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## Measurement of Polarization of GWs by 3G Detectors as a Test of Gravity

Investigating the capabilities of up to come facilities on measuring polarization modes of GWs and comparison with current generation ones

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DI TRIESTE

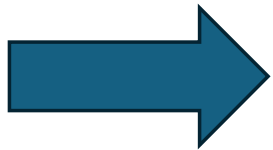


Max-Planck-Institut  
für Gravitationsphysik  
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# Testing GR with GWs

A broad subject:

- Parametrized post Einsteinian
- Propagation speed and Lorentz invariance
- Extra-polarization modes: ppE and null streams



Main focus → null streams

It has been already described with several degrees of complication

→ **Basic approach** and comparison of performances on simulated data over different networks of ground based detectors (2G and 3G)



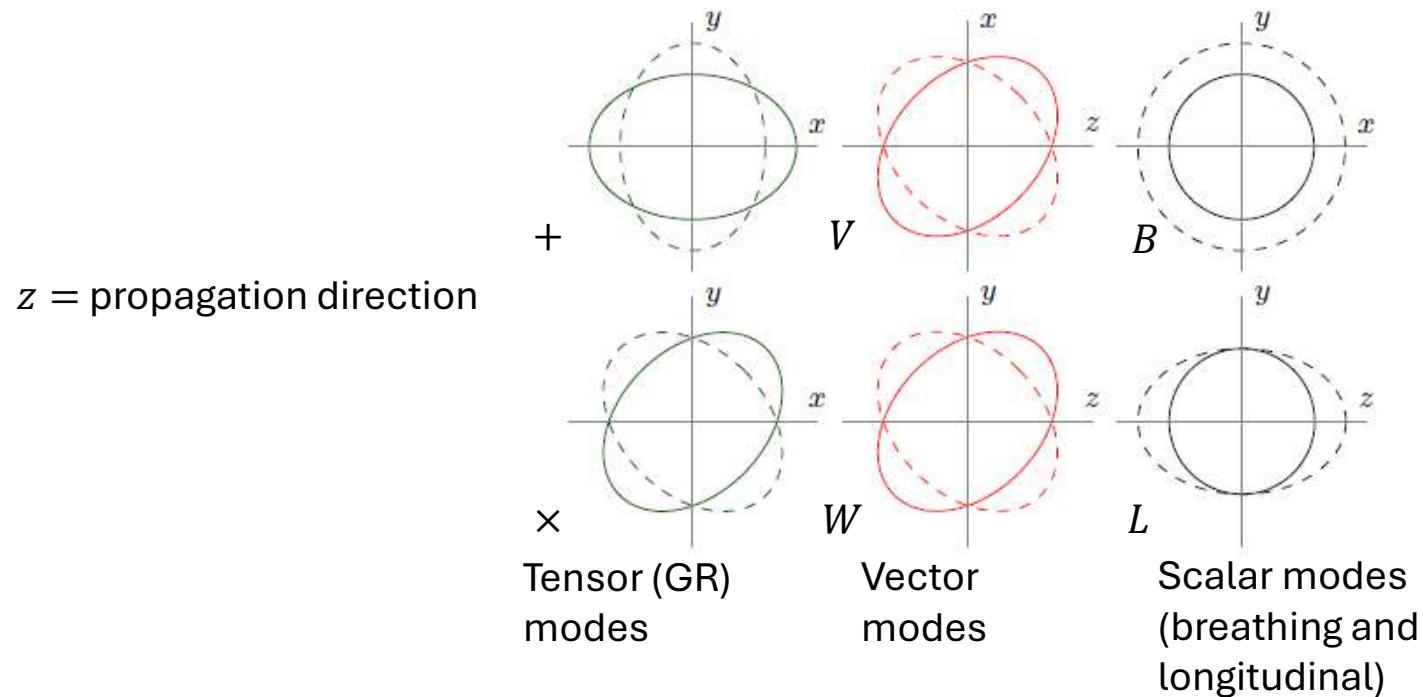
Outline of theoretical (null streams definition) and computational (HPC) methods and results

# GR and non-GR polarization modes

- GR only predicts tensor modes + and cross
- Modified theories of gravity → huge bestiary
- Number of polarizations is constrained to be  $\leq 6$  ([Newman&Penrose, 1962](#))
- Eardley's classification E(2) for a spin-2 particle ([Eardley et al, 1973](#))
- Such classification, and the effect on a ring of test masses, is general
- Massive gravity theories impose special conditions on the simultaneous presence of different modes → not taken into account ([Rham et al., 2011](#))

# GR and non-GR polarization modes

The effect of the 6 modes can always be represented as follows:



Picture from: [Isi&Weinstein, 2017](#)

# Short summary of Antenna Patterns

- GW detectors are not spherical antennas
- Relative displacement of test masses along IFO's arms:

$$\frac{\delta L}{L} = \sum_{pol} F^{pol}(\varphi, \vartheta, \psi) h_{pol}$$

$$F^{pol} = \frac{1}{2} \hat{\mathbf{d}}_{ij} \hat{\boldsymbol{\epsilon}}^{ij}_{pol} \rightarrow \text{a tensor expression}$$

Long wavelength approximation!

$$\hat{\mathbf{d}}_{ij} = \hat{e}_x^i \otimes \hat{e}_x^j - \hat{e}_y^i \otimes \hat{e}_y^j$$

$\hat{e}_x^i$  = unit vector along the  $x$ -th arm of the detector

# Measuring polarizations

- GR could be tested just by the observation of polarization modes
  - Single detector  $\rightarrow S_a = \sum_{pol} F_a^{pol}(\varphi, \vartheta, \psi) h_{pol}$
  - For the two scalar modes:  $F^B = -F^L$
- $\rightarrow$  Just five independent modes  $\rightarrow$  5 non-coaligned detectors are needed for full disentangling

The antenna patterns need be known with good precision:  $\rightarrow$  optimal localization needed

HLVK+India  $\rightarrow$  Hanford and Livingston are almost coaligned

Only very well-localized events could allow a polarization measure



ET+CE = no more than 4 non co-aligned detectors ☹️

# Testing GR

- Just checking for the presence of non-GR polarization (with no disentangling) would be enough  
→ Null stream formalism (localization, [Klimenko et al., 2011](#))
- Three-detector null stream:  $NS = \varepsilon_{ijk} F_i^+ F_j^\times S_k \rightarrow$  if signal is made of tensor modes only:  $NS = \text{AntiSym} \cdot \text{Sym} = 0$  (= *noise*)
- Otherwise, NS is generally non-zero

$$!! NS = \varepsilon_{ijk} F_i^+(\hat{\varphi}, \hat{\vartheta}, \hat{\psi}) F_j^\times(\hat{\varphi}, \hat{\vartheta}, \hat{\psi}) S_k(\varphi, \vartheta, \psi) !!$$

# Testing GR

Null stream-based tests of GR:

- No outcome so far
- Example application to GW170817: [Hagihara et al., 2019](#)

With a network of 4 non-coaligned detectors:

- More stringent analysis can be pursued
- 4-detector null stream:  $NS = \varepsilon_{ijkl} F_i^+ F_j^\times F_k^{pol} S_l$   
→ Allows getting rid of one extra polarization mode

**PRO:** Allows more testing

**CON:** Unsensitive to one non-GR mode



# Present work

- Testing GR with different networks of detectors
- 1) HLVKI, 2) ETCE
- Both 3 and 4-detector NSs
- Simulated pop. of BBH events (computational limits → no BNS yet)
- Localization (needed for NSs) → gwbench
- Computation of NSs' SNR for different polarization injections.

# Using gwbench



- It provides a vectorized function for computing antenna patterns  
Why vectorized? → The Earth rotates during the inspiral → antenna patterns change
- Makes use of Fisher Information formalism for error estimation (uncertainty on localization → key point):
- $\text{cov}^{-1} \approx \Gamma$  with maximum likelihood estimators
- Provides a useful tool for computing the outcome of a NS-based test of GR on several events

→ [Borhanian, 2020](#)

# Summary of workflow

- Given a network of detectors (HLVKI or ETCE)
  - Simulation of BBH population  $\rightarrow 10^5$  events
  - for event in the pop:
    - Retrieve coalescence parameters for that given event
    - Data acquisition by a network of dets simulated with gwbench: waveform=IMRPhenomXAS ([Pratten et al., 2020](#))
    - Injection of a non-tensor mode (as in [Takeda et al., 2018](#))
    - Uncertainty on the localization is computed by gwbench
    - A 2D-gaussian distribution of (RA, DEC) values is computed with  $\mu$  =injection parameters and  $\sigma$  = deviation given by gwbench
    - 100 samples are drawn from this distribution
    - For sample in (100 samples):
      - 3 and 4 dets-NSs are computed in the corresponding (RA, DEC) values for each sample
      - The noise of the NSs is computed is well propagating the noise of the detectors
      - NS power/noise = SNR of single sample
    - Averaging over the 100 samples: average SNR and its fluctuation
    - average SNR/fluctuation = significance of gravity test on single event
  - Cumulative sum of single event significances for different permutations of the order of events
- For which network does this sum provide a more meaningful test of GR?

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# BBH population

- Help provided by A. Dhani and Sathya
- Expected number of  $10^5$  events within  $z = 3$  following:

$$p(z) \propto \frac{dV_c}{dz} \frac{\psi(z)}{1+z} \longrightarrow \text{Madau\&Dickinson, 2014}$$

Binaries are uniform in  $\alpha$ ,  $\delta$  and  $\iota$ . Masses are distributed according to the PowerLaw + PeakModel of GWTC-3


GWTC-3  $\rightarrow$  small spins

Spin effects on localization measure  $\rightarrow$  negligible

} No spin in BBH population

Only high SNR events are chosen: network SNR > 12

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# Injection of non-tensor modes

- This part has been performed following non-GR injection in [Takeda et al., 2018](#)
- Each non-GR mode is obtained by:  $h_{pol} = \mathcal{A}_{pol} \mathcal{G}_{pol} h_+$  with  $\mathcal{G}_{pol} = \mathcal{G}_{pol}(\mathcal{M}, \iota)$  taken from [Takeda](#), and  $\mathcal{A}_{pol}$  being a dimensionless amplitude weighting the importance of non-GR modes in a signal with respect to GR modes
- $\mathcal{A}_{pol} = 1$  in this work  $\rightarrow$  unrealistic, used for benchmarking networks' sensitivity to the test

What about radiation reaction terms due to additional pol. modes (i.e. ppE)?

$\rightarrow$  Not considered here as this test depends on localization-ONLY!  
But the assumption of localization uncertainty being independent from any waveform variation is SIMPLISTIC

# Summary of workflow

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- For which network does this sum provide a more meaningful test of GR?



# Computing null streams

As previously pointed out:  $NS = \varepsilon_{ijk} F_i^+(\hat{\varphi}, \hat{\vartheta}, \hat{\psi}) F_j^\times(\hat{\varphi}, \hat{\vartheta}, \hat{\psi}) S_k(\varphi, \vartheta, \psi)$

→  $S_k(\varphi, \vartheta, \psi)$  is the k-th detector response as given by gwbench  
(depends on antenna patterns computed at the exact location)

→  $F_i^+(\hat{\varphi}, \hat{\vartheta}, \hat{\psi})$  is the i-th detector antenna pattern: depends on the values of  $\hat{\varphi}, \hat{\vartheta}, \hat{\psi}$  (which come from the uncertainty driven bi-Gaussian distribution)

→ gwbench: Stationary Phase Approx.:  $S_k(f) \approx \sum_{pol} F_k^{pol}(f) h_{pol}(f)$ ;  
same for NSs:  $NS(f) \approx \varepsilon_{ijk} F_i^+(f) F_j^\times(f) S_k(f)$

→  $PSD(f) = |NS(f)|^2$

→ NSs' uncertainty obtained propagating detector noise through

# One last issue

## Summary of workflow

- Given a network of detectors (HLVKI or ETCE)
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    - Data acquisition by a network of dets simulated with gwbench: waveform=IMRPhenomXAS ([Pratten et al., 2020](#))
    - Injection of a non-tensor mode (as in [Takeda et al., 2018](#))
    - Uncertainty on the localization is computed by gwbench
    - A bi-modal gaussian distribution of (RA, DEC) values is computed with  $\mu$  =injection parameters and  $\sigma$  = deviation given by gwbench
    - 100 samples are drawn from this distribution
    - For sample in (100 samples):
      - 3 and 4 dets-NSs are computed in the corresponding (RA, DEC) values for each sample
      - The noise of the NSs is computed is well propagating the noise of the detectors
      - NS power/noise = SNR of single sample
    - Averaging over the 100 samples: average SNR and its fluctuation
    - average SNR/fluctuation = significance of gravity test on single event
  - Cumulative sum of single event significances for different permutations of the order of events
- For which network does this sum provide a more meaningful test of GR? <sup>11</sup>

**330k computing hours**

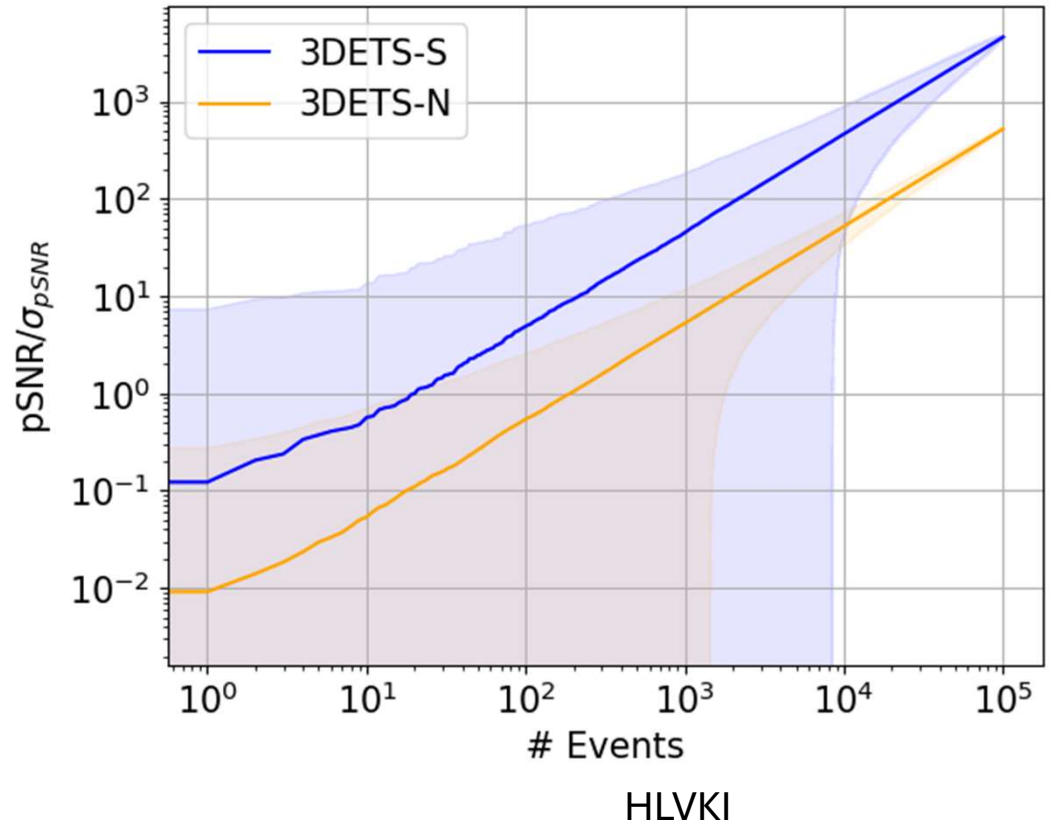
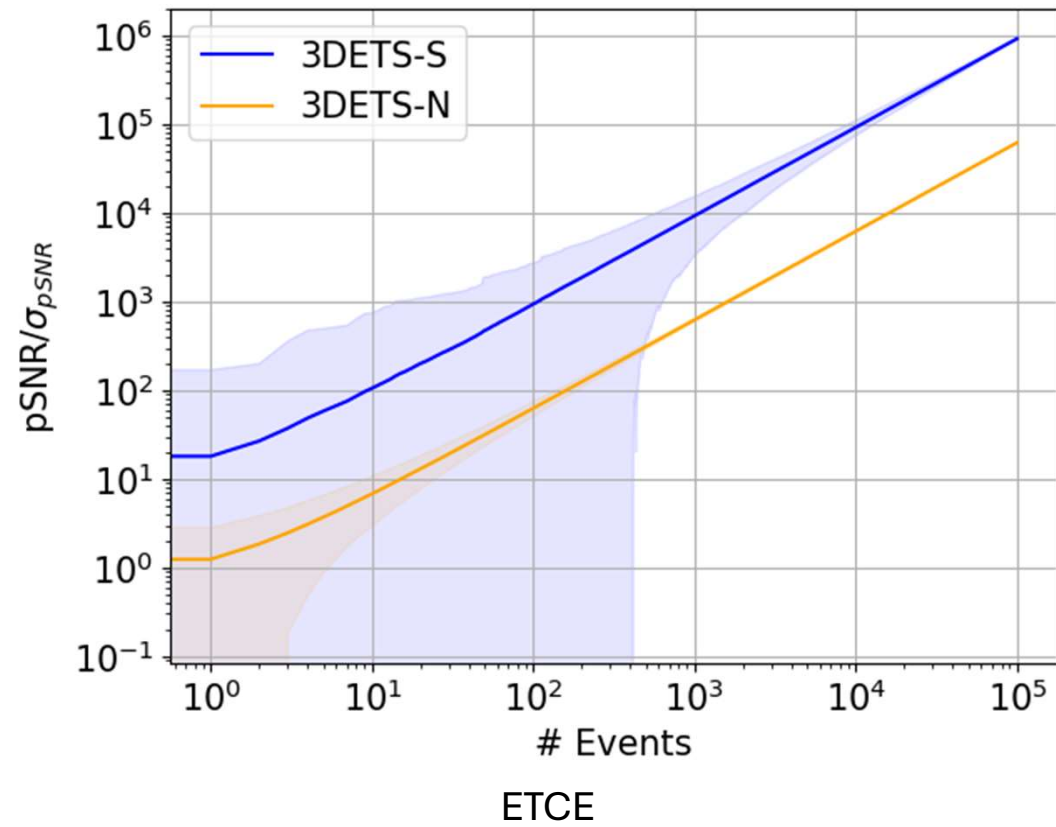


# Code for HPC structures

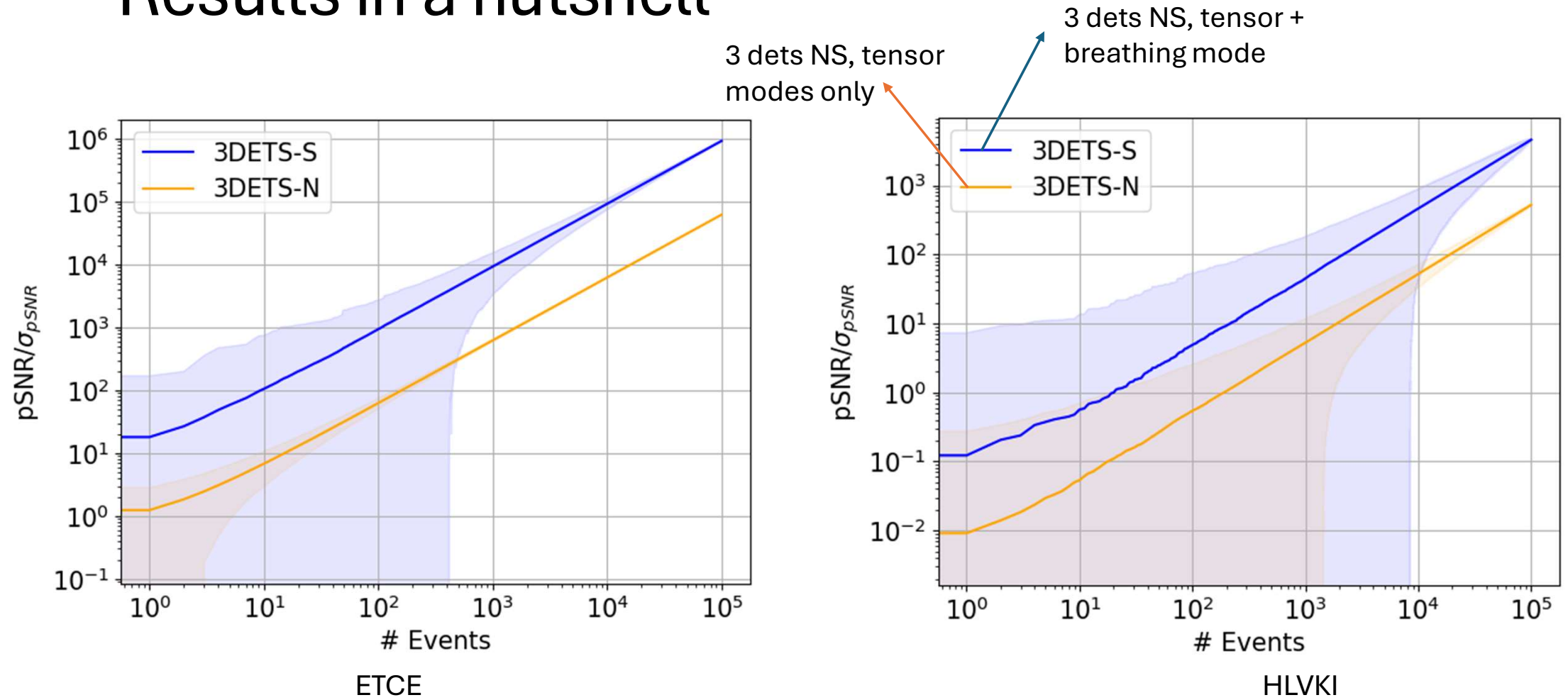
Gwave (PSU cluster) → <https://computing.docs.ligo.org/guide/computing-centres/psu/>

- In house version of gwbench for computing antenna patterns of several detectors for multiple events simultaneously
- Multiprocessing instance in the code: each of the 100 (RA,DEC) draws has a process which is assigned to the first available core → not a for loop
- Multiple batches of 1000 BBH events each are assigned to different nodes
- Just using multiple core architecture (no GPU) → 34 h ( $\ll$  330k)

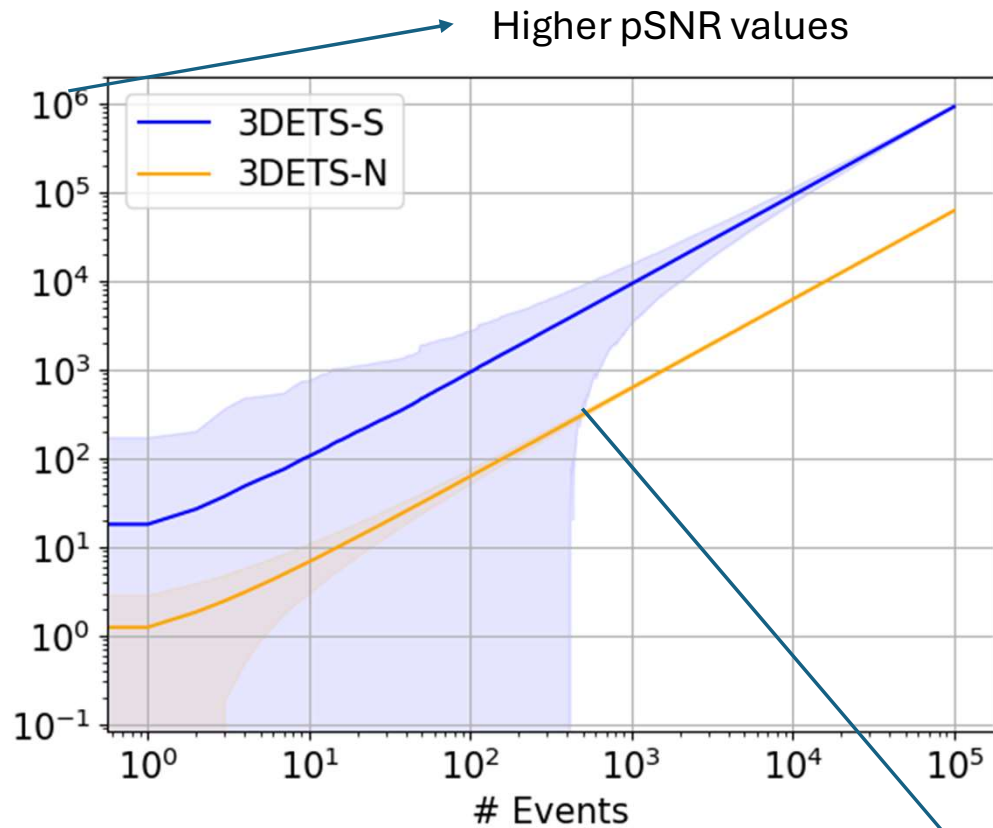
# Results in a nutshell



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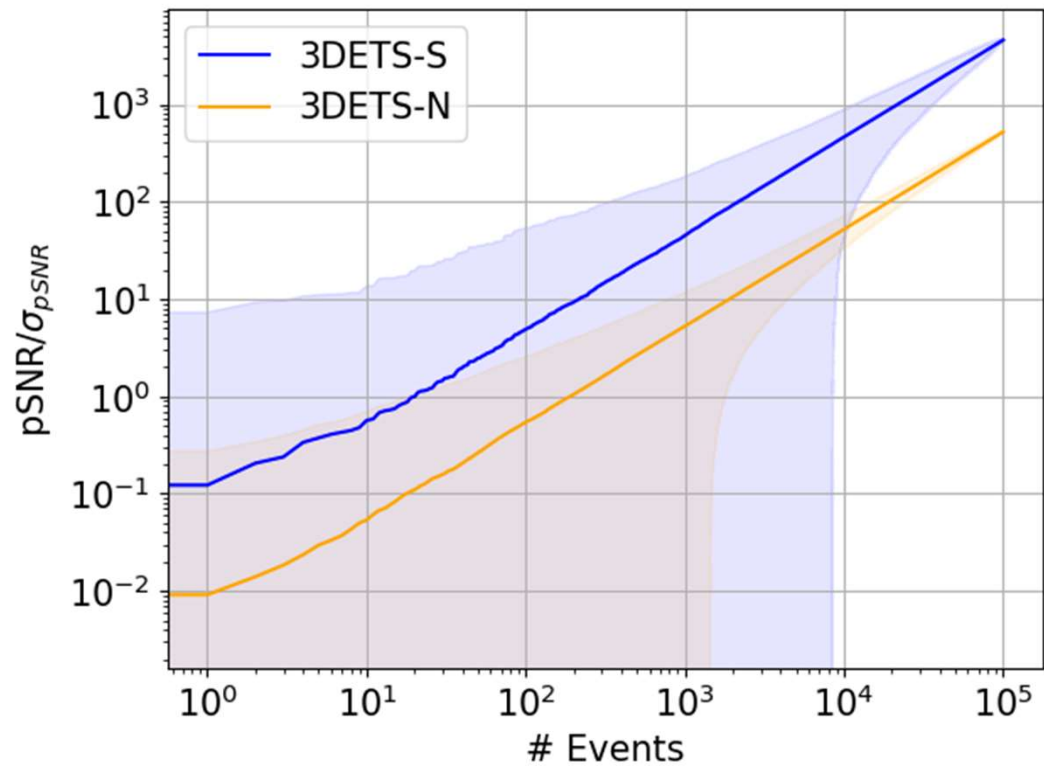


# Results in a nutshell



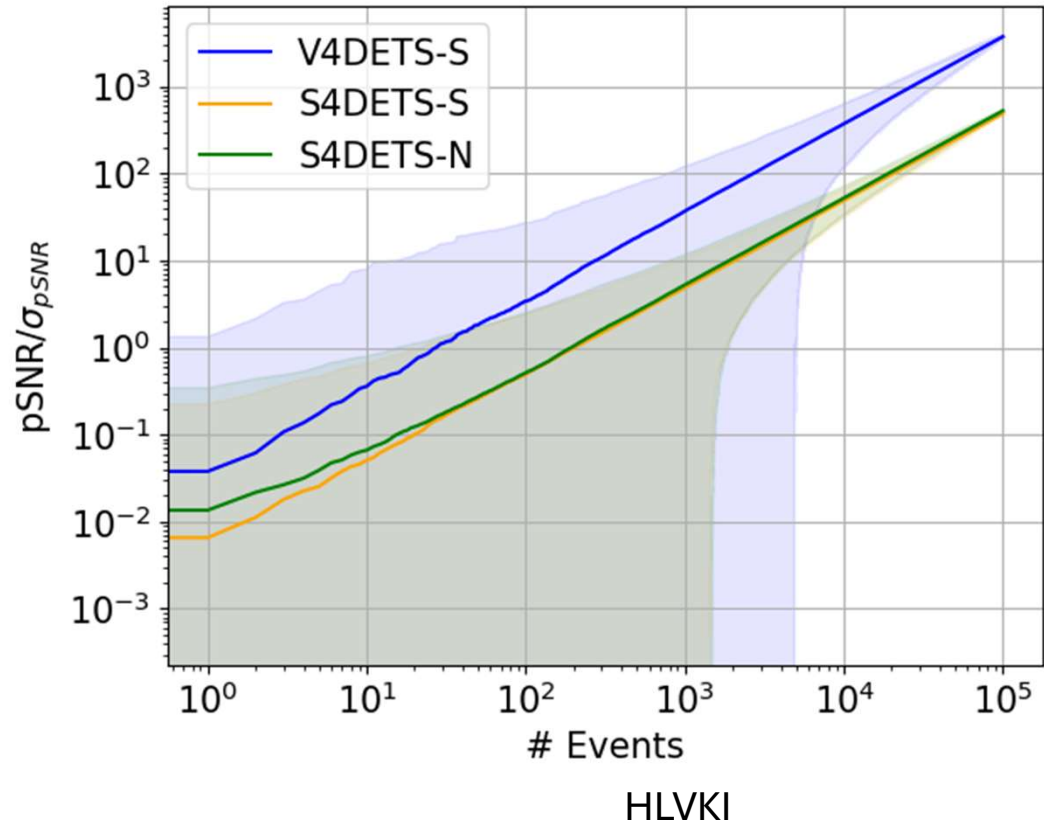
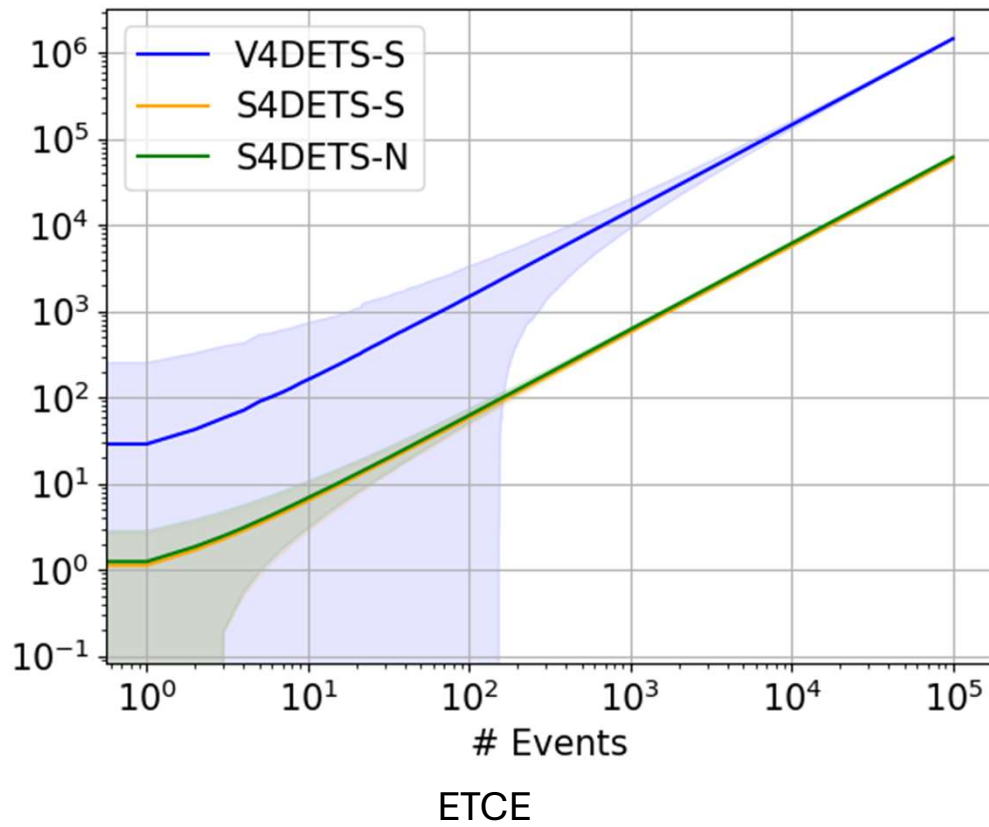
ETCE

Probability of fake claim < 1%  
after ~500 events



HLVKI

# Results in a nutshell

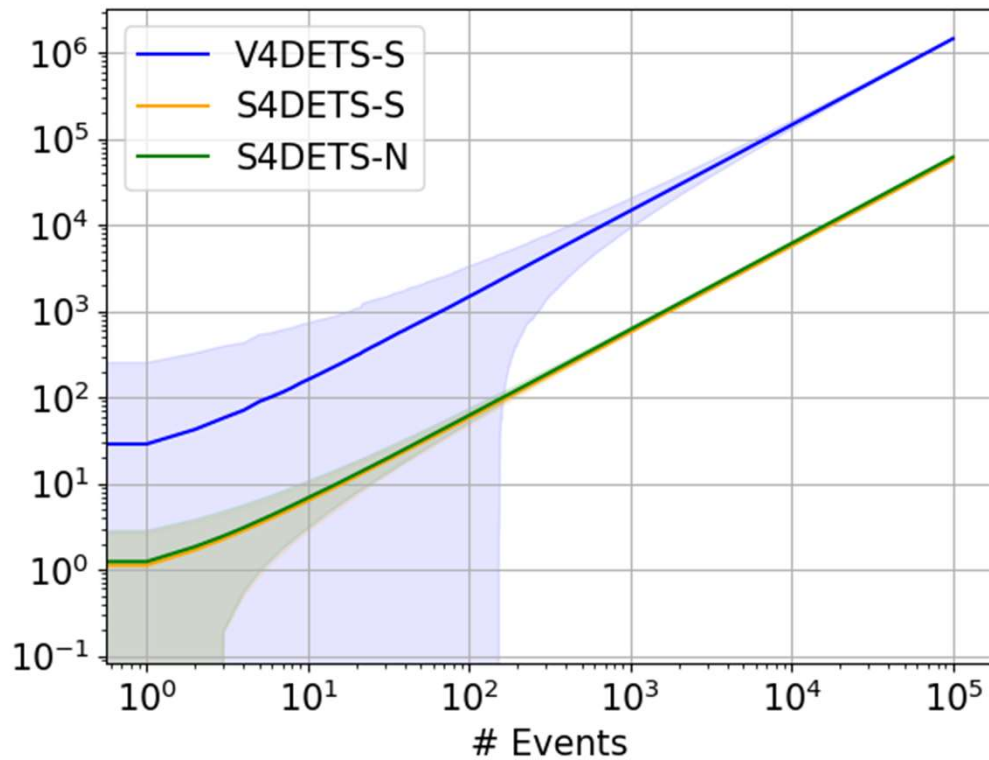


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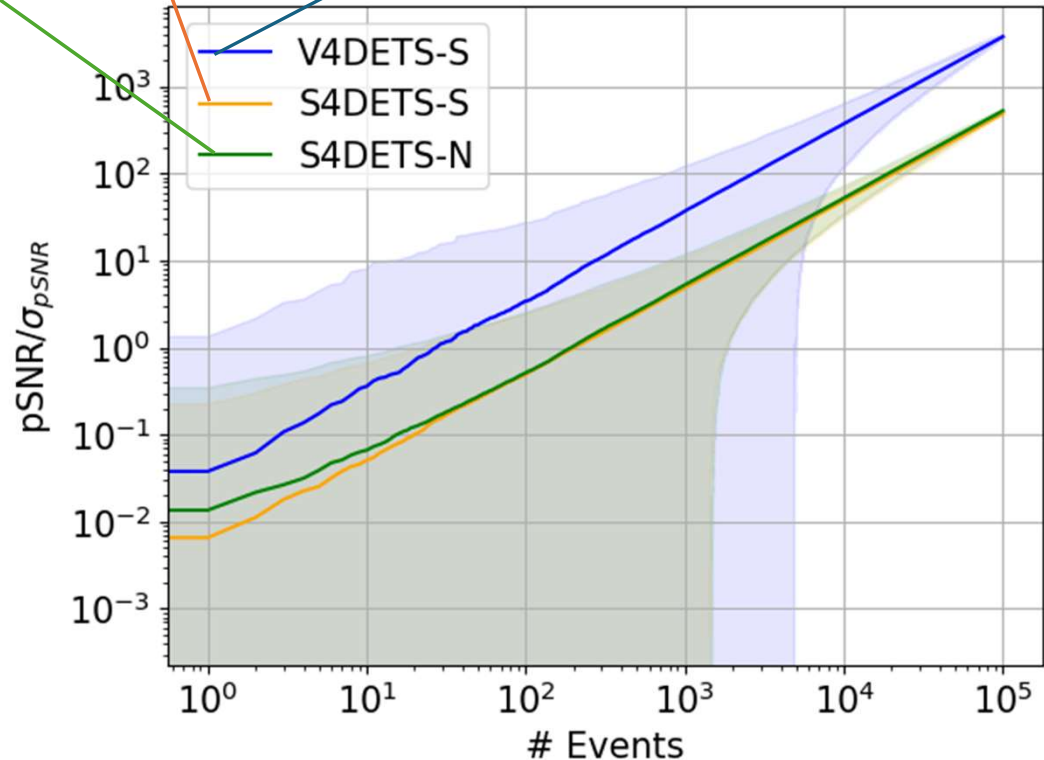
$\varepsilon_{ijkl} F_i^+ F_j^\times F_k^B S_l$ , tensor modes only

$\varepsilon_{ijkl} F_i^+ F_j^\times F_k^B S_l$ , tensor + breathing mode

$\varepsilon_{ijkl} F_i^+ F_j^\times F_k^V S_l$ , tensor + breathing mode



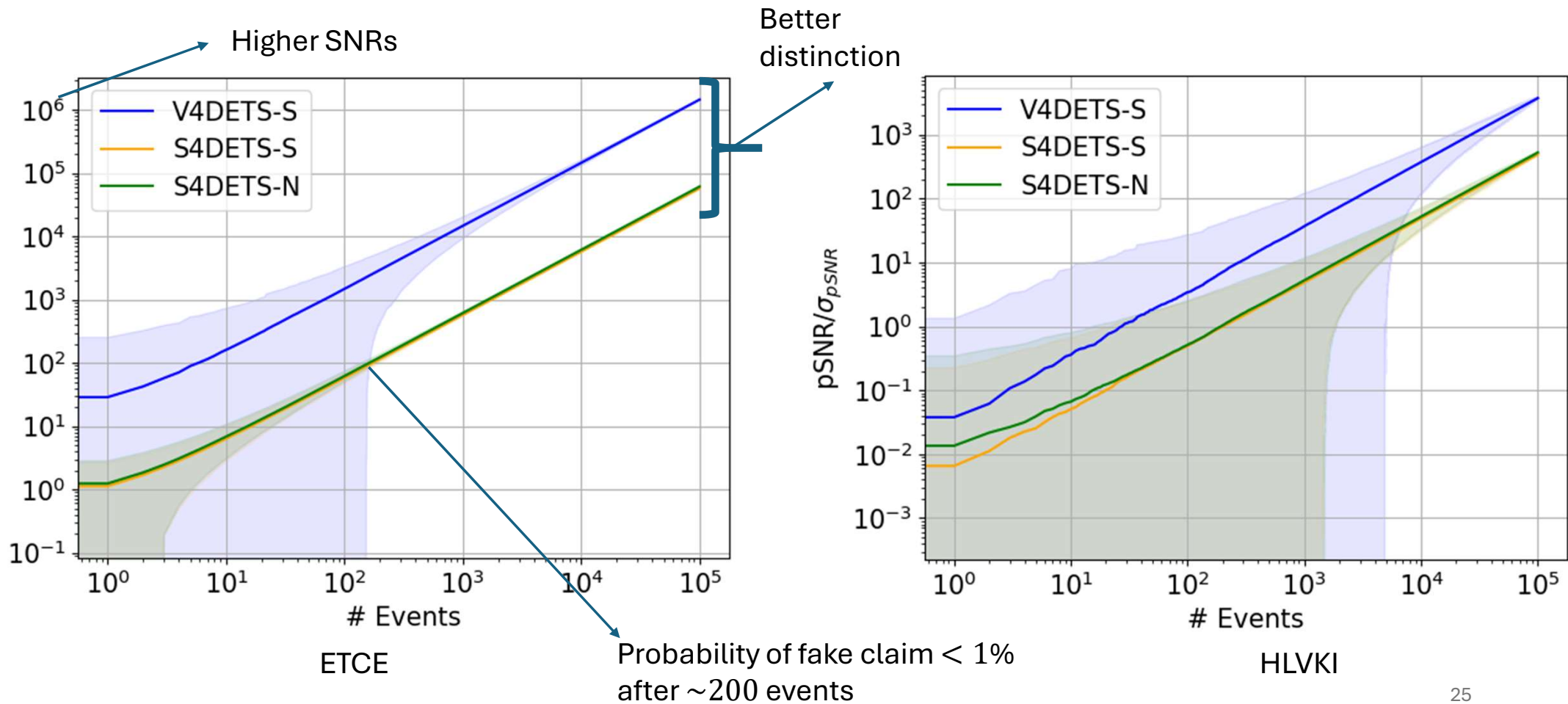
ETCE



HLVKI

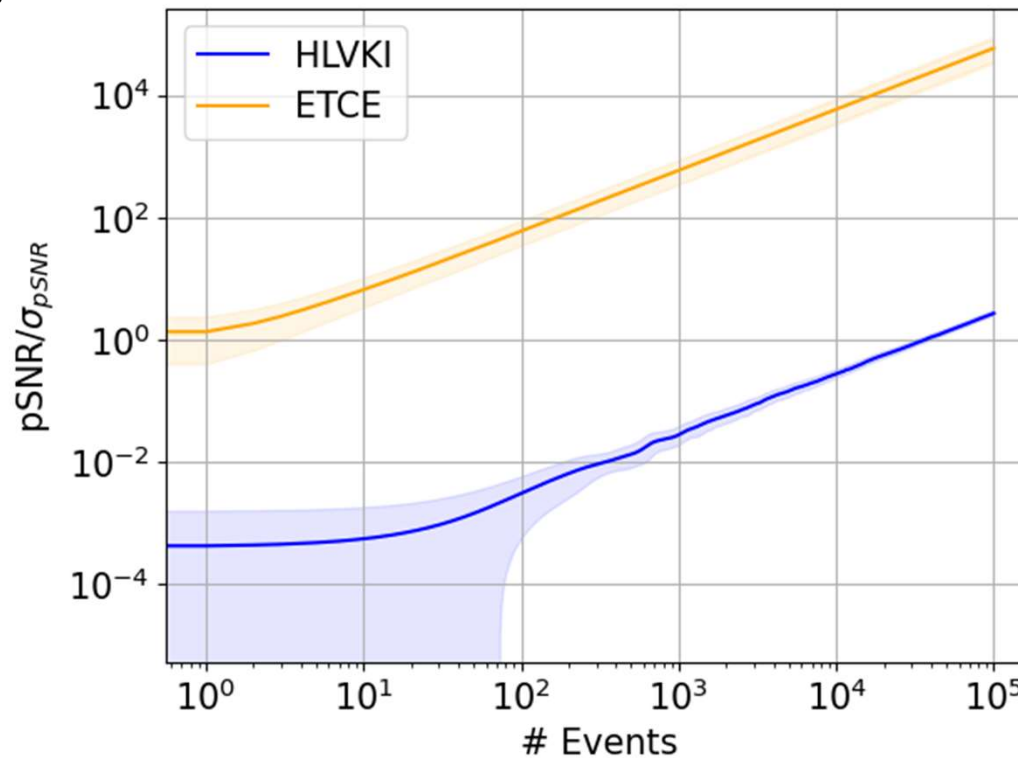
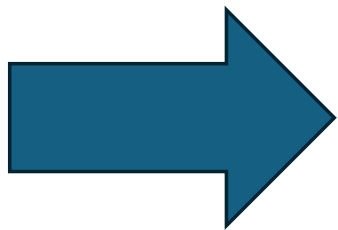


# Results in a nutshell



# Importance of 4-dets NSs

When observing GW, one can use all of the 4-dets NSs and see which one is minimized. Performing the ratio between one of the other NSs and the «silenced» one, one gets a more powerful test of gravity: silent NS is a proxy for tensor signals



For instance:

Scalar mode in signal.

→  $\varepsilon_{ijkl} F_i^+ F_j^\times F_k^B S_l$  is null

→ **Proxy for tensor signal**

→  $\varepsilon_{ijkl} F_i^+ F_j^\times F_k^V S_l$  is not null

→ Cumulative sum of their significances' ratio

4-dets NSs allow for a great enhancement in the precision of the test by ETCE:  $\mathcal{A}_{pol}^{ETCE} < 10^{-2} \mathcal{A}_{pol}^{HLVKI}$  26

# Conclusion

- ETCE would provide a significant claim in shorter time
- The precision of the test is slightly enhanced
  - Effect of the SNR>12 cut → when dropping the threshold to 8 one recovers a larger difference in the precision of the test
- The localization as achieved with CE and ET for loud and short events is only slightly better than that achieved by HLVKI for similarly high network SNRs
- → short events: localization error depends linearly on the inverse global SNR of the event.
- 4-dets null streams provide a stronger test for testing GR, but only when one non-tensor mode only is present

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*Thank you!*