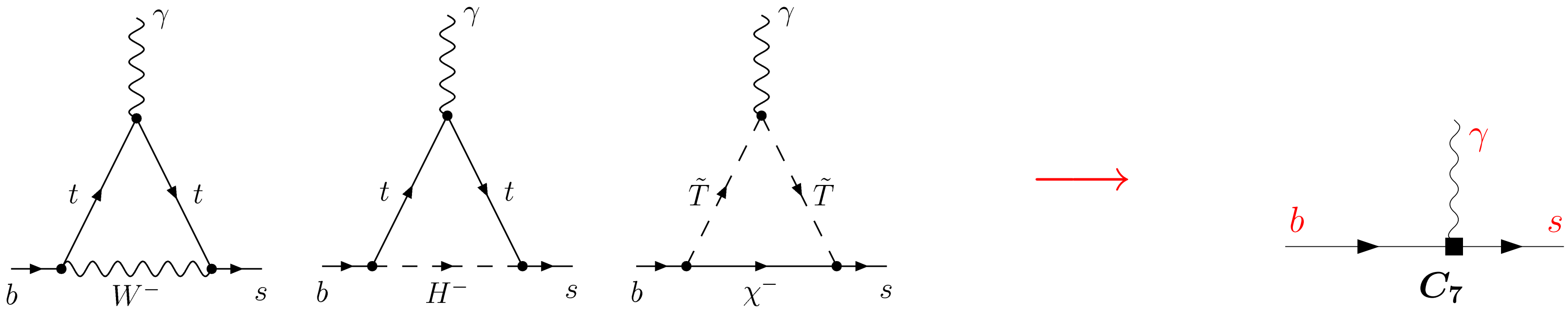
The inclusive $\bar{B} \to X_s \gamma$ photon spectrum and CP asymmetry

Mikołaj Misiak

(University of Warsaw)

1. **Introduction**
2. **Photon spectrum and cuts**
3. **Non-perturbative uncertainties in the decay rate**
4. **Isospin asymmetry**
5. **Direct CP asymmetry**
6. **Summary**

Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



$$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$$

The inclusive $\bar{B} \to X_s \gamma$ decay rate is well approximated by the corresponding perturbative decay rate of the b-quark:

$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p \gamma)_{E_\gamma > E_0} + \left(\begin{array}{c} \textbf{non-perturbative effects} \\ \mathbf{(2 \pm 5)\%} \\ \textbf{Benzke et al., arXiv:1003.5012} \end{array} \right)$$

provided E_0 is large $(E_0 \sim m_b/2)$
but not too close to the endpoint $(m_b - 2E_0 \gg \Lambda_{\rm QCD})$.

Conventionally, $E_0 = 1.6\,\text{GeV} \simeq m_b/3$ is chosen.

Results of the SM calculations:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{MM } et\ al.\text{, hep-ph/0609232, using the 1S scheme.} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kinetic scheme analysis of P. Gambino and P. Giordano in arXiv:0805.0271.} \end{cases}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, **3%** m_c-interpolation ambiguity at the NNLO (to be reduced soon),

3% higher order $\mathcal{O}(\alpha_s^3)$, **3%** parametric $\left(\alpha_s(M_Z),\ \mathcal{B}^{\rm exp}_{\rm semileptonic},\ m_c\ \&\ C,\ \ldots\right)$.

$\alpha_s(M_Z)$: 2.0%, $\mathcal{B}^{\rm exp}_{\rm semileptonic}$: 1.6%, m_c & C: 1.1% (1S), 2.5% (kin)

Experimental world averages:

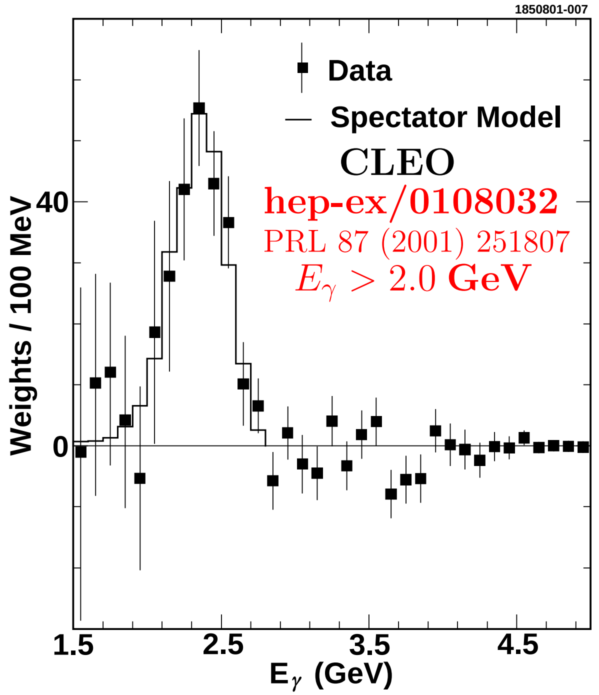
$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm EXP}_{E_\gamma > 1.6\ \mathrm{GeV}} = \begin{cases} (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}, & \text{[HFAG, arXiv:1010.1589],} \\ (3.50 \pm 0.17) \times 10^{-4}, & \text{[Artuso, Barberio, Stone, arXiv:0902.3743].} \end{cases}$$

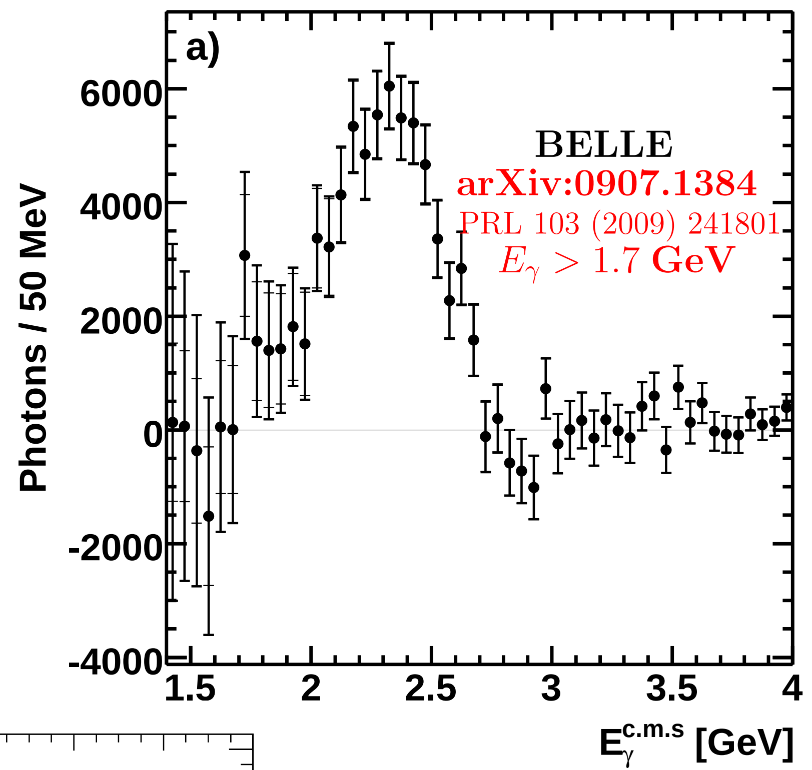
Experiment agrees with the SM at the $\sim 1.2\sigma$ level. Uncertainties: TH $\sim 7\%$, EXP $\sim 7\%$.

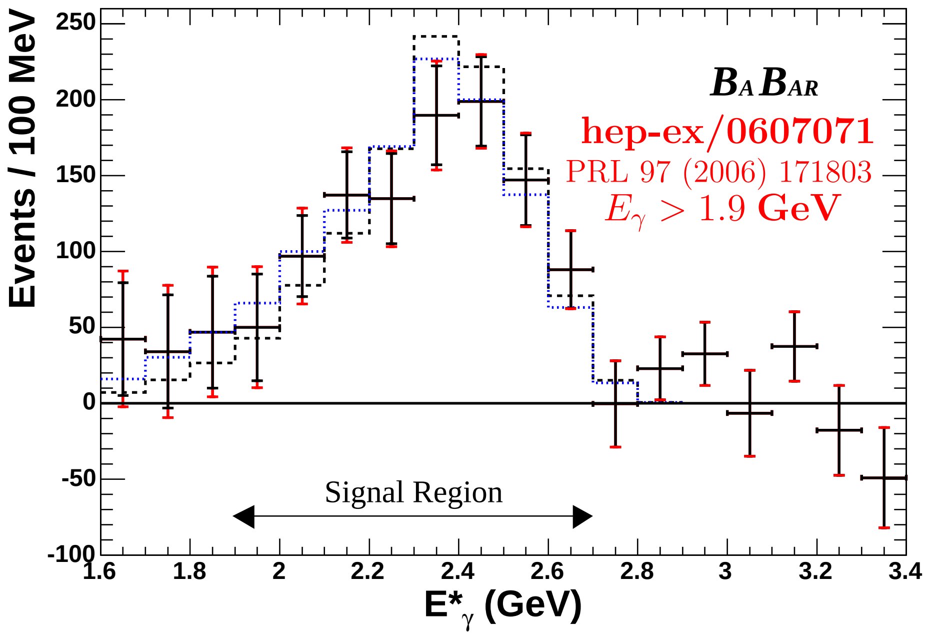
The HFAG average includes the following measurements:

Reference	Method	# of $B\bar{B}$	E_0 [GeV]	$\mathcal{B} \times 10^4$ at E_0
CLEO [PRL 87 (2001) 251807]	inclusive	9.70×10^6	2.0	$2.94 \pm 0.41 \pm 0.26$
BABAR [PRL 97 (2006) 171803]	inclusive	8.85×10^7	1.9 2.0 2.1 2.2	$3.67 \pm 0.29 \pm 0.34 \pm 0.29$ $3.41 \pm 0.27 \pm 0.29 \pm 0.23$ $2.97 \pm 0.24 \pm 0.25 \pm 0.17$ $2.42 \pm 0.21 \pm 0.20 \pm 0.13$
BELLE [PRL 103 (2009) 241801]	inclusive	6.57×10^8	1.7 1.8 1.9 2.0	$3.45 \pm 0.15 \pm 0.40$ $3.36 \pm 0.13 \pm 0.25$ $3.21 \pm 0.11 \pm 0.16$ $3.02 \pm 0.10 \pm 0.11$
BABAR [PRD 77 (2008) 051103]	inclusive with a hadronic tag (hadronic decay of the recoiling B ($\bar{B}$))	2.32×10^8, which gives 6.8×10^5 tagged events	1.9 2.0 2.1 2.2 2.3	$3.66 \pm 0.85 \pm 0.60$ $3.39 \pm 0.64 \pm 0.47$ $2.78 \pm 0.48 \pm 0.35$ $2.48 \pm 0.38 \pm 0.27$ $2.07 \pm 0.30 \pm 0.20$
BABAR [PRD 72 (2005) 052004]	semi-inclusive	8.89×10^7	1.9	$3.27 \pm 0.18^{+0.55+0.04}_{-0.40-0.09}$
BELLE [PLB 511 (2001) 151]	semi-inclusive	6.07×10^6	?	$3.36 \pm 0.53 \pm 0.42^{+0.50}_{-0.54}$

The “raw” photon energy spectra in the inclusive measurements







The peaks are centered around

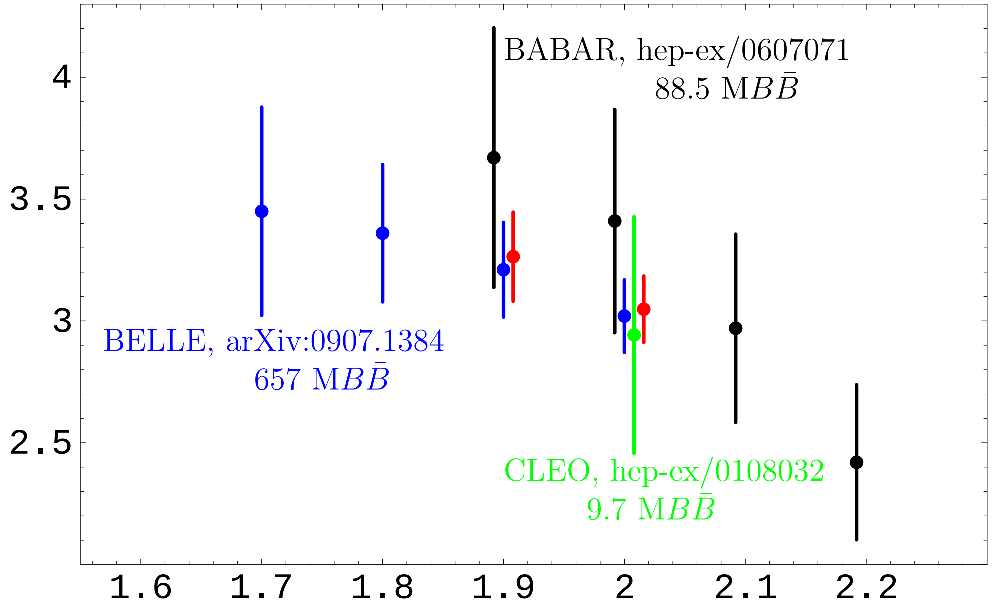
$$\tfrac{1}{2} m_b \simeq 2.35 \text{ GeV}$$

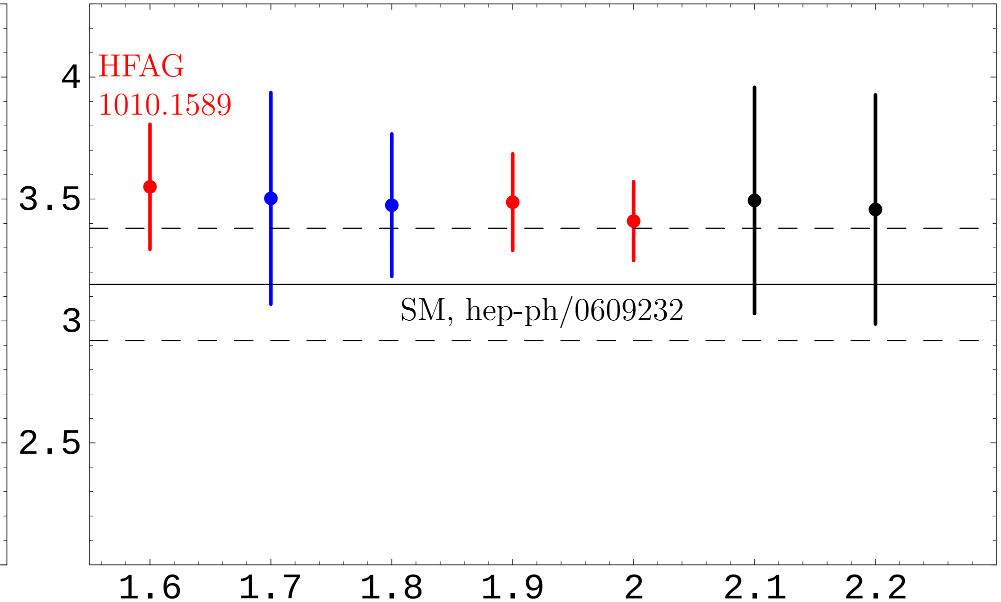
which corresponds to a two-body $b \to s\gamma$ decay.

Broadening is due to (mainly):

- perturbative gluon bremsstrahlung,
- motion of the b quark inside the $\bar{B}$ meson,
- motion of the $\bar{B}$ meson in the $\Upsilon(4S)$ frame.

Comparison of the inclusive measurements of $\mathcal{B}(\bar{B} \to X_s\gamma)$ by CLEO, BELLE and BABAR for each E_0 separately





The HFAG factors

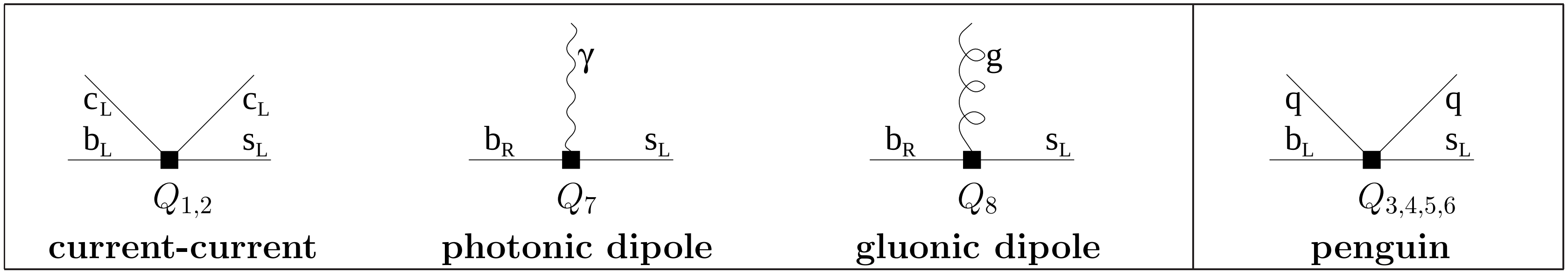
Scheme	$E_\gamma < 1.7$	$E_\gamma < 1.8$	$E_\gamma < 1.9$	$E_\gamma < 2.0$	$E_\gamma < 2.242$
Kinetic	0.986 ± 0.001	0.968 ± 0.002	0.939 ± 0.005	0.903 ± 0.009	0.656 ± 0.031
Neubert SF	0.982 ± 0.002	0.962 ± 0.004	0.930 ± 0.008	0.888 ± 0.014	0.665 ± 0.035
Kagan-Neubert	0.988 ± 0.002	0.970 ± 0.005	0.940 ± 0.009	0.892 ± 0.014	0.643 ± 0.033
Average	0.985 ± 0.004	0.967 ± 0.006	0.936 ± 0.010	0.894 ± 0.016	0.655 ± 0.037

- **Why do we need to extrapolate to lower E_0?**
- **Are the HFAG factors trustworthy?**

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\text{weak}} \sim \Sigma\; C_i(\mu_b)\, Q_i$$

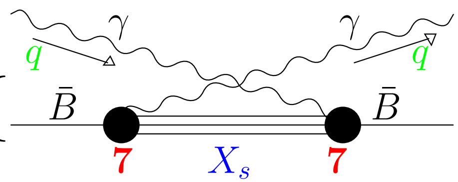
8 operators matter in the SM when the higher-order EW and/or CKM-suppressed effects are neglected:



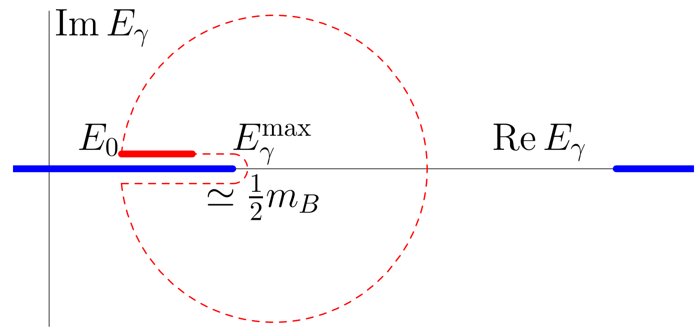
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = |C_7|^2\, \Gamma_{77}(E_0) + (\text{other})$$

Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \text{Im}\{\ \ldots\ \} \equiv \text{Im}\, A$$



Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\text{QCD}}}{m_b}$ and α_s that begins with

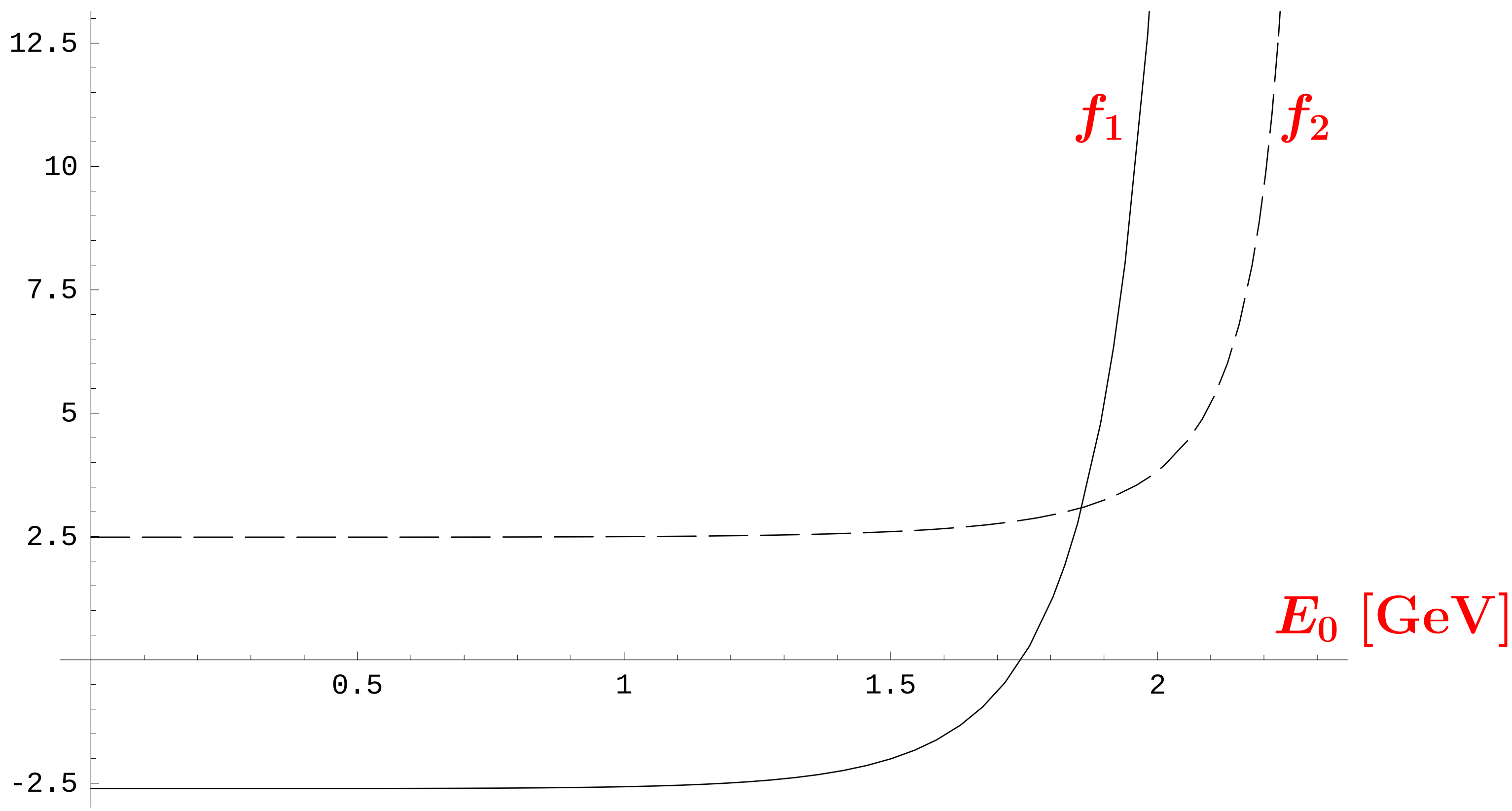
$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3},\ldots;\ \frac{\alpha_s\mu_\pi^2}{(m_b - 2E_0)^2},\ \frac{\alpha_s\mu_G^2}{m_b(m_b - 2E_0)};\ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\text{QCD}})$ are extracted from the semileptonic $\bar{B} \to X_c e\bar{\nu}$ spectra and the B–$B^\star$ mass difference.

The $\mathcal{O}\left(\frac{\alpha_s \mu_\pi^2}{(m_b-2E_0)^2}\right)$ and $\mathcal{O}\left(\frac{\alpha_s \mu_G^2}{m_b(m_b-2E_0)}\right)$ corrections

[T. Ewerth, P. Gambino and S. Nandi, arXiv:0911.2175]

$$\Gamma_{77}(E_0) = \Gamma_{77}^{\text{tree}} \Big\{ 1 + (\text{pert. corrections}) - \frac{\mu_\pi^2}{2m_b^2}\left[1 + \frac{\alpha_s}{\pi}\left(f_1(E_0) - \frac{4}{3}\ln\frac{\mu}{m_b}\right)\right] - \frac{3\mu_G^2(\mu)}{2m_b^2}\left[1 + \frac{\alpha_s}{\pi}\left(f_2(E_0) + \frac{1}{6}\ln\frac{\mu}{m_b}\right)\right]\Big\}$$



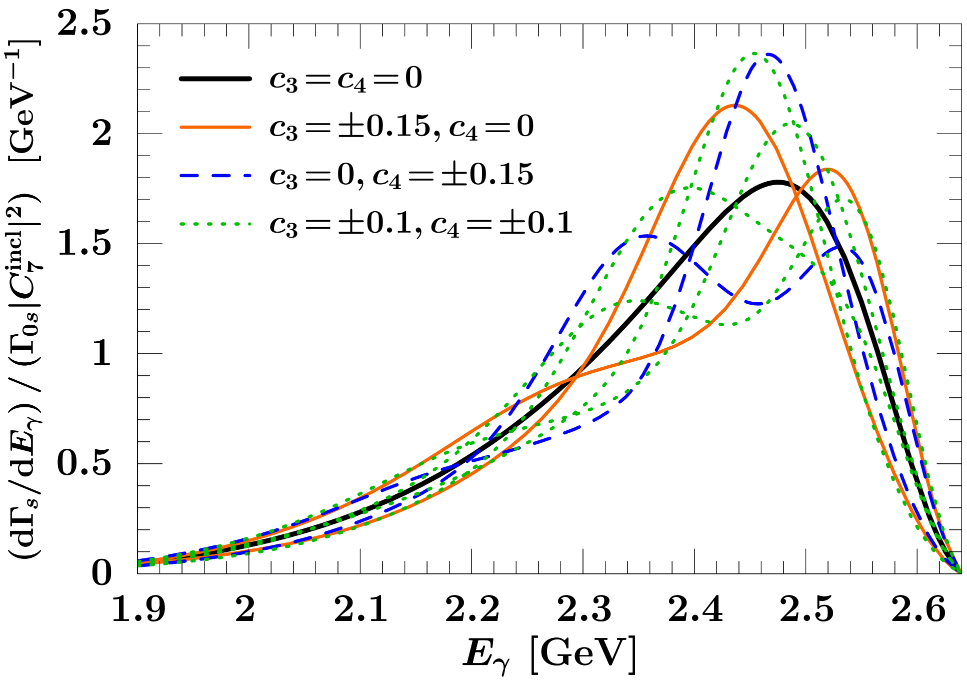
When $(m_b - 2E_0) \sim \Lambda \equiv \Lambda_{\rm QCD}$, no OPE can be applied.

Local operators $\longrightarrow$ Non-local operators

Non-perturbative parameters $\longrightarrow$ Non-perturbative functions

$$\frac{d}{dE_\gamma}\Gamma_{77} = \underbrace{N\, H(E_\gamma)}_{\text{pert.}} \int_0^{M_B-2E_\gamma} dk\ \underbrace{P(M_B-2E_\gamma-k)}_{\text{pert.}}\ \underbrace{F(k)}_{\text{non-pert.}} + \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

Photon spectra from models of $F(k)$ [Ligeti, Stewart, Tackmann, arXiv:0807.1926]



The function $F(k)$ is:

- perturbatively related to the standard shape function $S(\omega)$,
- exponentially suppressed for $k \gg \Lambda$,
- positive definite,
- constrained by measured moments of the $\bar{B} \to X_c e\bar{\nu}$ spectrum (local OPE),
- constrained by measured properties of the $\bar{B} \to X_u e\bar{\nu}$ and $\bar{B} \to X_s\gamma$ spectra (not imposed in the plot).

Upgrading the HFAG factors by fitting $F(k)$ to data:

- The SIMBA Collaboration [arXiv:1101.3310] (work in progress)

$$F(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n f_n\left(\frac{k}{\lambda}\right)\right]^2, \qquad f_n - \text{basis functions. Truncate and fit.}$$

- Another way: $F(k) = \underset{\text{perfect fit}}{A(k)}B(k)$ and use the SIMBA approach for $B(k)$.

Why do we need to upgrade the HFAG factors?

- The old models (Kagan-Neubert 1998, ...) are not generic enough (too few parameters).
- Inclusion of $\mathcal{O}\left(\frac{\Lambda}{m_b}\right)$ effects and and taking other operators $(Q_i \neq Q_7)$ into account is necessary [Benzke, Lee, Neubert, Paz, arXiv:1003.5012].

What about just fitting C_7 without extrapolation any particular E_0?

- Fine, but measurements at low E_0 (even less precise) are still going to be crucial for constraining the parameter space.
- The fits are going to give the extrapolation factors anyway. Publishing them is necessary for cross-checks/upgrades by other groups.

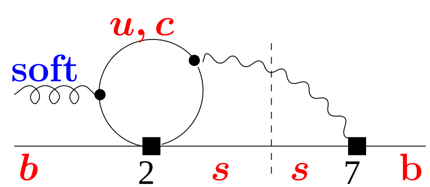
Non-perturbative effects in the presence of other operators $(Q_i \neq Q_7)$

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012].

$$\frac{d}{dE_\gamma}\Gamma(\bar{B} \to X_s\gamma) = (\Gamma_{77}\text{-like term}) \; + \; \tilde{N} E_\gamma^3 \sum_{i \le j} \mathrm{Re}\left(C_i^* C_j\right) F_{ij}(E_\gamma).$$

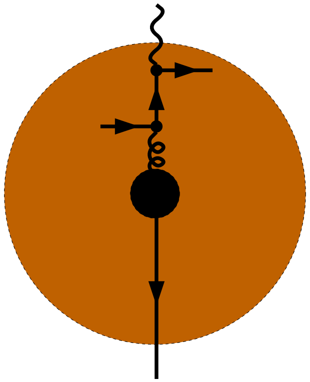
Remarks:

- The SCET approach is valid for large E_γ only. It is fine for $E_\gamma > E_0 \sim \frac{1}{3} m_b \simeq 1.6$ GeV. Lower cutoffs are academic anyway.
- For such E_0, non-perturbative effects in the integrated decay rate are estimated to remain within 5%. They scale like:
  - $\frac{\Lambda^2}{m_b^2}$, $\frac{\Lambda^2}{m_c^2}$ (known),
  - $\frac{\Lambda}{m_b}\frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}$ (negligible),
  - $\frac{\Lambda}{m_b}$, $\frac{\Lambda^2}{m_b^2}$, $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by tails of subleading shape functions ("27"),
  - $\alpha_s \frac{\Lambda}{m_b}$ to be constrained by future measurements of the isospin asymmetry ("78"),
  - $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by $Q_d^2 = \frac{1}{9}$ ("88").



- **Extrapolation factors?** Tails of subleading functions are less important for them.

Importance of the isospin asymmetry



A hard gluon scatters on the valence quark or a "sea" quark and produces an energetic photon. The quark that undergoes this Compton-like scattering is assumed to remain soft in the $\bar{B}$-meson rest frame to ensure effective interference with the leading "hard" amplitude. Without interference the contribution would be negligible ($\mathcal{O}(\alpha_s^2\Lambda^2/m_b^2)$).

Suppression by Λ can be understood as originating from dilution of the target (size of the $\bar{B}$-meson $\sim \Lambda^{-1}$).

A rough estimate using vacuum insertion approximation gives

$$\Delta\Gamma/\Gamma \in [-2.8\%, -0.3\%] \qquad (\mathcal{O}(\alpha_s\Lambda/m_b)).$$

[Lee, Neubert, Paz, hep-ph/0609224]
[Benzke, Lee, Neubert, Paz, arXiv:1003.5012]

However:

1. Contribution to the interference from scattering on the "sea" quarks vanishes in the $SU(3)_{\mathrm{flavour}}$ limit because $Q_u + Q_d + Q_s = 0$.

2. If the valence quark dominates, then the isospin-averaged $\Delta\Gamma/\Gamma$ is given by:

$$\frac{\Delta\Gamma}{\Gamma} \simeq \frac{Q_d+Q_u}{Q_d-Q_u}\Delta_{0-} = \left(+0.2 \pm 1.9_{\mathrm{stat}} \pm 0.3_{\mathrm{sys}} \pm 0.8_{\mathrm{ident}}\right)\%,$$

using the BABAR semi-inclusive measurement (hep-ex/0508004) of the isospin asymmetry

$$\Delta_{0-} = [\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)]/[\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^- \to X_s\gamma)],$$

for $E_\gamma > 1.9$ GeV.

Quark-to-photon conversion gives a soft s-quark and poorly interferes with the "hard" $b \to s\gamma g$ amplitude.

The direct CP asymmetry

$$A_{X_s\gamma} = \frac{\Gamma(\bar{B}\to X_s\gamma) - \Gamma(B\to X_{\bar{s}}\gamma)}{\Gamma(\bar{B}\to X_s\gamma) + \Gamma(B\to X_{\bar{s}}\gamma)}$$

Semi inclusive measurements $\Rightarrow$ $A^{\rm exp}_{X_s\gamma} = -(1.2 \pm 2.8)\%$ (HFAG average)

SM estimate [Benzke, Lee, Neubert, Paz, arXiv:1012.3167]:

$$A^{\rm SM}_{X_s\gamma} \simeq \mathrm{Im}\left(\frac{V^*_{us}V_{ub}}{V^*_{ts}V_{tb}}\right)\pi\left|\frac{C_1^{\rm their}}{C_7}\right|\left[\frac{\tilde{\Lambda}^u_{17}-\tilde{\Lambda}^c_{17}}{m_b} + \frac{40\alpha_s}{9\pi}\frac{m_c^2}{m_b^2}\left(1-\frac{2}{5}\ln\frac{m_b}{m_c}+\frac{4}{5}\ln^2\frac{m_b}{m_c}-\frac{\pi^2}{15}\right)\right]$$

$$\simeq \left(1.15\,\frac{\tilde{\Lambda}^u_{17}-\tilde{\Lambda}^c_{17}}{300\ \mathrm{MeV}} + 0.71\right)\% \in [-0.6\%, +2.8\%] \quad \text{using} \begin{cases} -330\ \mathrm{MeV} < \tilde{\Lambda}^u_{17} < +525\ \mathrm{MeV} \\ -9\ \mathrm{MeV} < \tilde{\Lambda}^u_{17} < +11\ \mathrm{MeV} \end{cases}$$

Despite the uncertainties, $A_{X_s\gamma}$ provides constraints on models with non-minimal flavour violation. Such models are also constrained by:

$$A_{X_{(s+d)}\gamma} = \frac{\Gamma(\bar{B}\to X_{(s+d)}\gamma) - \Gamma(B\to X_{(\bar{s}+\bar{d})}\gamma)}{\Gamma(\bar{B}\to X_{(s+d)}\gamma) + \Gamma(B\to X_{(\bar{s}+\bar{d})}\gamma)} \qquad \left(A^{\rm SM}_{X_{(s+d)}\gamma} \simeq 0\right)$$

Summary

- For the $\bar{B} \to X_s\gamma$ branching ratio and moments of the photon spectrum, measurements at all the photon energy cutoffs $E_0 \in [1.6, 2.1]$ GeV are relevant (with correlation matrices) for getting constraints on C_7.

- A coordinated effort of theorists and experimentalists can lead to significant reduction of TH/EXP errors and making them reliable.

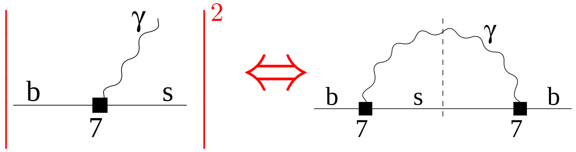
- The direct CP asymmetry $A_{X_s\gamma}$ in the SM is likely to be dominated by unknown non-perturbative contributions. Nevertheless, it can still provide constraints on non-MFV models, in parallel to $A_{X_{(s+d)}\gamma}$.

BACKUP SLIDES

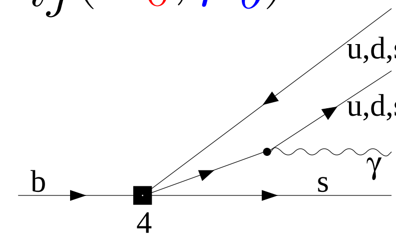
Perturbative evaluation of $\Gamma(b \to X_s^{\mathrm{p}}\gamma)$ at $\mu_b \sim \frac{m_b}{2}$.

$$\Gamma(b \to X_s^{\mathrm{p}}\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\mathrm{em}}}{32\pi^4} \left| V_{ts}^* V_{tb} \right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

LO: $G_{77} = 1$



Other LO are small, e.g.,:



[Kamiński, Poradziński, MM, in preparation]

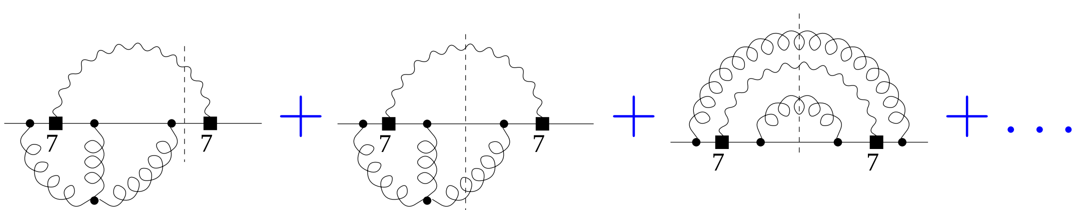
NLO: **1996: Quasi-complete** G_{ij} { [Greub, Hurth, Wyler, 1996] [Ali, Greub, 1991-1995]

2002: Complete$^{(*)}$ G_{ij} { [Buras, Czarnecki, Urban, MM, 2002] [Pott, 1995]

$^{(*)}$ **Up to** $b \to sq\bar{q}\gamma$ **channel contributions involving diagrams similar to the above LO one.**
They get suppressed by $\alpha_s C_{3,4,5,6}$ **and phase-space for** $E_0 \sim m_b/3$.

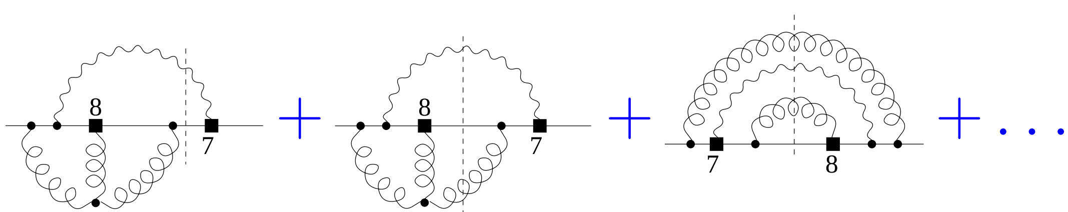
NNLO: **We are still on the way to the quasi-complete case:**

G_{77} **is fully known:**



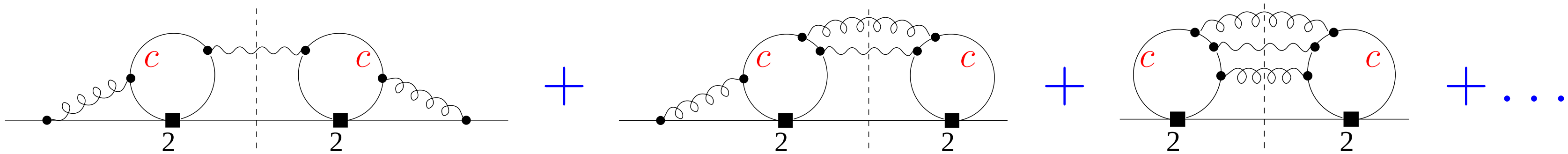
{ [Blokland et al., 2005] [Melnikov, Mitov, 2005] [Asatrian et al., 2006-2007]

G_{78} **is fully known:**

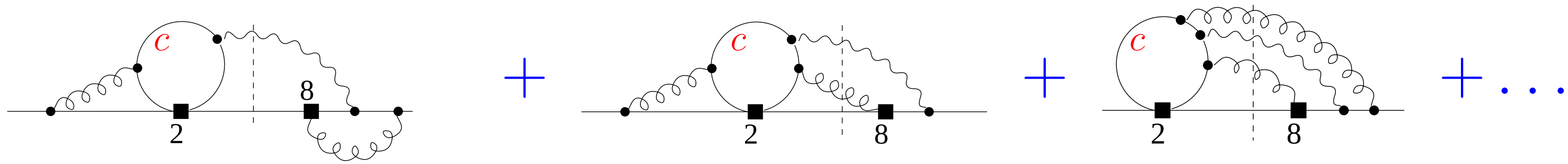


[Asatrian et al., arXiv:1005.5587]

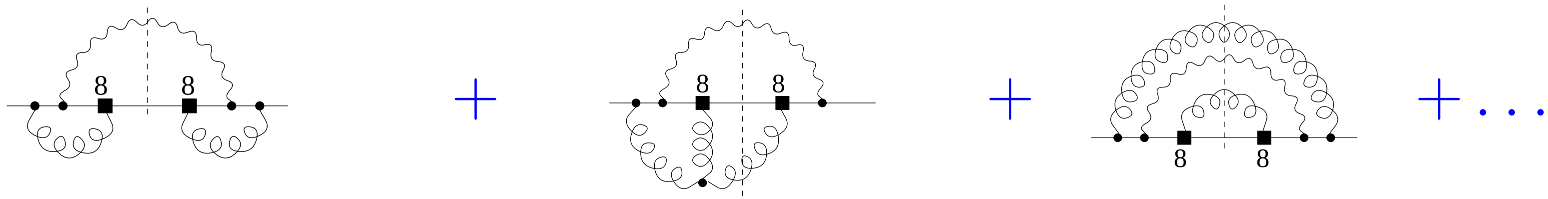
G_{22}:
(and analogous G_{11} & G_{12})



G_{28}:
(and analogous G_{18})



G_{88}:

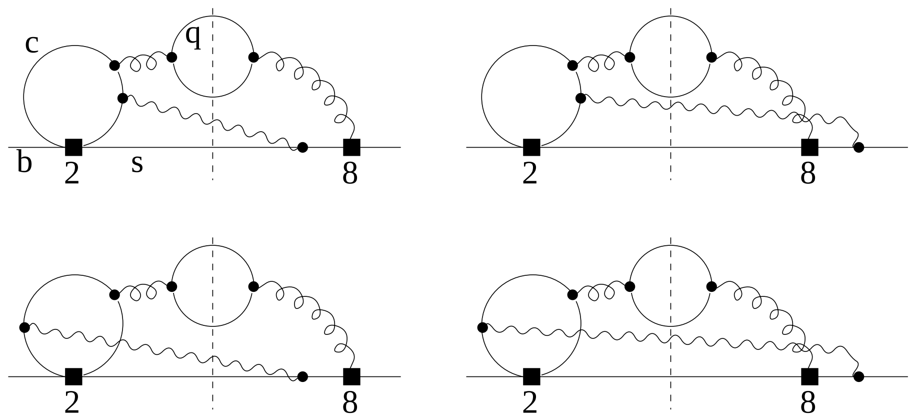


Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only: [Ligeti, Luke, Manohar, Wise, 1999], [Ferroglia, Haisch, arXiv:1009.2144], [Poradziński, MM, arXiv:1009.5685].
NLO+(NNLO BLM) corrections are not big $(+3.8\%)$**.**

Example:

Evaluation of the $(n > 2)$**-particle cut contributions to** G_{28} **in the Brodsky-Lepage-Mackienzie (BLM) approximation ("naive nonabelianization", large-**β_0 **approximation)** [Poradziński, MM, arXiv:1009.5685]:



q **– massless quark,**

N_q **– number of massless flavours (equals to 3 in practice because masses of** u, d, s **are neglected).**

Replacement in the final result:

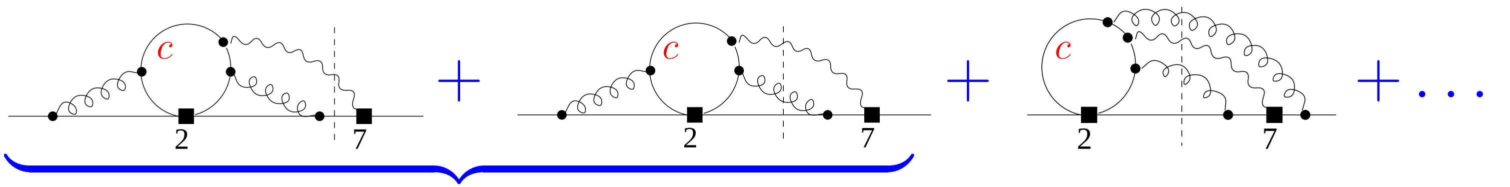
$$-\tfrac{2}{3}N_q \quad \longrightarrow \quad \beta_0 = 11 - \tfrac{2}{3}(N_q + 2).$$

The diagrams have been evaluated using the method of Smith and Voloshin [hep-ph/9405204].

Non-BLM contributions to G_{ij} **from quark loops on the gluon lines are quasi-completely known.**
[Boughezal, Czakon, Schutzmeier, 2007], [Asatrian, Ewerth, Gabrielyan, Greub, 2007], [Ewerth, 2008].

The only important but still missing NNLO contribution to G_{ij}:

G_{27}:
(and analogous G_{17})



$m_c = 0$: [Boughezal, Czakon, Schutzmeier, to be published]
[T. Schutzmeier, Ph.D. thesis, 2010]
$\mathcal{O}(200)$ massive 4-loop on-shell master integrals.

$m_c = 0$: [Czakon, Huber, Schutzmeier, Fiedler]
in progress...

The $m_c \gg m_b/2$ limit is known [Steinhauser, MM, 2006].

The BLM approximation is known for arbitrary m_c: [Bieri, Greub, Steinhauser, 2003], [Ligeti, Luke, Manohar, Wise, 1999].

The non-BLM correction to G_{27} has been interpolated in m_c assuming BLM in Γ at $m_c = 0$.

Towards G_{27} at the NNLO for arbitrary m_c.

[M. Czakon, R.N. Lee, M. Steinhauser, A.V. Smirnov, V.A. Smirnov, MM] **in progress.**

1. **Generation of diagrams and performing the Dirac algebra to express everything in terms of four-loop two-scale scalar integrals with unitarity cuts.**

2. **Reduction to master integrals with the help of Integration By Parts (IBP).**
   Available `C++` codes:

FIRE	[A.V. Smirnov, arXiv:0807.3243] (public in the *Mathematica* version only),
REDUZE	[C. Studerus, arXiv:0912.2546],
DiaGen/IdSolver	[M. Czakon, unpublished (2004)].

   The IBP for 2-particle cuts has just been completed with the help of FIRE: ~ 0.5 TB RAM has been used ~ 1 month at CERN and KIT.
   Number of master integrals: around 500.

3. **Extending the set of master integrals I_n so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations**

$$\frac{d}{dz} I_n = \Sigma_k \, w_{nk}(z, \epsilon) \, I_k, \qquad (*)$$

where w_{nk} are rational functions of their arguments.

4. **Calculating boundary conditions for $(*)$ using automatized asymptotic expansions at $m_c \gg m_b$.**

5. **Calculating three-loop single-scale master integrals for the boundary conditions using dimensional recurrence relations** [R.N. Lee, arXiv:0911.0252].

6. **Solving the system $(*)$ numerically** [A.C. Hindmarsch, `http://www.netlib.org/odepack`] **along an ellipse in the complex z plane. Doing so along several different ellipses allows us to estimate the numerical error.**

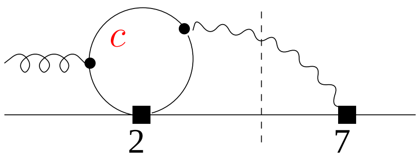
This algorithm has already been successfully applied for diagrams with (massless and massive) quark loops on the gluon lines where $18 + 47 + 38 = 103$ master integrals were present.

[R. Boughezal, M. Czakon, T. Schutzmeier, arXiv:0707.3090]

Non-perturbative contributions from the photonic dipole operator alone ("77" term) are well controlled for $E_0 = 1.6$ GeV:

$\mathcal{O}\left(\frac{\alpha_s^n \Lambda}{m_b}\right)_{n=0,1,2,\ldots}$ vanish, $\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)$ [Bigi, Blok, Shifman, Uraltsev, Vainshtein, 1992], [Falk, Luke, Savage, 1993], $\mathcal{O}\left(\frac{\Lambda^3}{m_b^3}\right)$ [Bauer, 1997], $\mathcal{O}\left(\frac{\alpha_s \Lambda^2}{m_b^2}\right)$ [Ewerth, Gambino, Nandi, 2009].

The dominant non-perturbative uncertainty originates from the "27" interference term:



$$\frac{\Delta\mathcal{B}}{\mathcal{B}} = -\frac{6C_2 - C_1}{54 C_7}\left[\frac{\lambda_2}{m_c^2} + \sum_n b_n \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\left(\frac{m_b \Lambda}{m_c^2}\right)^n\right)\right]$$

$\lambda_2 \simeq 0.12\,\mathrm{GeV}^2$

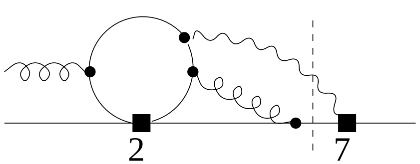
from B–B^* mass splitting

The coefficients b_n decrease fast with n.

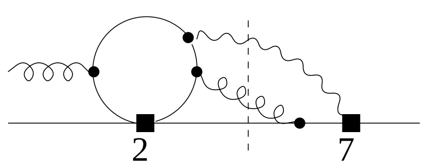
[Voloshin, 1996], [Khodjamirian, Rückl, Stoll, Wyler, 1997]
[Grant, Morgan, Nussinov, Peccei, 1997]
[Ligeti, Randall, Wise, 1997], [Buchalla, Isidori, Rey, 1997]

New claims by Benzke, Lee, Neubert and Paz in arXiv:1003.5012:

One cannot really expand in $m_b\Lambda/m_c^2$. All such corrections should be treated as Λ/m_b ones and estimated using models of subleading shape functions. Dominant contributions to the estimated $\pm 5\%$ non-perturbative uncertainty in $\mathcal{B}$ are found this way, with the help of alternating-sign shape functions that undergo weaker suppression at large gluon momenta.



correction to the above



phase-space suppressed

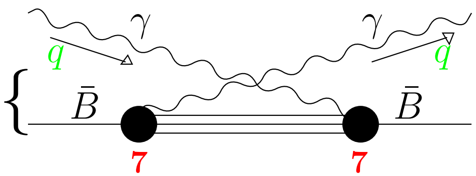
$\mathcal{O}\left(\frac{\alpha_s \Lambda}{m_b}\right)$ **Main worry in hep-ph/0609232, and reason for the $\pm 5\%$ non-perturbative uncertainty.**

The "hard" contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b) \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2(\mu_b) \langle X_s \gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$

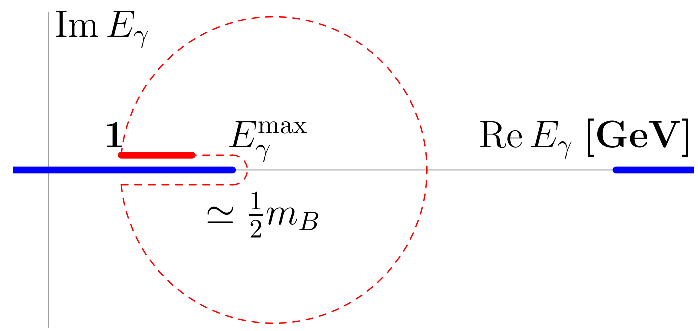
The "77" term in this sum is purely "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$**:**

$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**
However, the $\bar{B} \to X_s \gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$**.**

Once $A(E_\gamma)$ **is considered as a function of arbitrary complex** E_γ**,** $\mathrm{Im} A$ **turns out to be proportional to the discontinuity of** A **at the physical cut. Consequently,**

$$\int_{1\,\mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \; \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \; A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ **is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1 - 2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0) | Q^{(j)}_{\mathrm{local\ operator}} | \bar{B}(\vec{p}=0) \rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b**.**

At $(\Lambda/m_b)^0$**:** $\langle \bar{B}(\vec{p}) | \bar{b}\gamma^\mu b | \bar{B}(\vec{p}) \rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s \gamma) = \Gamma(b \to X_s^{\mathrm{parton}} \gamma) + \mathcal{O}(\Lambda/m_b).$

At $(\Lambda/m_b)^1$**:** **Nothing! All the possible operators vanish by the equations of motion.**

At $(\Lambda/m_b)^2$**:** $\langle \bar{B}(\vec{p}) | \bar{h} D^\mu D_\mu h | \bar{B}(\vec{p}) \rangle = -2 m_B \lambda_1,$ $\quad \lambda_1 = (-0.27 \pm 0.04)\mathrm{GeV}^2$ **from** $\bar{B} \to X \ell^- \nu$ **spectrum.**

$\langle \bar{B}(\vec{p}) | \bar{h} \sigma^{\mu\nu} G_{\mu\nu} h | \bar{B}(\vec{p}) \rangle = 6 m_B \lambda_2,$ $\quad \lambda_2 \simeq \frac{1}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.12\,\mathrm{GeV}^2.$

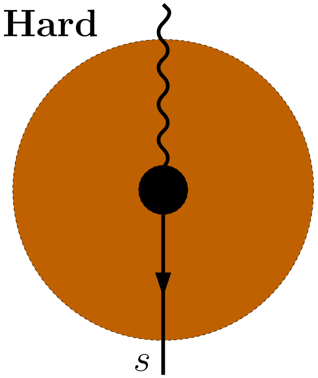
The HQET heavy-quark field $h(x)$ **is defined by** $h(x) = \frac{1}{2}(1 + \not{v}) b(x) \exp(i m_b\, v \cdot x)$ **with** $v = p/m_B$**.**

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

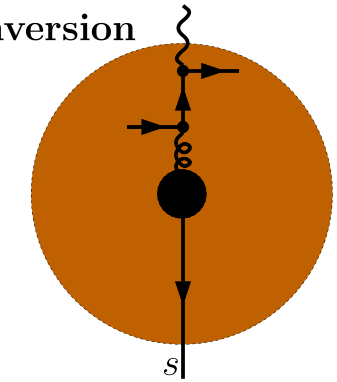
A. Without long-distance charm loops:

1. **Hard**



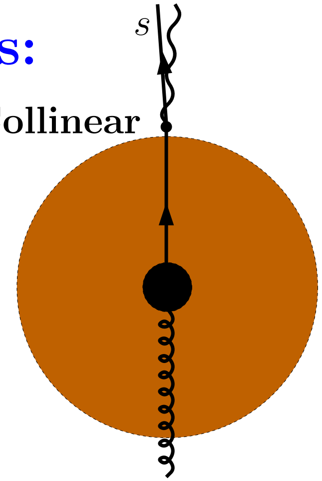
Dominant, well-controlled.

2. **Conversion**



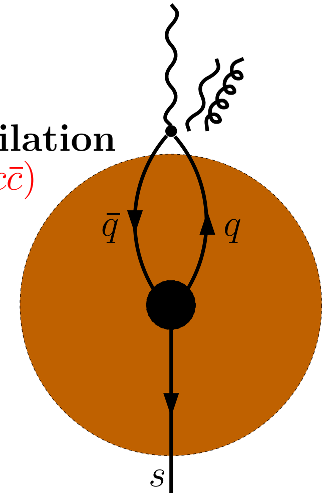
$\mathcal{O}(\alpha_s\Lambda/m_b)$, $(-1.6 \pm 1.2)\%$.
[Benzke, Lee, Neubert, Paz, 2010]

3. **Collinear**



$\sim -0.2\%$ or $(+0.8 \pm 1.1)\%$.
[Kapustin,Ligeti,Politzer, 1995]
[Benzke, Lee, Neubert, Paz, 2010]

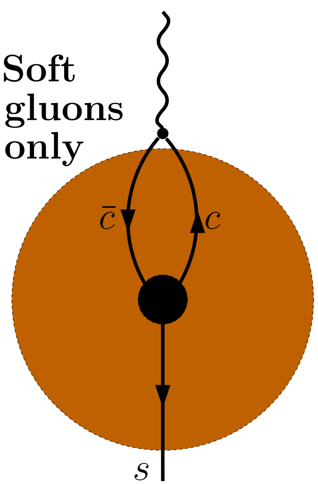
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

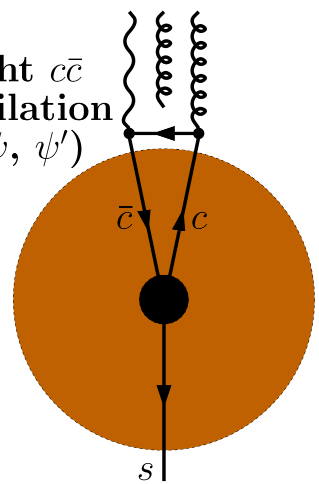
B. With long-distance charm loops:

5. **Soft gluons only**



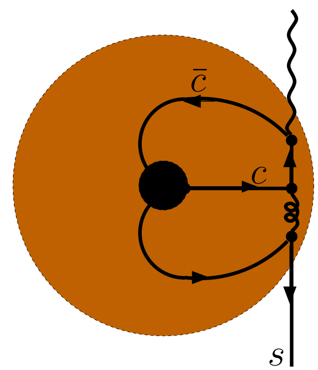
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]
[Benzke, Lee, Neubert, Paz, 2010]: add $(+1.1 \pm 2.9)\%$

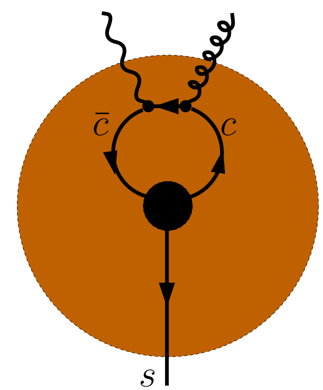
6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**



Exp. J/ψ subtracted ($< 1\%$).
Perturbatively (including hard): $\sim +3.6\%$.

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**





$\mathcal{O}(\alpha_s(\Lambda/M)^2)$ $\mathcal{O}(\alpha_s\Lambda/M)$

$M \sim 2m_c, 2E_\gamma, m_b$.

e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \, D^*(2007)^0 \;\;] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \, \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.