The tale of Navier and Stokes meeting Heisenberg at Hawking's place (with some disappointment for Einstein)

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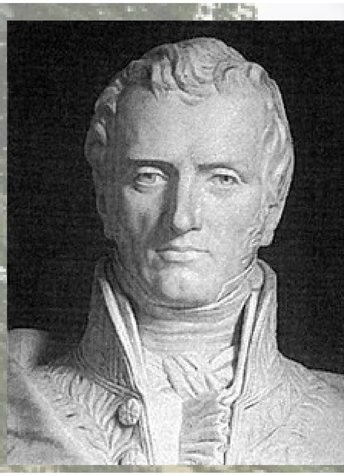
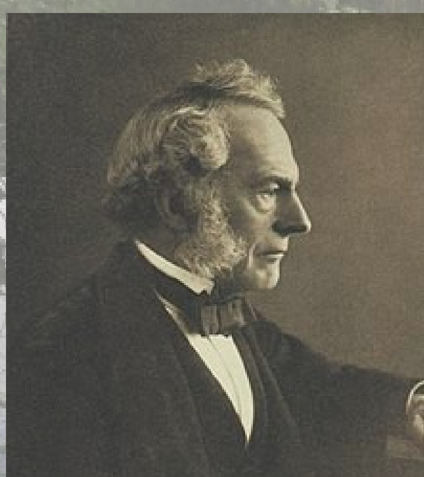
A science-fiction tale with celebrated characters...



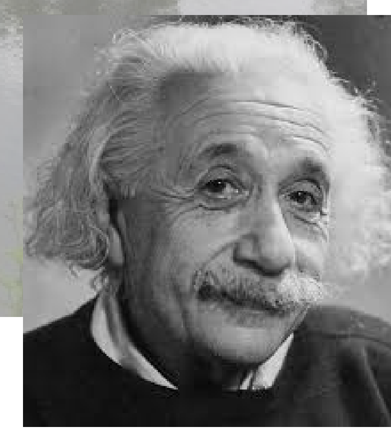
Claude-Louis Navier
1785-1836

Werner Heisenberg 1901-1976



George Stokes 1819-1903

Stephen Hawking 1942-2018



Albert Einstein 1879-1955

From analog black holes to quantum simulation of gravitational problems

a fruitful bidirectional synergy of gravity and quantum optics

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Part 1

Basics of analog models

Quantum simulation before quantum simulation

Analog Hawking radiation in atomic superfluids

"Fishic" (but not fishy!) horizon

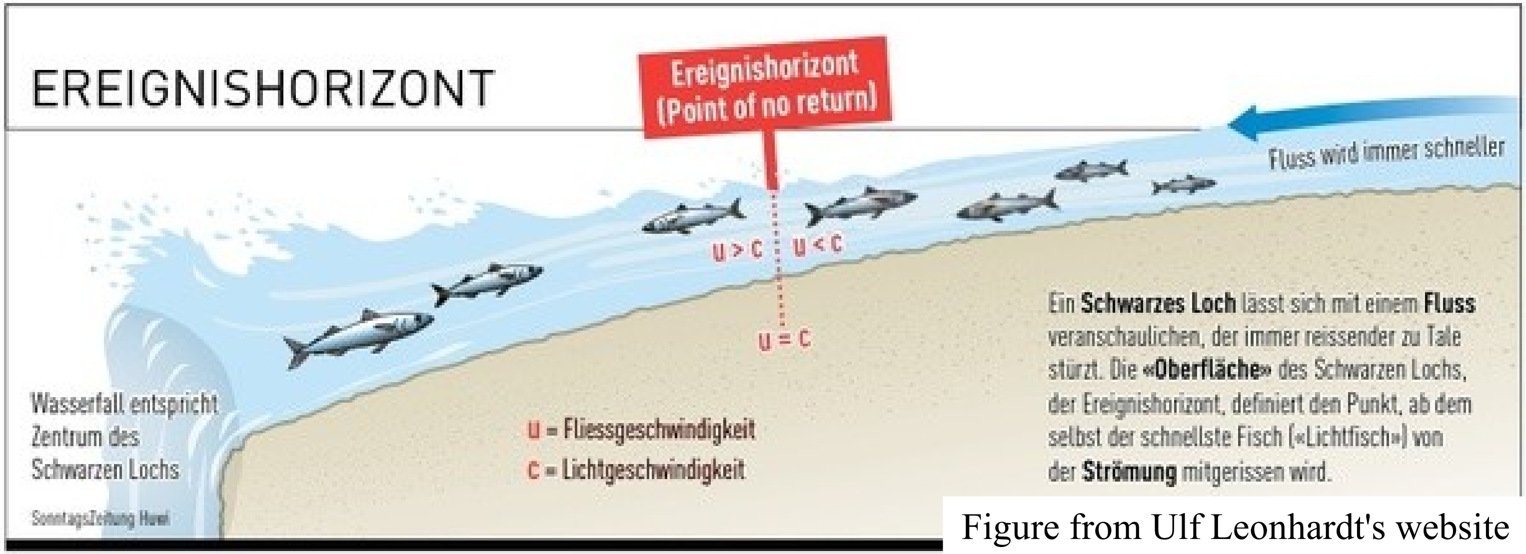


Figure from Ulf Leonhardt's website

Fish swim at $v=c_s$ in the river's frame i.e. at $c_s \pm v_{flow}$ in the land's frame

- Horizon (where $c_s=v_{flow}$) separates sub-fishic flow (upstream) from super-fishic flow (downstream)
- Fish in super-fishic region can not swim back through fishic horizon

} Behavior analogous to light around astrophysical black hole horizon

Acoustic horizon

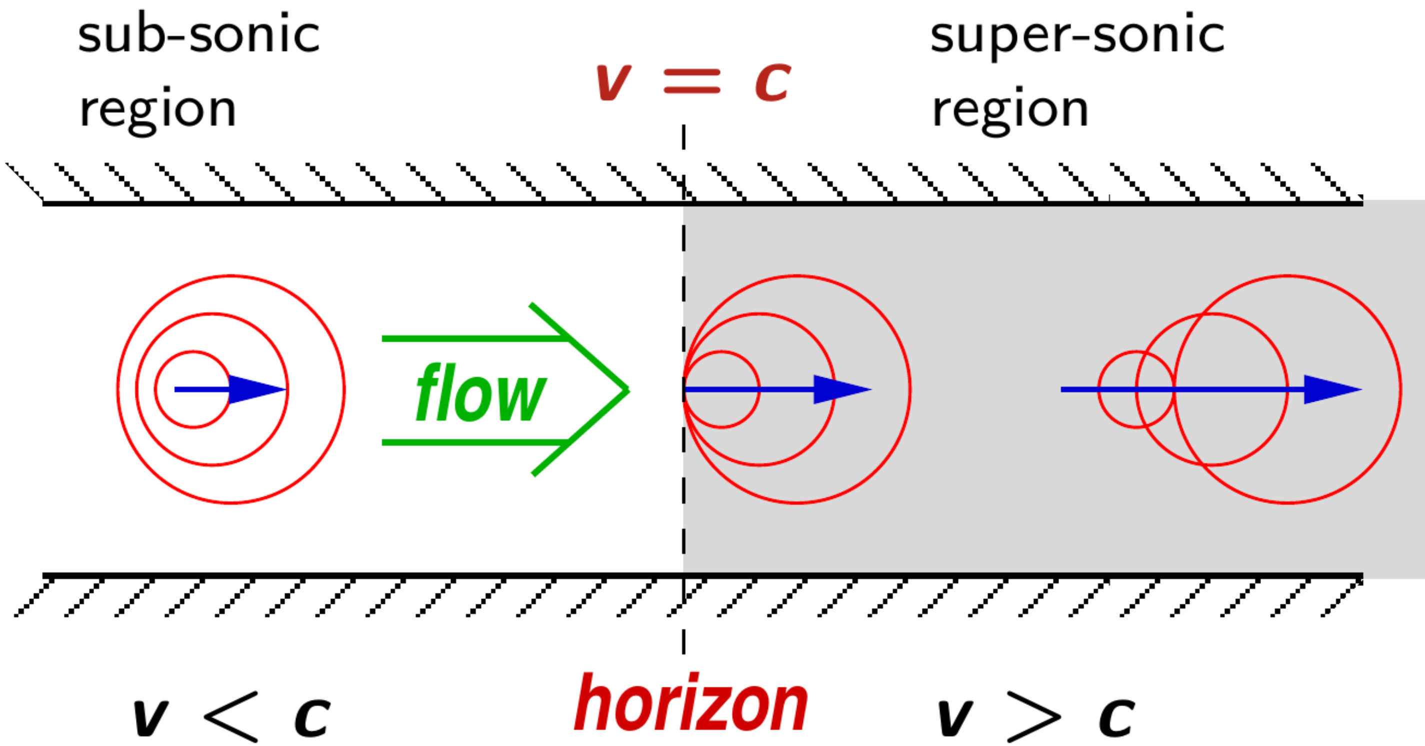


Figure from N. Pavloff's website

- Sound emitted in super-sonic region is dragged by the flow in the downstream direction
- Excitations in super-sonic region can not travel back through horizon
- Acoustic analog of black hole horizon in gravity

- What happens with quantized radiation field ? Hawking radiation of sound ?

Unruh, *Experimental Black-Hole Evaporation?,* PRL 1981 (earlier than 1982 celebrated Feynman's work)
Barceló, Liberati, Visser, Liv. Rev. Relativity **14**, 3 (2011)

Mathematical framework

Superfluid hydrodynamics of dilute BEC, e.g. ultracold atomic gas

Gross-Pitaevskii equation for BEC order parameter $\Psi(x,t)$:

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Psi(x,t)+V(x)\Psi(x,t)+g\,|\Psi(x,t)|^2\Psi(x,t)$$

- Sonic dispersion of low-k phonons $\omega=c\,|k|$.
- Doppler shifted $\omega=c\,|k| - v\cdot k$ in moving fluid at v

Modulus-phase picture $\Psi(x,t)= n(x)^{1/2}\, e^{\,i\,\Phi(x,t)}$ → relativistic eq for $\Phi(x,t)$

$$\frac{1}{\sqrt{-G}}\,\partial_\mu\left[\sqrt{-G}\,G^{\mu\nu}\partial_\nu\right]\,\phi(x,t)=0$$

Equivalent to light propagation in curved space-time metric

$$ds^2=G_{\mu\nu}dx^\mu dx^\nu=\frac{n(x)}{c_s(x)}\left[-c_s(x)^2dt^2+(d\vec{x}-\vec{v}(x)dt)(d\vec{x}-\vec{v}(x)dt)\right]$$

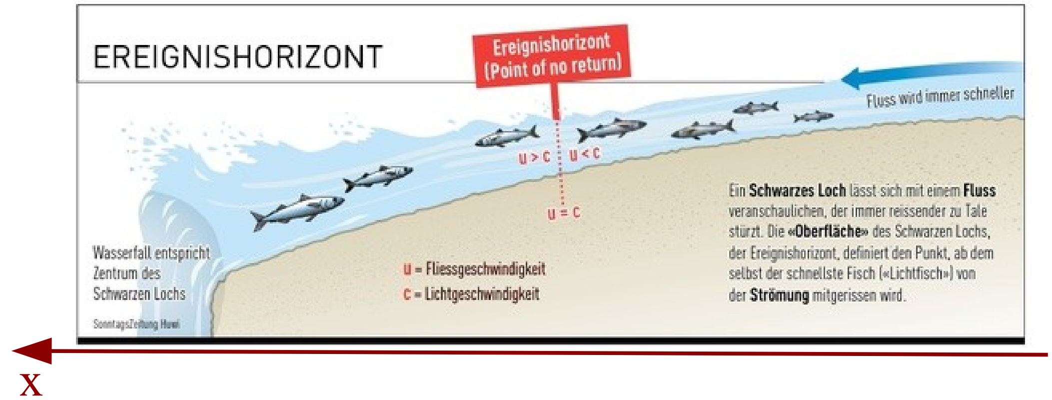
Once quantized → quantum field theory in a curved space time → Hawking emission?

Unruh, PRL 1981; Barceló, Liberati, Visser, Liv. Rev. Relativity **14**, 3 (2011)

Acoustic Black Hole

Simplest analog black hole geometry:

- one-dimensional geometry
- flow in the +x direction
- $v(x)/c_s(x)$ increases along +x direction
- horizon where $v(x_H) / c_s(x_H) = 1$



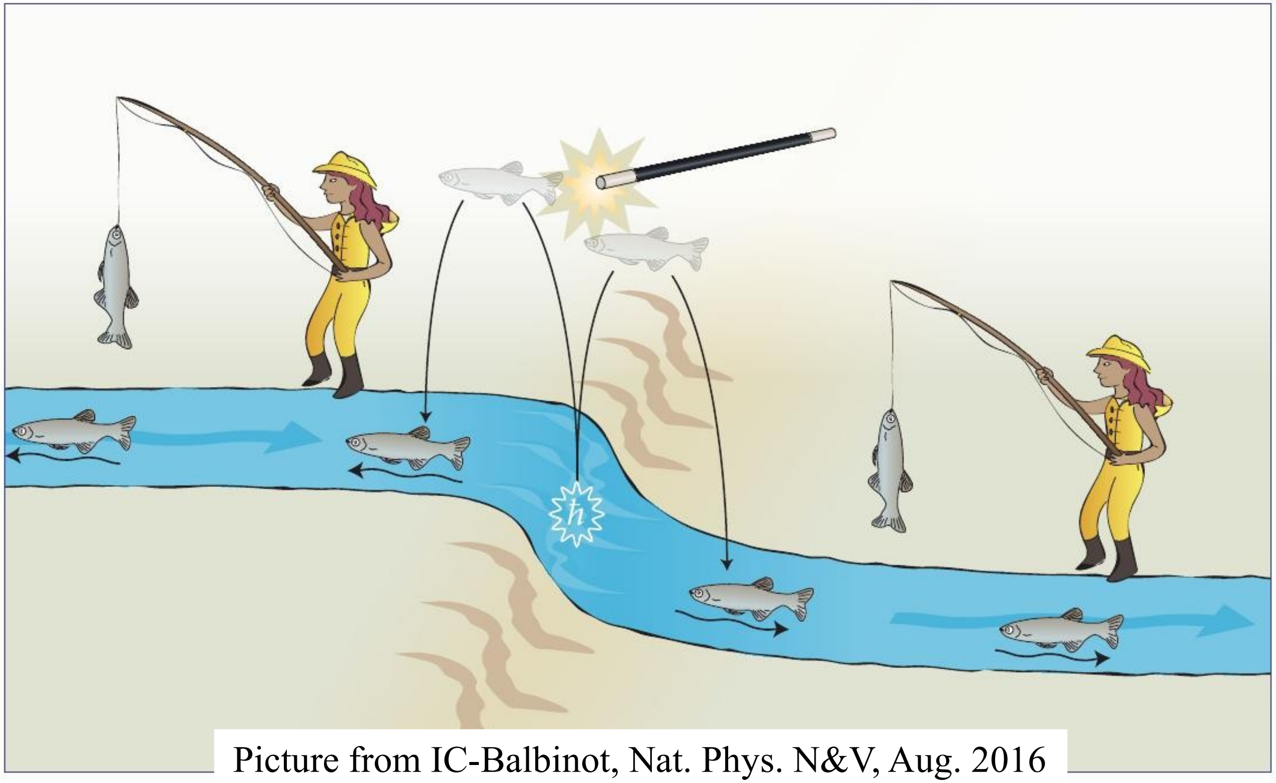
Astrophysical black holes → Hawking emission at $T_H = \frac{\hbar c^3}{8\pi k_B G M}$

- ≈ fraction of μK for solar mass BHs, even lower for supermassive BH
- to be compared to 2.7K of Cosmic Microwave Background

Analog models → Hawking emission of sound at $T_H = \frac{\hbar}{4\pi k_B v} \frac{d}{dx}(c_s^2 - v^2)\Big|_{x=x_h}$

- $T_H \sim$ nK, to be compared with $T_{BEC} \sim$ nK as well
- but also something new and exciting...

How to detect Hawking radiation?



Picture from IC-Balbinot, Nat. Phys. N&V, Aug. 2016

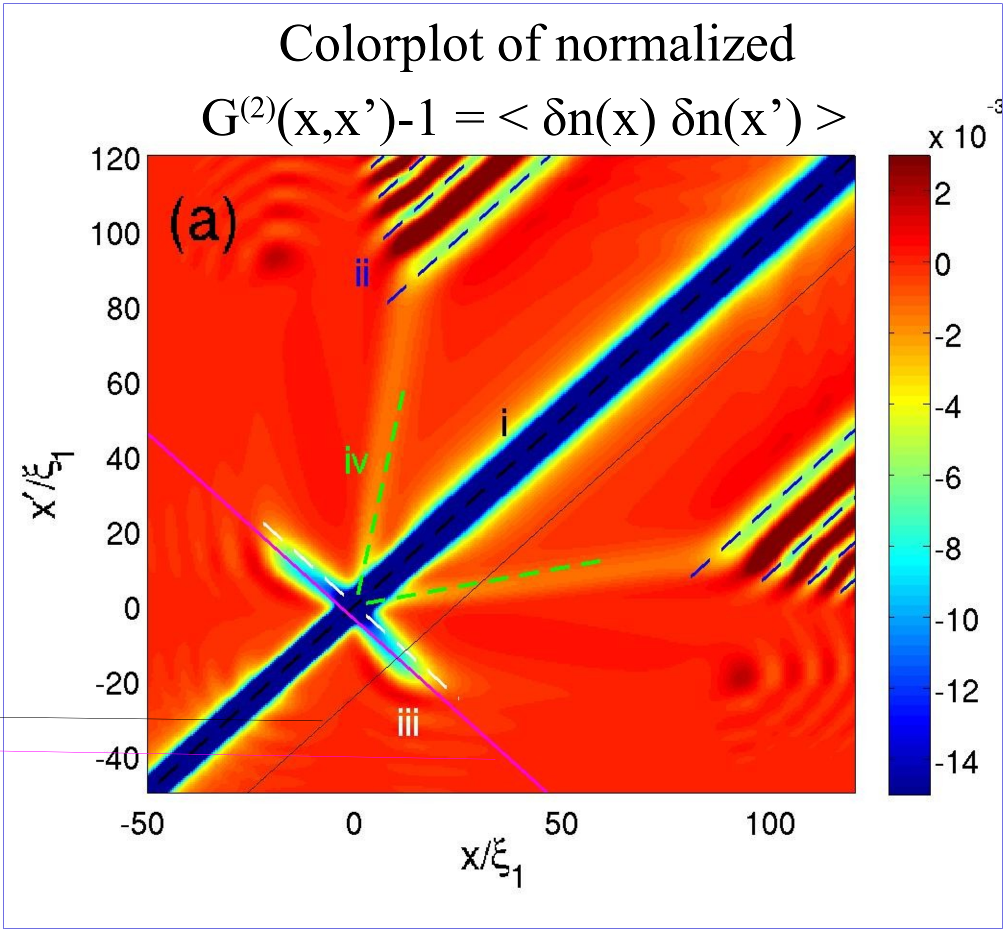
- Hawking radiation → correlated pairs generated simultaneously at the horizon
- Of course not detectable in astrophysics
- In analog models, HR isolated from background of thermal and noise phonons by measuring correlations on opposite sides of horizon
- In the picture: Hawking fish are caught simultaneously by the two fisherwomen!

The Hawking signal: theoretical prediction

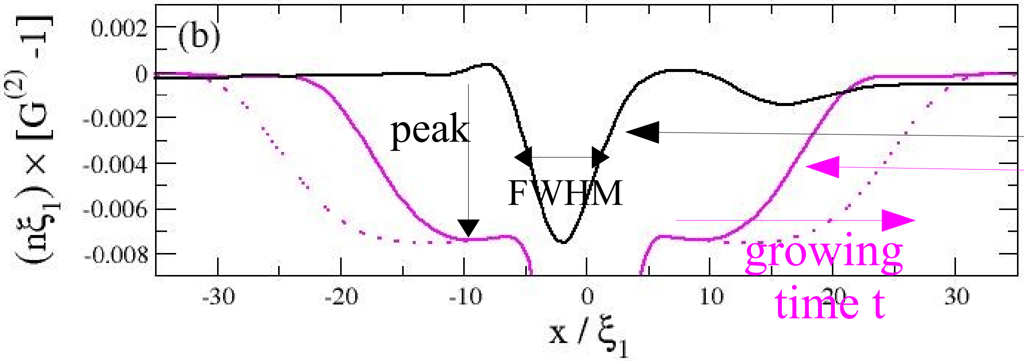
Wigner Monte Carlo simulations of the quantum fluctuations

Negative correlation tongue extending from the horizon x=x'=0

- long-range in/out density correlation which disappears if both $c_{1,2} < v_0$
- length grows linearly in t
- peak height, FWHM constant in t
- slope $\frac{v_0 - c_2}{v_0 - c_1}$ agrees with theory
  - pairs emitted at all t from horizon
  - propagate at sound speed



cuts of $G^{(2)}(x,x') - 1$

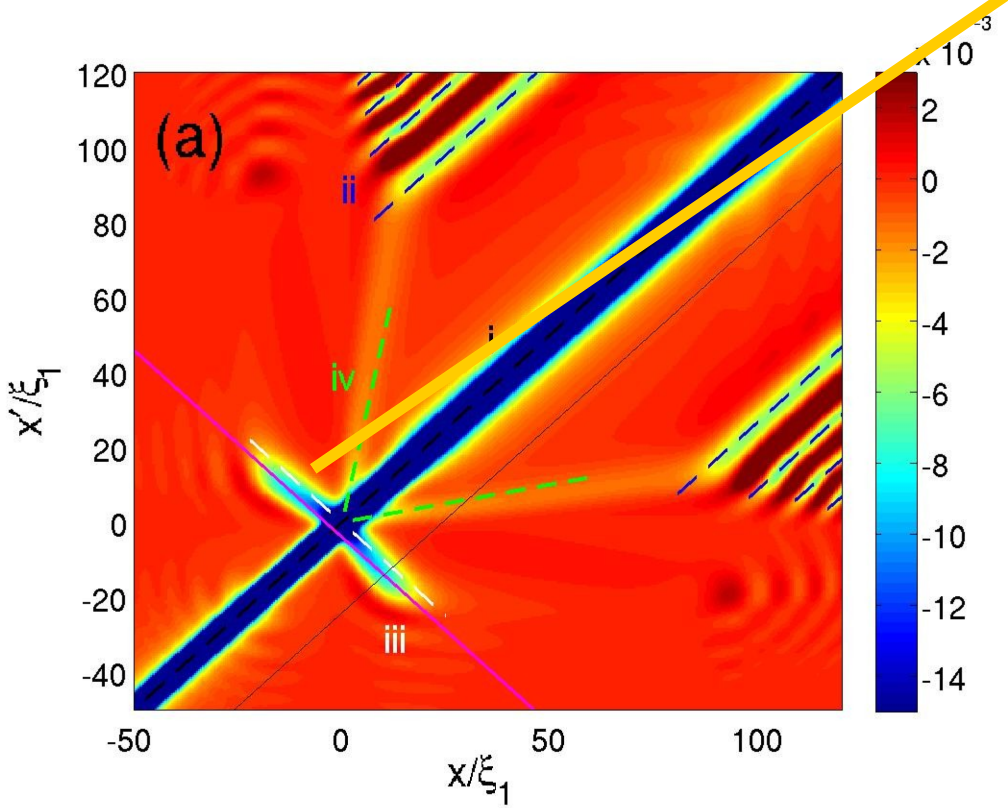


Numerics in: IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)
In agreement with theory in: R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, IC, PRA 78, 021603 (2008).

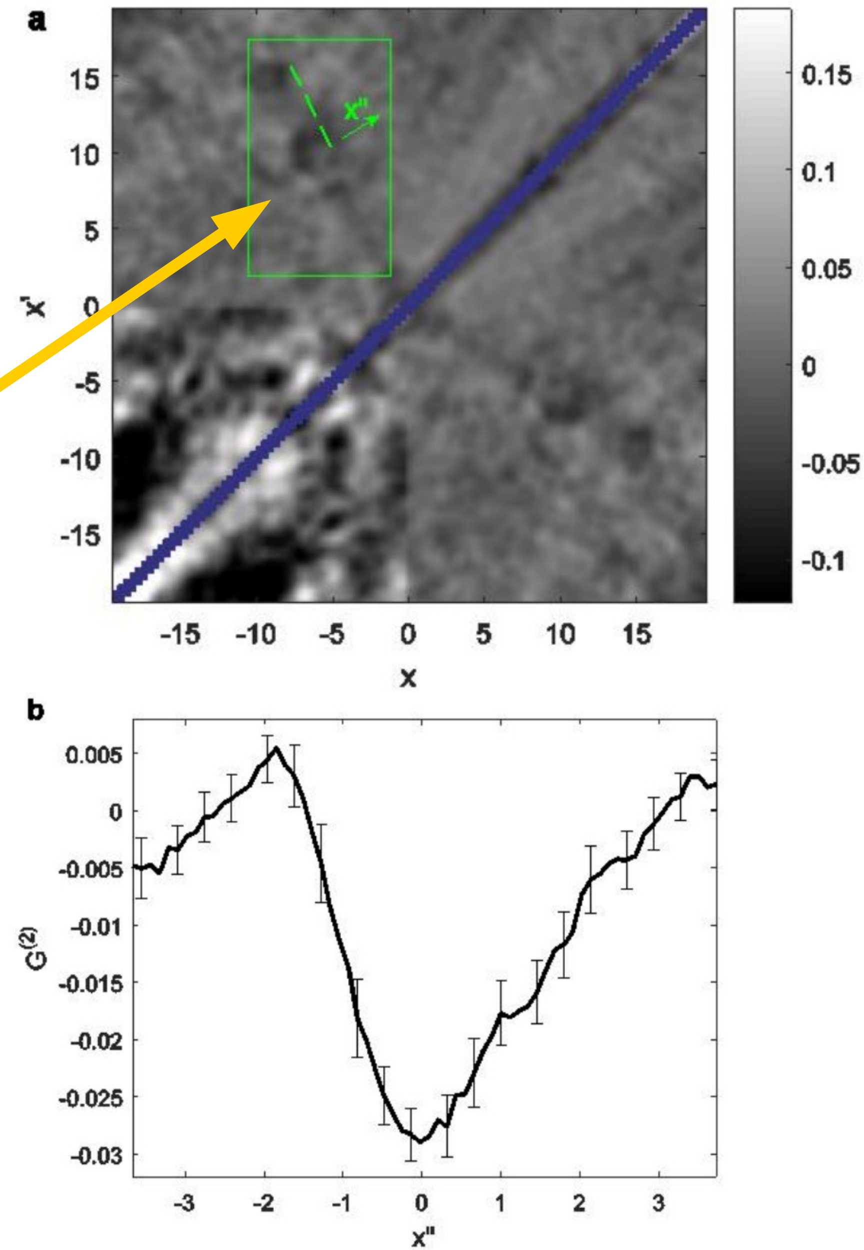
Analog Hawking radiation detected in the lab!

Analog black hole configuration obtained by sending 1D atomic BEC against optical potential

Experimental evidence of HR based on Balbinot-Fabbri moustache in correlation function of density fluctuations



Theory: IC et al., NJP 2008



Expt: Steinhauer Nat. Phys. '16

New (better) pictures from De Nova et al., Nature ‘19

Experimental evidence of entanglement ?[N]

Peres-Horodecki criterion for non-separability

Fourier transforms of $G^{(2)}(x,x')$ along suitable lines

$$\Delta \equiv \langle \hat{b}^\dagger_{k_{HR}} \hat{b}_{k_{HR}} \rangle \langle \hat{b}^\dagger_{k_P} \hat{b}_{k_P} \rangle - |\langle \hat{b}_{k_{HR}} \hat{b}_{k_P} \rangle|^2 < 0$$

Population — Anomalous correlation

Entanglement visible in intermediate k-range

→ HR from zero-point fluctuations

→ produces entangled phonon pairs

Are data statistically significant?
Any other alternative explanation?

} Harsh on-going debate

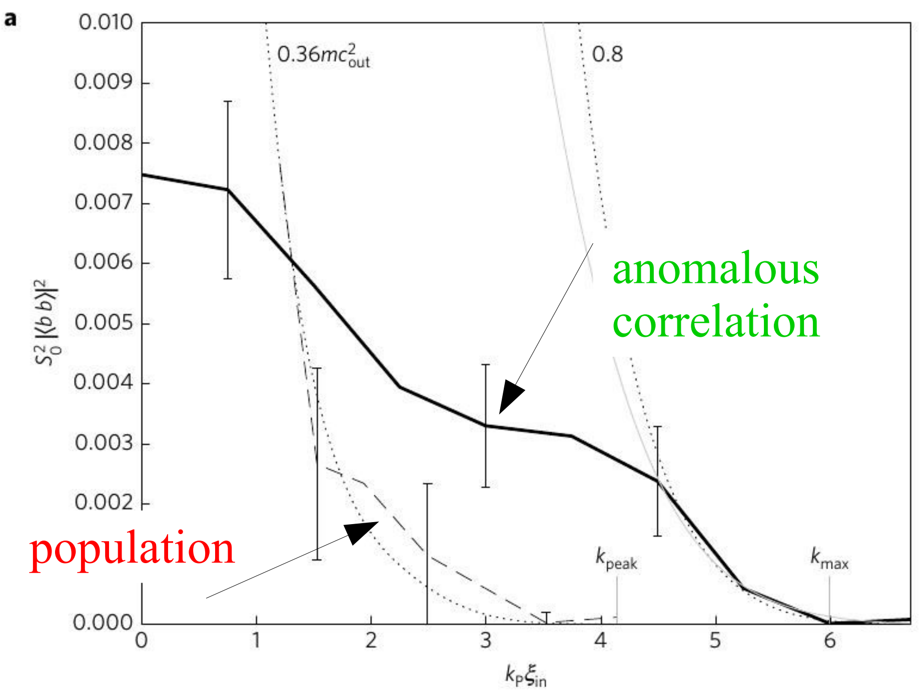


Figure from Steinhauer, Nat. Phys. '16

Much more theoretical work by de Nova, Sols, Parentani, Bruschi, Fuentes, etc.

Long-term perspectives

- Quantum Hydrodynamics: Navier-Stokes eqs. with hats on macroscopic hydrodynamic variables
- Entangled states of a macroscopic fluid

What physics can be learnt on ‘real’ black holes ?

Standard derivations of Hawking radiation assume:

- linear dispersion $\omega(k) = c\,|k|$ at all length scales
- infinite blue shift of modes at horizon, GR and QFT valid up to arbitrary energies

These assumptions violated in analogs:

- closer look: microscopic mechanism of HR very different
- key role of deviations from hydrodynamics at high energies

What do we learn from the observation of analog HR?

- thermal HR robust to “Planck-scale” physics and Lorentz-violations
- Peculiar features imprinted onto HR spectrum

- Observable @ LHC ? explanation why we survived 2008 switch-on?

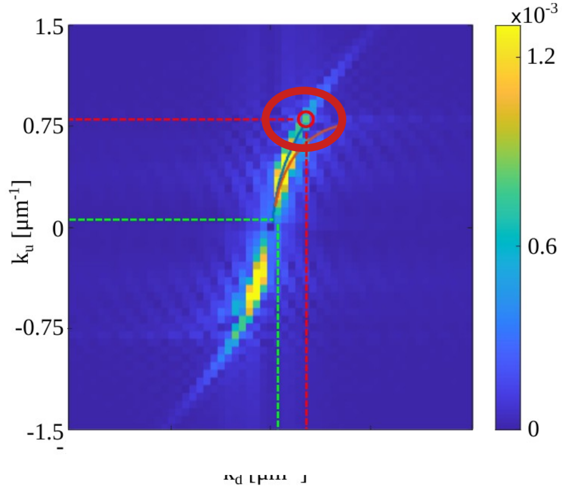
pseudo-La Repubblica 11/9/08

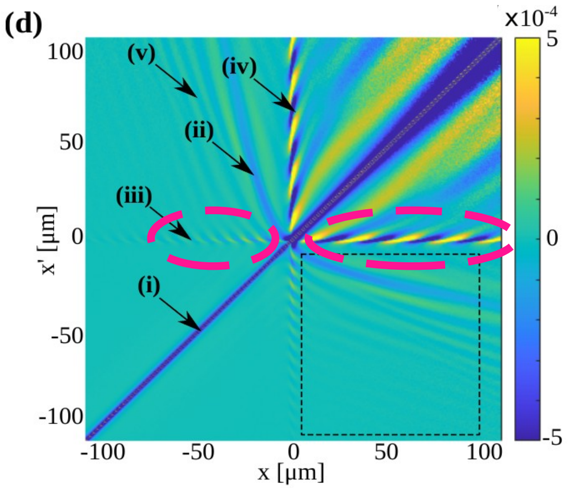
W. G. Unruh, Phys. Rev. D 51, 2827 (1995); R. Brout et al., Phys. Rev. D 52, 4559 (1995)
T. A. Jacobson and R. Parentani, *An echo of Black Holes*, Scientific American, Dec. 2005.

An unexpected new direction...

During preliminary study for black hole expt:

- BH may feature a localized quasi normal mode
- spontaneous Hawking-like emission displays peak @ QNM
- hosts finite zero-point quantum excitation
  by same mechanism responsible of Hawking emission
- correlation between emission and QNM excitation
- result generic to conservative and driven-dissipative analog models





QNMs are common feature of astrophysical BHs

- general wave modes (e.m., gravitational, …) orbiting around BH
- decaying eigenmodes of BH dynamics; classically excited
  during astrophysical processes, e.g. mergers
- radiatively decay into emitted gravitational waves

So… what we may dare to guess about astrophysical BHs ?

- Not only BHs are not fully "black" because they emit Hawking radiation...
- ...but also their shape "fluctuates" under the effect of zero-point fluctuations of space-time
- ...and Hawking emiss. (gravitational and e.m.) → specific spectral features (beyond grey-body factor)

Part 2:

False vacuum decay via bubble formation in ferromagnetic superfluids

theory & experiments at the Pitaevskii BEC Center

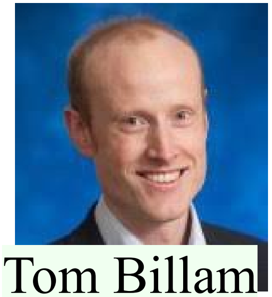
Similar work is going on in the (mostly UK-based) Quantum Simulator for Fundamental Physics (qSimFP) consortium

The dream team

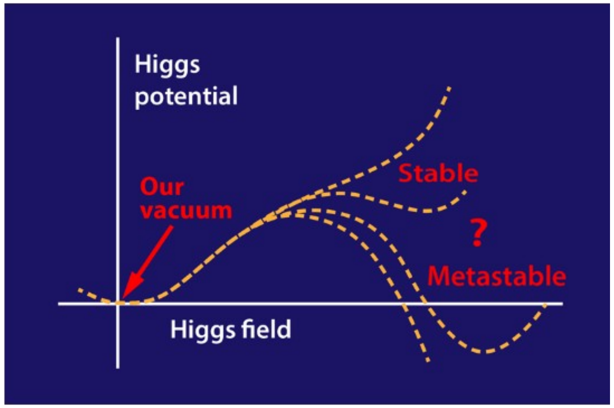
Experimentalists
@ Pitaevskii BEC Center

Theorists @ Pitaevskii BEC Center

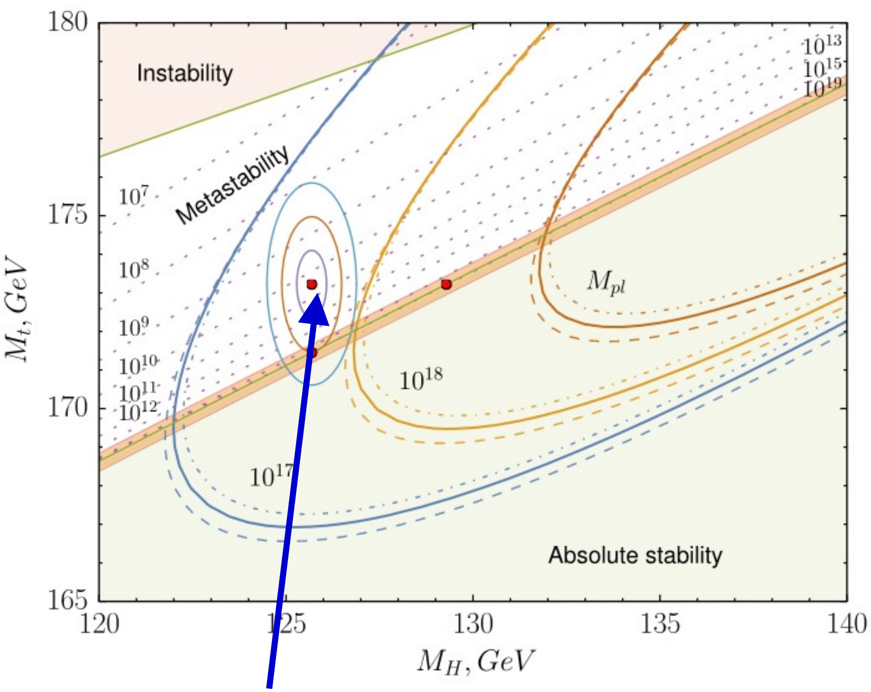
Our collaborators @ Newcastle
on theory of false vacuum decay



Are we on the brink of the Higgs abyss?



Figures & Title from Bednyakov et al., PRL 2015 and associated viewpoint on Physics by Kusenko



Our Universe is expected to be metastable but not far from stability boundary
(many hypothesis and approximations behind calculation)

According to standard model
Higgs field may be in metastable state

Tunneling event towards global energy minimum
→ totally different physics and huge energy release
The END of our world !

Theoretical open questions:

- Existing calcul.→ very slow decay (~10^{65} yrs), but...
- Standard Model correct up to Planck scale?
- Astrophysical & gravity effects, e.g. seed by BHs ?

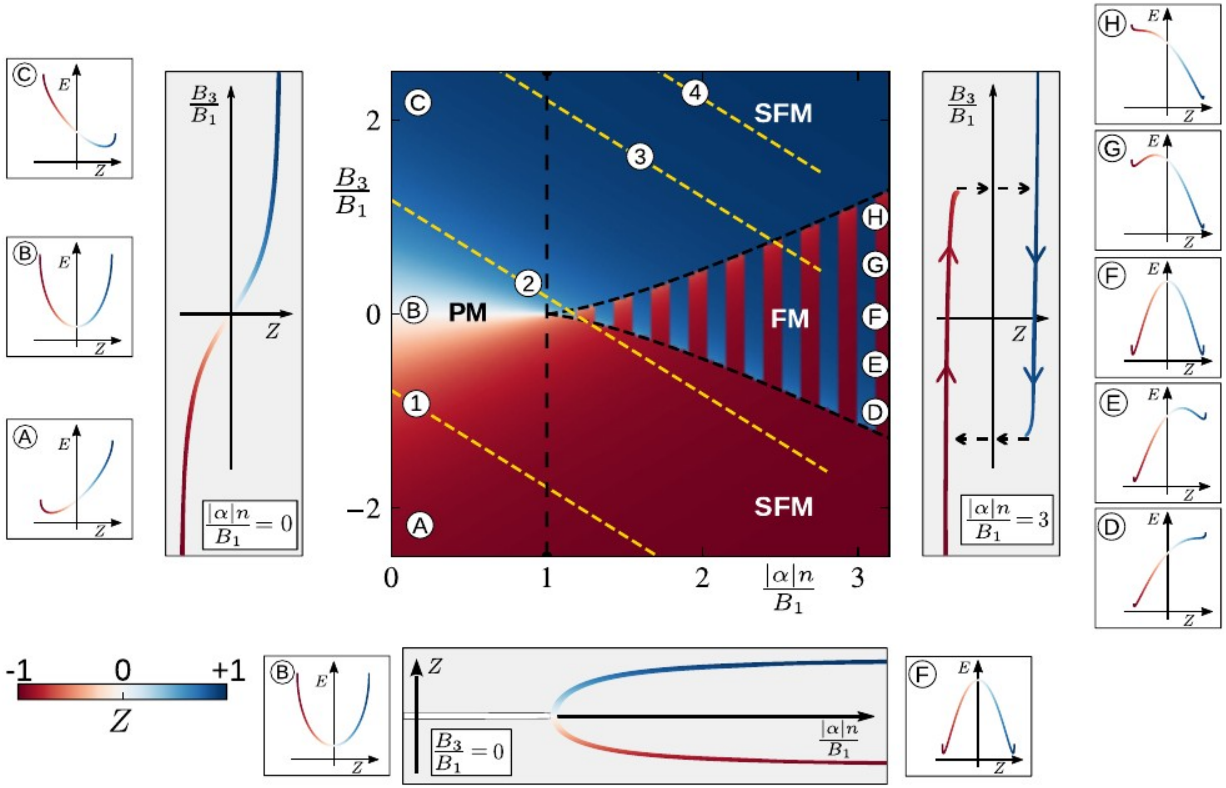
- Instanton calcul. only tractable under serious approx
- Many degrees of freedom at play simultaneously

→ Physical insight from simplified models very useful !

General questions:

- why Universe in metastable rather than stable state?
- bounds on dark energy/mass from (meta)stability?

Ferromagnetic two-component BEC



Atomic Bose-Einstein condensate:

- Two component Na atoms |↑> =|2,-2>, |↓>=|1,-1>
- Immiscible regime $a^2_{\uparrow\downarrow} > a_{\uparrow\uparrow}\, a_{\downarrow\downarrow}$
- Mixture stabilized by coherent coupling Ω_R

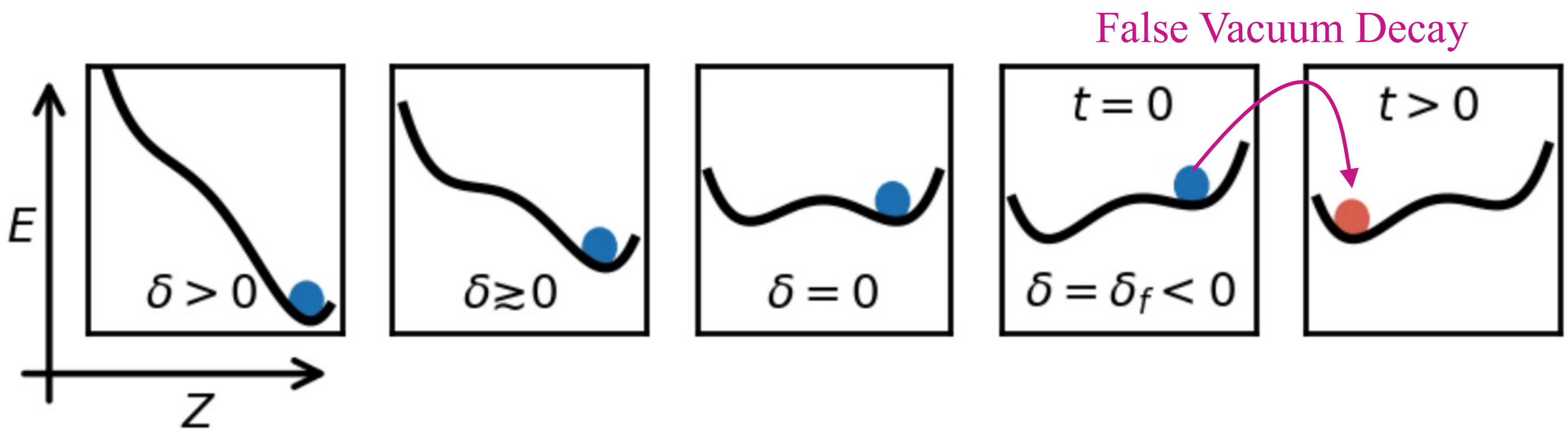
→ Energy landscape resemble Higgs abyss

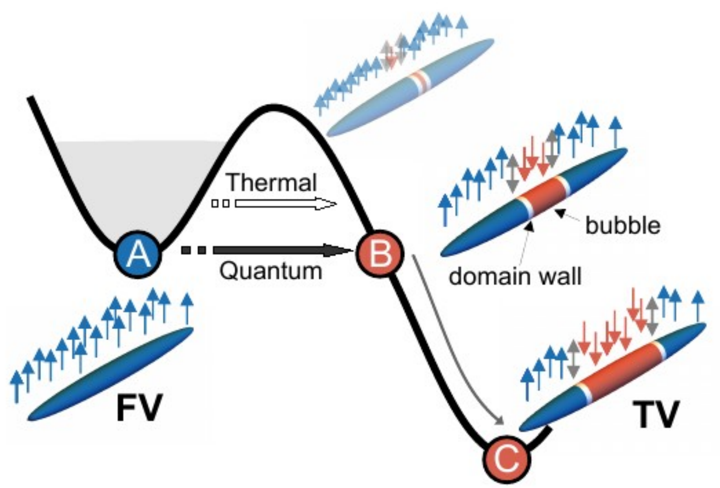
Mapped on Ising model in transv.-B

- Coherent coupling Ω_R → transverse field B_t
- Zeeman detuning δ → longitudinal B_z
- Atomic interactions → Ising z-z coupling
- Mean-field picture: $\mathbf{H}_{\text{eff}} = (\Omega_R, 0, \delta_{\text{eff}} - \kappa n Z)$

Cominotti et al., *Ferromagnetism in an Extended Coherently Coupled Atomic Superfluid*, PRX **13**, 021037 (2023)

1st order phase transition and metastability





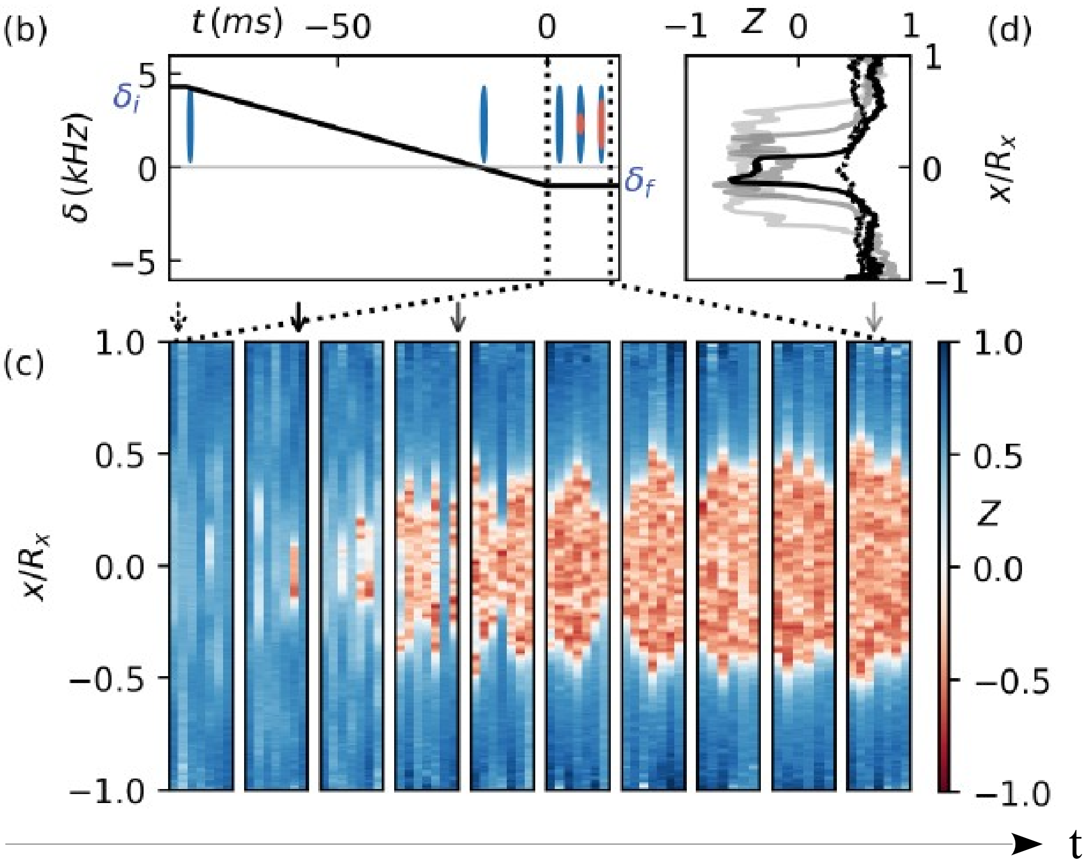
- Start from BEC in ground |↑> state @ $\delta > 0$
- Slowly decrease δ to $\delta < 0$
- |↑> state is metastable local minimum
  -- False Vacuum
- We expect:
  - Eventually decays to ground |↓> state
    -- True Vacuum
  - Slow process via bubble formation,
    activated by quantum/thermal fluctuations

Zenesini et al., *Observation of false vacuum decay via bubble formation in ferromagnetic superfluids*, Nature Physics (2024).

Bubble formation & False Vacuum Decay

Experimental observation of bubbles

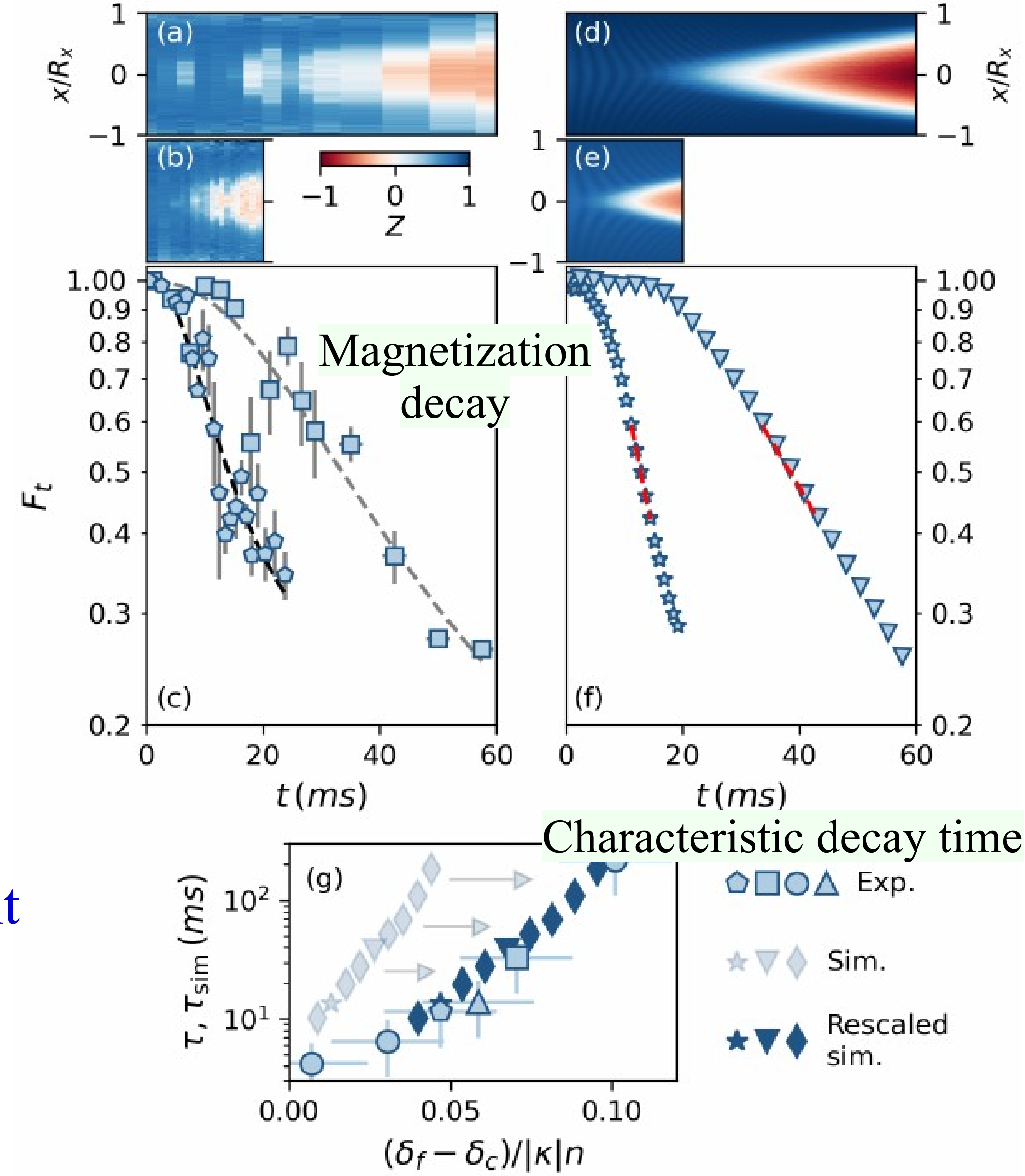
- Wait different times t
- Separated images of |↓,↑> states
  → extract magnetization profile
- Observe:
  - Stochastic bubble formation
  - Most likely at center of sample
  - On average, bubble size grows with t



Zenesini et al., *Observation of false vacuum decay via bubble formation in ferromagnetic superfluids*, Nature Physics (2024).

Quantitative analysis: magnetization decay time

Average of magnetization profile as a function of time



<u>Numerical simulations:</u>

- Truncated Wigner method unreliable (UV cut-off dependent, thermalizes to classical state before FVD,…)
- Stochastic Gross-Pitaevskii → good description of thermal effects
- Follow all stages of preparation and then evolution

Results in good quantitative agreement with experiment

Zenesini et al., *Observation of false vacuum decay via bubble formation in ferromagnetic superfluids*, Nature Physics (2024); Anna Berti, PhD Thesis, UniTN (2024).

Quantum- or thermally- activated process?

- Significant thermal component in the outer part of cloud
- Experiment reproduced by stochastic GPE simulations @ finite T
  - initial thermal fluctuations modified during preparation into non-equilibrium state
  - effective temperature of spin modes in numerics
    estimated around 100nK - 1μK

Our QFT specialists
Ian Moss and Tom Billam

- Instanton calculations in QFT formalism
  → quantum tunneling dominant
  only at much lower nK temperatures

Strong indication that our false vacuum decay in our experiments is triggered by classical thermal fluctuations

False Vacuum Decay in monitored systems

Quantum tunneling requires much lower T

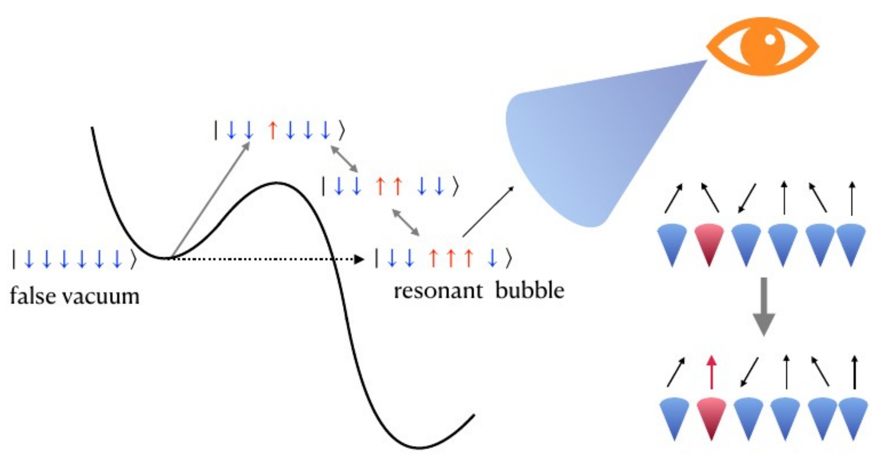
This might be experimentally possible but one more subtle hurdle on the way...

Any realistic system:

- Coupled to environment (external and/or other DoF and/or atom losses)
- Information transferred to environment via different couplings
- Effect on quantum-induced false vacuum decay?

Simplest theoretical model:

- Spin-1/2 Ising chain in transverse B_t. At weak B_t: ferromagnetic state
- Start from |↓> state, continuously monitor sites in |↑> state
- Measurement projects spin onto |↑>. Deposited energy favors bubble formation



Jeff Maki

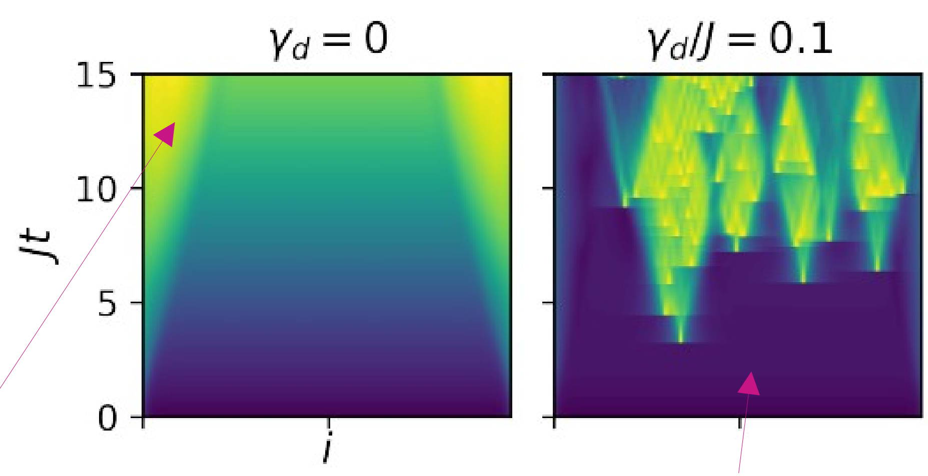
Alberto Biella

Maki… IC, Biella, *Monte Carlo matrix-product-state approach to the false vacuum decay in the monitored quantum Ising chain*, SciPost Physics (2023)

Numerical calculations

Monte Carlo Matrix product states (MCMPS) technique:

- MPS representation of many-body states and (non-Hermitian) evolution
- Measurements → Quantum jumps at random times



γ_d=0 → dominated by edge processes

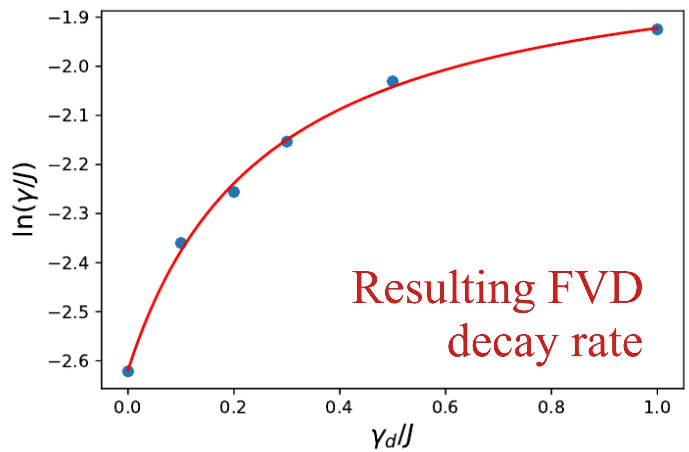
γ_d>0 → dominated by random measurement events

Maki… IC, Biella, *Monte Carlo matrix-product-state approach to the false vacuum decay in the monitored quantum Ising chain*, arXiv:2306.01067 (2023)

Coupling to other DoF
→ similar effect as measurement ?

On-going work: Spin-1 chain

- accessible to ab-initio numerics
- hosts phonon modes like in BECs
- generic model for other DoF



Interest goes beyond quantum simulators/analog models

- Crucial to assess if *quantum* tunneling is fundamentally possible
- May induce radical revision of our estimates for (non-human contribution to) doomsday...

Conclusions & perspectives

superfluids of atoms & light ↔ gravitational phenomena: a fruitful bidirectional synergy

Hawking radiation from Black Holes:

- Original theoretical predictions + $G^{(2)}(x,x')$ correlations → experimentally observed
- robust to UV corrections, solves trans-Planckian problem of infinite frequency @ horizon
- On-going challenge → robust evidence of quantum correlations in HR → *quantum hydrodynamics*

"The tale of Navier and Stokes meeting Heisenberg at Hawking's place (with some disappointment for Einstein)"

Quantum simulation of false vacuum decay:

- Decay of metastable state via (random) formation of bubbles.
- Thermally activated mechanism. Ultralow (sub nK) temperatures needed to go quantum.
- Extra channels: FVD triggered by monitoring. Key role of other DoF (phonons, atom losses,…) ?

Astrophysical/cosmological/gravity implications of fluctuations to be explored:

- Coupling to other DoF → radical revisions on decay rate of Universe ?
- Analog Hawking radiation from BHs → QNMs give intrinsic fluctuations of space-time around BH ?
- Cosmological pre-heating → quantum fluctuation of inflaton field. Observable in CMB ?

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Living Reviews in Relativity
December 2011, 14:3 | Cite as

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First Online: 11 May 2011

Superradiant phenomena
Lessons from and for Bose–Einstein condensates

Luca Giacomelli

Ph.D. thesis submitted to Dipartimento di Fisica
Università degli studi di Trento

Under the supervision of
Dr. Iacopo Carusotto
Prof. Massimiliano Rinaldi

news & views

QUANTUM HYDRODYNAMICS

Acoustic Hawking radiation

A milestone for quantum hydrodynamics may have been reached, with experiments on a black hole-like event horizon for sound waves providing strong evidence for a sonic analogue of Hawking radiation.

Iacopo Carusotto and Roberto Balbinot

Nat. Phys., Aug.15h, 2016

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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I. Carusotto, C. Ciuti, Rev. Mod. Phys. **85**, 299 (2013)

Dedicated to a friend and a master

Renaud Parentani, July 31, 1962 - May 20, 2020