Machine-Learning (ML) enhanced Quantum State Tomography (QST) and its applications to the Gravitational Wave Detectors

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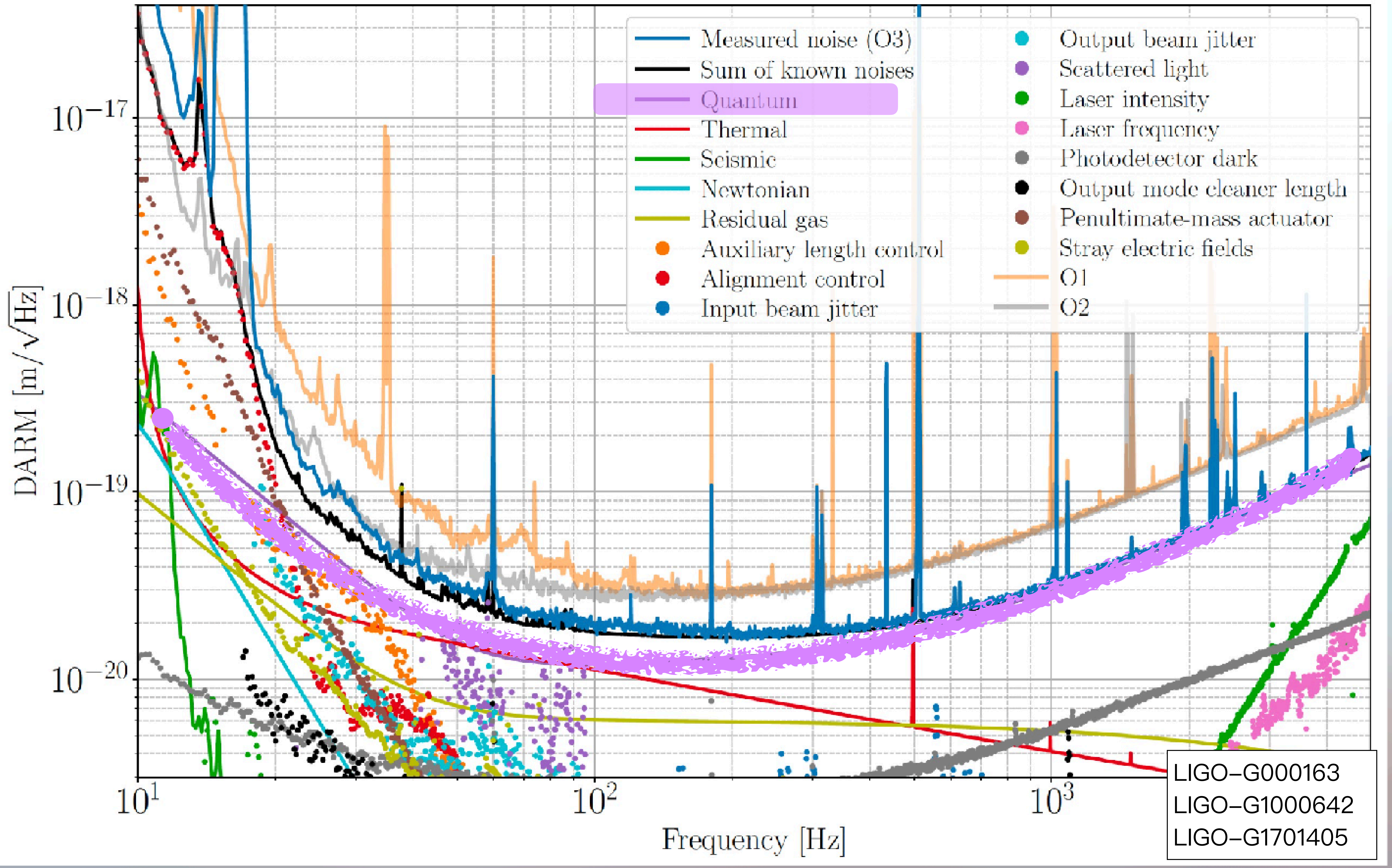
LIGO-Virgo-KAGRA collaboration


ICSSUR 2023
International Conference on Squeezes States and Uncertainty Relations
26 - 30 June 2023 Taipei, Taiwan
Zi-Hao Shi
Yi-Ru Chen
Chien-Ming Wu
Hsien-Yi Hsieh
Hua Li Chen
2023 ICSSUR 26 - 30 June 2023 in Taipei, Taiwan
Ray-Kuang Lee's LAB
Institute of Photonic Technology, National Tsing Hua University

Noise Budget:



LIGO–G000163
LIGO–G1000642
LIGO–G1701405

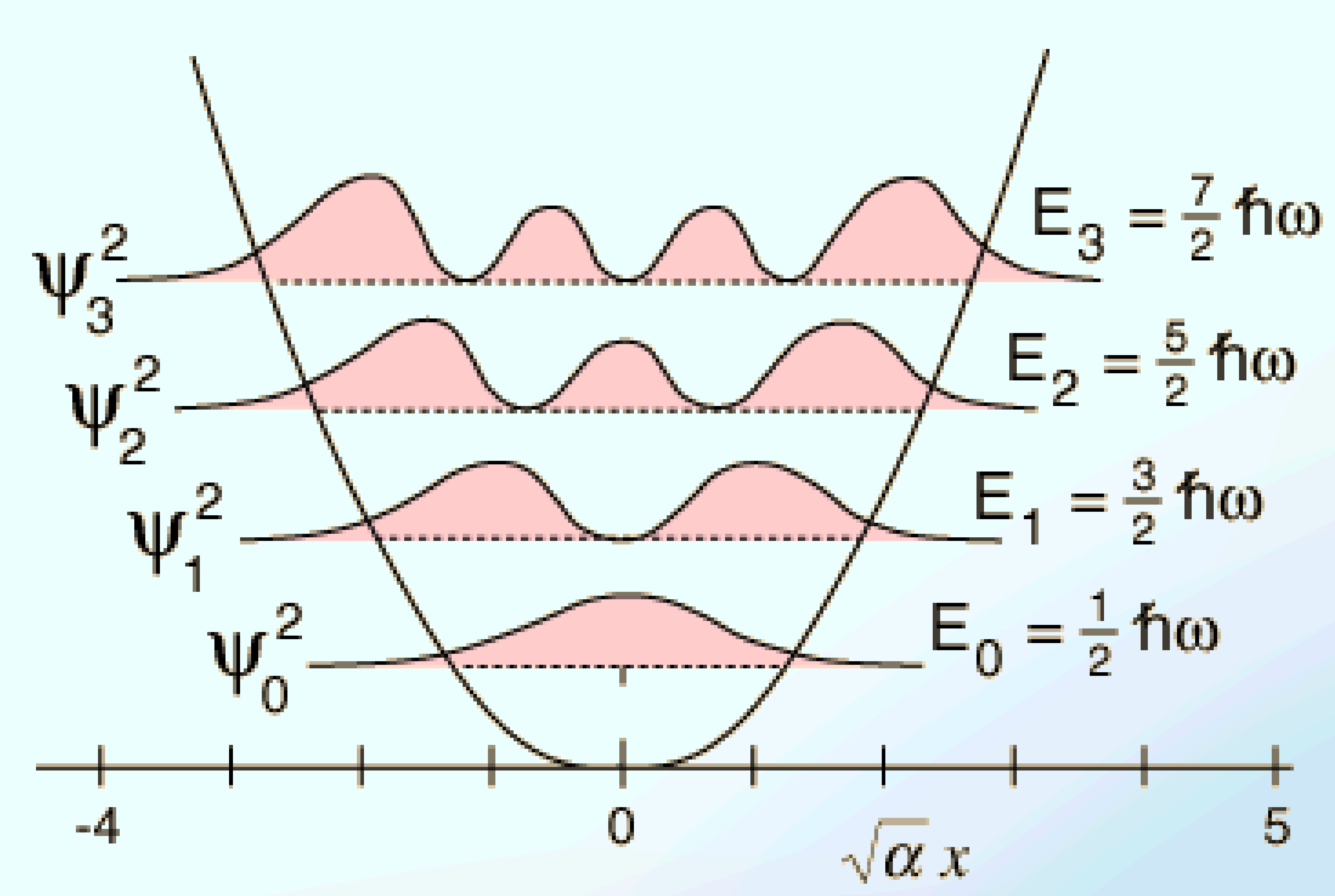
Quantum Simple Harmonic Oscillator (SHO):

$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\hat{x}^2, \quad [\hat{x},\hat{p}] = i\hbar,$$
$$= \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}), \quad [\hat{a},\hat{a}^\dagger] = 1,$$

$$\hat{N}|n\rangle = n|n\rangle,$$
$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle,$$
$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle,$$
$$E_n = \hbar\omega(n+\frac{1}{2}).$$

annihilation subtraction

creation addition



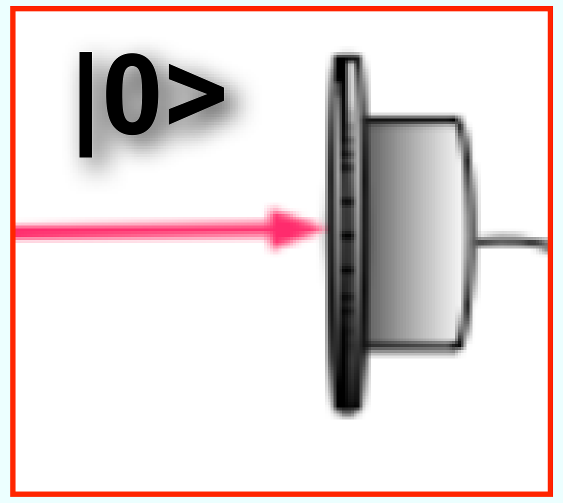
Number (Fock) States |n>

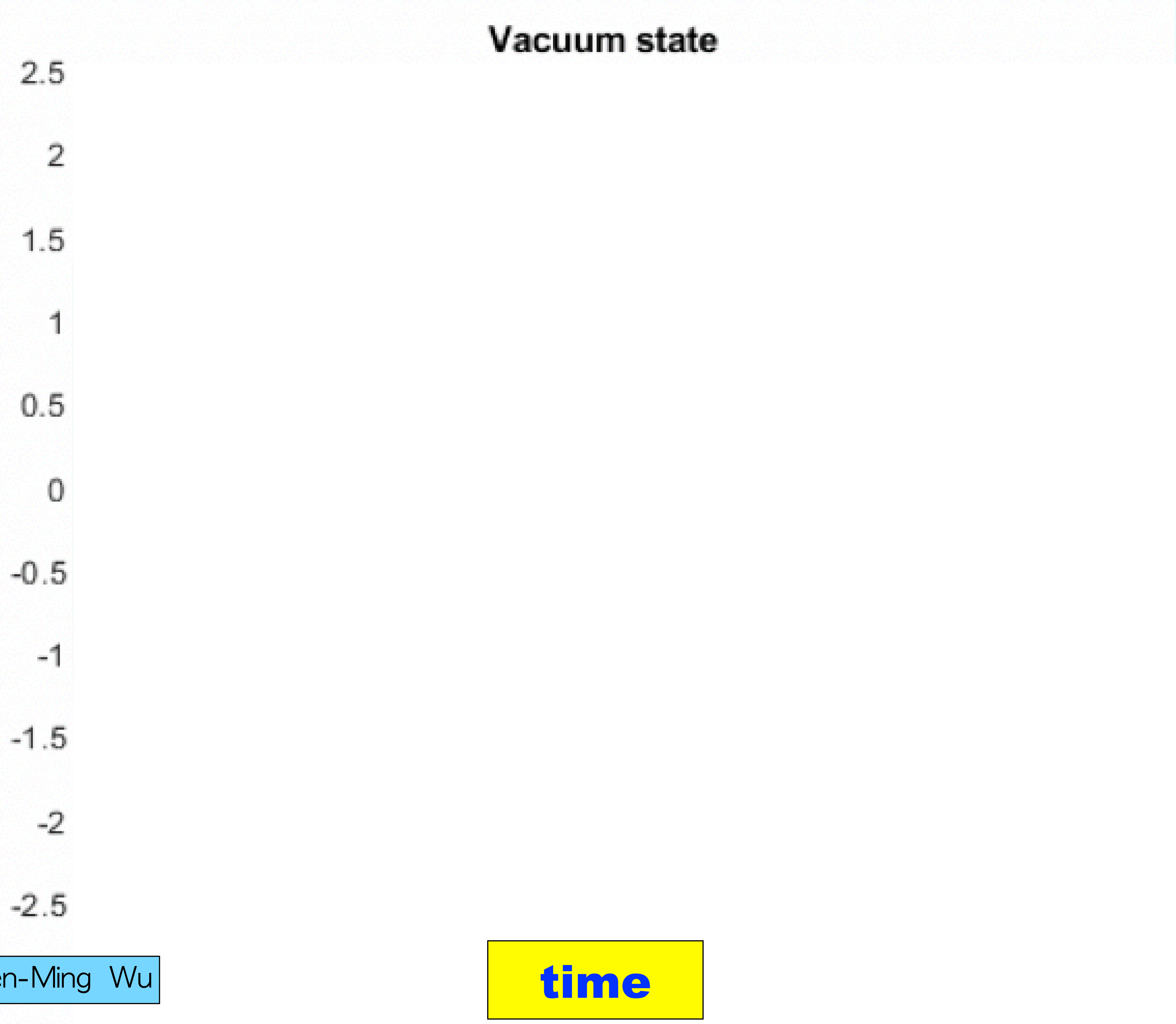
Vacuum States |0>

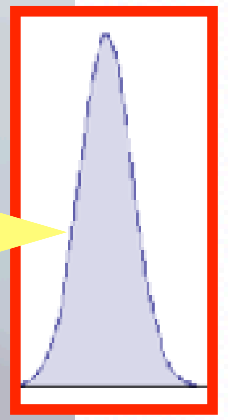
$$\langle x|n\rangle = \psi(x) = H_n(x)\exp[-x^2/2],$$

- Energy quantization
- Equally spacing in energy difference
- Zero-point energy $\neq 0$

Vacuum State: |0>



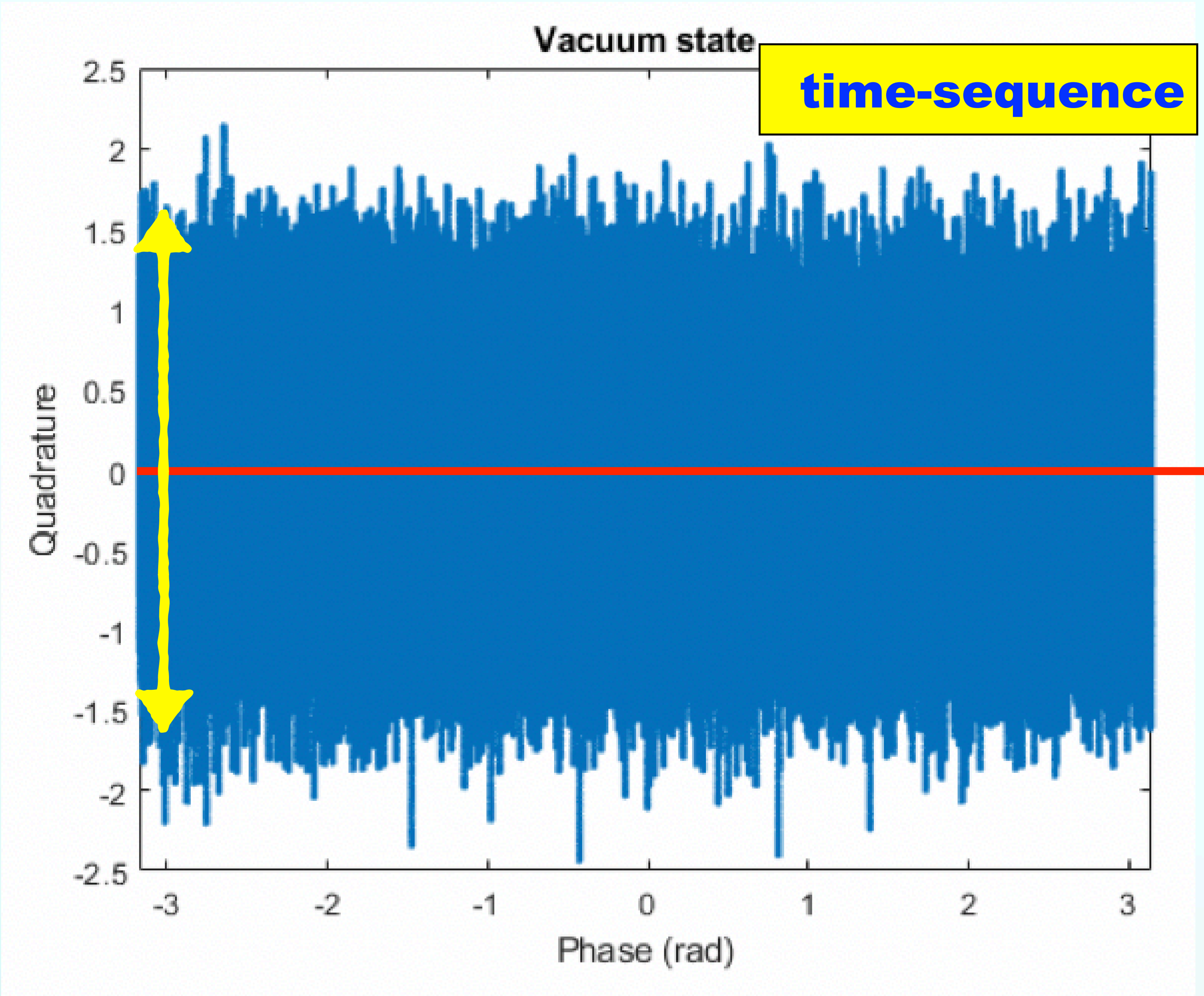


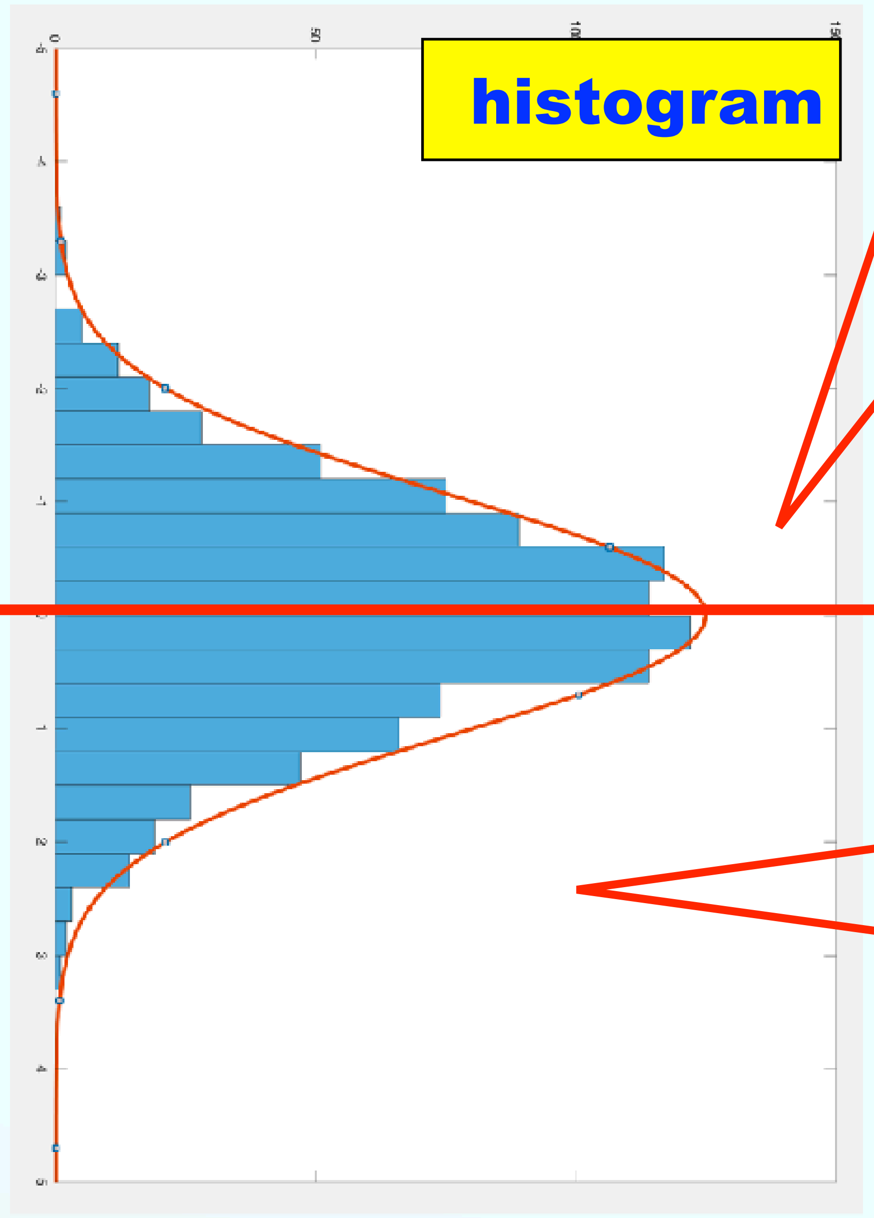


wave-nature

with Yi-Ru Chen, Chien-Ming Wu

Vacuum State: |0>





with Zero Mean

$$E_0 = \hbar\omega/2$$

Zero-Point Energy

Gaussian wave-package

$$\Psi(x) \quad = \quad < x|0 >= C\, exp[-x^2/\Delta x^2]$$

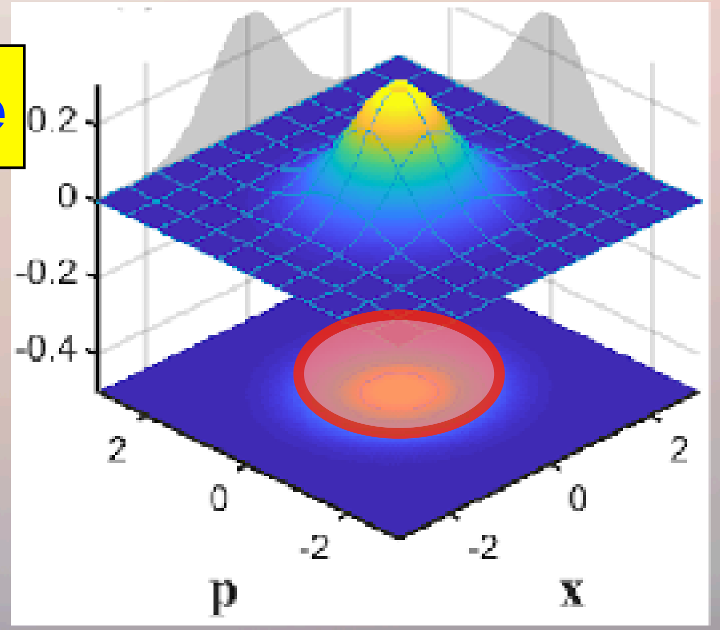
$$\tilde{\Psi}(p) \quad = \quad < p|0 >= C\, exp[-\Delta x^2\, p^2]$$

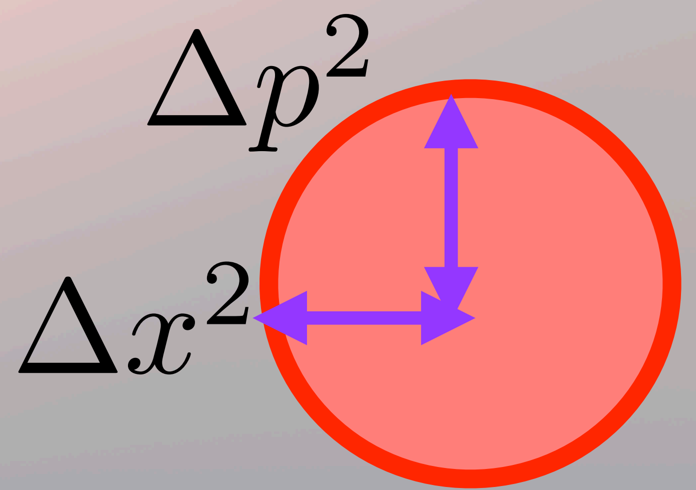
Uncertainty-Relation

$$\Delta x^2 \times \Delta p^2 \geq \frac{\hbar^2}{4}$$

- Planck constant: $\hbar$
- Discrete Energy levels:
- Quantum states：| **Ψ** >
- Wave-function
- Probability distribution
- Wave-Particle Duality
- Uncertainty Relation
- Vacuum fluctuation

Phase Space

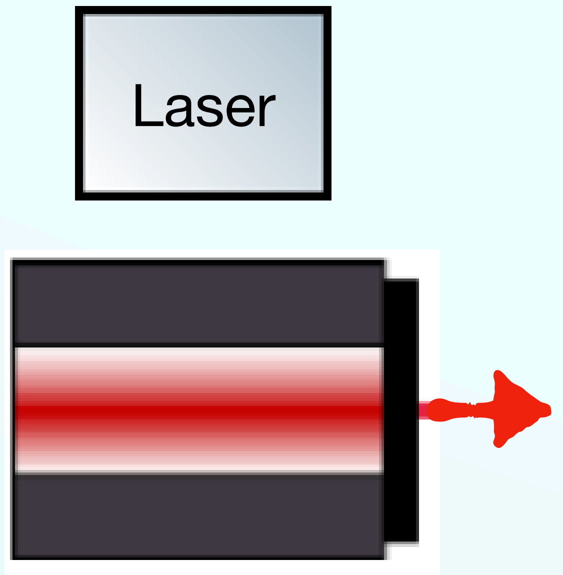


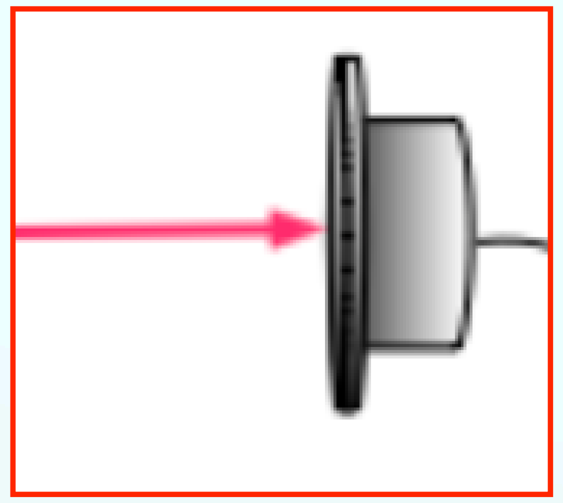


circle

Poisson Photon Number Distributions:

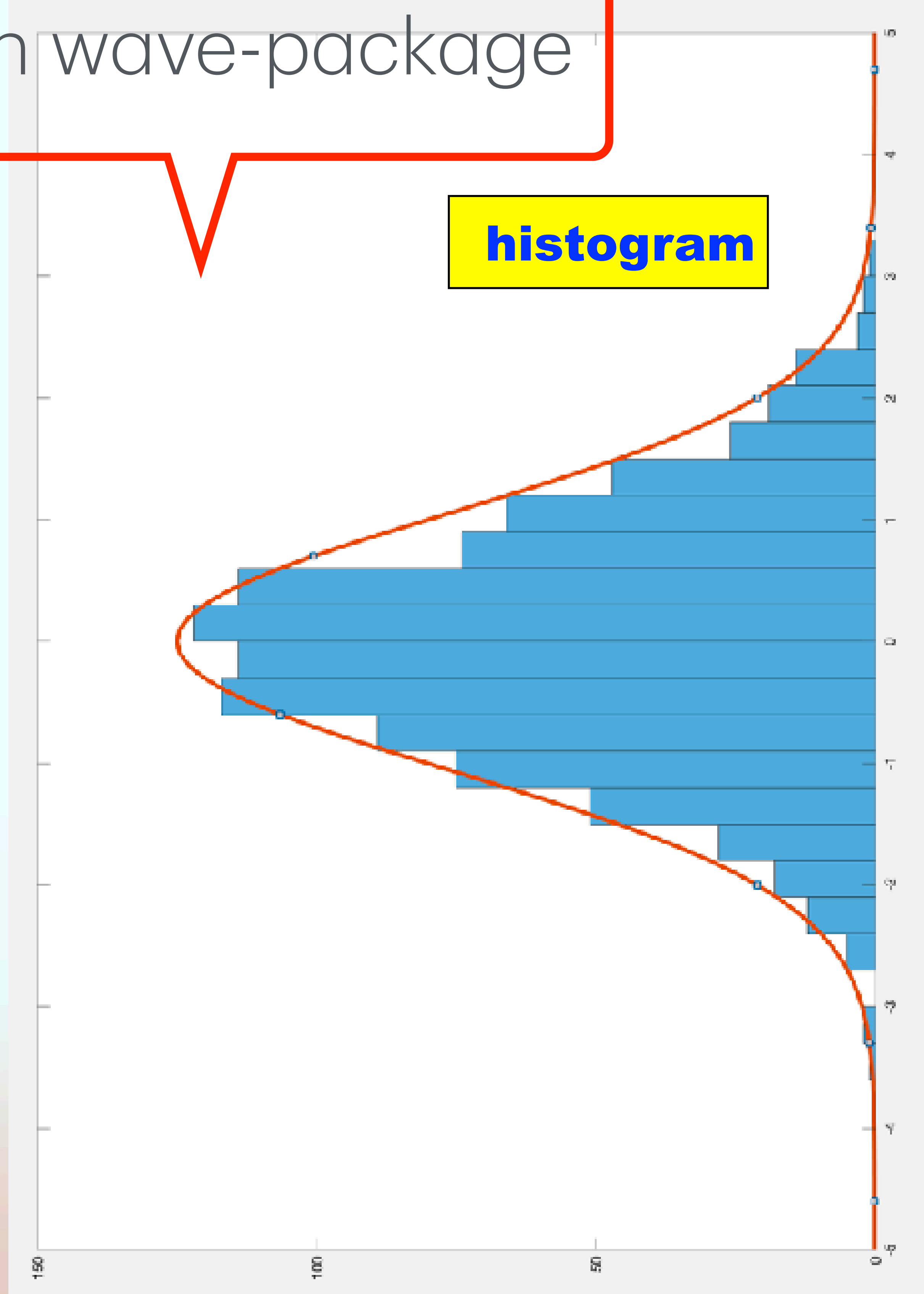
coherent states



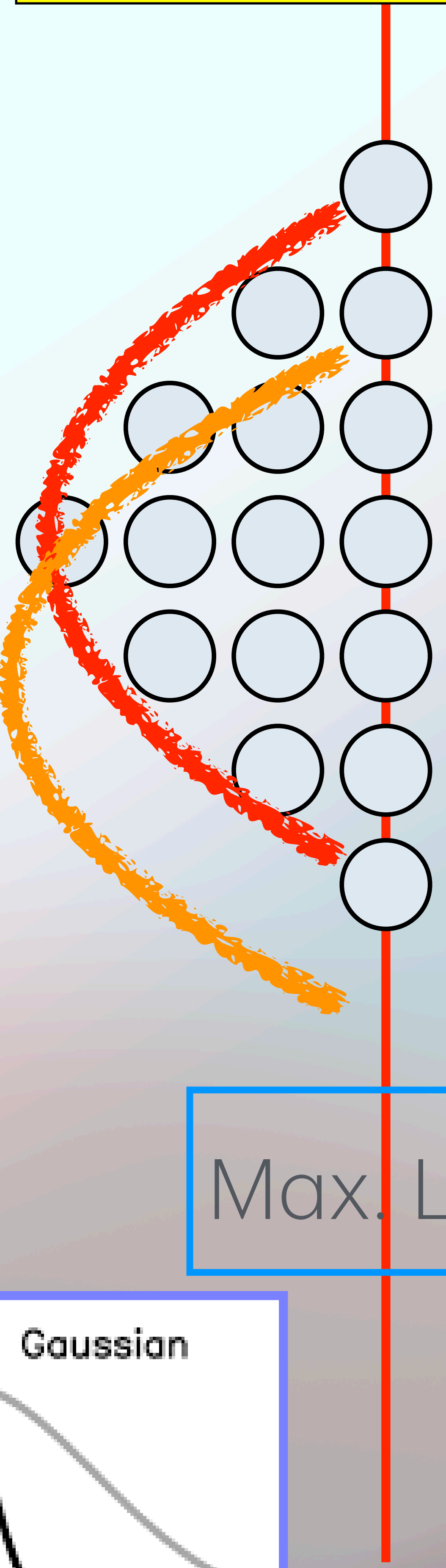


Gaussian wave-package

histogram



x, Position
p, Momentum
phase/time



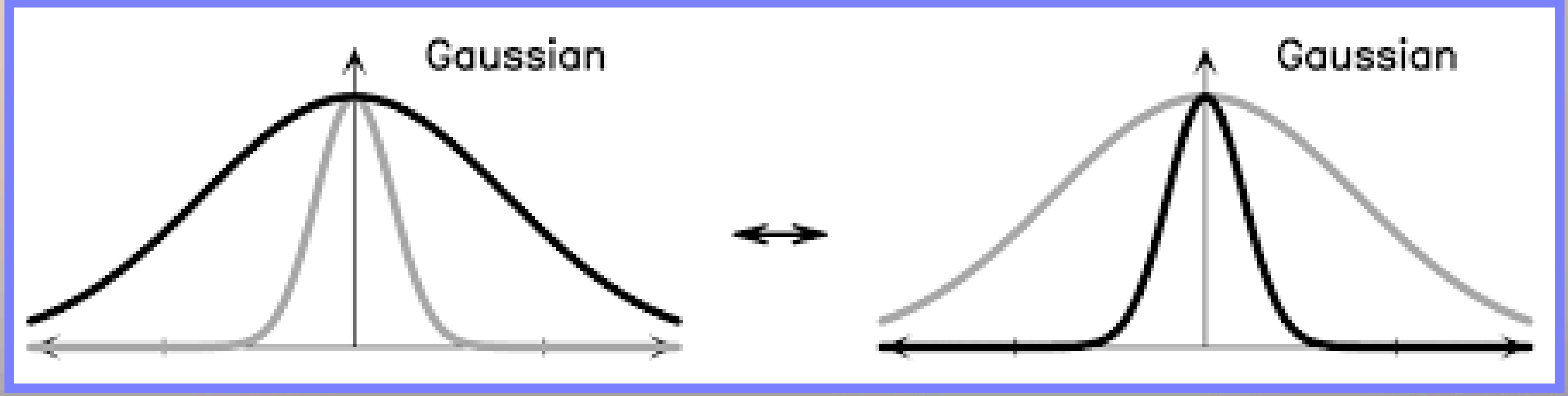
Poisson (Normal) Distribution

Max. Likelihood

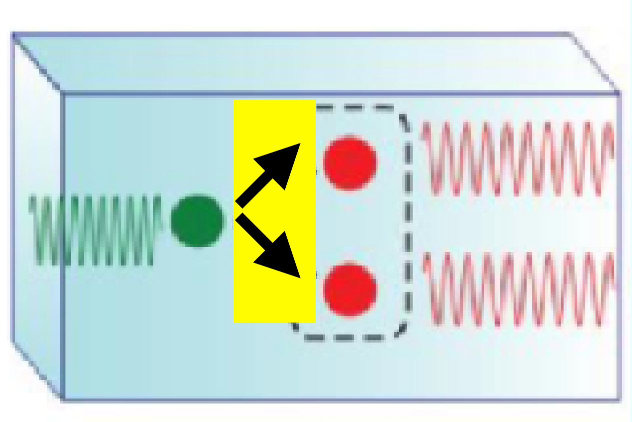
uncertainty in momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

uncertainty in position

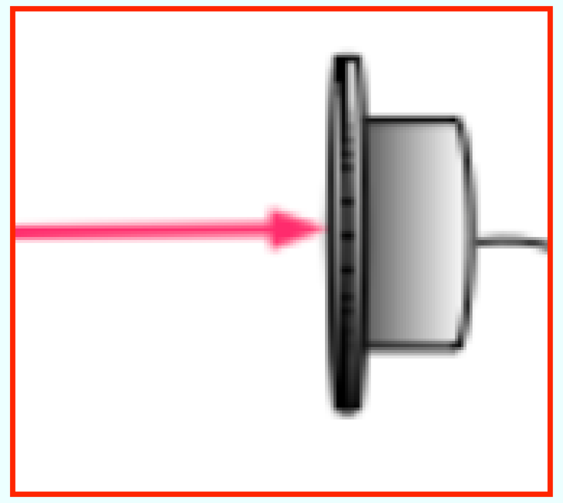


Sub-Super Poisson Distributions:

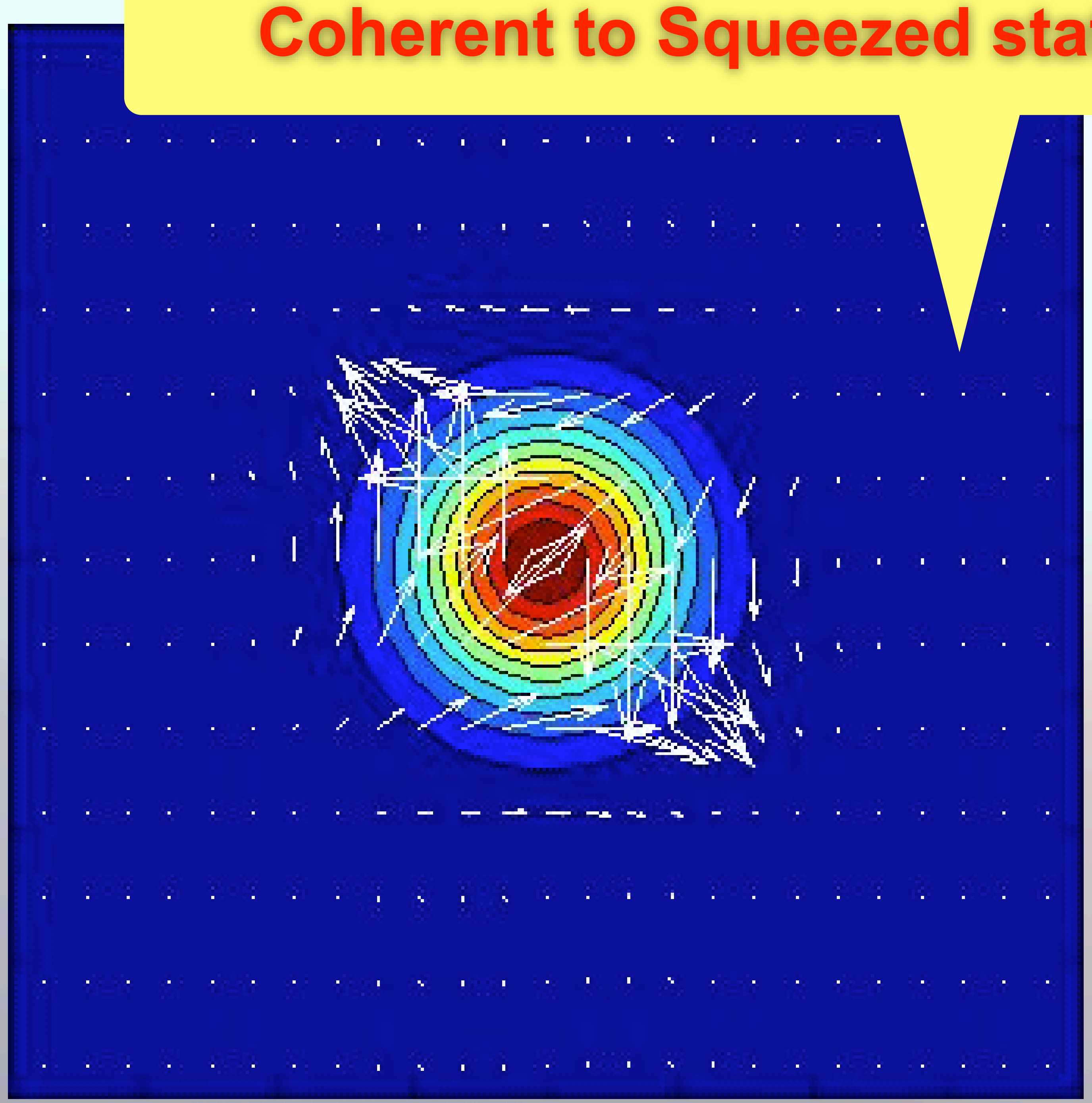


Photon pairs

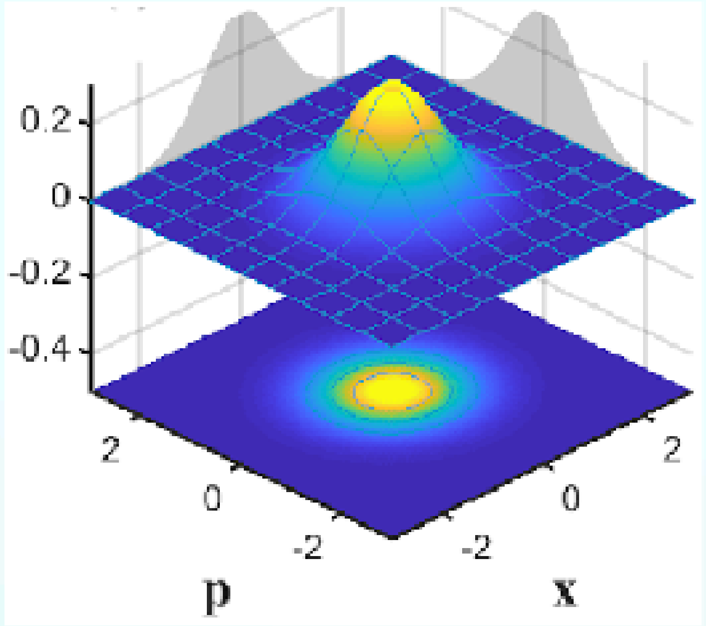
Optical Parametric Oscillator, OPO



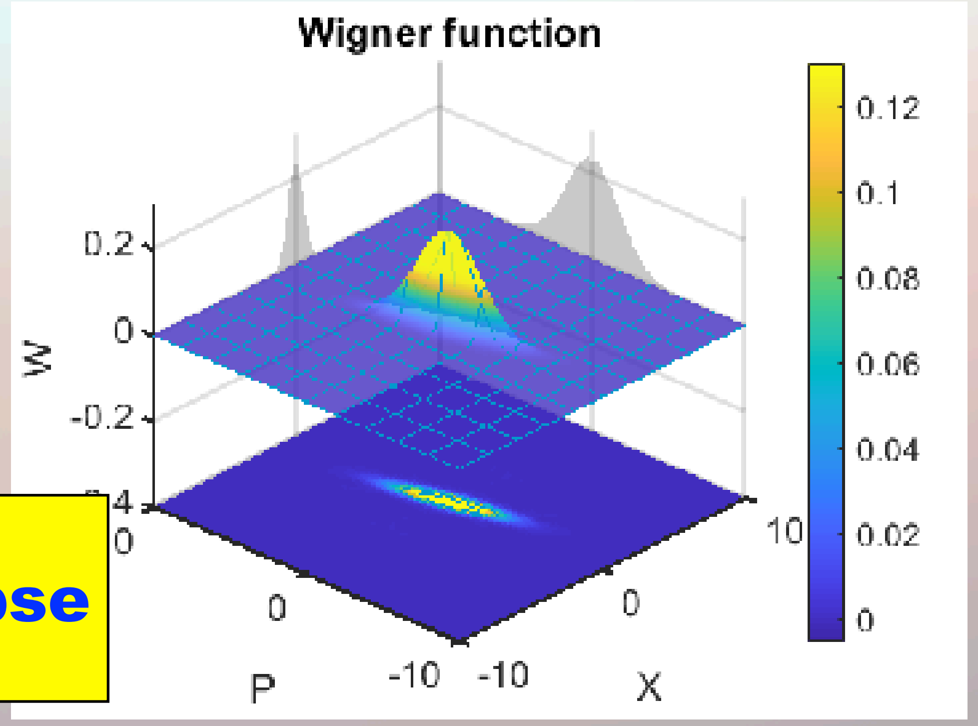
Coherent to Squeezed states



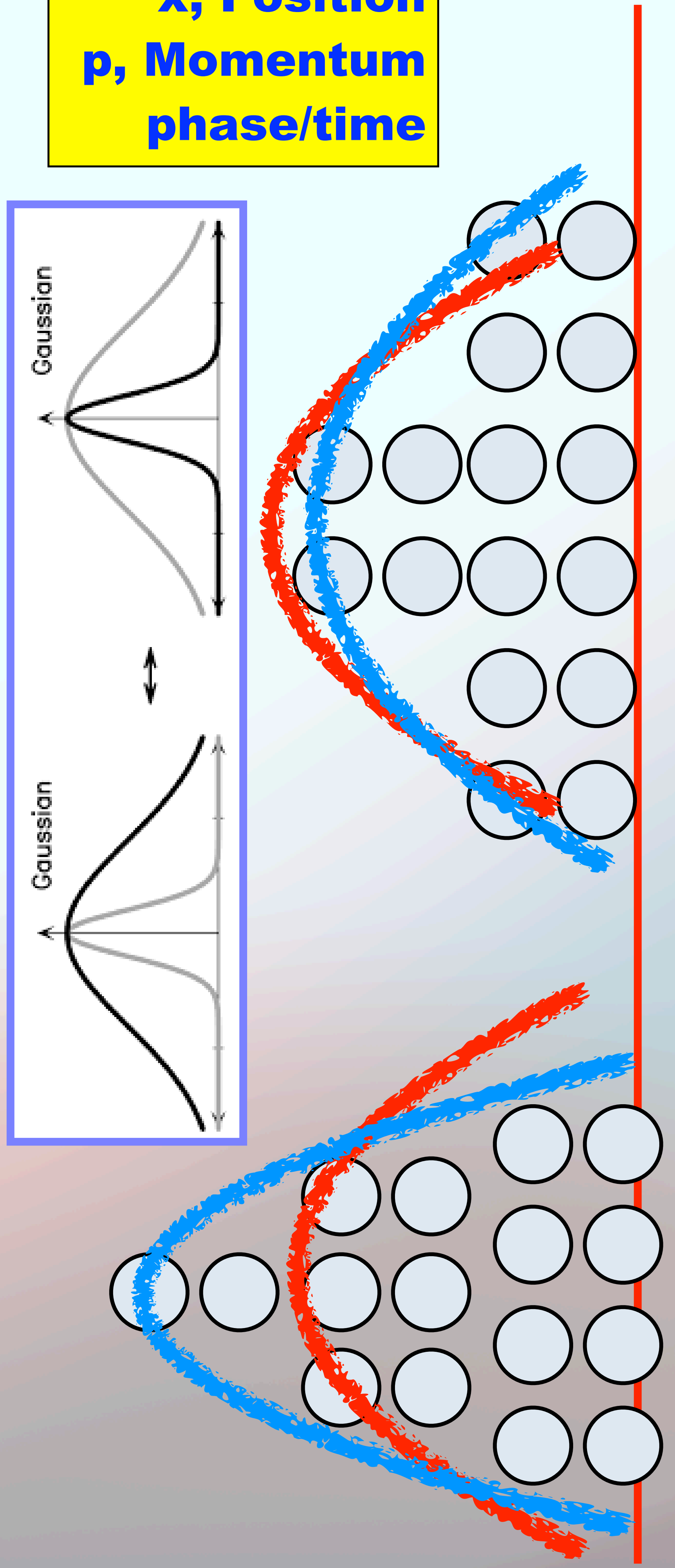
circle



ellipse



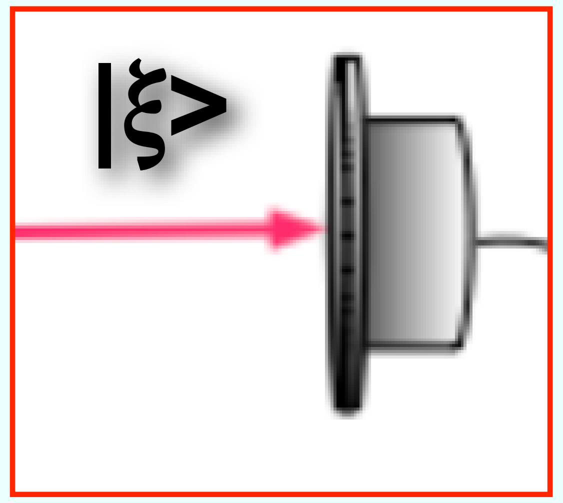
x, Position
p, Momentum
phase/time

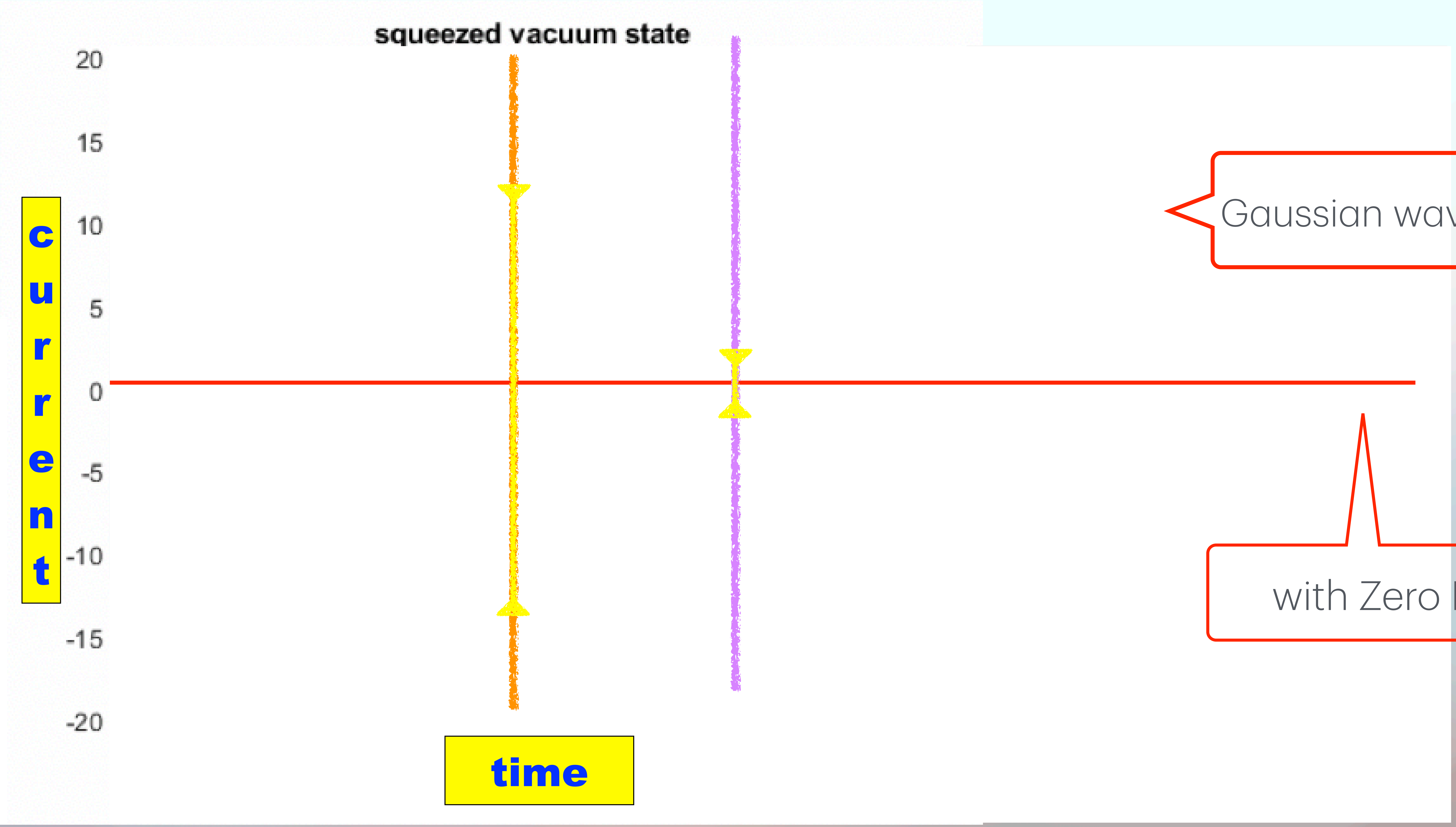


Super-Poisson

Sub-Poisson

Squeezed Vacuum State: |ξ>





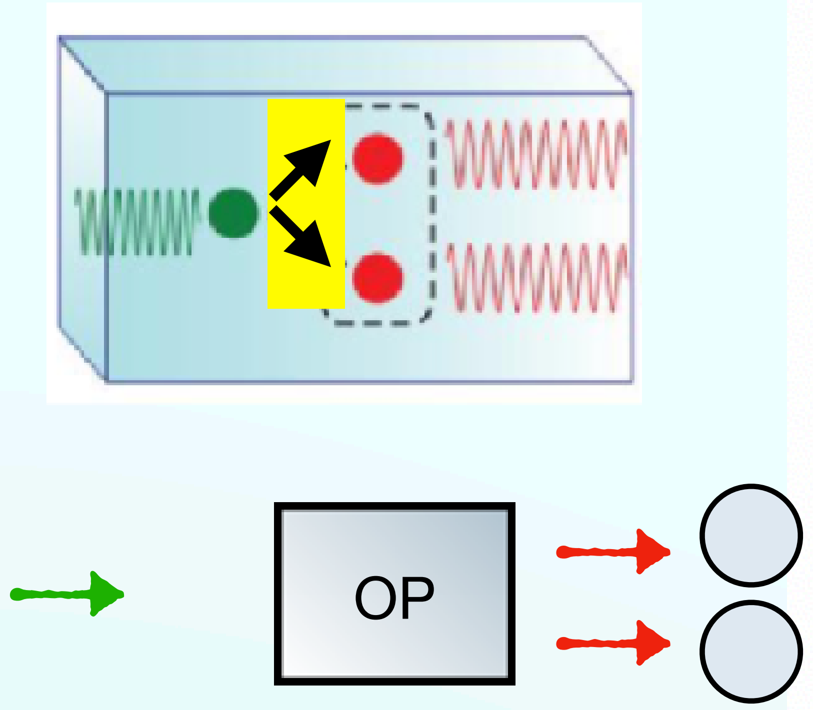
Gaussian wave-package

with Zero Mean

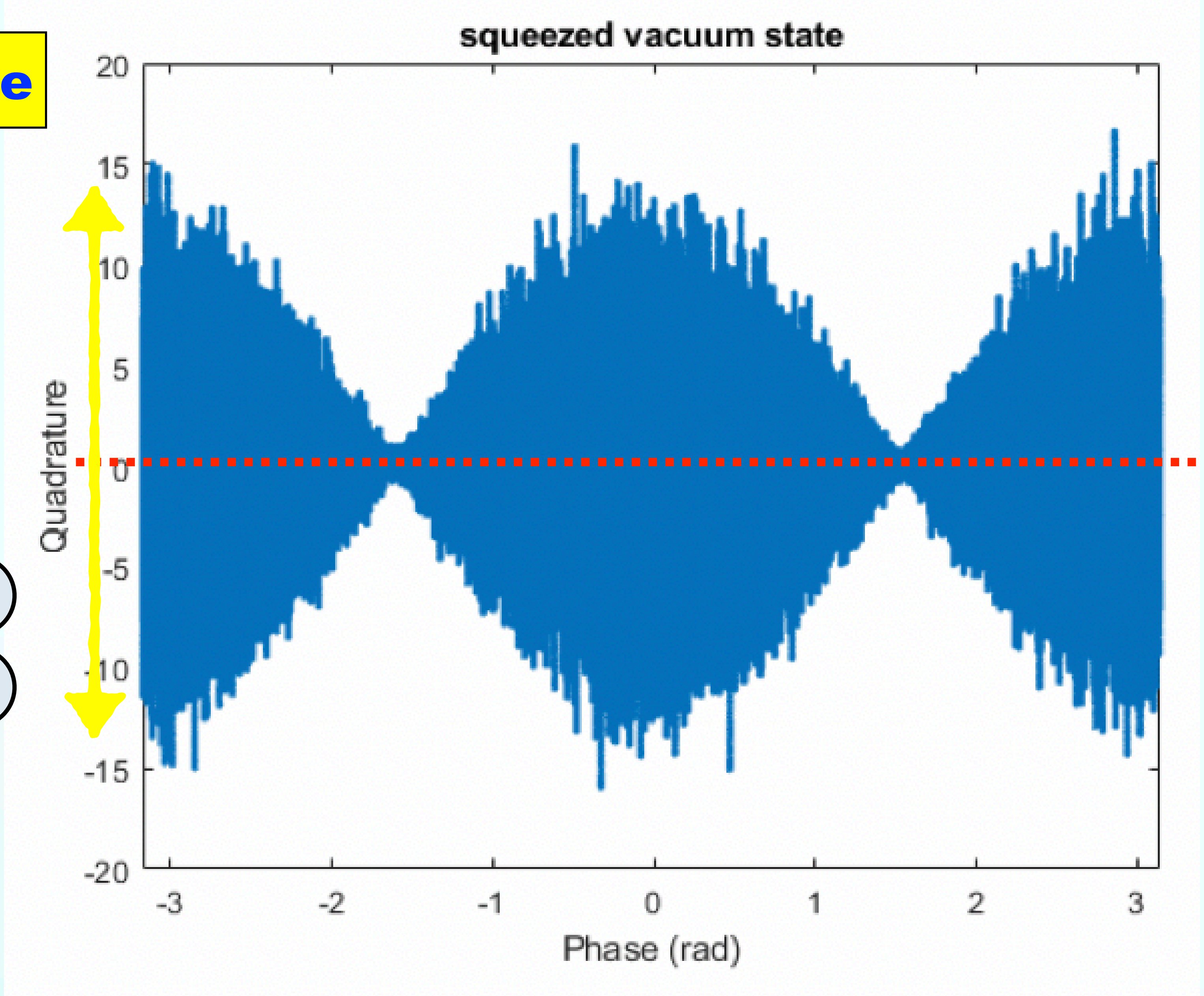
with Chien-Ming Wu

Squeezed Vacuum State: |ξ>

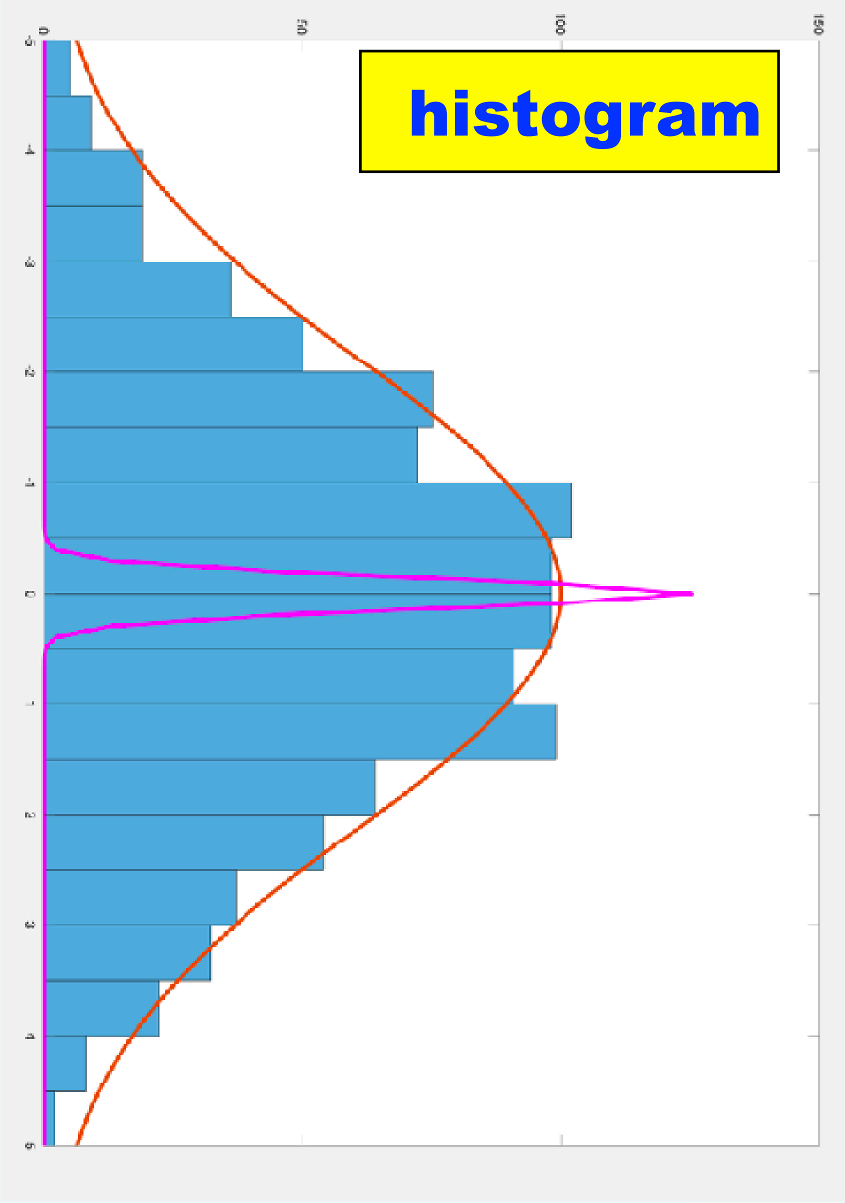
time-sequence



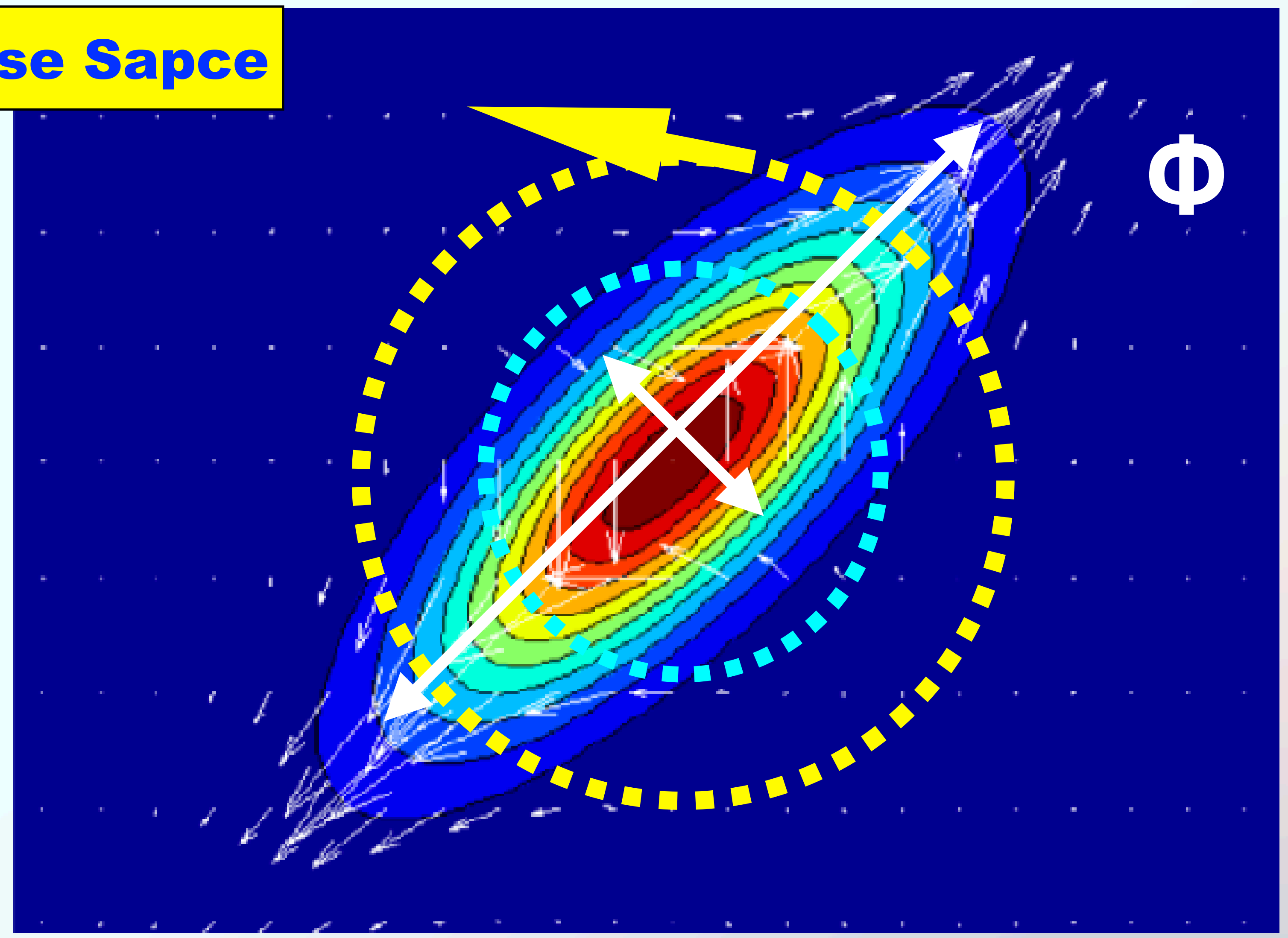
Photon pairs



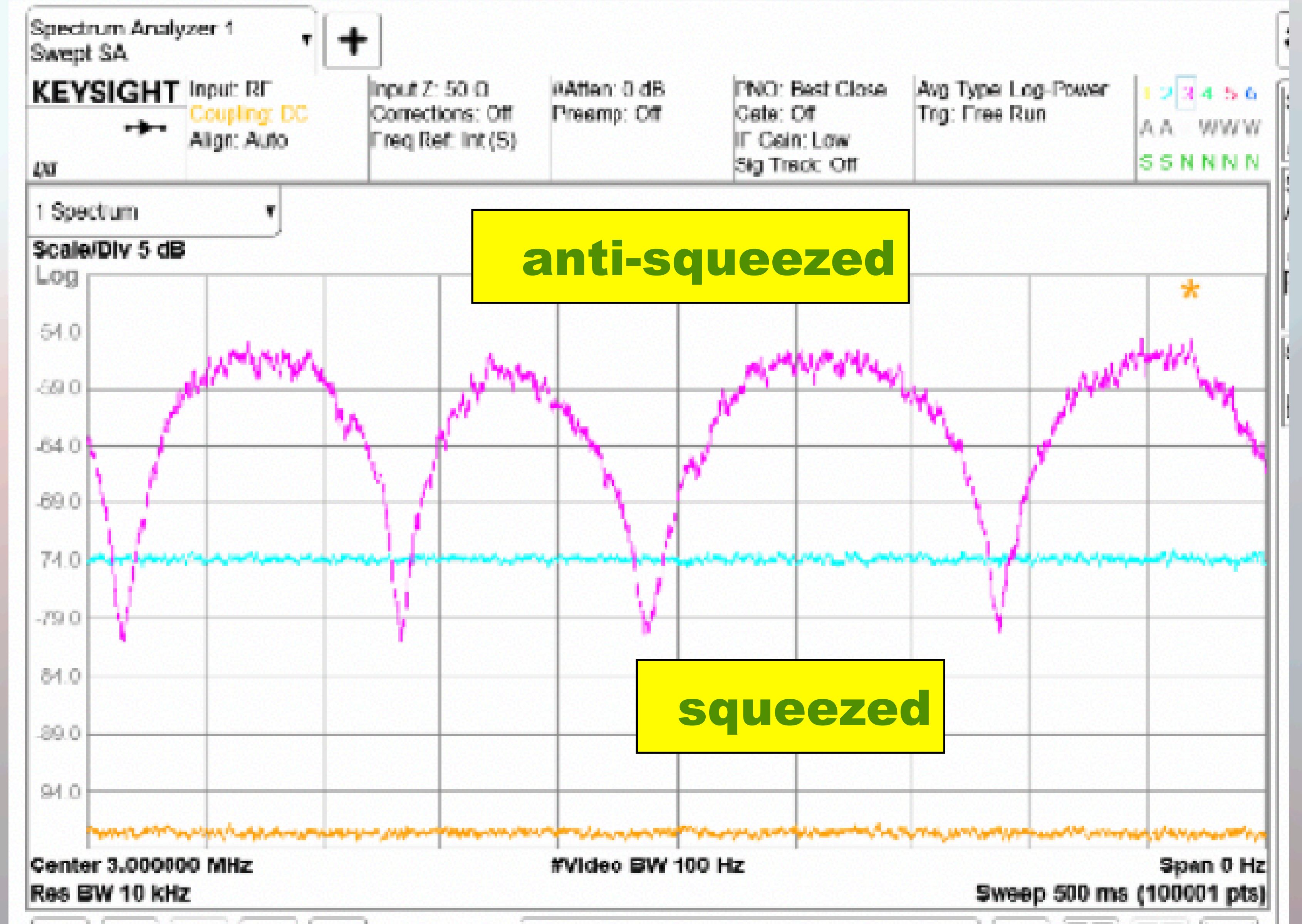
histogram



Phase Sapce



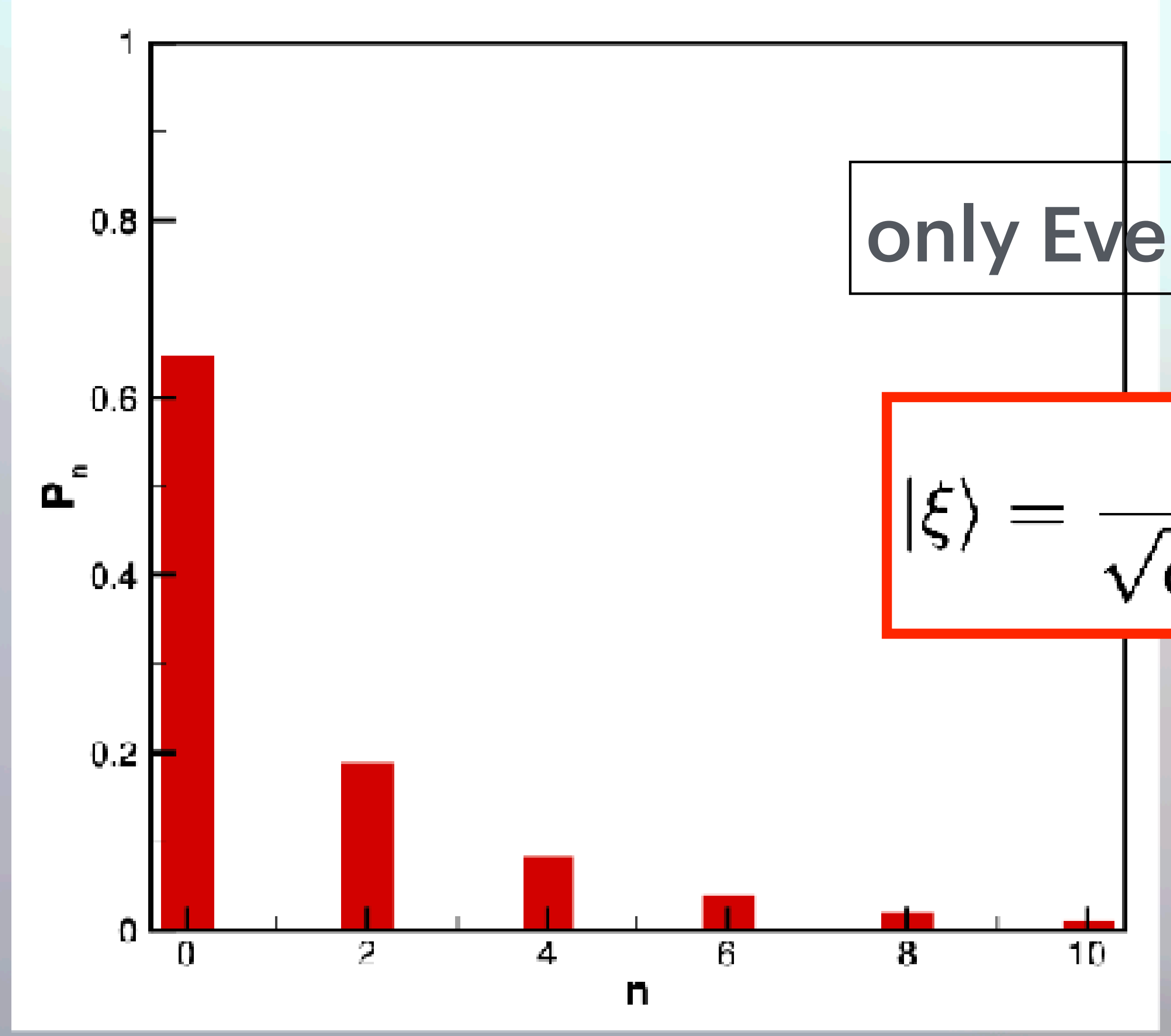
BHD signal



anti-squeezed

squeezed

LO phase

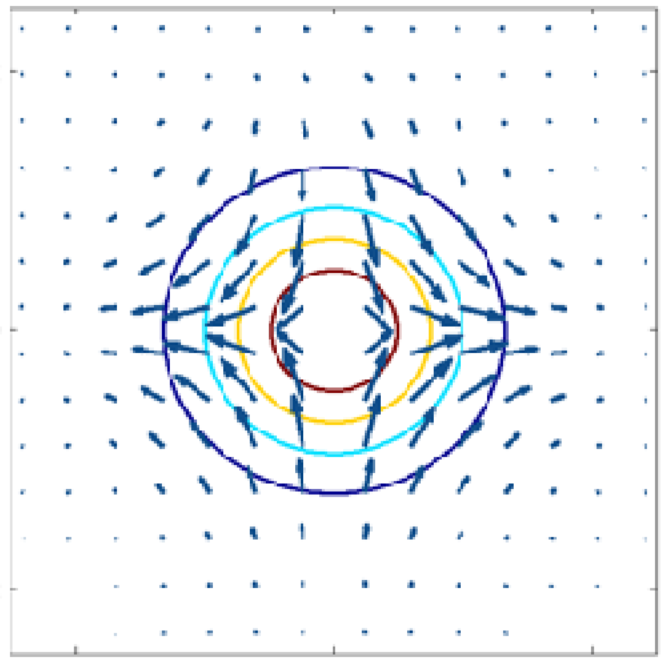


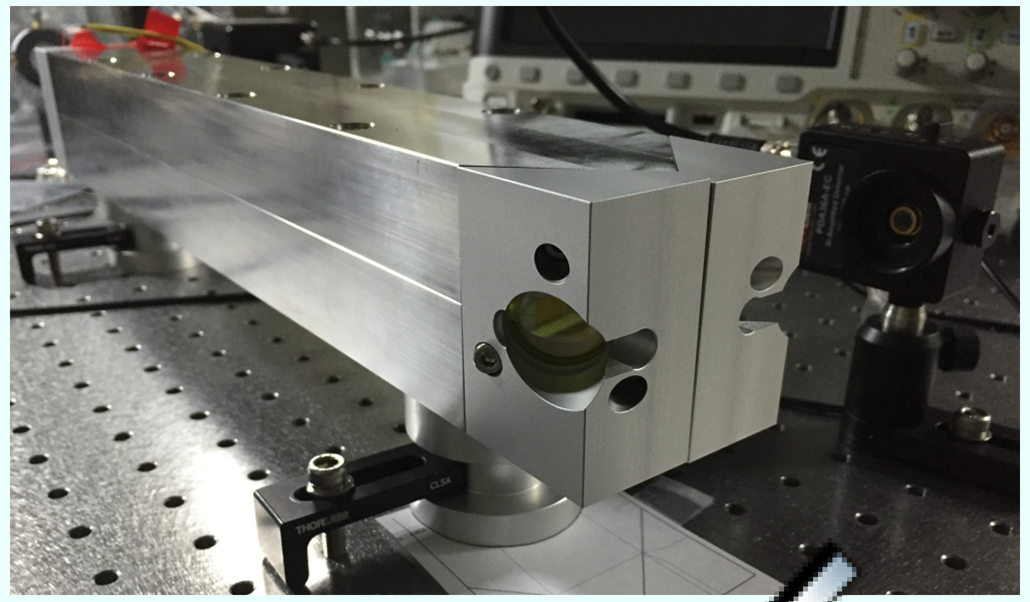
only Even Number of photons

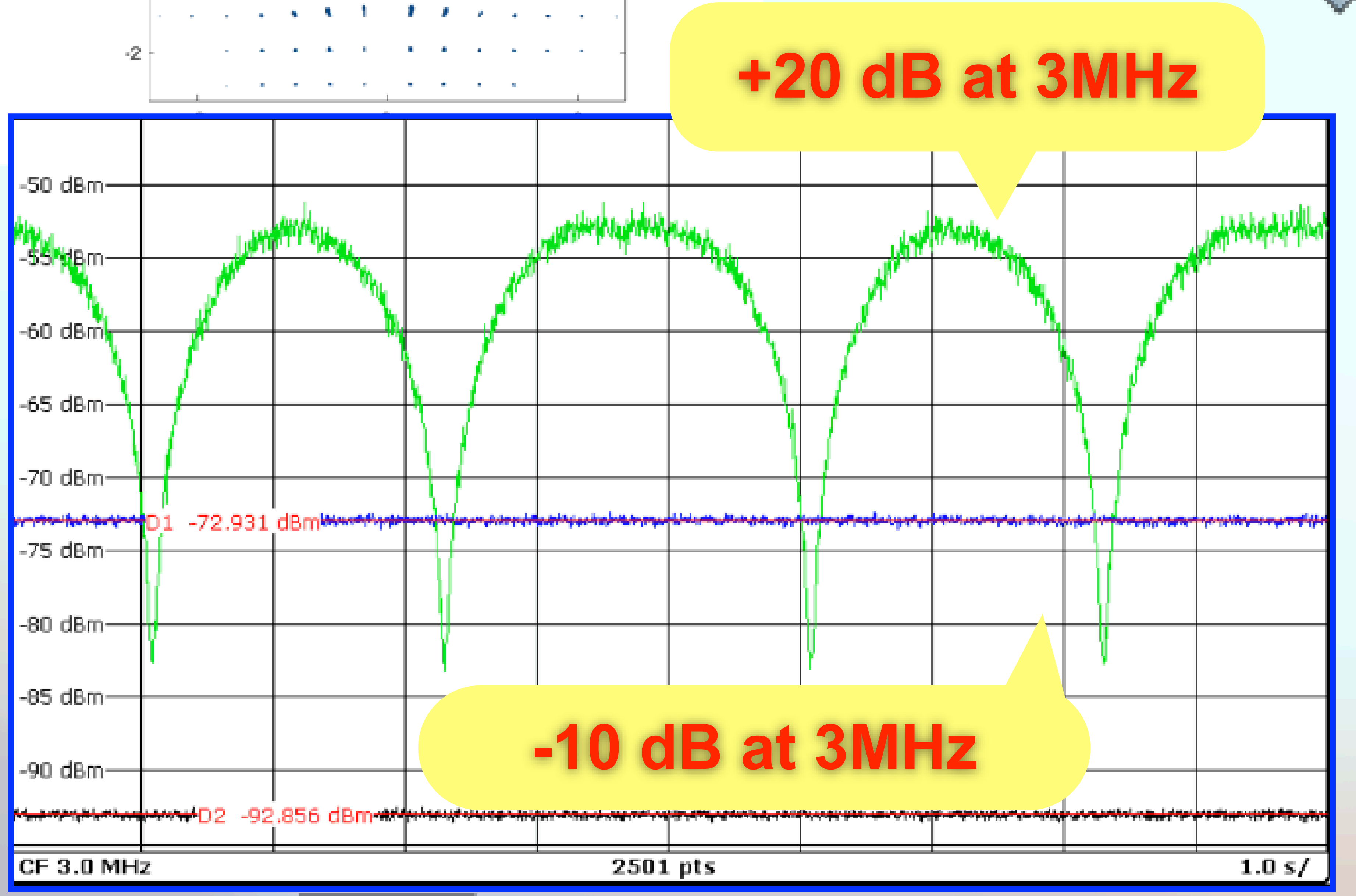
$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} e^{im\theta} \tanh^m r |2m\rangle$$

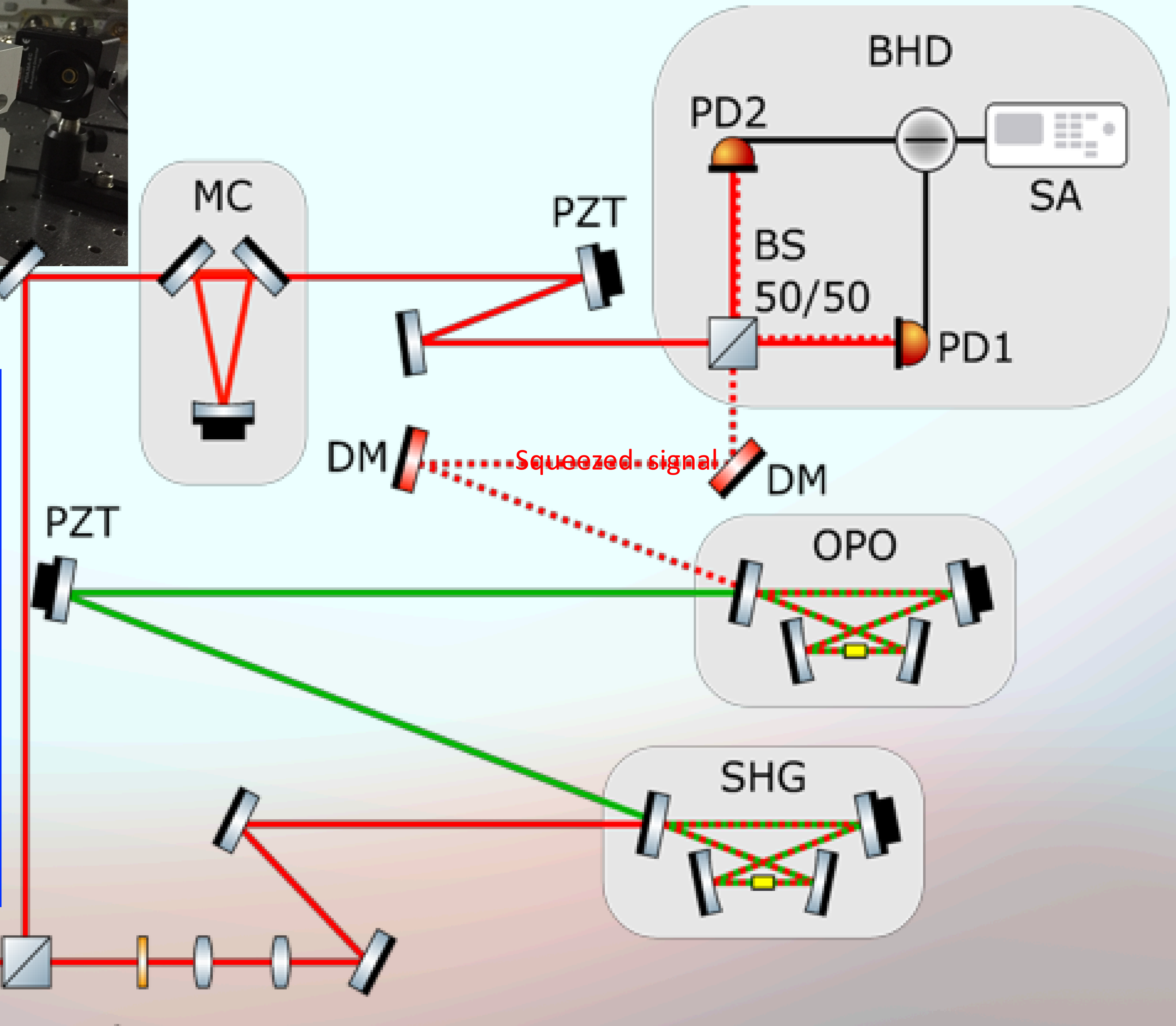
Squeezer @ NTHU, Taiwan

Exp. Reconstruction

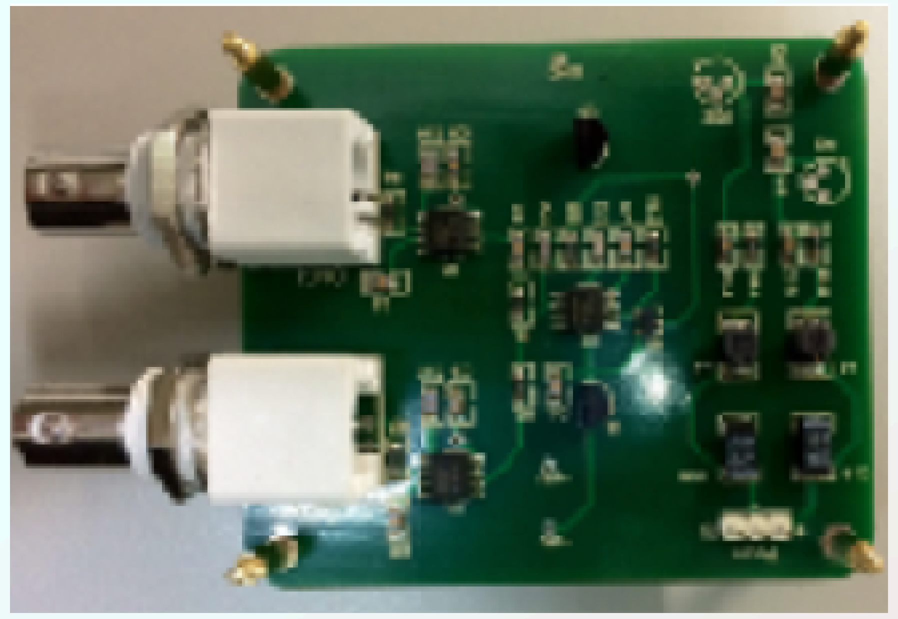


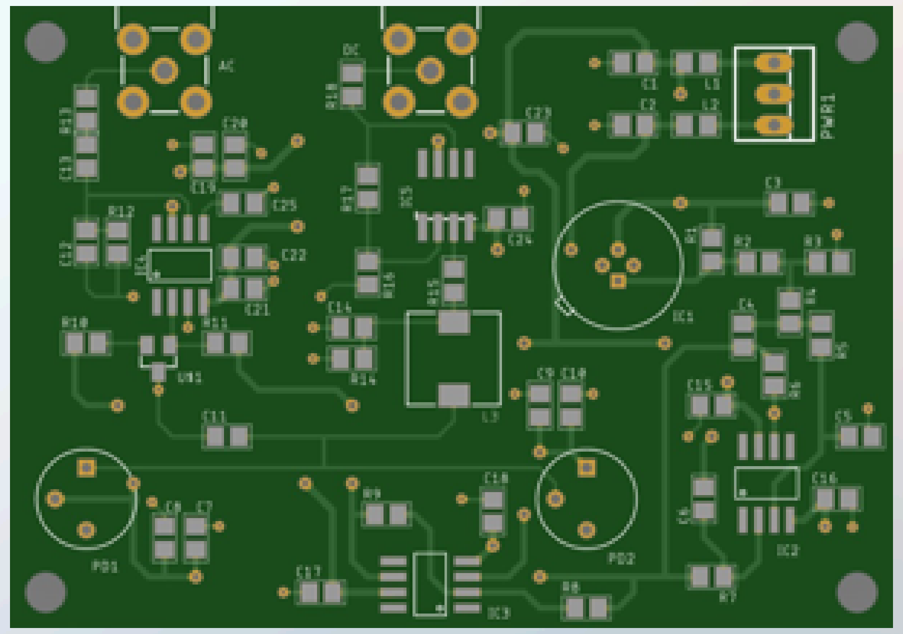


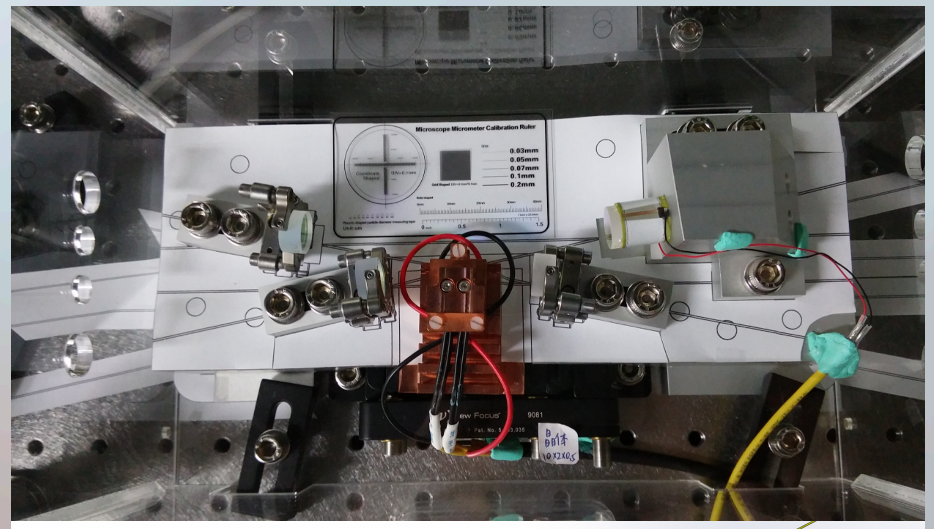


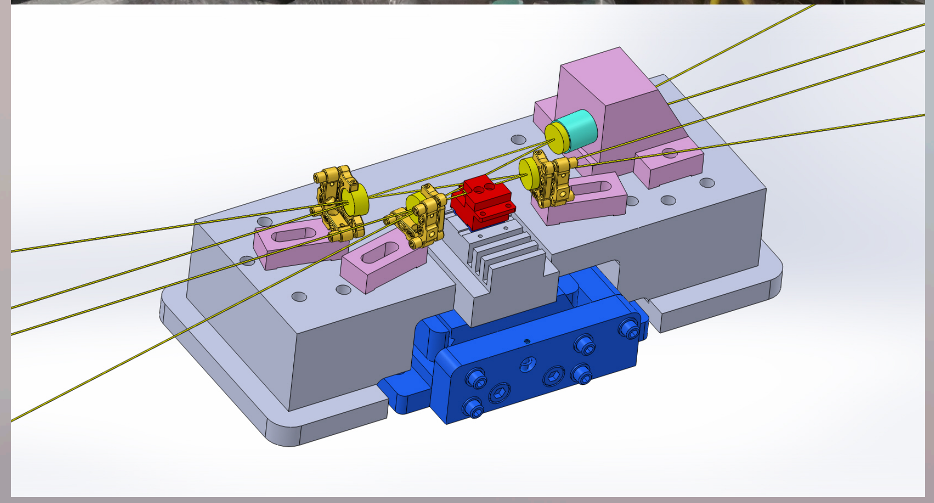


CW @ 1064 nm









Phys. Rev. Lett. 128, 073604 (2022);
Phys. Rev. A 108, 023729 (2023);

Bala

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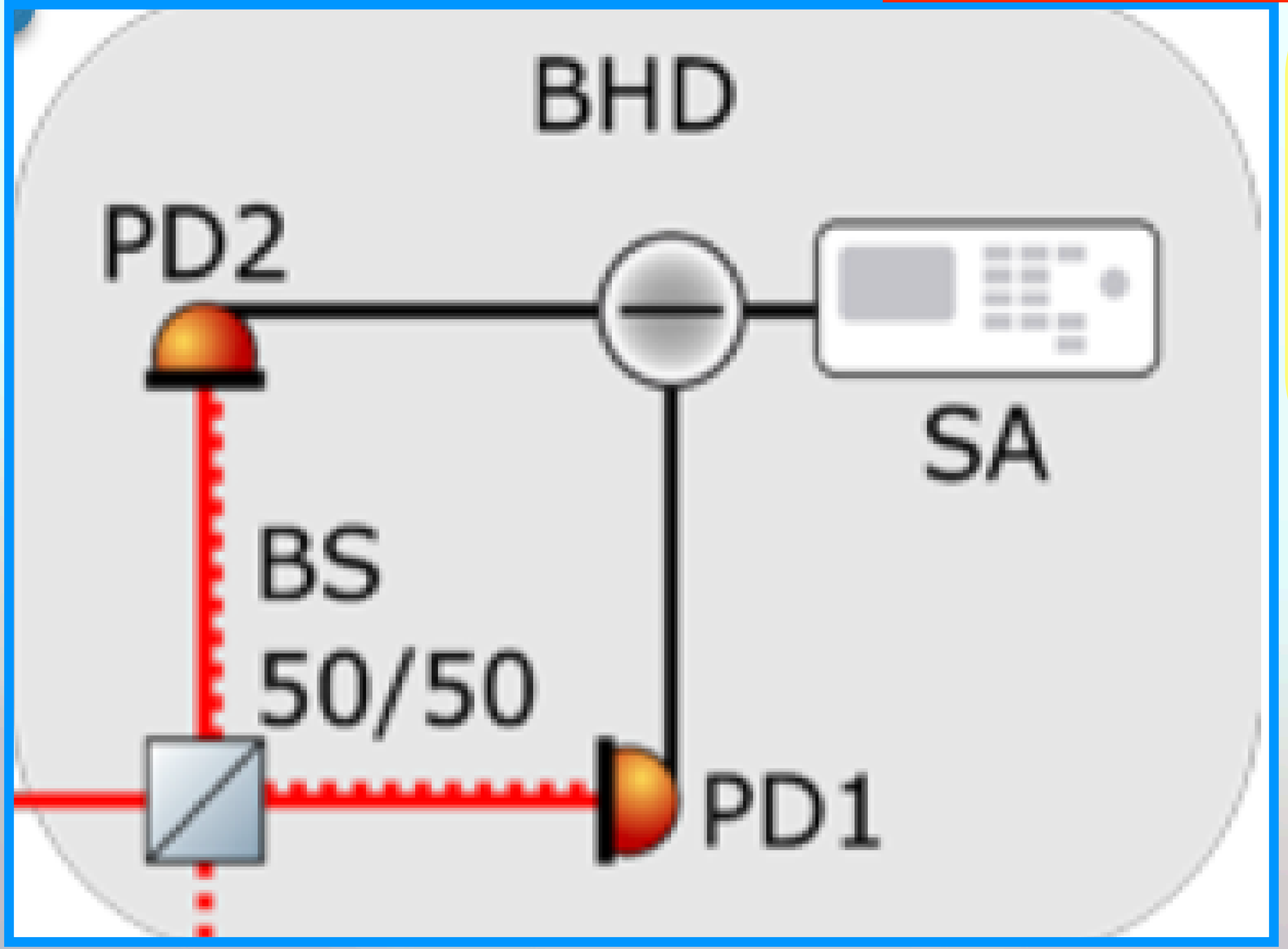
Products / Light Analysis / Optical Receivers / Balanced Optical Receivers

Balanced Optical Receivers

For noise sensitive experiments, we offer balanced photoreceivers that suppress up to 50 dB of laser / source intensity noise in experimental set-ups. These balanced optical receivers with two well-matched photodetectors can eliminate the need for lock-in amplifiers and can make all of the difference when you are trying to detect a small signal in applications like absorption spectroscopy, or heterodyne detection. Please see our Balanced Detector Guide for additional information.

Clearance:
1st -> 2nd generation

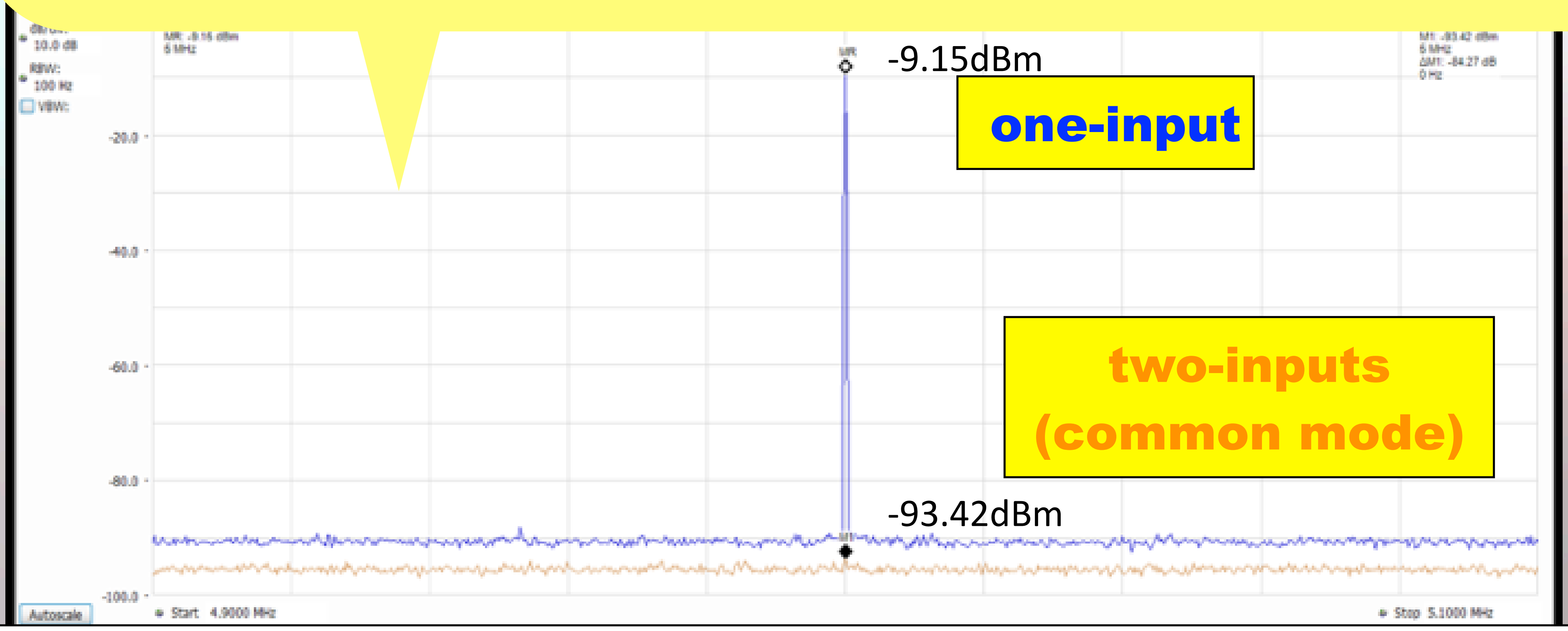
15->24 dB at 6mW
20->30 dB at 30mW

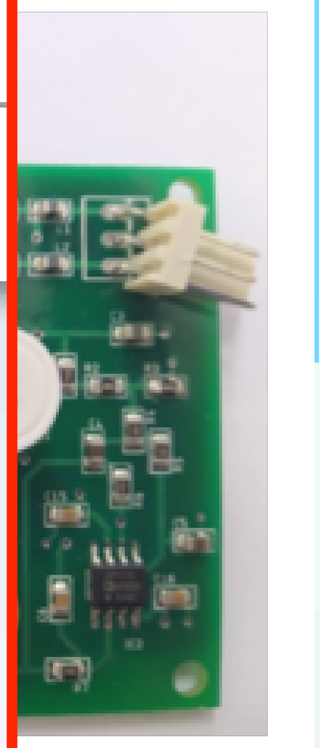


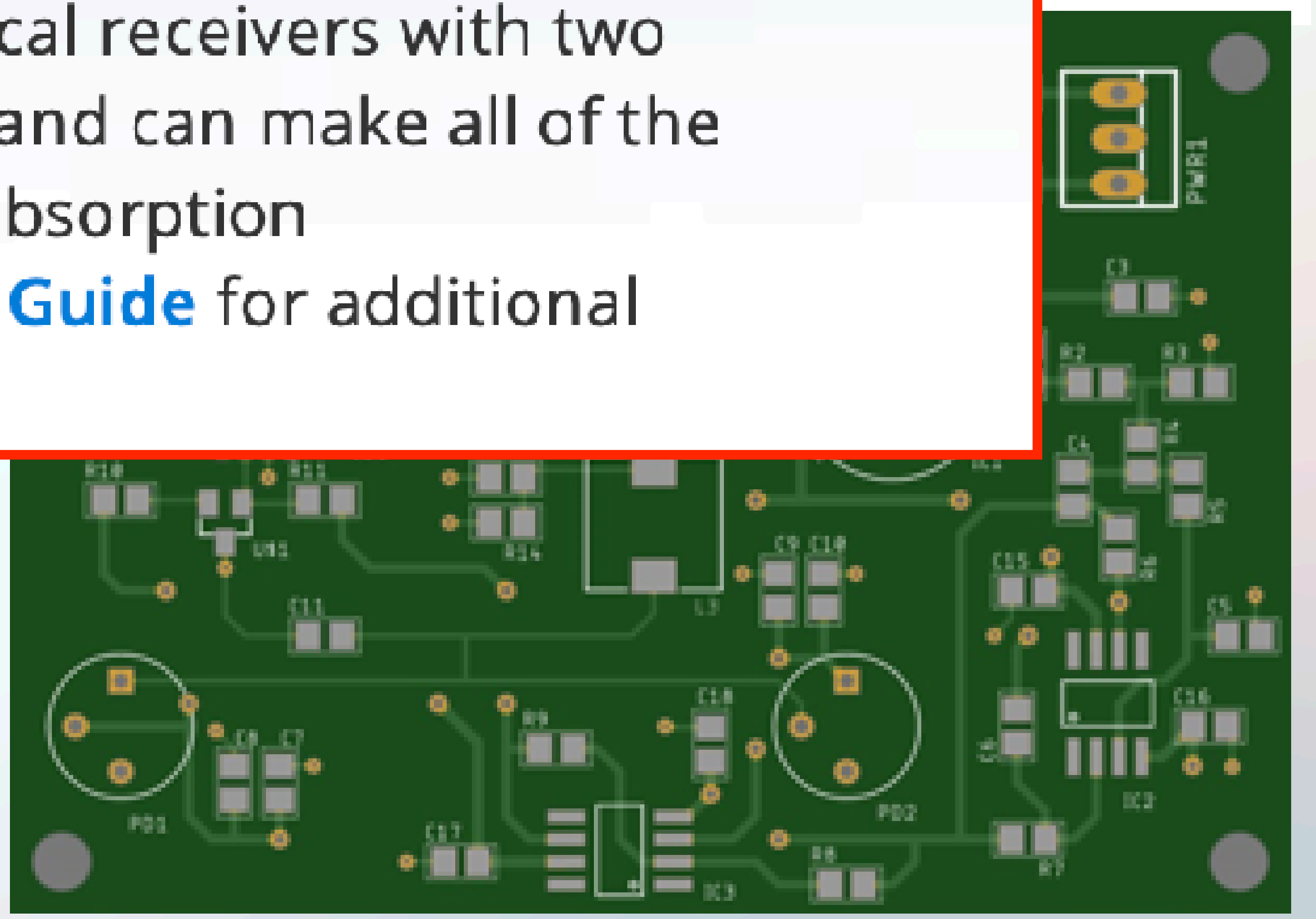
Courtesy: Ping-Koy Lam (ANU)

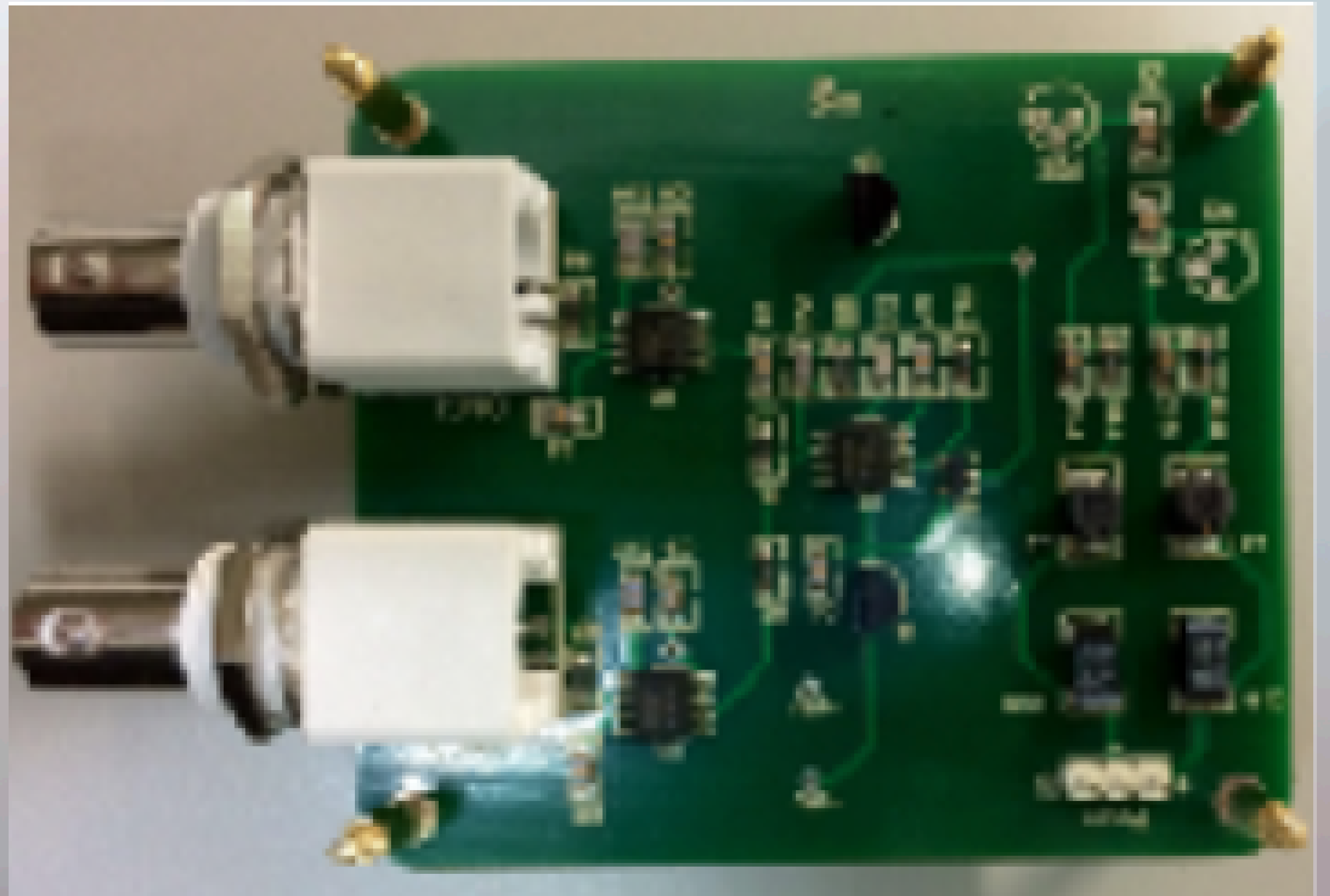
CMRR (common mode rejection ratio) > 84.27 dB

EOM driving frequency :5MHz,
Span: 200kHz, RBW:100Hz







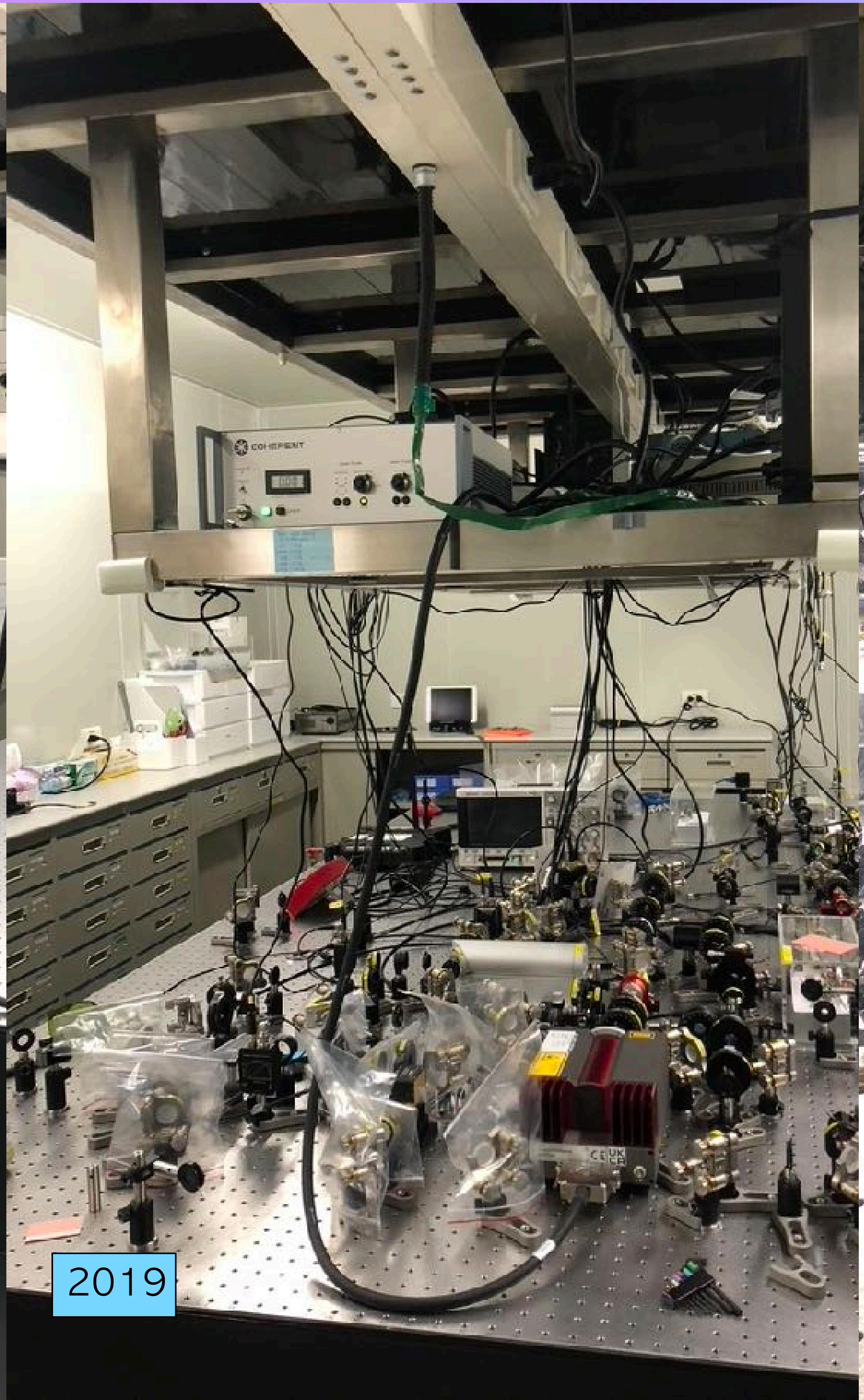


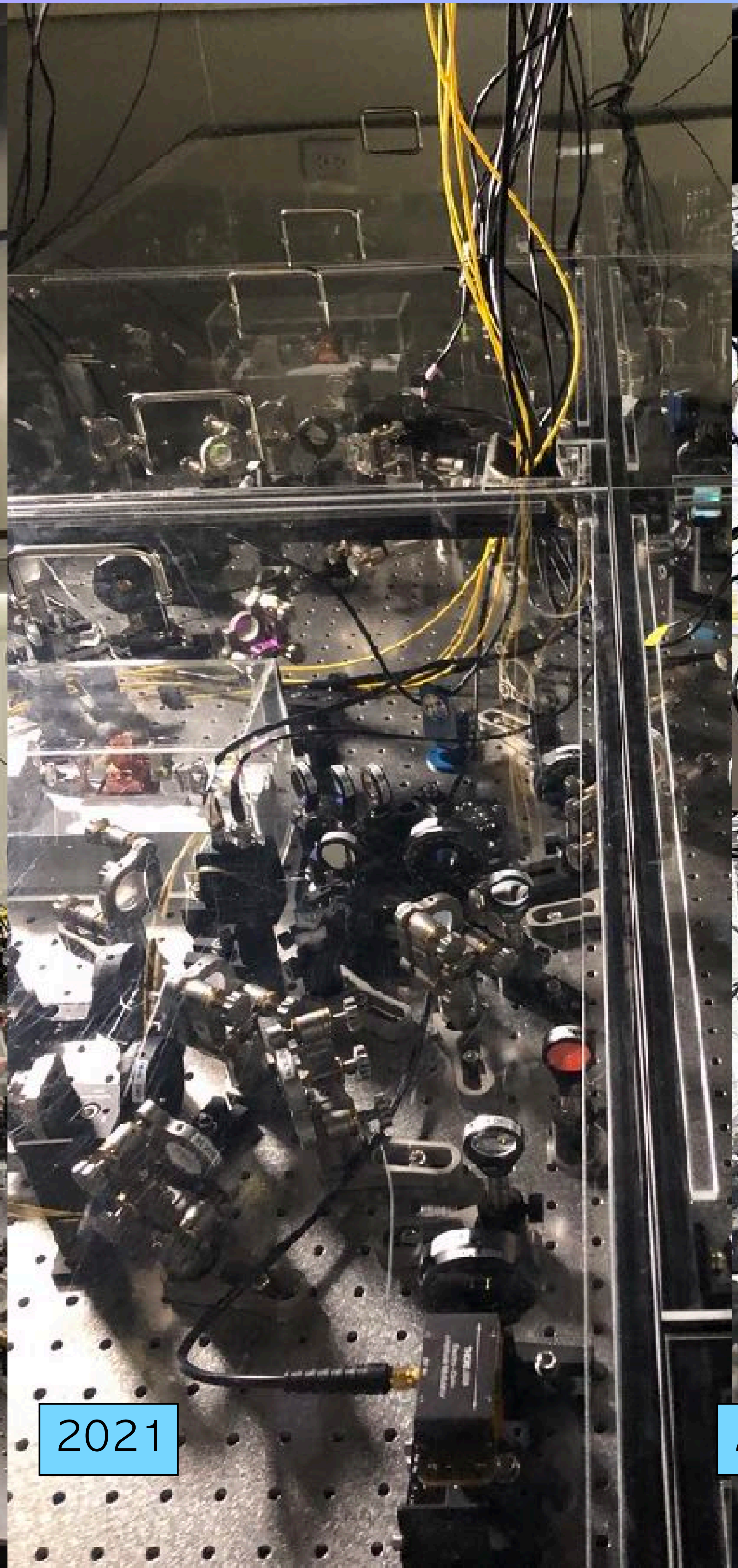
Chien-Ming Wu, et al., "Detection of 10 dB vacuum noise squeezing at 1064 nm by balanced homodyne detectors with a common mode rejection ratio more than 80 dB," Conference on Lasers and Electro-Optics (CLEO), JTu2A.38 (2019).

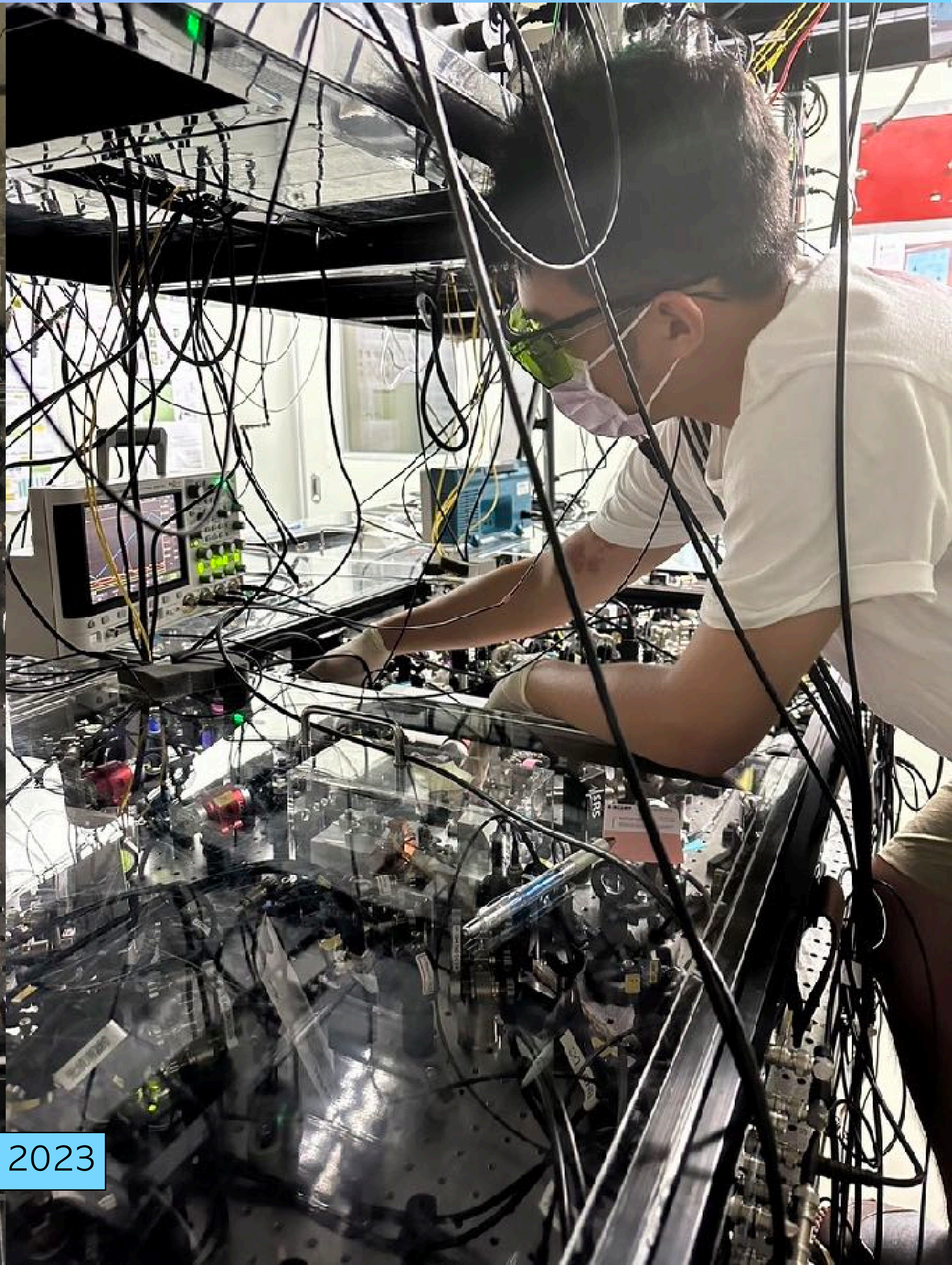
Ray-Kuang Lee's LAB
Institute of Photonics Technologies, National Tsing Hua University

Squeezer @ NTHU, Taiwan


2017

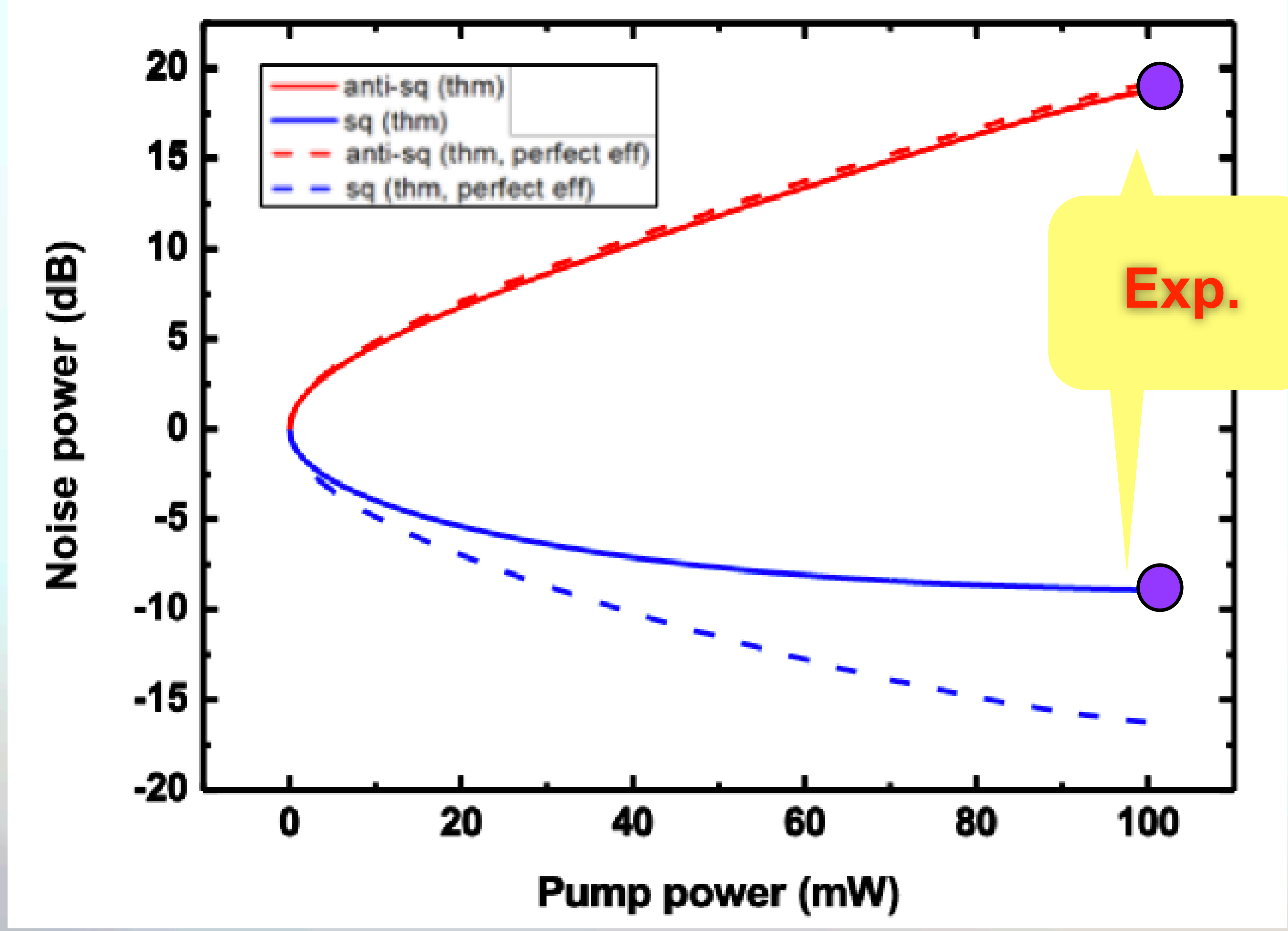

2019


2021

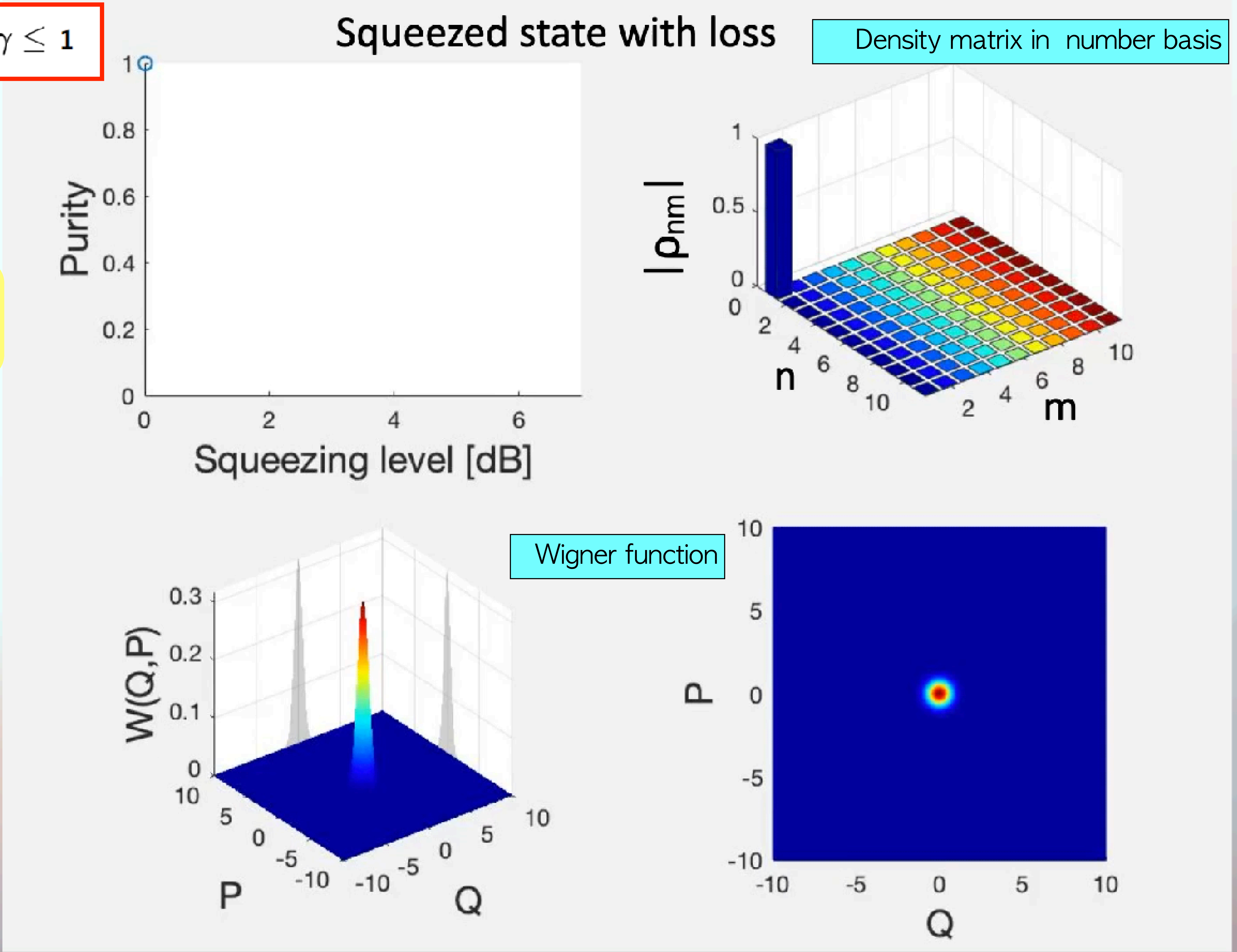

2023

Toward Real-Time QST to Extract Degradation information

$$\gamma \equiv \mathrm{tr}(\rho^2) \;,\; 0 < \gamma \leq 1$$



Hsieh-Yi Hsieh, Yi-Ru Chen, et al., Phys. Rev. Lett. 128, 073604 (2022).

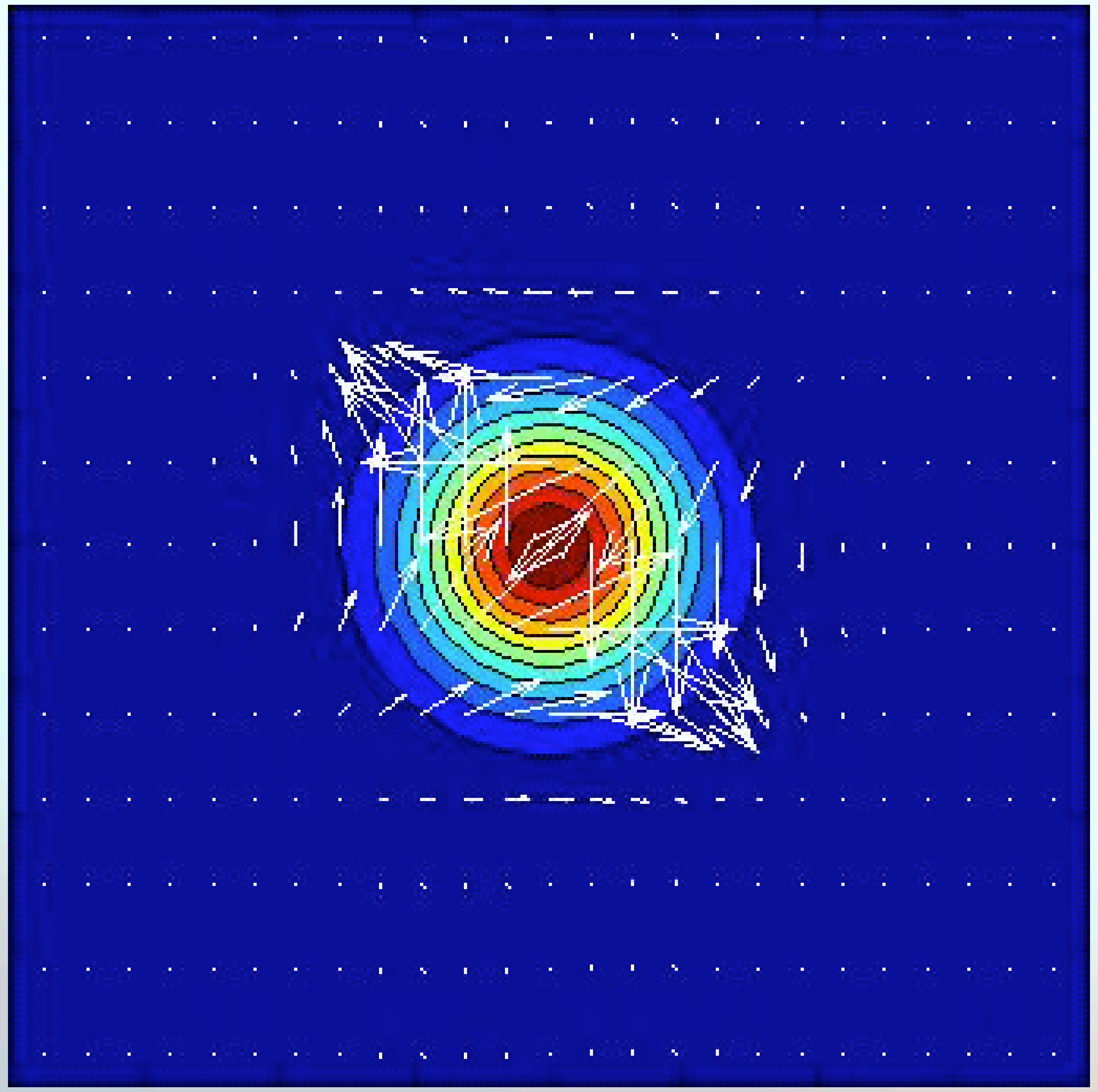


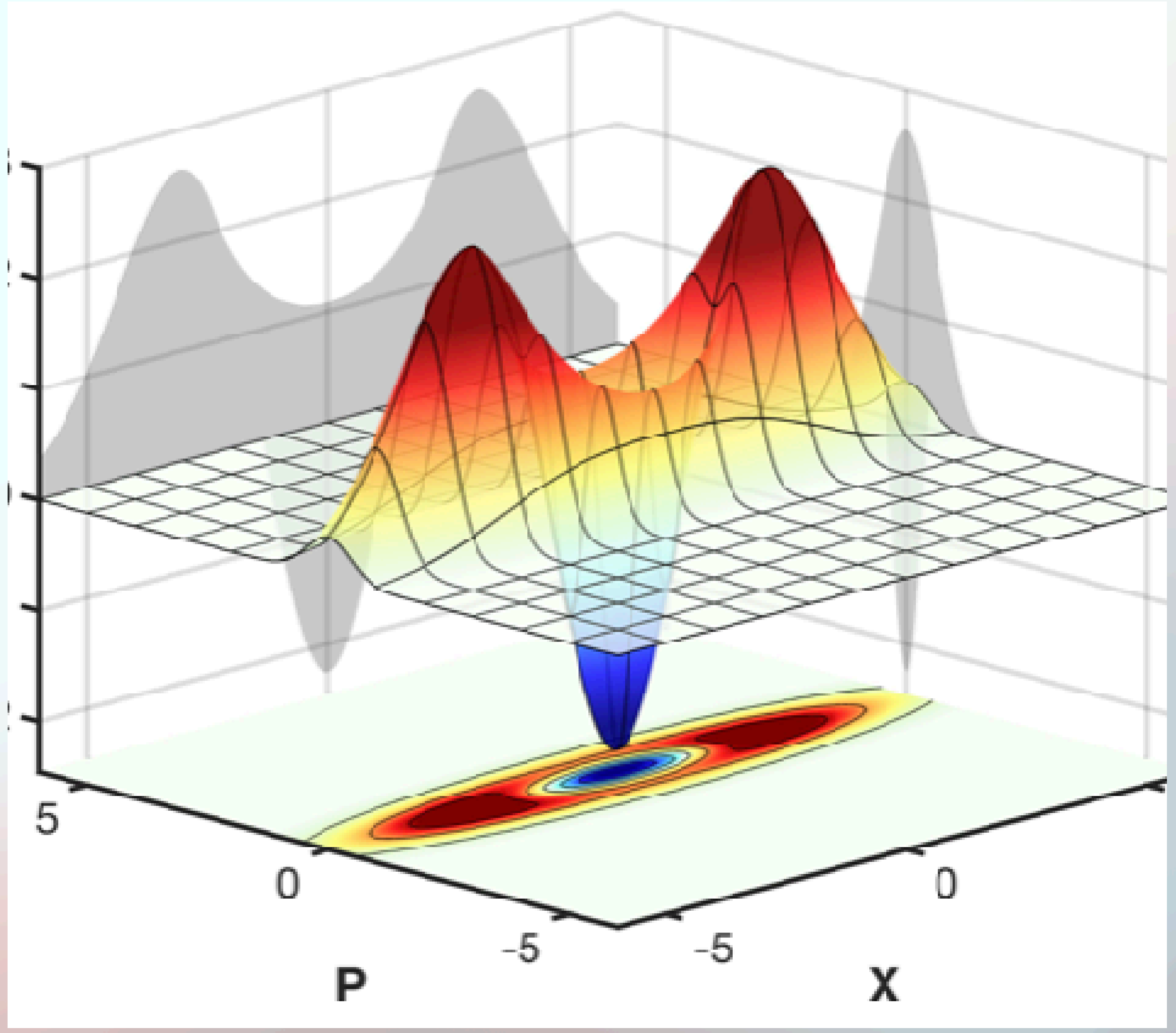
Outline:

Quantum State Tomography

Machine-Learning

Non-Gaussian: Fock and Cat States





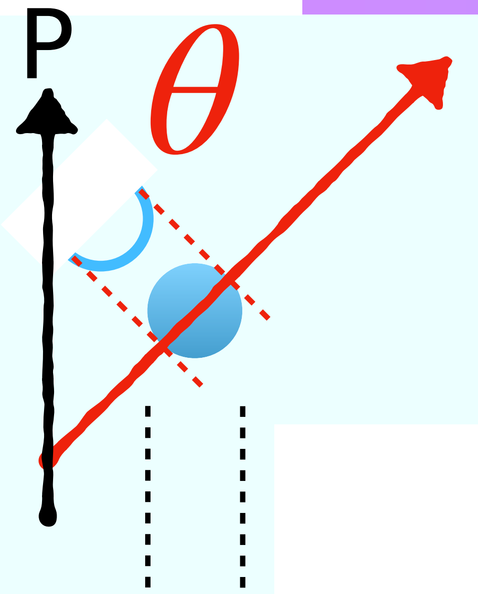
ML-Quant. State Tomography, Phys. Rev. Lett. 128, 073604 (2022);
Wigner Current, Phys. Rev. A 108, 023729 (2023);

Optical Cat States by Photon-Addition, Phys. Rev. A 110, 023703 (2024);
Machine-Learning-Fock State Tomography, Phys. Rev. A [arXiv: 2405.02812];
Quantumness Measure from phase space distributions, [arXiv: 2311.17399];

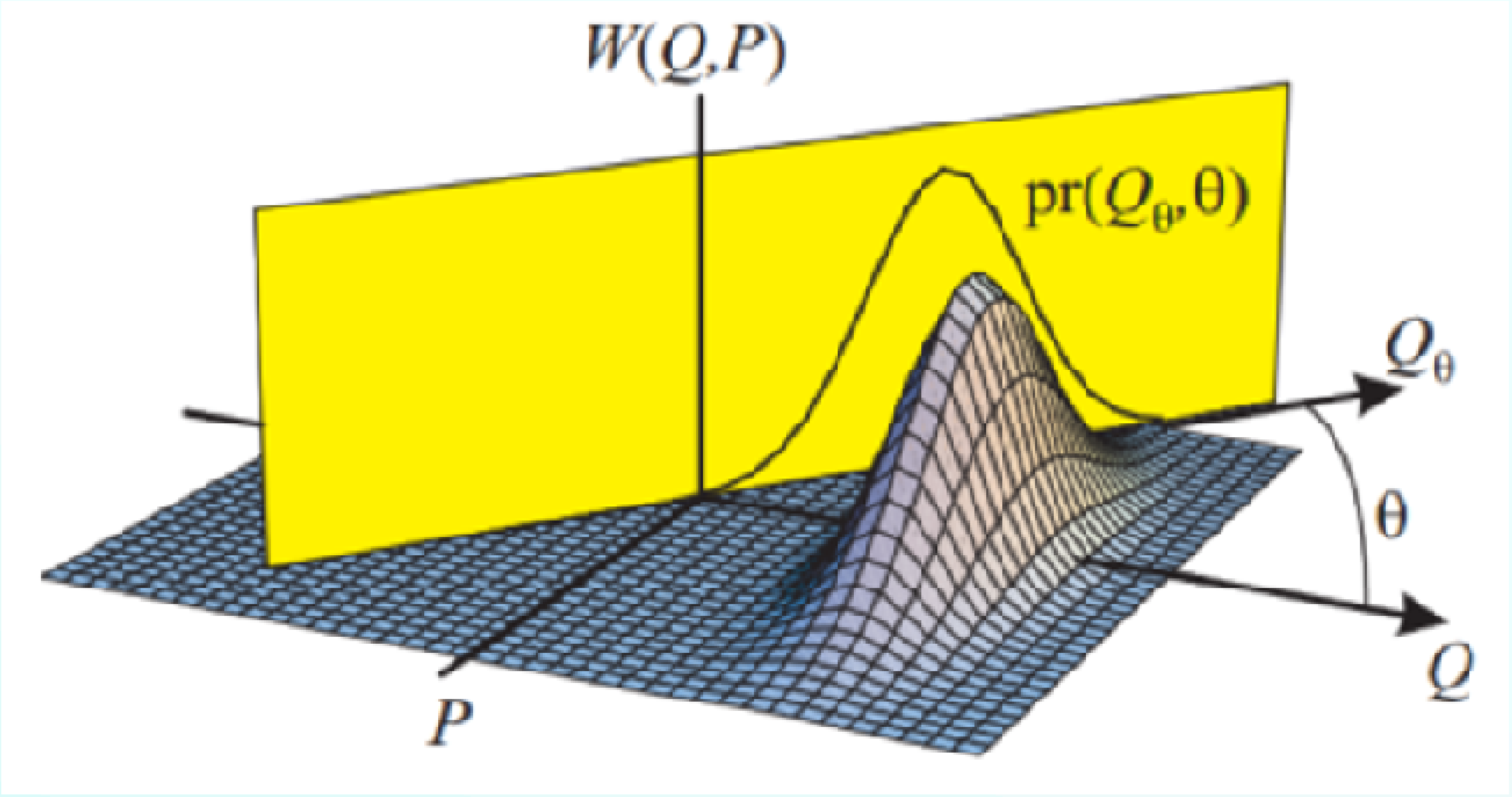
Review on Quantum ML, Advances in Phys. X 8, 2165452 (2023);

Quantum State Tomography:

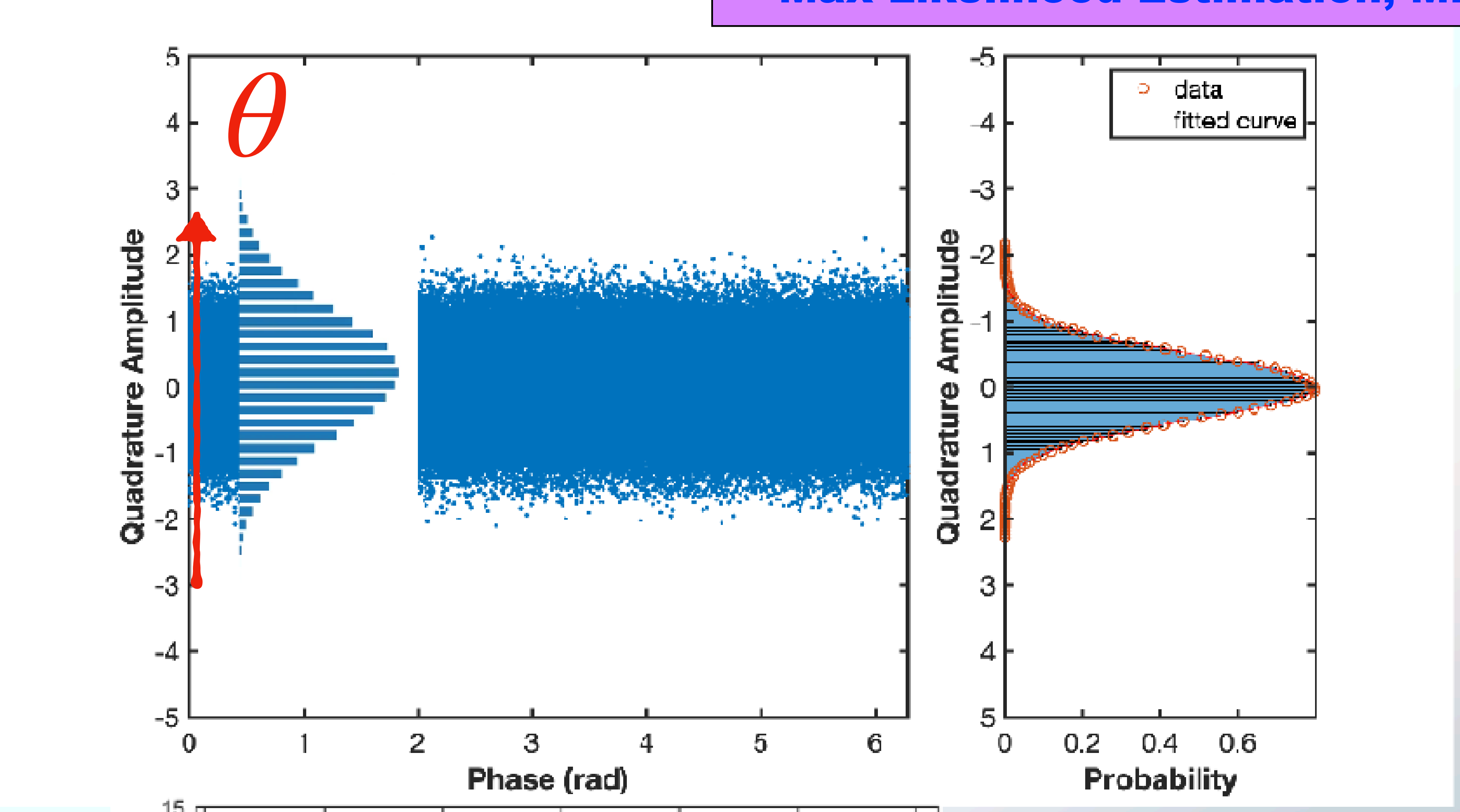
Max Likelihood Estimation, MLE

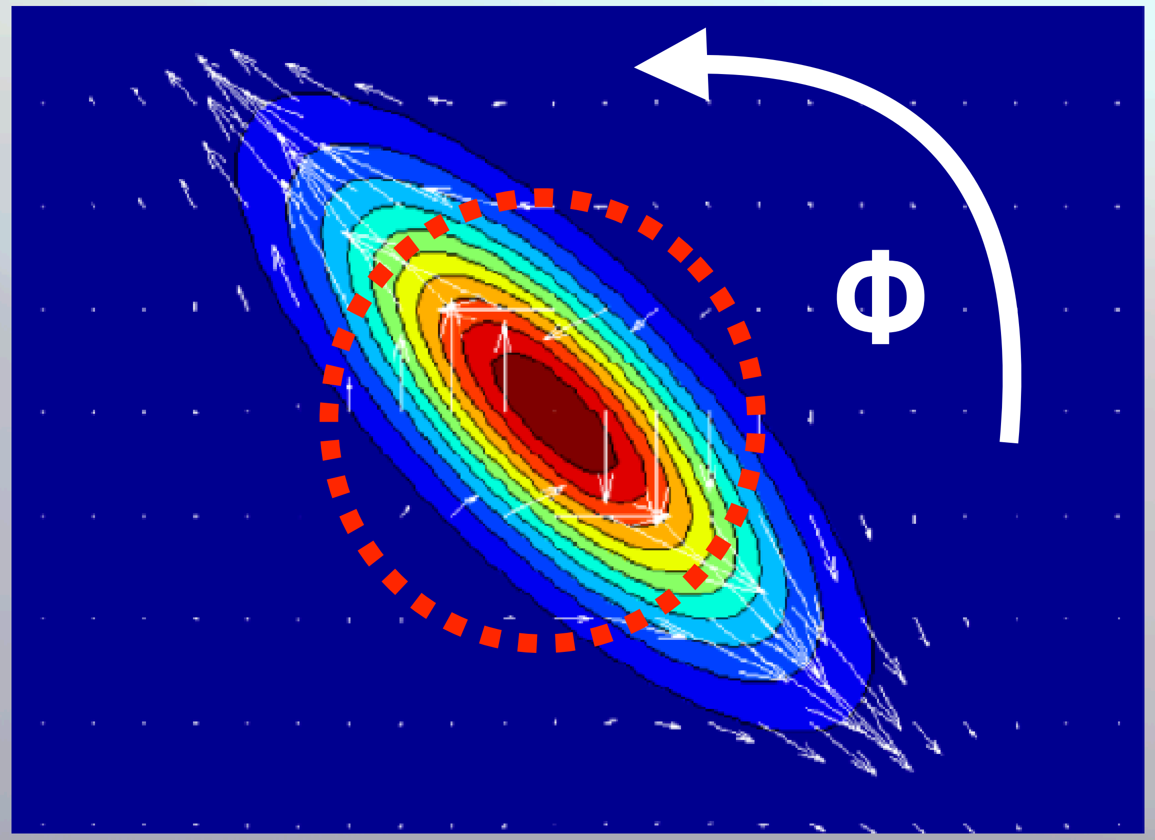


Inverse-Radon transformation



A. I. Lvovsky and M. G. Raymer, Rev. Mod. Phys. 81, 299 (2009).





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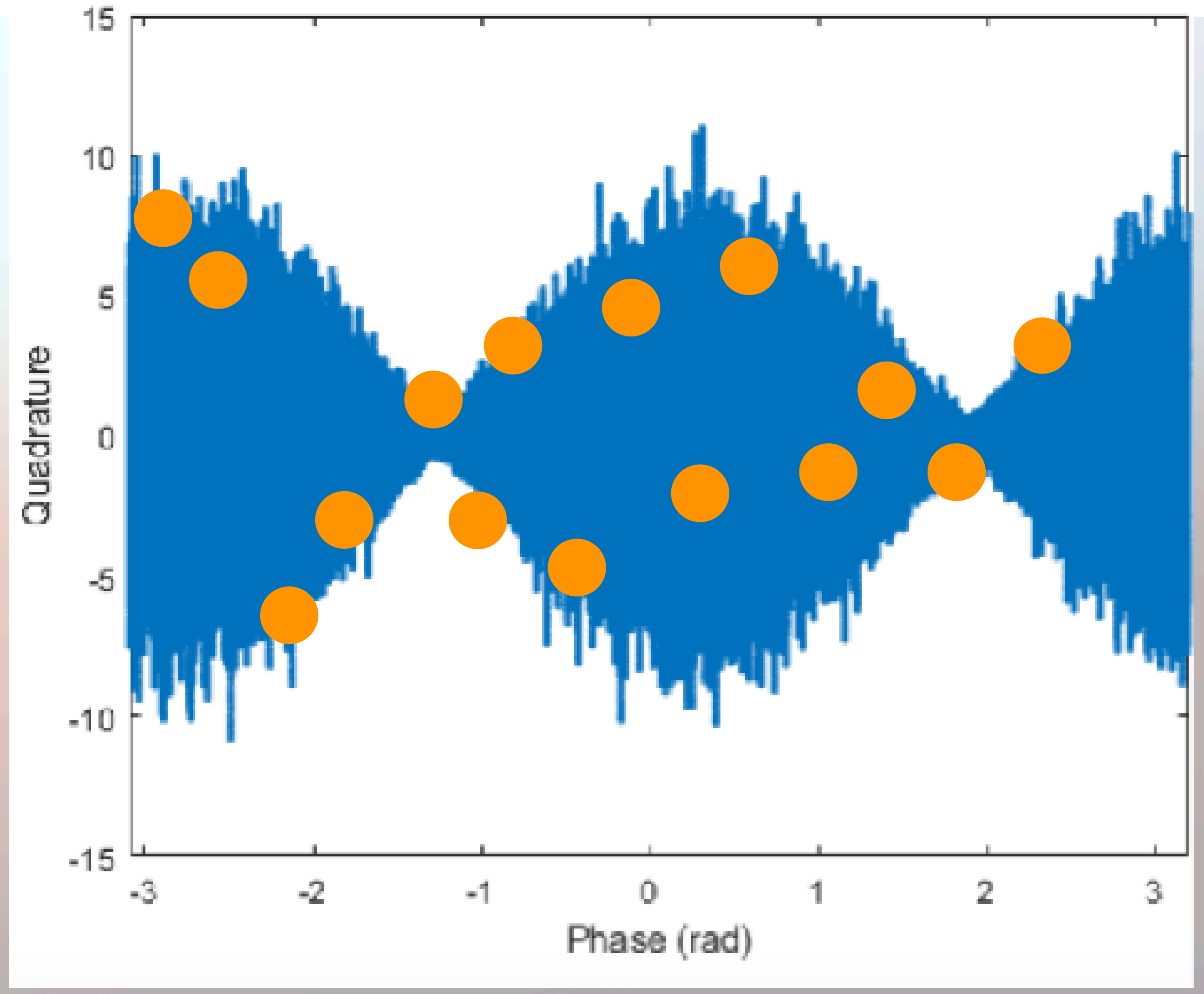
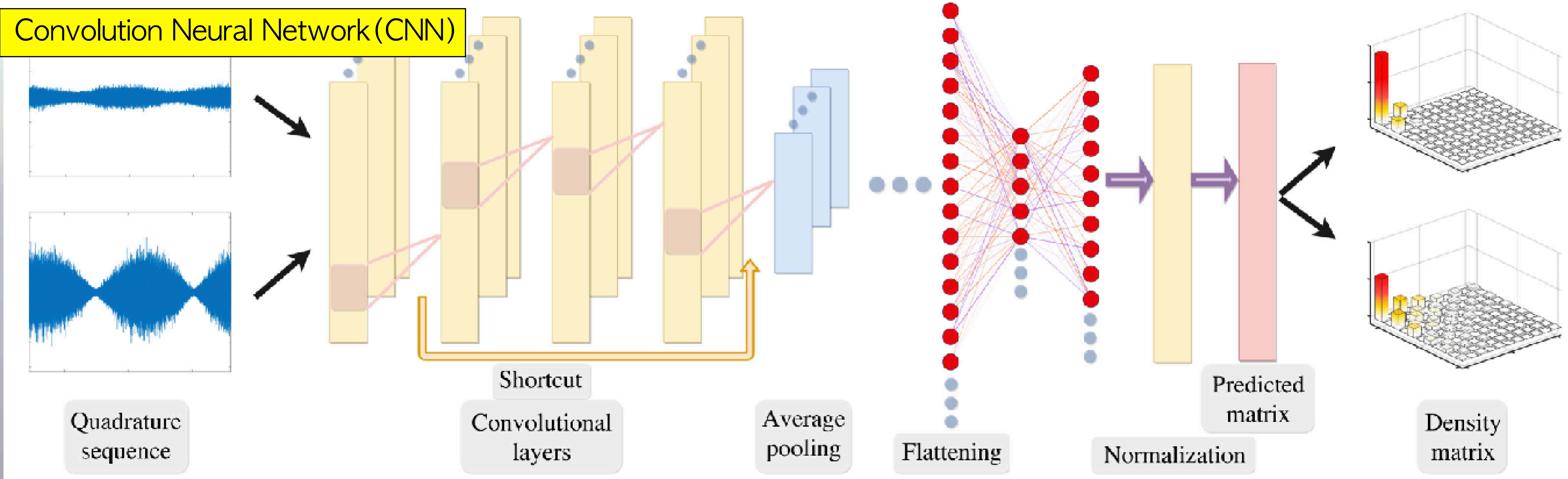
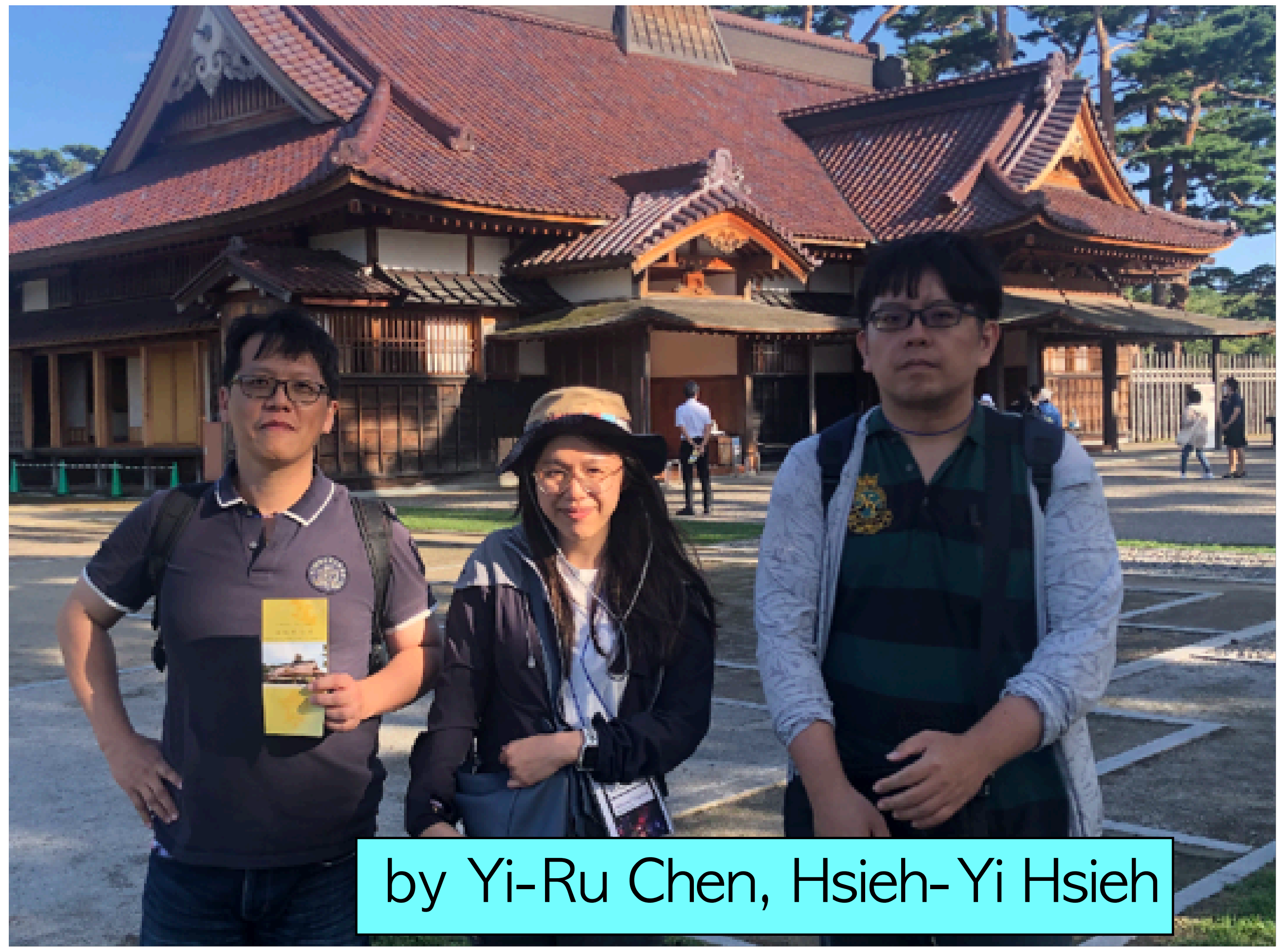


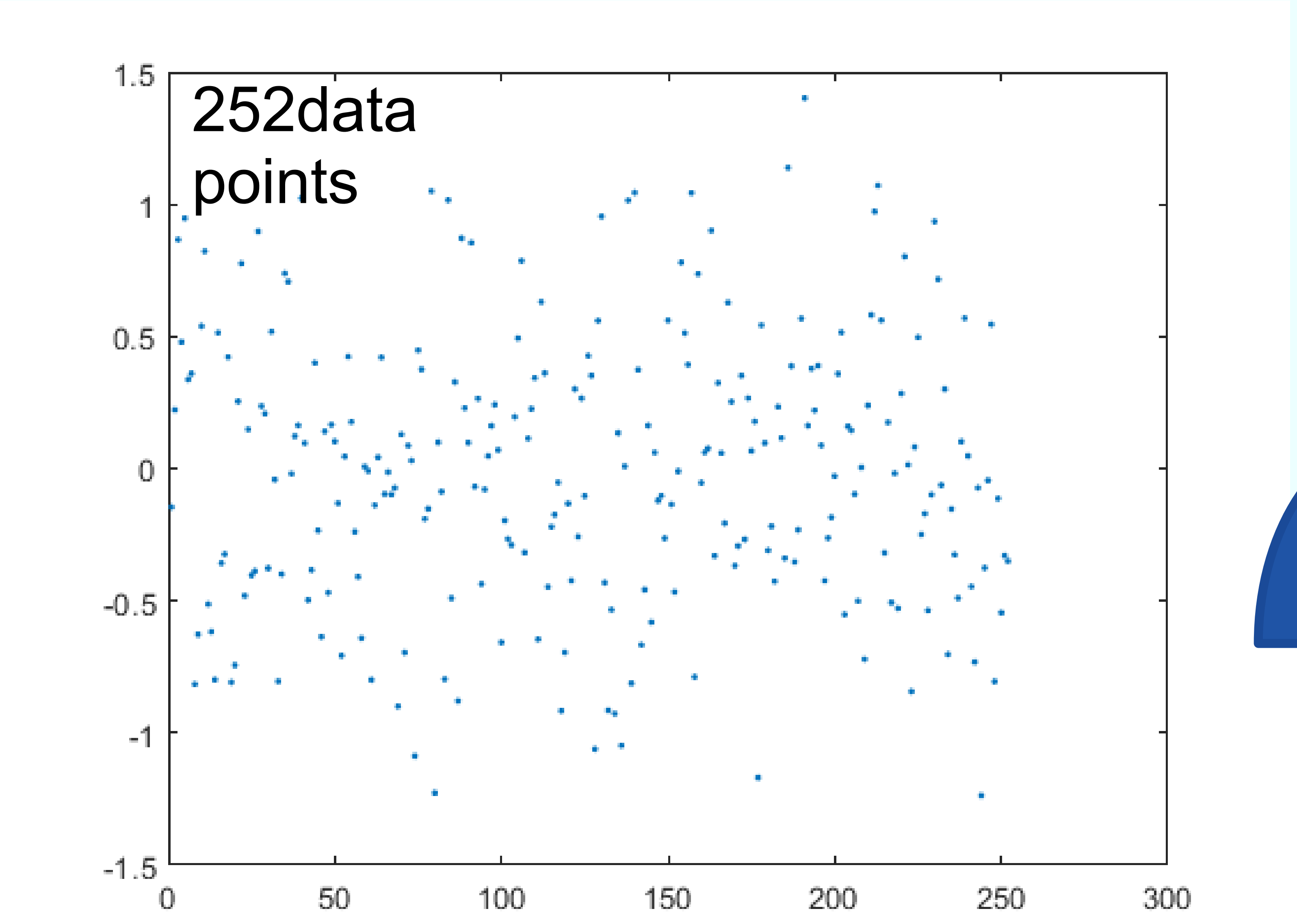
One Scan !?

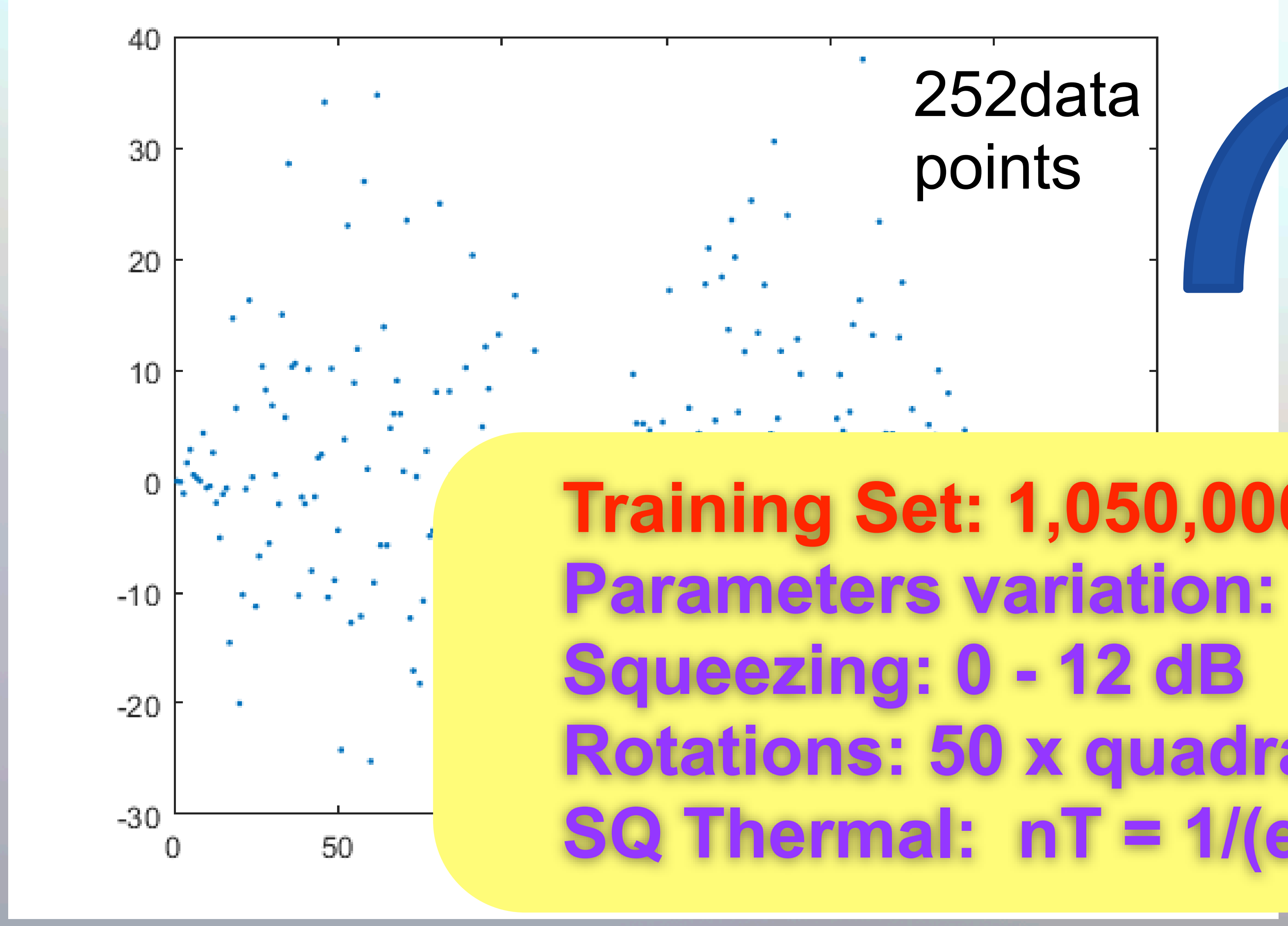
Pattern Recognition & Machine Learning:




by Yi-Ru Chen, Hsieh-Yi Hsieh

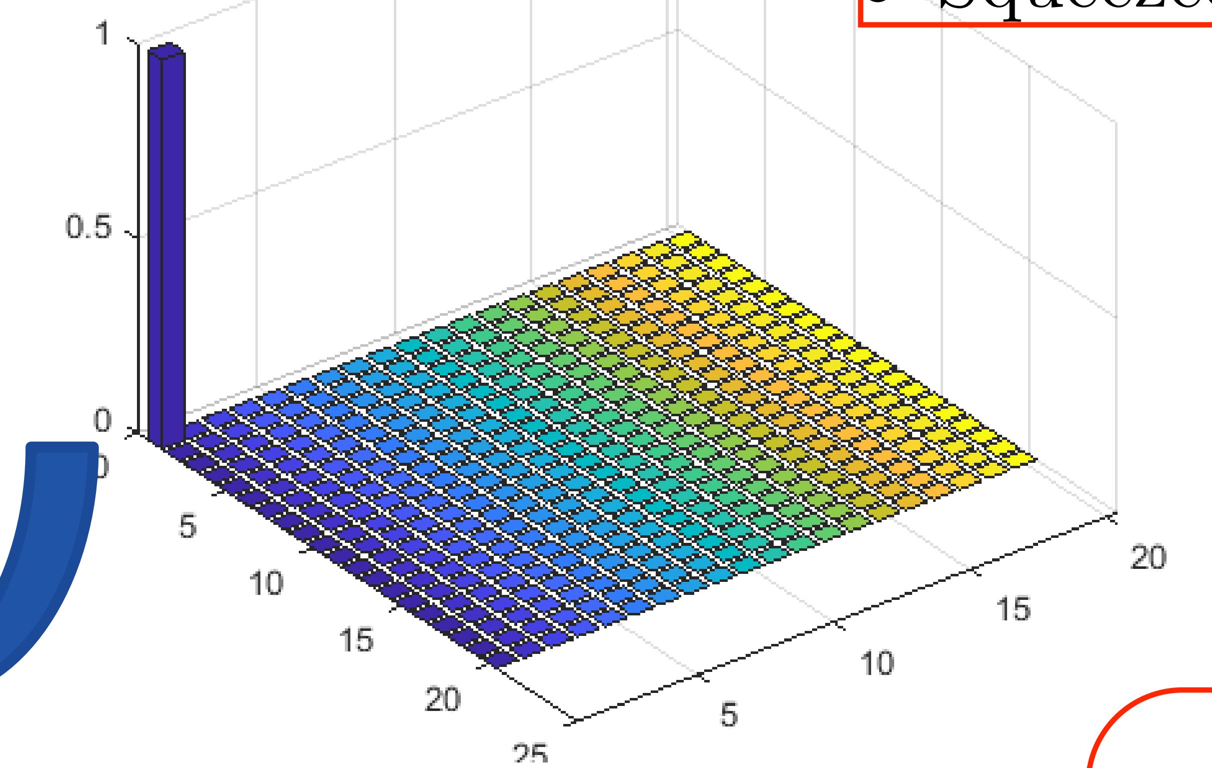
Machine Learning (SQ Learner):

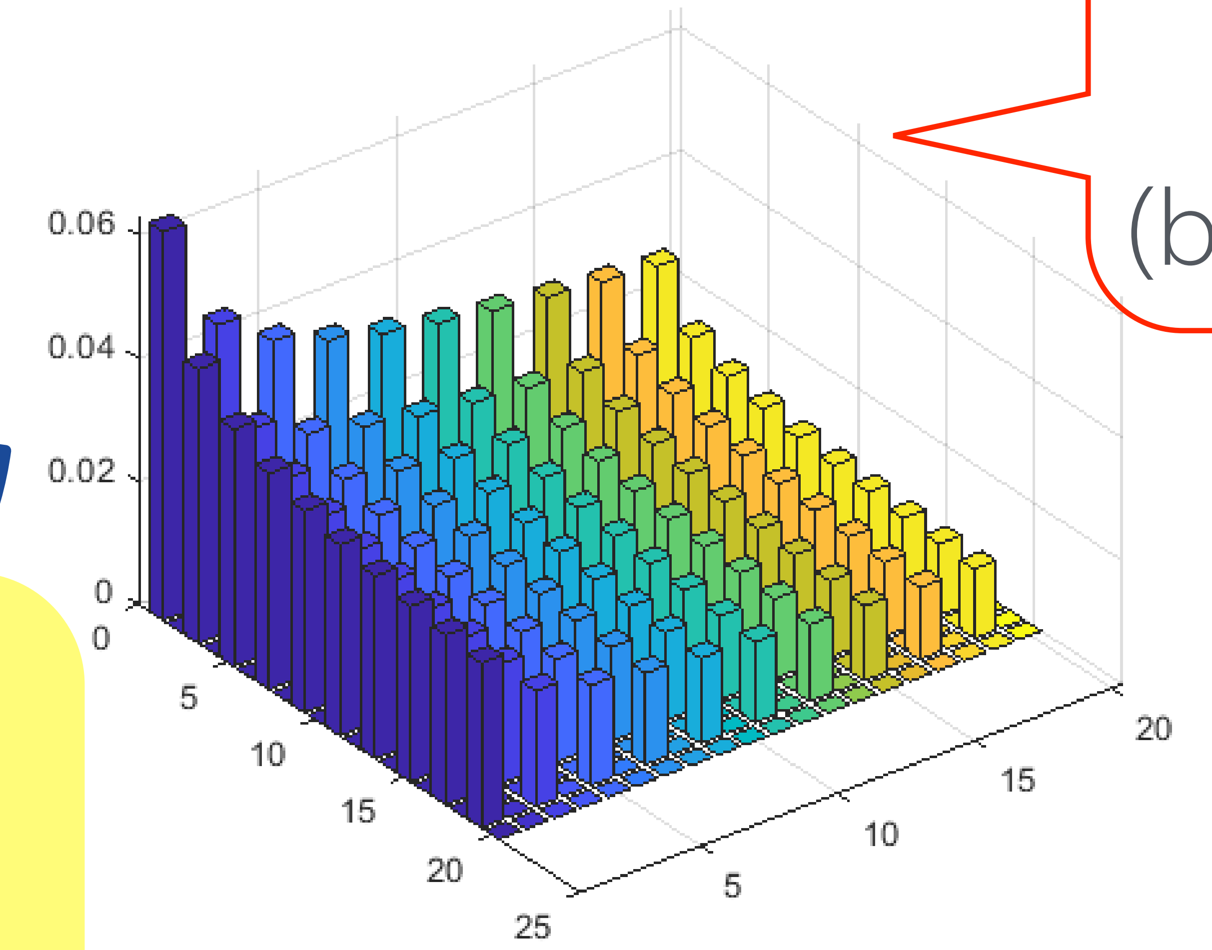




SQ learner

- Squeezed Vacuum State: $\rho_{SQ} = \hat{S}\rho_0\hat{S}^\dagger$
- Squeezed Therma State: $\rho_{th} = \hat{S}\rho_{th}\hat{S}^\dagger$





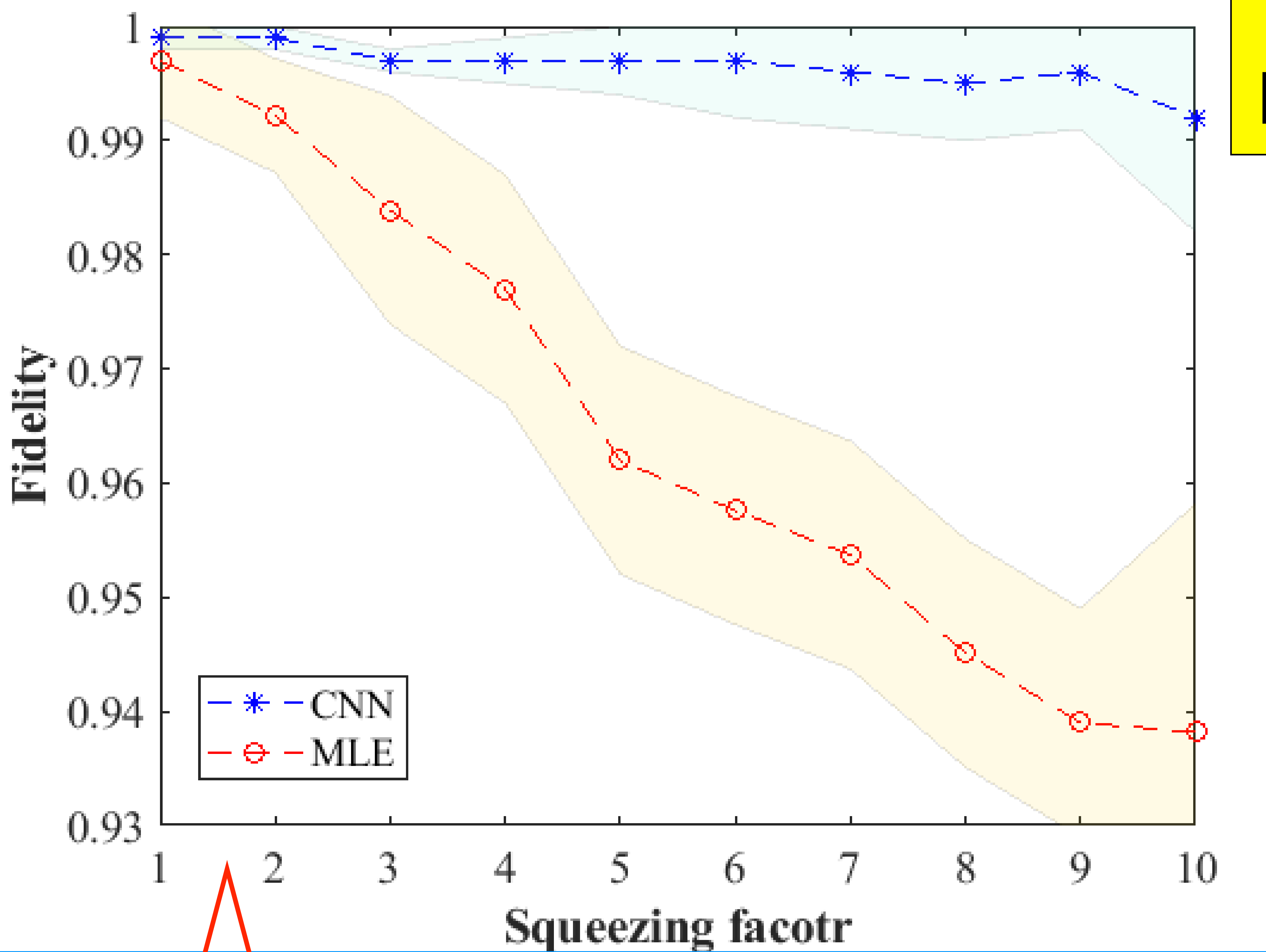
10 Hours

(but one-time) training

Training Set: 1,050,000 (1M) samples
Parameters variation:
Squeezing: 0 - 12 dB
Rotations: 50 x quadrature
SQ Thermal: nT = 1/(exp(h*f/k_BT)-1), nT : 0.1-0.6

by Hsieh-Yi Hsieh

Machine Learning (SQ Learner) vs MLE



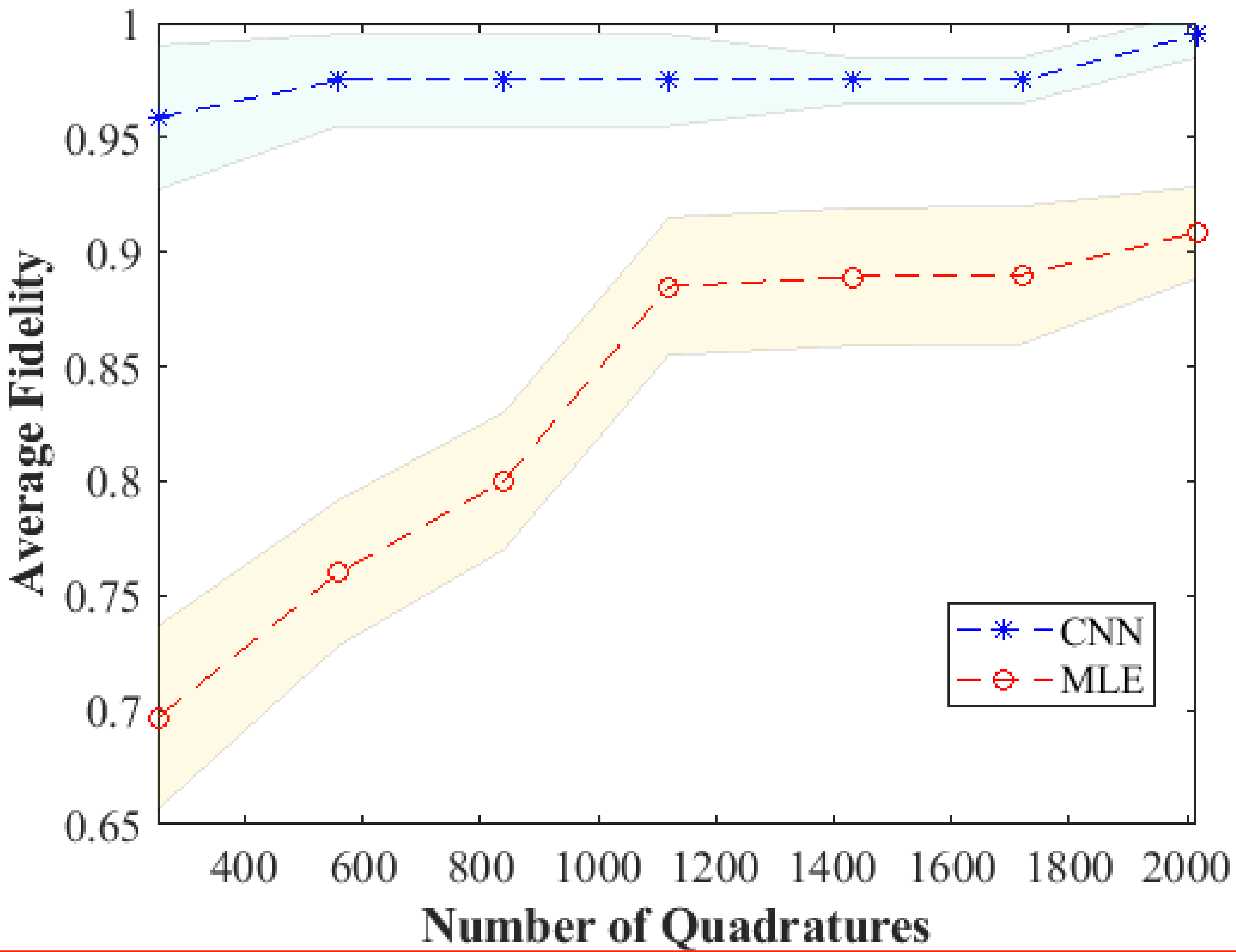
2K Data Points

at Least **several Hours** to reconstruct wavefunction

Fidelity:
$$F(\rho,\sigma) \equiv [\mathrm{Tr}\{\sqrt{\sqrt{\rho},\sigma\sqrt{\rho}}\}]^2$$

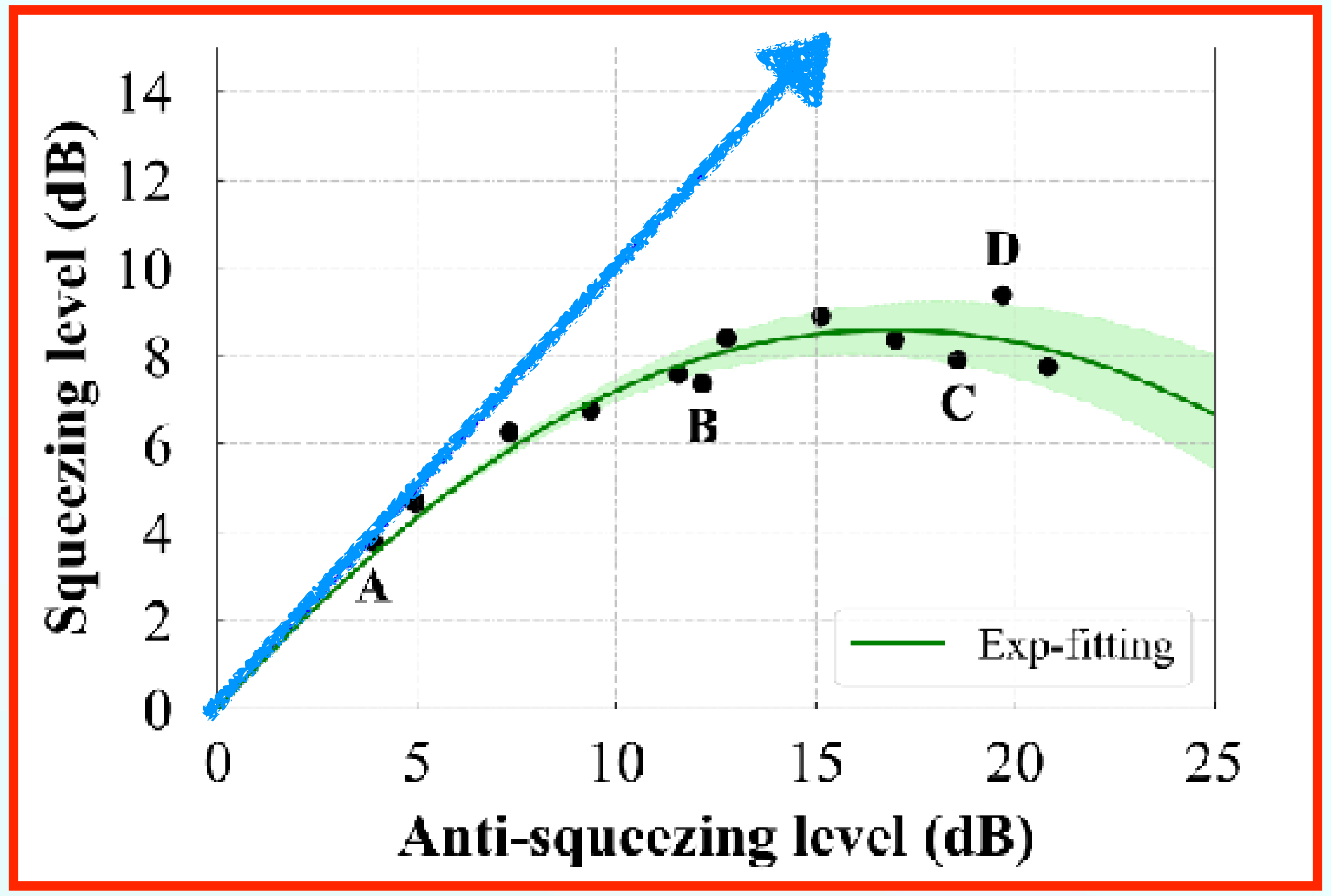
in **less than milli seconds (ms)**

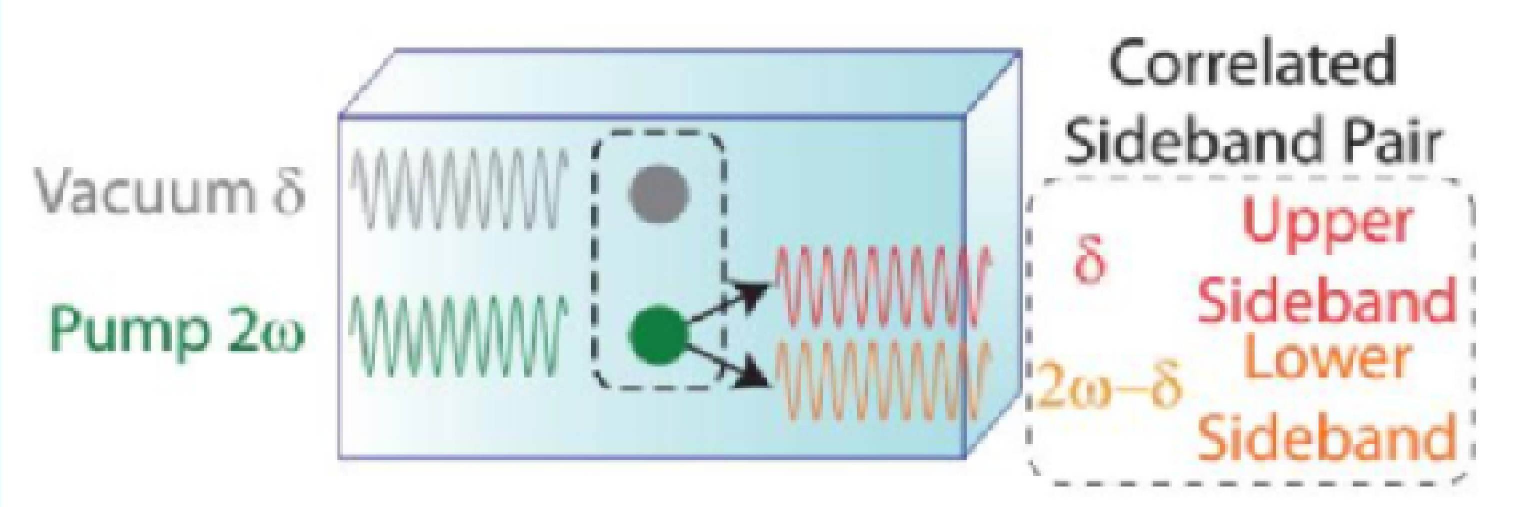
8 dB Squeezing



Hsieh-Yi Hsieh, Yi-Ru Chen, et al., Phys. Rev. Lett. 128, 073604 (2022).

Degradation: Loss and Phase noise



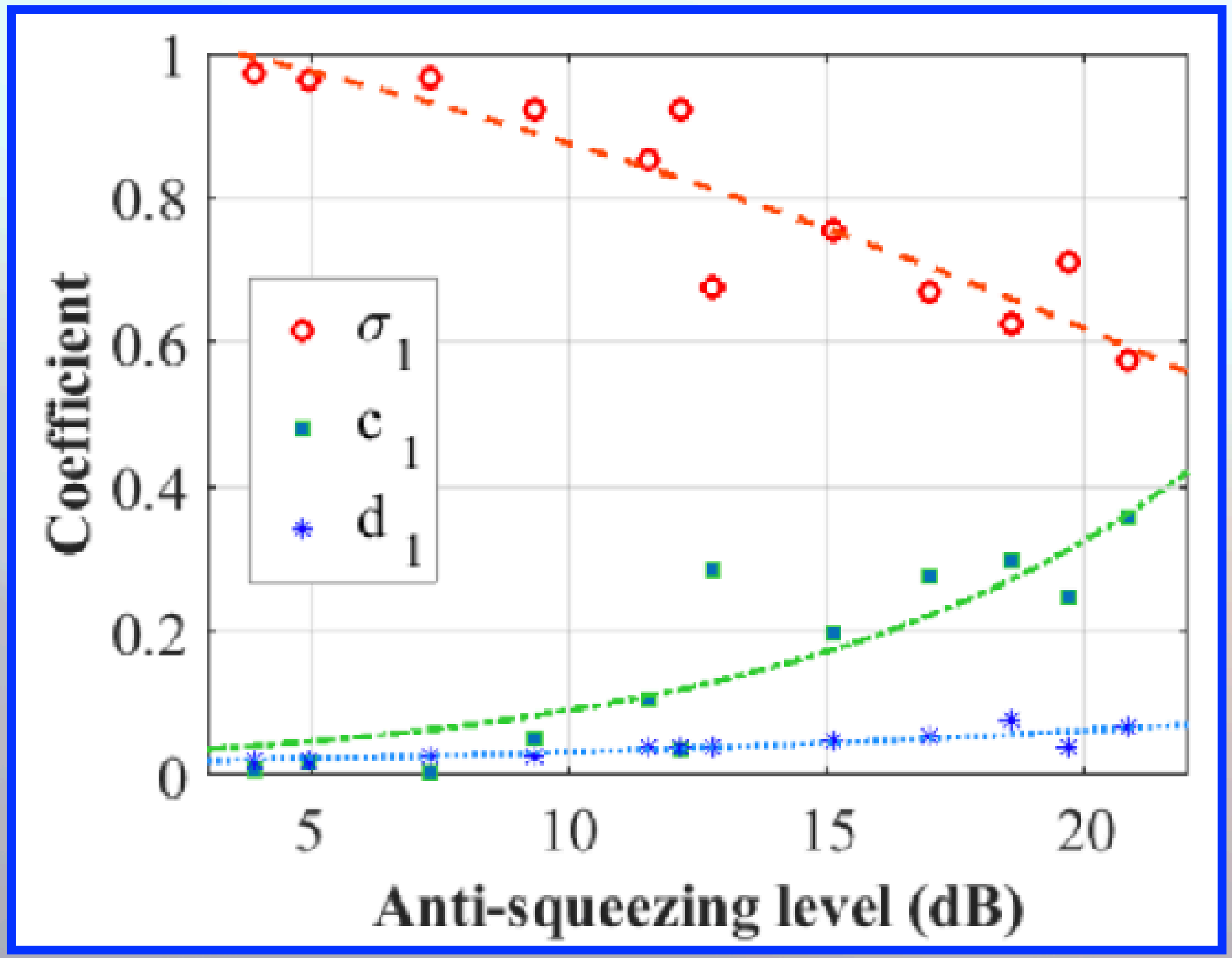


Loss: L (17.80+/-0.35%)

Phase noise: θ (34.50+/-1.26 mrad)

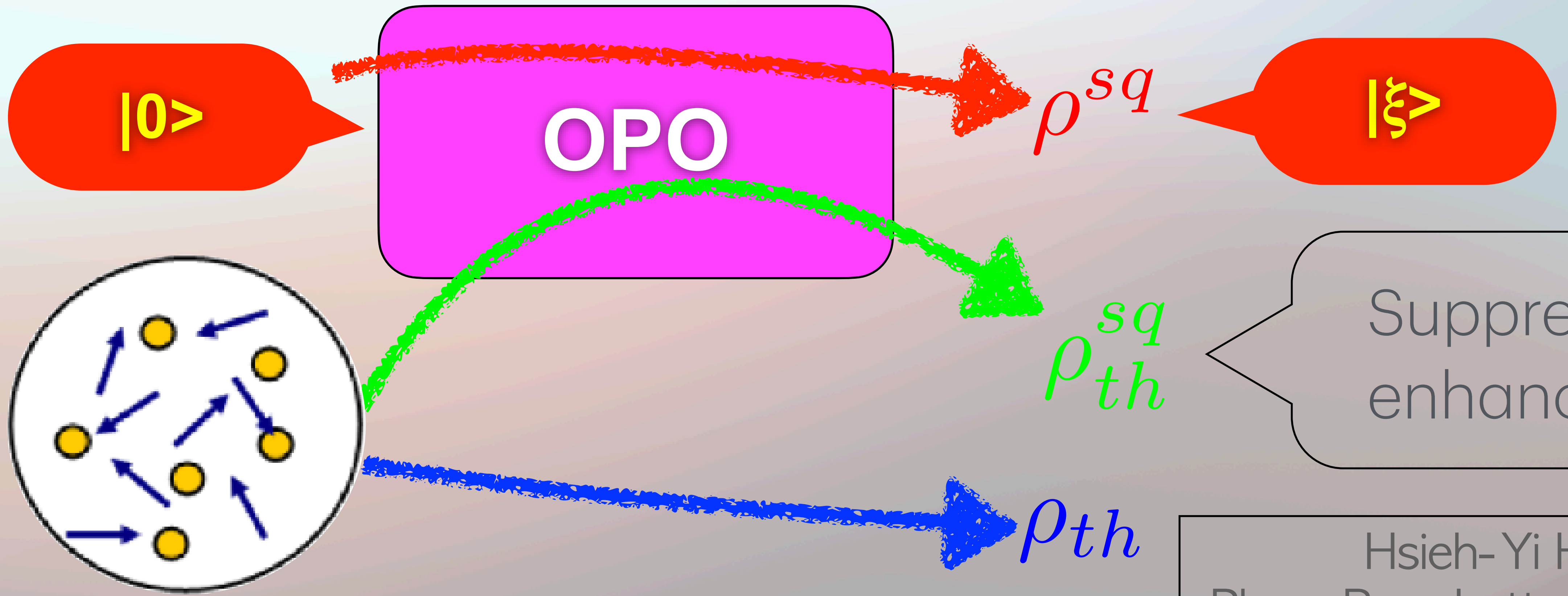
$$V^{\mathrm{sq}} = (1-L)[V^{\mathrm{sq}}_{\mathrm{id}} \times \cos^2\theta + V^{\mathrm{as}}_{\mathrm{id}} \times \sin^2\theta] + L,$$

$$V^{\mathrm{as}} = (1-L)[V^{\mathrm{as}}_{\mathrm{id}} \times \cos^2\theta + V^{\mathrm{sq}}_{\mathrm{id}} \times \sin^2\theta] + L,$$



$$\rho = \sigma_1\, \rho^{sq} + c_1\, \rho^{sq}_{th} + d_1\, \rho_{th}$$

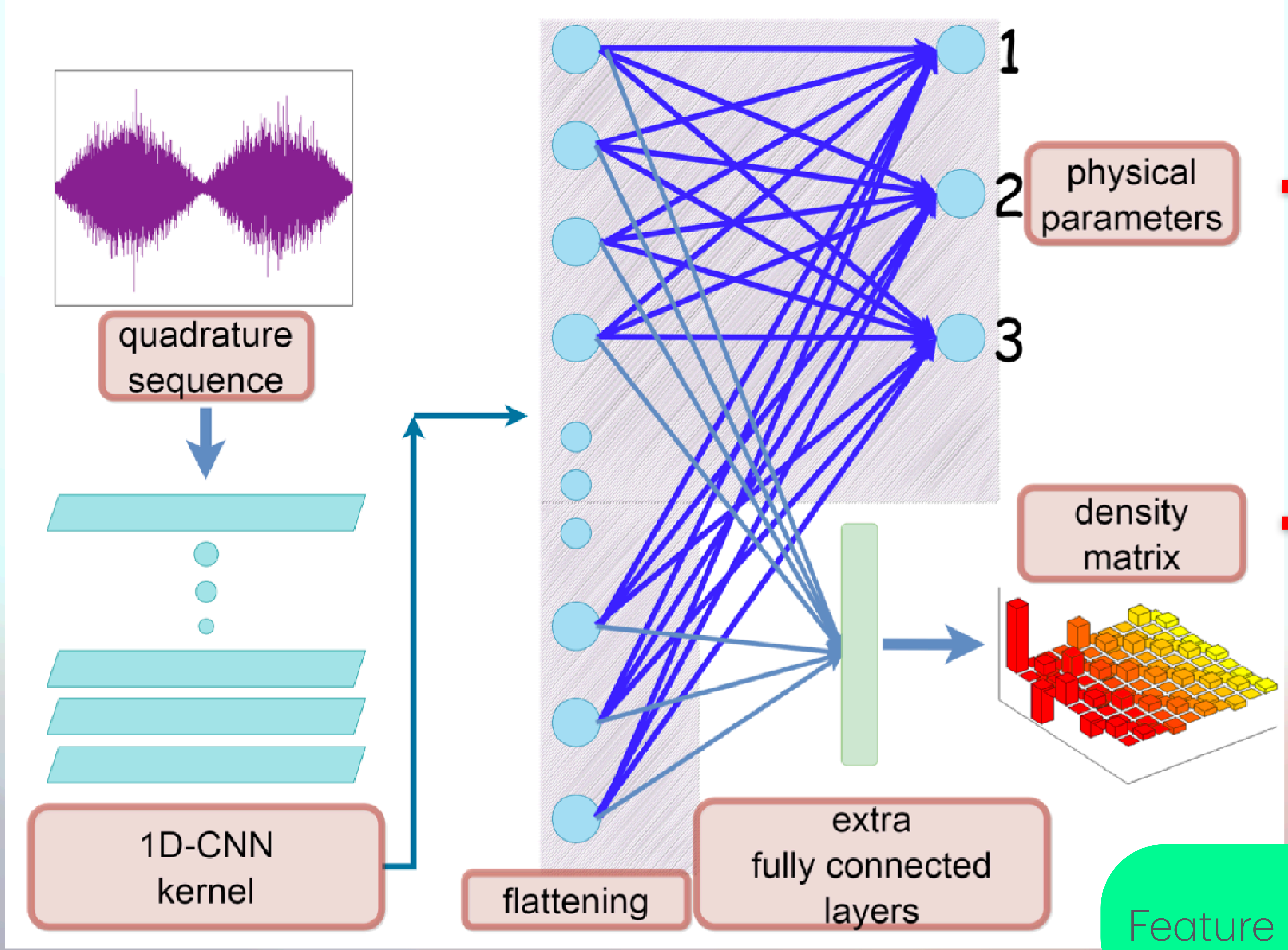
SVD

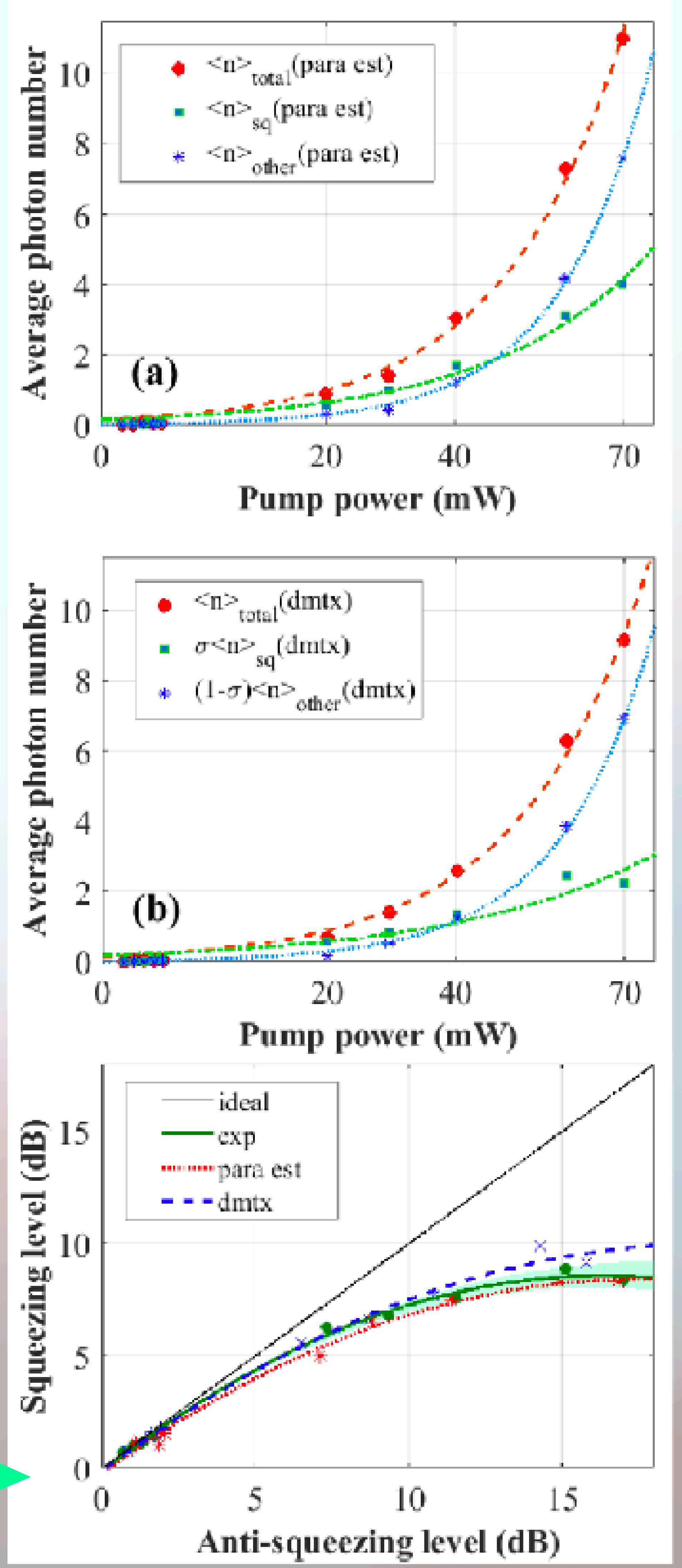


Suppress it to enhance Squeezing

Hsieh-Yi Hsieh, Yi-Ru Chen, et al., Phys. Rev. Lett. 128, 073604 (2022).

ML-QST: Direct Parameter Estimations

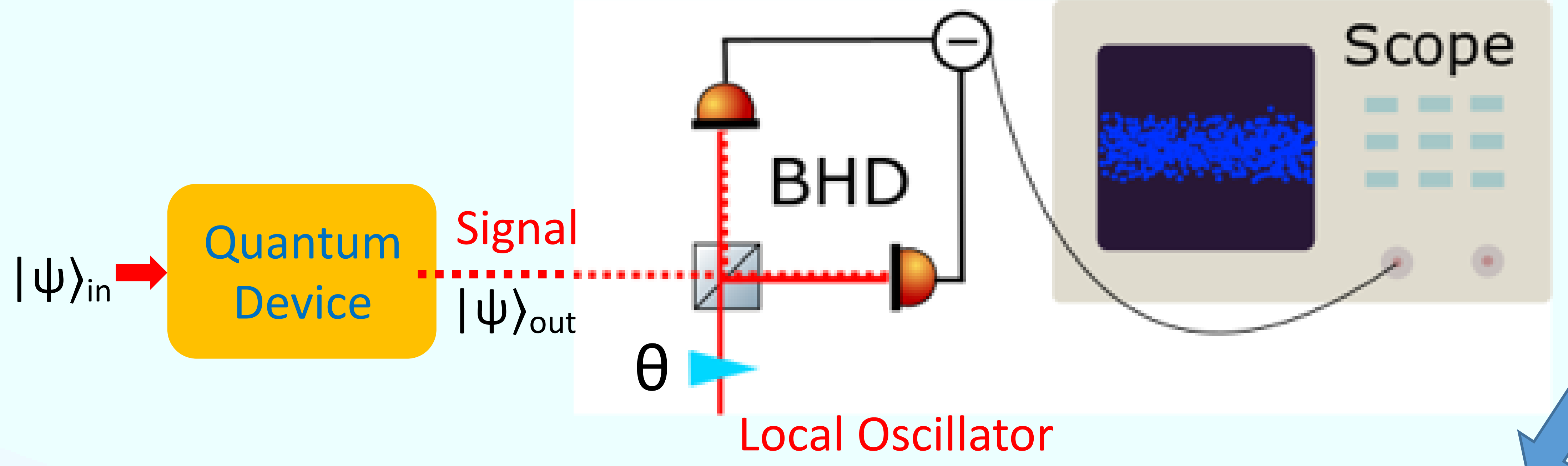


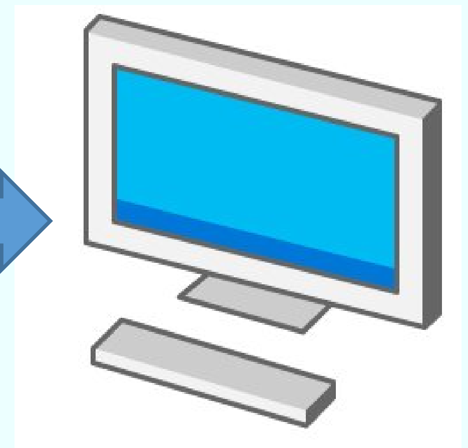


Feature Extraction

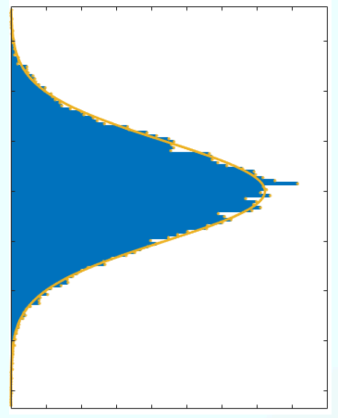
H.-Y. Hsieh, et al., Symmetry 14, 874(2022);
Special Issue "Quantum Optimization & Machine Learning";

ML-enhanced QST:



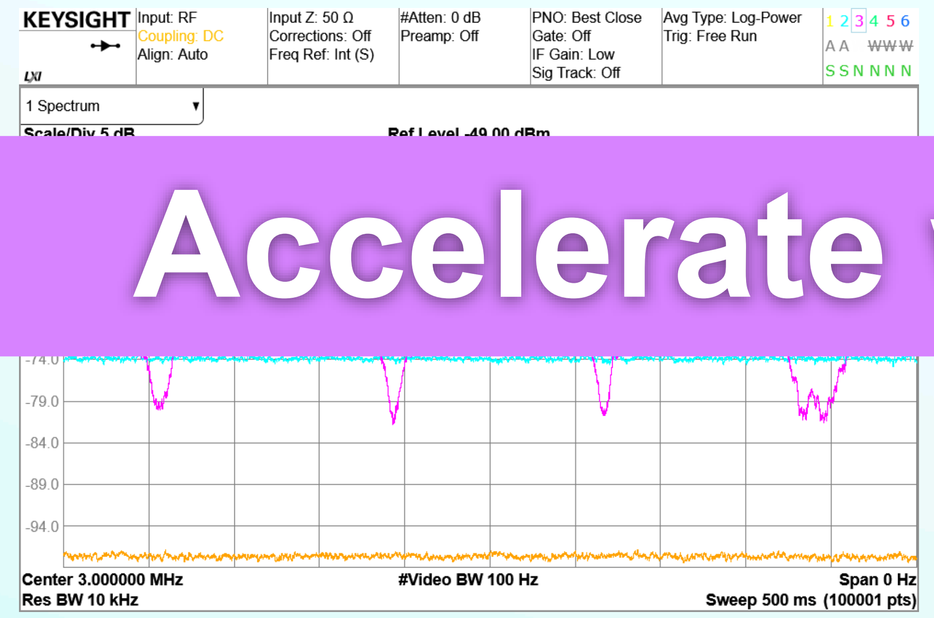


Probability distribution
Pr(q, θ)



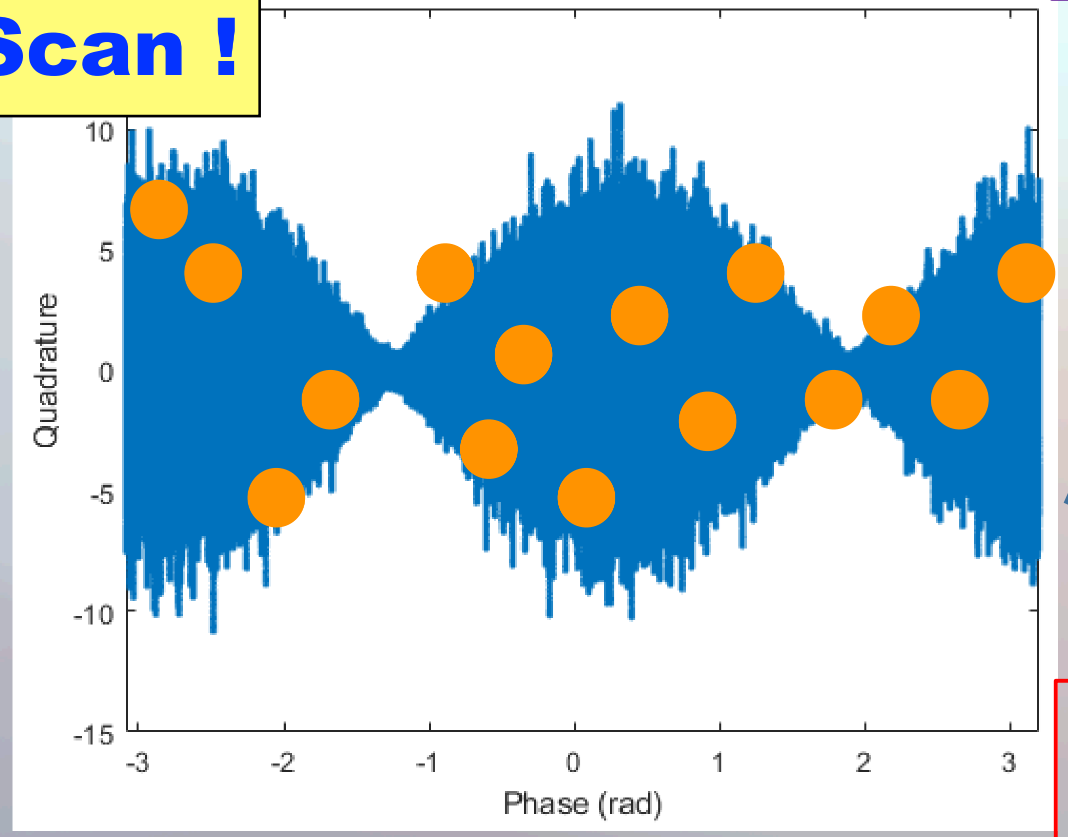
by Hsieh-Yi Hsieh

Reconstruction Algorithms



Accelerate with ML, but also Re-use training data

One Single Scan !

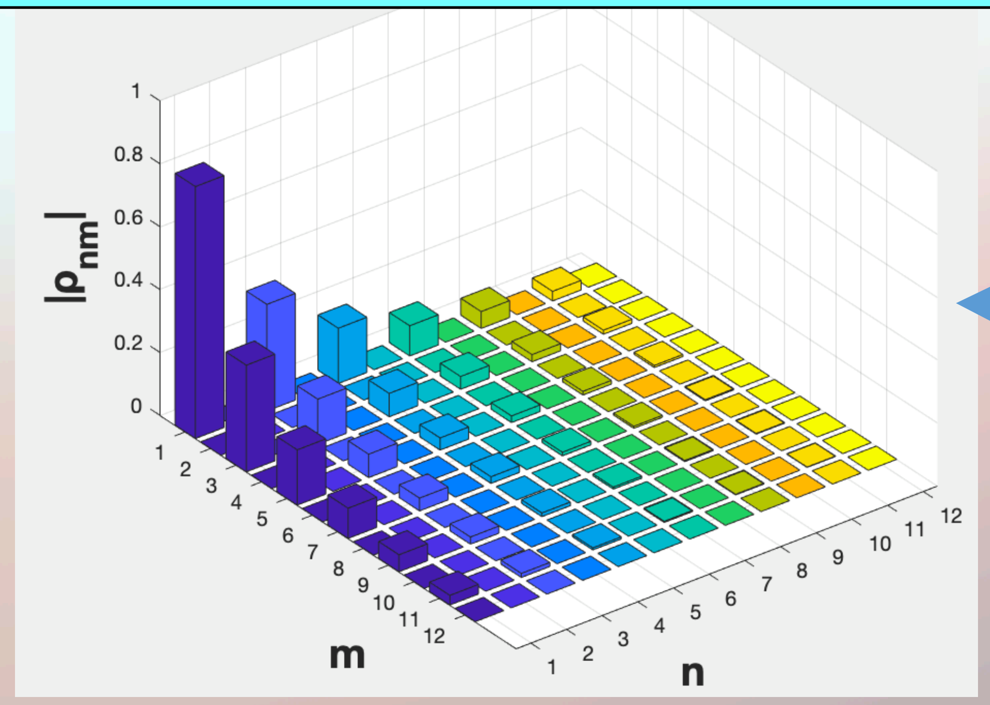


few minutes to reconstruct

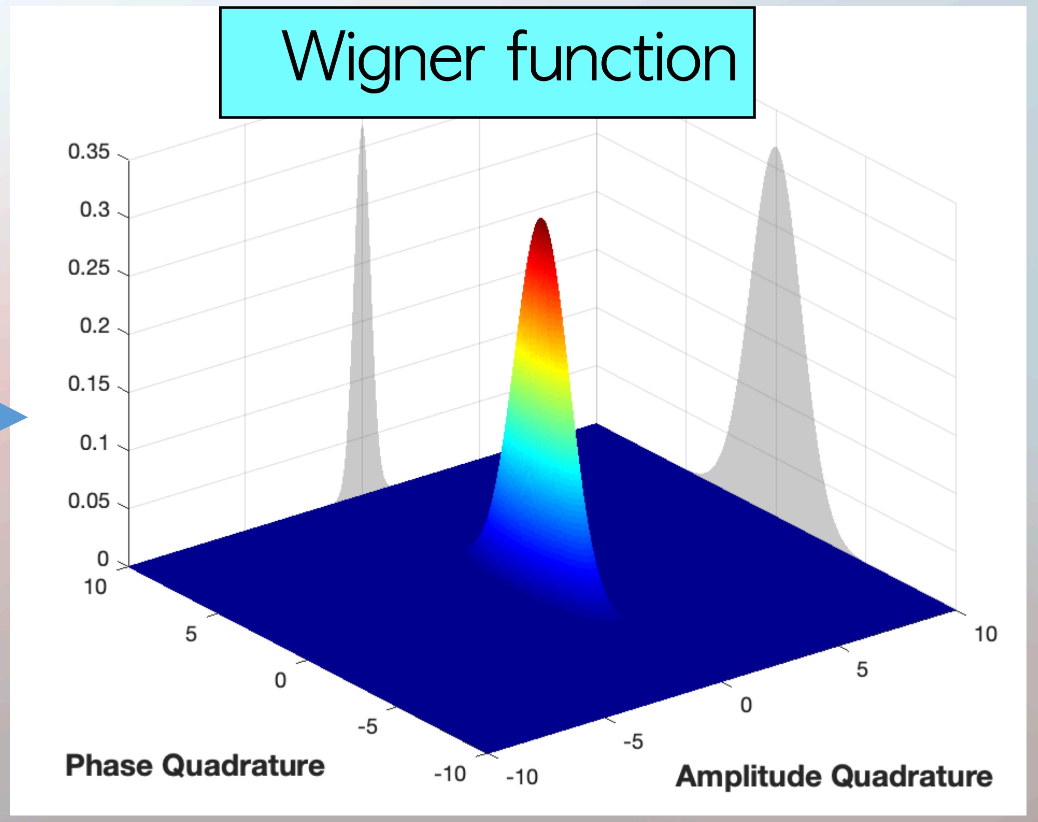
< 1s to reconstruct

Max. Likelihood Estimation, MLE

Density matrix in number basis



Wigner function



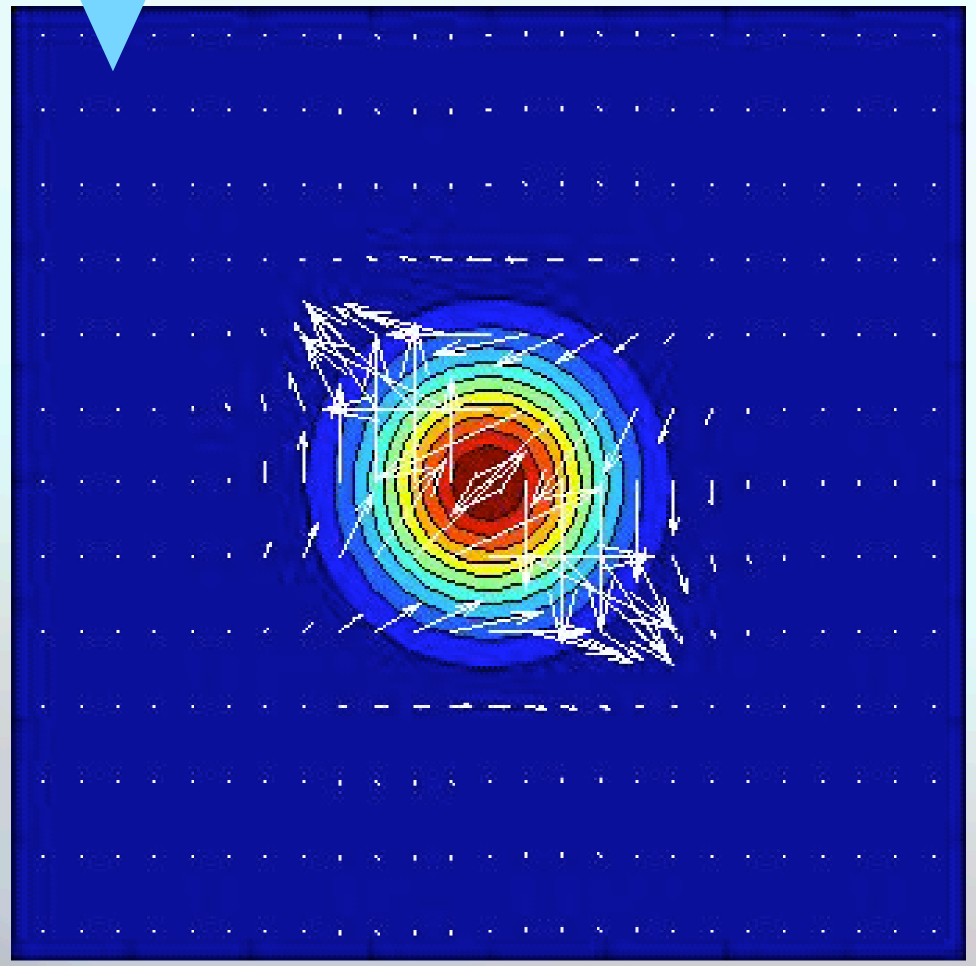
Machine Learning (SQ Learner), CNN

Copenhagen interpretation:

Wigner quasi-probability distribution:

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \langle x - y|\hat{\rho}|x + y\rangle e^{2ipy/\hbar}\, dy$$

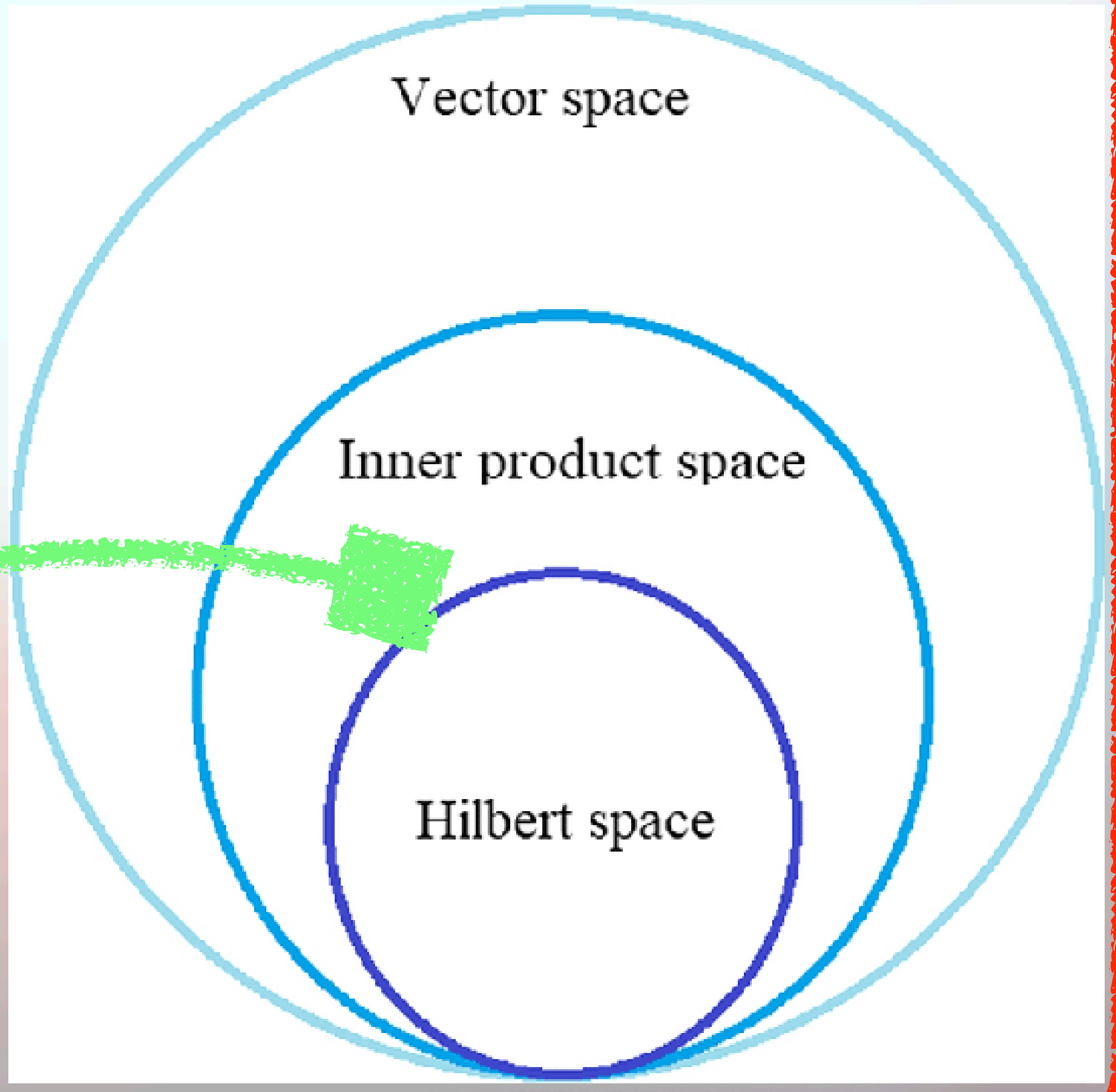
Squeezed States (CV)
continuous variables



Let’s Take a Video !!

物理現實
Physical Reality

抽象數學
Abstract Mathematics



$|0\rangle$

$|1\rangle$

$\vdots$

$|n\rangle$

$|\alpha\rangle$

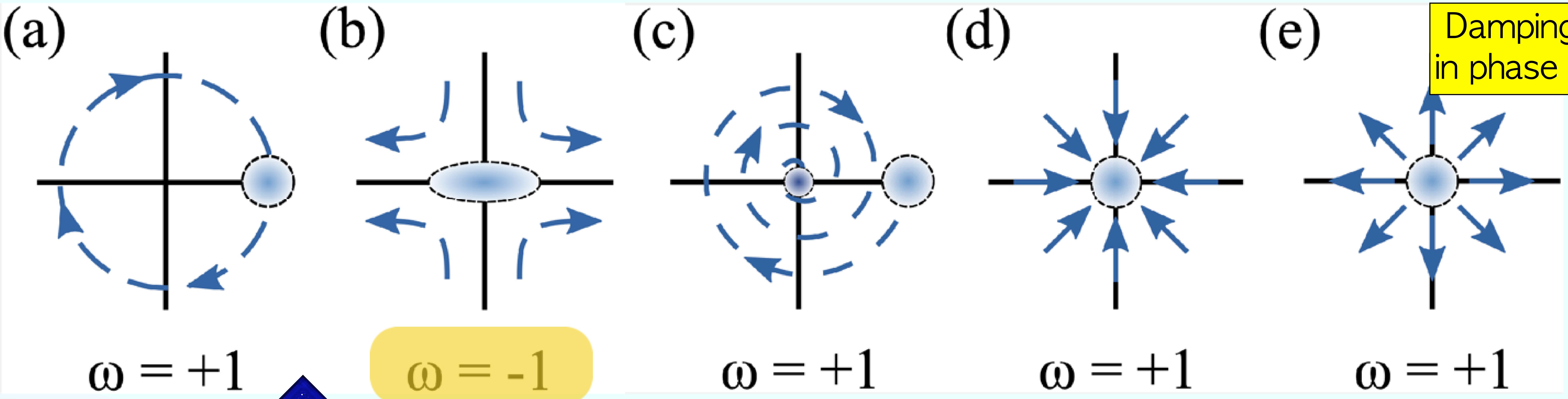
$|\xi\rangle$

$|\alpha\rangle \pm |-\alpha\rangle$

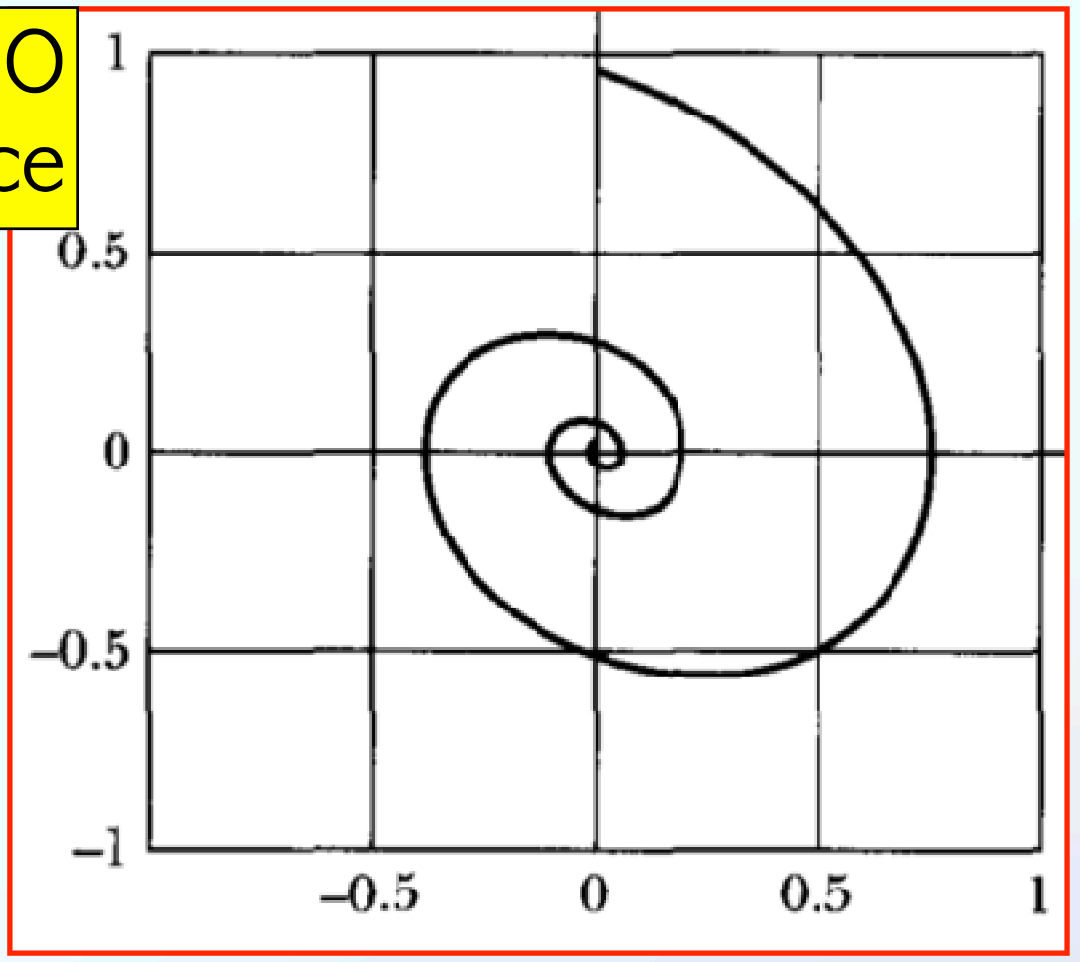
$\vdots$

ρ_{th}

Damped SHO: Wigner Currents in Decoherence



Damping SHO in phase space

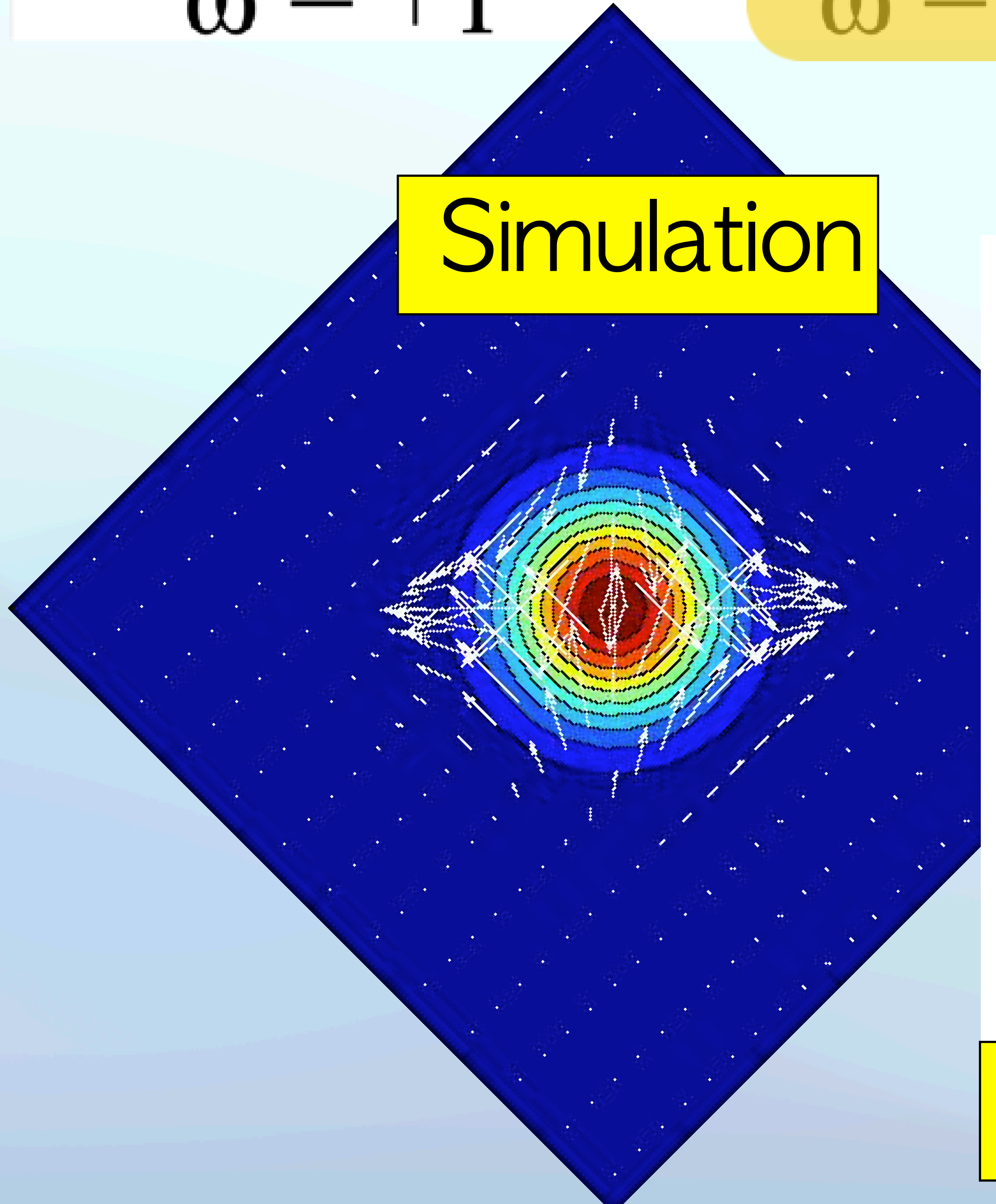


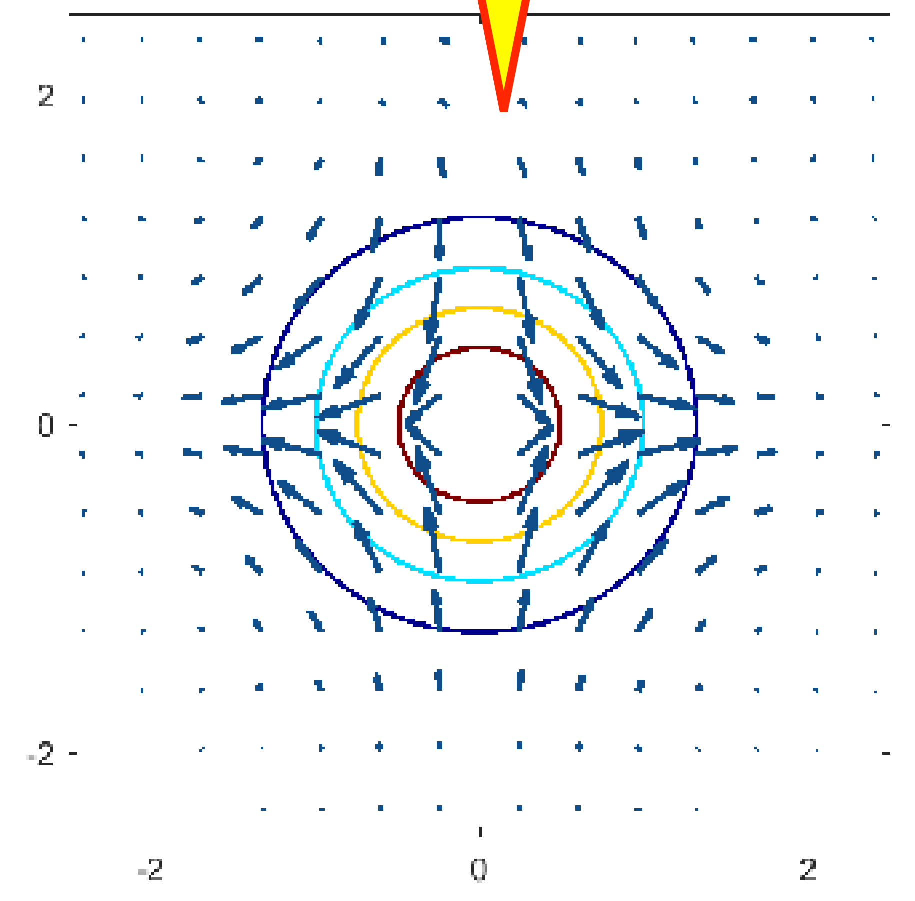
Exp. Reconstruction

Damping Current

Diffusive Current

Simulation





The First Experimental Reconstruction

Yi-Ru Chen et al., Phys. Rev. A 108, 023729 (2023).

Damped SHO: Wigner Currents in Decoherence

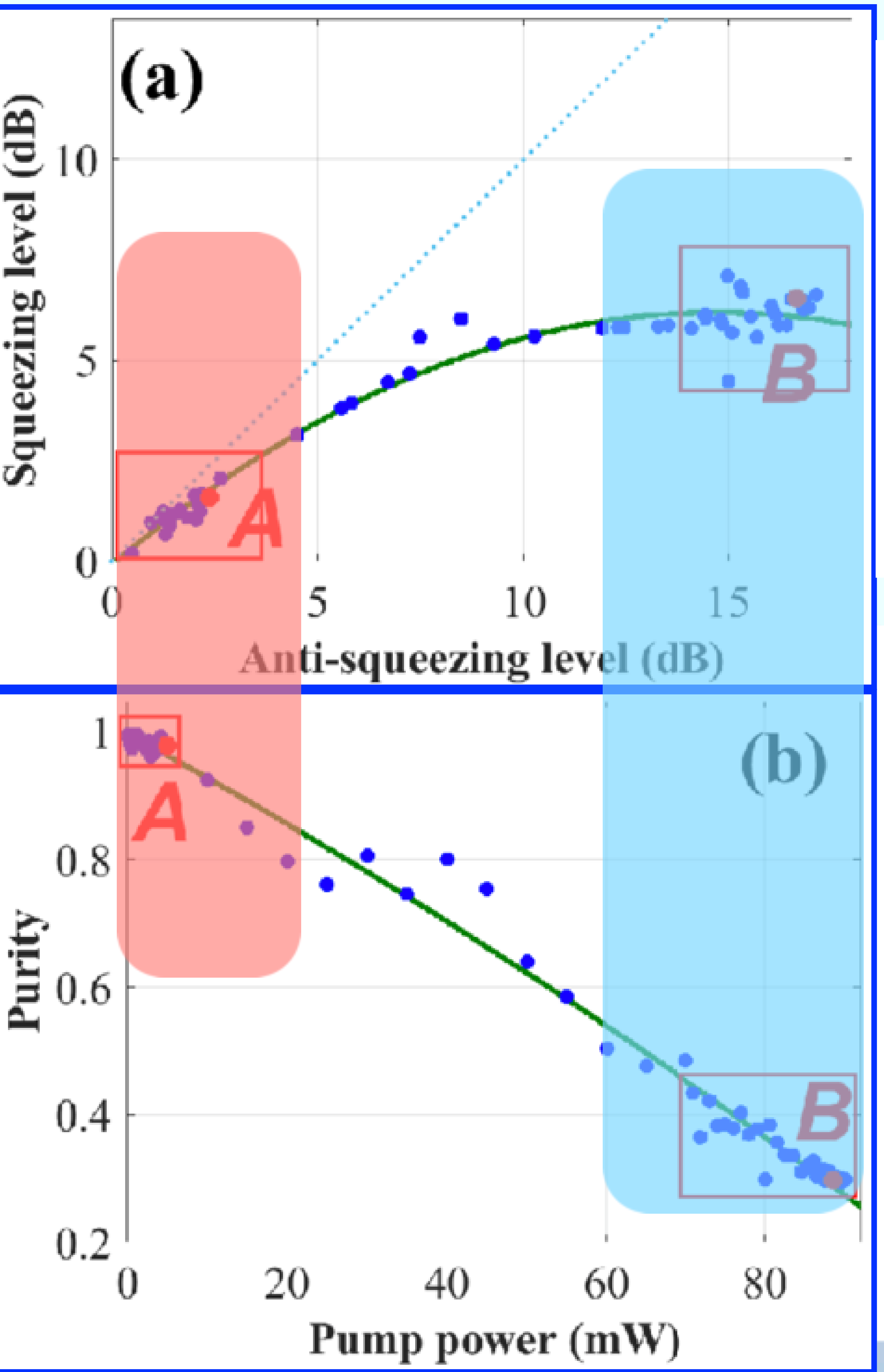
$$\hat{H} = \frac{i\hbar\chi^{(2)}}{2}(|\alpha|\hat{a}^2 - |\alpha|\hat{a}^{\dagger 2}),$$

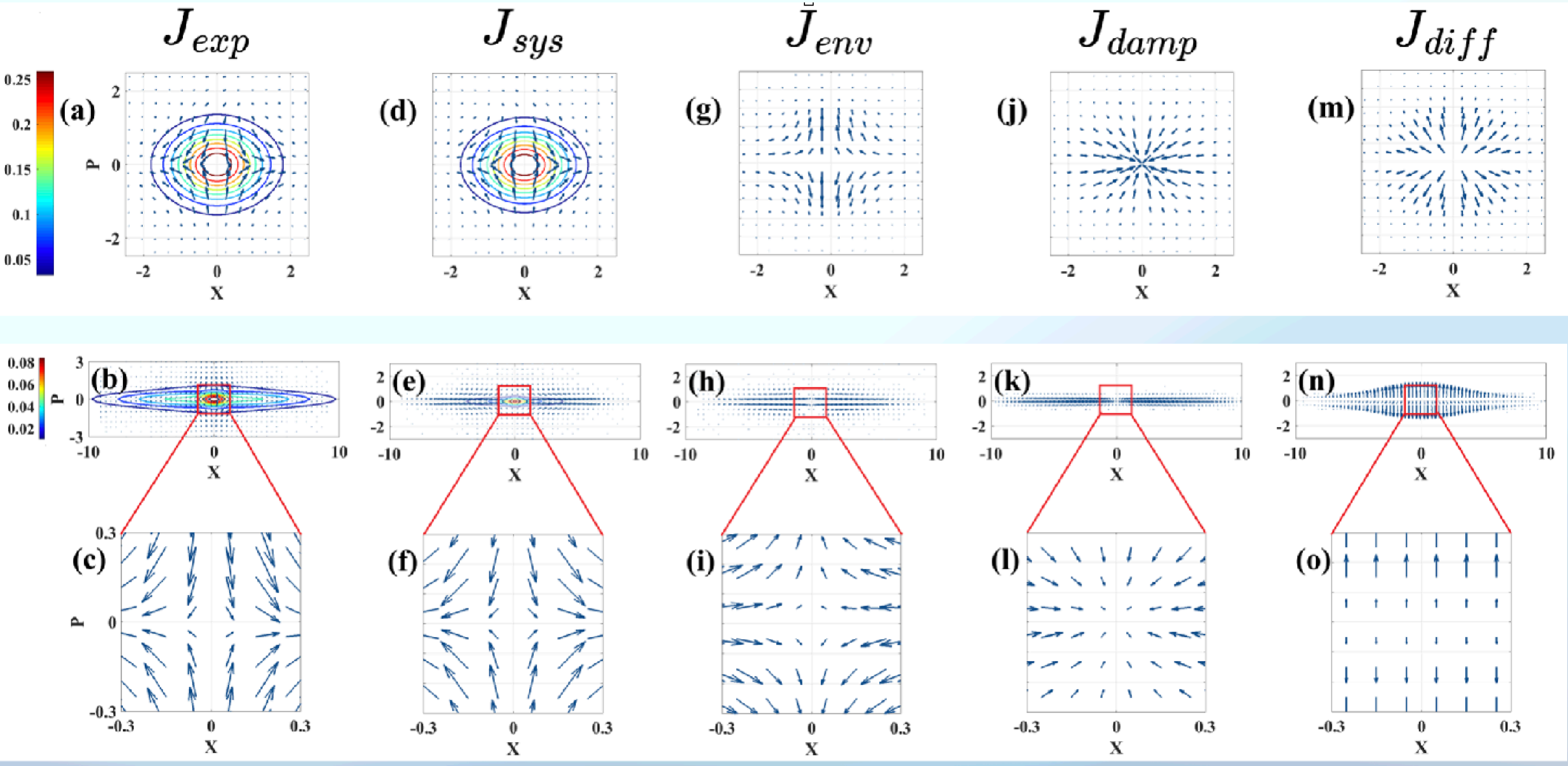
$$\hat{U}(t) = \exp[\frac{-i\hat{H}t}{\hbar}] = \exp[\frac{\chi^{(2)}|\alpha|t}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})],$$

$$\tau_{\text{eff}} \propto \chi^{(2)}|\alpha| \equiv |\xi|,$$

OPO: effective time (via Pump)

Yi-Ru Chen et al., Phys. Rev. A 108, 023729 (2023).

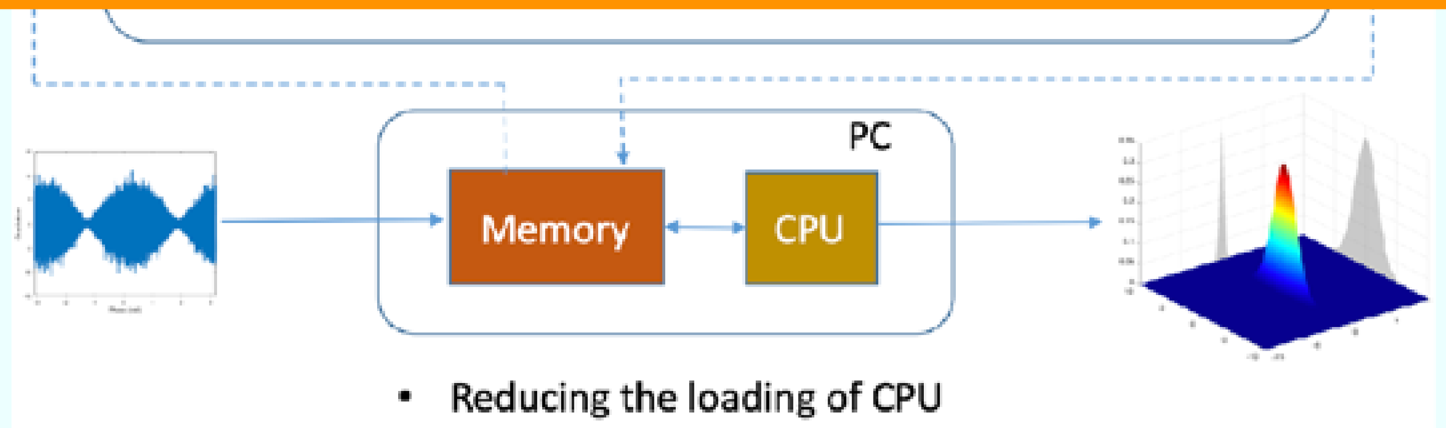


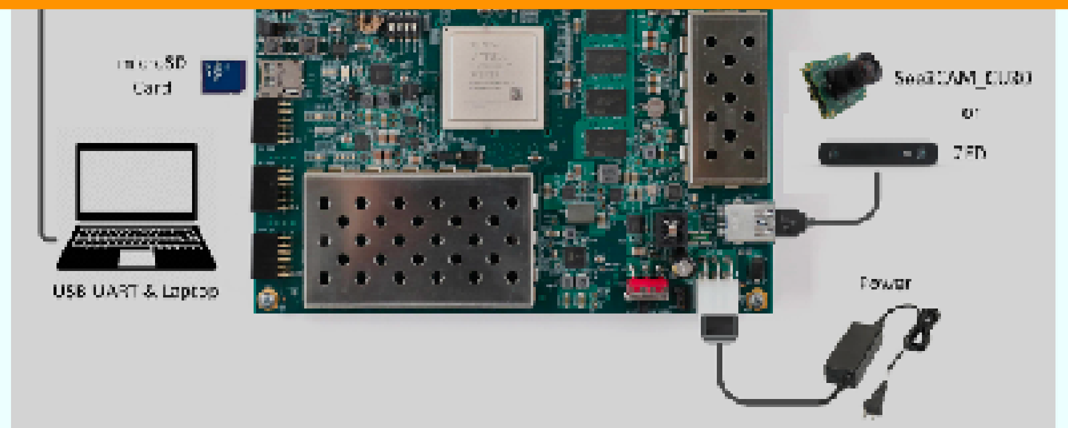


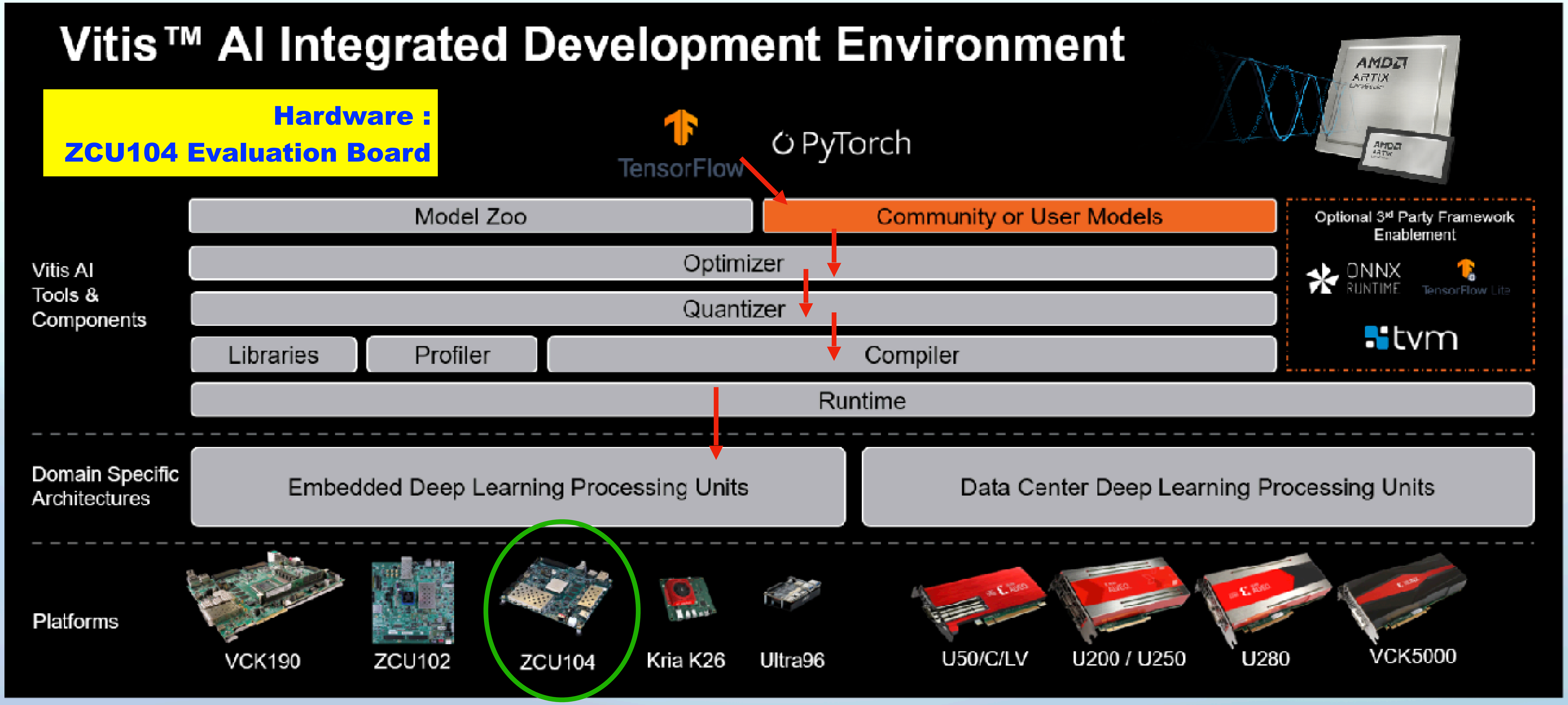
Real-time QST: with FPGA Acceleration

by Hsun-Chung Wu

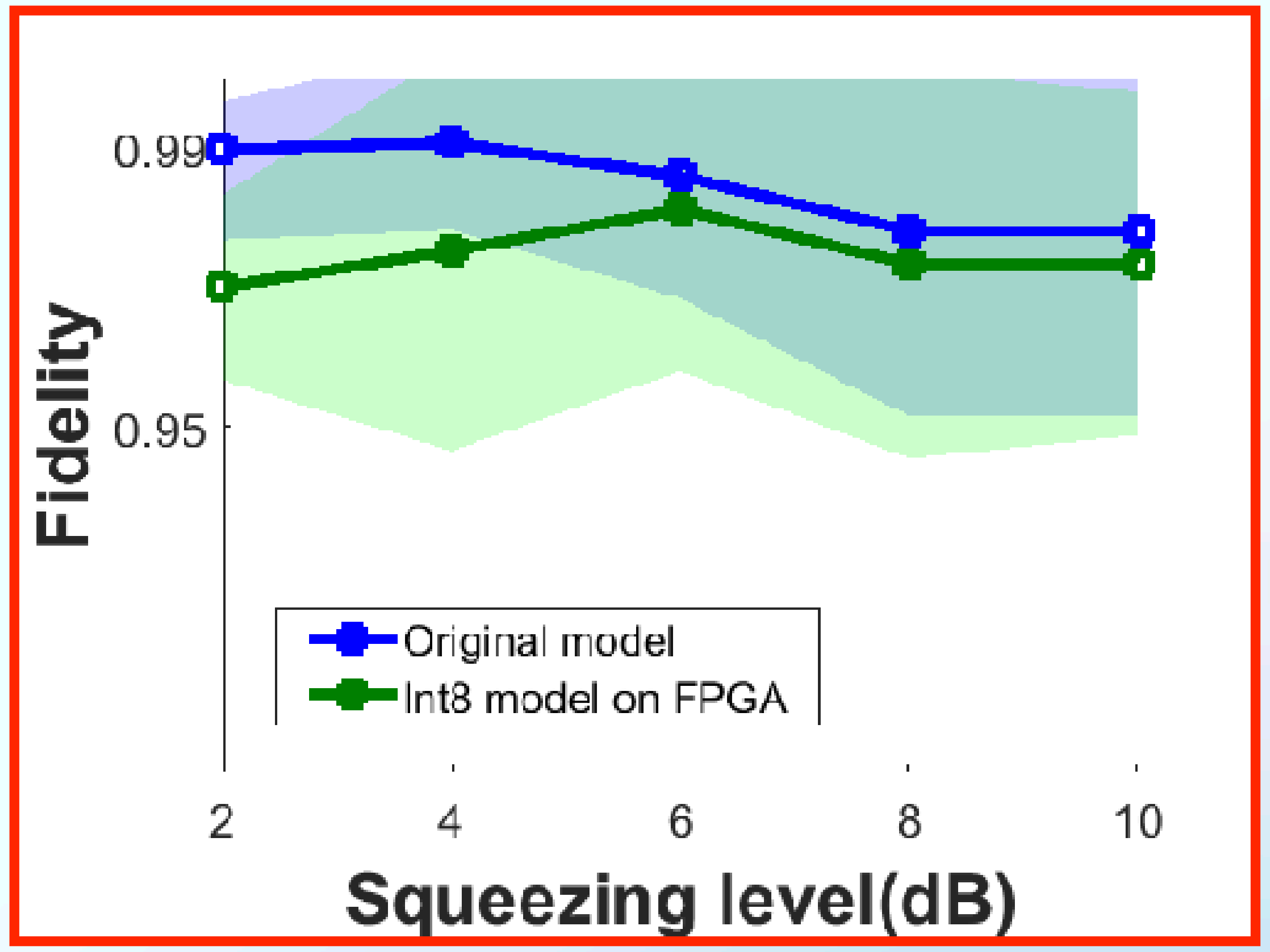
Software + Hardware 軟硬兼施

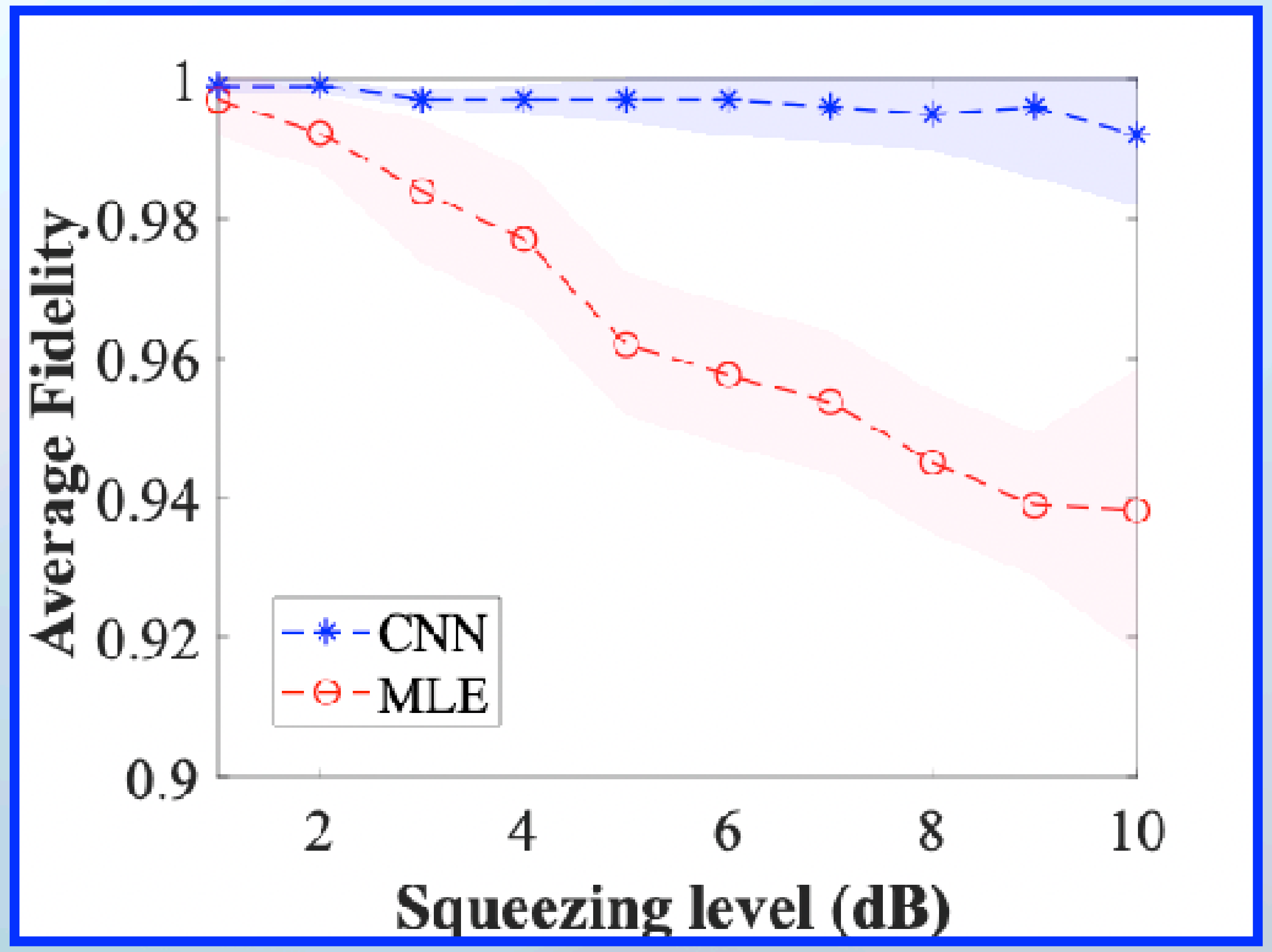






Hsun-Chung Wu et al., (in preparation, 2024).



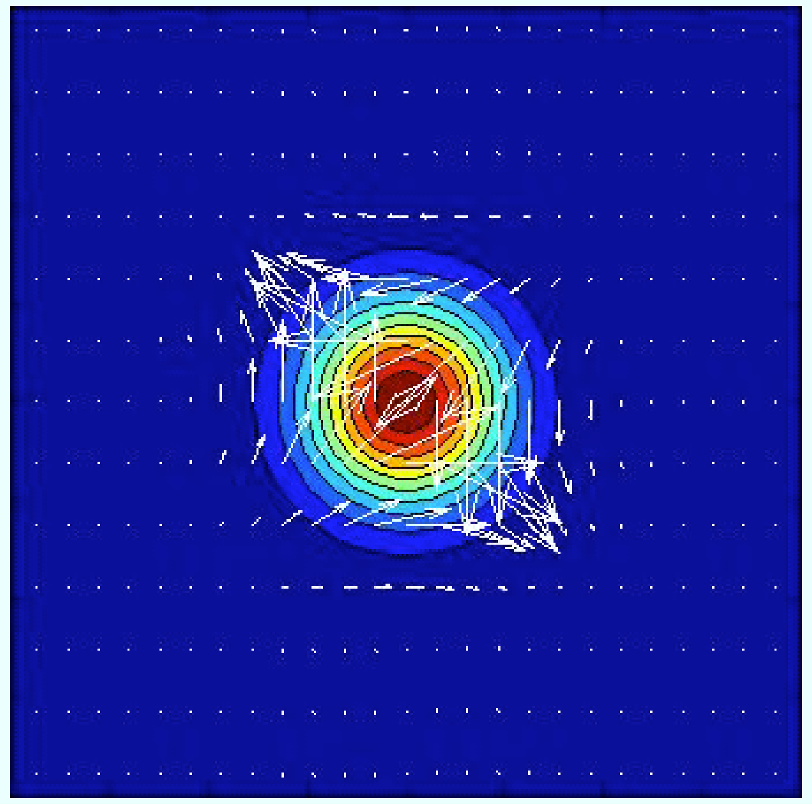


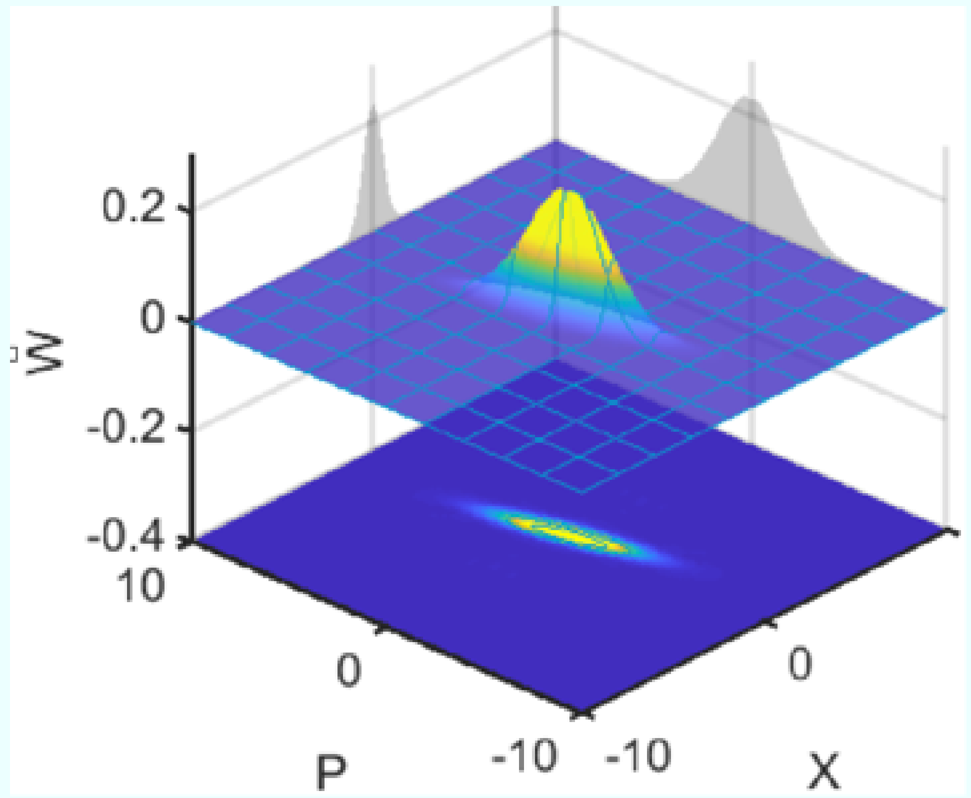
H.-Y. Hsieh, et al., Phys. Rev. Lett. 128, 073604 (2022).

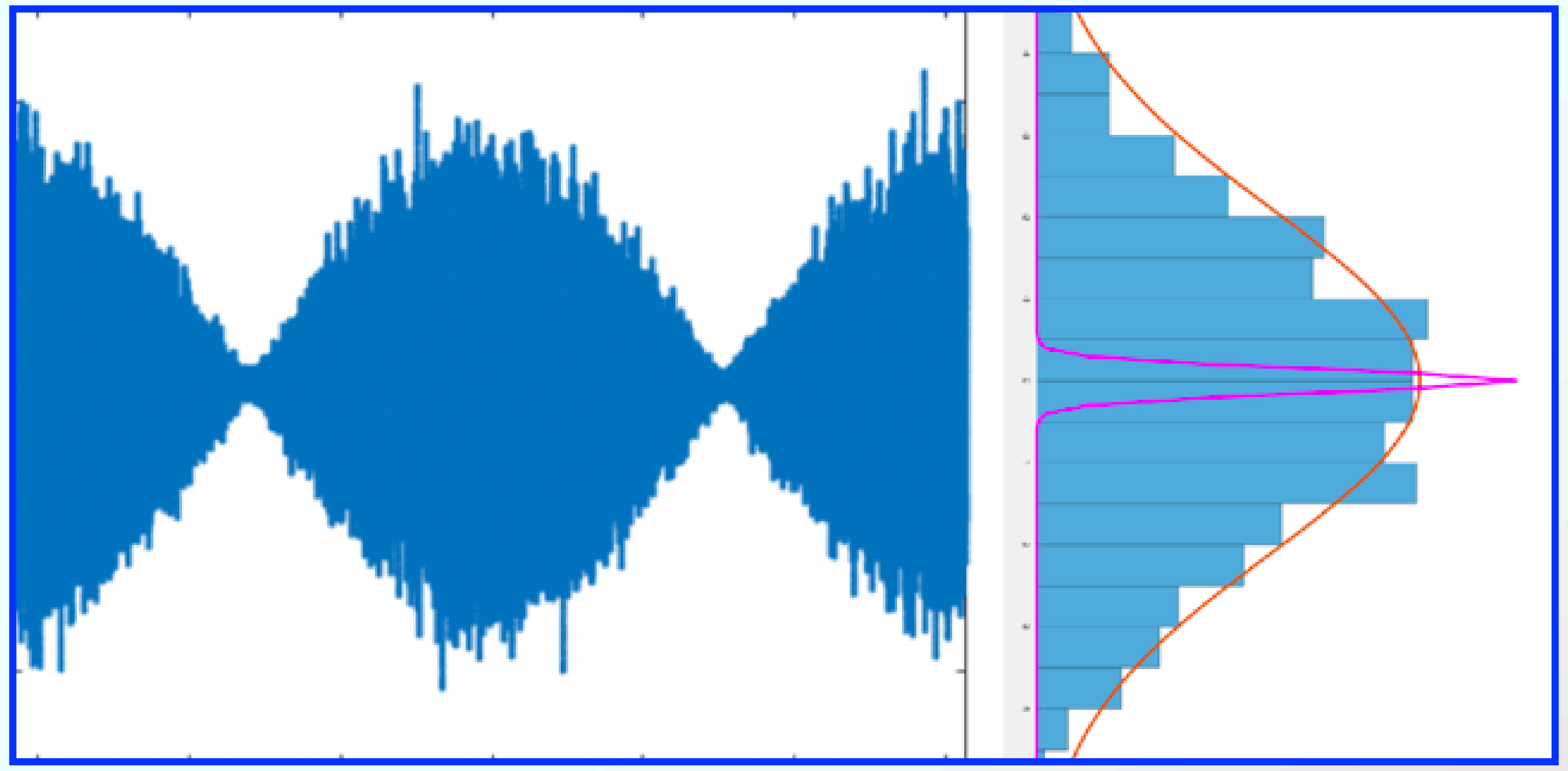
Non-Gaussian States:

Squeezed States (CV)
continuous variables

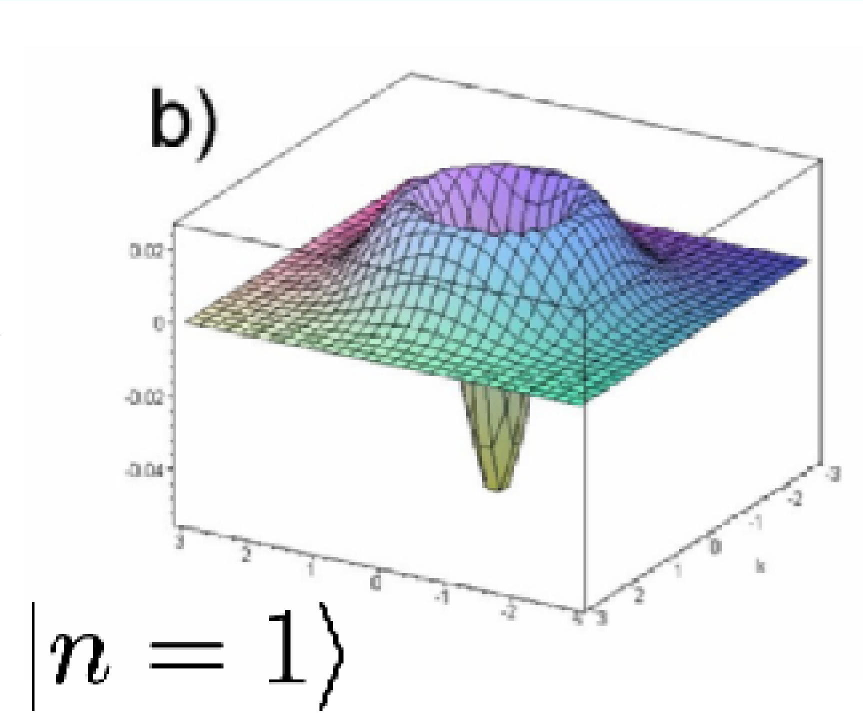
non-classicality



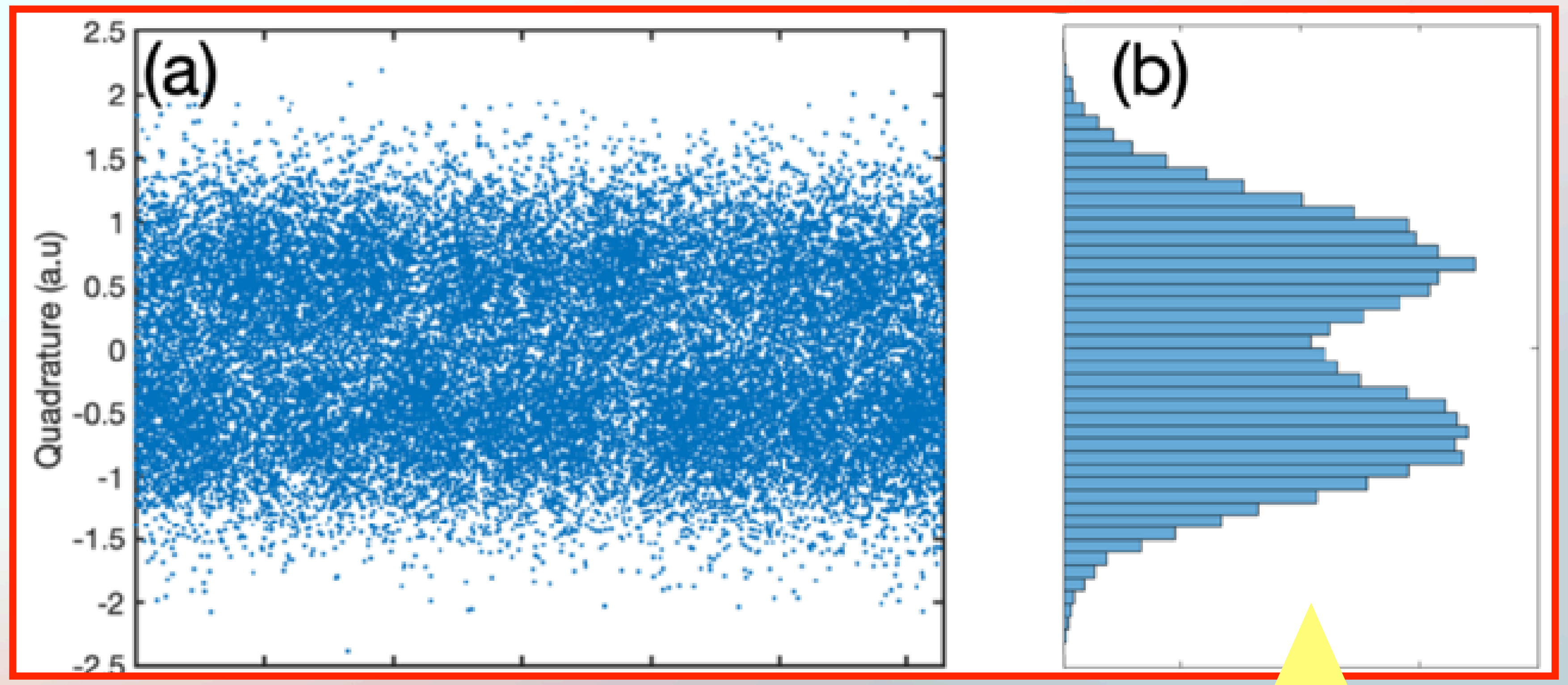




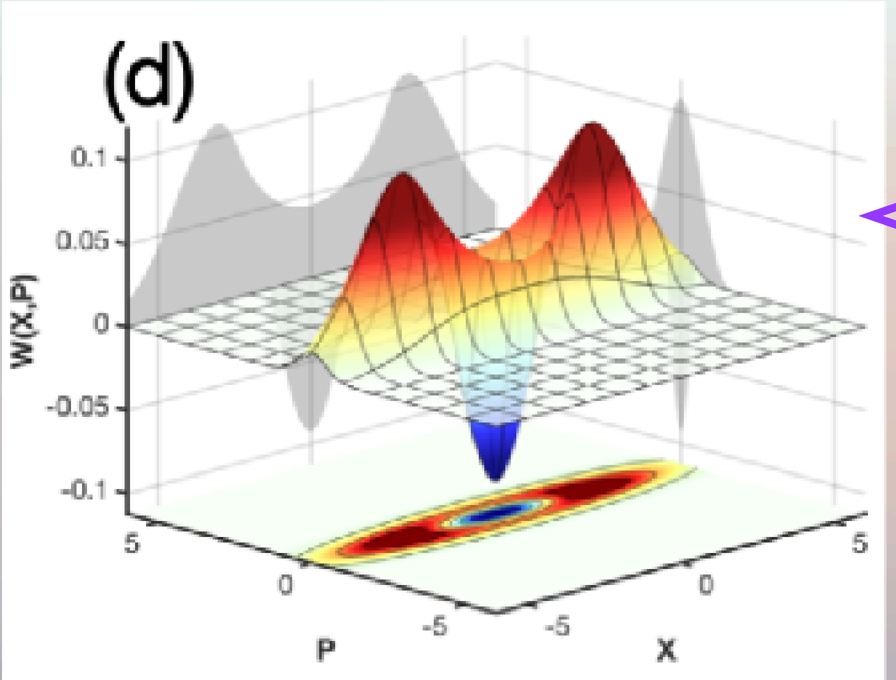
single-photon (DV)
discrete variable



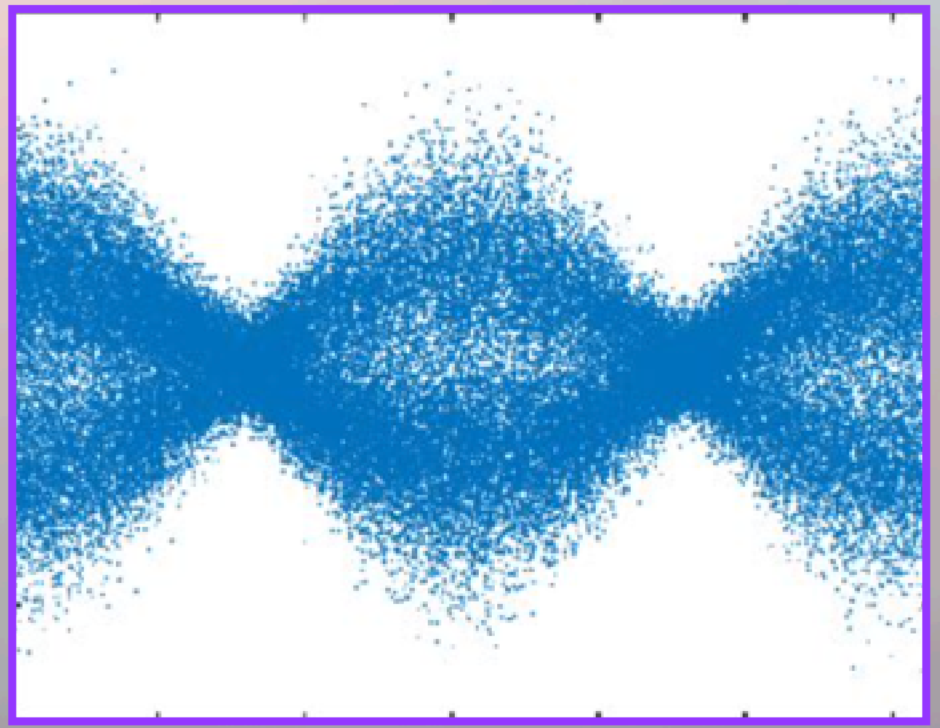
negative
probability



Cat States (CV)



CV:
Negativity
Non-Gaussianity



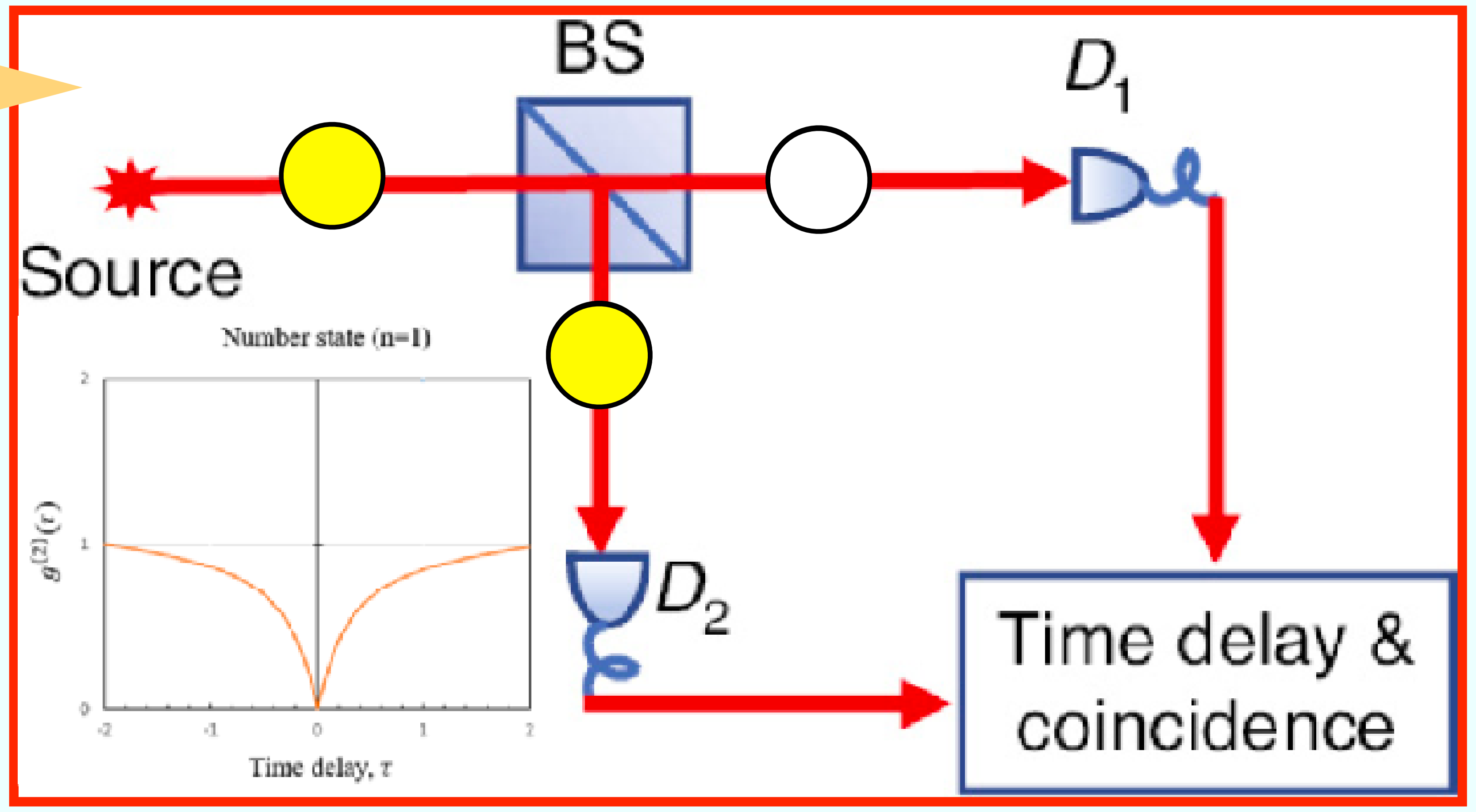
non-Gaussian

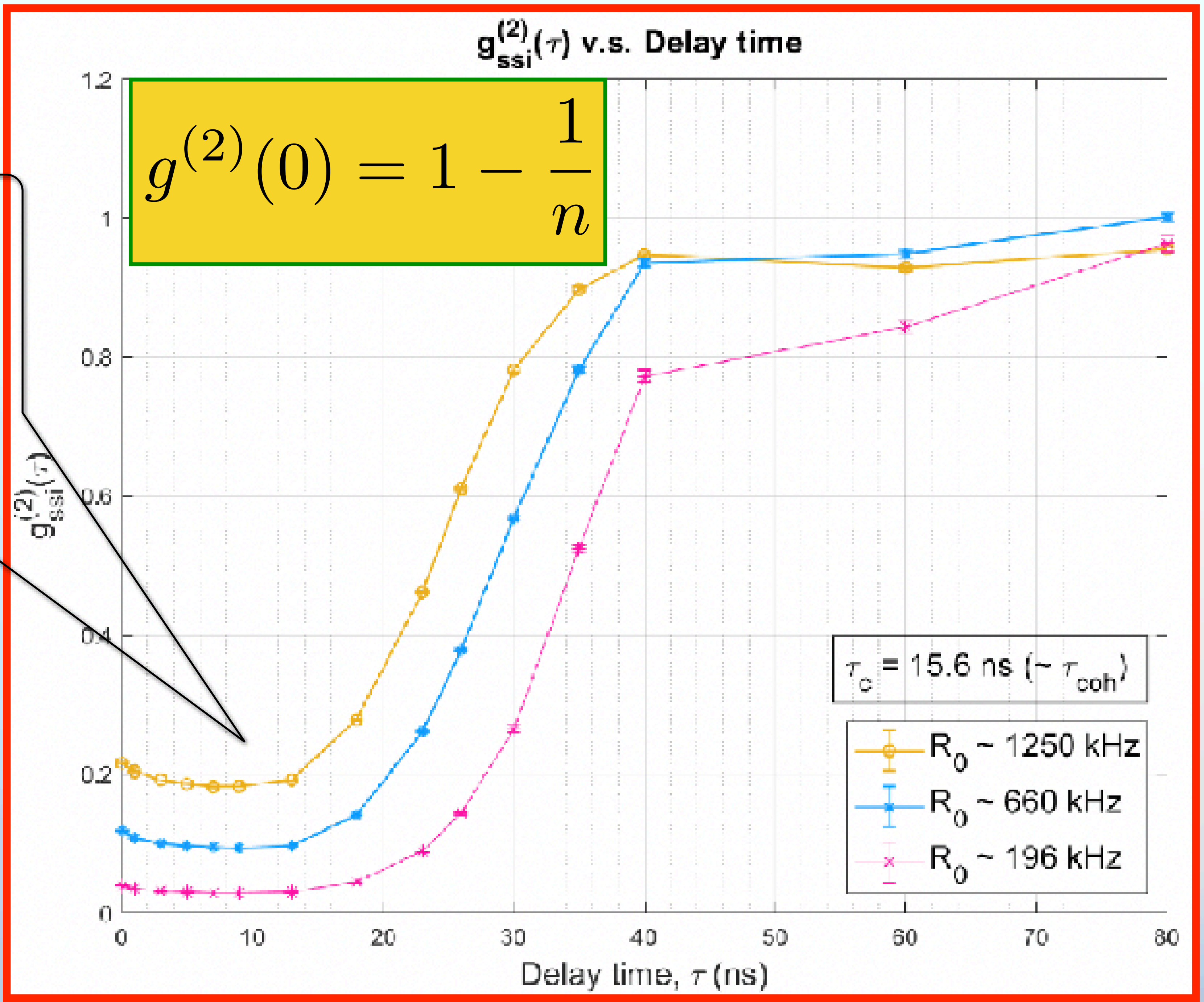
Ole Steuernagel and RKL,
Quantumness Measure from Phase
Space Distributions,
(arXiv: 2311.17399).

Non-Gaussian States: |n=1>

Hsien-Yi Hsieh et al., arXiv: 2405.02812(2024).

HBT interferometry



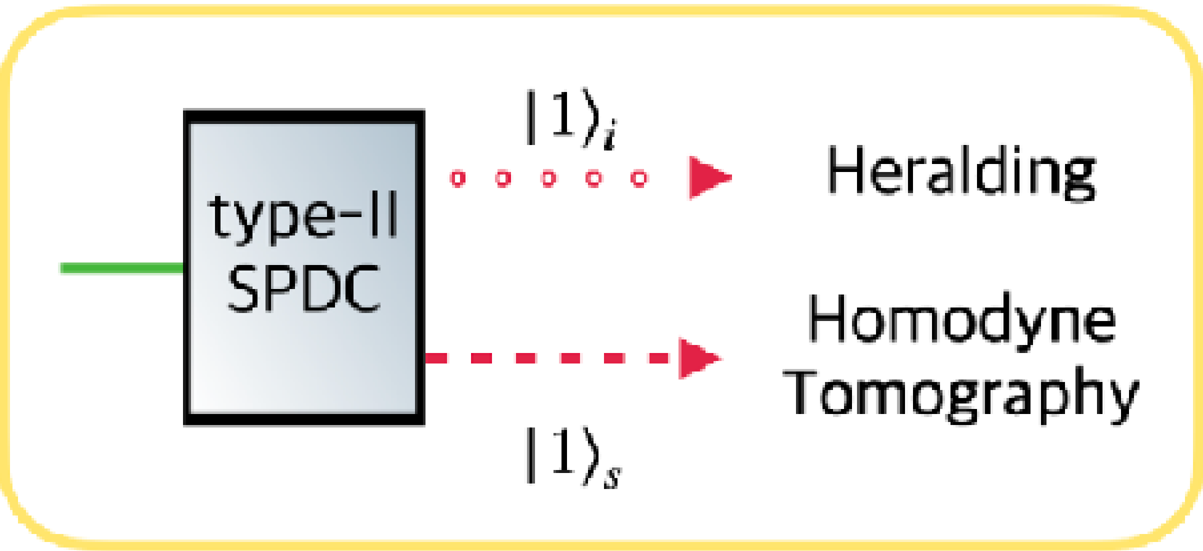


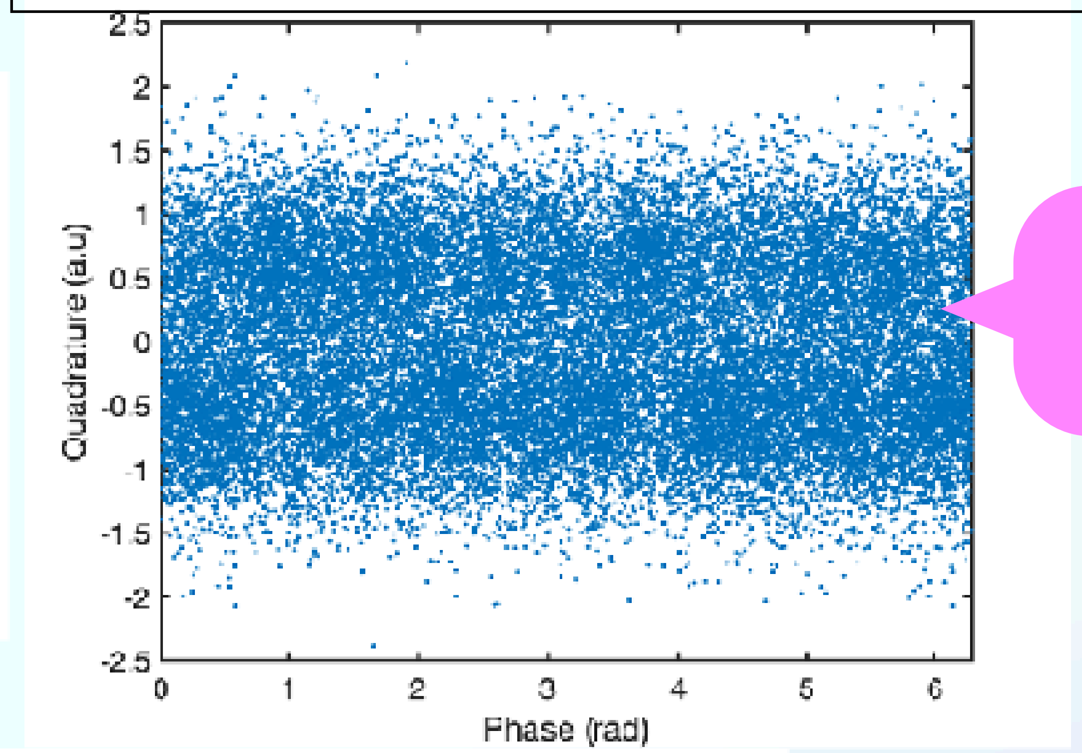
$$g^{(2)}(0) = 1 - \frac{1}{n}$$

g²(0)=0.20 @ 1250 kHz

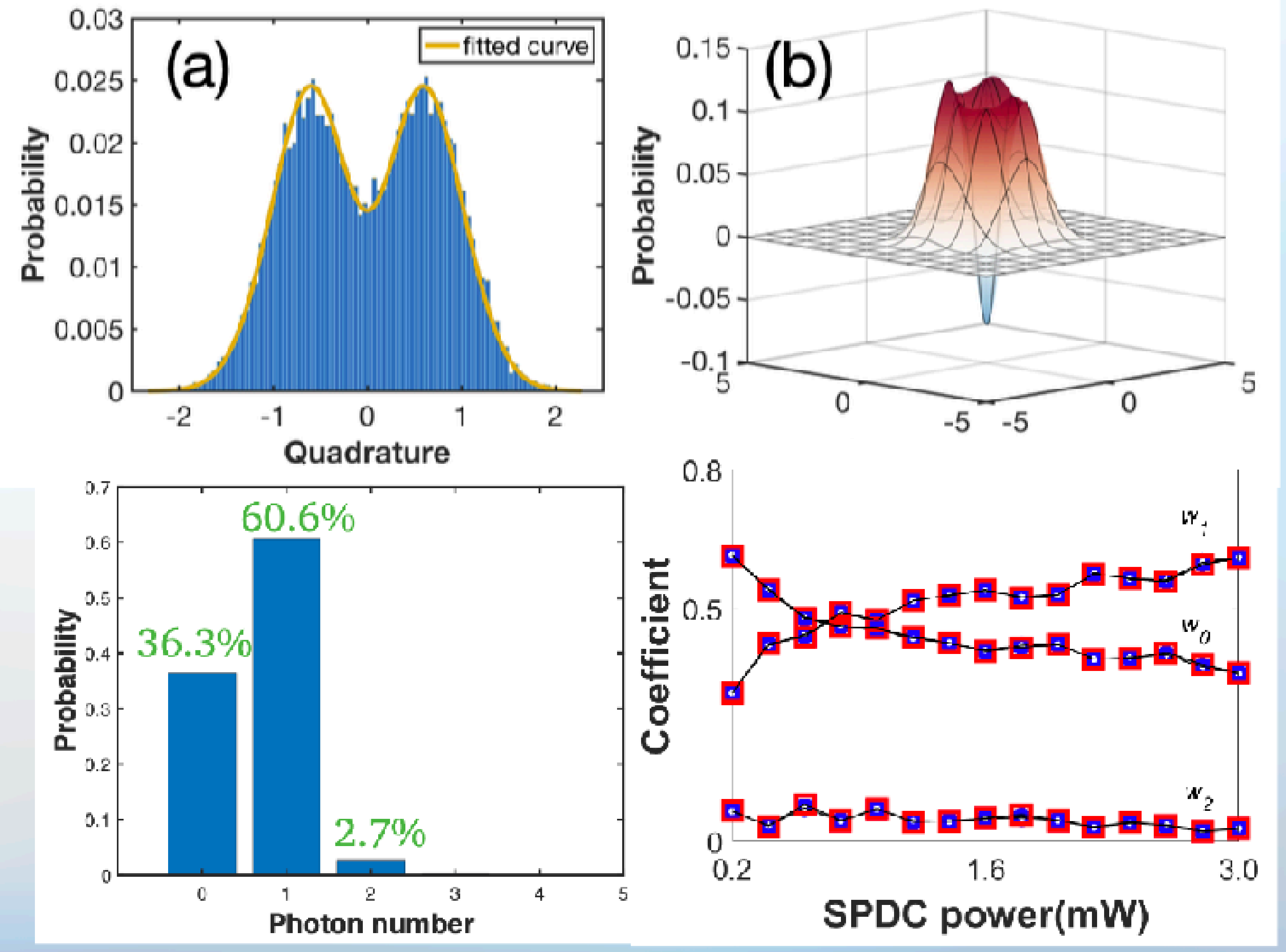
g²(0)=0.10 @ 660 kHz

g²(0)=0.04 @ 196 kHz

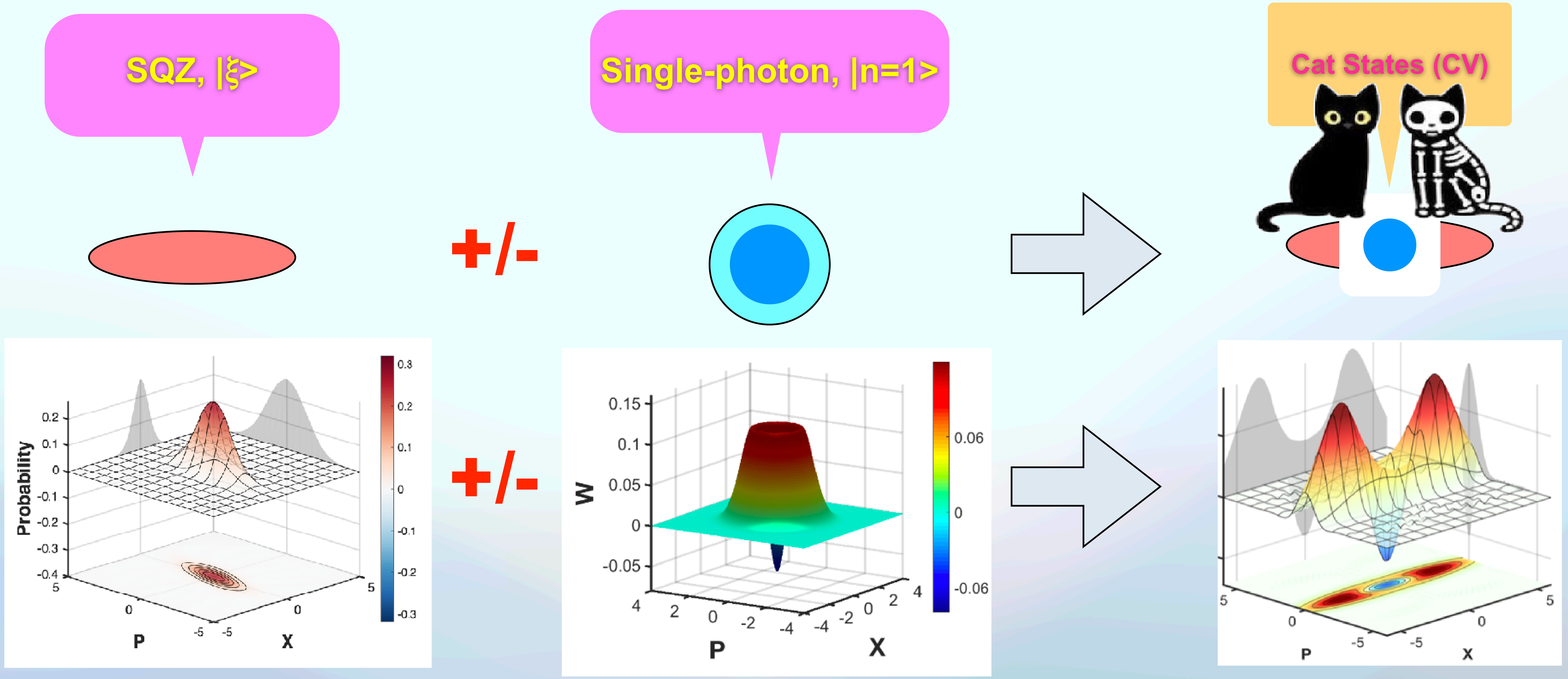




noisy n=1



SQZ +/- Single-Photon:



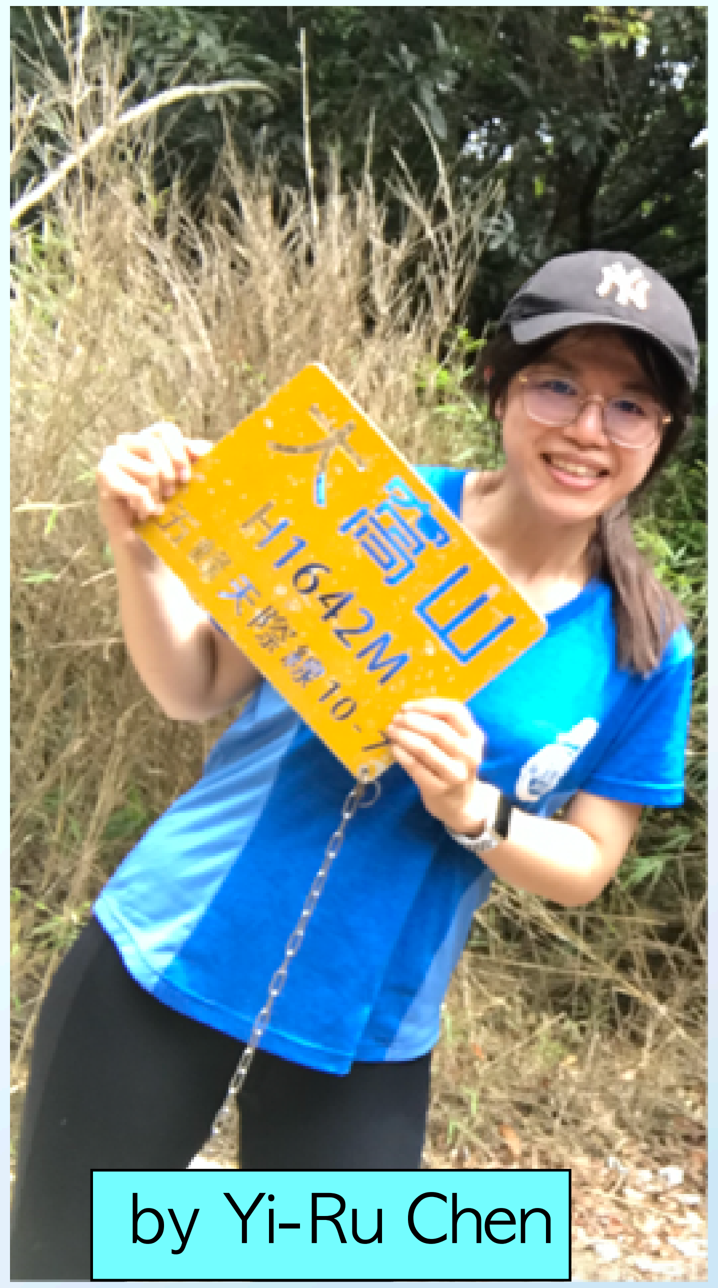
Optical Cats: Photon-Addition

$\hat{a}^\dagger \hat{S}\,|0\rangle$

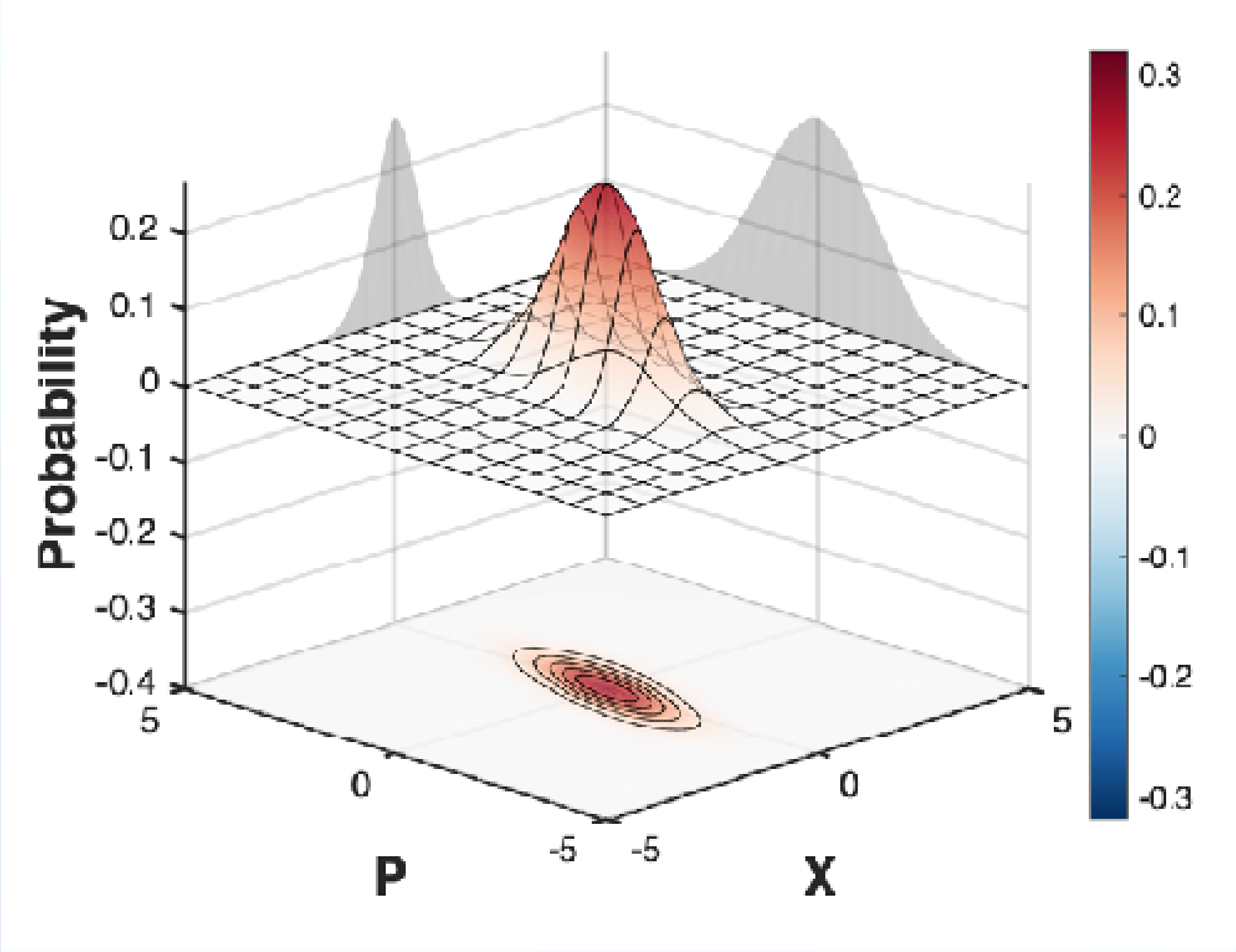
SPDC

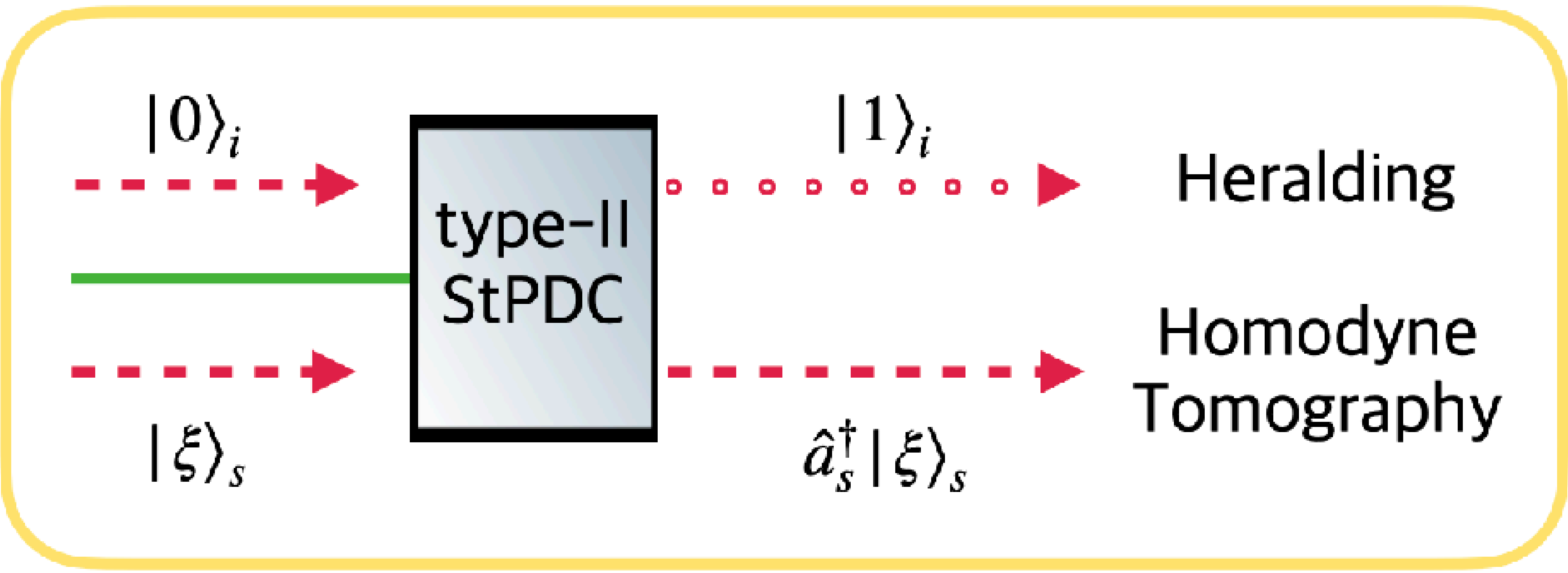
$$\hat{S}(g) = e^{g(\hat{a}_1\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_2^\dagger)} \approx 1 + g(\hat{a}_1\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_2^\dagger)$$

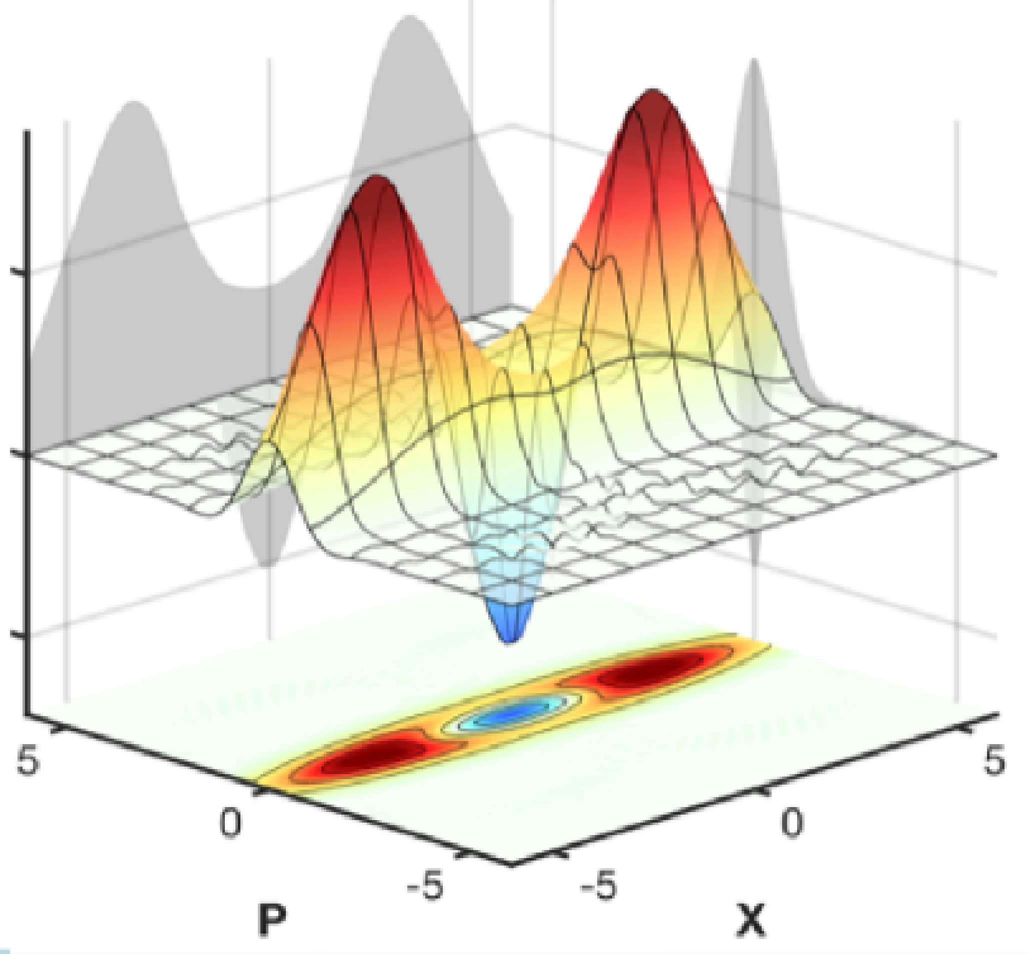
$$\hat{S}(g)\hat{S}|0\rangle_1|0\rangle_2 \approx \hat{S}|0\rangle_1|0\rangle_2 - g\hat{a}_1^\dagger\hat{S}|0\rangle_1|1\rangle_2$$


by Yi-Ru Chen

Yi-Ru Chen et al., arXiv:2306.13011 (2023).

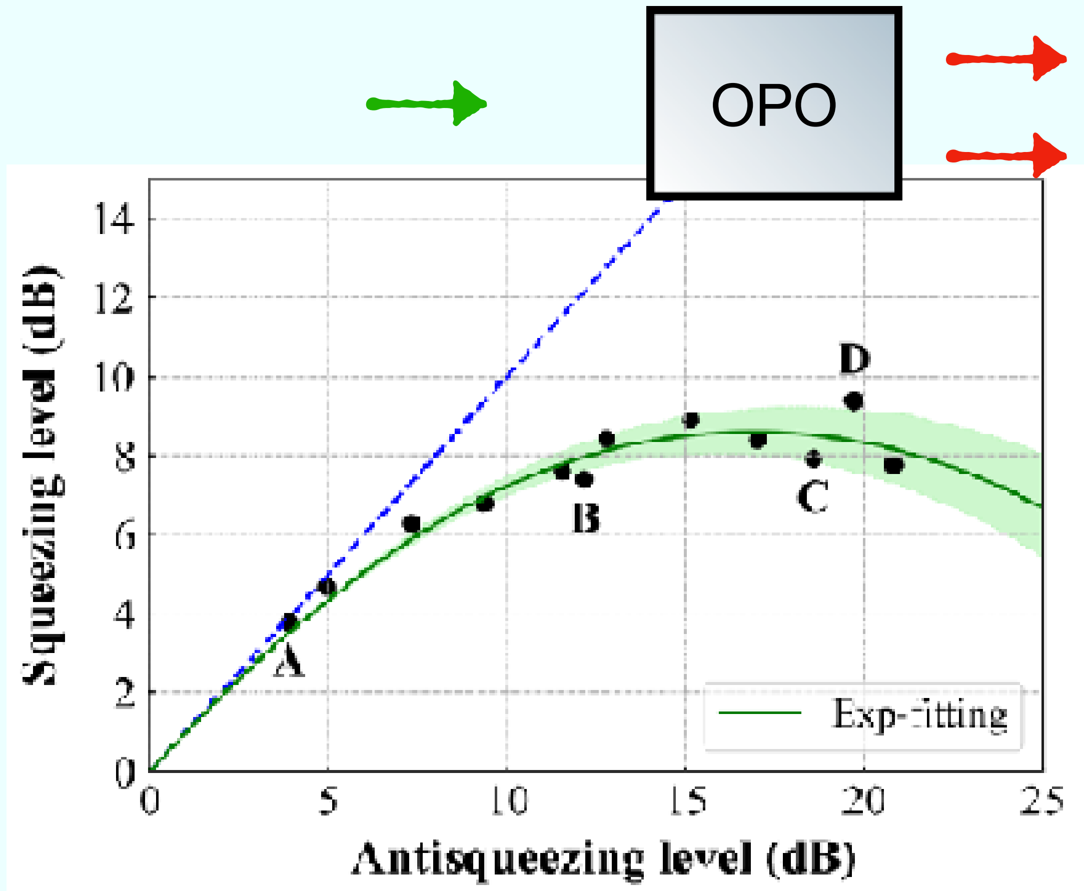


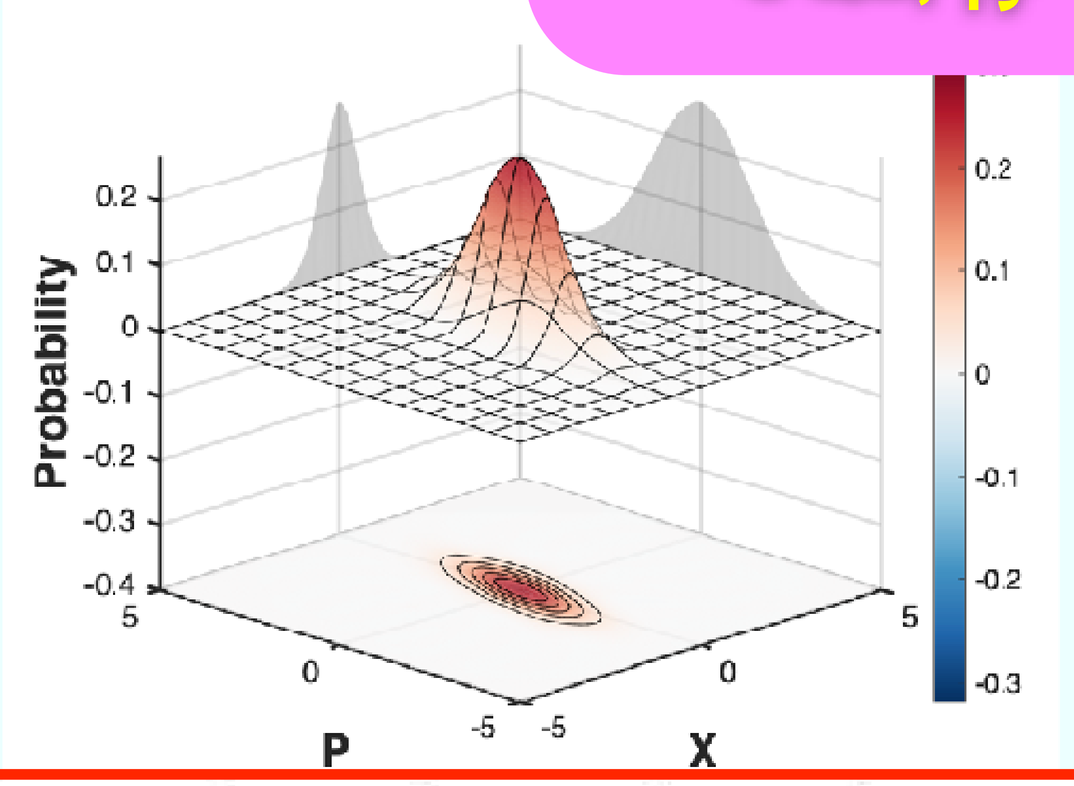




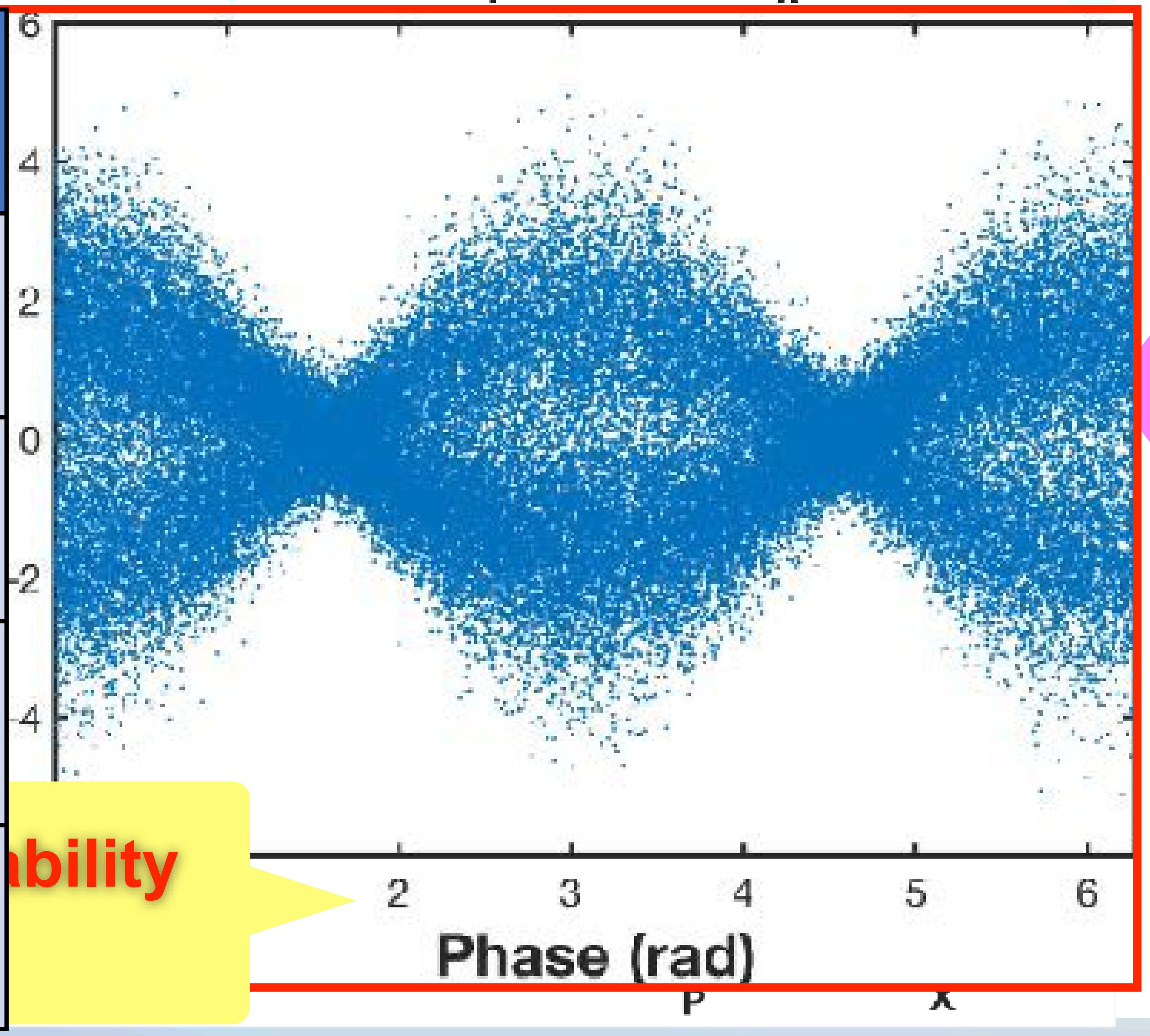
Optical Cats: Photon-Addition

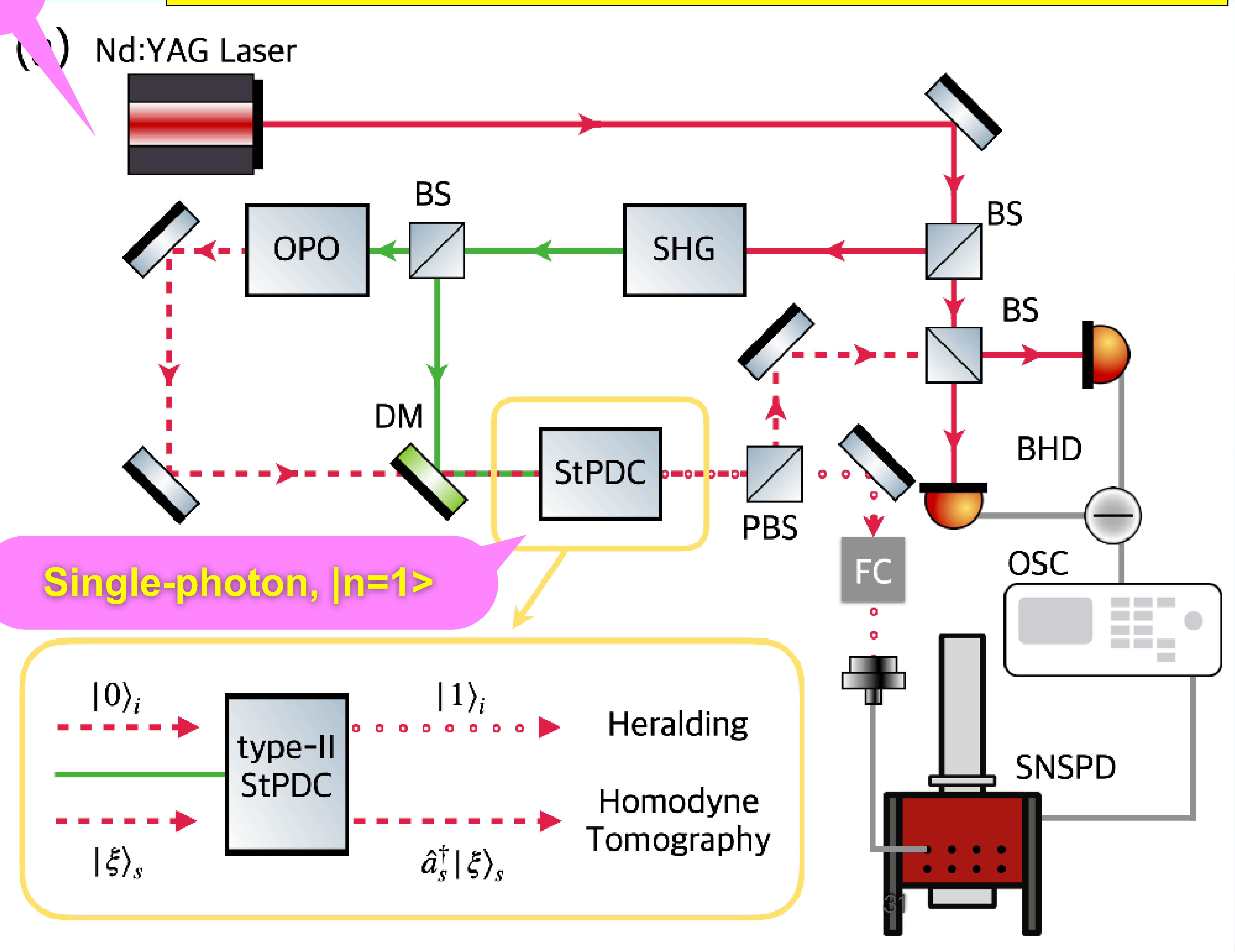
The First Experimental Realization with photo-addition.





	Our Work	State-of-the-art
Cat size |α|	1.77	1.85 [1]
Wigner negativity	-0.094	−0.184 ± 0.001 [2]
Generation rate	235 kHz	~10 kHz [3]
Fidelity	66%	78.2% [2]

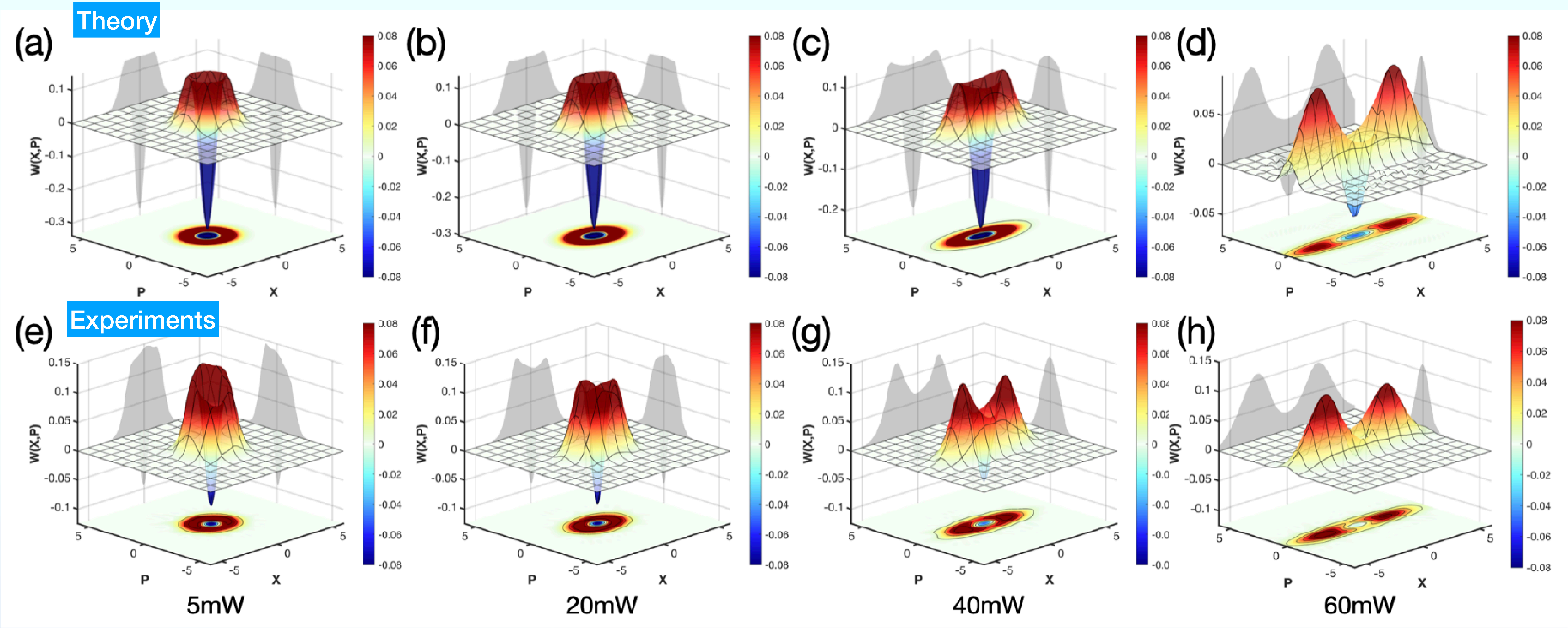




Hsien-Yi Hsieh et al., arXiv: 2405.02812(2024).

Yi-Ru Chen et al., PRA 110 023703 (2024).

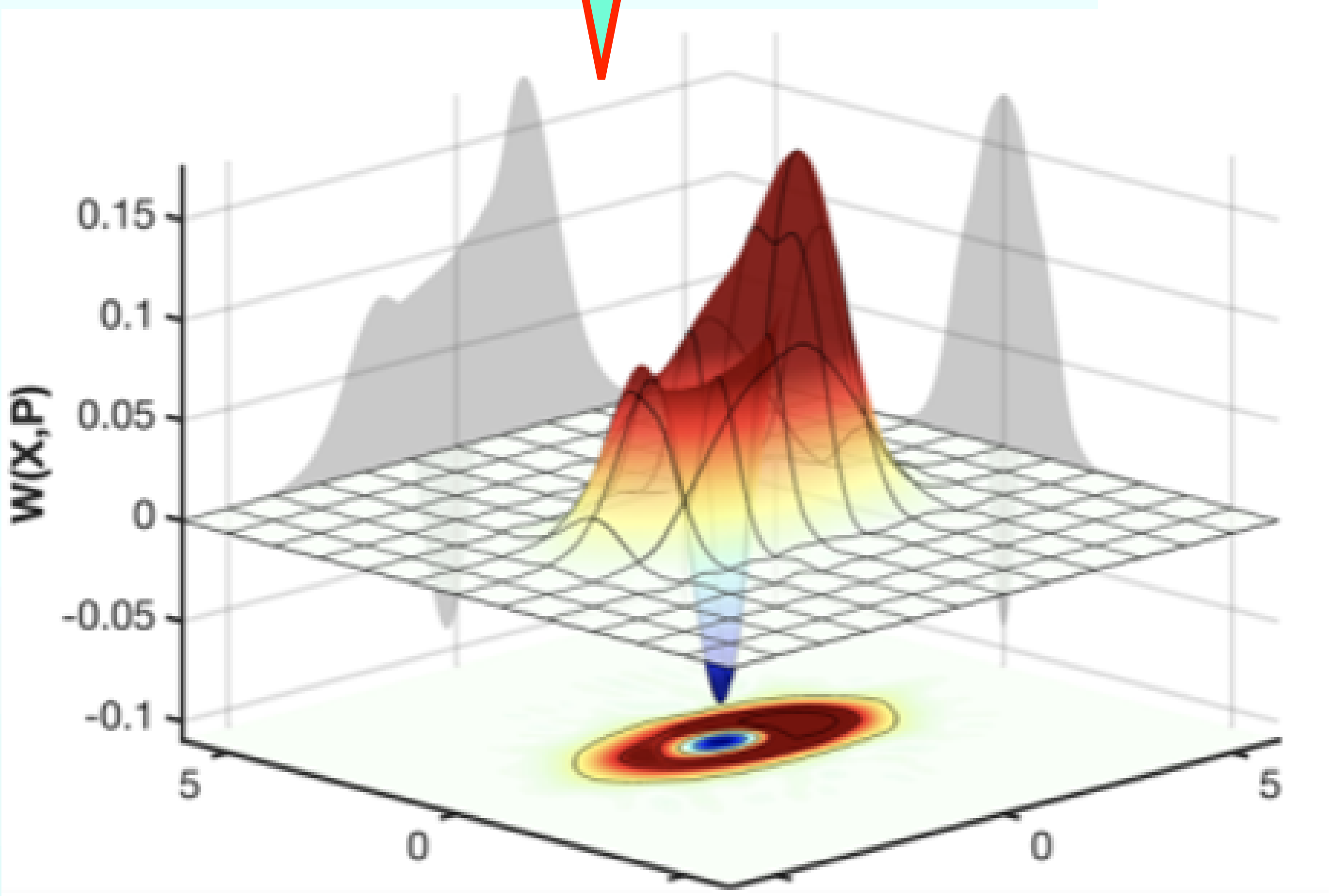
Optical Cats: Photon-Addition

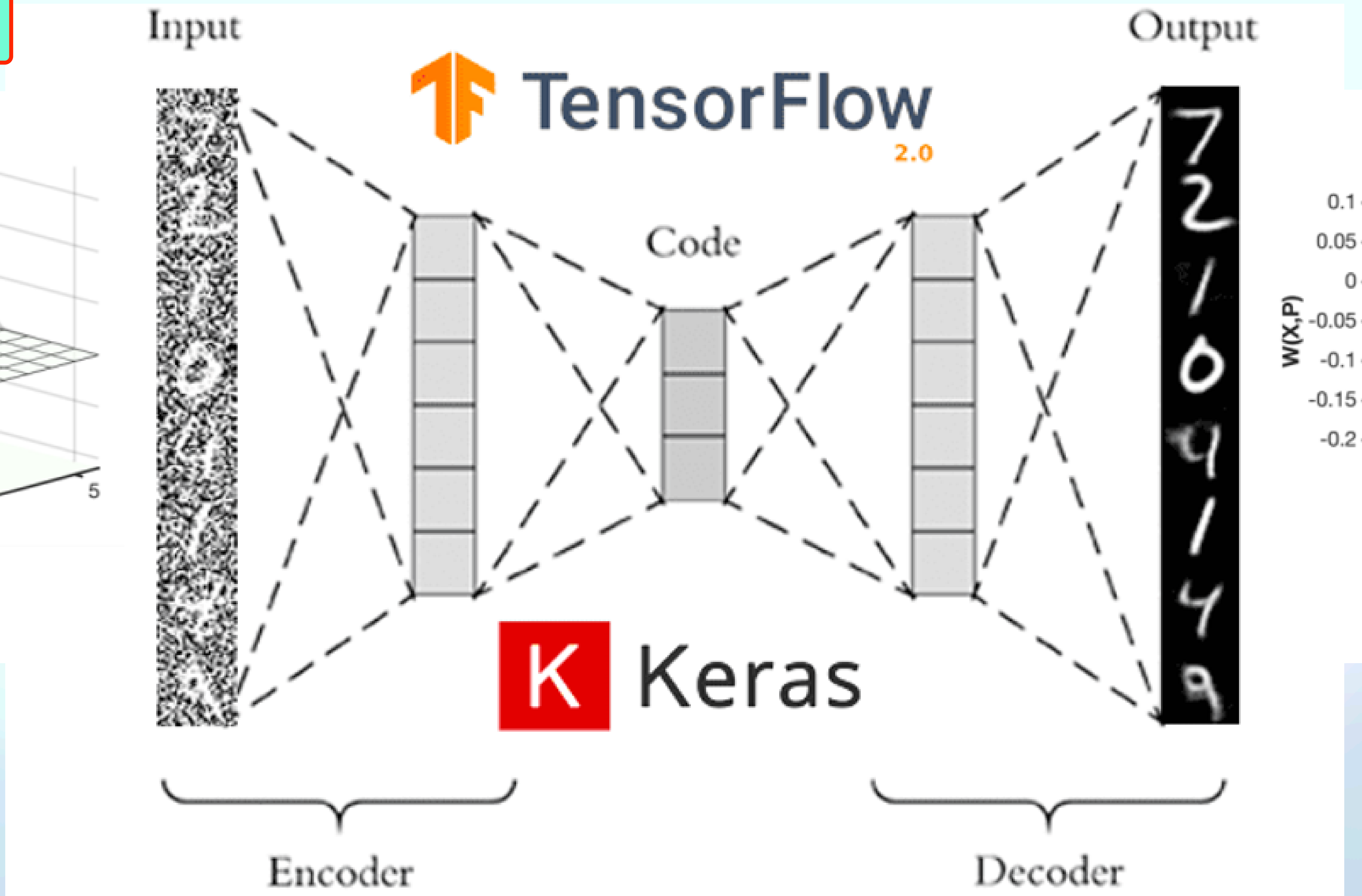


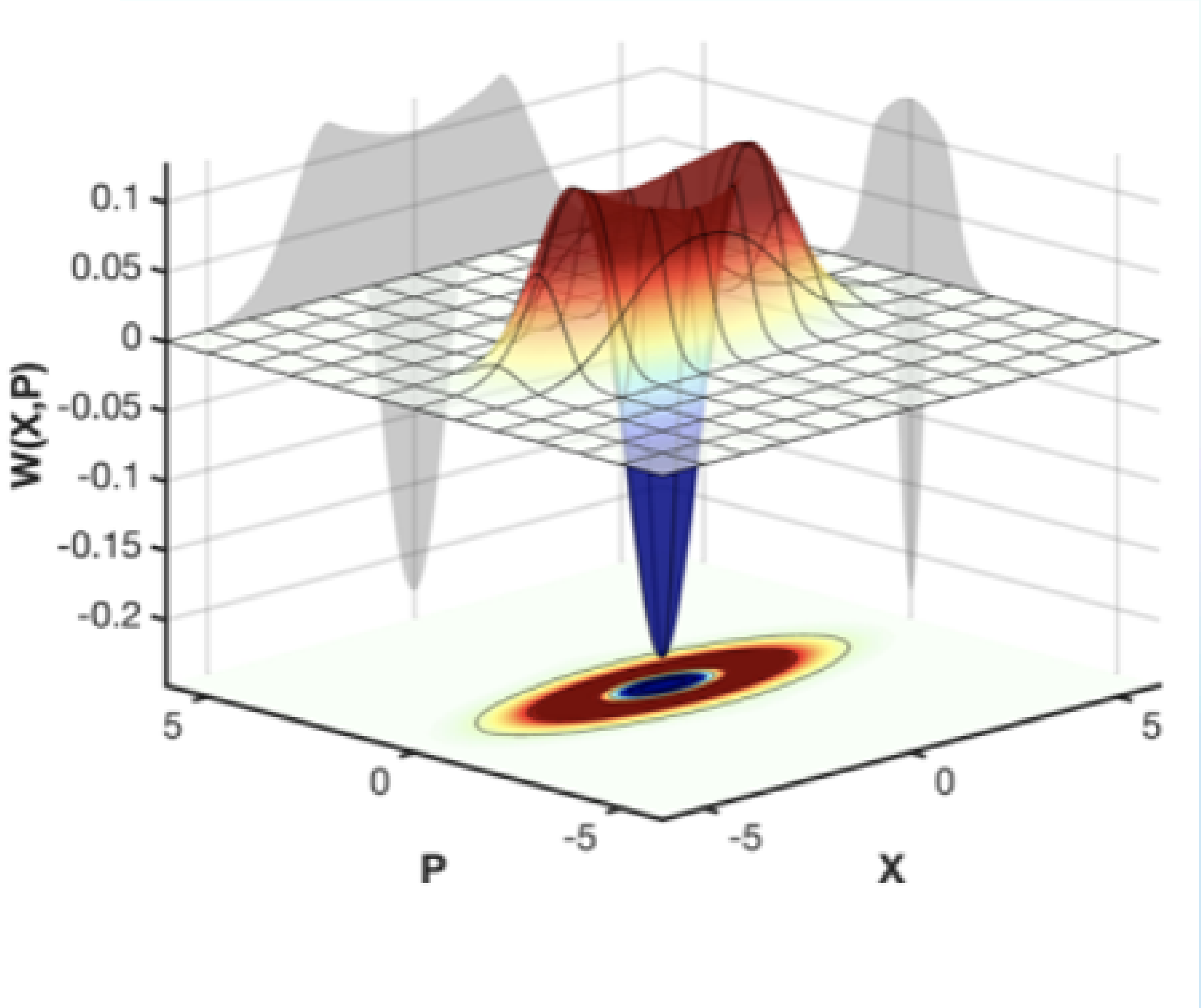
Yi-Ru Chen et al., PRA 110 023703 (2024).

ML-enhanced QST: de-noise

Back-Scattering Light





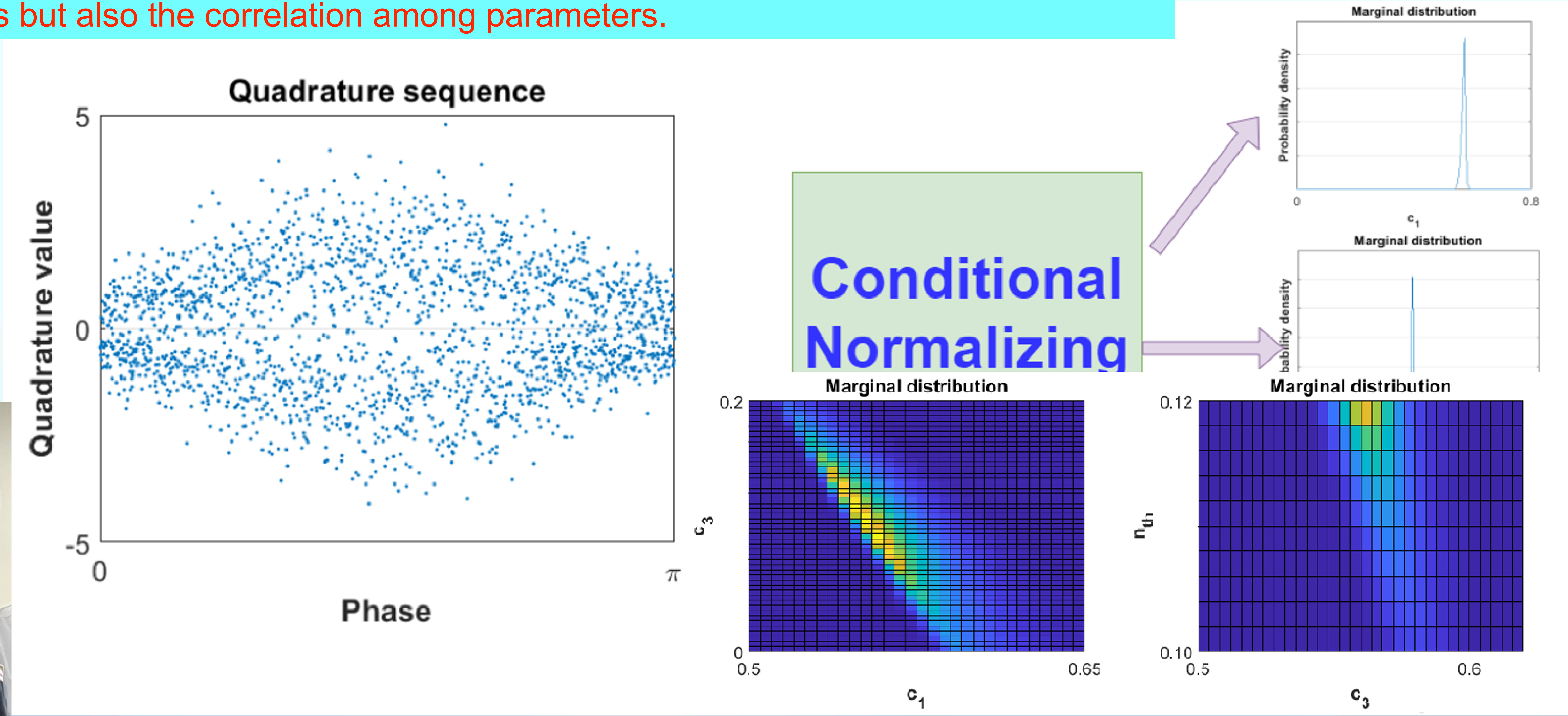


by Hsieh-Yi Hsieh

Hsieh-Yi Hsieh et al., (in preparation, 2024).

ML-enhanced QST: Bayesian inversion

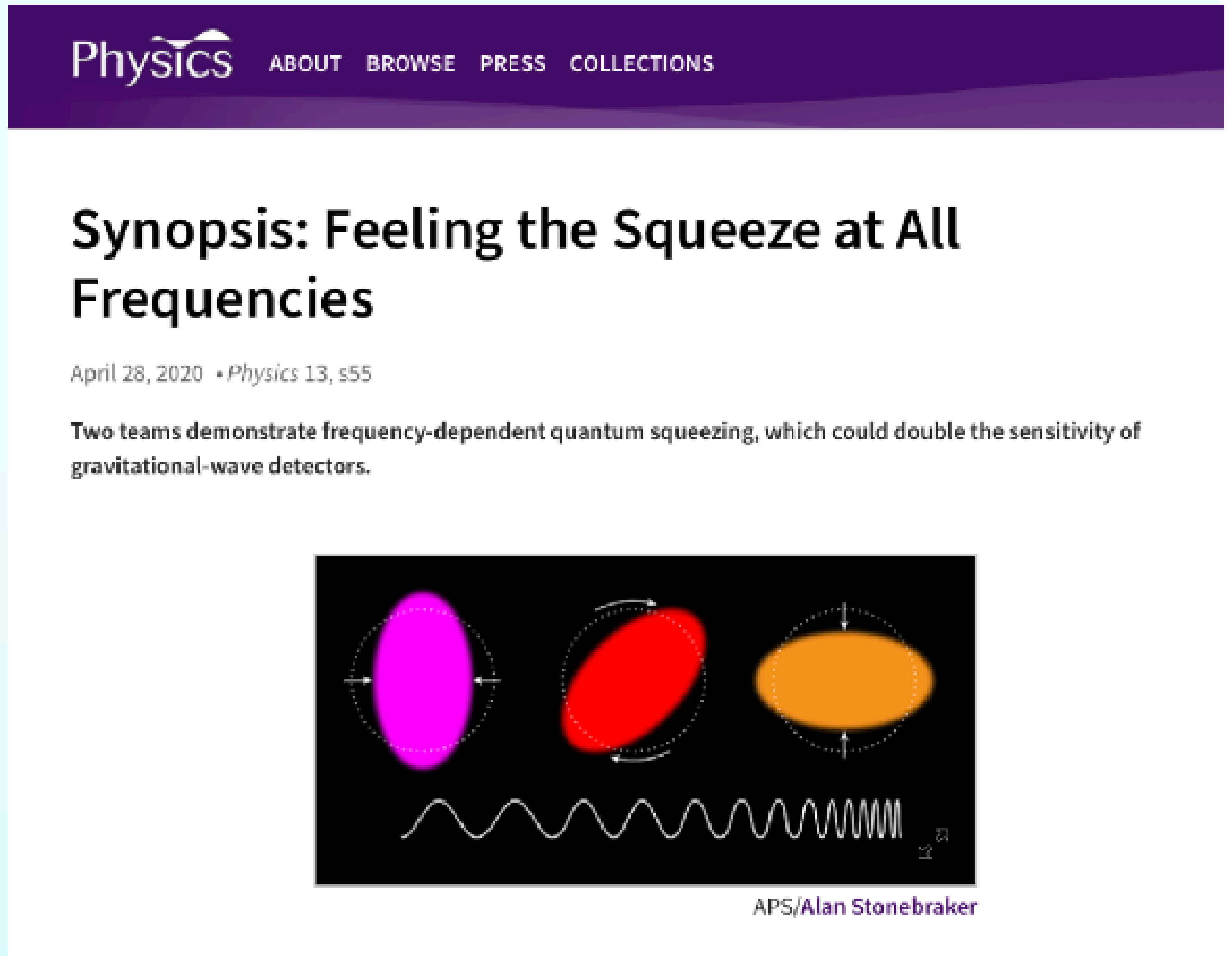
Bayesian quantum state tomography not only provides uncertainty in the estimation of parameters but also the correlation among parameters.



by Hsieh-Yi Hsieh

Hsieh-Yi Hsieh et al., (in preparation, 2024).

Frequency Dependent Squeezing (FDS)


Physics ABOUT BROWSE PRESS COLLECTIONS

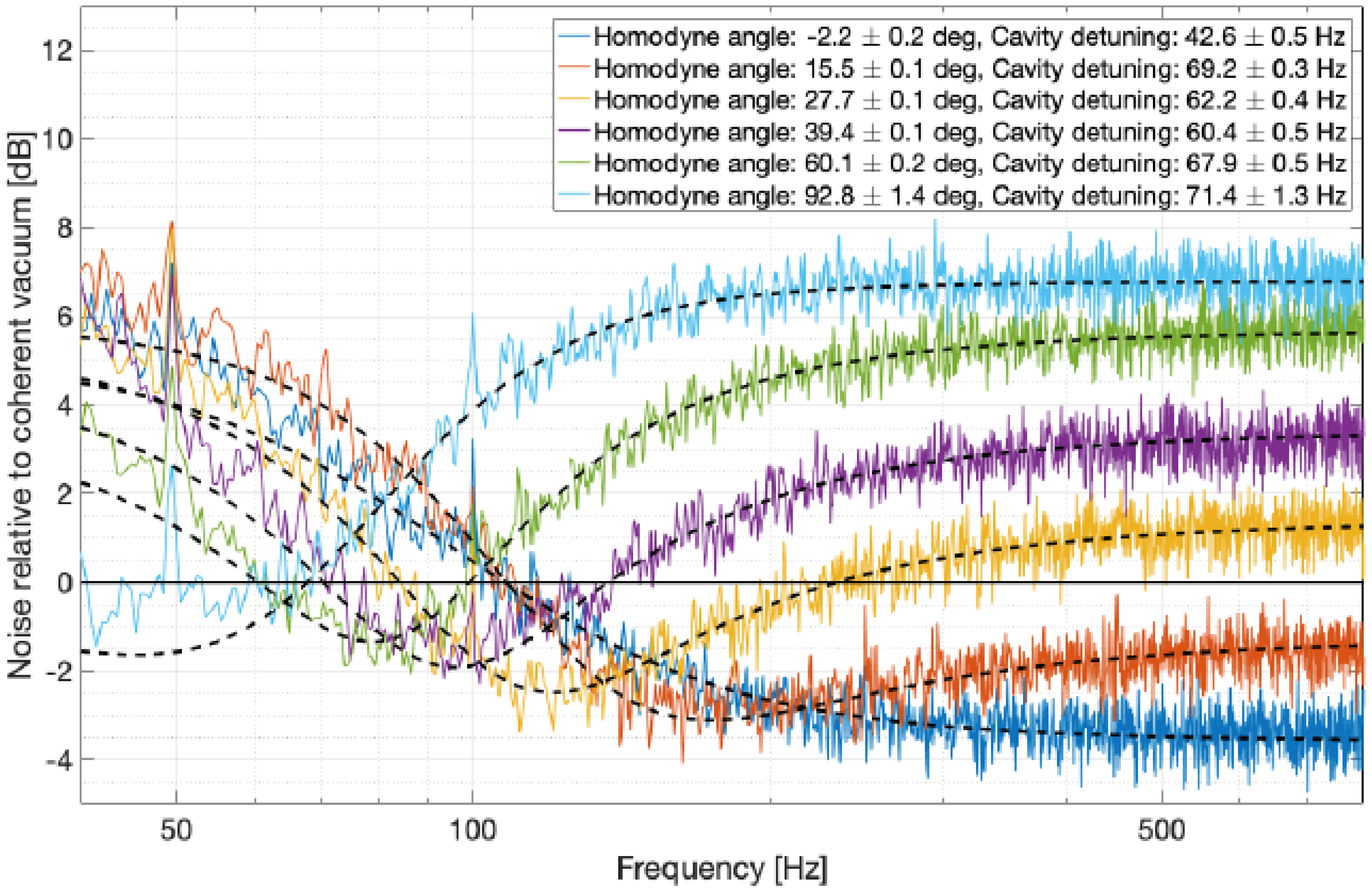
Synopsis: Feeling the Squeeze at All Frequencies

April 28, 2020 • *Physics* 13, s55

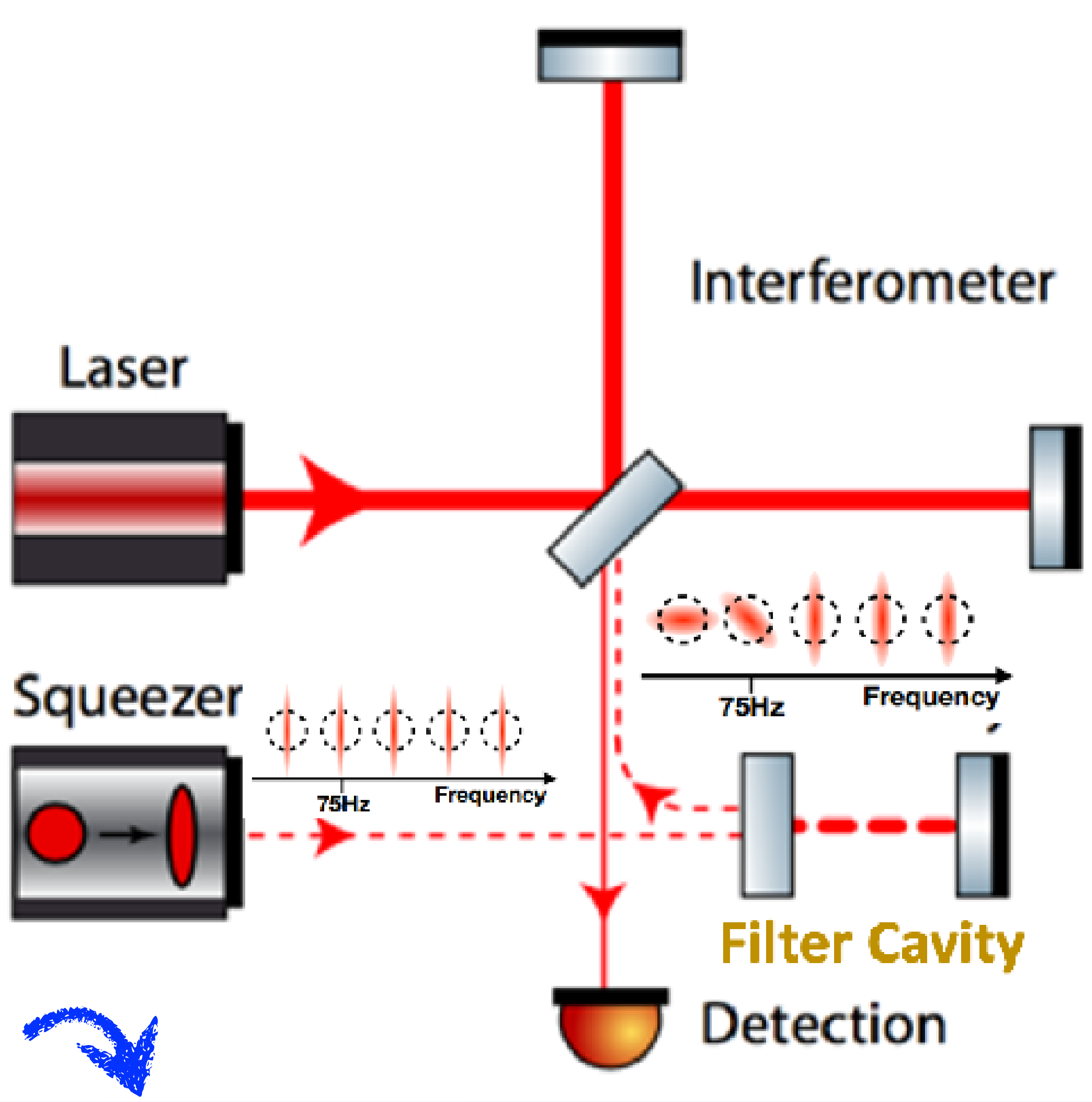
Two teams demonstrate frequency-dependent quantum squeezing, which could double the sensitivity of gravitational-wave detectors.

APS/Alan Stonebraker

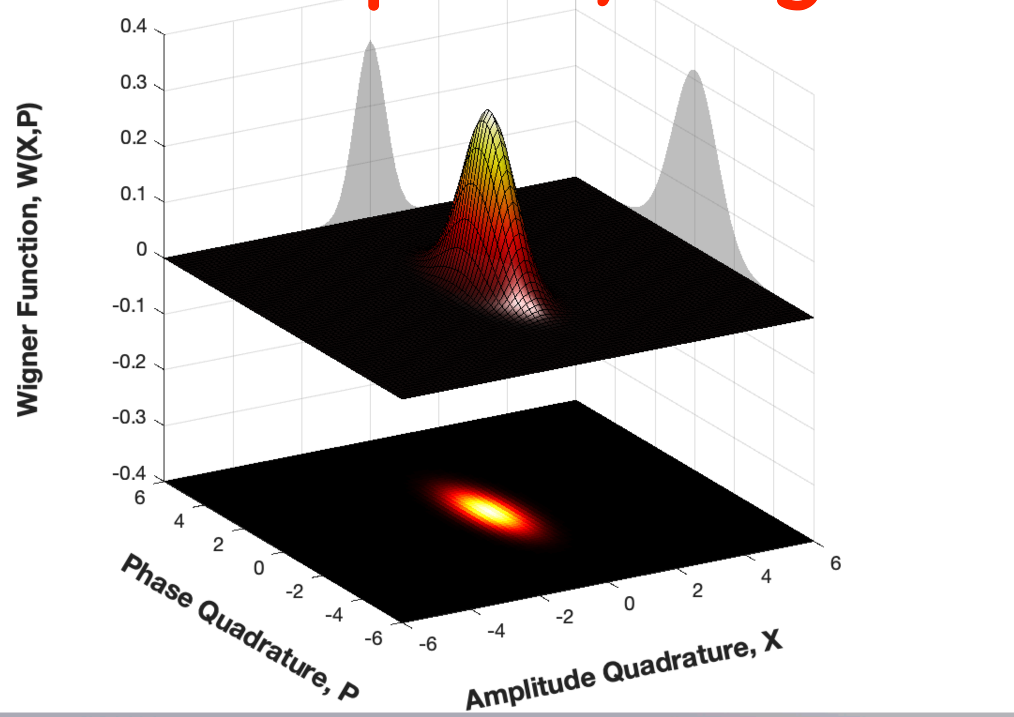
Yuhang Zhao et al.,
Phys. Rew. Lett. 124, 171101 (2020).



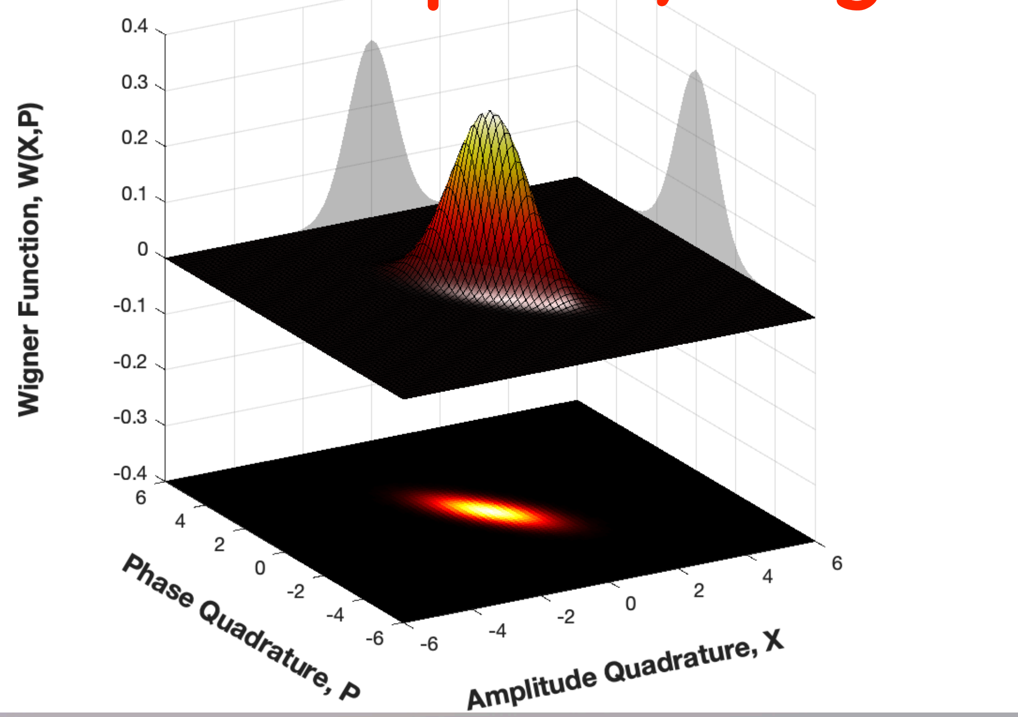
with NAOJ, 日本國立天文台



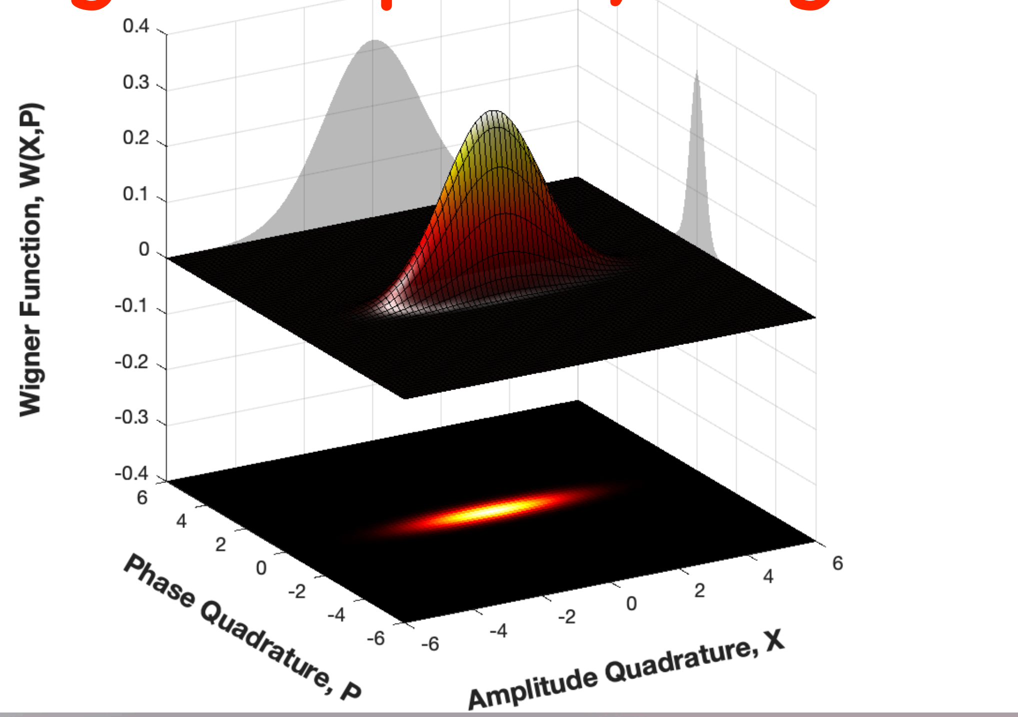
Low frequency region



Middle frequency region



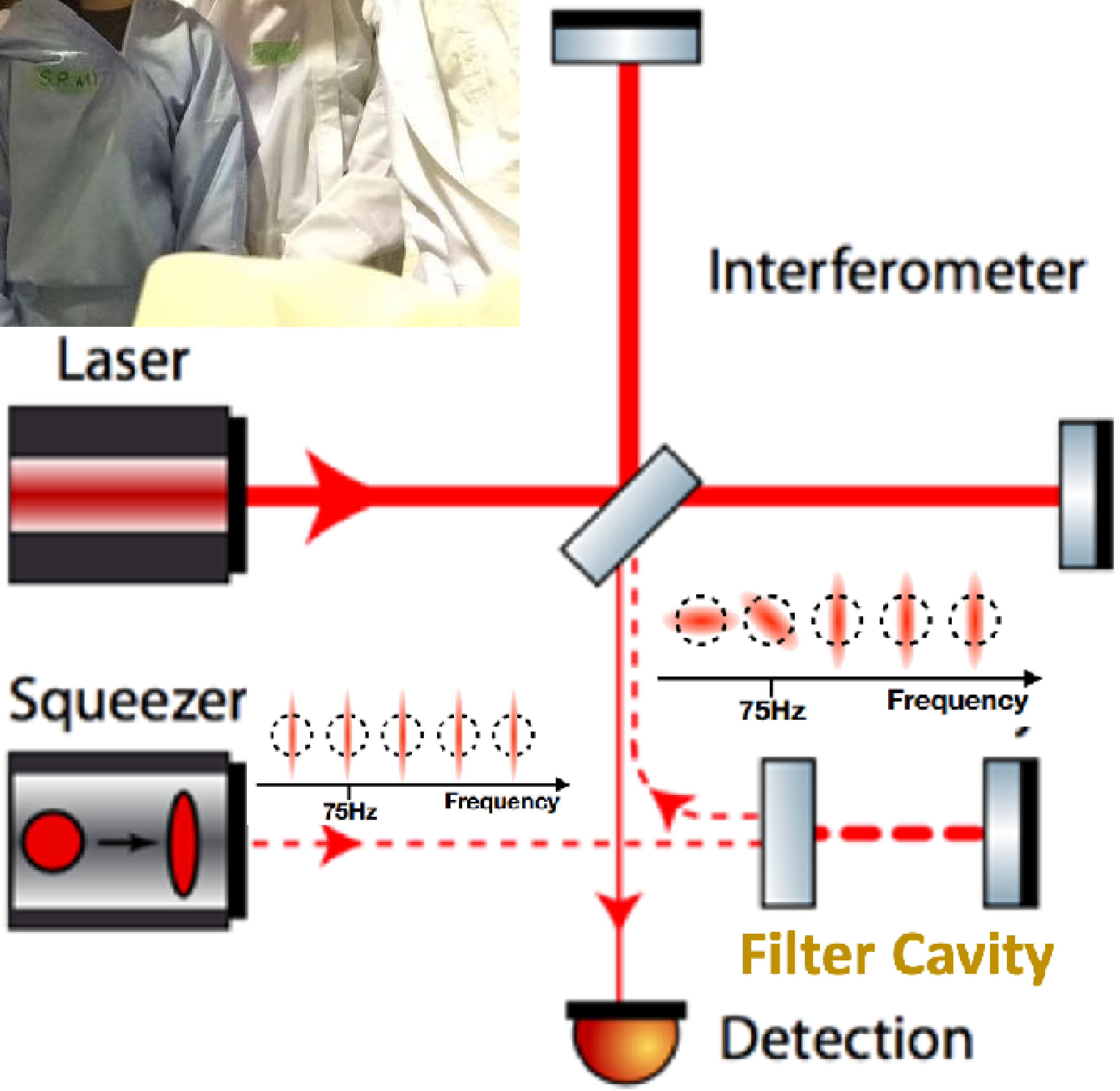
High frequency region

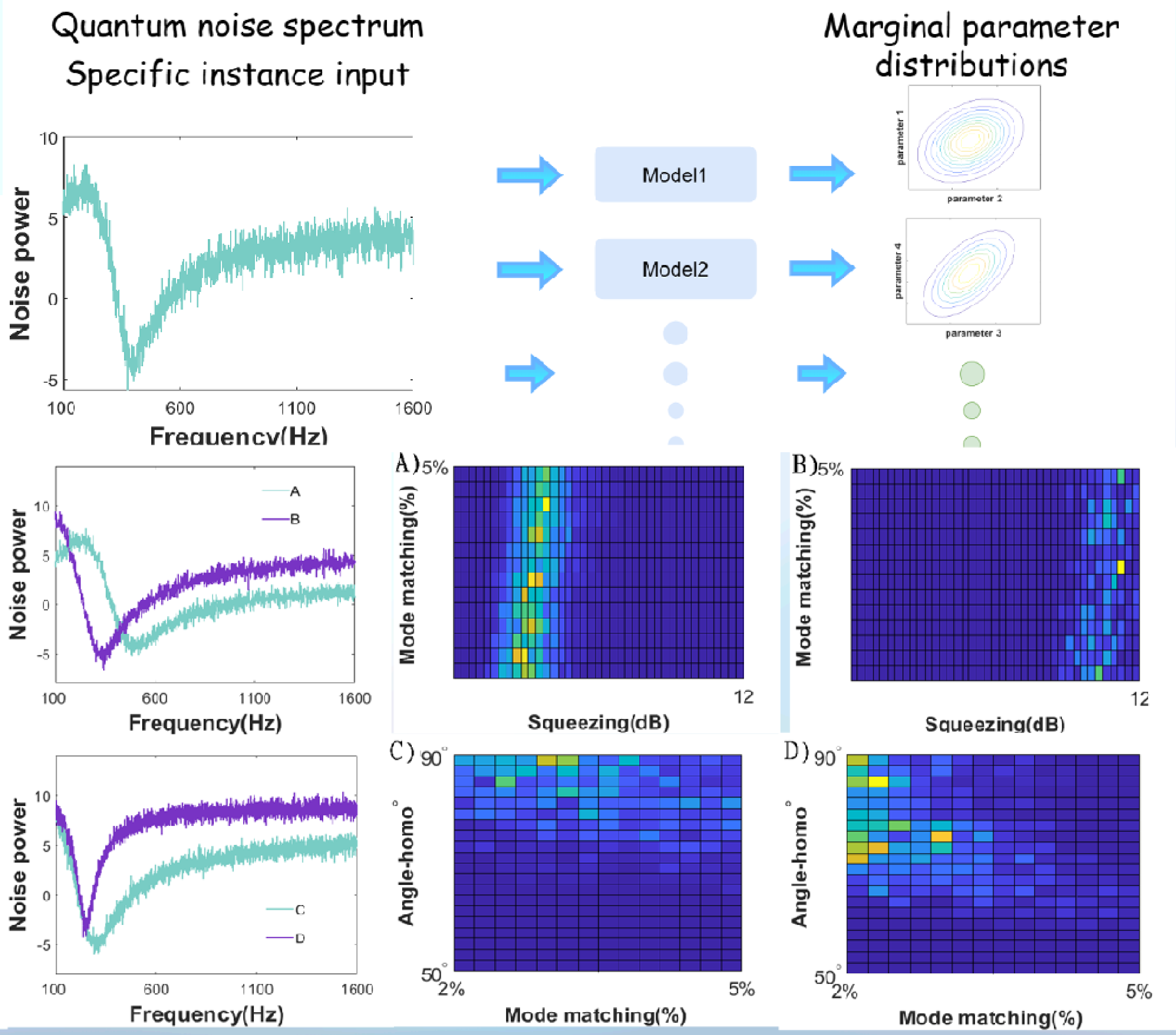


ML-enhanced QST: Gravitational Wave Detectors

Multi-Parameters Bayesian Estimation

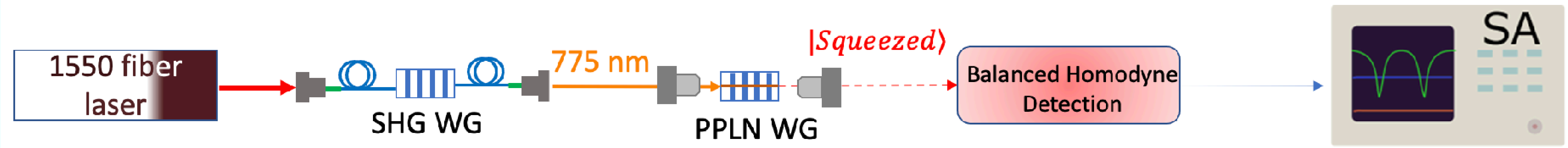
with NAOJ, 日本國立天文台

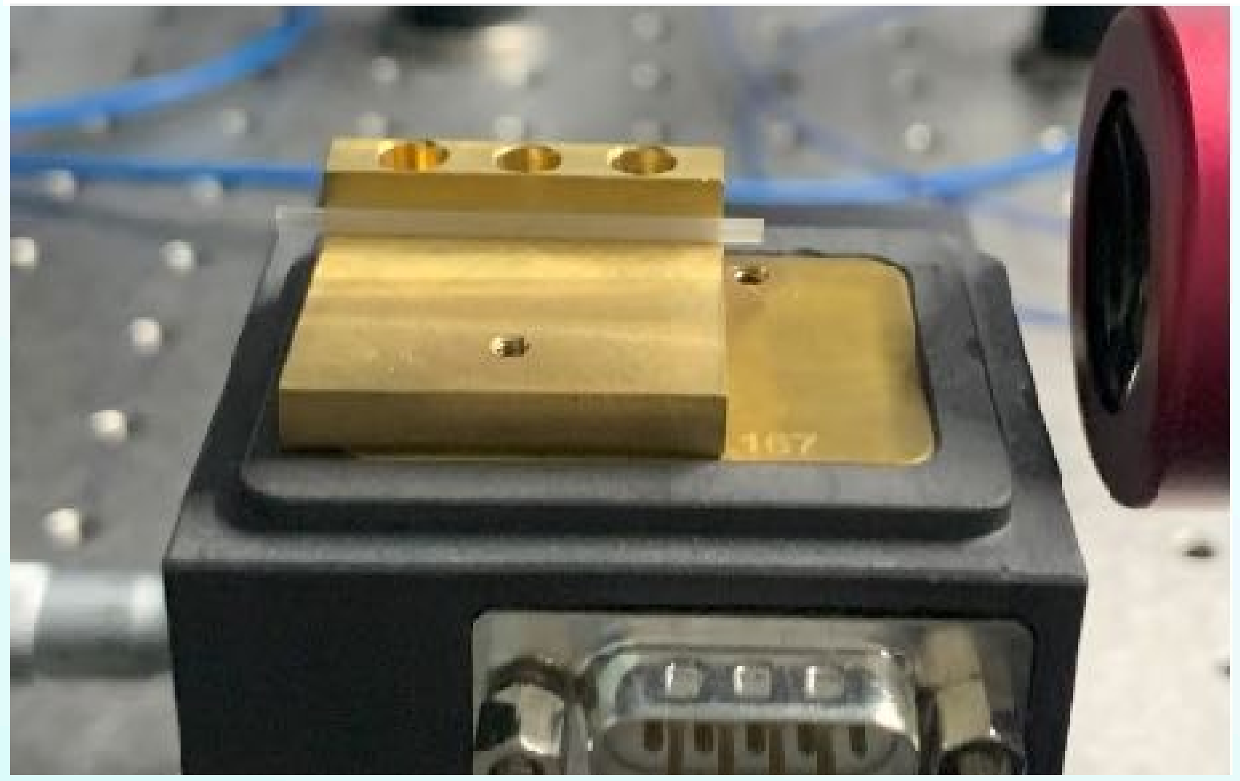


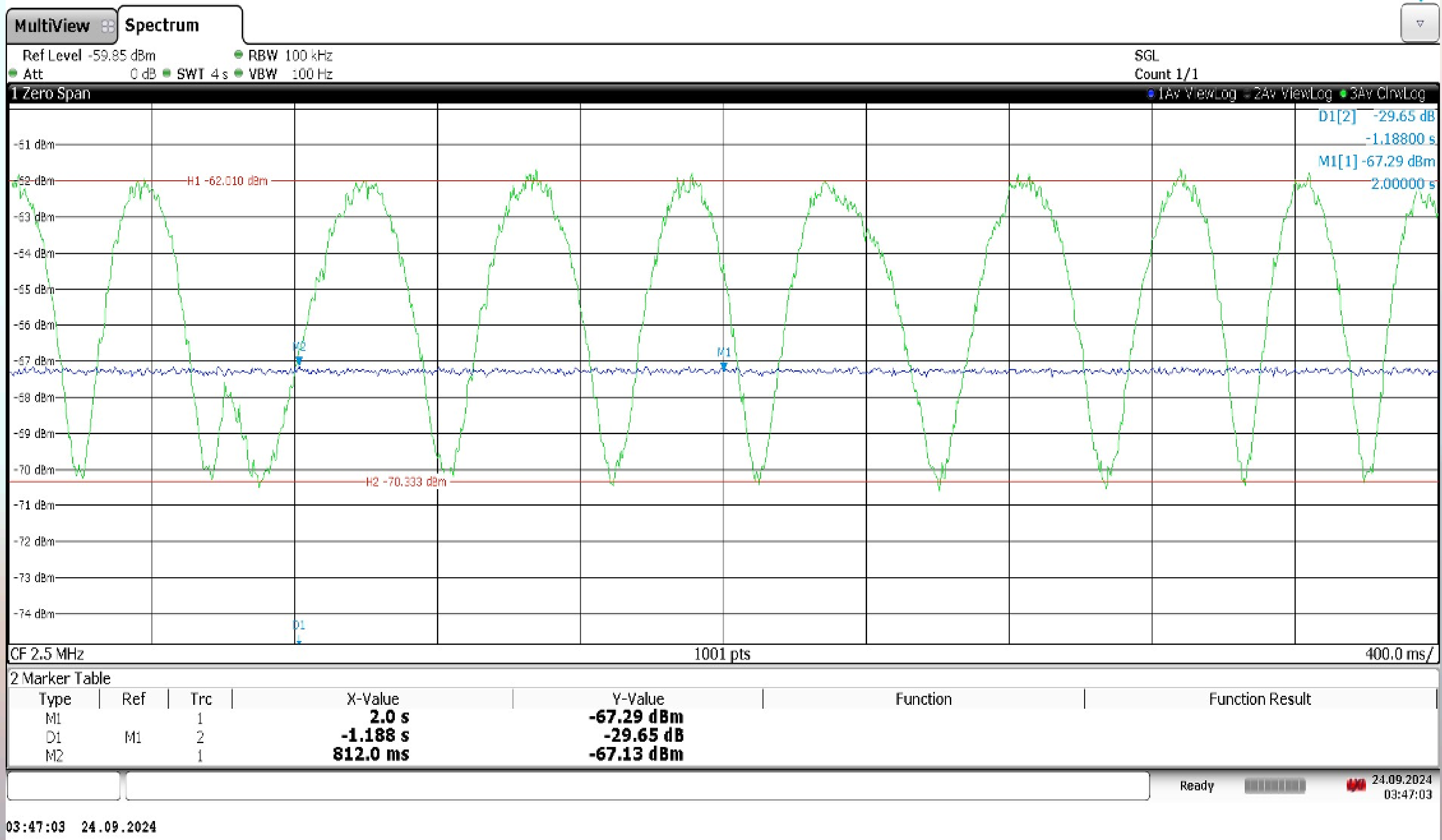


by Hsieh-Yi Hsieh

Squeezer on Chip: PPLN



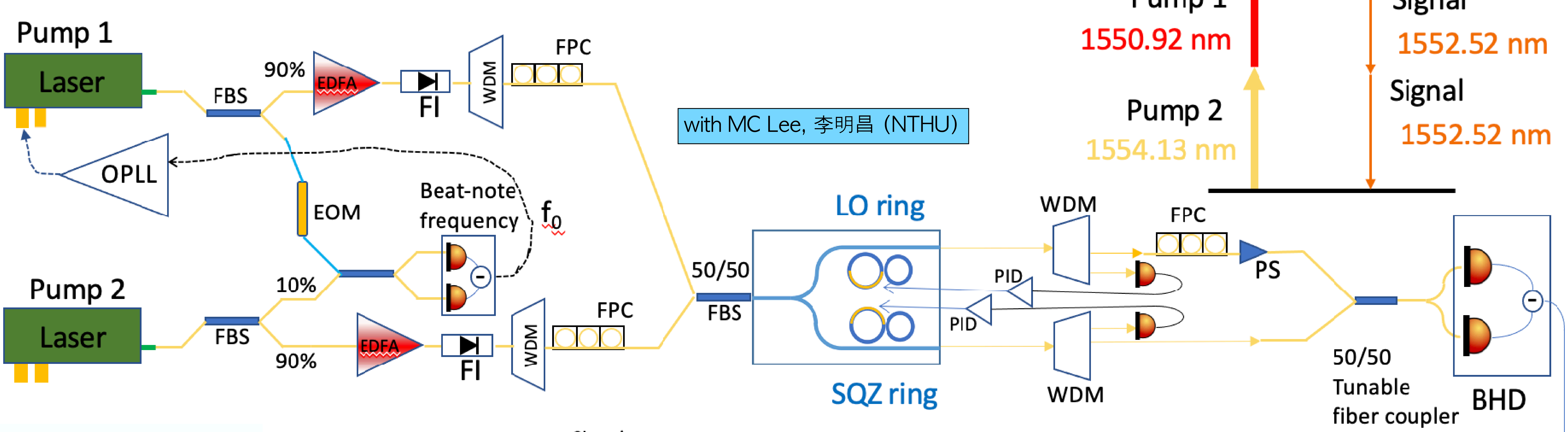


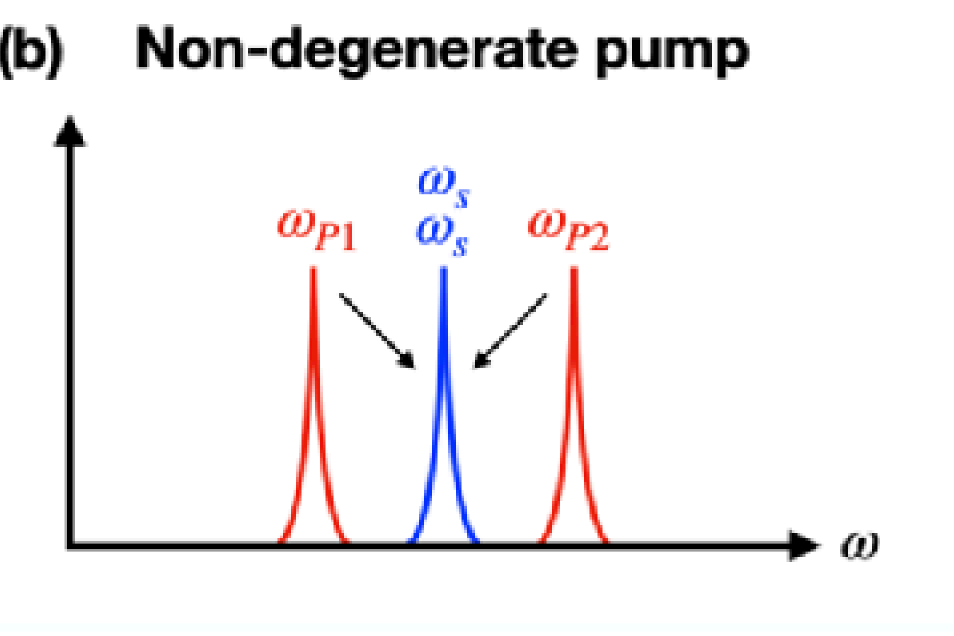


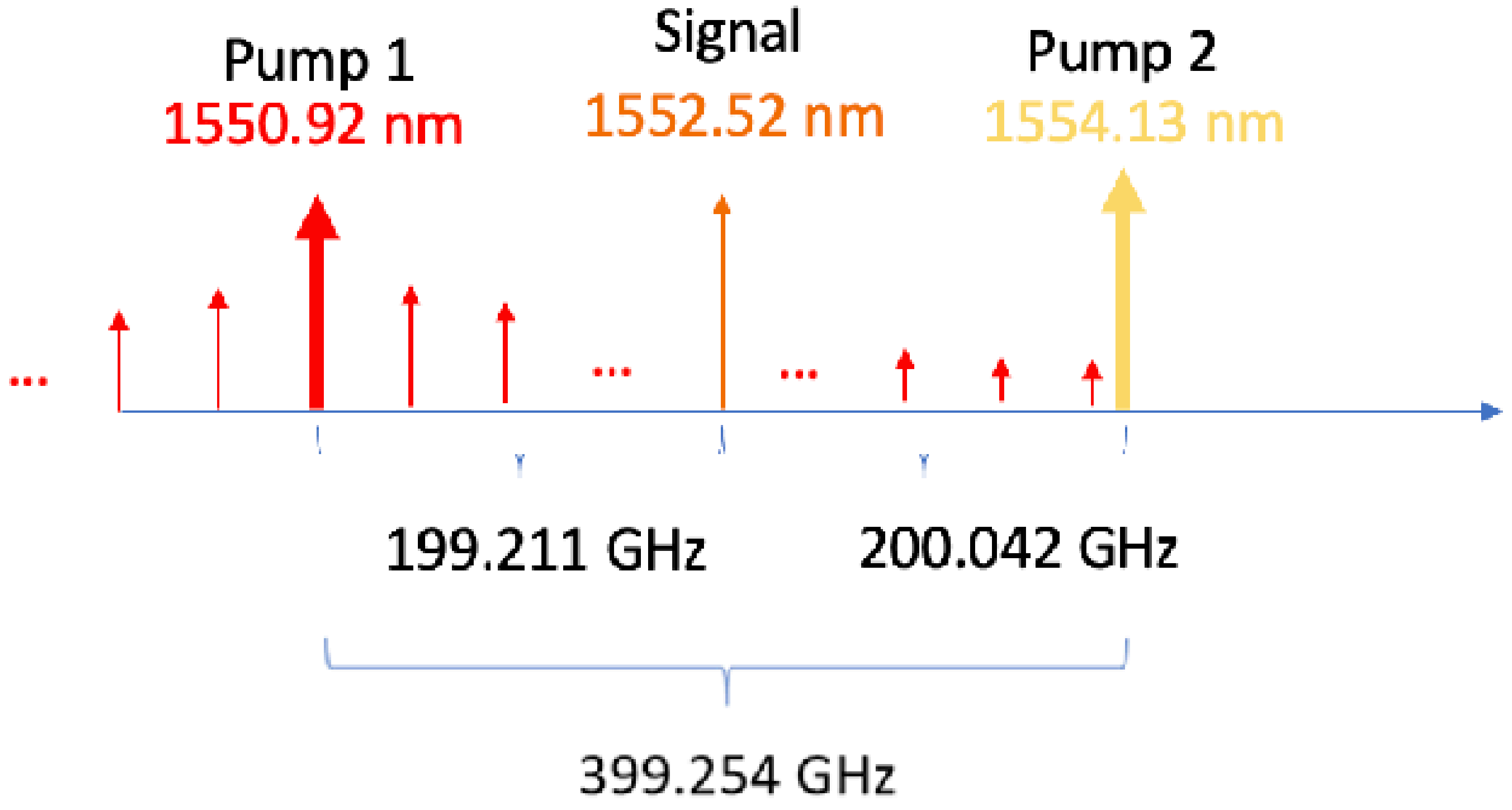
Waveguide type	**Ridged PPLN**
Squeezing level	-3.1 dB
Anti-squeezing level	5.2 dB
Waveguide length	30 mm
Waveguide loss	≤ 0.54 dB/cm

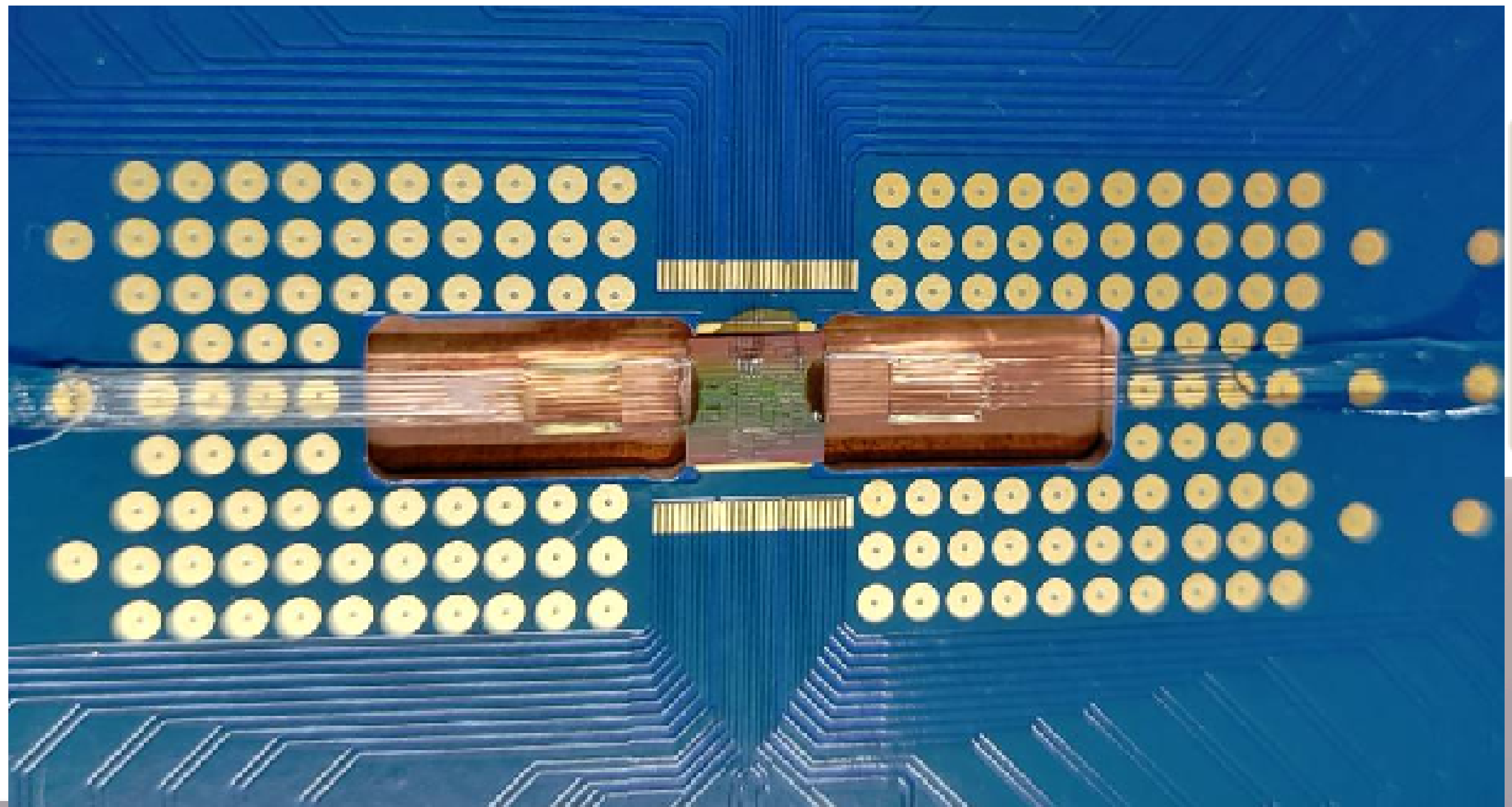
Squeezer on Chip: SiN

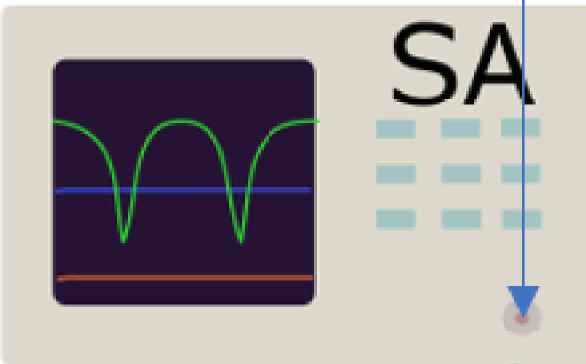
Plan : Use Optical Phase Locked Loop to lock the frequency difference between two lasers



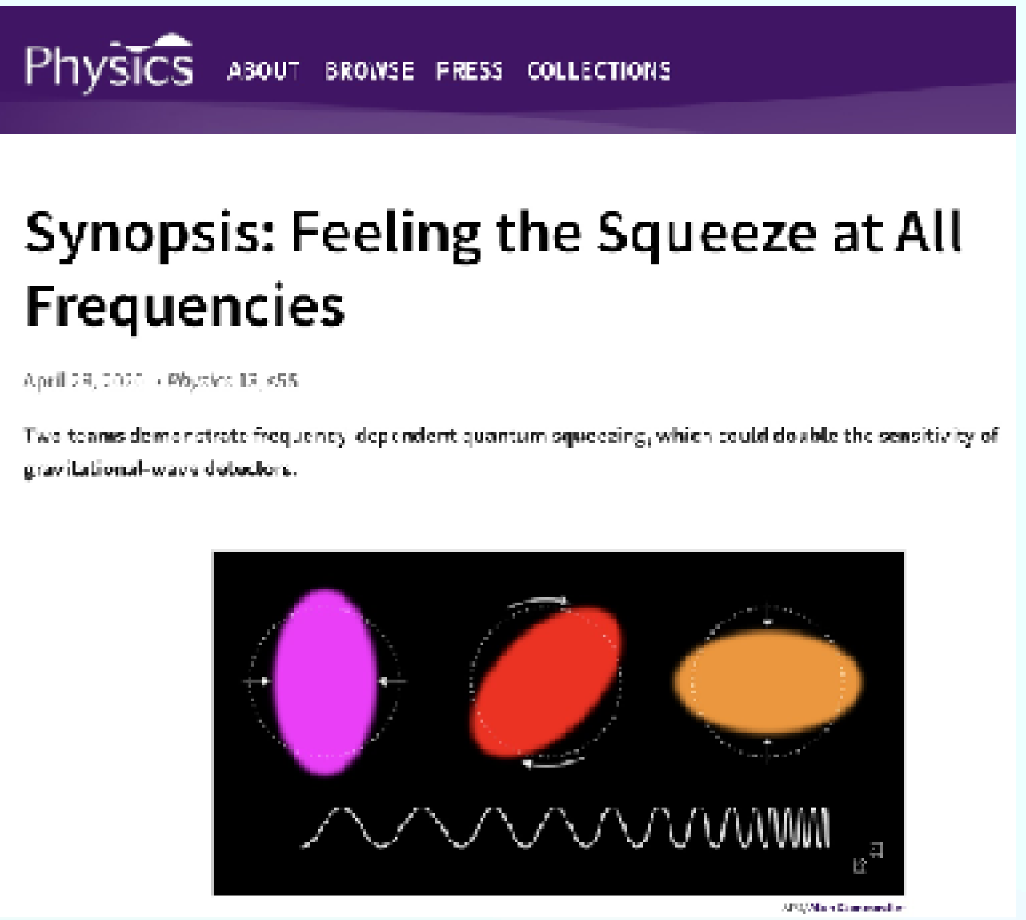






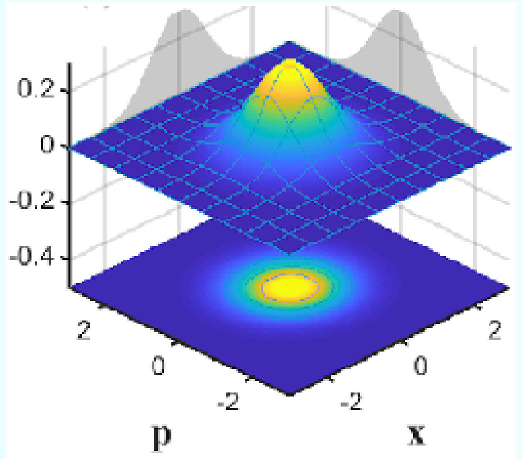


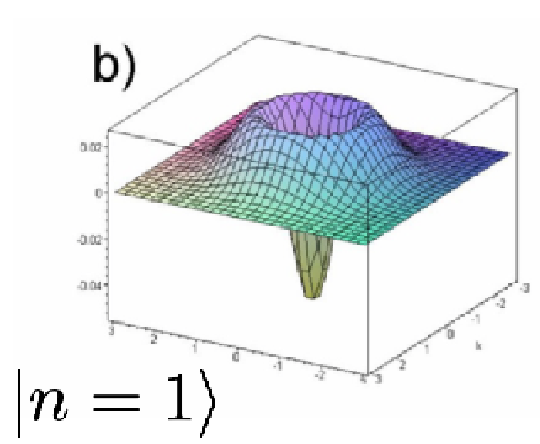
Quantum 2.0:

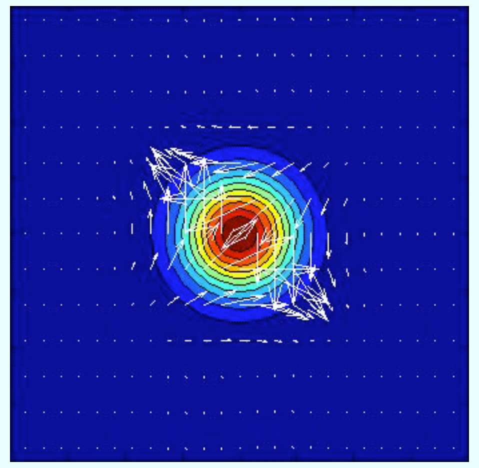


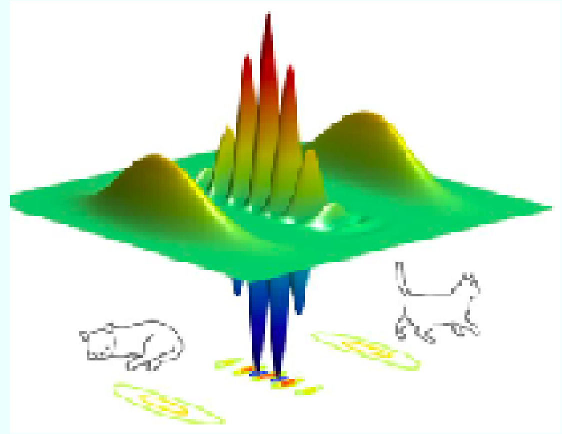
Phys. Rev. Lett. 124, 171101 (2020);

- Quantum Sensor
  - ✓FDS for GWD
  - EPR-SQZ
- Entanglement
  - Two-mode SQZ
- Teleportation
- Quantum Gate
- Error-Correcting
- Quantum Resource





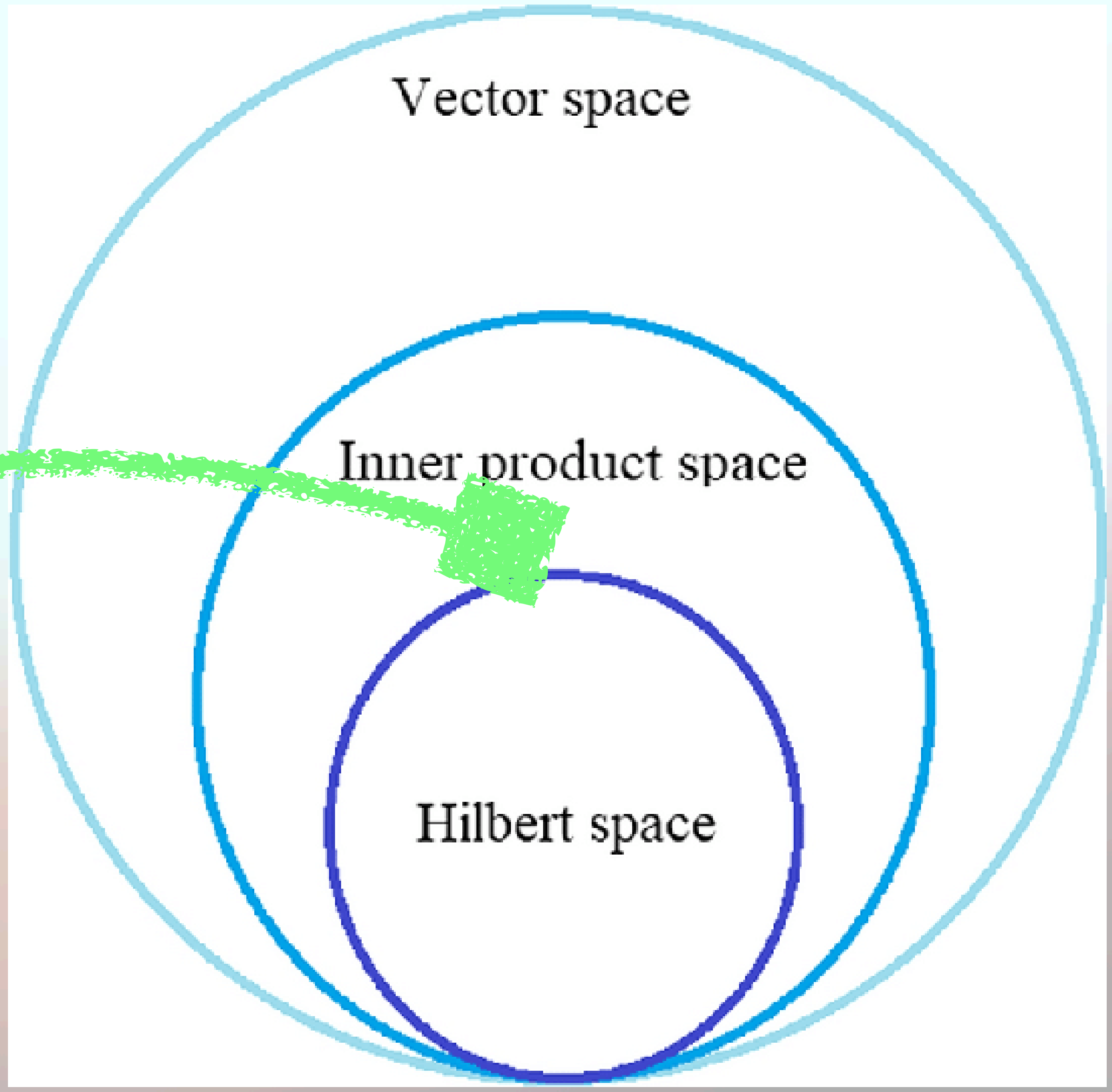




物理現實
Physical Reality

測量問題
Measurement Problem (Wavefunction Collapse)

抽象數學
Abstract Mathematics



$\hat{\rho} = \hat{\rho}_A \otimes \hat{\rho}_B$

$|0\rangle$

$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$

$|1\rangle$

$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$

$|\mathrm{W}\rangle$ $|n\rangle$

$|\mathrm{GHZ}\rangle$ $|\alpha\rangle$

$|N00N\rangle$ $|\xi\rangle$

$\vdots$ $|\alpha\rangle \pm |-\alpha\rangle$

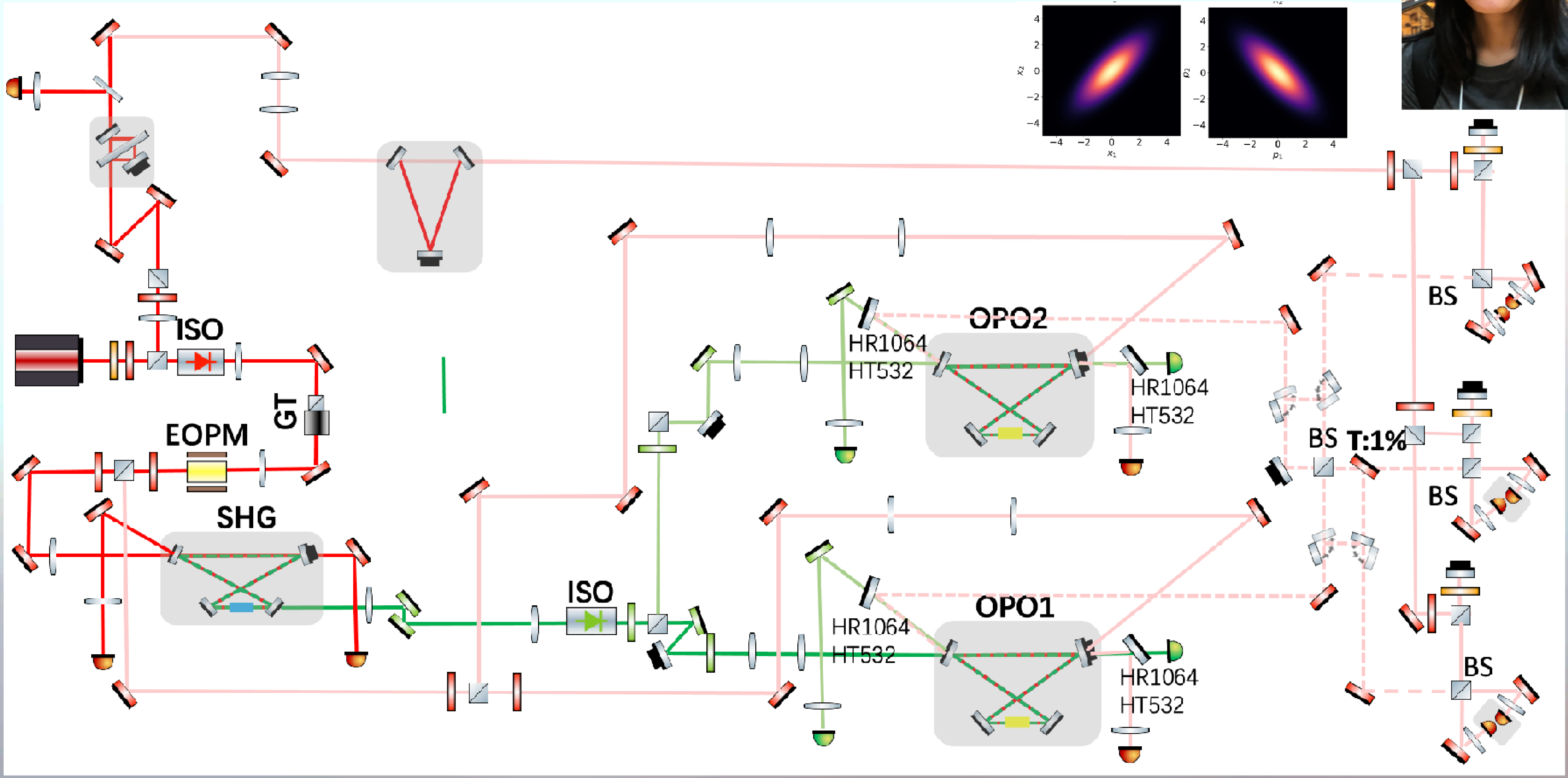
$|\xi_1, \xi_2\rangle$ $\vdots$

$|\mathrm{cluster}\rangle$ ρ_{th}

$|\mathrm{GKP}\rangle$

$\vdots$

Two-mode SQZ:


ISO
GT
EOPM
SHG
ISO
OPO2
HR1064
HT532
HR1064
HT532
OPO1
HR1064
HT532
HR1064
HT532
BS
BS
T:1%
BS
BS

Ray-Kuang Lee's LAB
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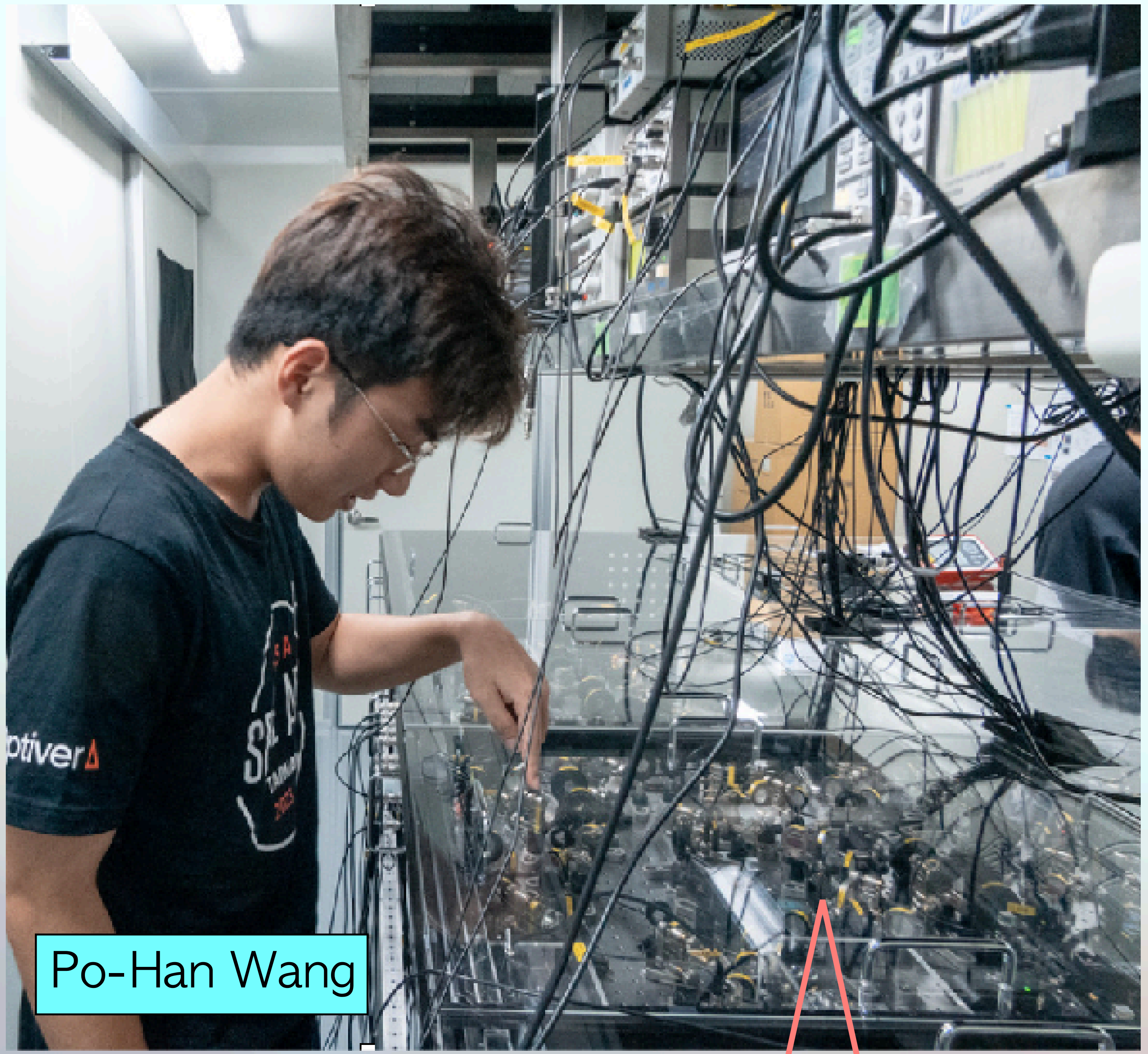
Current Activities:

Optical Cat States:
Non-Gaussianity

ML-QST:
Entanglement

Quantum Sensors:
Gravitational Wave Detector



- Largest Big Optical Cat (two-photon addition)
- GKP states as Error-Correcting Code
- Probe the Decoherence

- ML-QST for Entanglement in 2 mode SQZ
- Quantum Teleportation/Gate
- Entangled Cats

- Frequency-Dependent Squeezing (FDS)
- EPR-SQZ
- Quantum Filter

Summary:

Unavoidable coupling from the noisy environment makes the quantum light in a mixed state with **Degradation** embedded.

- ✓ **Extract the Degradation Information** in Squeezed States with **Machine Learning**, Phys. Rev. Lett. 128, 073604 (2022).
- ✓ Direct parameter estimations from ML-enhanced Quantum State Tomography, Symmetry, 14, 874, (2022).
- ✓ Neural network enhanced single-photon Fock state tomography, [arXiv: 2405.02812] (2024).

- ✓ **Video:** Wigner current in Decoherence, Phys. Rev. A 108, 023729 (2023)
- ✓ **Quantum Jump:** No Wigner current for single-photon, [arXiv: 2307.16510].
- ✓ **FPGA:** in-line ML, in preparation (2024).
- ✓ **Utilize: Heralded optical `Schrodinger cat' states by photon-addition,** Phys. Rev. A 110, 023703 (2024).

- ✓ **Reinforce Learning-QST** Non-Gaussian States, in preparation (2024).
- ✓ **Quantumness Measure** [arXiv: 2311.17399] (2023).
- ✓ **Quantum Machine Learning** Review, Advances in Phys. X 8, 2165452 (2023).

Thank you for your attention.