Birefringence measurements of substrate materials and coatings for Einstein Telescope

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Summary

- Brief background
- Birefringence measurements in transmission
- Birefringence measurements in reflection
- Comments and questions

Birefringence and ellipticity

The index of refraction is a complex number: $\tilde{n} = n + i\kappa$

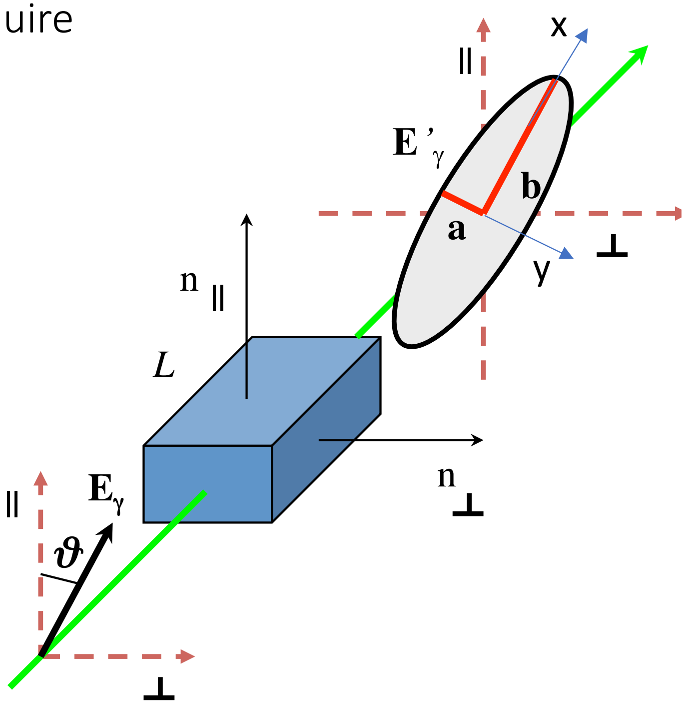
- In a birefringent medium $n_{\parallel} \neq n_{\perp}$
- A linearly polarized beam passing through a birefringent medium will acquire an ellipticity $\psi = \pm a/b$ (the sign determines the rotation direction of $\mathbf{E}_\gamma$)

$$\mathbf{E}_\gamma = E_\gamma \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \Delta\phi = \frac{2\pi(n_{\parallel} - n_{\perp})L}{\lambda}$$

$$\mathbf{E}'_\gamma = E_\gamma \begin{pmatrix} 1 + i\frac{\Delta\phi}{2}\cos 2\vartheta \\ i\frac{\Delta\phi}{2}\sin 2\vartheta \end{pmatrix}, \qquad \Delta\phi \ll 1$$

Immaginary

$$\psi = \pm\frac{a}{b} \approx \frac{\Delta\phi}{2}\sin 2\theta = \frac{\pi(n_{\parallel} - n_{\perp})L}{\lambda}\sin 2\vartheta$$



Dichroism and rotation

The index of refraction is a complex number: $\tilde{n} = n + i\kappa$

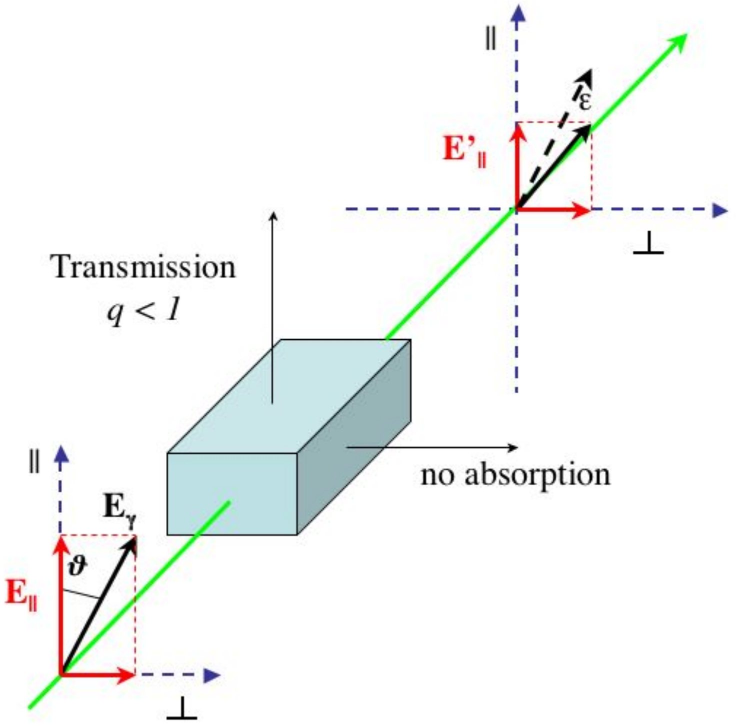
- In a dichroic medium $\kappa_{\|} \neq \kappa_{\perp}$
- A linearly polarized beam passing through a dichroic medium will acquire a rotation ε

$$\mathbf{E}_\gamma = E_\gamma \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad 1 - q = \frac{2\pi}{\lambda}(\kappa_{\|} - \kappa_{\perp})L$$

$$\vec{E}_\gamma = E_\gamma \begin{pmatrix} 1 + \left(\frac{1-q}{2}\right)\cos 2\vartheta \\ \left(\frac{1-q}{2}\right)\sin 2\vartheta \end{pmatrix}$$

Real

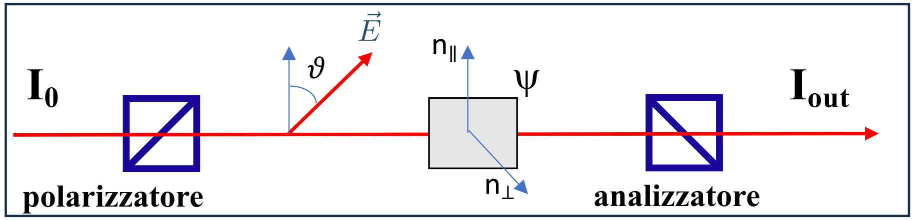
$$\epsilon \approx \left(\frac{1-q}{2}\right)\sin 2\vartheta = \frac{\pi(\kappa_{\|} - \kappa_{\perp})L}{\lambda}\sin 2\vartheta$$



If $|\psi| = \varepsilon$, both will give the same ouput power in a direction perpendicular to E_γ but have very different origins

Measuring the ellipticity of a sample

The typical elipticity one would like to measre is $\psi \lesssim 10^{-4}$

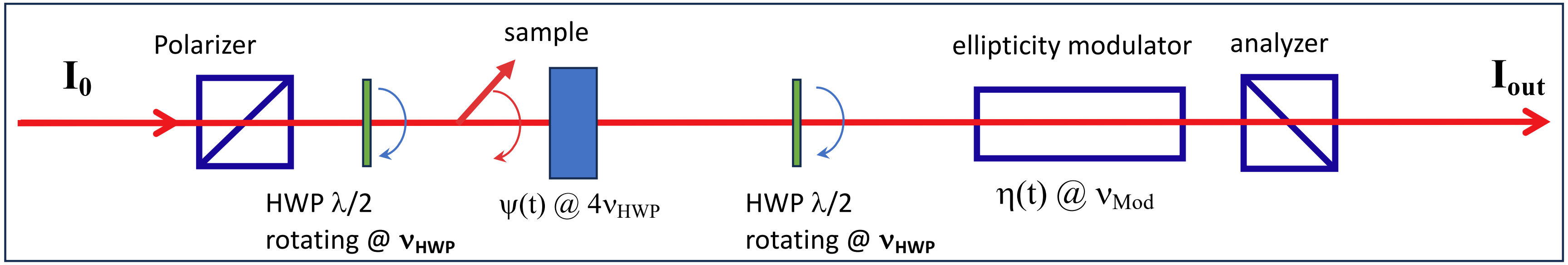


DC direct detection is excluced

$$I_{\text{out}} = I_0 \left[\sigma^2 + \psi^2 \sin^2 2\vartheta\right]$$

Extinction $\sigma^2 \approx 10^{-7}$, $\psi^2 \lesssim 10^{-8}$

- Add a known time varying ellipticity $\eta(t)$ to ψ. With η, $\psi << 1$, these add algebraically
- Also make the ellipticity ψ(t) time dependent by rotating the polarization in the sample using a HWP



$$I_{\text{out}} = I_0 \left\{\sigma^2 + \left|i\psi(t) + i\eta(t)\right|^2\right\} = I_0 \left\{\sigma^2 + \eta^2(t) + 2\eta(t)\psi(t) + \ldots\right\}$$

The output power is now linear in the ellipticity $\psi(t)$.

Ellipiticity vs Rotation

- Ellipticities ψ,η are imaginary numbers whereas rotations φ are real. If small, they also add up algebraically.
- After the analyzer, the electric field and the power will be

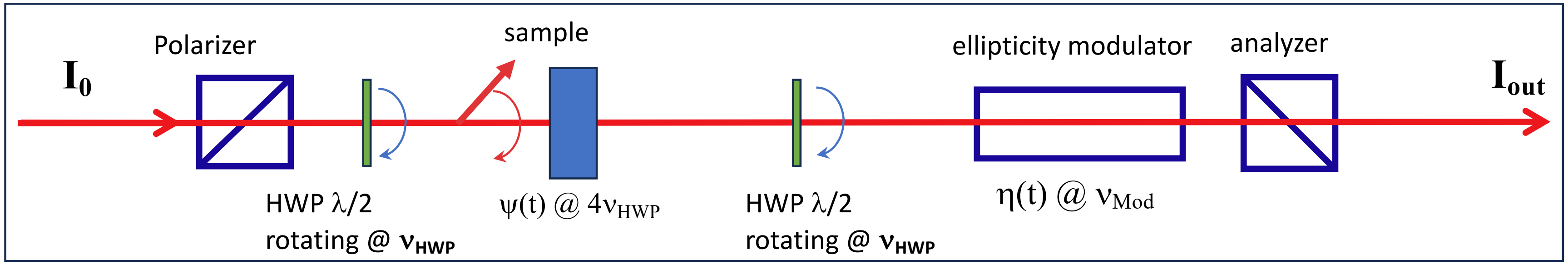
$$\vec{E}_{\mathrm{out}} = E_0 \begin{pmatrix} 0 \\ \varphi(t) + i\psi(t) + i\eta(t) \end{pmatrix}$$

$$\Rightarrow \quad I_{\mathrm{out}} = I_0 |\varphi(t) + i\psi(t) + i\eta(t)|^2 = I_0 \left[\varphi(t)^2 + \eta(t)^2 + \psi(t)^2 + 2\eta(t)\psi(t)\right]$$

- There is no product between φ and η. Rotations do not beat with ellipticities. With an ellipticity modulator (time dependent) one measures only ellipticities.

Measuring the ellipticity of a sample

- Add a known time varying ellipticity $\eta(t)$ to ψ. With η, $\psi << 1$, these add algebraically
- Also make the ellipticity $\psi(t)$ time dependent by rotating the polarization in the sample using a HWP



$$I_{\text{out}} = I_0 \left\{\sigma^2 + |i\psi(t) + i\eta(t)|^2\right\} = I_0 \left\{\sigma^2 + \eta^2(t) + 2\eta(t)\psi(t) + \ldots\right\}$$

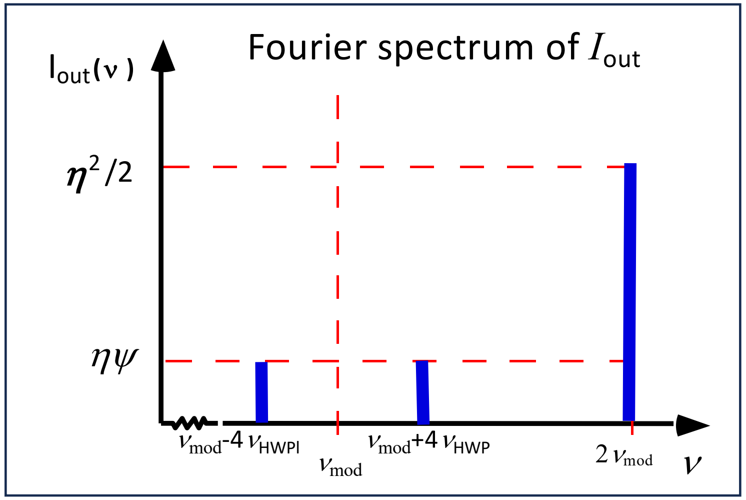
The output power is now linear in the ellipticity $\psi(t)$.

$$\eta(t)^2 = \eta^2 \cos^2 \omega_{\text{mod}} t = \frac{\eta^2}{2}\left(1 + \cos 2\omega_{\text{mod}} t\right)$$

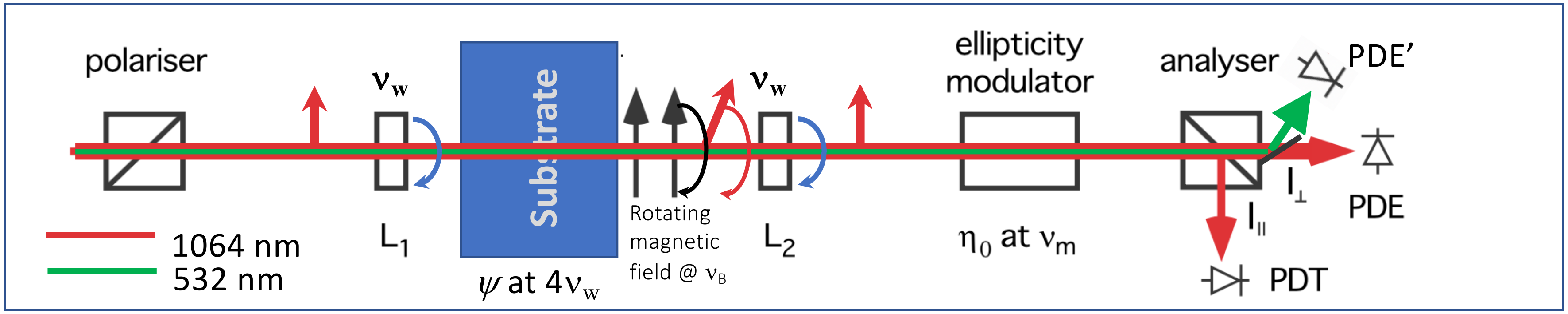
$$2\eta(t)\psi(t) = 2\eta\psi \cos\omega_{\text{mod}} t \cdot \cos 4\omega_{\text{HWP}} t =$$

$$= \eta\psi \left[\cos(\omega_{\text{mod}} + 4\omega_{\text{HWP}})t + \cos(\omega_{\text{mod}} - 4\omega_{\text{HWP}})t\right]$$

Ideally



Baseline scheme for substrate birefringence measurements



$$\psi(t) = \psi_0 \sin 4\phi(t) + \frac{\alpha_1(t)}{2} \sin 2\phi(t) + \frac{\alpha_2(t)}{2} \sin[2\phi(t) + 2\Delta\phi(t)]$$

Signal @ $4\nu_w$

Spurious signals
Contain harmonics of ν_w

Relative rotation phase error
Degrades extinction

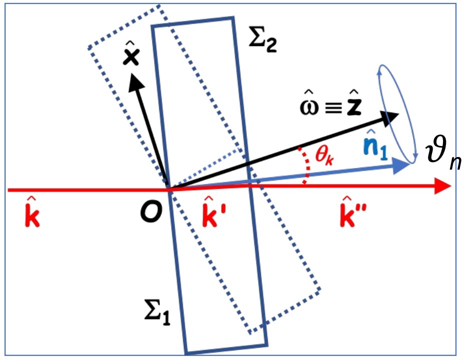
$\alpha_{1,2}$ are the phase errors from π of the two HWPs and $\phi(t)$ is their rotation angle

- ✓ 532 nm beam (HWP -> FWP) allows independent alignment of the rotating HWPs to reduce 1st, 3rd and 4th harm.
- ✓ At 1064 nm, control the temperature of the wave-plates to reduce the dominating 2nd harmonic
- ✓ Reduced systematic peaks such that $\alpha_{1,2}^{(1,2,3)} \lesssim 10^{-4}$ at all relevant harmonics and in particular for the 4th harmonic, $\alpha_{1,2}^{(4)} \lesssim 10^{-5}$. Can be subtracted vectorially ➔ Ellipticity sensitivity $\psi_0 \approx 10^{-6}$
- ✓ Can produce X-Y 'maps' of the static average birefringence of a substrate: $\Delta n = \frac{\psi_0 \lambda}{\pi L}$
- ✓ Optical path difference sensitivity $S_{OPD} \lesssim 10^{-12}$ m
- ✓ Calibration with the Cotton-Mouton effect in air using a rotating 2.5 T permanent magnet

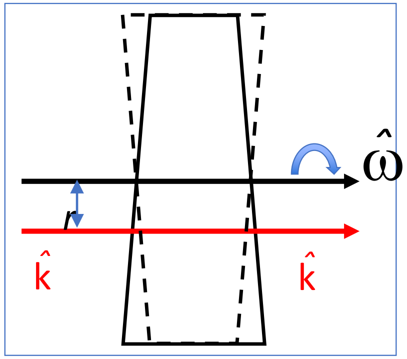
Generation of spurious harmonics from rotating HWPs

$$\alpha_{1,2}(\phi, \mathrm{T}, r) = \alpha_{1,2}^{(0)}(\mathrm{T}) + \alpha_{1,2}^{(1)} \cos\phi(\mathrm{t}) + \alpha_{1,2}^{(2)} \cos 2\phi(\mathrm{t}) + \ldots$$

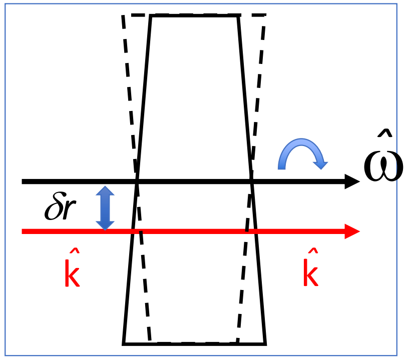
Temperature dependence of $\alpha_{1,2}^{(0)}(\mathrm{T}) = \frac{2\pi}{\lambda}\int \Delta\mathrm{n}\,\mathrm{dL}$



ALIGNMENT



WEDGE β



WEDGE + OSCILLATION @ ν_w

$$\alpha_{1,2}^{(1)} \approx \frac{2\pi}{\lambda}\Delta n \frac{D}{n^2}\vartheta_n\vartheta_k$$

$$\alpha_{1,2}^{(2)} \approx \frac{2\pi}{\lambda}\Delta n \frac{D}{4n^2}\vartheta_n^2\vartheta_k^2$$

$$\alpha_{1,2}^{(1)} \approx \frac{2\pi}{\lambda}\Delta n\ \Delta r_0\ \beta$$

$$\alpha_{1,2}^{(2)} \approx \frac{2\pi}{\lambda}\Delta n\ \delta r\ \beta$$

Generate 4th harmonic but can be controlled to < 10^{-5} level corresponding to an optical path difference $\int \Delta n\, dL \lesssim 10^{-12}$ m

✓ The HWPs can be aligned separately using a frequency doubled laser @ 532 nm

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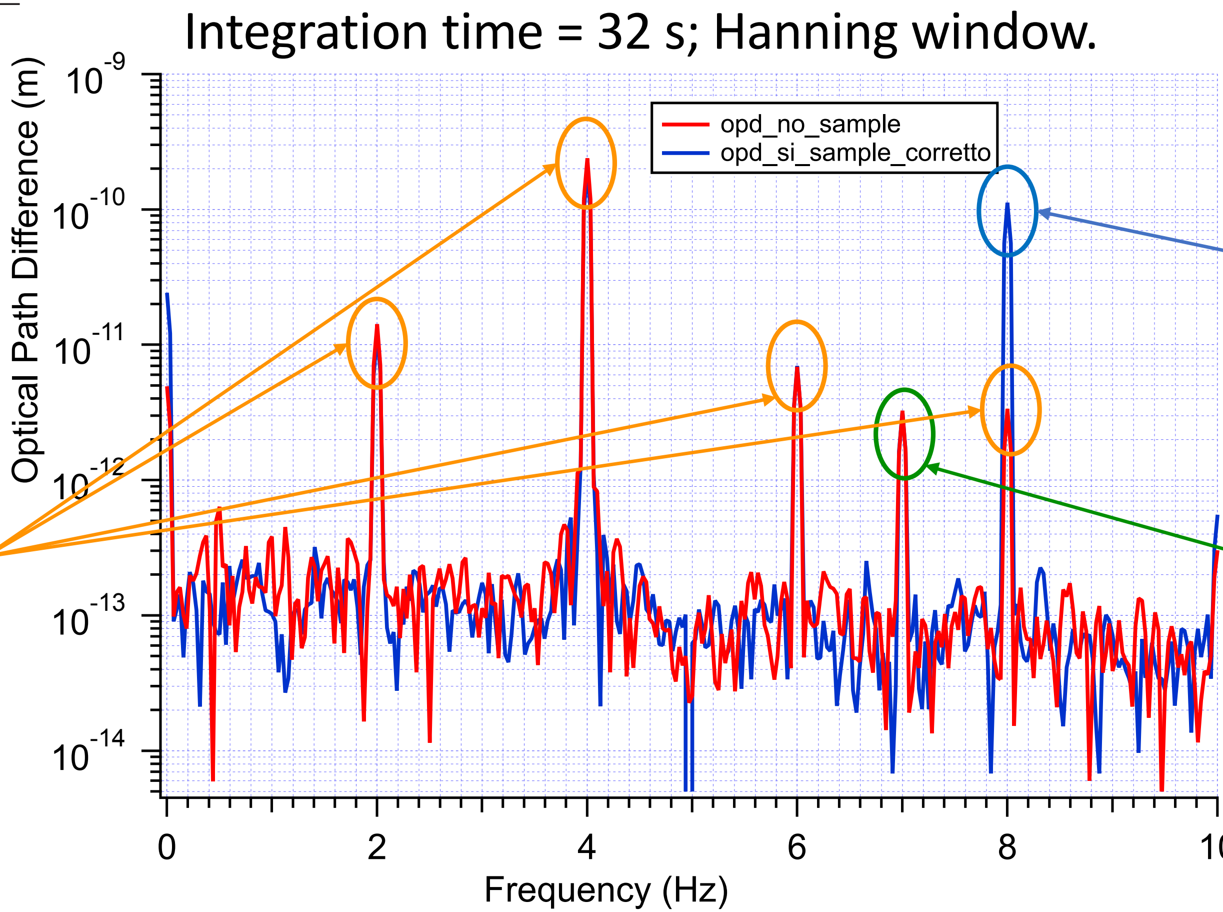
Example of a demodulated spectrum

$$OPD = \Delta n L = \frac{\psi_0 \lambda}{\pi}$$



- Spurious harmonics from temperature and misalignment.

- Peak due to silicon birefringence: $\Delta n = 1.1 \times 10^{-7}$; L = 1 mm

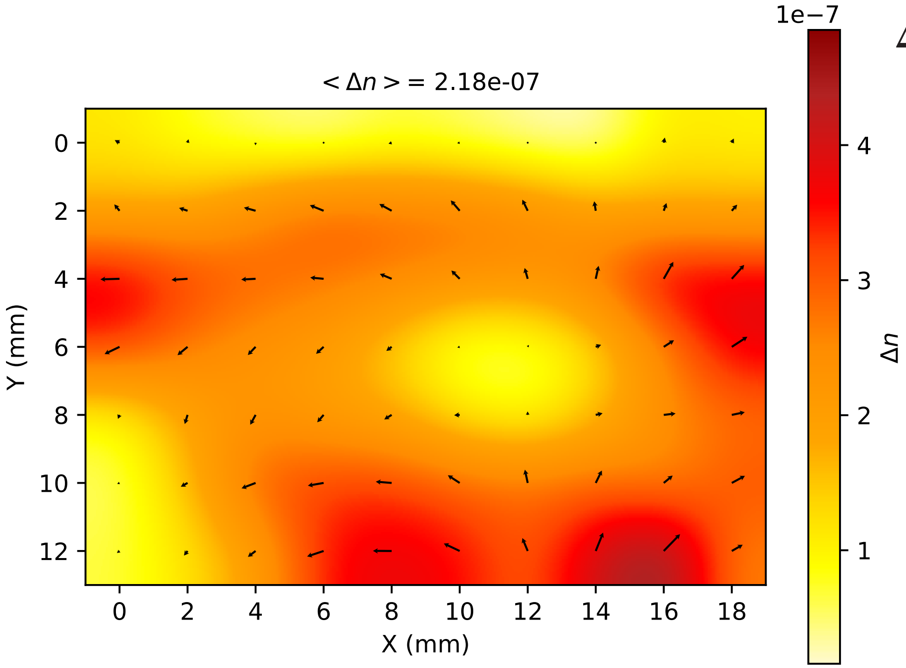
- Calibration Cotton-Mouton peak of air. $\Delta n = 3.9 \times 10^{-12}$; L = 0.84 m

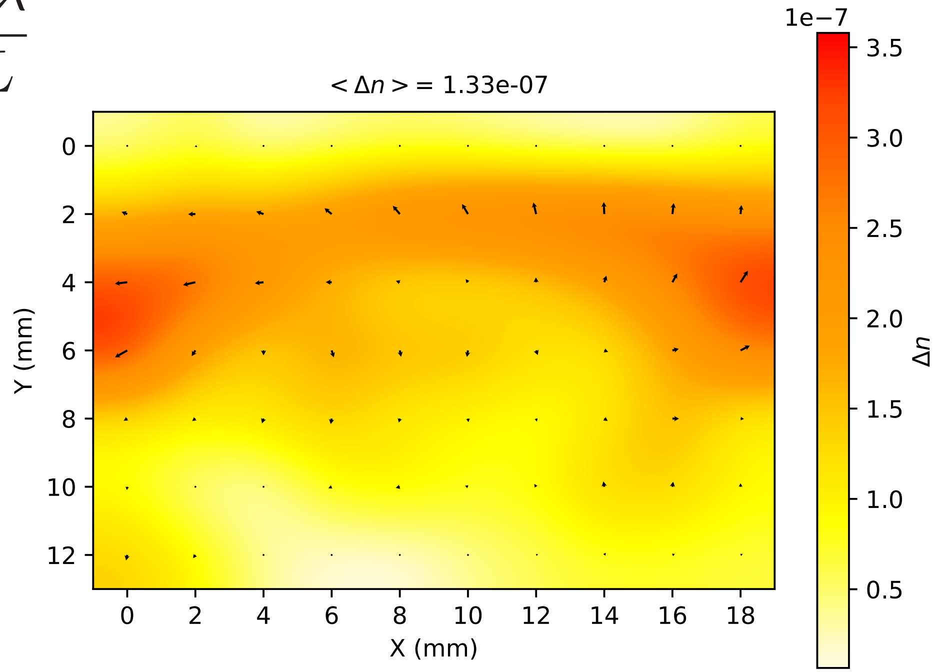
Subtraction of the 4th harmonics with and without the sample is done vectorially.

Example of birefringent map of silicon

- Silicon crystal samples (100), L = 1 mm thick, 2.5 cm x 2.5 cm, cut in house from larger sample
- Measurements using 1064nm (significant absorption). Will be repeated with 1550nm
- Subtracted vectorially the waveplate contribution (small effect)
- Held with clamp from bottom edge (left): extra stress can be seen due to clamp.
- Held without clamp (right). Upper half maintains same birefringence.
- <u>Non uniform birefringence</u>.

$$\Delta n = \frac{\psi_0 \lambda}{\pi L}$$

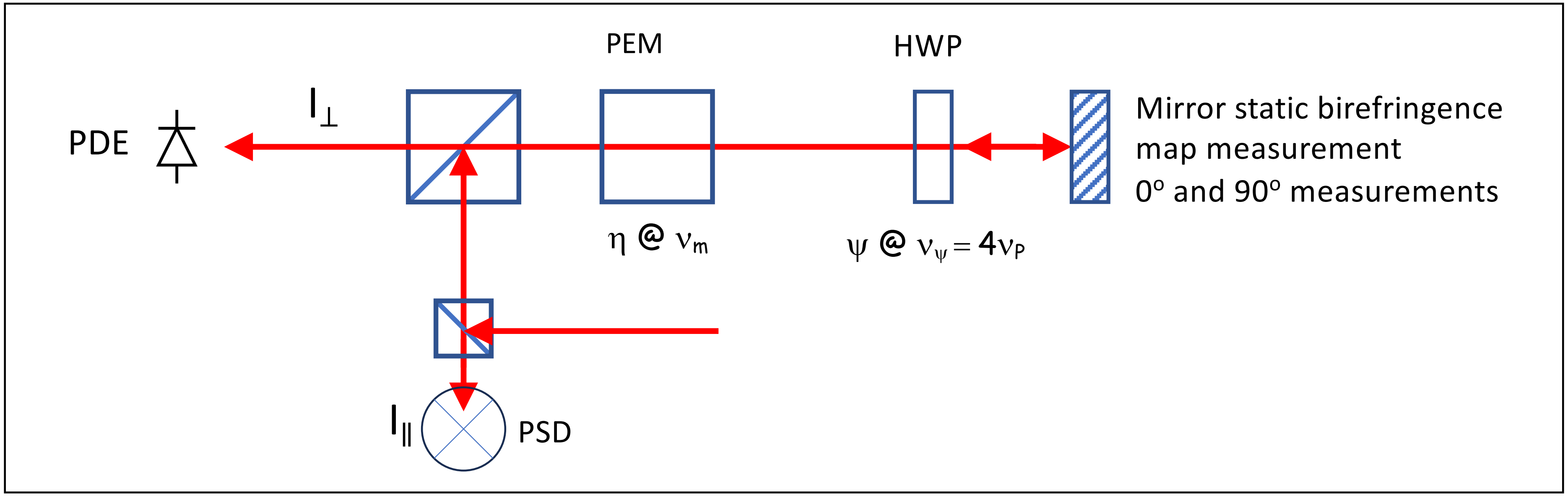




Reflective coating birefringence measurements

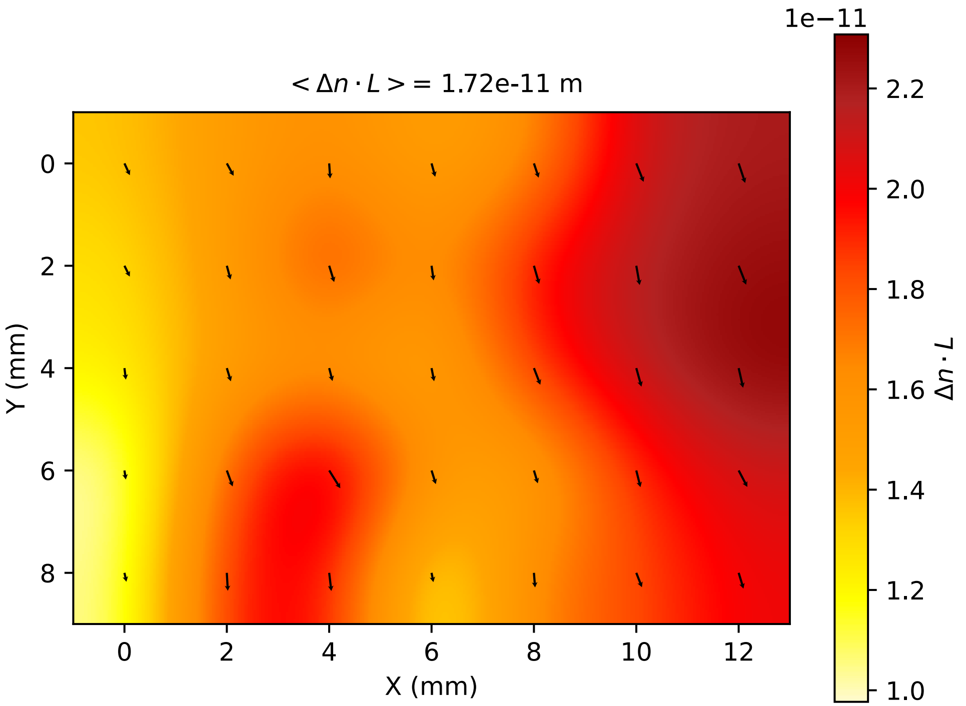
Reflection scheme for static birefringence maps of reflective coatings:

- At present we have a 1064 nm and 532 nm beams aligned.
- When mapping the sample, the reflected beam must be re-aligned on the PSD.
- Zero measurements cannot be made without the sample.
- $0^o - 90^o$ measurements allow the separation of the sample and HWP contributions.
- Installed a rotating magnet for calibration.

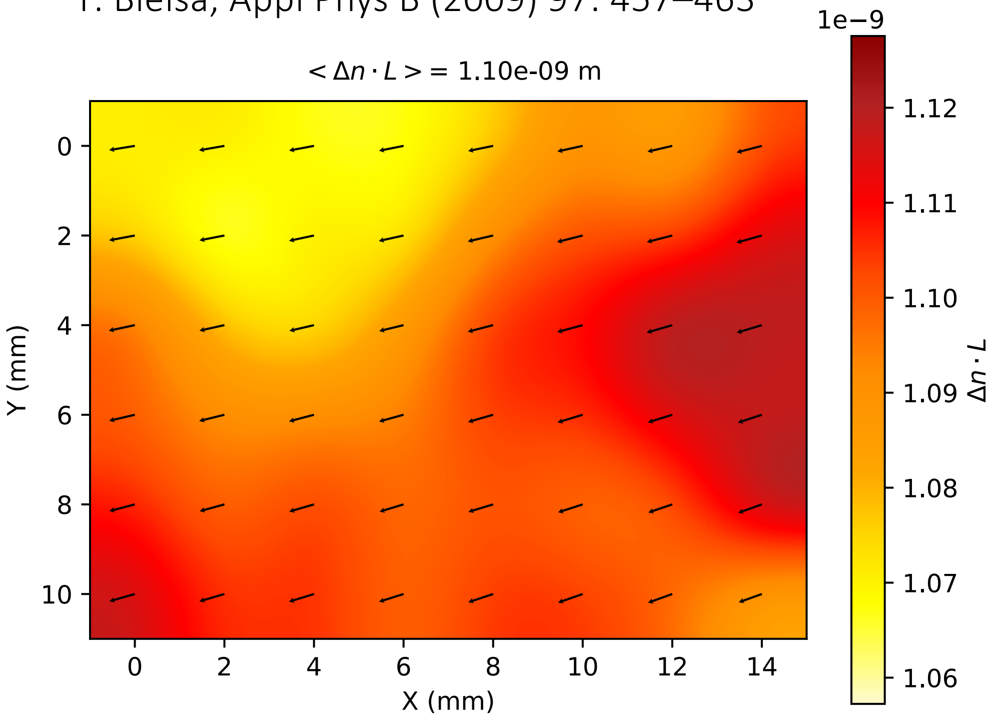


Example of birefringent map of coatings: first samples

- Silver mirror.
- Very low birefringence.
- Measured ellipticity is dominated by the rotating half-waveplate.



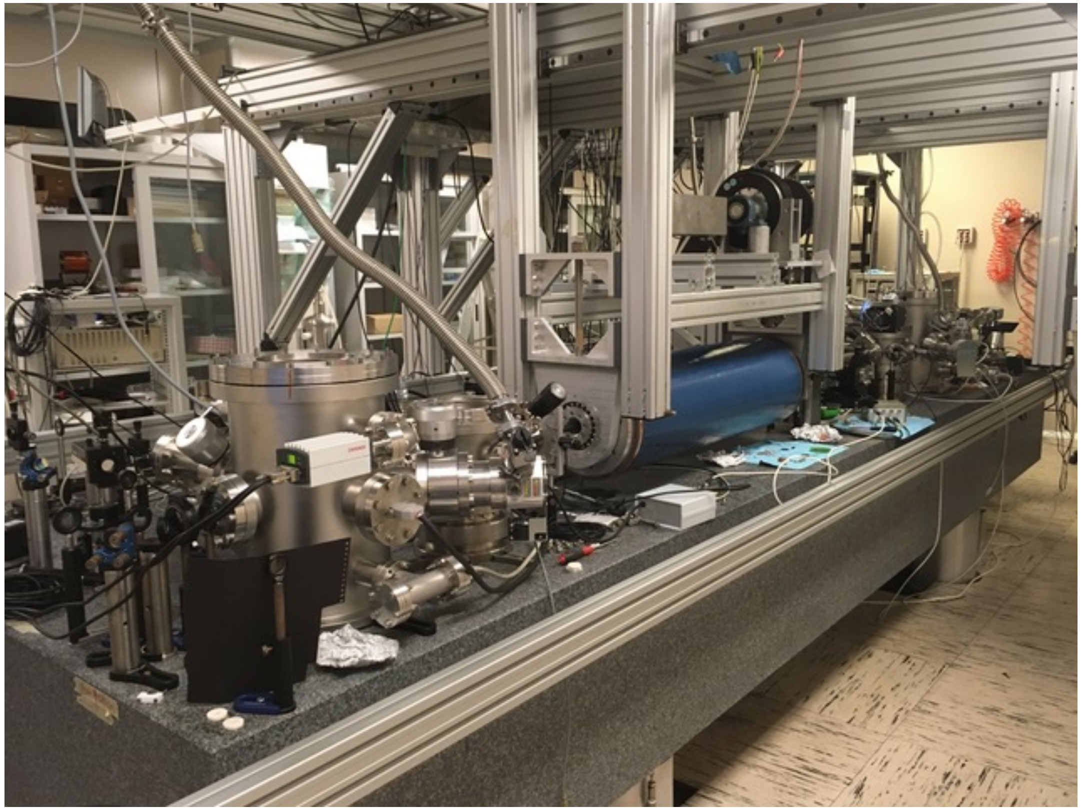
- Dielectric mirror with T ≈ 10^{-3}. 'Uniform'.
- Polarization can be aligned in cavities.
- Higher reflectivity, lower birefringence. For F ≈ 10^5, $\Delta n \cdot L \approx 3 \times 10^{-13}$ m.
- Brandi et al. Appl. Phys. B 65, 351–355 (1997); F. Bielsa, Appl Phys B (2009) 97: 457–463



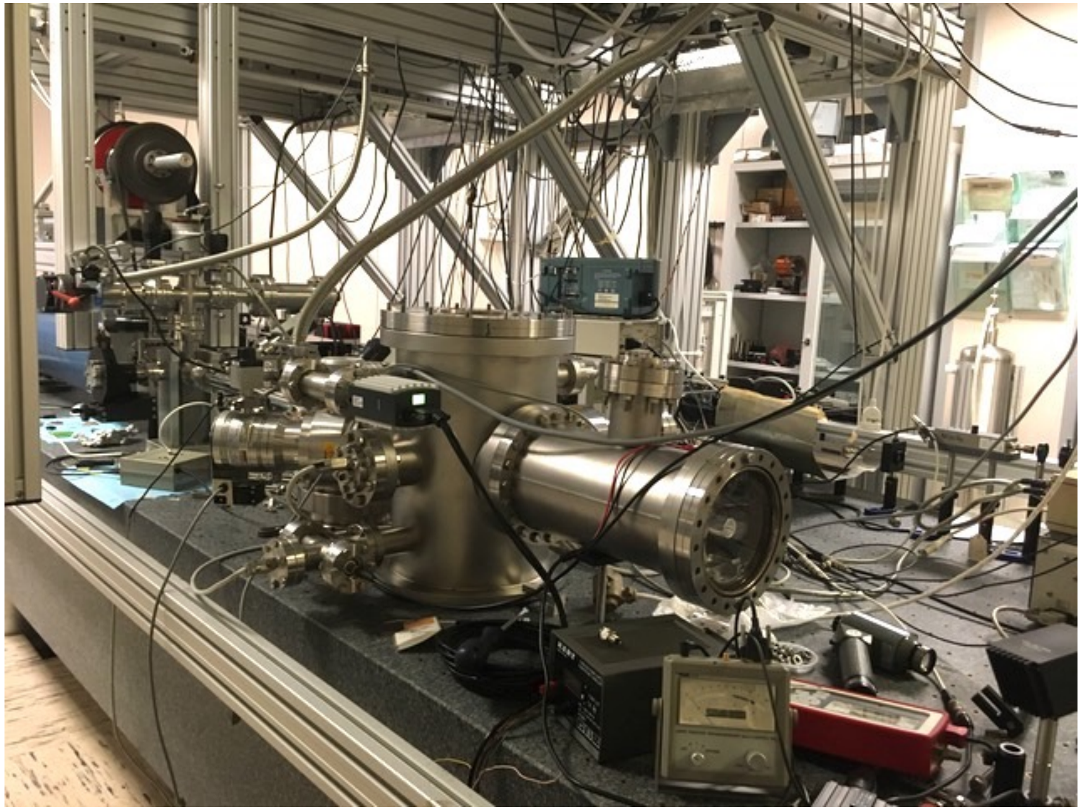
Pictures

At present being used with rotating HWPs.

General view from input side



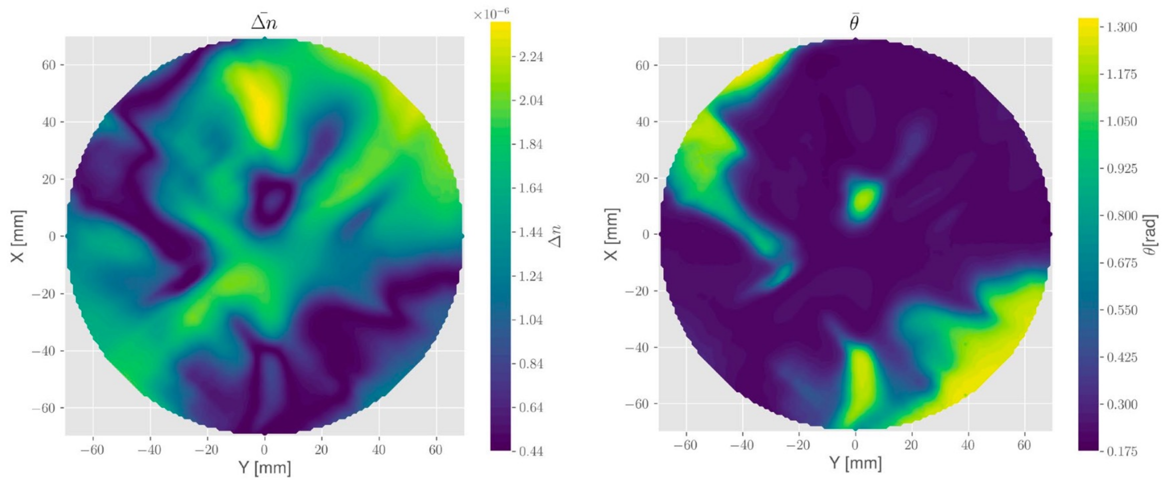
General view from output side

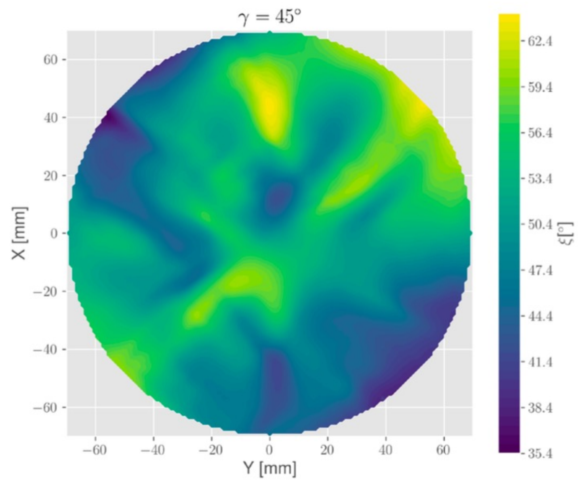


Comments and questions: 1

KAGRA

- The measured residual birefringence of sapphire perpendicular to the C-axis is $\Delta n \approx 10^{-6}$ with 15 cm thick substrates.
- Non uniform birefringence map of substrate (amplitude and direction). Phase shifts of ≈ 1 rad.
- $\Delta n \approx 10^{-7}$ in silicon*. Non uniform here too. For ET the desired thickness is 67 cm.
- ➔ Phaser shift ≈ 0.3 rad. ET would like a X10 better sensitivity
- Is $\Delta n \lesssim 10^{-8}$ necessary? If Δn is uniform ➔ align polarization with axis of system birefringence including coatings? How?
- If non uniform...





$$\xi = \tan^{-1}\left(\sqrt{\frac{P_S}{P_P}}\right)$$

Rotation or ellipticity?

Figure 4. Mean distribution of both birefringence Δn and θ-angle, calculated from the six input-polarization combinations which led to no miscalculations.

Zeidler, S., *et al.* Correlation between birefringence and absorption mapping in large-size Sapphire substrates for gravitational-wave interferometry. *Sci Rep* **13**, 21393 (2023)(https://doi.org/10.1038/s41598-023-45928-0)

*see also C. Krüger et al. Class. Quantum Grav. 33 (2016) 015012

Comments and questions: 2

MIRRORS

1. Our experience and other's too (Toulouse BMV group) have found that for the static birefringence of coatings:

$$\Delta n_{\text{high finesse}} < \Delta n_{\text{low finesse}}$$

2. There seems to be a 'more' uniform map compared to substrates (over ≈ few centimeters).

- The origin of this birefringence is not clear. C. Rizzo's group, Toulouse, attribute the birefringence to the first layer near to the substrate (F. Bielsa, Appl Phys B (2009) 97: 457–463). The cause is the stress between the substrate and first layer of the coating?
- In our Fabry-Perot based polarimeter for VMB measurements with a finesse = 700000 the static mirror birefringences were oriented to subtract each other and the polarisation aligned to the axis of the cavity as a whole. In this way the two eigenmodes of the cavity are almost superimposed.

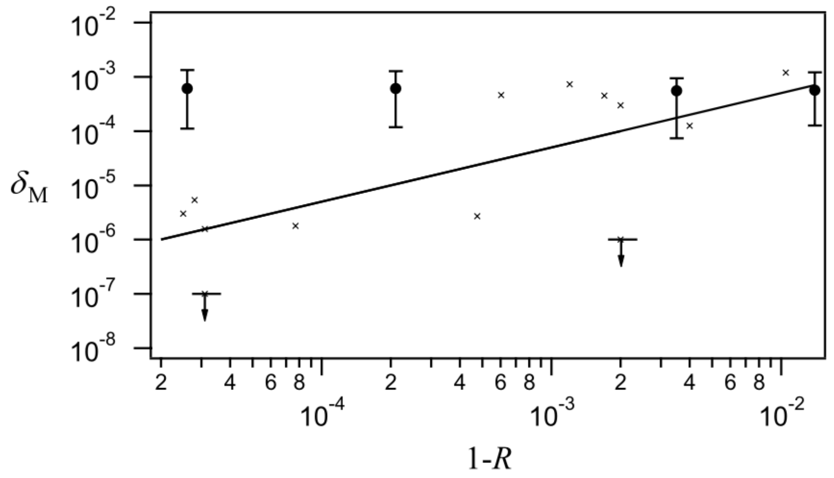


Fig. 6 Two different numerical calculations for the induced phase retardation per reflection as a function of $(1 - R)$. *Solid curve*: birefringence only for the first layer just after the substrate. *Dots with error bars*: calculation with random birefringence per each layer. *Crosses*: measurements plotted in Fig. 3

Thank you for your attention

Induced birefringence from stress

- Residual stress will generate a (static) birefringence map inside the sample
- External stress will also generate a birefringence

$$\Delta n = C_{\mathrm{SOC}} \, (\sigma_1 - \sigma_2)$$

- C_{SOC} = Stess optic coefficiente [Pa^{-1}], σ_1 and σ_2 stress along perpendicular directions [Pa]
- Typical values of stress optic coefficient: $C_{\mathrm{SOC}} \approx 10^{-12}\ Pa^{-1}$
- Fused silica: $3.4 \times 10^{-12}\ Pa^{-1}$
- Crystalline Silicon (axes): $(0.6 \div 1) \times 10^{-12}\ Pa^{-1}$
- Some initial work done for stress induced birefringence in Silicon as ET-LF substrate: C. Krüger et al. Class. Quantum Grav. 33 (2016) 015012
- Sapphire: could not find a value for C_{SOC}.

Alignment of sample in refelction

Disentangle HWP effect from sample signal: dual phase de-modulation at $4\nu_w$

- Sample at 0°: $\vec{V}_{\psi_{\rm HWP}} + \vec{V}_{\psi_{\rm sample}}$
- Sample at 90°: $\vec{V}_{\psi_{\rm HWP}} - \vec{V}_{\psi_{\rm sample}}$
- From semi-sum and semi-difference, one separates the two effects
- Graphs of the X and Y components of $\vec{V}_{\psi_{\rm HWP}}$ and $\vec{V}_{\psi_{\rm sample}}$ as a function of beam incident angle on the sample (silver mirror).
- The HWP signals depend significantly on the incident angle: the reflected beam passes through the HWP in a different point