Resonant behavior of linear three-mirror cavities in the context of quantum noise reduction

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ÉCOLE DOCTORALE PHENIICS

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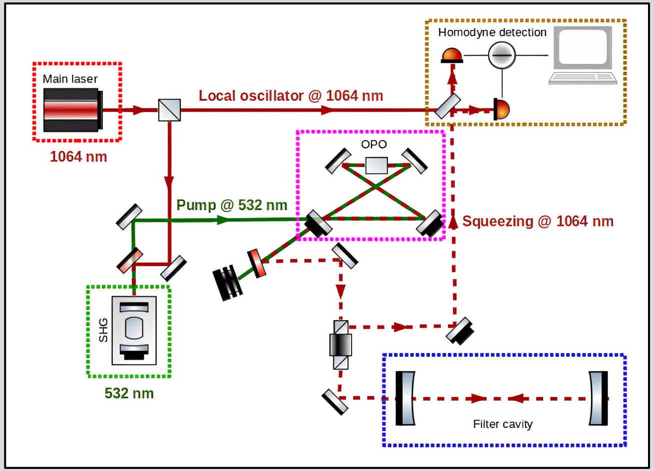
Outline

1. **Context: Frequency Dependant Squeezing**
2. **Three-mirror cavity - model**
3. **Three-mirror cavity - prototype at *IJCLab (Orsay, France)***
4. **Summary**

Context: Frequency dependant squeezing

Summary of current frequency dependant squeezing setup



Simplified scheme of a frequency dependant squeezing system

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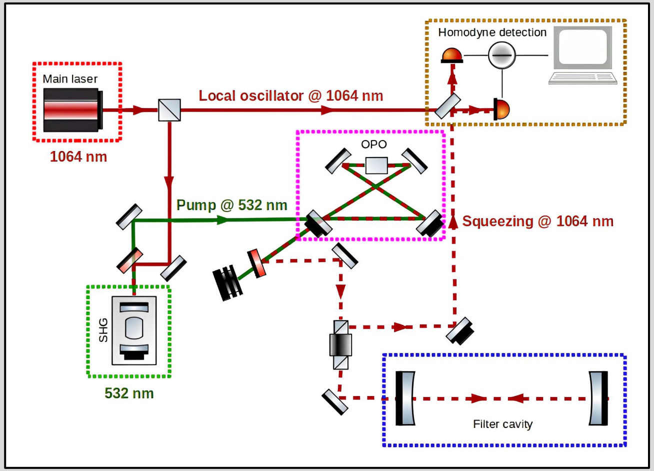

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The filter cavity allow to introduce a **frequency Ω_t around which the squeezing ellipse rotates**

$$\Omega_t = \frac{\pi c}{\sqrt{2} L F(r_i)}$$

L: cavity length

F: finesse of the cavity (depends on mirrors reflectivities r_i)

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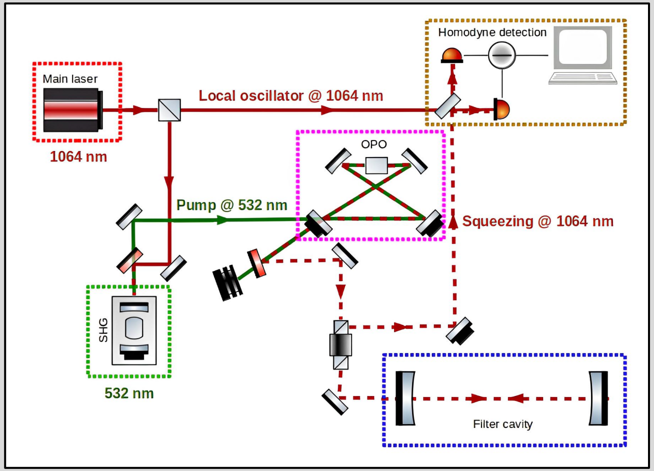

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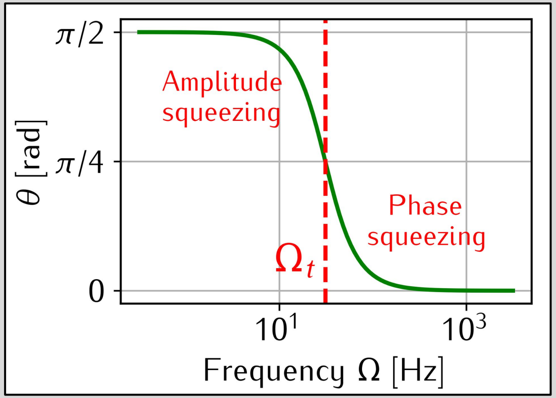

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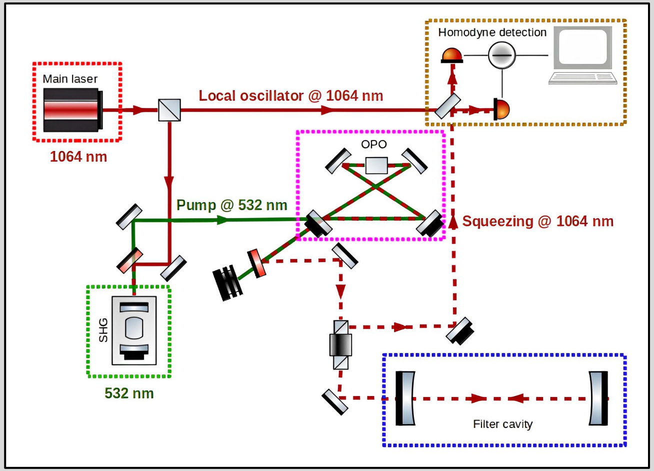

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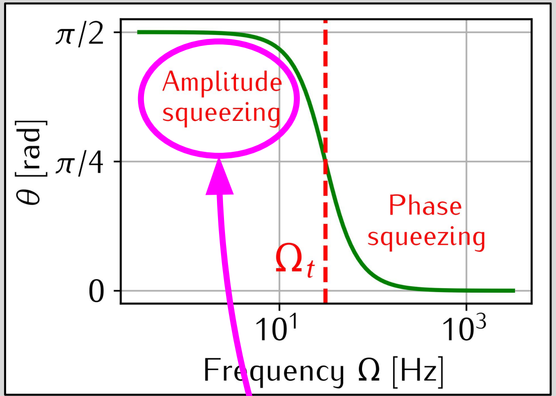

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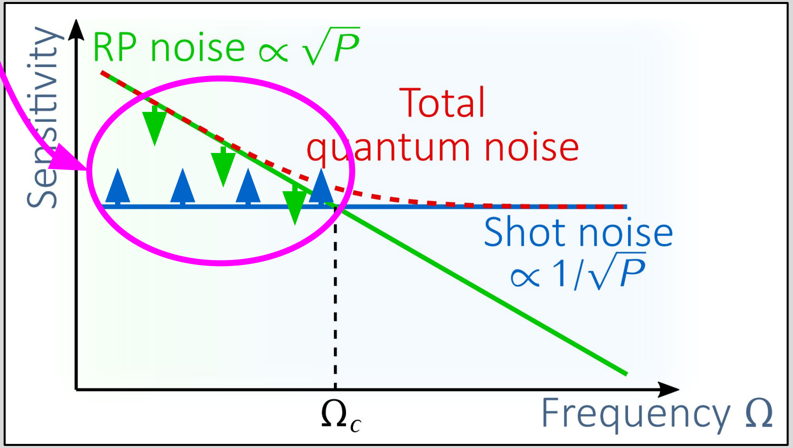

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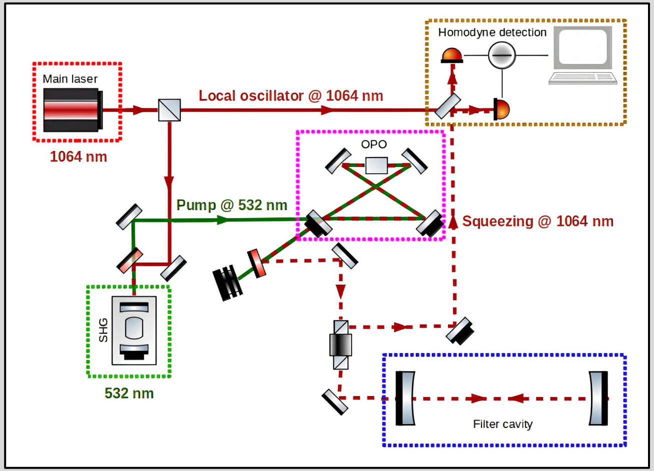

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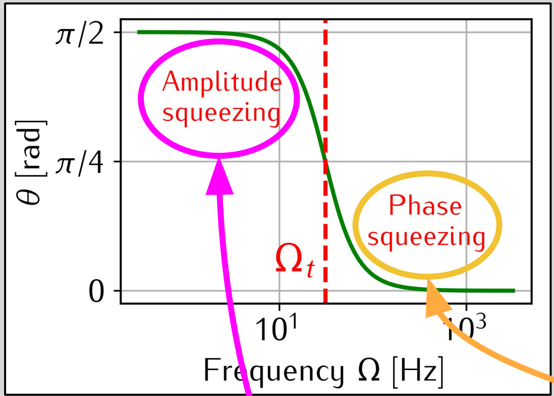

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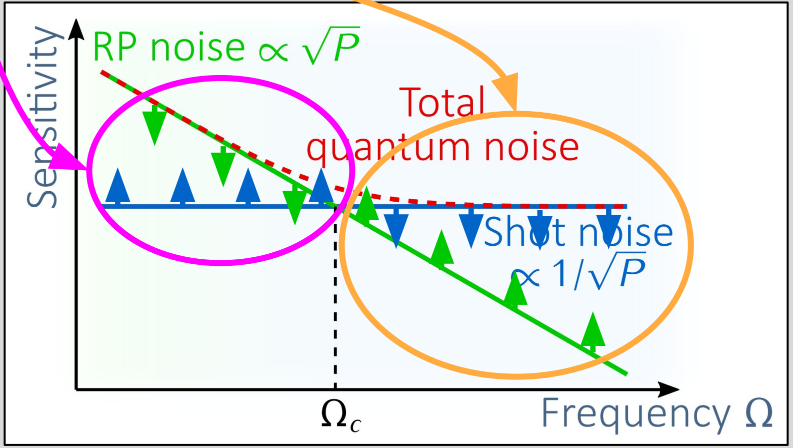
Reduction of radiation pressure (RP) noise below Ω_c,

Reduction of shot noise above Ω_c

⇒ QN reduced at all frequencies



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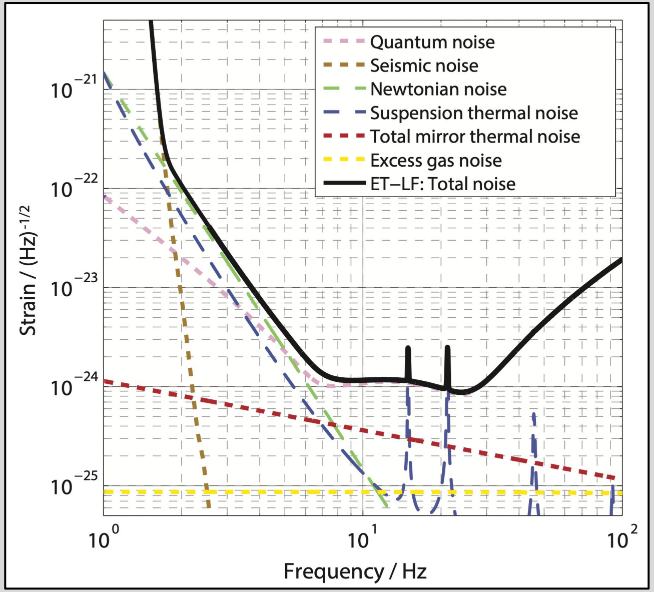
The example of Einstein Telescope - Low Frequency (ET-LF):



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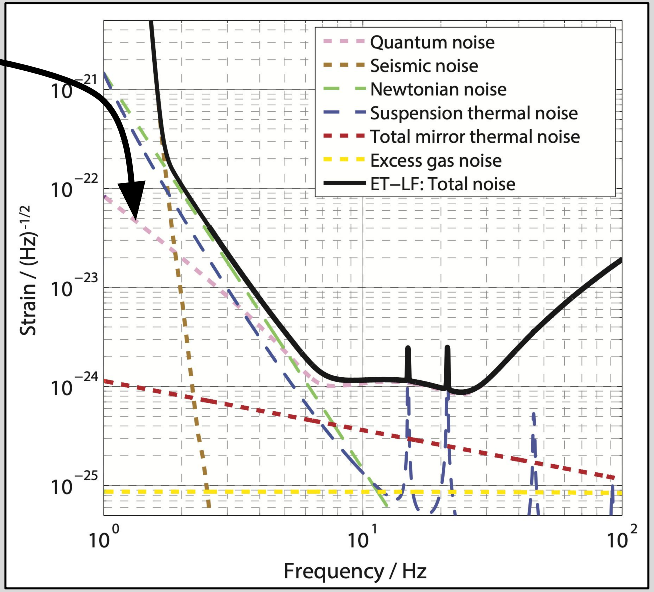
More **complex frequency dependance of quantum noise**
The rotation curve allowed by **a simple cavity is not enough**
Current proposition: **two two-mirror cavities in series**



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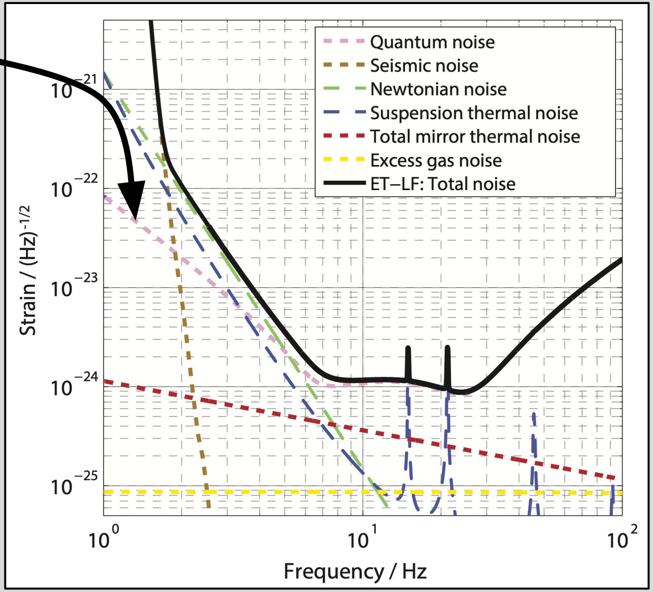

What could be the role of three-mirror cavities (3MC) ?

At this stage, the use of three-mirror cavity for squeezing filtering is an open questions.

To provide answers, we need to understand the squeezing properties in the three-mirror cavity ⇒ First step is to have a complete understanding of three-mirror cavity optical behavior

Three-mirror cavity - model

Modelisation of the three-mirror cavity

Problematic

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Problematic	• Linear three-mirror cavity: two sub-cavities interact together ⇒ despite simple configuration change compared to a two-mirror cavity, **it does not have a trivial/intuitive behavior**

Modelisation of the three-mirror cavity

arXiv: 2406.07752

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Step 1 - Fields propagation through the system

Step 2 - Simulations

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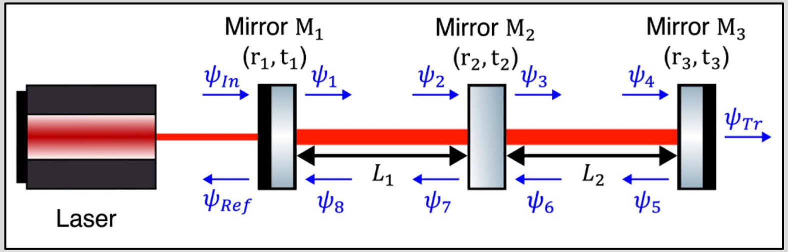

$$\begin{cases} \psi_1 = it_1\psi_{In} + r_1\psi_8 & \psi_2 = \psi_1 e^{-ikL_1} \\ \psi_3 = it_2\psi_2 + r_2\psi_6 & \psi_4 = \psi_3 e^{-ikL_2} \\ \psi_5 = r_3\psi_4 & \psi_6 = \psi_5 e^{-ikL_2} \\ \psi_7 = it_2\psi_6 + r_2\psi_2 & \psi_8 = \psi_7 e^{-ikL_1} \\ \psi_{Ref} = it_1\psi_8 + r_1\psi_{In} & \psi_{Trans} = it_3\psi_4 \end{cases}$$

k: wave-vector; r_i and t_i: reflection and transmission coefficients of mirror "i"

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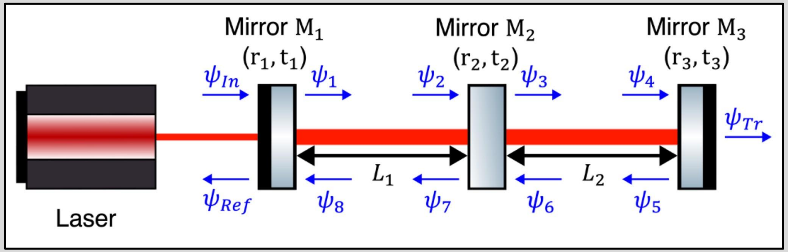

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B - Global reflection and transmission coefficients:

No simple dependency on cavity parameters

$$r = \frac{\psi_{Ref}}{\psi_{In}} = \frac{r_1 e^{2ik(L_1+L_2)} - r_1 r_2 r_3 e^{2ikL_1} - r_2(r_1^2+t_1^2)e^{2ikL_2} + r_3(r_1^2+t_1^2)(r_2^2+t_2^2)}{e^{2ik(L_1+L_2)} - r_1 r_2 e^{2ikL_2} - r_2 r_3 e^{2ikL_1} + r_1 r_3 (r_2^2+t_2^2)}$$

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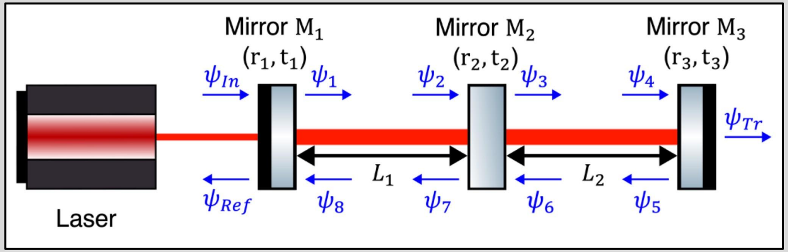

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Step 2 - Simulations

Implement **global transmission coefficient** in a code

Parameters to vary:

- **Laser wavelength (wave-vector)**
- **First, second and third mirrors transmission coefficients**
- **L_1 and L_2 distances**

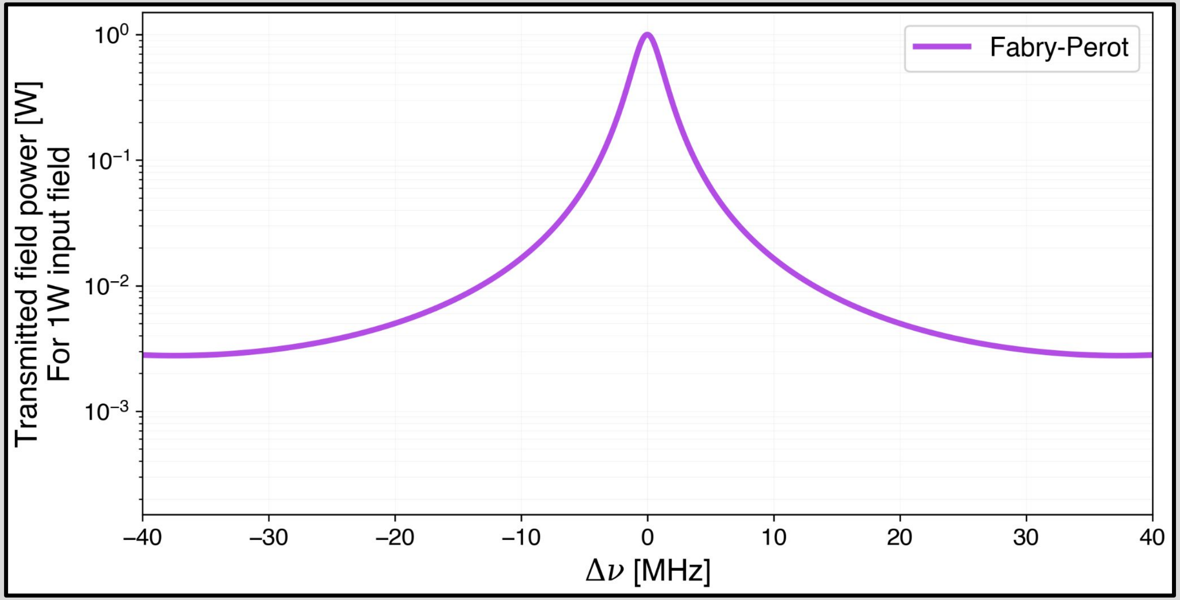
Three-mirror cavity resonance properties

arXiv: 2406.07752

Three-mirror cavity - model

Study of the **transmitted field as function of laser detuning** compared to resonant frequency

Reference: simulation of a two-mirror Fabry-Perot cavity



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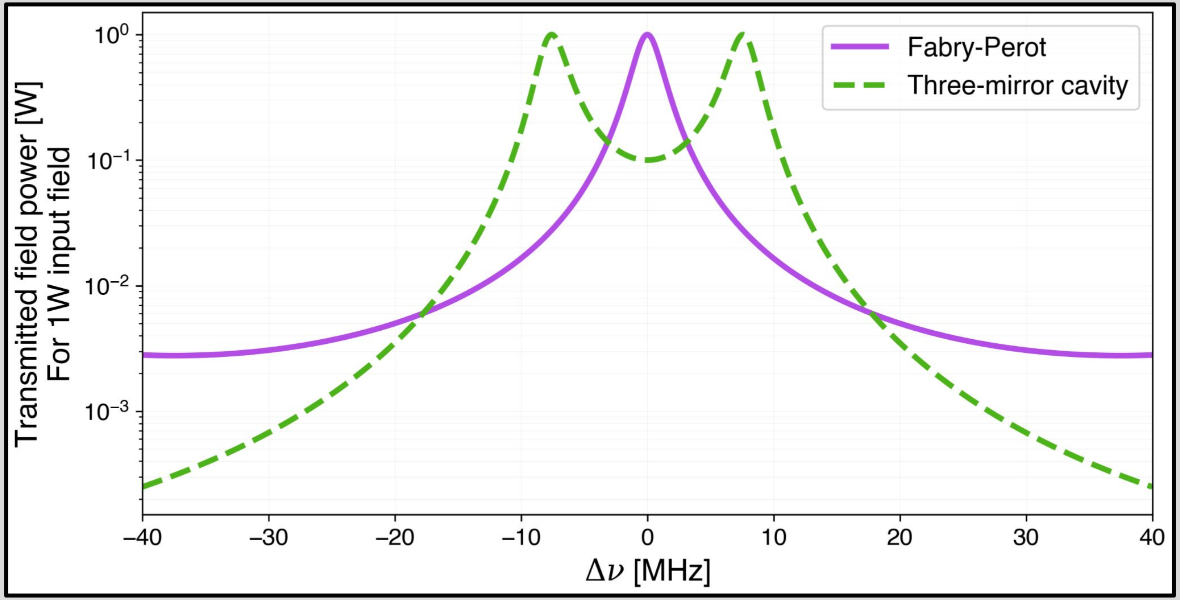
Result: simulation of the three-mirror cavity shows a **doubling of the transmission peak** caused by the coupling between sub-cavities



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Three-mirror cavity $R_1 = R_2 = R_3 = 0.9$, $L_1 = L_2 = 0.5$m.

How this "double-peak" shape varies as function of cavity parameters ?

Double-peak modulation

Simulation

Initial configuration :
$\lambda_{Laser} = 1064nm$
$R_1 = R_2 = R_3 = 0.9$
$L_1 = L_2 = 0.5m$

From initial configuration, we can vary:

Middle mirror transmissivity

Input/end mirror transmissivity

Microscopic spacing of one sub-cavity

Three-mirror cavity - model

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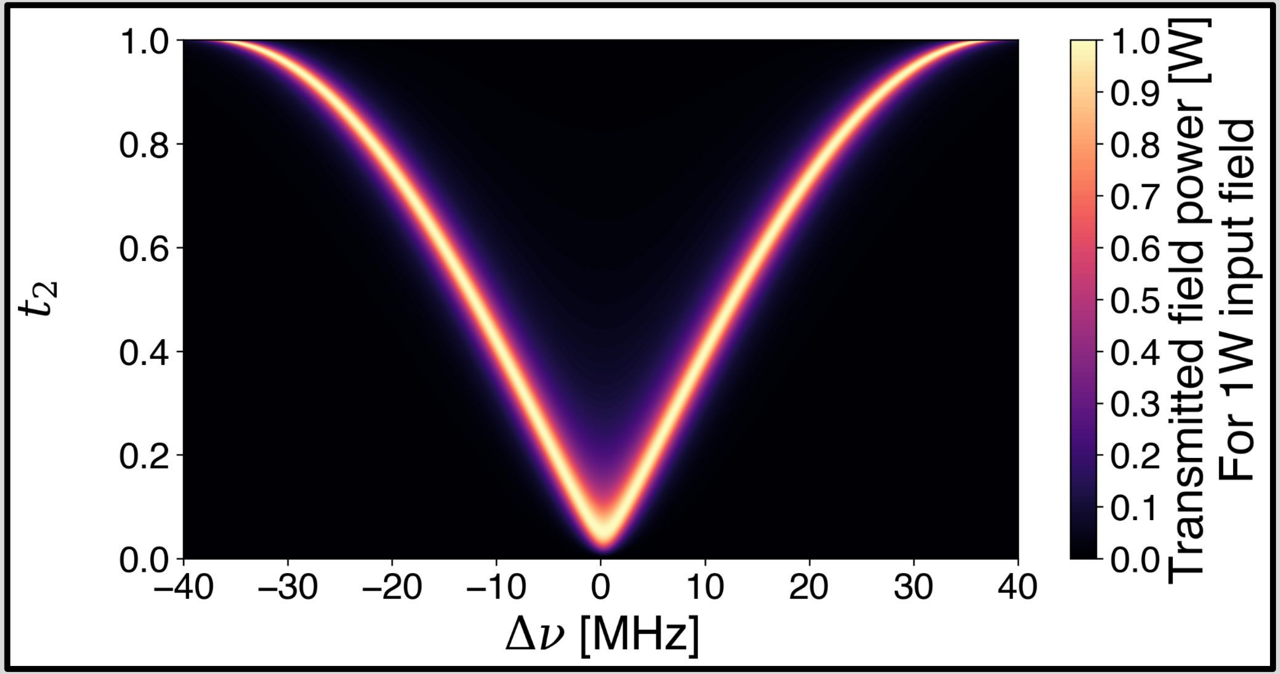

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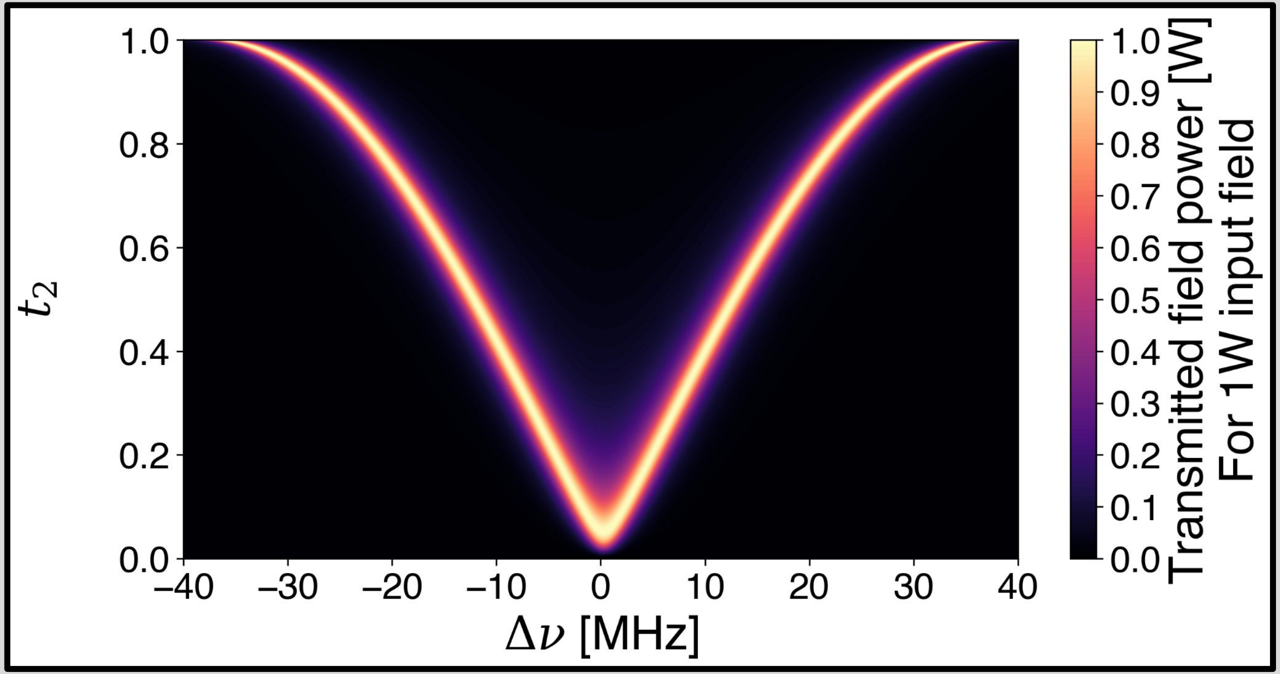

⇒ Symmetrical variation of maxima spacing

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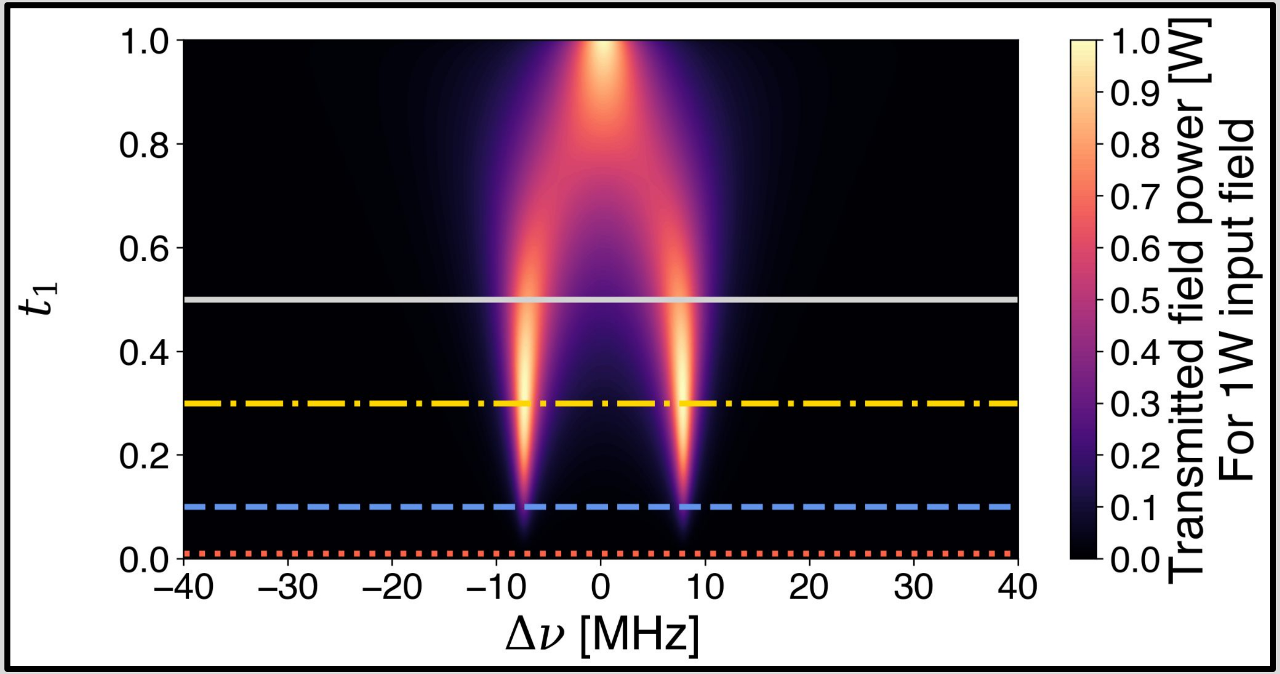

⇒ Intrinsic maxima with variation

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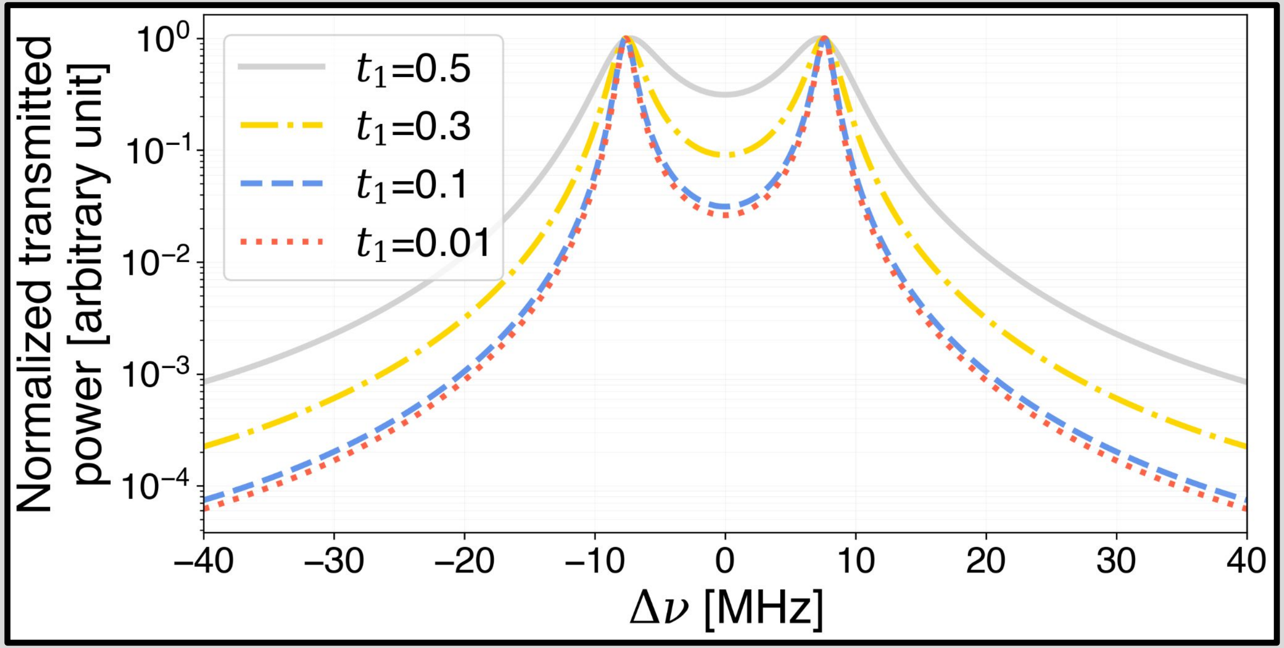

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Complementary role: middle mirror change position without changing the maxima with/input and end mirror do the exact opposite

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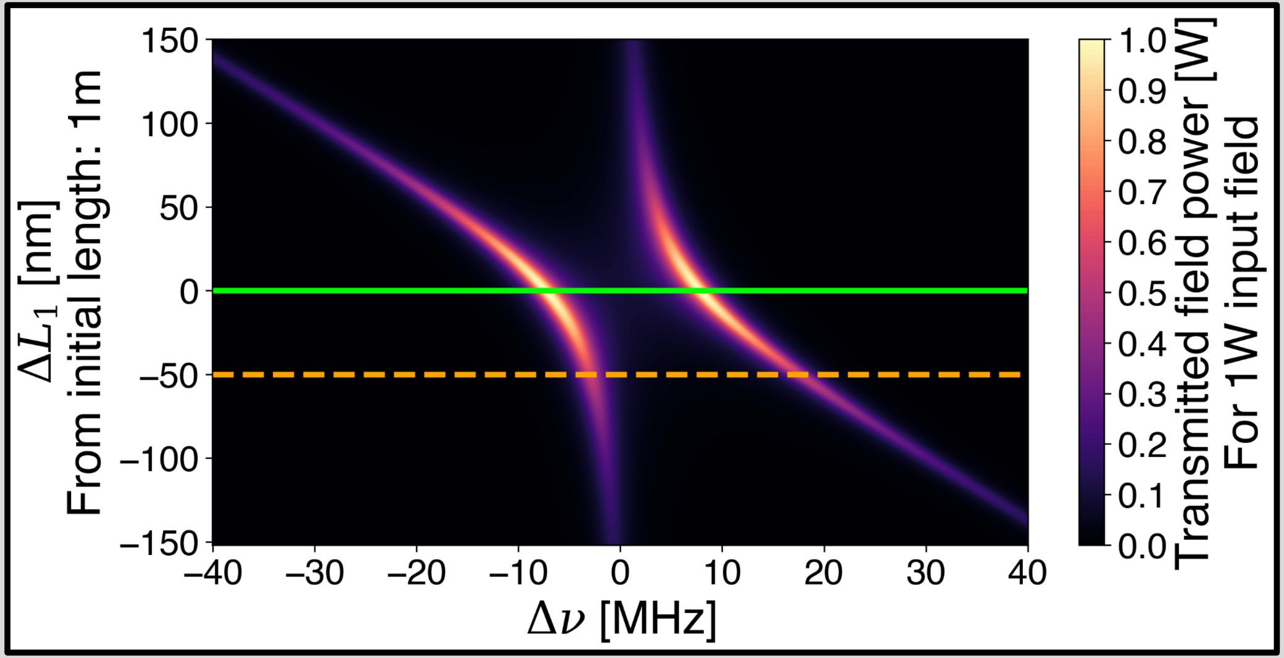

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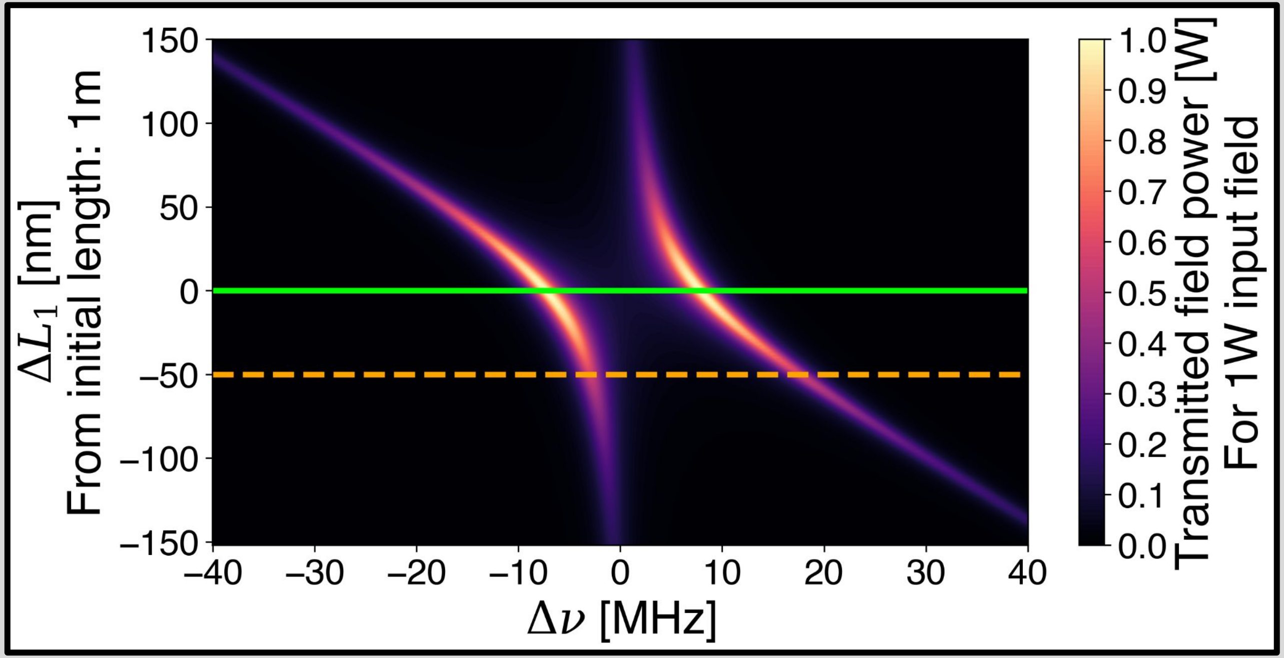

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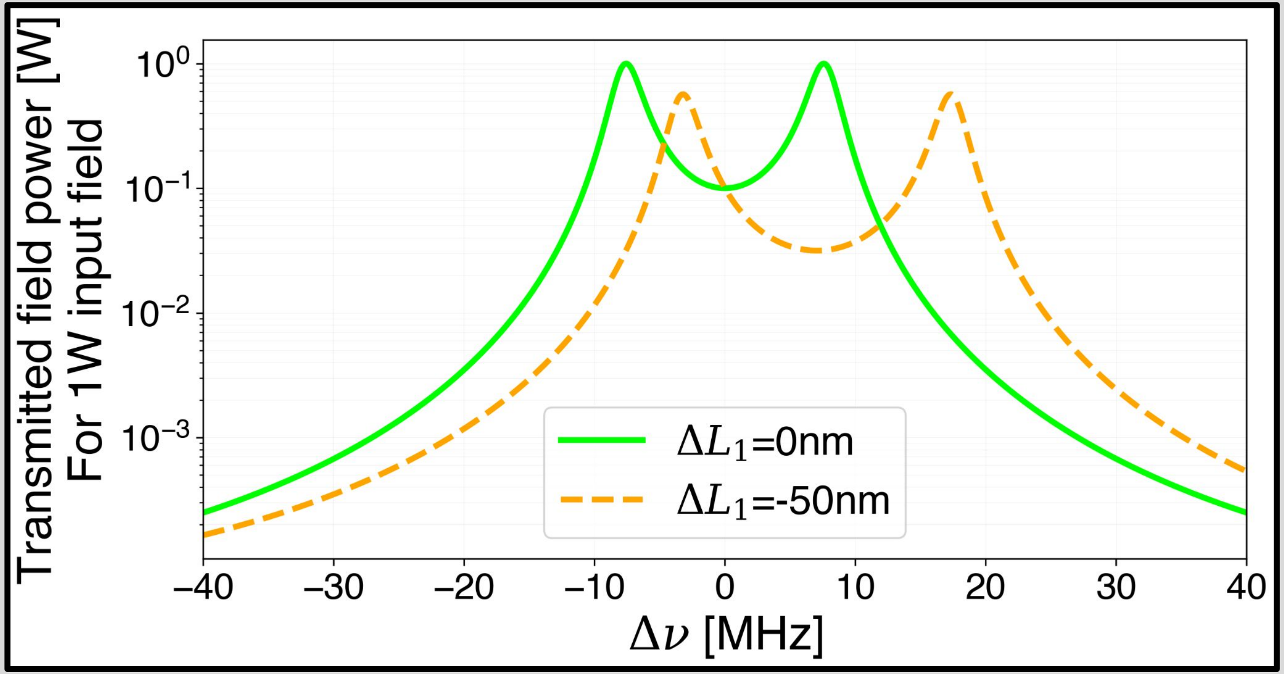

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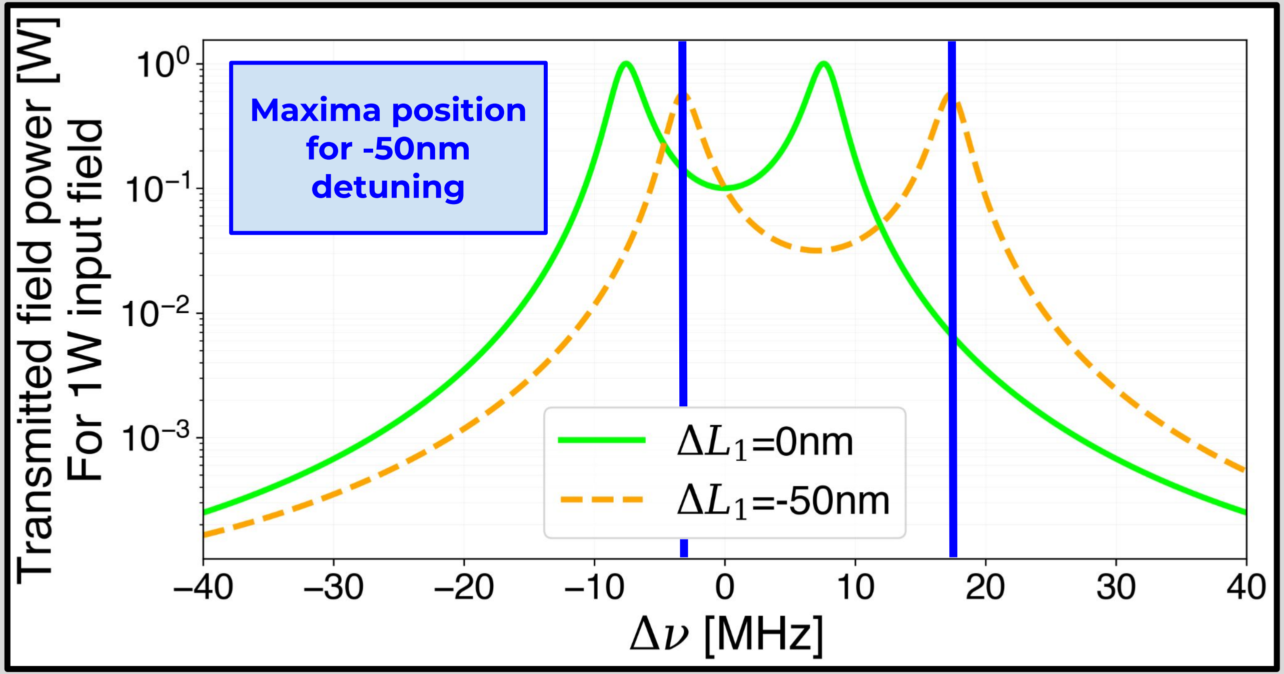

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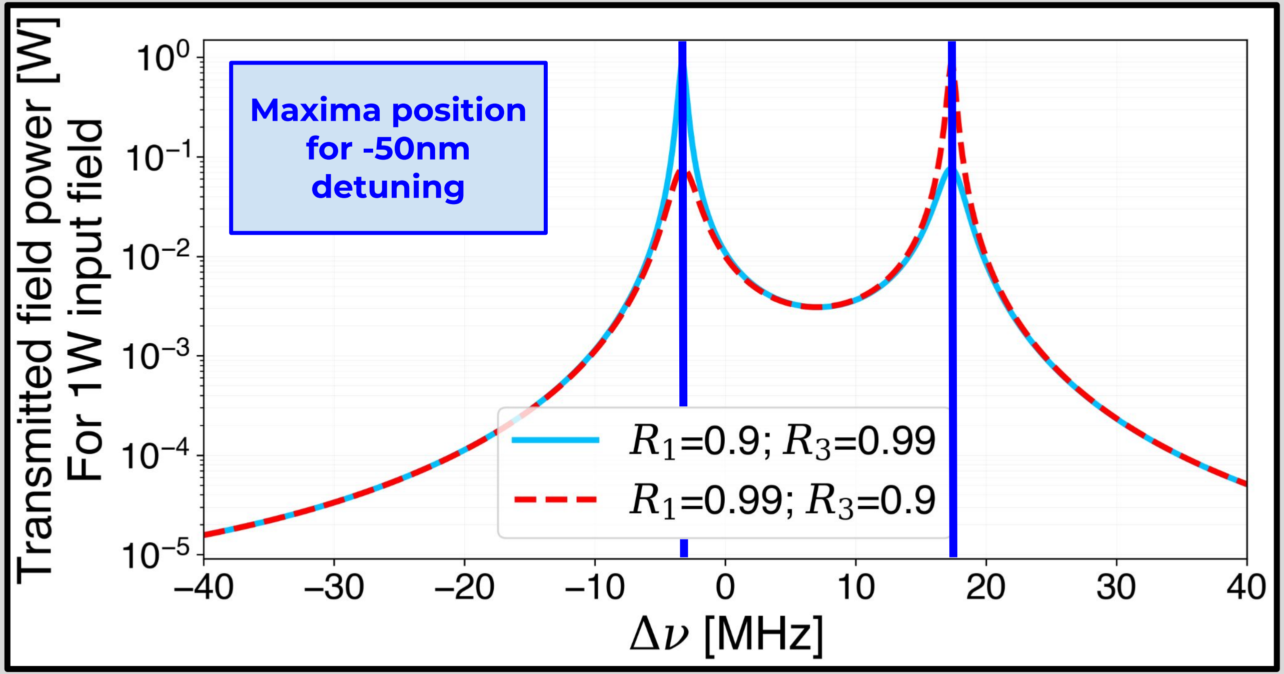

⇒ From a microscopically asymmetrical cavity, it is possible to induce an asymmetry between maxima height by <u>changing the ratio R1/R3</u>

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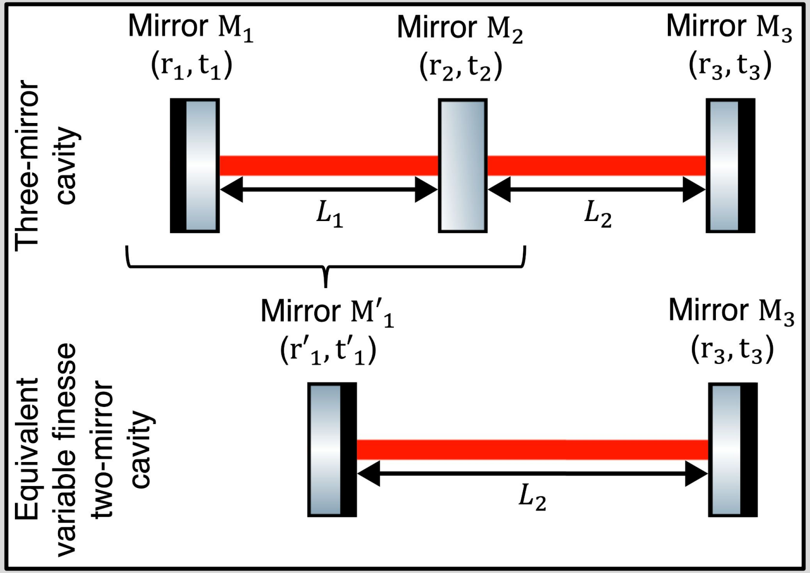
Combining mirrors transmissivity/cavity length it is possible to drastically change the shape of the double-peak: almost complete control on resonance properties

Variable finesse

We can consider the first sub cavity as an **equivalent mirror having properties (r'_1,t'_1) that depend on L_1**



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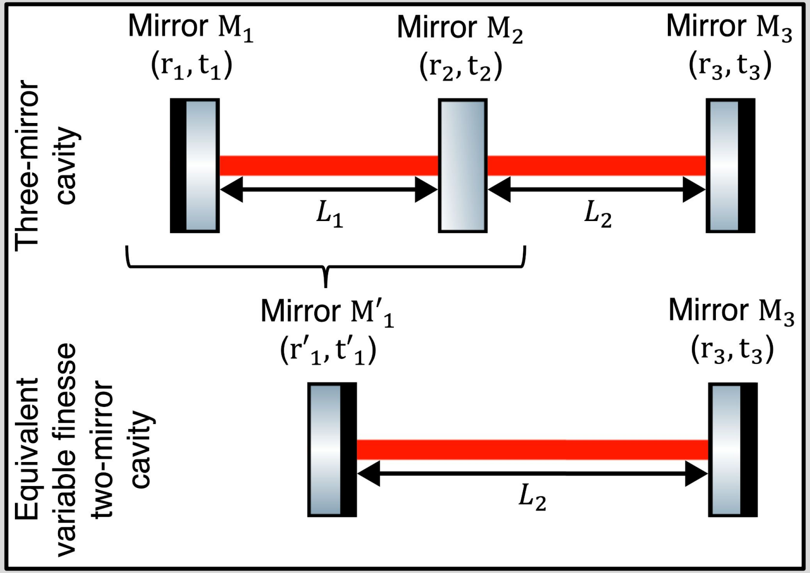
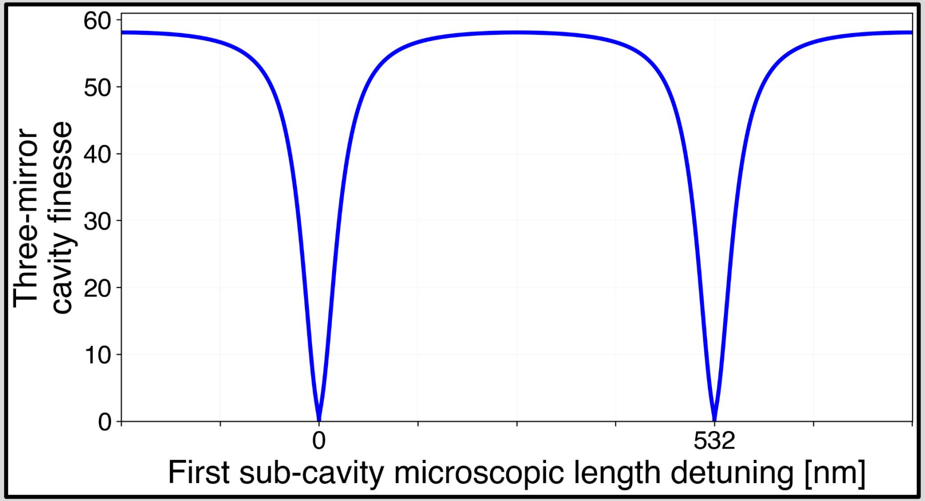




Simulation parameters: $R_1 = R_2 = R_3 = 0.9$, $\lambda_{Laser} = 1064nm$

First sub-cavity:

Resonant ⇒ Finesse minimum
Anti-resonant ⇒ Finesse maximum

Enough theory…

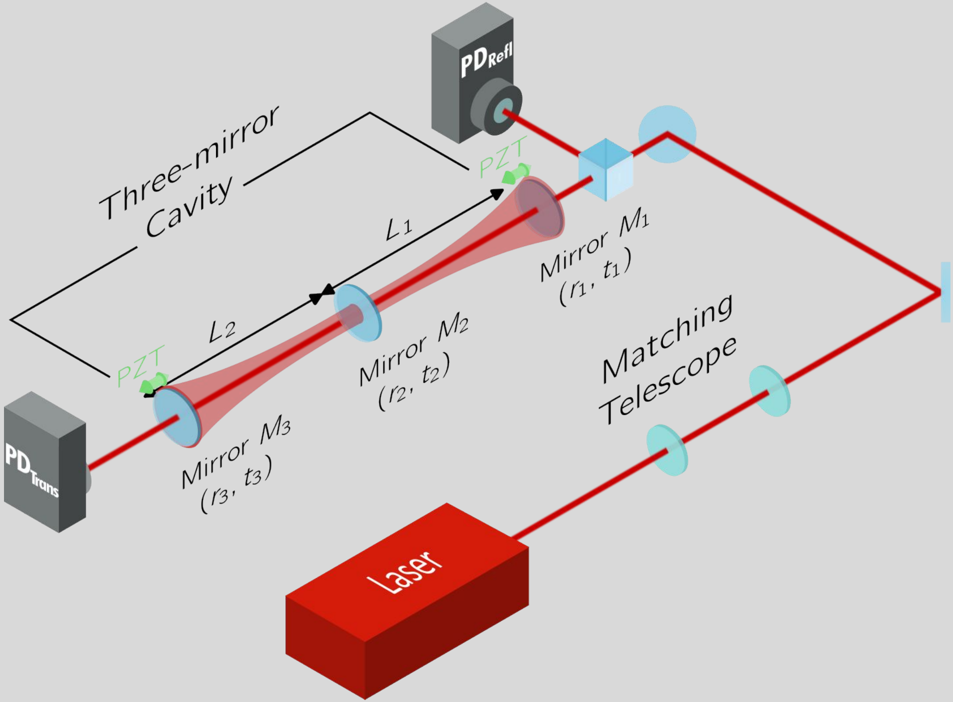
Does it work in practice ?

Three-mirror cavity - prototype at *IJCLab (Orsay - France)*

Prototype configuration

Laser source: λ_{Laser} = 1064nm

Each extremal mirror is placed on an independant piezoelectric stage



Prototype configuration

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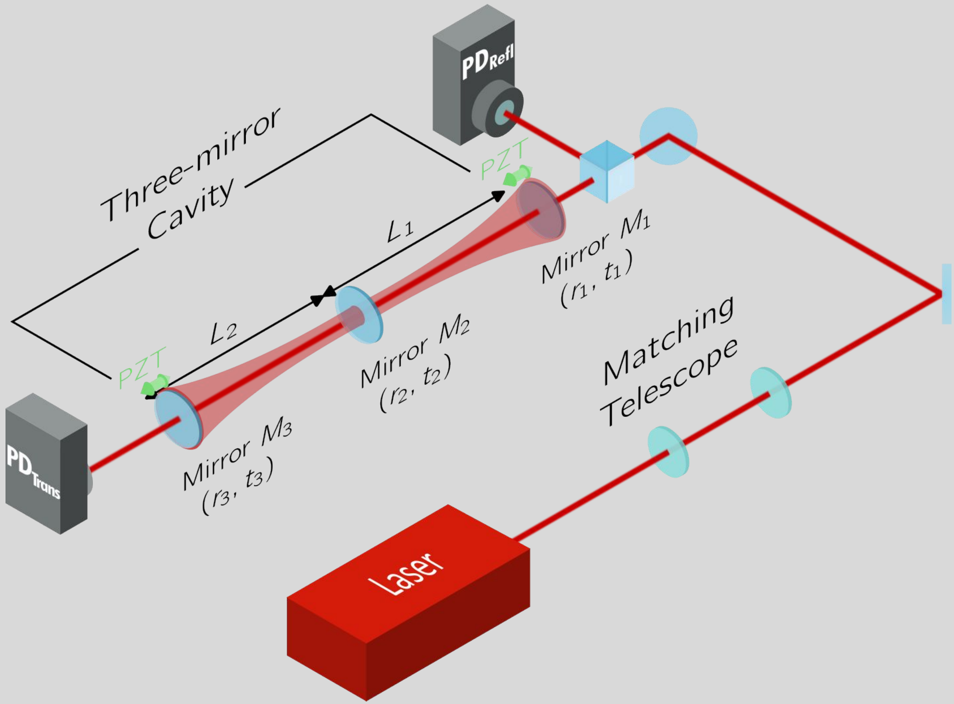
Each extremal mirror is placed on an independant piezoelectric stage

We test **two different three-mirror cavity setup** having symmetrical/asymmetrical sub-cavities length

Setup A	Setup B
Symmetrical configuration	**Asymmetrical configuration**
$R_1 = R_2 = R_3 = 0.9$	$R_1 = R_2 = R_3 = 0.9$
$L_1 = L_2 = 500$mm	$L_1 = 256$mm, $L_2 = 161$mm
$RoC_1 = RoC_3 = 1000$mm	$RoC_1 = RoC_2 = 150$mm
$RoC_2 = \infty$	$RoC_3 = 250$mm



Ongoing experiment... A few preliminary results

Three-mirror cavity - prototype

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Double peak

Method: Send the exact same ramp to each piezo so that each sub-cavity contracts/expands by the same amount ⇒ equivalent to scan the laser frequency

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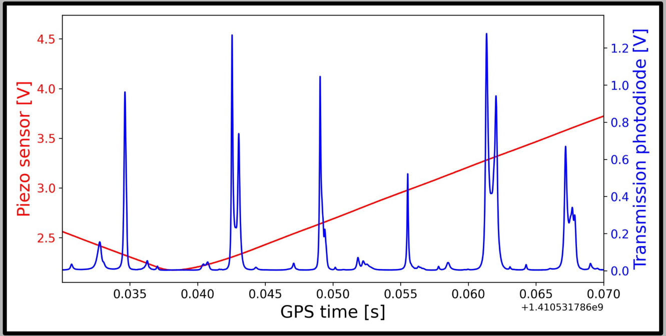

Double-peaks have been seen !

We are currently analysing the data

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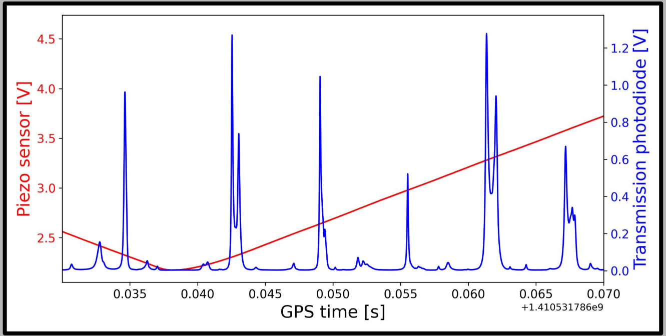

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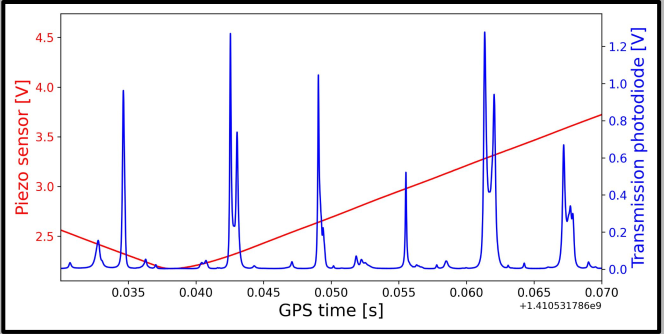

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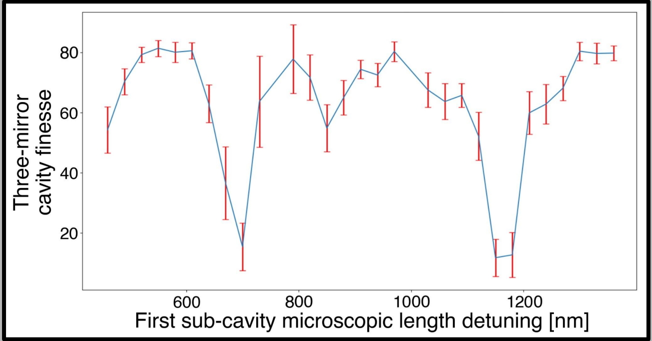

Results from 6 weekends internship by 3rd year students: two regions separated by ~500nm where the finesse is dropping

We are doing a second data analysis to reduce uncertainties

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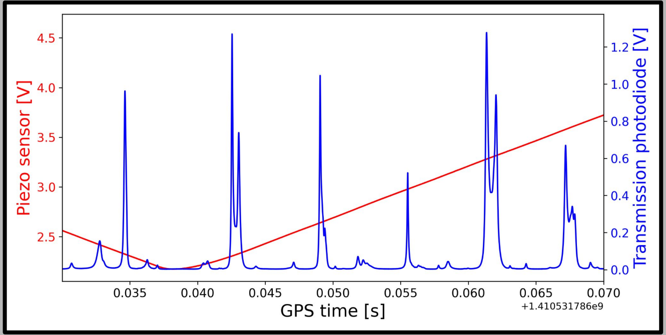

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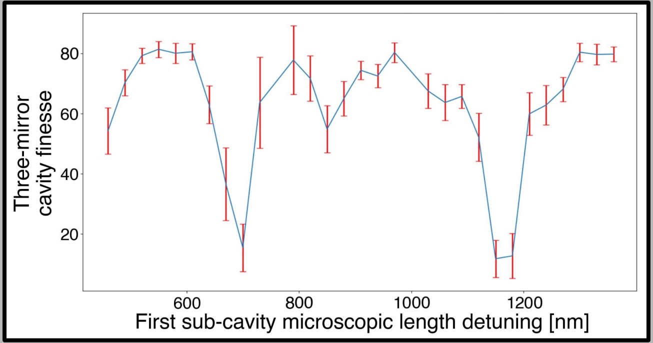

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Paper in preparation

Summary

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Three-mirror cavity model

- **Three-mirror cavities have resonant properties that are more modulable** by design than two-mirror cavities
- **Real-time tuning** of the system
- Need to pay attention to the cavity geometry for stability reasons (not presented here)
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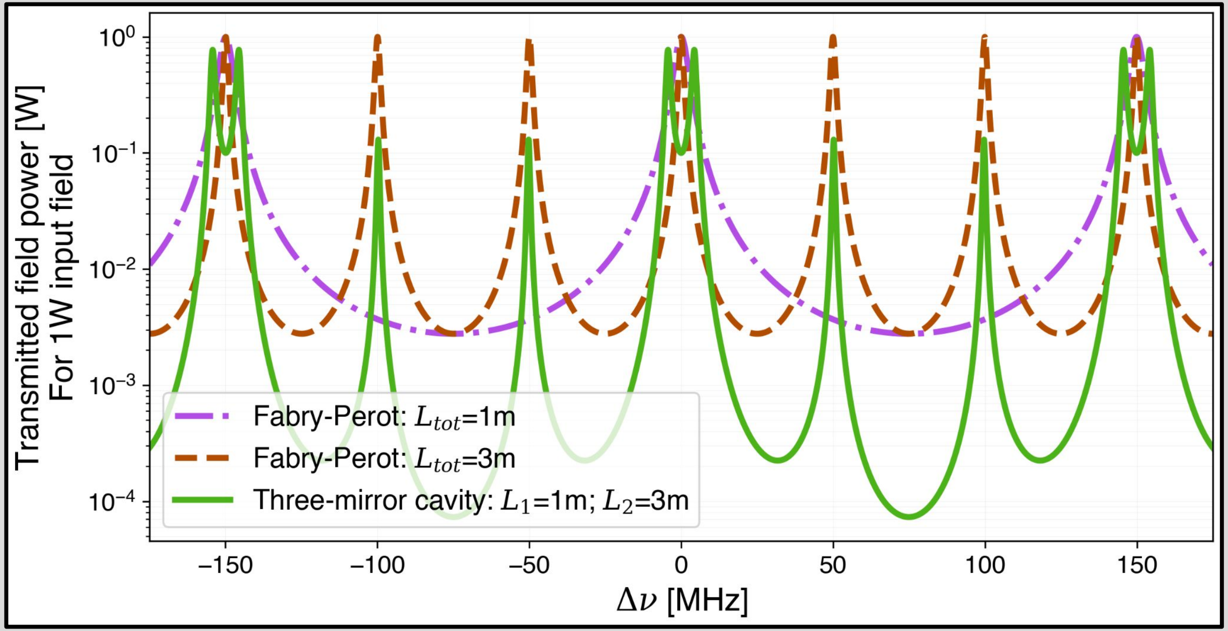
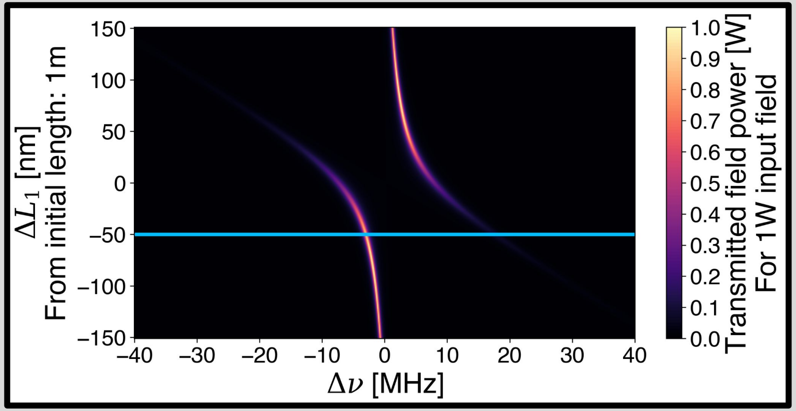
Thanks to its adaptability, three-mirror cavity could be a powerful device for the optimization of squeezing filtering especially in the case of Einstein Telescope

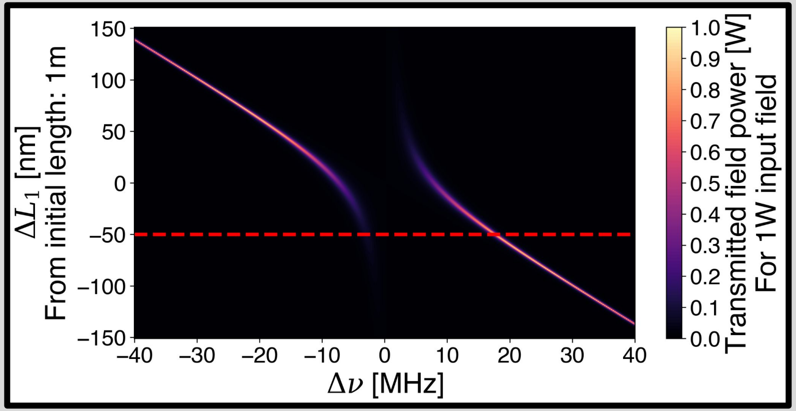
Backup slides

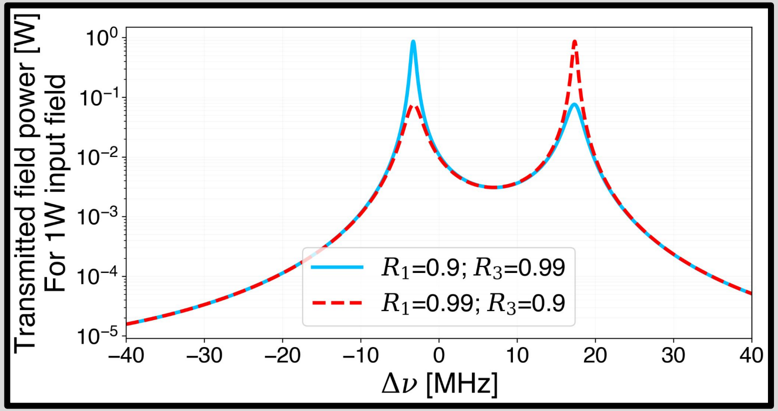
Condition for doubling of transmission peak



Double-peak modulation







Macroscopic mirrors spacing