UNIVERSITY OF TRENTO

Assessing the impact of metallic adhesion on the injection of a proof mass into a geodesic trajectory

GRAvitational - wave Science & technology Symposia

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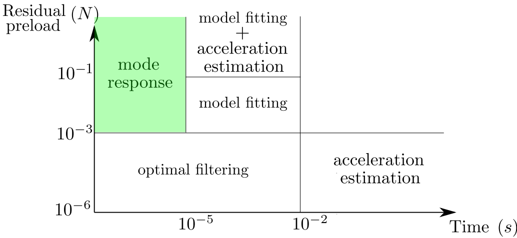
Characterization of metallic adhesion

Goal

Estimate the adhesive impulse at the separation of two metallic surfaces

In the literature, the techniques are based on:

- Static measure of adhesion force
- Low residual preload and separation velocities



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The proposed technique exploits the multi-mode response of the sensing body

Novelty

- Technique relies on independent observables
- Impulse time duration is also estimated
- Focus on the impulse developed by the adhesion force

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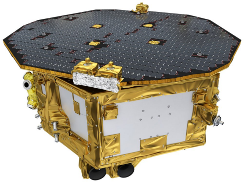
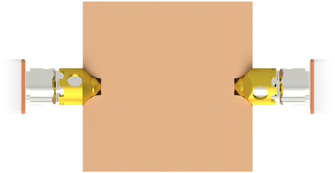
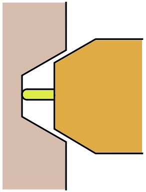
Prediction of the momentum acquired by a body released into free-fall



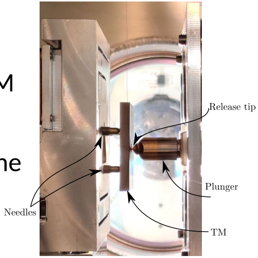


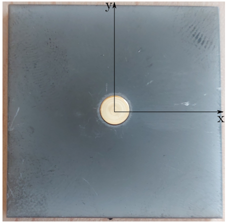


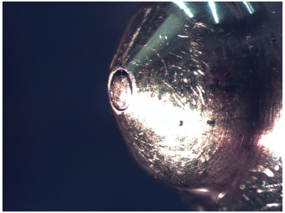
On-ground testing

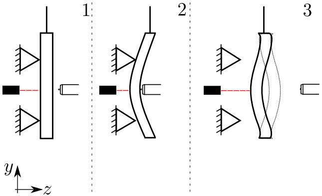
Performed tests applying different preload:

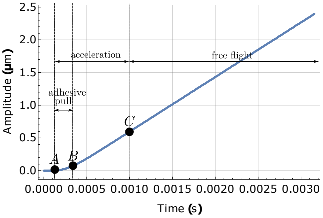
- Needles moved toward the TM
- Tip pushed towards the TM
- Tip retraction (≈ 80 μs) and the adhesive bonds are broken

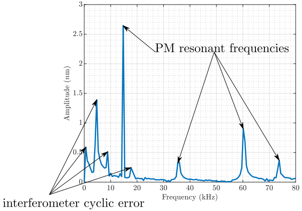












Dynamic modeling

Motion of a linear elastic plate body subjected to a force:

$$w(x, y, t) = \sum_{m=1}^{\infty} W_m(x, y) q_m(t)$$

$$w(0, 0, t) \approx \alpha_1 q_1(t) + \alpha_2 q_2(t) + \alpha_3 q_3(t) + \alpha_4 q_4(t) \qquad \text{where } \alpha_m = W_m(0, 0)$$

Each modal coordinate obeys the differential equation of a simple oscillator:

$$q''_m(t) + \omega_m^2 q_m(t) = \frac{Q_m(t)}{b_m} \qquad \text{given} \qquad q_m(0) = \frac{Q_m(0)}{\omega_m^2 b_m} \ \text{ and } \ q'_m(0) = 0$$

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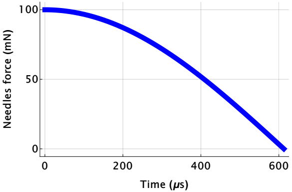
Each mode represents an independent dynamical system whose steady-state vibrations represent our measurement technique

Dynamic modeling: forces and initial conditios

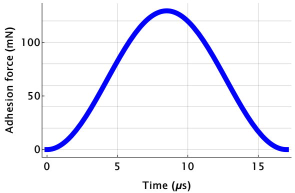
Needles: one-sided preloaded spring

$$f_{ndl}(t) = \frac{p}{3}\cos(\omega_p t)\left(1 - H\left(t - \frac{\pi}{2\omega_p}\right)\right)$$



Adhesion: squared sine[1]

$$f_{tip}(t) = \frac{2l}{\tau}\sin^2\left(\frac{\pi t}{\tau}\right)H(t)(1 - H(t - \tau))$$



Modal projection of the forces:

$$Q_{m,tip}(t) = \int_S W_m(x,y) f_{tip}(t)\delta(x - x_{tip}, y - y_{tip})\, dxdy = \alpha_{m,tip}\, f_{tip}(t)$$

$$Q_{m,ndl,i}(t) = \int_S W_m(x,y) f_{ndl}(t)\delta(x - x_{ndl,i}, y - y_{ndl,i})\, dxdy = \alpha_{m,ndl,i}\, f_{ndl}(t)$$

$$\Longrightarrow$$

$$q_m(0) = \frac{Q_{m,tip}(0) - Q_{m,ndl,1}(0) - Q_{m,ndl,2}(0) - Q_{m,ndl,3}(0)}{b_m\, \omega_m^2} = \frac{\alpha_{m,tip}\, p - \alpha_{m,ndl,1}\, \frac{p}{3} - \alpha_{m,ndl,2}\, \frac{p}{3} - \alpha_{m,ndl,3}\, \frac{p}{3}}{b_m\, \omega_m^2}$$

$f_{tip}(t)$, $f_{ndl}(t)$ →

- → $Q_{1,tip}(t), Q_{1,ndl,1}(t), Q_{1,ndl,2}(t), Q_{1,ndl,3}(t)$ → $q_{0,1}$ → $q_1''(t) + \omega_1^2 q_1(t) = \frac{Q_1(t)}{b_1}$
- → $Q_{2,tip}(t), Q_{2,ndl,1}(t), Q_{2,ndl,2}(t), Q_{2,ndl,3}(t)$ → $q_{0,2}$ → $q_2''(t) + \omega_2^2 q_2(t) = \frac{Q_2(t)}{b_2}$
- → ●●● → ●●● → ●●●

[1] D. Bortoluzzi et al., “Improvements in the measurement of metallic adhesion dynamics,” *Mechanical Systems and Signal Processing*

Dynamic modeling: tip preload

Analytical model of the TM displacement:

$$
\begin{cases}
z_1 = 0 & t \leq t_0 \\
z_2 = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3 + A_1 sin(\omega_1 t) + B_1 cos(\omega_1 t) + A_2 sin(\omega_2 t) + & \\
+ B_2 cos(\omega_2 t) + A_3 sin(\omega_3 t) + B_3 cos(\omega_3 t) + A_4 sin(\omega_4 t) + B_4 cos(\omega_4 t) & t_0 \leq t \leq t_1 \\
z_3 = mt + q + A_1 sin(\omega_1 t) + B_1 cos(\omega_1 t) + A_2 sin(\omega_2 t) + B_2 cos(\omega_2 t) + & \\
+ A_3 sin(\omega_3 t) + B_3 cos(\omega_3 t) + A_4 sin(\omega_4 t) + B_4 cos(\omega_4 t) & t_1 \leq t
\end{cases}
$$

Constraints:

$$
\begin{aligned}
z_2(t_0) &= 0 \\
z_2'(t_0) &= d \\
z_2''(t_0) &= p \\
z_2(t_1) &= z_3(t_1) \\
z_2'(t_1) &= z_3'(t_1) \\
z_2''(t_1) &= 0
\end{aligned}
$$

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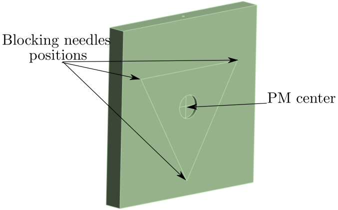
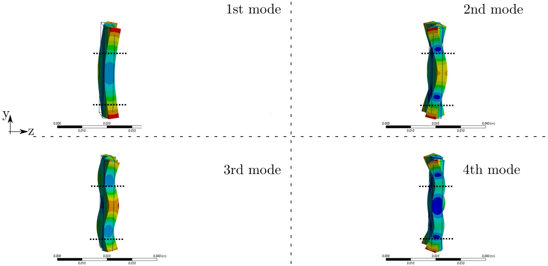
Minimization algorithm performed to get estimation of all fitting parameters

Dynamic modeling: modal parameters





$\Longrightarrow$

Parameter	Value
$\alpha_{1,tip}$	0.636
$\alpha_{2,tip}$	0.572
$\alpha_{3,tip}$	-0.925
$\alpha_{4,tip}$	-0.652
$\alpha_{1,ndl}$	-0.209
$\alpha_{2,ndl}$	-0.823
$\alpha_{3,ndl}$	0.349
$\alpha_{4,ndl}$	-0.120
b_1	0.011 kg
b_2	0.010 kg
b_3	0.011 kg
b_4	0.005 kg

Frequency (Hz)	Modes
$\approx$ 14 700	1st
$\approx$ 35 700	2nd
$\approx$ 60 900	3rd
$\approx$ 74 300	4th

Steady-state oscillations amplitude

The technique relies on the amplitude of the steady-state modes oscillations[2]

Steady-state solution to m$-th$ oscillator:

$$q_m(t) = A_m sin(\omega_m t) + B_m cos(\omega_m t)$$

Oscillation amplitude of the m$-th$ mode:

$$c_m^2 = A_m^2 + B_m^2$$

$$\Longrightarrow$$

Normalized squared oscillations amplitude:

$$\frac{c_m^2}{\alpha_{m,tip}^2} = \frac{A_m^2 + B_m^2}{\alpha_{m,tip}^2} = f(I, \tau, p, \omega_p, \omega_m, b_m, \alpha_{m,tip}, \alpha_{m,ndl})$$

Substituting FEM and fitting parameters and initial conditions:

$$c_m^2 - \alpha_{m,tip}^2\, f(I, \tau) = 0$$

[1] E. Dalla Ricca et al., “An improved vibration multi mode-based technique for the characterization of metallic adhesion impulses”, *AIAA Journal*, 2024

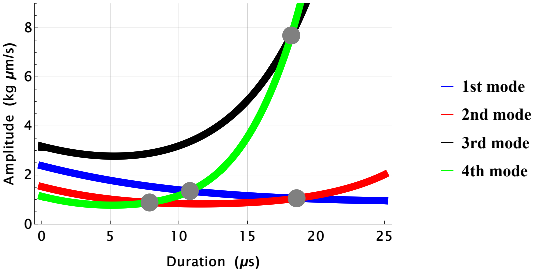
Experimental results

Hybrid estimation approach: $\Longrightarrow$ Taking into account the four detected modes:

- FEM model $\rightarrow \alpha_m, b_m, \omega_m$ first guess
- Fit $\rightarrow p, A_m, B_m, \omega_m, c_m^2$
- Filter $\rightarrow c_m^2$

$$\begin{cases} c_1^2 - \alpha_{1,tip}^2 f(I,\tau) = 0 \\ c_2^2 - \alpha_{2,tip}^2 f(I,\tau) = 0 \\ c_3^2 - \alpha_{3,tip}^2 f(I,\tau) = 0 \\ c_4^2 - \alpha_{4,tip}^2 f(I,\tau) = 0 \end{cases}$$

Contour plot as function of solely impulse amplitude and duration:

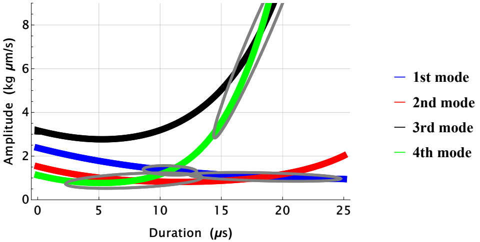


- Locus of points compatible with c_m^2
- Intersections gives the impulse amplitdue I and duration τ
- **Redundant set of measurement**

Optimal solution

Given two modes (i and j), the solution $(\bar{I}, \bar{\tau})$ is found solving for I and τ:

$$\begin{cases} c_i^2 - \alpha_{i,tip}^2 f(I,\tau) = 0 \\ c_j^2 - \alpha_{j,tip}^2 f(I,\tau) = 0 \end{cases} \implies$$

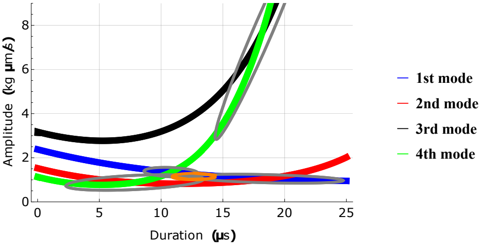


$$\begin{pmatrix} \sigma_{\bar{I}} \\ \sigma_{\bar{\tau}} \end{pmatrix} = -J_{I,\tau}^{-1}(\bar{I},\bar{\tau},\bar{v}) J_v(\bar{I},\bar{\tau},\bar{v})\delta v = Q(\bar{I},\bar{\tau},\bar{v})\delta v$$

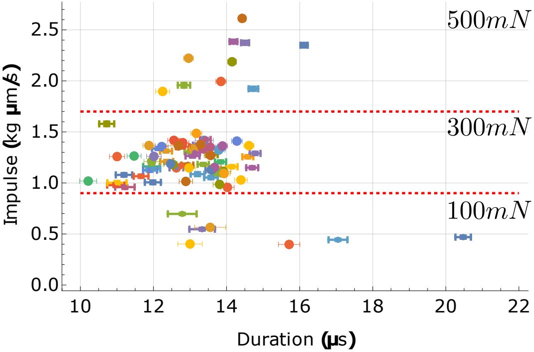
Optimal estimation:

$$\tilde{I} = \sum_{n=1}^{k} w_{I,n}\bar{I}_n \qquad \sigma_{\tilde{I}}^2 = \sum_{n=1}^{k} w_{I,n}^2 \sigma_{\bar{I},n}^2$$

$$\nabla\sigma_{\tilde{I}}^2 = \begin{bmatrix} \frac{\partial\sigma_{\tilde{I}}^2}{\partial w_{I,1}} \\ \vdots \\ \frac{\partial\sigma_{\tilde{I}}^2}{\partial w_{I,k}} \end{bmatrix} = 0 \qquad \tilde{w}_{I,n} = \frac{1}{\sigma_{\bar{I},n}^2}\left(\sum_{n=1}^{k}\frac{1}{\sigma_{\bar{I},n}^2}\right)^{-1} \implies$$



Conclusions



- Impulse amplitude increases with preload
- Impulse duration not affected by preload
- Repeatability of the phenomenon at same preload

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Increasing the preload strengthens the adhesive bonds leaving their elongation nearly unaffected

Preload	Mean adhesion impulse
100 mN tests	0.45 ± 0.10 kg µm s^{-1}
300 mN tests	1.16 ± 0.16 kg µm s^{-1}
500 mN tests	2.03 ± 0.26 kg µm s^{-1}

Momentum transferred to TM due to adhesion $\approx$ 10% of LPF momentum requirement