Measuring coating loss angle in thermoelastic-dominated crystalline substrates for new generation gravitational wave detectors

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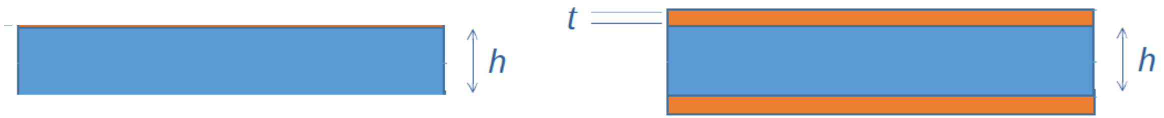
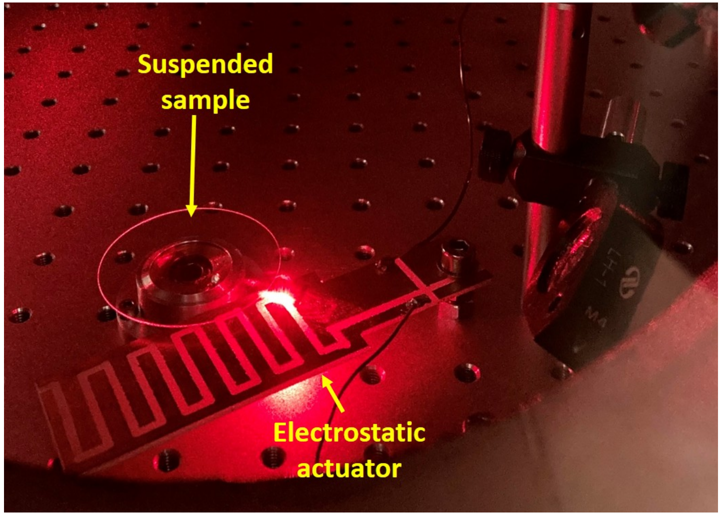
iLM Institut Lumière Matière

LMA Laboratoire Matériaux Avancés

measuring the coating loss angle





Coating measurement made by comparison b/w the losses of bare substrate and substrate + (much thinner) coating

- seek substrate with **lowest possible dissipation** to accentuate the contribution of the coating
- seek substrate in which the dissipation is **stable** before / after coating deposition, to allow for a meaningful comparison

Room Temperature measurement:

- synthetic fused silica

Cryo Temperature measurement:

- in sillica as the temperature drops the dissipation increases by orders of magnitude
- in crystalline materials (e.g. silicon) work very well at low T, but when the temperature rises, the loss increases: dominated by **thermoelastic mechanism**
- the thermoelastic loss depends not only on the substrate but also on the properties and thickness of the coating: change after coating deposition difficult to estimate exactly
- However: while nothing can be done to reduce dissipation in fused silica, the thermoelastic loss depends on the physical size of the resonator: **geometry as a path to improvement**

thermoelastic loss

Heat transfer b/w expanded & compressed regions of the vibrating sample

Debye peak in frequency:

$$\phi_{TE}(\omega) = D^{TE} \cdot \Delta \frac{\omega \tau}{1+(\omega \tau)^2}$$

$$\Delta = \frac{(3\alpha)^2 BT}{\rho C_v}$$

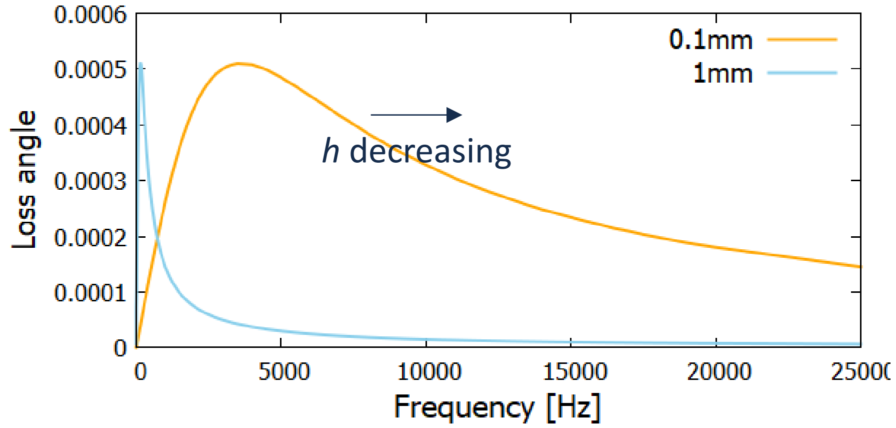
Max Intensity of peak
- linear thermal expansion
- temperature
- specific heat capacity

$$f_{peak} = \frac{1}{2\pi\tau} = \frac{\pi}{2} \frac{k}{\rho C_v h^2}$$

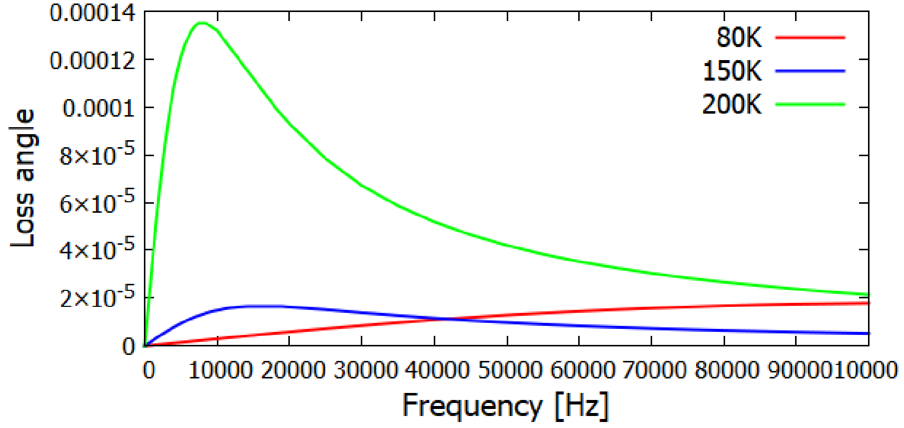
Frequency Position of peak
- thermal conductivity vs thickness
- specific heat capacity

- Less intense at low temperature. because of explicit temperature dependence
- Temperature dependence of α(T) and k(T) for a specific material affect behaviour at low temperature

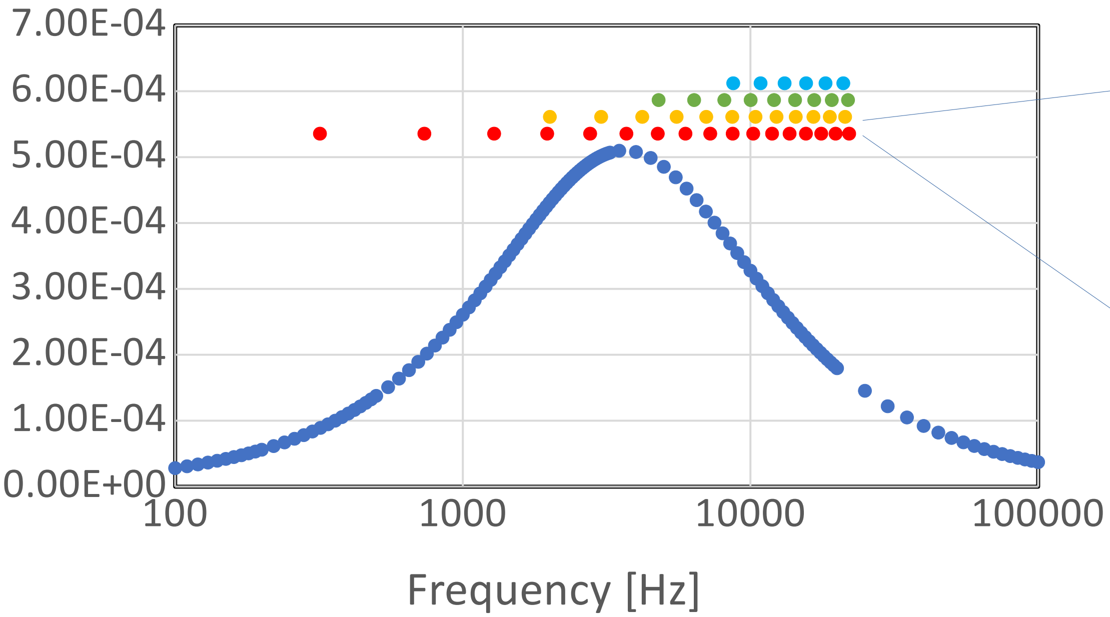
dependence on thickness:

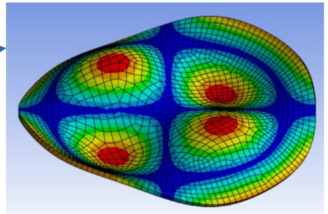


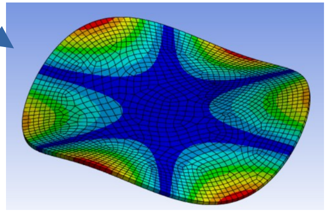
dependence on temperature (on Si):



thermoelastic loss







Characteristic frequencies of the resonant modes

$$f_{modes} \propto \frac{thickness}{diameter^2}$$

for each mode

$$\phi_{TE}(\omega) = D^{TE} \cdot \Delta \frac{\omega\tau}{1+(\omega\tau)^2}$$

$$D^{TE} = \frac{E_{dil}}{E_{dil}+E_{shear}}$$

Mode-dependent dilution factor
Elastic energy = dilation + shear
(strong dependence on the family)

thermoelastic shift when adding coating

Mechanical losses:

$$
\begin{cases}
\phi_{meas}(0) = \phi_s^{intr} + D^{TE}(0)\phi_s^{TE}(0) & \text{for } t = 0 \text{ (no coating)} \\
\phi_{meas}(t) = \left[1 - D_c(t)\right]\left[\phi_s^{intr} + D^{TE}(t)\phi_s^{TE}(t)\right] + D_c(t)\phi_c^{intr} & \text{for } t \neq 0 \text{ (coating)}
\end{cases}
$$

not the same!

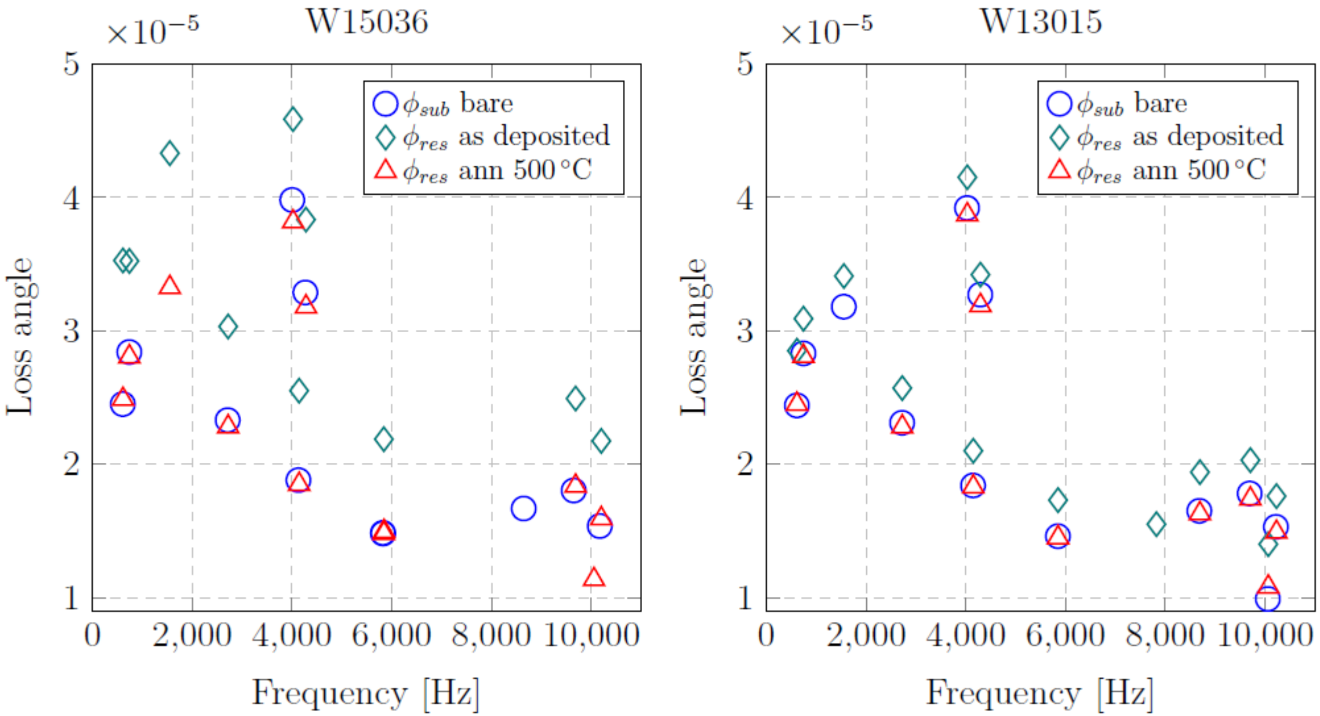
what we want

- The thermoelastic losses **of the substrate** change because of the presence **of the coating** (boundary conditions)

- This change (“thermoelastic shift”) depends on the **unknown** thermo-mechanical parameters of the thin film coating, and their temperature dependence

- Shift has been seen by M. Granata and L. Mereni at LMA in silica coating on Si (unexpected loss reduction after coating)

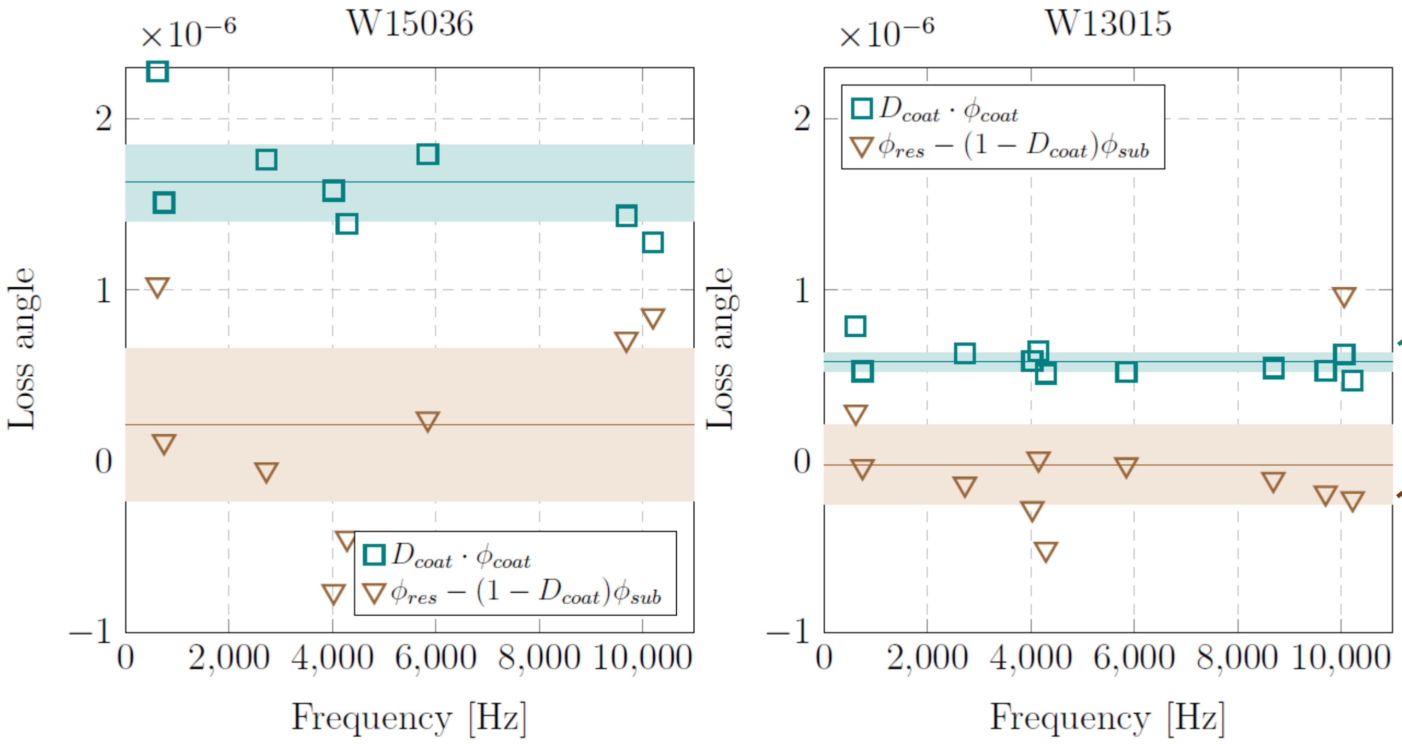
experimental evidence



	W15036	W13015
Material	Si [0 0 1] no flats	Si [0 0 1] no flats
Substrate diameter [mm]	76.03 ± 0.05	76.00 ± 0.05
Substrate thickness d [µm]	451.7 ± 0.4	453.8 ± 0.4
Substrate mass m [mg]	4777.9 ± 0.1	4796.2 ± 0.4
Coating type	SiO_2	SiO_2
Film thickness as deposited h [nm]	2964 ± 14	1112 ± 11

- Two silicon substrates were coated with silica
- Thermal treatment @ 500°C in ambient atmosphere, for 10 hours
- The measurements on the annealed samples unexpectedly show that the coated resonators have mechanical losses lower than the bare ones in some of the vibration modes

experimental evidence



The type of silica deposited was characterized in a previous work on amorphous substrate

Expected coating contribution from previous data

Measured coating contribution by subtracting bare substrate from coated

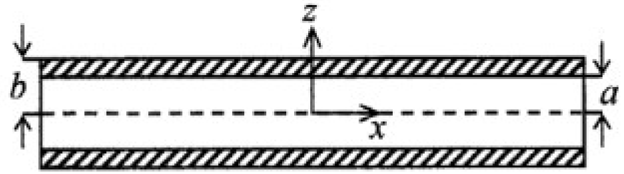
Systematic difference = change in substrate background

Due to:

- annealing effect on coating? ✗ from previous work on this coating
- annealing effect on substrate? ✗ from our tests on similar substrates
- thermoelastic shift in substrate, after coating deposition ✓

Vengallatore's model

$$\psi^{TE}_{s+c} = D^{TE}\phi^{TE}_{s+c} = D^{TE}\frac{(3\alpha_L)^2 BT}{C_v}\frac{1}{\frac{1}{3}\left[\frac{a^2}{b^2}+\frac{E_2}{E_1}\left(1-\frac{a^2}{b^2}\right)\right]}\sum_{n=1}^{\infty} Q_n \frac{\omega\,\omega_{peak,n}}{\omega^2+\omega^2_{peak,n}}$$



$$\psi^{TE}_{s} = D^{TE}\phi^{TE}_{s} = D^{TE}\frac{(3\alpha_L)^2 BT}{C_v}\, 6\sum_{n=1}^{\infty}\frac{\omega\,\omega_{peak,n}}{\omega^2+\omega^2_{peak,n}}\frac{1}{\gamma_n^4}$$

- Series of Debye peaks (consider here only the first term as significant)
- Reduces to usual Zener's formula for b → a (= covers the case of bare substrate)
- In principle, the thermoelastic loss with and without coating could be calculated **if** the coatings parameters were known & the mode-dependent dilution factors were found numerically

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parametrising the thermoelastic shift

Thermoelastic shift = 1) Frequency shift to the left + 2) Peak intensity change
Combination 1) + 2) determines which side of the thermoelastic curve (= *to the left or to the right of the peak*)
is more favourable for measurement,
and this in turn determines the best geometry

1) Frequency shift to the left:

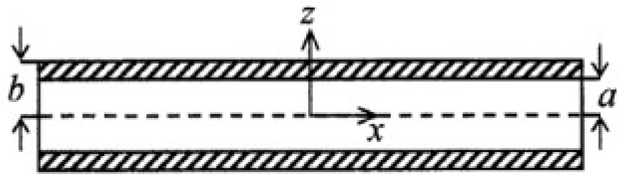
- always present with coating addition (because thickness increases: e.g. for crystalline Si [0 0 1], diameter = 3'', thickness = 0.2 mm, coating thickness *of the same material* = 1 um, the frequency peak shifts by about 70 Hz)
- can be enhanced/suppressed by the physical parameters of the coating, if different from substrate, namely C = specific heat capacity * density and alpha = linear thermal expansion

2) Peak intensity:

- remains the same if physical parameters of coating and substrate are very close;
- changes (*increases* or *decreases)* if the physical parameters are different, because the temperature profile across the thickness & the effect of this on deformation are modified

Vengallatore's formula linearized in parameter (coating thickness vs substrate thickness)

$$\epsilon = \frac{b-a}{a}$$



shift in peak frequency

1) Shift of peak frequency towards the left, after coating deposition:

If coating material were to be = substrate material:

$$\Delta\omega_0 = \frac{\pi^2}{3}\frac{k_s}{C_s}\left[\frac{1}{b^2} - \frac{1}{a^2}\right]$$

- main contribution due to increase in thickness
- what we would expect if coating material = substrate material
- only depends on geometry (= a, b)

In the most general case:

- Correction to the main contribution
- Depends on density and specific heat capacity of the coating
- Independent (in the first order) of thermal conductivity of the coating

(subscripts c, s denoting coating, substrate)

$$\Delta\omega_p = -2\frac{\rho_{coat}\, c_{v,coat}}{\rho_{sub}\, c_{v,sub}}\,\omega_p \cdot \epsilon + O(\epsilon^2)$$

shift in intensity

2) Change of peak intensity, after coating deposition:

≈ 1

$$\phi_{th.el.V} = D_{th.el.} \Delta \frac{\omega\tau}{1+(\omega\tau)^2} I$$

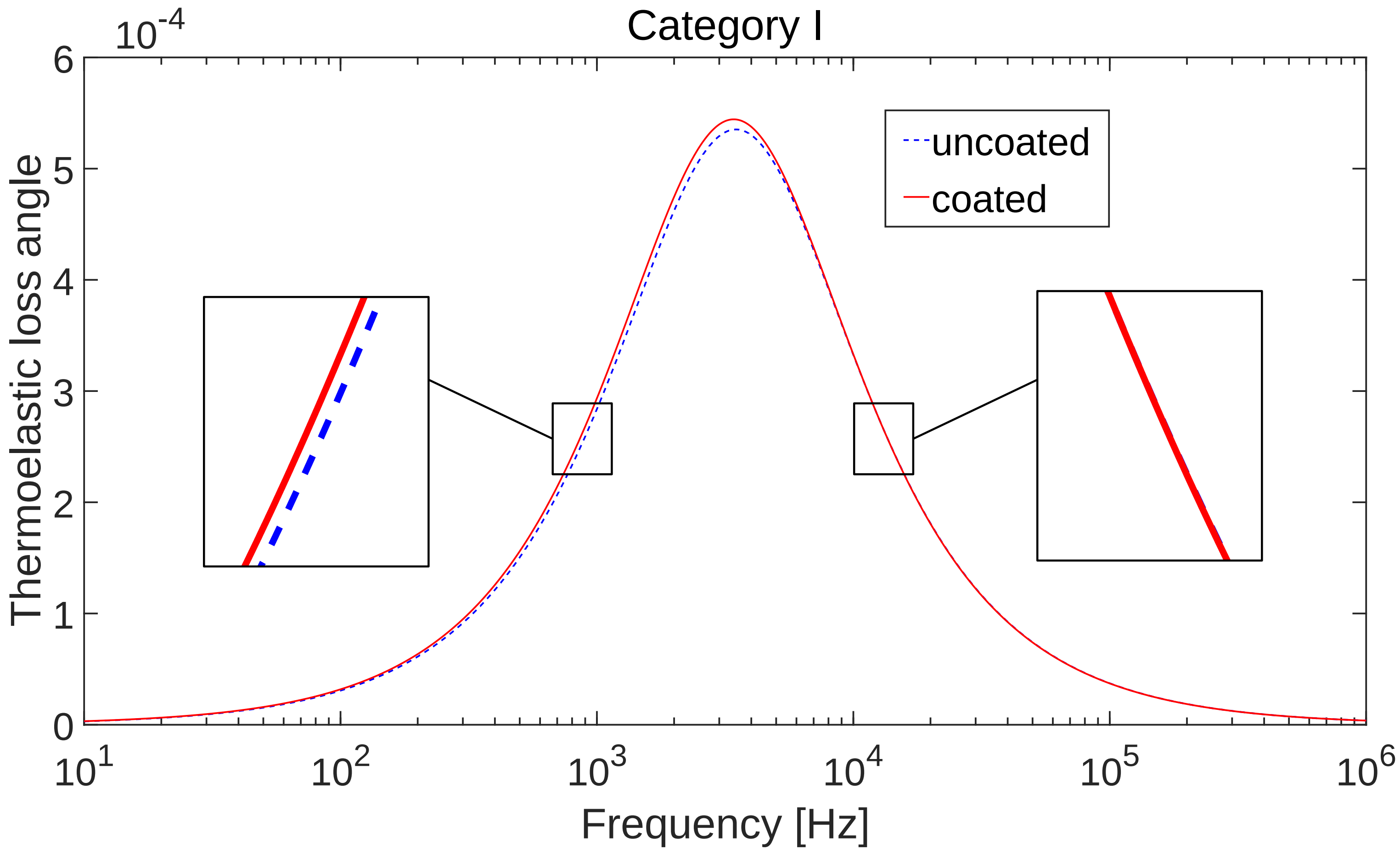
$$I = \frac{96}{\pi^4} + \frac{48}{\pi^2} \left[\frac{Y_{coat}}{Y_{sub}} \left(\frac{\alpha_{coat}}{\alpha_{sub}} - \frac{6}{\pi^2} \right) - \frac{\rho_{coat}\, c_{v,coat}}{\rho_{sub}\, c_{v,sub}} \left(1 - \frac{6}{\pi^2} \right) \right] \cdot \epsilon + O(\epsilon^2)$$

Dependence on the coating physical parameters:

- If coating material ~ substrate, I is the same (independent on geometry)
- If the physical parameters are significantly different:
  depends on ratios of Young's modulus, linear thermal expansion, density and specific heat capacity

Consider **3 categories of materials**, with intensity decreasing / increasing / unchanged
And determine optimal region to perform the measurement

category I



Increased Intensity:

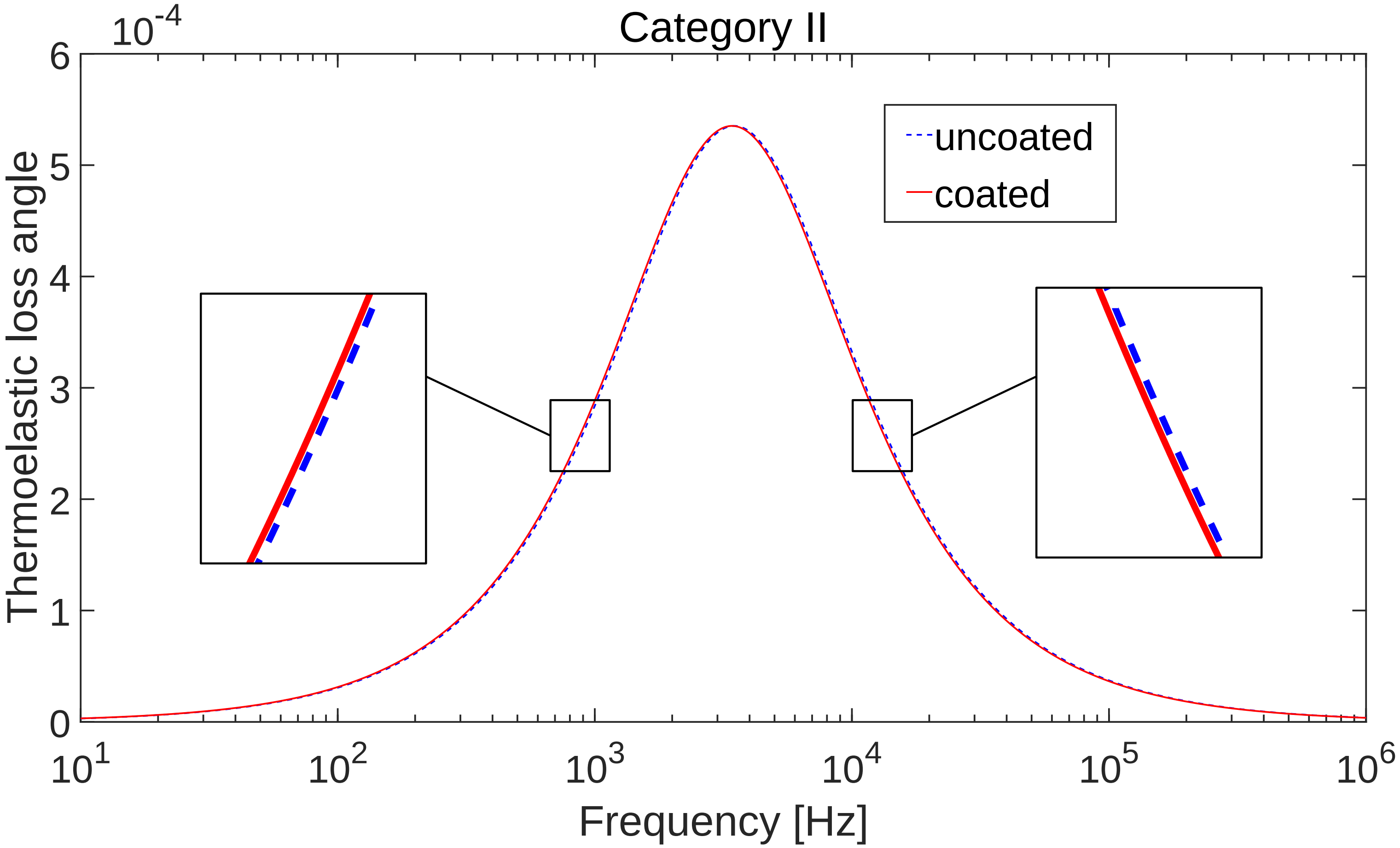
- Max Position shift to left, due to increased thickness + small correction
- Max Intensity **increases**

Impact of shift: Better on right side

$$\omega_p \propto \frac{1}{d^2} \qquad f_{modes} \propto \frac{d}{r^2}$$

Thick & narrow geometry

category II

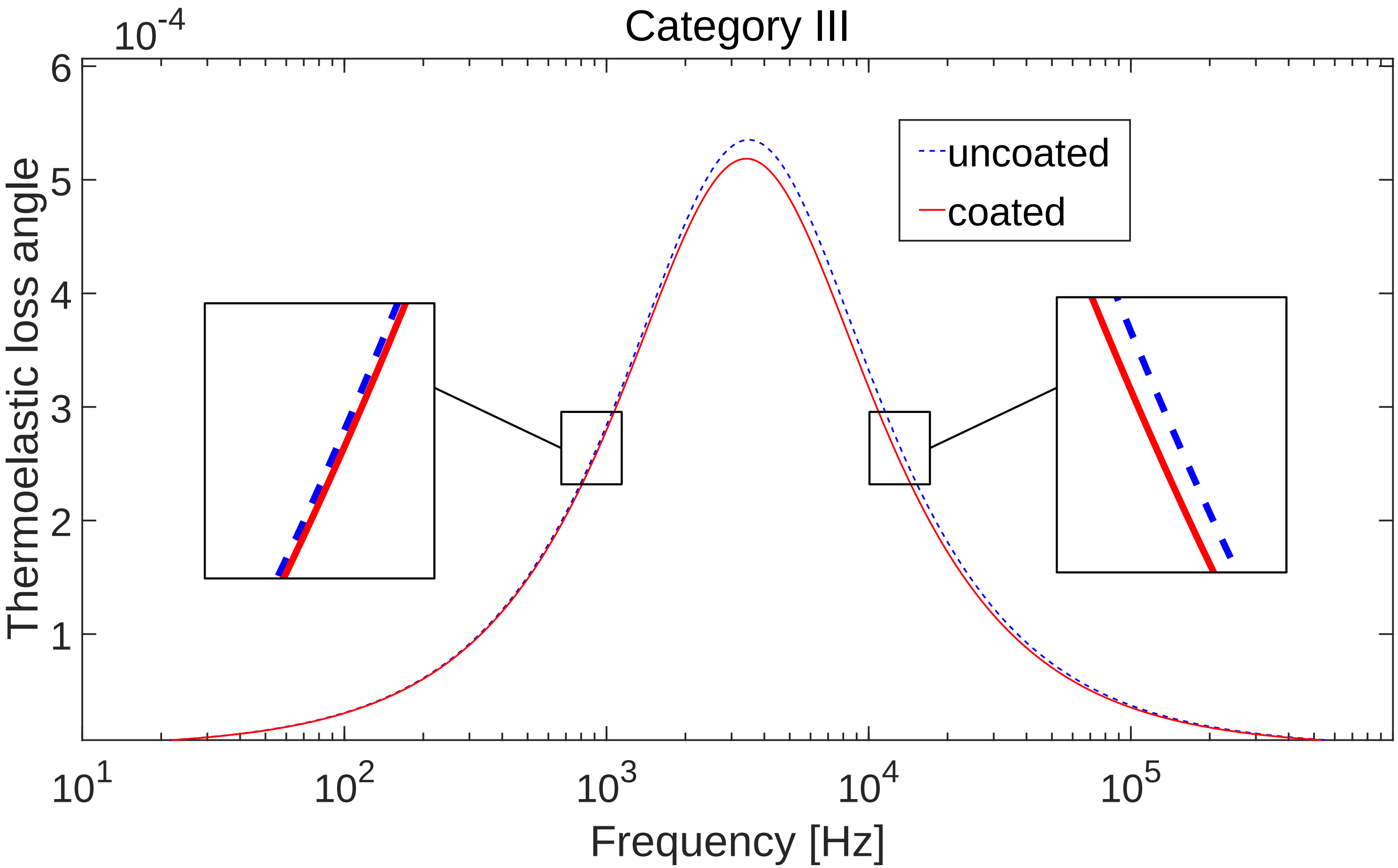


Equal Intensity:

- Max Position shift to left, due to increased thickness + small correction
- Max Intensity **unaffected** (e.g. because thermo-mechanical parameters of coating close to substrate)

Impact of shift:
No side is better

category III



Decreased Intensity:

- Max Position shift to left, due to increased thickness + small correction
- Max Intensity **decreases**

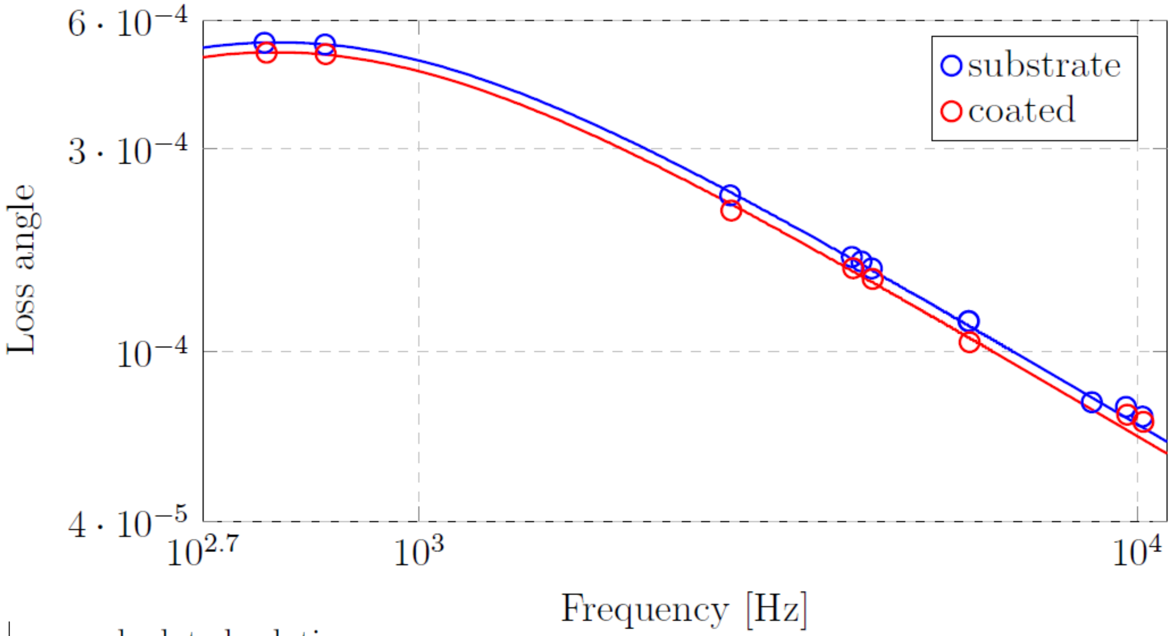
Impact of shift: Better on left side

$$\omega_p \propto \frac{1}{d^2} \qquad f_{modes} \propto \frac{d}{r^2}$$

Thin & large geometry

testing our model

- **Bare substrate** & **Coated sample** **data**: Divide data by mode-dependent dilution factor to get thermoelastic Debye curve
- **Coated sample** **data**: Subtract coating contribution (from measurement of the same coating deposited on silica) to get substrate contribution only
- Fit to Debye curves

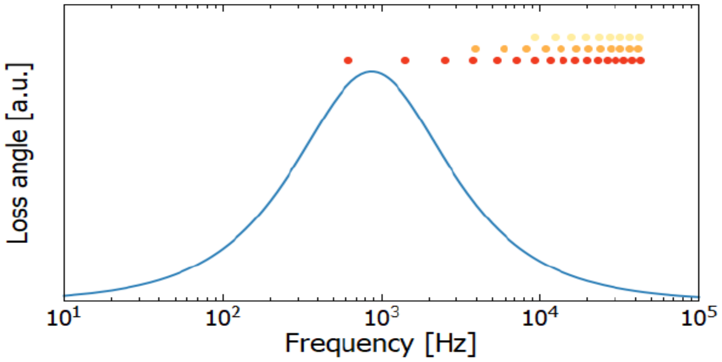


	bare	coated	measured relative variation	calculated relative variation **(our model)**
W15036				
f_p [Hz]	636 ± 13	629 ± 30	(-1 ± 7) %	- 2.9%
Δ [$\times 10^{-3}$]	1.07 ± 0.01	1.01 ± 0.02	(-5.6 ± 2.8) %	- 4.2%
W13015				
f_p [Hz]	637 ± 12	629 ± 15	(-1.3 ± 4.3) %	- 1%
Δ [$\times 10^{-3}$]	1.06 ± 0.01	1.04 ± 0.01	(-1.9 ± 1.9) %	- 1.4%

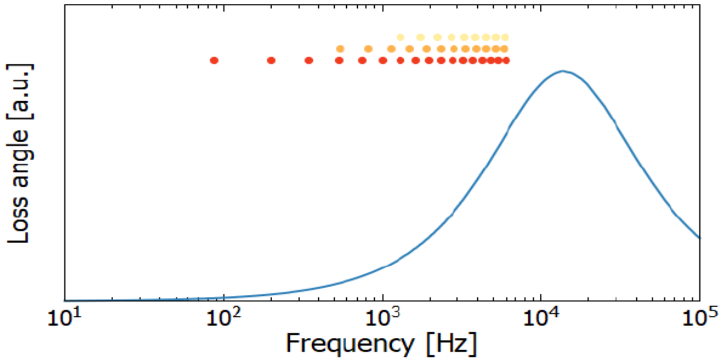
- **Bare substrate** **data**: absolute values compatible with silicon parameters
- **Shift**: compatible with model for both coating thicknesses (but needs more data on both sides of the peak to improve uncertainties)

an example of application

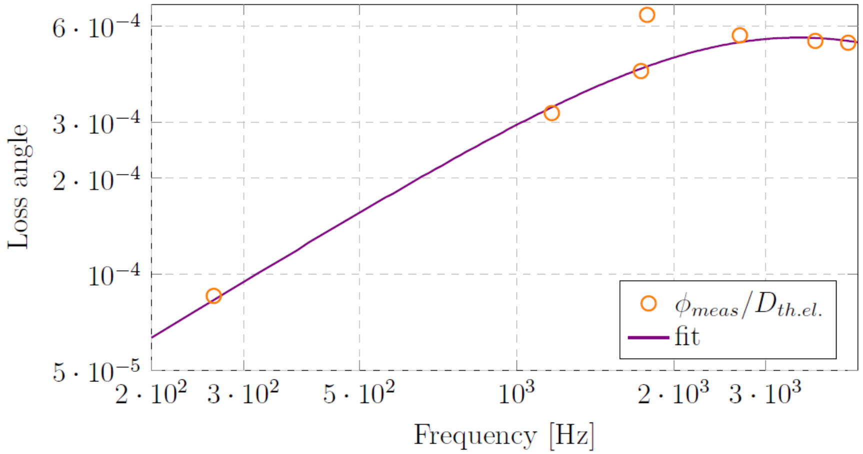
Looking for a suitable substrate for silica coating at room T (category III) = thin & large



Diameter of 3" and thickness 0.4 mm
(geometry used so far)



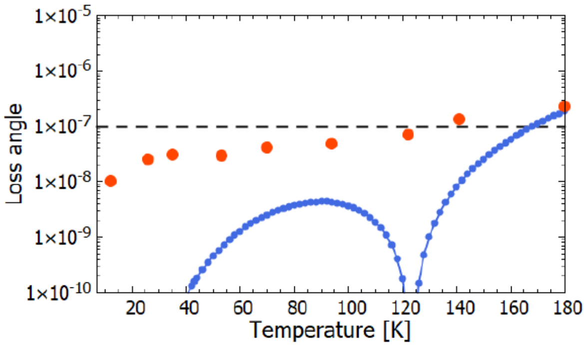
Diameter of 4" and thickness 0.2 mm
Practical considerations: limit at 3''



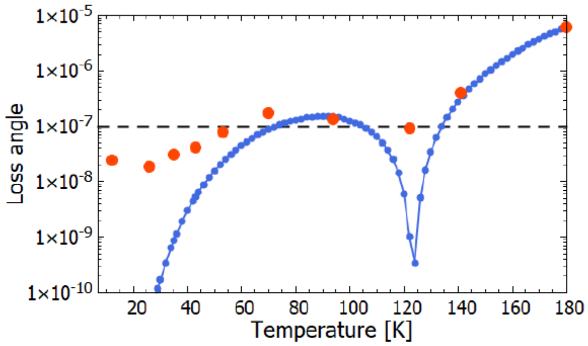
- Data are well fitted by the Debye function → we conclude that the thermoelastic damping is dominant
- As desired, most modes fall on the left side of the peak

an example of application

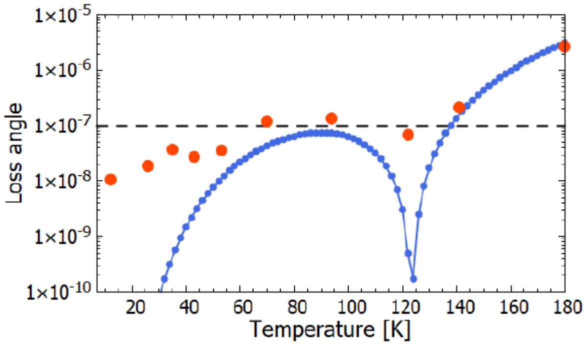
Temperature dependence: should be favourable with this geometry as the thermoelastic peak in silicon further moves to higher frequencies



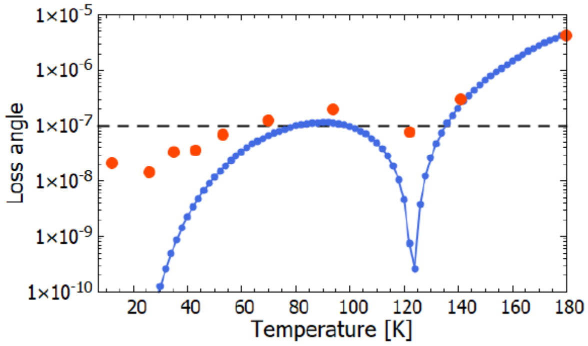
(a) Mode (2,0)



(b) Mode (2,1)



(c) Mode (5,0)

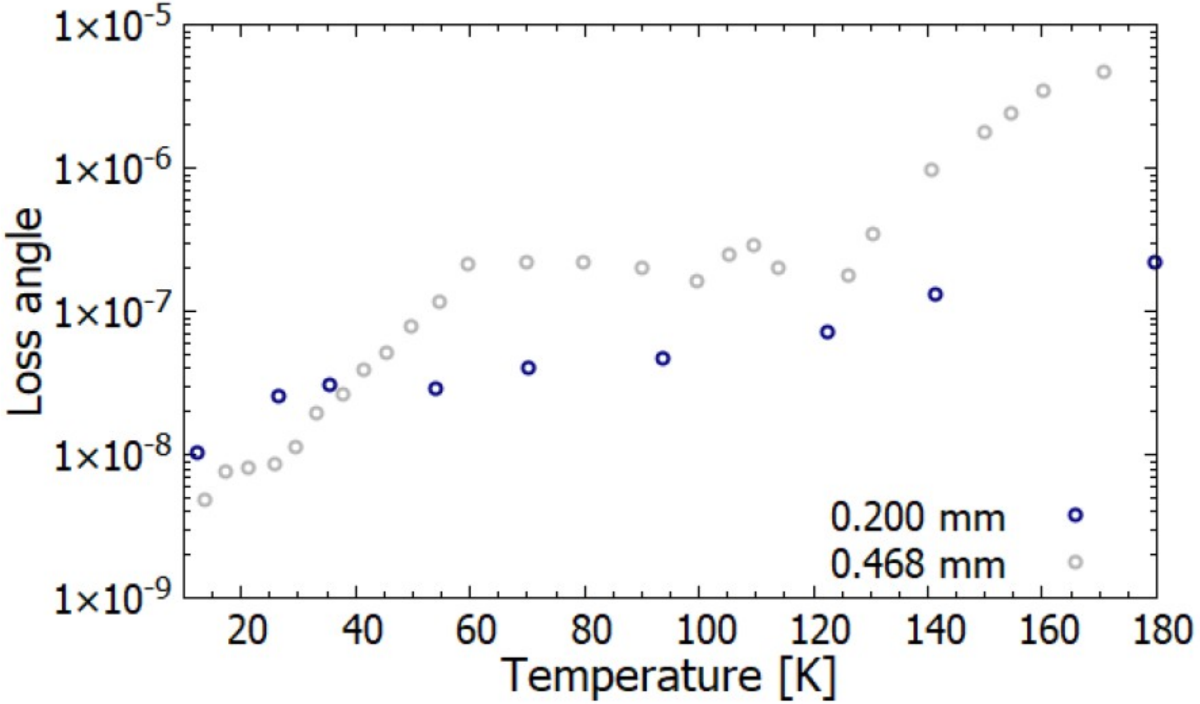


(d) Mode (6,0)

Typical values:
coating $\phi = 10^{-5}$ dilution factor 10^{-2} → loss angle of 10^{-7}
Substrate having the same losses = degradation of a factor 2 in the quality factor after deposition (the higher the degradation the better) →
Consider the factor of 2 degradation as a threshold value **(dashed line)**

- butterfly mode (2,0) exhibits notably low values in the loss angle
- in the region between 50 K - 120 K, the curve is almost linear, without any peak of thermoelastic damping → suggests this phenomenon is no longer dominant
- Conversely, the higher frequency modes display a slight impact from thermoelastic damping, as expected

an example of application



Comparison of the loss angle of the mode (2,0) between the **0.2 mm** Sil'Tronix substrate and a **0.468 mm** disc of a past work

Confirms thin & large geometry is favourable @ low T

conclusions

Joint requirements of bringing the absolute value of the thermoelastic noise below the acceptable threshold + minimising the thermoelastic shift

- Experimental evidence of thermoelastic shift

- Simple model obtained by linearization of Vengallatore’s formula in the case of thin films

- **Geometry approach**
  Knowing **only roughly** the physical parameters of the coating, manipulate the sample geometry (thickness, diameter) to minimise the shift (even if not quantitatively determined, we can assume it is negligible within the experimental uncertainties that we can expect)

- **Quantitative approach**
  With the aid of the linearized model, try and devise a series of measurement (on different samples with different geometries, coating thicknesses, …)
  that allow for a measurement of the thermoelastic shift induced by a particular coating