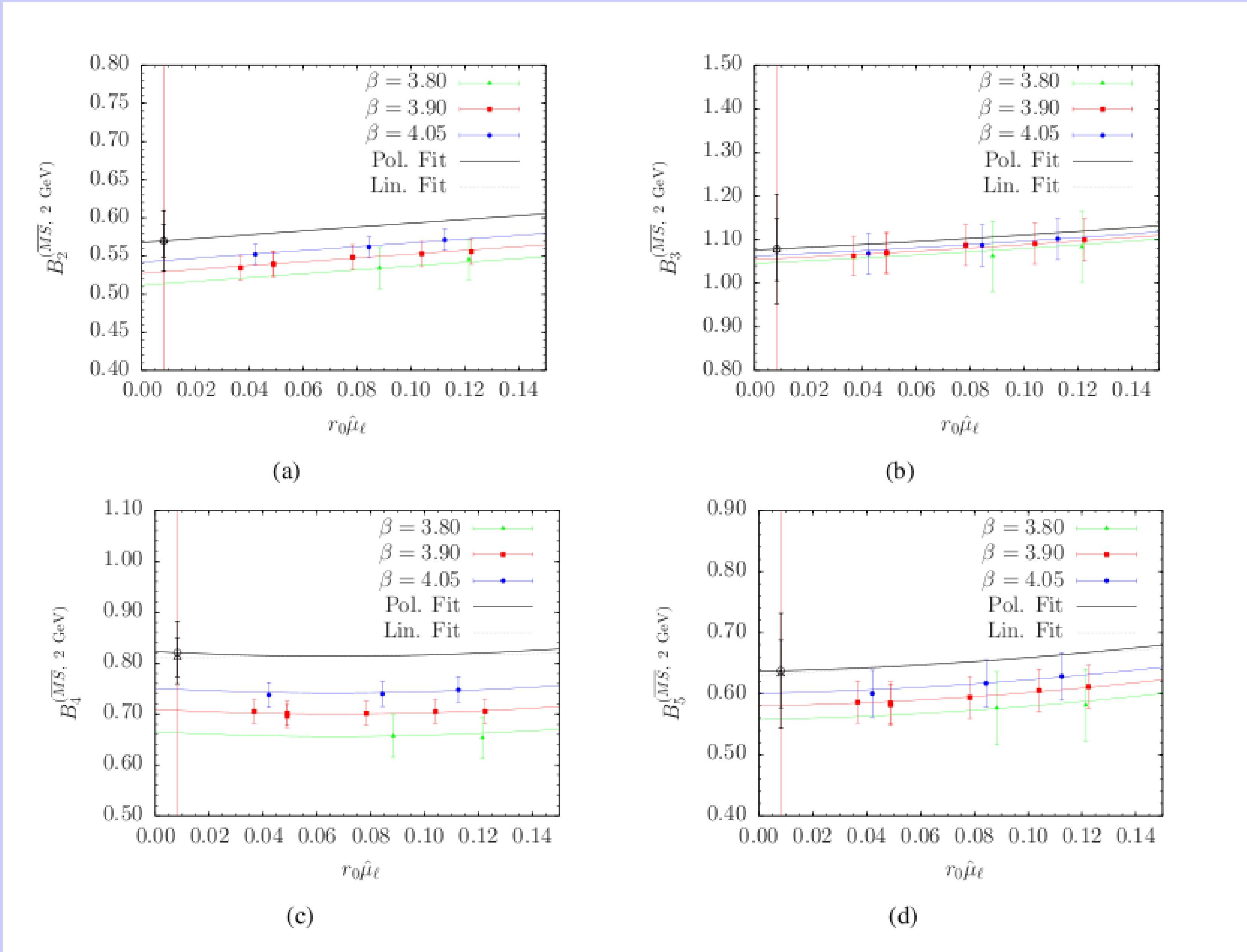

- **Results from Linear and Polynomial Fit are compatible within 1 σ.**

Results & Comparisons

B_i (RI/MOM at 2 GeV)					
This work		Babich *et al.* 2006		Donini *et al.* 1999	
	C.L.	$a = 0.09$ fm	$a = 0.13$ fm	$a = 0.07$ fm	$a = 0.09$ fm
1	0.51(02)	0.56(05)	0.53(04)	0.68(21)	0.70(15)
2	0.74(05)	0.87(07)	0.90(10)	0.67(07)	0.72(09)
3	1.31(12)	1.41(12)	1.53(40)	0.95(15)	1.21(10)
4	1.04(08)	0.94(05)	0.90(13)	1.00(09)	1.15(05)
5	0.77(10)	0.62(05)	0.56(14)	0.66(11)	0.88(06)

[Babich *et al*. PRD 2006] : Overlap fermions (quenched)

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B_K overestimated

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1	1	1	1	1	1
2	-13.6(06)	-16.1(30)	-15.8(29)	-6.7(18)	-6.6(11)
3	4.9(03)	5.2(09)	5.4(08)	1.9(05)	2.3(04)
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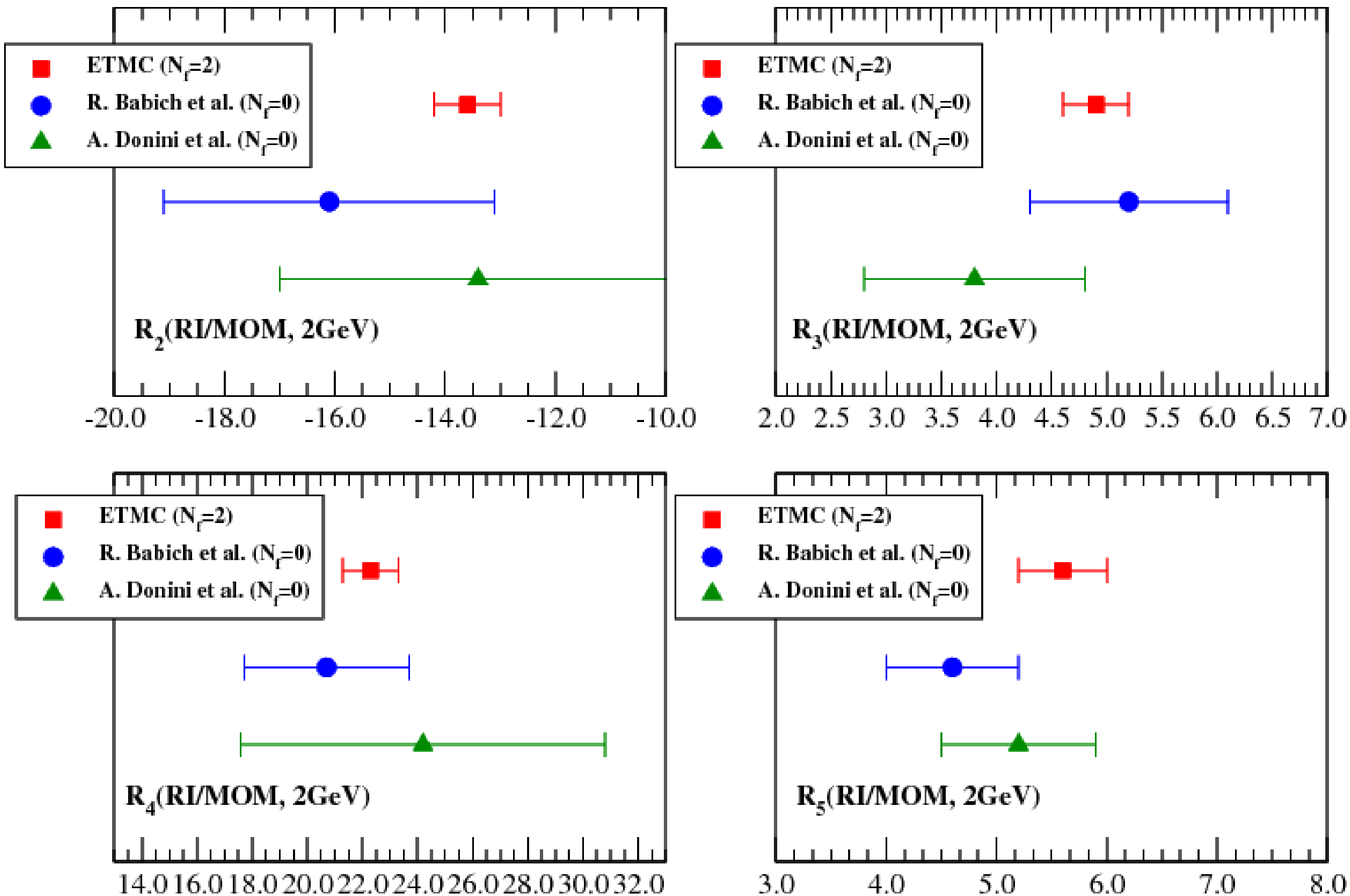
B_K ↑ ~ x **1.2**

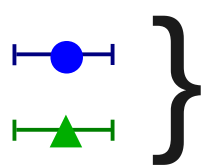
m_s^2 ↑ ~ x **1.3^2**

} ~ x **2**

Results & Comparisons



x 2 (*correction factor!!*)



Values and errorbars, those of the finest lattice result.

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3	4.9(03)	5.2(09)	5.4(08)	1.9(05)	2.3(04)
4	22.3(10)	20.7(30)	18.8(28)	12.1(33)	12.6(21)
5	5.6(04)	4.6(06)	3.9(13)	2.6(07)	3.3(05)

Total uncertainty estimates

[ETMC 2011] : ~ 4 -7 %

[Babich *et al*. PRD 2006] : ~ 13 -19 %

[Donini *et al*. PLB 1999] : ~ 20 - 25 %

Conclusions

- First results for K-neutral mixing BSM ME from unquenched simulations (Nf=2) in the CL.

- Results from unquenched and quenched simulations on the same ballpark.

- Great improvement in the final results' total uncertainties.

- Phenomenological analysis and implications still to be done.