Non-perturbative computation of renormalization constants with 4 dynamical flavours

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Outline

1. Introduction
2. Status
   - $\beta = 1.95$
   - $\beta = 2.10$
   - $\beta = 1.90$
3. Conclusions & Outlook

The problem

Computing (RI′-MOM) RCs of various operators (e.g. P, $O_j^{\Delta F=2}$) for the action used by ETMC in the 2+1+1 project

Mass independent scheme $\Leftrightarrow$ Extrapolation to zero quark mass

2+1+1 ensembles are not well suited

$\Rightarrow$ dedicated simulations with 4 degenerate light quarks are needed

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Action

For this study we consider the action:

$$S_L = S_{Iwa}^{\mathrm{YM}} + a^4 \sum_{x,f} \bar{\chi}_f \left[\gamma \cdot \widetilde{\nabla} - \frac{a}{2} \nabla^* \nabla + m_0 + i r_f \mu_q \gamma_5 \right] \chi_f(x)$$

or, by passing from twisted to physical quark basis via

$$\chi_f \to q_f = \exp[\tfrac{i}{2}(\tfrac{\pi}{2} - \theta_{0f})\gamma_5]\chi_f \,, \quad \bar{\chi}_f \to \bar{q}_f = \bar{\chi}_f \exp[\tfrac{i}{2}(\tfrac{\pi}{2} - \theta_{0f})\gamma_5]$$

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quark mass parameters

$$M_0 = \sqrt{(m_0 - m_{\rm cr})^2 + \mu_q^2}\,, \ \sin\theta_{0f} = \frac{m_0 - m_{\rm cr}}{M_0}\,, \ \cos\theta_{0f} = \frac{\mu_q r_f}{M_0}$$

- Renormalized parameters conveniently chosen as

$$M = Z_P \hat{M} = \sqrt{Z_A^2 m_{\rm PCAC}^2 + \mu_q^2}\,, \quad \tan\theta = \frac{Z_A m_{\rm PCAC}}{\mu_q}$$

- $d = 4$ term of Symanzik LEL involves only M, not θ.
- Partially quenched setup (convenient for RC studies)

$$(M, \theta) \implies (M_{\rm sea}, \theta_{\rm sea}; M_{\rm val}, \theta_{\rm val})$$

- $O(a)$ improvement via $\theta-$average (Frezzotti-Rossi 04)

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Practical features of the actual computation

- Tune the κ-value in order to obtain opposite values for $m_{\rm PCAC}$ (safely determined from smooth simulations far from κ_c)
- Hence, double the number of necessary $n_f = 4$ ensembles (positive and negative $m_{\rm PCAC}$)
- still keep the same μ in the twin ensembles $\rightarrow$ same M
- Since we are interested in relatively small values of $M_{\rm PS}$ for subtracting carefully the Goldstone boson pole, we may need to re-tune the κ-value in the valence sector. In those cases

$$m_{\rm PCAC}^{\rm sea} \neq m_{\rm PCAC}^{\rm val}$$

But

$$|m_{\rm PCAC}^{\rm sea,p}| = |m_{\rm PCAC}^{\rm sea,m}|\,;\ |m_{\rm PCAC}^{\rm val,p}| = |m_{\rm PCAC}^{\rm val,m}|$$

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Analysis strategy

On the ensembles Ep/m (E=1,2,...) with $(M_{\rm sea}^{\rm E}, \theta_{\rm sea}^{\rm Ep/m})$ we compute RC-estimators for several $(M_{\rm val}, \theta_{\rm val})$'s and $\tilde{p}^2$'s

1. remove $O(a)$ artifacts in RCs via θ-average
2. valence chiral limit: via linear fit to the RC-estimator dependence on $M_{\rm val}$, with term $\sim (M_{\rm val})^{-1}$ for $\Gamma = P$ (ignoring $\theta^{\rm val}$-depend; good χ^2)
3. sea chiral limit: $(M_{\rm sea})^2 \to 0$ taking θ-dependence into account
4. remove $O(g^2 a^2)$ artifacts in RCs via perturbative computation

(M1) build (with PT-evolution in E_Γ) RC-estimators at ren. scale $1/a$, i.e.

$$Z_\Gamma((a\Lambda)^{-2}; (a\tilde{p})^2) = Z_\Gamma(\tilde{p}^2/\Lambda^2; (a\tilde{p})^2)\, E_\Gamma^{PT}(\tilde{p}^2 \to a^{-2})$$

and get $Z_\Gamma((a\Lambda)^{-2}; 0)$ by extrapolation in $(a\tilde{p})^2$

(M2) build RC-estimator at ren. scale $\tilde{p}^2_{\rm M2} \sim 12.2\ {\rm GeV}^2$ i.e.

$$Z_\Gamma((a\Lambda)^{-2}; (a\tilde{p}_{\rm M2})^2) = Z_\Gamma(\tilde{p}^2_{\rm M2}/\Lambda^2; (a\tilde{p}_{\rm M2})^2)\, E_\Gamma^{PT}(\tilde{p}^2_{\rm M2} \to a^{-2})$$

and get $Z_\Gamma((a\Lambda)^{-2}; 0)$ by averaging around $\tilde{p}^2_{\rm M2}$

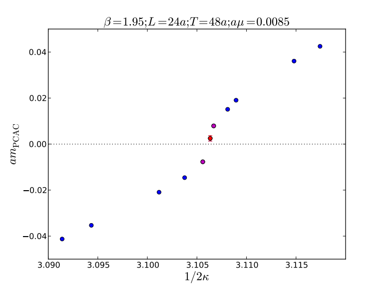
Sea Runs

($\beta = 1.95 \leftrightarrow a = 0.078\,\mathrm{fm}$; $L = 24a$; $T = 48a$)

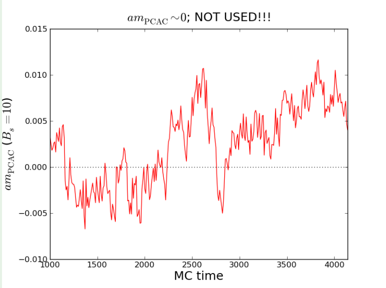
ensemble	$a\mu_{\mathrm{sea}}$	$am^{\mathrm{sea}}_{\mathrm{PCAC}}$	aM^{sea}_{0}	θ^{sea}
1m	0.0085	-0.04125(13)	0.03308(10)	-1.3109(08)
1p	0.0085	+0.04249(13)	0.03286(09)	1.3091(08)
7m	0.0085	-0.03530(13)	0.02851(10)	-1.2681(10)
7p	0.0085	+0.03608(11)	0.02854(08)	1.2683(09)
8m	0.0020	-0.03627(11)	0.02804(08)	-1.4994(02)
8p	0.0020	+0.03624(13)	0.02743(10)	1.4978(03)
3m	0.0180	-0.0160(2)	0.02191(09)	-0.6068(59)
3p	0.0180	+0.0163(2)	0.02183(09)	0.6015(57)
2m	0.0085	-0.02091(16)	0.01815(11)	-1.0834(32)
2p	0.0085	+0.0191(2)	0.01692(13)	1.0445(45)
4m	0.0085	-0.01459(13)	0.01404(08)	-0.9206(43)
4p	0.0085	+0.0151(2)	0.01420(12)	0.9289(64)
5m	0.0085	-0.008(5?)	?	?
5p	0.0085	+0.008(5?)	?	?
6m	0.0085	~ 0	?	?
6p	0.0085	~ 0	?	?

- stats = $O(4k - 5k)$

Reaching κ_C



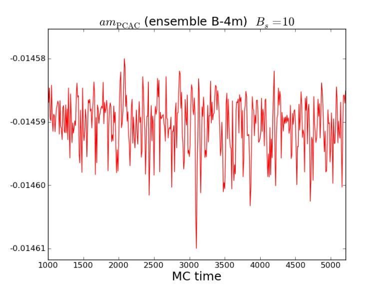
(a)



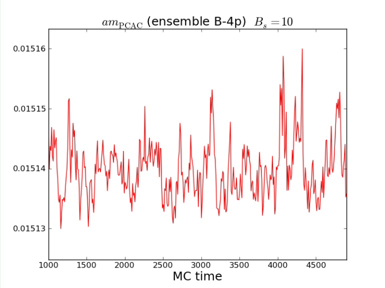
(b)

- Tuning done using $L = 16a$ ensembles

MC histories of ensembles with smallest masses used @ $\beta = 1.95$



(c)



(d)

Valence Runs ($\beta = 1.95$)

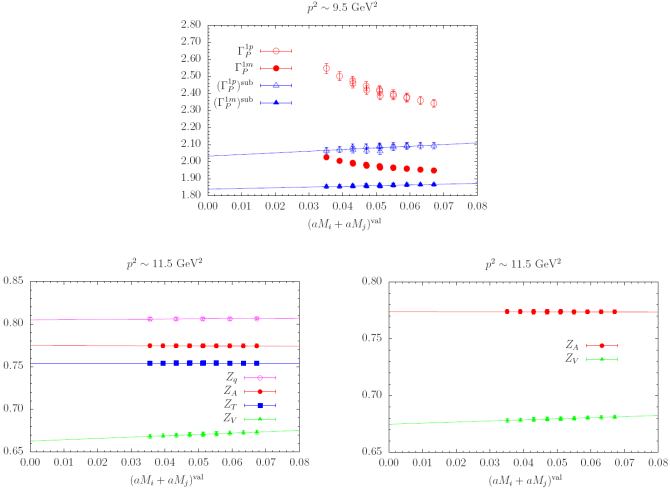
ensemble	κ^{sea}	κ^{val}	$a\mu^{\rm sea}$	$a\mu^{\rm val}$	$am^{\rm sea}_{\rm PCAC}$	$am^{\rm val}_{\rm PCAC}$	stats
1m**	0.161739	0.160754	0.0085	set1	-0.0413(01)	-0.0216(02)	304
1p**	0.160389	0.162145	0.0085	set1	+0.0425(01)	+0.0195(02)	304
2m	0.161229	0.161229	0.0085	set1	-0.0209(02)	-0.0213(02)	352
2p	0.160826	0.160826	0.0085	set1	+0.0191(02)	+0.0191(02)	352
3m	0.161229	0.161229	0.0180	set1	-0.0160(02)	-0.0160(02)	352
3p	0.160826	0.160826	0.0180	set1	0.0163(02)	+0.0162(02)	352
4m	0.161095	0.161095	0.0085	set2	-0.0146(01)	-0.0146(02)	224
4p	0.160870	0.160870	0.0085	set2	+0.0151(01)	+0.0154(02)	224
7m*	0.161585	0.160681	0.0085	set1	-0.0353(01)	-0.0179(02)	736
7p*	0.160524	0.161925	0.0085	set1	+0.0361(01)	+0.0178(02)	736
8m*	0.161585	0.160681	0.0020	set2	-0.0363(01)	-0.0193(02)	624
8p*	0.160524	0.161925	0.0020	set2	+0.0362(01)	+0.0181(02)	624

set1 $\rightarrow$ $a\mu^{\rm val} \in \{0.0085, 0.0150, 0.0203, 0.0252, 0.0298\}$

set2 $\rightarrow$ $a\mu^{\rm val} \in \{0.0060, 0.0085, 0.0120, 0.0150, 0.0180, 0.0203, 0.0252, 0.0298\}$

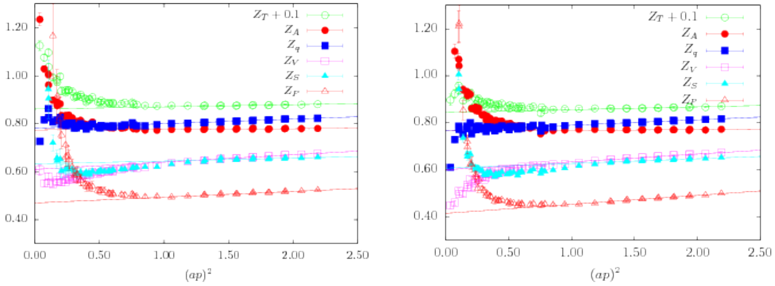
The $(M^{\rm val})^2$-dependence of RC-estimators @ $(a\tilde{p})^2 = 1.5$

Figure: $Z = Z_0 + \frac{A}{(aM_{\rm ps})^2} + B(aM_{\rm val})$



The $\tilde{p}^2$-dependence of RC-estimators @ $M^{\rm val} = 0$

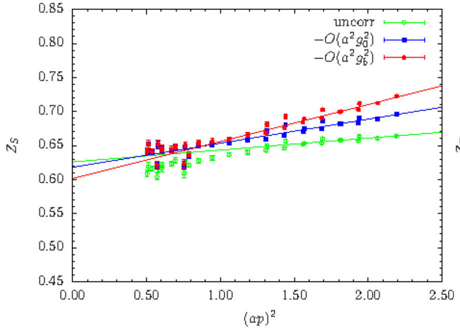
Figure: ensembles 1m and 2p

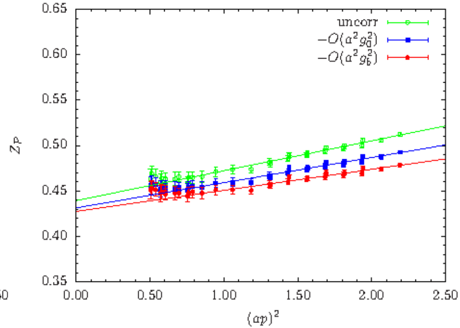


M1: intercept at $\tilde{p}^2 = 0$ of the shown best fit lines

M2: values at $\tilde{p}^2 = 12.2\ {\rm GeV}^2$, here corresponding to $a^2\tilde{p}^2 = 1.9$

Removing $O(g^2 a^2)$ cutoffs





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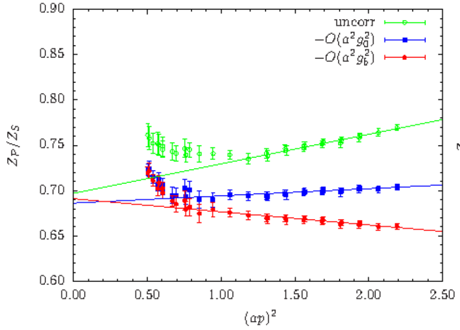

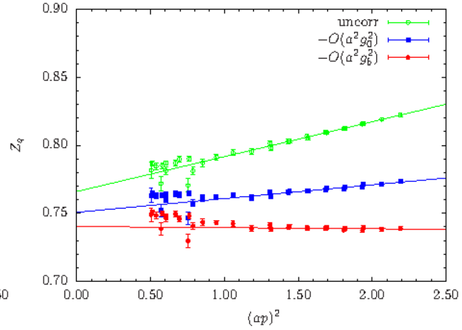

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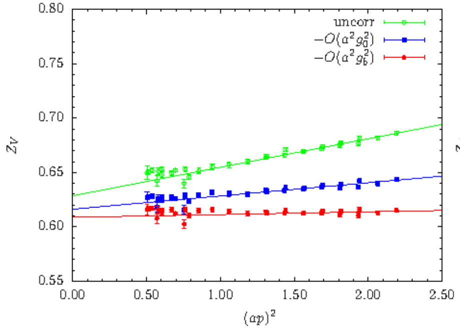

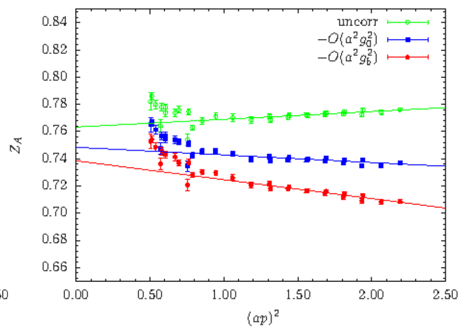

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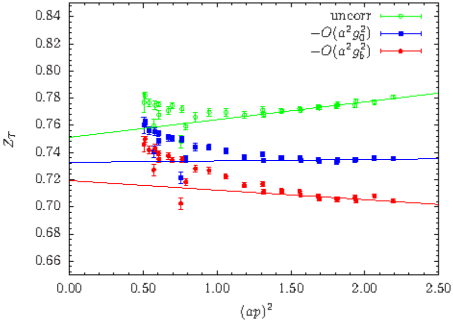
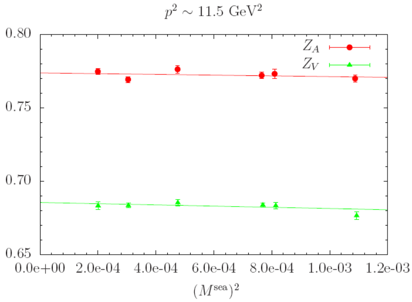
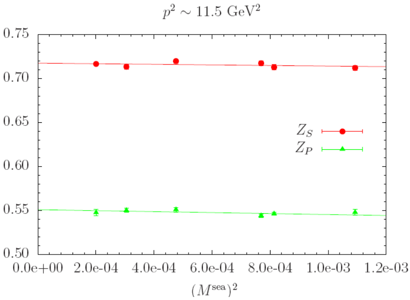


The M^{sea}-dependence ($M^{\text{val}} = 0$)





Results for RCs of bilinears (M1/M2 @ $M^{\rm sea} = M^{\rm val} = 0$)

$\beta = 1.95$ (Iwasaki)

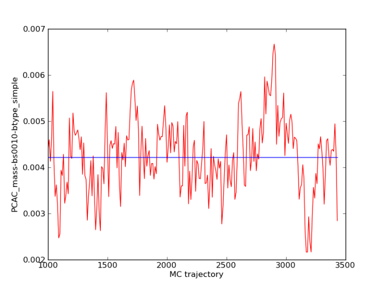
	Uncor/ed		$-O(a^2 g_b^2)$		$-O(a^2 g_0^2)$	
RC	M1	M2	M1	M2	M1	M2
Z_A	0.763(03)	0.774(01)	0.739(03)	0.712(01)	0.748(03)	0.738(01)
slope ($\times 10^{-3}$)	06(02)	-	-14(02)	-	-06(02)	-
Z_V	0.629(03)	0.678(02)	0.609(03)	0.613(02)	0.616(03)	0.639(02)
slope ($\times 10^{-3}$)	26(02)	-	03(02)	-	12(02)	-
$Z_P(1/a)$	0.439(06)	0.501(02)	0.427(06)	0.471(02)	0.431(06)	0.483(02)
slope ($\times 10^{-3}$)	33(02)	-	23(02)	-	28(02)	-
$Z_S(1/a)$	0.626(08)	0.659(01)	0.602(08)	0.704(01)	0.618(08)	0.684(01)
slope ($\times 10^{-3}$)	17(03)	-	54(04)	-	35(04)	-
$Z_{P/S}$	0.697(07)	0.760(03)	0.709(07)	0.669(043)	0.697(07)	0.706(03)
slope ($\times 10^{-3}$)	33(03)	-	-14(3)	-	08(03)	-
$Z_T(1/a)$	0.751(03)	0.776(01)	0.719(03)	0.706(01)	0.733(03)	0.734(01)
slope ($\times 10^{-3}$)	13(02)	-	-07(02)	-	01(02)	-
$Z_q(1/a)$	0.766(04)	0.814(01)	0.740(04)	0.738(01)	0.751(04)	0.769(01)
slope ($\times 10^{-3}$)	26(02)	-	00(02)	-	10(02)	-

Sea Runs
($\beta = 2.10 \leftrightarrow a = 0.06\,\mathrm{fm};\ L = 32a\,;\ T = 64a$)

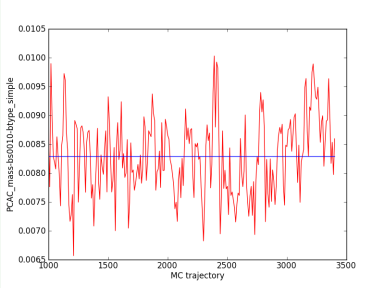
ensemble	$a\mu_{\rm sea}$	$am_{\rm PCAC}^{\rm sea}$	$aM_0^{\rm sea}$	$\theta^{\rm sea}$
5m	0.0078	-0.00823(9)	0.01018(8)	-0.6981(9)
5p	0.0078	+0.00829(7)	0.01021(4)	0.7011(5)
4m	0.0064	-0.00682(13)	0.00841(7)	-0.7059(9)
4p	0.0064	?		
3am	0.0046	-0.00585(8)	0.00656(5)	0.794(7)
3ap	0.0046	+0.00559(14)	0.00642(8)	0.7713(13)
3bm	-	-	"	$-\pi/10$
3bp	-	-	"	$\pi/10$
2am	0.0030	-0.00403(14)	0.00440(8)	-0.8214(17)
2ap	0.0030	+0.00421(13)	0.00451(8)	0.8431(15)

- stats = $O(2.5k - 3k)$
- $a\mu$ and $am_{\rm PCAC}$ chosen such that $\theta \sim \pi/4$

am_{PCAC} MC histories @ $\beta = 2.10$



(a) 2am



(b) 5p

Valence Runs ($\beta = 2, 10$)

Ensemble	κ_{sea}	μ_{sea}	μ_{val}	index	Stats
D-2ap-L32	0.156042	0.0030	{0.0013, 0.0030, 0.0080, 0.0143, 0.0195, 0.0247, 0.0298}	0500-0700	50
D-2am-L32	0.156157	0.0030	{0.0013, 0.0030, 0.0080, 0.0143, 0.0195, 0.0247, 0.0298}	0716-0896	50
D-3p-L32	0.156017	0.0046	{0.0025, 0.0046, 0.0090, 0.0152, 0.0201, 0.0249, 0.0297}	0220-1240	250
D-3m-L32	0.156209	0.0046	{0.0025, 0.0046, 0.0090, 0.0152, 0.0201, 0.0249, 0.0297}	0114-1522	350
D-4p-L32	0.155983	0.0064	{0.0039, 0.0064, 0.0112, 0.0184, 0.0240, 0.0295}		
D-4m-L32	0.156250	0.0064	{0.0039, 0.0064, 0.0112, 0.0184, 0.0240, 0.0295}	0500-0716	55
D-5p-L32	0.155949	0.0078	{0.0048, 0.0078, 0.0119, 0.0190, 0.0242, 0.0293}	0512-1424	230
D-5m-L32	0.156291	0.0078	{0.0048, 0.0078, 0.0119, 0.0190, 0.0242, 0.0293}	0500-1704	300

First analysis results for RCs (No sea chiral limit)

Only 2 ensembles (2am, 2ap), 50 confs each. VERY PRELIMINARY!

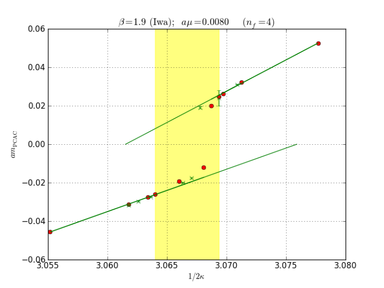
RC(RI′)	M1
Z_A	0.773(13)
Z_V	0.669(10)
$Z_P(1/a)$	0.498(08)
$Z_S(1/a)$	0.687(11)
$Z_T(1/a)$	0.756(12)
$Z_q(1/a)$	0.793(13)

Sea Runs
($\beta = 1.90 \leftrightarrow a = 0.086\,\mathrm{fm};\ L = 24a;\ T = 48a$)

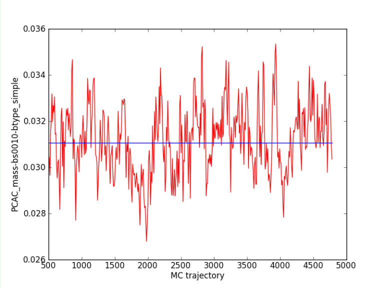
ensemble	$a\mu_{\rm sea}$	$am^{\rm sea}_{\rm PCAC}$	$aM^{\rm sea}_{0}$	$\theta^{\rm sea}$
1m	0.0080	-0.0274(2)	0.02078(13)	-1.176(3)
1p	0.0080	+0.0276(2)	0.02091(13)	1.178(3)
2m	0.0080	-0.03182(14)	0.02367(09)	-1.2260(14)
2p	0.0080	+0.0311(2)	0.02319(13)	1.219(2)
3m	0.0080	-0.0358(4)	0.0263(3)	-1.262(3)
3p	?	?	?	?
4m	?	?	?	?
4p	?	?	?	?
5m?	?	?	?	?
5p?	?	?	?	?

- aiming at stats = $O(4k - 5k)$

Around κ_C



(c)



(d) 2p

- Tuning done using $L = 16a$ ensembles

What remains

- Completing chain of programs for valence sector
  - $\beta = 1.95$ was done using apeNEXT $\Rightarrow$ apeNEXT died
  - Mariane and Xining are working hard to invert $\beta = 2.10$ (Thanks a lot!)
- Cross-checking some parts of analysis program
- $\beta = 1.95$
  - complete statistics in sea (ensemble 4m is corrupted!)
  - $\rightarrow$ complete statistics in valence
- $\beta = 2.10$
  - finish ensemble 4p
  - 3bp/3bm (different θ)?
  - check 2am (how corrupted is it?)
  - ... or redo it?
  - complete inversions

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- $\beta = 1.90$
  - conclude production (4-6 ensembles more)
  - start inversions asap!
- Put all together: global analysis

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$n_f = 2 + 1 + 1$	$n_f = 2$
20 $\times$ L=24a ensembles	9 $\times$ L=24a ensembles
8+ $\times$ L=32a ensembles	4 $\times$ L=32a ensembles

Bonus tracks

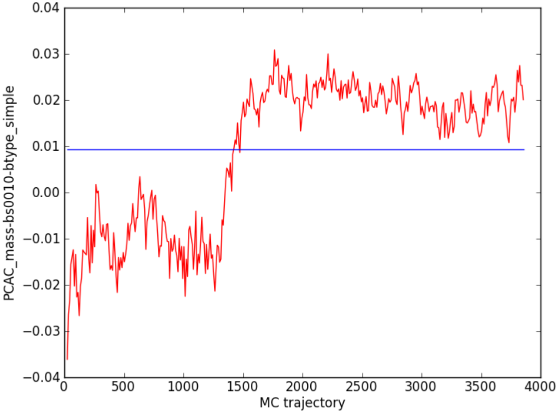
"Democratic" momenta

$$\Delta_4(p) \equiv \frac{\sum_\rho \tilde{p}_\rho^4}{(\sum_\rho \tilde{p}_\rho^2)^2} < 0.28$$

with

$$\tilde{p}_\nu \equiv \frac{1}{a} \sin(a p_\nu)$$

Within the yellow world



continue

$$\mathcal{D}_d = \begin{cases} U_\mu(x) & \rightarrow & U_\mu^\dagger(-x - a\hat{\mu}) \\ \psi(x) & \rightarrow & e^{i3\pi/2}\psi(-x) \\ \bar{\psi}(x) & \rightarrow & e^{i3\pi/2}\bar{\psi}(-x) \end{cases}$$

Action and quark mass parameters⋆

The lattice action before allows to compute RCs relevant for...

- operators made out of quark fields with ETMC $2+1+1$ action, which in the twisted basis reads

$$S^{\rm YM}_{Iwa} + a^4 \sum_x (\bar\chi_{\ell 1}, \bar\chi_{\ell 2}) \left[\gamma \cdot \widetilde{\nabla} - \frac{a}{2}\nabla^*\nabla + m_0 + i\mu_\ell \gamma_5 \tau^3\right] \begin{pmatrix}\chi_{\ell 1}\\ \chi_{\ell 2}\end{pmatrix}(x)$$

$$+a^4 \sum_x (\bar\chi_{h1}, \bar\chi_{h2}) \left[\gamma \cdot \widetilde{\nabla} - \frac{a}{2}\nabla^*\nabla + m_0 + i\mu_\sigma \gamma_5 \tau^3 - \mu_\delta \tau^1\right] \begin{pmatrix}\chi_{h1}\\ \chi_{h2}\end{pmatrix}(x)$$

- operators involving Osterwalder-Seiler valence quarks: S_L above with $m_0 = m_{\rm cr}$, $\mu_q > 0$ (maximal twist)

(Frezzotti-Rossi'04)

Bilinears: $\Gamma \;\Leftrightarrow\; S, P, V, A, T$

RI′MOM scheme

- $Z_q^{-1} \frac{-i}{12N(p)} \sum_\rho' \left[\frac{\mathrm{Tr}(\gamma_\rho S_f(p)^{-1})}{\tilde{p}_\rho}\right]_{\tilde{p}^2=\mu^2} = 1\,,$ any f (and r_f)
- $Z_q^{-1} Z_O^{(ff')} \mathrm{Tr}\left[\Lambda_\Gamma^{(ff')}(p,p) P_\Gamma\right]_{\tilde{p}^2=\mu^2} = 1$ $\quad f \neq f'$ $(r_{f'} = -r_f)$

$\tilde{p}^2 = \sum_\mu \tilde{p}_\mu^2$; $\tilde{p}_\mu \equiv \frac{1}{a}\sin ap_\mu$; $\sum_\rho' \to \forall\rho \mid p_\rho \neq 0$; $N(p) = \sum_\rho' 1$

We need to compute:

- The quark propagator: $S_f(p) = a^4 \sum_x e^{-ipx} \langle \chi_f(x) \bar{\chi}_f(0) \rangle$
- the Green function: $G_\Gamma^{(ff')}(p,p) = a^8 \sum_{x,y} e^{-ip(x-y)} \langle \chi_f(x) (\bar{\chi}_f \Gamma \chi_{f'})(0) \bar{\chi}_{f'}(y) \rangle$
- and the amputated vertex: $\Lambda_\Gamma^{(ff')}(p,p) = S_f^{-1}(p) G_\Gamma^{(ff')}(p,p) S_{f'}^{-1}(p)$

$O(a)$ improvement via θ-average (Frezzotti-Rossi'04)

Based on the symmetry of the lattice action S_L under

$$\mathcal{P} \times (\theta_0 \to -\theta_0) \times \mathcal{D}_d \times (M_0 \to -M_0)$$

one can prove that the $O(a^{2k+1})$ artifacts occurring in the vev of (multi)local operators O that are invariant under $\mathcal{P} \times (\theta_0 \to -\theta_0)$

- are quantities that change sign upon sign change of θ_0 (or θ)
- are absent in θ-averages: $\frac{1}{2}\left[\langle O\rangle|_{\hat{M},\theta} + \langle O\rangle|_{\hat{M},-\theta}\right]$

That holds for form factors invariant under $\mathcal{P} \times (\theta_0 \to -\theta_0)$ ...e.g.

for the RC-estimators at all $\hat{M}$'s (and $\tilde{p}^2$'s in RI-MOM)

In PQ setup: $(M, \theta) \quad \Rightarrow \quad (M^{\rm sea}, \theta^{\rm sea}; M^{\rm val}, \theta^{\rm val})$ (θ's referred to $f = 1$)

Numerical strategy at $N_f = 4$ "light" sea quarks

If $a\mu_q \lesssim 0.01$ at $0.08[0.09] fm \Leftrightarrow \beta = 1.95[1.90]$

- considerable fine tuning in $1/2\kappa$ is needed to work at maximal twist
- $\sigma_{\rm stat}[am_{\rm PCAC}]$ difficult to evaluate when $am_{\rm PCAC} \ll 0.01$

Various setups still possible:

A) maximal twist, at larger M's $\longrightarrow$ need some fine tuning work,

B) out of maximal twist, at larger M's $\longrightarrow$ remove $O(a)$ effects by $\theta_{\rm val/sea}$-average

We have chosen B)

θ-dependence in chiral limit extrapolation of RC estimators

The analysis of the Symanzik LEL of our RC estimators

- computed with $r_{f_1} = -r_{f_2}$
- ignoring terms $\mathcal{O}(\, M_{\mathrm{val}}^k,\, M_{\mathrm{sea}}^k\,)\,, \quad k \geq 3$
- and after θ-average is performed

implies:

- Sea: $Z = Z_0 + B(\theta_{\mathrm{sea}})M_{\mathrm{sea}} + C(\theta_{\mathrm{sea}})M_{\mathrm{sea}}^2$
  - with $B(\theta_{\mathrm{sea}}) = B_0 + B_1 \cos(2\theta_{\mathrm{sea}})$
  - and $C(\theta_{\mathrm{sea}}) = C_0 + C_1 \cos(2\theta_{\mathrm{sea}})$

continue

θ-dependence in chiral limit extrapolation of RC estimators

- Valence: $Z = Z_0 + B(\theta_{\rm val})M_{\rm val} + C(\theta_{\rm val})M_{\rm val}^2$
  - Z_q, Z_S, Z_P, Z_T:
    - $B(\theta_{\rm sea}) = B_0 + B_1 \cos(2\theta_{\rm sea})$
    - $C(\theta_{\rm sea}) = C_0 + C_1 \cos(2\theta_{\rm sea})$
  - Z_A, Z_V, Z_{VA+AV}:
    - $B(\theta_{\rm sea}) = B_0 + B_1 \cos(2\theta_{\rm sea}) + B_2(\cos(2\theta_{\rm sea}))^2$
    - $C(\theta_{\rm sea}) = C_0 + C_1 \cos(2\theta_{\rm sea}) + C_2(\cos(2\theta_{\rm sea}))^2$

continue