The Ghost Story: $\Lambda_{\bar{MS}}$ and α_s

Konstantin Petrov for France/Spain alliance

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B. Blossier, Ph. Boucaud, M. Brinet, F. De Soto, X. Du,
V. Morenas, O. Pène, J. Rodríguez-Quintero

Basics

The Goal is to determine $\Lambda_{\overline{\mathrm{MS}}}$ from lattice simulations with $2+1+1$ twisted-mass dynamical flavours.

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2,\Lambda^2) F^2(\mu^2,\Lambda^2) \ ,$$

where F and G stand for the ghost and gluon dressing functions

Diagonal Thinking

- Democracy is a popular choice (Leinweber'98)
- Pick momenta p such that
- $\frac{p^{[4]}}{(p^2)^2} < 0.3$ where
- $p^{[n]} = \sum_\mu p^n$, and $a^2 p^2 < 3$.
- Works quite well, but as usual in democracy
- Voter turnout is small
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- Works quite well, but as usual in democracy
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- We loose information from many momenta
- Also, democracy is mathematically impossible (Arrow'50)

H4

$$\alpha_T^{\rm Latt}\left(a^2p^2, a^2\frac{p^{[4]}}{p^2},\ldots\right) = \widehat{\alpha}_T(a^2p^2) + \left.\frac{\partial \alpha_T^{\rm Latt}}{\partial\left(a^2\frac{p^{[4]}}{p^2}\right)}\right|_{a^2\frac{p^{[4]}}{p^2}=0} a^2\frac{p^{[4]}}{p^2} + \ldots$$

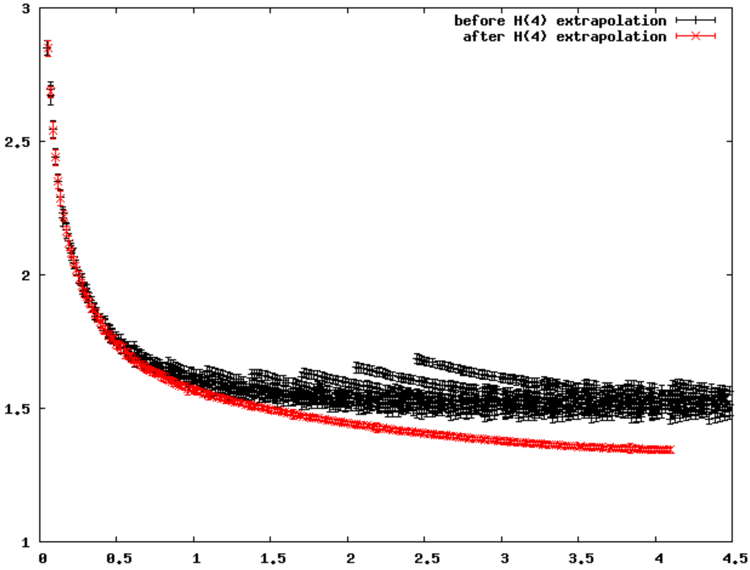
where $p^{[4]} = \sum_i p_i^4$ is the first $H(4)$-invariant (and the only one indeed relevant in our analysis).

H4

- average over any combination of momenta being invariant under $H(4)$ ($H(4)$ orbit)
- extrapolate then to the "continuum case"
- the effect of $a^2 p^{[4]}$ must vanish, by applying H4 for all the orbits sharing the same value of p^2
- with the only assumption that the slope depends smoothly on $a^2 p^2$
- $H(4)$-artefact-free lattice coupling, $\widehat{\alpha}_T(a^2p^2)$ might differ from the continuum coupling by some $O(4)$-invariants artefacts,

$$\widehat{\alpha}_T(a^2p^2) \;=\; \alpha_T(p^2) \;+\; c_{a2p2}\; a^2p^2 \;+\; \mathcal{O}(a^4)\;, \qquad (1)$$

Ghost Dressing Function



Running with the Coupling

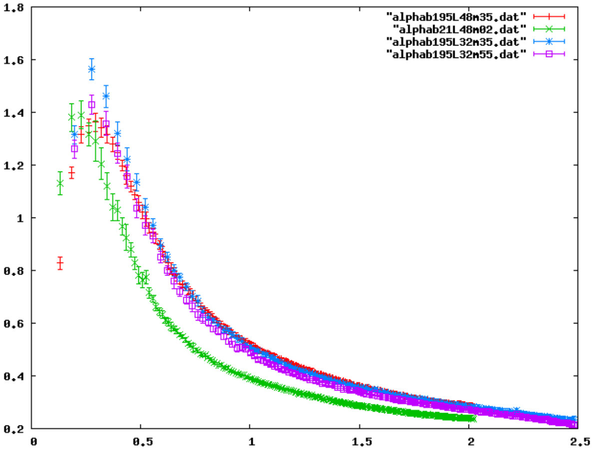
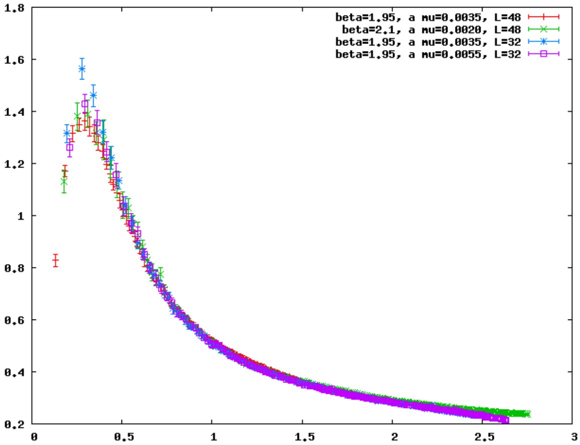
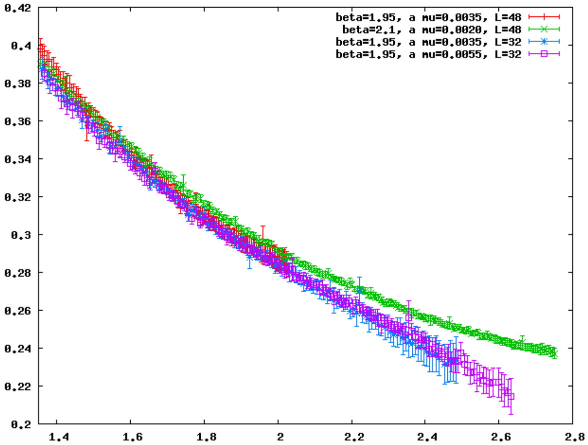


Figure: $\beta = 1.95$ with $a\mu_l = 0.0035$ at $48^3 \times 96$ (red) and $32^3 \times 64$ lattices (blue), $a\mu_l = 0.0055$ at $32^3 \times 64$ (violet) and $\beta = 2.1$ with $a\mu_l = 0.0020$ at $48^3 \times 96$ (green).

superimposing



zooming


beta=1.95, a mu=0.0035, L=48
beta=2.1, a mu=0.0020, L=48
beta=1.95, a mu=0.0035, L=32
beta=1.95, a mu=0.0055, L=32
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8

Very Long Formula

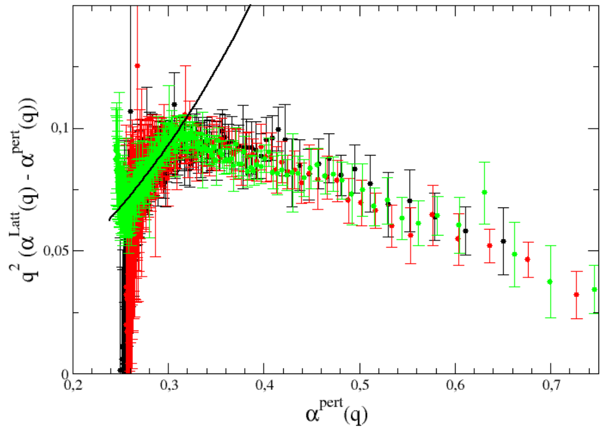
$$\alpha_T(\mu^2) \ = \ \alpha_T^{\rm pert}(\mu^2) \ \left(1 + \frac{9}{\mu^2} \ R\left(\alpha_T^{\rm pert}(\mu^2), \alpha_T^{\rm pert}(q_0^2)\right) \left(\frac{\alpha_T^{\rm pert}(\mu^2)}{\alpha_T^{\rm pert}(q_0^2)} \right)^{1-\gamma_0^{A^2}/\beta_0} \right.$$

where $\gamma_0^{A^2}$ can be taken from Gracey, Chetyrkin to give for $N_f = 4$

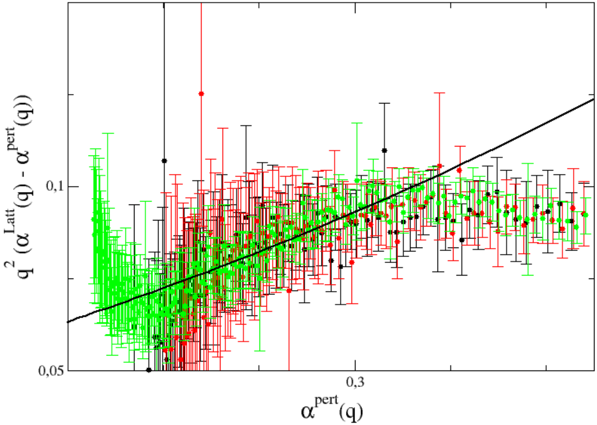
$$1 - \gamma_0^{A^2}/\beta_0 = \frac{27}{132 - 8N_f} = \frac{27}{100} \ ;$$

$$\frac{\Lambda_{\overline{\rm MS}}}{\Lambda_T} = e^{-\dfrac{507 - 40N_f}{792 - 48N_f}} \quad . \qquad (2)$$

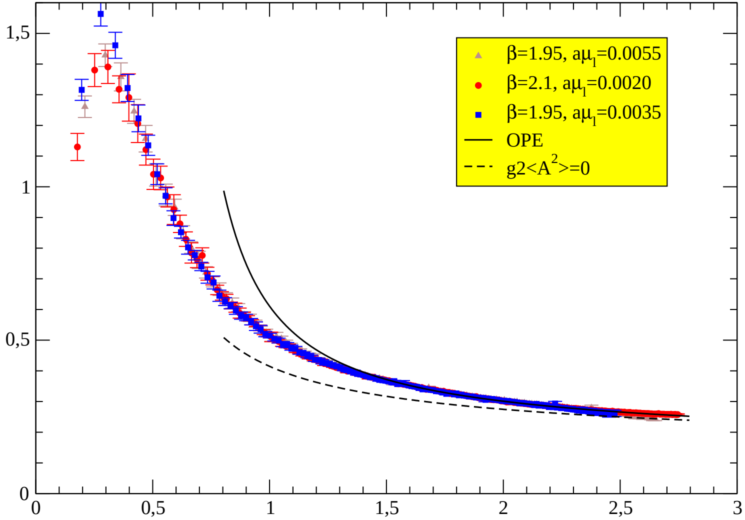
Checking the Wilson Coefficient



zooming



Correcting



β=1.95, aμl=0.0055
β=2.1, aμl=0.0020
β=1.95, aμl=0.0035
OPE
g2<A2>=0
0
0,5
1
1,5
2
2,5
3

Numbers

$$a(2.1) = 0.0607(2)\ \text{fm}$$

$$\Lambda_{\overline{\mathrm{MS}}}^{N_f=4} \;=\; 298 \pm 13\ \mathrm{MeV}$$

$$\alpha_S\,(M_{Z^0}) \;=\; 0.1187(9)\ ;$$

Delusions and Hope

- CINES SGI ICE - Jade 6.5M (source smearing and dilution study)
- IDRIS IBM SP Vargas 2.75M (nucleon and 48 propagators)
- IBM BG/P - Babel 24M (gauge fixing and contractions)
- TGCC Bull - Curie noeuds hybrides GPU 72k $B \longrightarrow D^{**}$
- Some PetaQCD time dedicated to multiGPU tests/development
- which will be wasted on inverting $64^3 \times 128$