Ratio Method and B-physics

P. Dimopoulos , R. Frezzotti , G. Herdoíza

ETM Collaboration

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Ratio Method

Build ratios at successive masses of an observable $\Upsilon(m_Q) \xrightarrow[m_Q \to \infty]{} \mathrm{cst}$

- $\Upsilon(m_Q) = f_{h\ell}\sqrt{m_Q}/C_A^{stat}(m_Q)$
- $\Upsilon(m_Q) = M_{h\ell}/m_Q$

$$y(m_Q^{(n)}, \lambda) = \frac{\frac{M_{h\ell}(m_Q^{(n)})}{m_Q^{(n)}}}{\frac{M_{h\ell}(m_Q^{(n-1)})}{m_Q^{(n-1)}}} = \frac{\Upsilon(m_Q^{(n)})}{\Upsilon(m_Q^{(n-1)})}$$

where $m_{Q,\mathrm{pole}} = \rho(\log \bar{\mu}_h, p) \cdot \bar{\mu}_h[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}]$ and $\lambda = \bar{\mu}_h^{(n)}/\bar{\mu}_h^{(n-1)}$

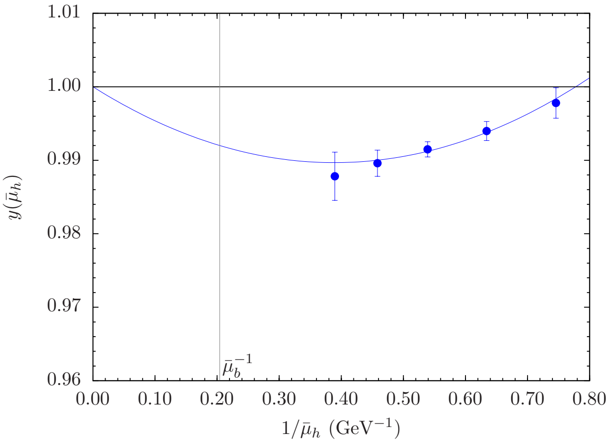
Fit the mass dependence of y :

$$y(x, \lambda)\Big|_p = 1 + \eta_1(\log x, \lambda)\, x + \eta_2(\log x, \lambda)\, x^2$$

where $x = 1/\bar{\mu}_h$

Ratio y vs. $x = 1/\bar{\mu}_h$

ρ at NLO



Quark Mass

HQET

$$M_{H\ell} = m_Q + \bar{\Lambda} - \frac{(\lambda_1 + 3\lambda_2)}{2m_Q} + O\left(\frac{\Lambda_{\rm QCD}}{m_Q^2}\right)$$

where

$$\lambda_1 = \frac{1}{2M_{H\ell}} \langle B(v) \,|\, \bar{h}\,(iD)^2\, h \,|\, B(v)\rangle$$
$$\lambda_2 = \frac{1}{6M_{H\ell}} \langle B(v) \,|\, \bar{h}\, \frac{g}{2}\, \sigma_{\mu\nu}\, G^{\mu\nu}\, h \,|\, B(v)\rangle$$

- Renormalisation — R. Sommer [Nara, 1996]
- $\bar{\Lambda}$ is an effective parameter
- Non-perturbative HQET [ALPHA]

$$M_B = m_{\rm b} + \widehat{\delta m} + E_{\rm stat} + \omega_{\rm kin} E_{\rm kin} + \omega_{\rm spin} E_{\rm spin}$$
$$E_{\rm stat} = - \lim_{x_0 \to \infty} \widetilde{\partial_0} \ln C_{\rm AA}^{\rm stat}(x_0)\Big|_{\delta m = 0}$$

Quark Mass

HQET

$$M_{H\ell} = m_Q + \bar{\Lambda} - \frac{(\lambda_1 + 3\lambda_2)}{2m_Q} + O\left(\frac{\Lambda_{QCD}}{m_Q^2}\right)$$

Phenomenological estimates :

$$M_{B*} - M_B = -\frac{2}{m_b}\lambda_2 + O\left(\frac{\Lambda_{QCD}}{m_Q^2}\right) \quad \rightsquigarrow \quad \lambda_2 \approx \frac{M_{B^*}^2 - M_B^2}{4} = 0.12(2)\,\mathrm{GeV}^2$$

[Grem, Kapustin, Ligeti, Wise, 1996] :

- determine $\bar{\Lambda}$ and λ_1 from CLEO data for $B \to X\ell\bar{\nu}_\ell$
- use pole mass at NLO

Ref.	Scheme	$\bar{\Lambda}$ (GeV)	λ_1 (GeV^2)
Grem et al. (1996)	pole	0.39(11)	-0.19(10)
Neubert (1994)	pole (?)	0.50(20)	-0.25(20)
Giménez et al. (1996)	pole (NP subt.)	0.18(03)	–
Kronfeld & Simone (2000)	pole	0.68(12)	-0.45(12)
Bali & Pineda (2003)	RS	0.46(06)	–
McNeile et al. (2004)	pole (PT subt.)	0.65(10)	–

Quark Mass

HQET

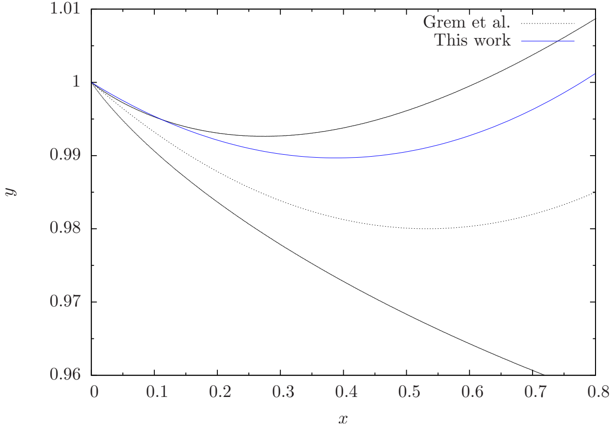
$$M_{H\ell} = m_Q + \bar{\Lambda} - \frac{(\lambda_1+3\lambda_2)}{2m_Q} + O\left(\frac{\Lambda_{\rm QCD}}{m_Q^2}\right)$$

$$\begin{aligned} y(X,\lambda) &= \frac{M_{h\ell}(m_Q^{(n)})}{m_Q^{(n)}} \times \frac{m_Q^{(n-1)}}{M_{h\ell}(m_Q^{(n-1)})} \\ &= 1 + \bar{\Lambda}(1-\lambda)\,X + \left[-\frac{\lambda_1+3\lambda_2}{2}(1-\lambda^2) + \bar{\Lambda}^2(\lambda-1)\lambda\right] X^2 + O(X^3) \end{aligned}$$

where $X = 1/m_Q^{(n)}$

Ratio y vs. $x = 1/\bar{\mu}_h$

ρ at NLO



ETMC :	$\eta_1 = -0.053(12)\,\text{GeV}$	$\eta_2 = 0.068(20)\,\text{GeV}^2$	$\bar{\Lambda} \approx 0.37\ \text{GeV}$	$\lambda_1 \approx 0$
Grem et al. :	$\eta_1 = -0.075(20)\,\text{GeV}$	$\eta_2 = 0.070(55)\,\text{GeV}^2$	$\bar{\Lambda} \approx 0.39(11)\ \text{GeV}$	$\lambda_1 \approx -0.19(10)\,\text{GeV}^2$

Bag parameters of $\Delta B = 2$ operators

- B–$\bar{B}$ mixing in the SM

$$O_1(m_b) = \bar{b}\gamma_\mu(1-\gamma_5)q\,\bar{b}\gamma_\mu(1-\gamma_5)q$$

- After matching to HQET

$$O_1(m_b) \;=\; \mathcal{M}_{11}[m_b,\mu]\,\widetilde{O}_1(\mu) \;+\; \mathcal{M}_{12}[m_b,\mu]\,\widetilde{O}_2(\mu) \;+\; \mathcal{O}\left(\frac{1}{m_b}\right)$$

where $\widetilde{O}_{1,2}$ are HQET operators

$$\widetilde{O}_1 = \bar{h}\gamma_\mu(1-\gamma_5)q\,\bar{h}\gamma_\mu(1-\gamma_5)q$$
$$\widetilde{O}_2 = \bar{h}(1-\gamma_5)q\,\bar{h}(1-\gamma_5)q$$

- But . . . it is $\widetilde{O}_{1,2}$ that fulfil the scaling laws of HQET . . .

$\rightsquigarrow$ expand the set of operators

Bag parameters of $\Delta B = 2$ operators

- Extended set of operators

$$
\begin{aligned}
O_1 &= \bar{b}^i \gamma_\mu (1-\gamma_5) q^i \, \bar{b}^j \gamma_\mu (1-\gamma_5) q^j \\
O_2 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1-\gamma_5) q^j \\
O_3 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1-\gamma_5) q^i
\end{aligned}
$$

- HQET operators

$$
\begin{aligned}
\widetilde{O}_1 &= \bar{h} \gamma_\mu (1-\gamma_5) q \, \bar{h} \gamma_\mu (1-\gamma_5) q \\
\widetilde{O}_2 &= \bar{h} (1-\gamma_5) q \, \bar{h} (1-\gamma_5) q \\
\widetilde{O}_3 &= -(\widetilde{O}_2 - \widetilde{O}_1/2)
\end{aligned}
$$

- Define vectors $\vec{O} = (O_1, O_2, O_3)$ and $\vec{\widetilde{O}}$
- The matching to HQET now involves 3×3 matrices

Bag parameters of $\Delta B = 2$ operators

- Connection to HQET [J. Reyes, 2001; D. Becirevic et al., 2002]

$$\mathbf{W}^T_{QCD}[m_h,\mu]^{-1}\,\langle\vec{O}(\mu)\rangle_{m_h} \;=\; C(m_h)\,\mathbf{W}^T_{HQET}[m_h,\mu]^{-1}\,\langle\vec{\tilde{O}}(\mu)\rangle \;+\; \mathcal{O}\left(\frac{1}{m_h}\right)$$

- Scaling laws of HQET apply for

$$\vec{\Theta}[m_h,\mu] \equiv \left(W^T_{QCD}[m_h,\mu]\,C(m_h)\,W^T_{HQET}[m_h,\mu]^{-1}\right)^{-1}\langle\vec{O}(\mu)\rangle_{m_h}$$
$$= \langle\vec{\tilde{O}}(\mu)\rangle \;+\; \mathcal{O}\left(\frac{1}{m_h}\right)$$

- Build the ratio method on these observables

$$w_k(m_h^{(n)},\mu,\lambda;m_\ell,a) = \frac{\Theta_k[m_h^{(n)},\mu]}{\Theta_k[m_h^{(n-1)},\mu]}$$

where $k \in \{1,2,3\}$ and $\lambda = m_h^{(n)}/m_h^{(n-1)}$