$B \to X_s\gamma$: normalization and parametric uncertainties

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Parameters in the radiative BR

$$\boxed{\mathrm{BR}[B \to X_s\gamma]_{E_\gamma > E_0} \sim \alpha\, G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2 |C_7^{eff}(m_b)|^2}$$

The CKM factor is essentially $|V_{cb}|^2$

$$|V_{ts}^* V_{tb}|^2 = |V_{cb}|^2 \left[1 + \lambda^2(2\bar\rho - 1) + O(\lambda^4)\right] = 0.961(2)|V_{cb}|^2$$ (UTFit 2011)

Relevant parameters:

$$\alpha \quad |V_{cb}| \quad \underset{2\%}{\alpha_s} \quad m_b \quad m_c \quad \underset{0.5\%}{m_t} \quad M_W \quad \boxed{\mu_\pi^2 \sim -\lambda_1 \quad \mu_G^2 \sim 3\lambda_2 \quad \rho_D^3 \quad \rho_{LS}^3}$$

in EW corr & subleading terms also $\sin^2\theta_W \quad M_H \quad M_Z \quad V_{ub} \quad \ldots$

$$\frac{\delta V_{cb}}{V_{cb}} \sim 2\% \Rightarrow \frac{\delta \mathrm{BR}_\gamma}{\mathrm{BR}_\gamma} \sim 4\% \qquad \frac{\delta m_b}{m_b} \sim 0.5\% \Rightarrow \frac{\delta \mathrm{BR}_\gamma}{\mathrm{BR}_\gamma} \sim 2.5\%$$

Non-pert parameters, b,c masses and V_{cb} from inclusive sl B decays,
also important for extrapolation to E_0=1.6GeV

The phase space factor

$$\mathrm{BR}_\gamma(E_0) \equiv \mathrm{BR}[B \to X_s\gamma]_{E_\gamma > E_0} = \frac{\mathrm{BR}_{c\ell\nu}}{C}\left(\frac{\Gamma[B \to X_s\gamma]_{E_\gamma>E_0}}{|V_{cb}/V_{ub}|^2\,\Gamma[B \to X_u e\bar\nu]}\right)$$

$$\boxed{C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[B \to X_c e\bar\nu]}{\Gamma[B \to X_u e\bar\nu]}}$$

phase space factor given by OPE

$$C = g\left(\frac{m_c}{m_b}\right)\left[1 + c_1\frac{\alpha_s}{\pi} + c_2(\frac{\alpha_s}{\pi})^2 + c_\pi\frac{\mu_\pi^2}{m_b^2} + c_G\frac{\mu_G^2}{m_b^2} + c_D\frac{\rho_D^3}{m_b^3} + c_L\frac{\rho_{LS}^3}{m_b^3} + O(\alpha_s^3, \frac{\Lambda^3}{m_b^3})\right]$$

neglecting WA which cancel out in F

$$\boxed{F \equiv \frac{1+\delta_{NP}}{C}}$$

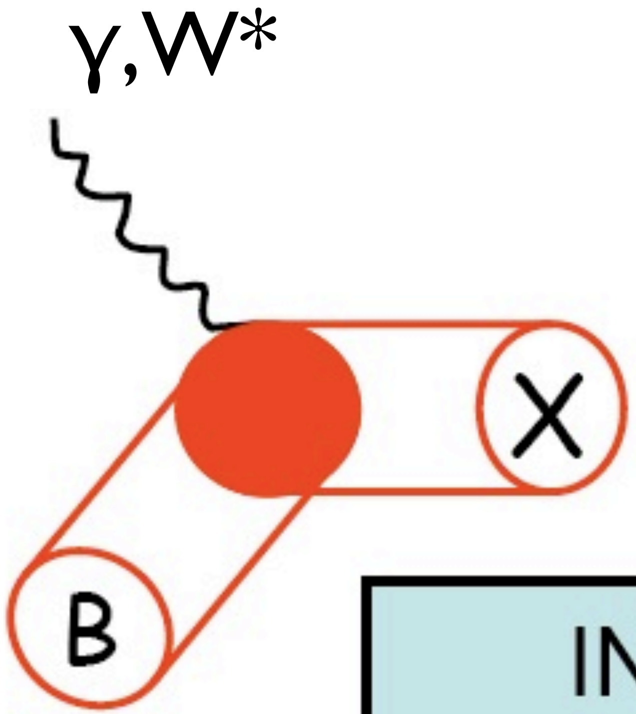
is actually what enters BRγ

~3%

$$\left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\,\alpha_{\rm em}}{\pi}\,[1+\delta_{NP}]\,P(E_0)$$

NNLO QCD
its m_c dependence partially cancels that of C

Inclusive B decays



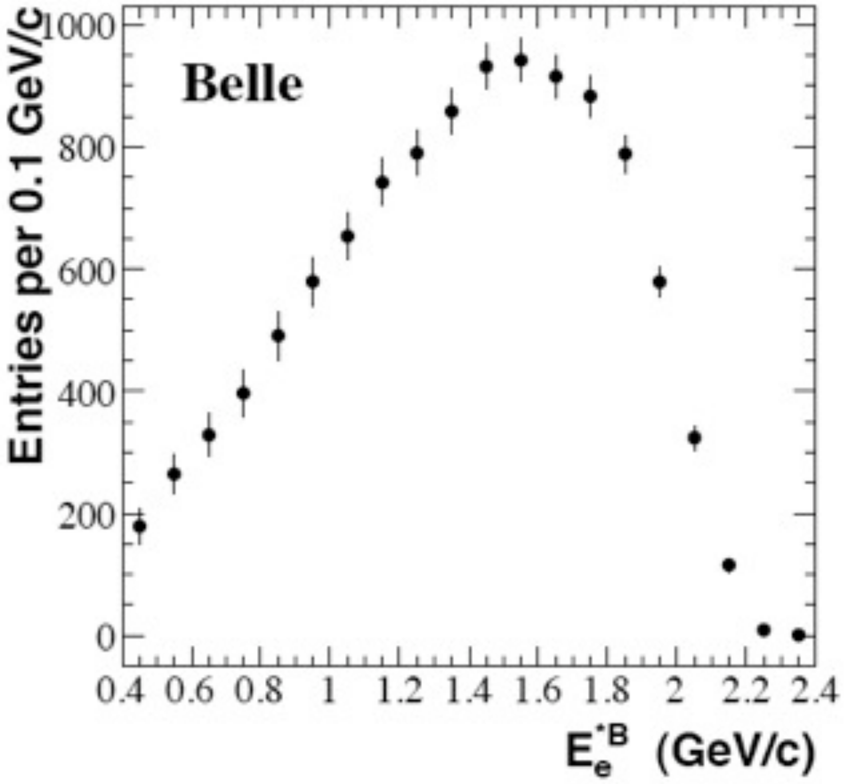
BR_{clv} C δ_{NP}

can all be extracted from s.l. B decays are correlated

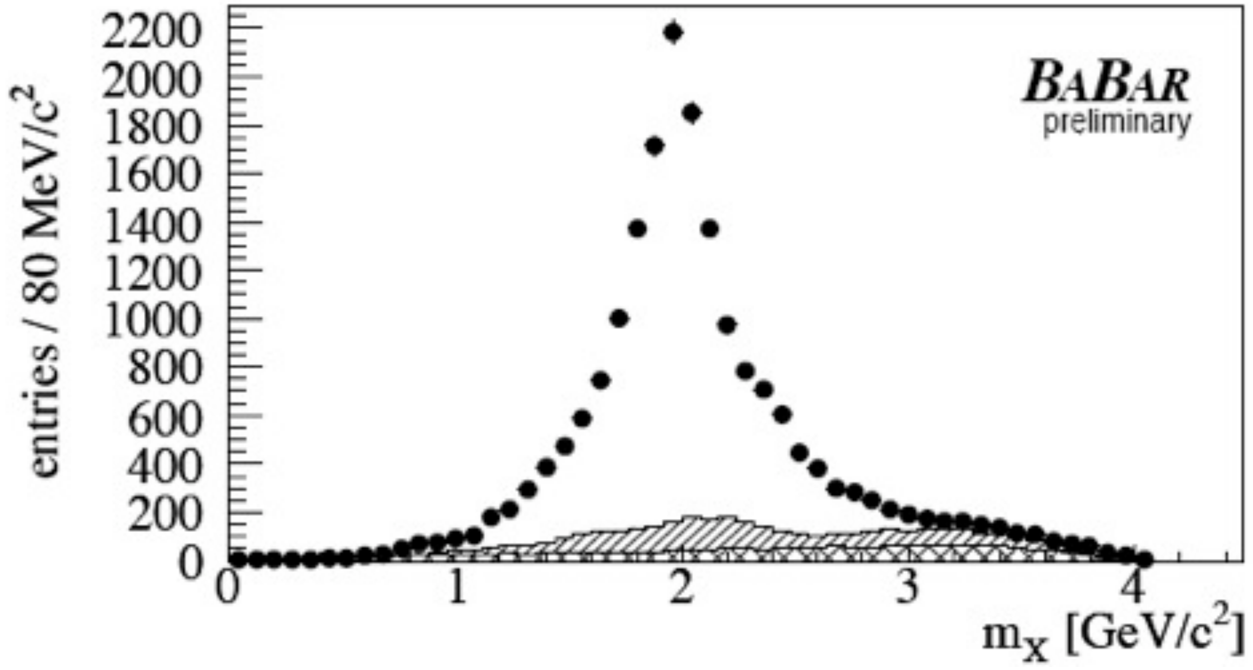
INCLUSIVE	EXCLUSIVE
OPE: non-pert physics described by B matrix elemnts of local operators can be extracted by exp suppressed by $1/m_b^2$	**Form factors:** in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization

Fitting OPE parameters to the moments

E_l spectrum



m_X spectrum



Total ***rate*** *gives* $|V_{cb}|$*, global* ***shape*** *parameters (moments of the distributions) tell us about B structure,* m_b *and* m_c

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications

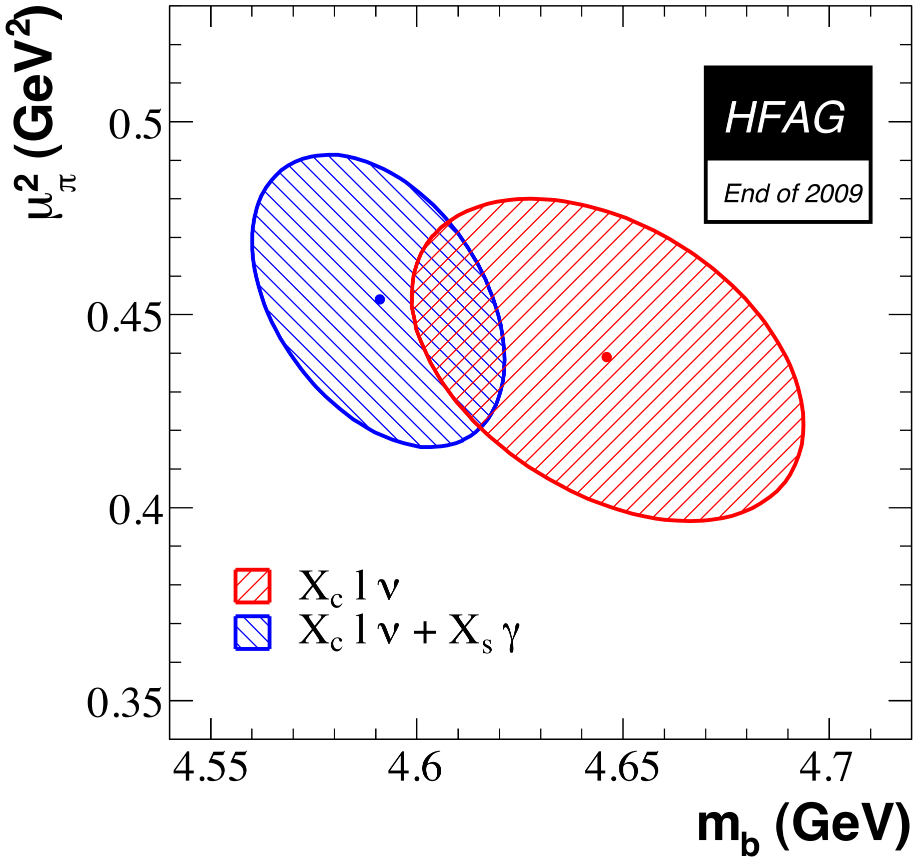
Present implementations include all terms through $O(\alpha_s^2\beta_0, 1/m_b^3)$: $m_{b,c}, \mu^2_{\pi,G}, \rho^3_{D,LS}$ *6 parameters*

Global HFAG fit

Inputs	$\|V_{cb}\|\ 10^3$	m_b^{kin}	χ^2/ndf
b→c & b→sγ	41.85(44)(58)	4.590(31)	29.7/59
b→c only	41.68(48)(58)	4.646(47)	24.2/48

Based on PG, Uraltsev, Benson et al

These results refer to the kinetic scheme, *where the contributions of gluons with energy below $\mu \approx 1$GeV are absorbed in the OPE parameters*

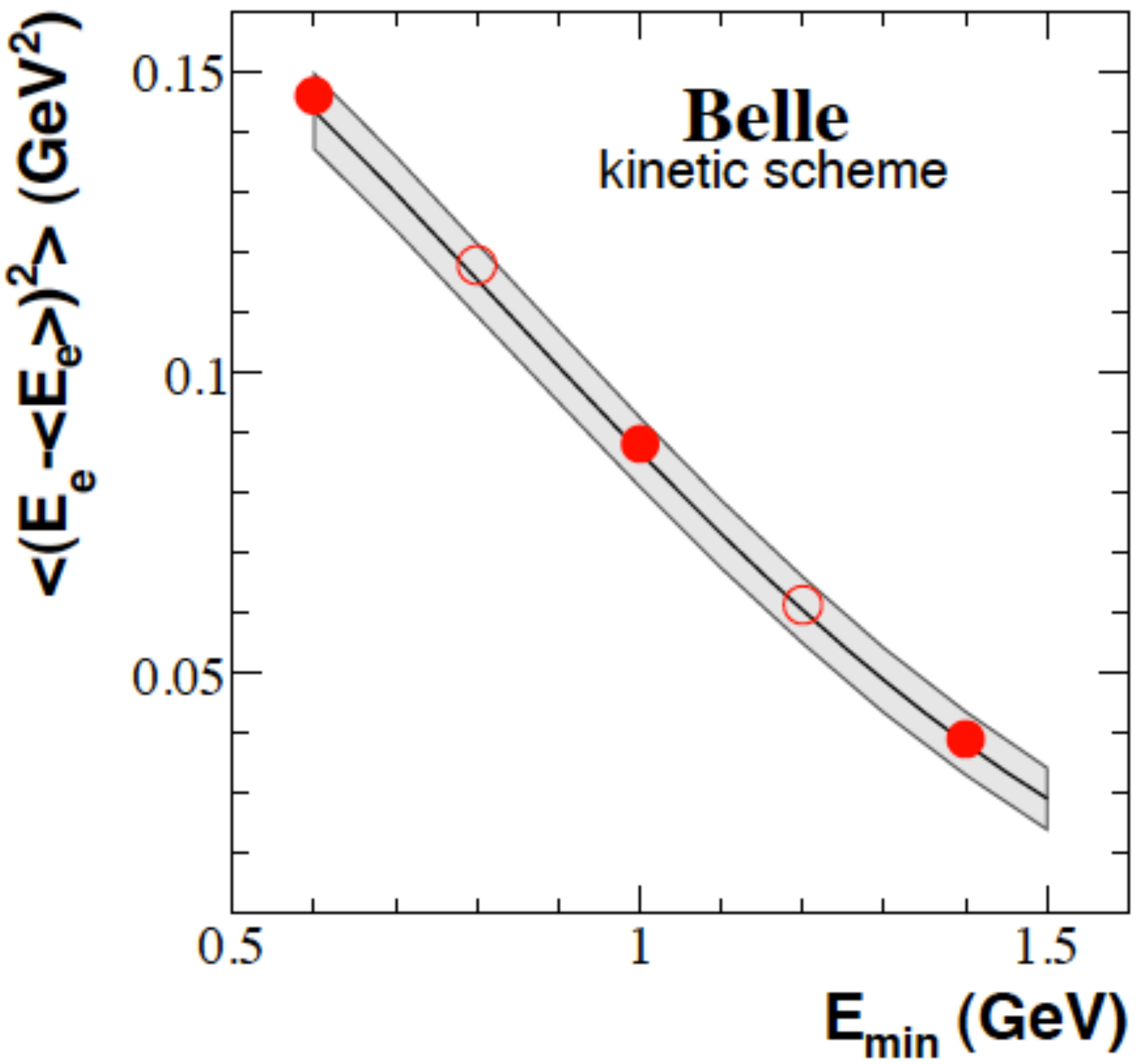


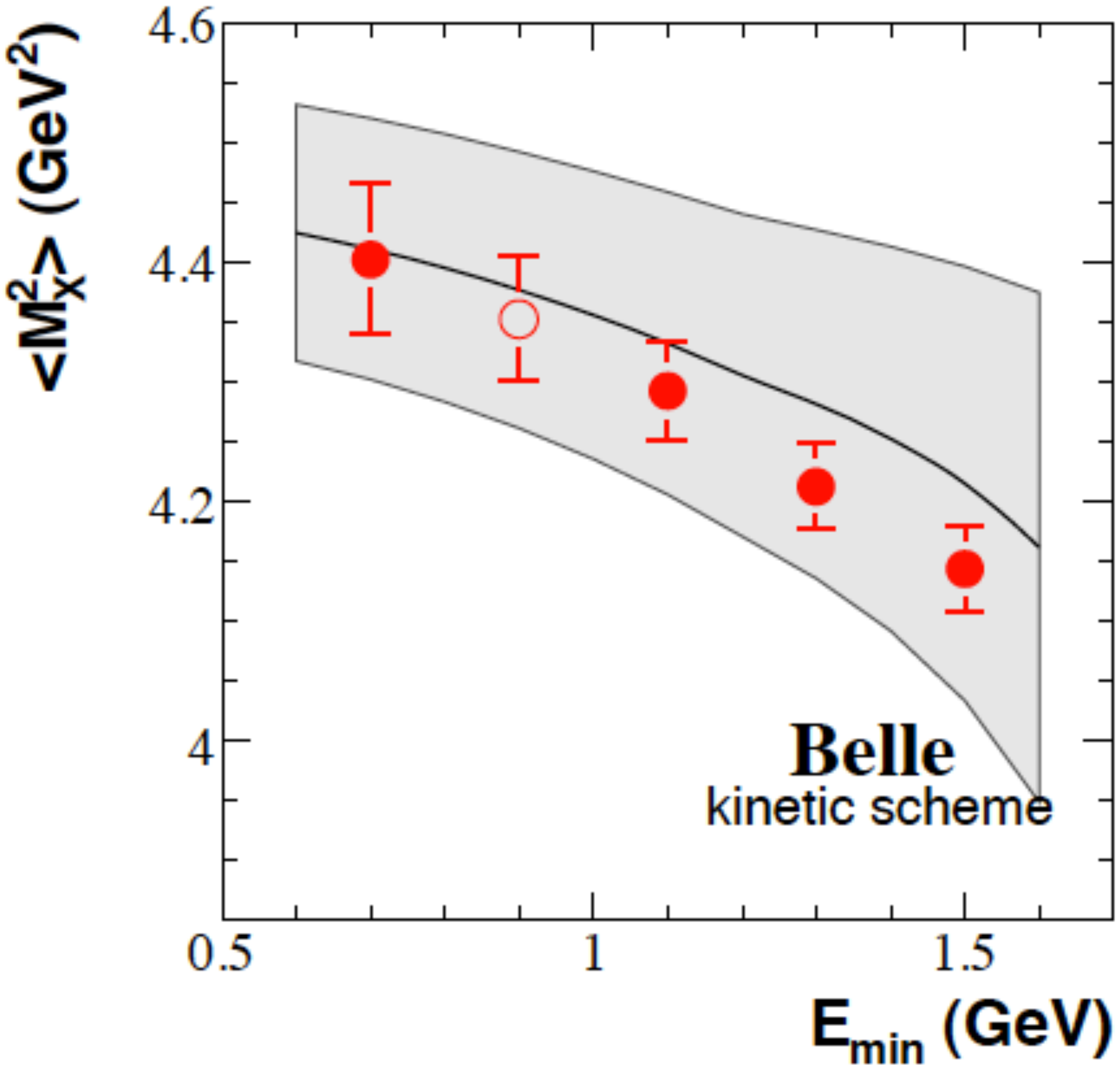
A number of different assumptions are also important: which data are included, how theory errors are computed...

Very close result for $|V_{cb}|$ in 1S scheme

Bauer Ligeti Luke Manohar Trott

Theoretical errors dominate





C from inclusive s.l. B decays

- Bauer et al, Manohar (2004-06): **C=0.580(16)**

  NNLO BR_γ=3.15 10^{-4} total parametric error 3%

- Giordano, PG (2008,HFAG fit): **C=0.546(17)(16)**
  exp th

  NNLO BR_γ=3.28 10^{-4} total parametric error 3.7%

Uncomfortable discrepancy: why?

Where the two determinations differ

1. 1S (Bauer et al) vs kinetic scheme (GG)
2. different treatment of higher power corrections in hadronic moments
3. use of additional constraints: $\overline{M}_B$-$\overline{M}_D$ etc (Bauer et al)
4. slightly different data sets
5. different treatment of radiative moments & their uncertainties
6. different estimates of theory uncertainties and correlations among them

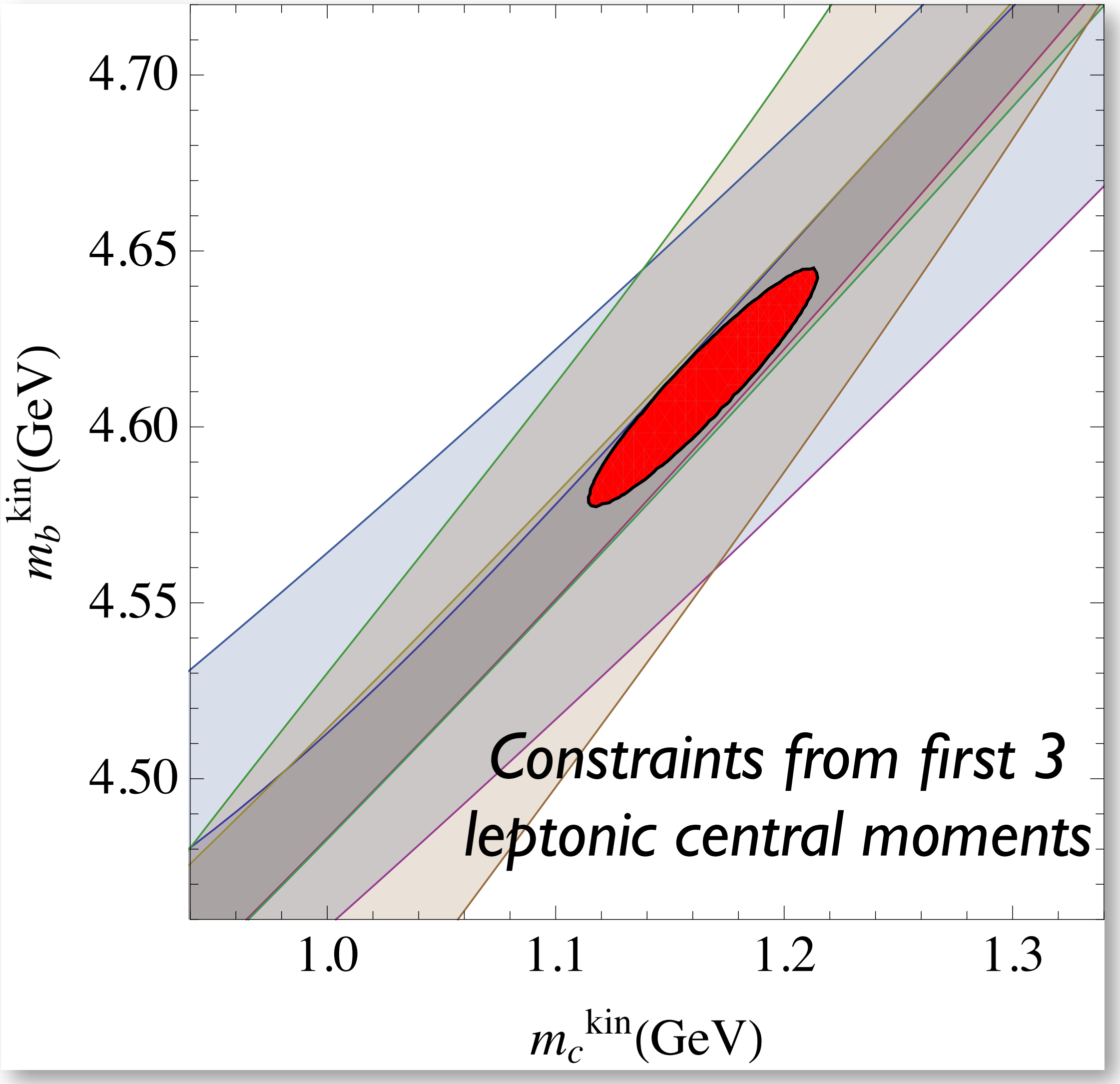
Once m_c/m_b is fixed, C receives small corrections: look at the masses

$$\begin{aligned} C &= 0.625 - 0.028_{\alpha_s} - 0.022_{\alpha_s^2} - 0.004_{\mu_G^2} \\ &\quad - 0.025_{\rho_D^3} - 0.001_{\rho_{LS}^3} - 2.0\, B_{\rm WA}(m_b/2) \end{aligned}$$

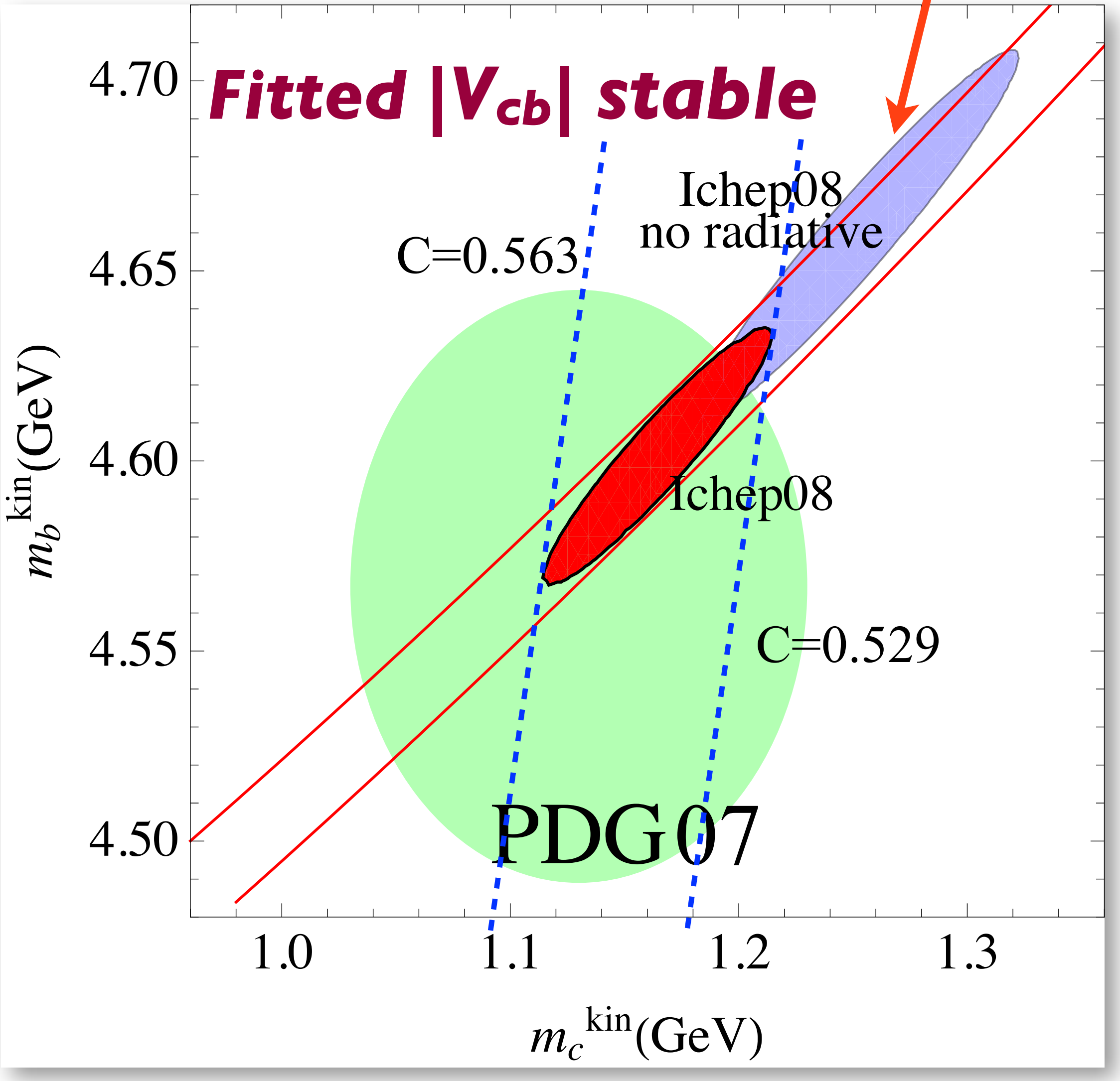
Weak Annihilation cancels out in F!

A strip in the m_b-m_c plane

Constant values of s.l. width at fixed V_{cb}



Constraints from first 3 leptonic central moments



Fitted $|V_{cb}|$ stable

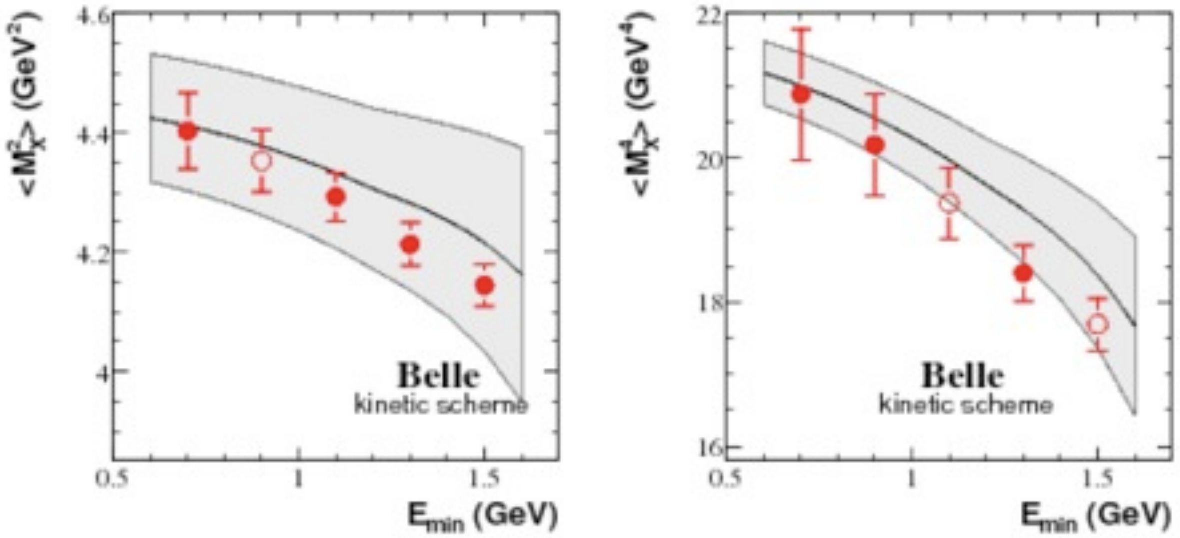
Semileptonic moments do not measure m_b well. They rather identify a strip in (m_b,m_c) plane along which the minimum is shallow.

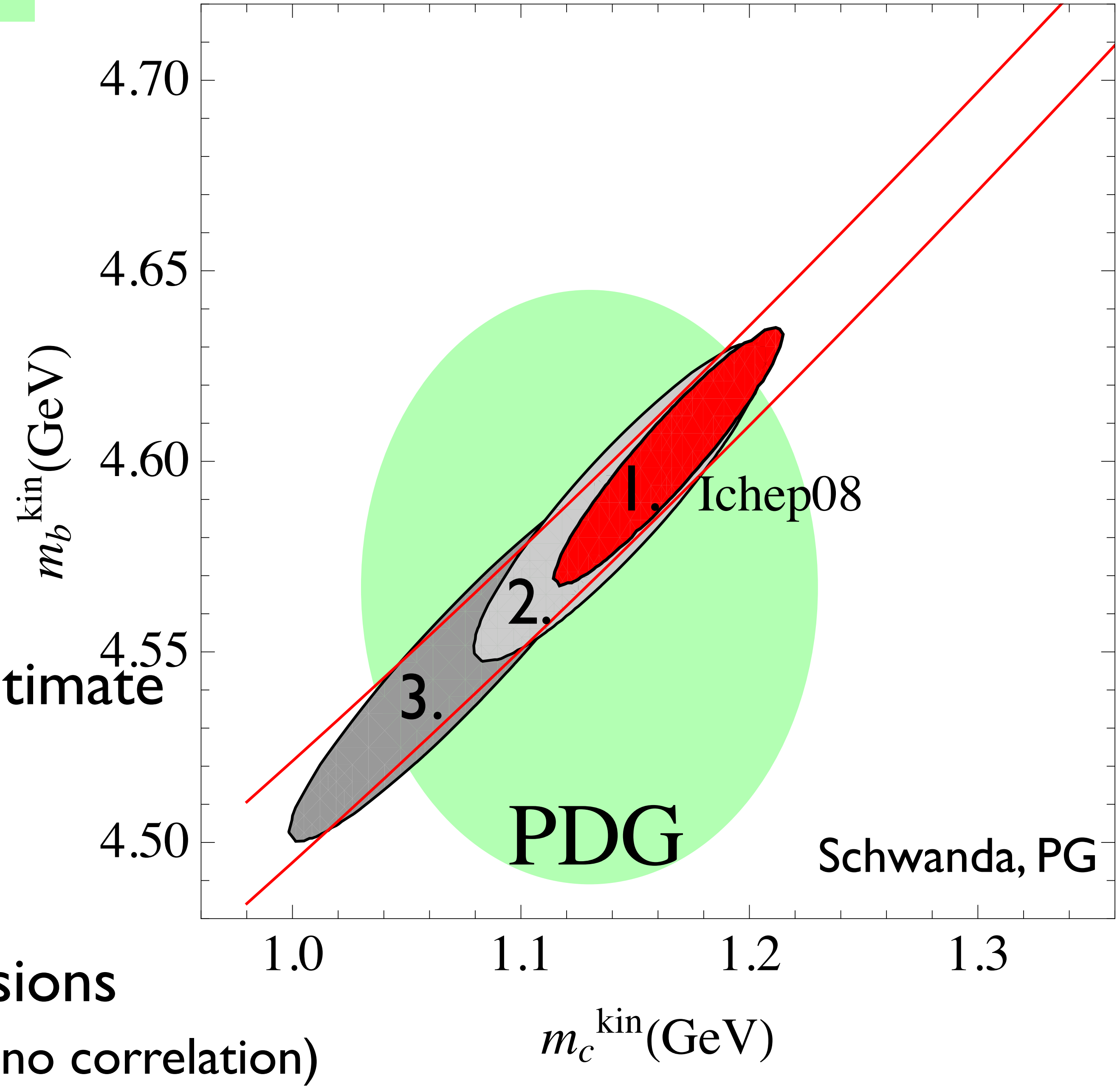
Unknown non-pert $O(\alpha_s/m_b)$ effects in radiative moments. Possibly irrelevant here but must be studied. But role of radiative moments in the fits is equivalent to using loose bound $m_b(m_b)$=4.20(7)GeV

How reliable are mass determinations?

Collaboration with C. Schwanda, in progress

Theoretical correlations





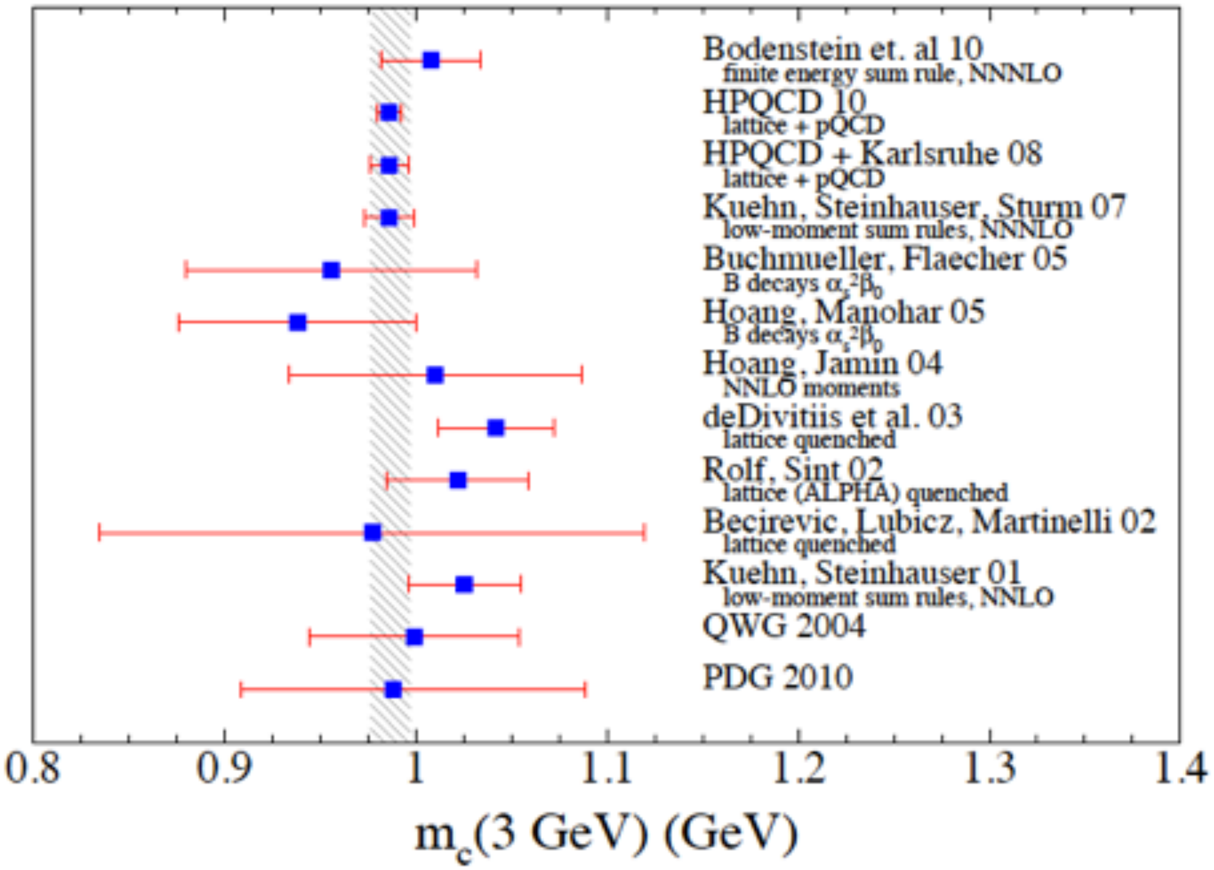
Correlations between theory errors of moments with different cuts difficult to estimate

Examples:

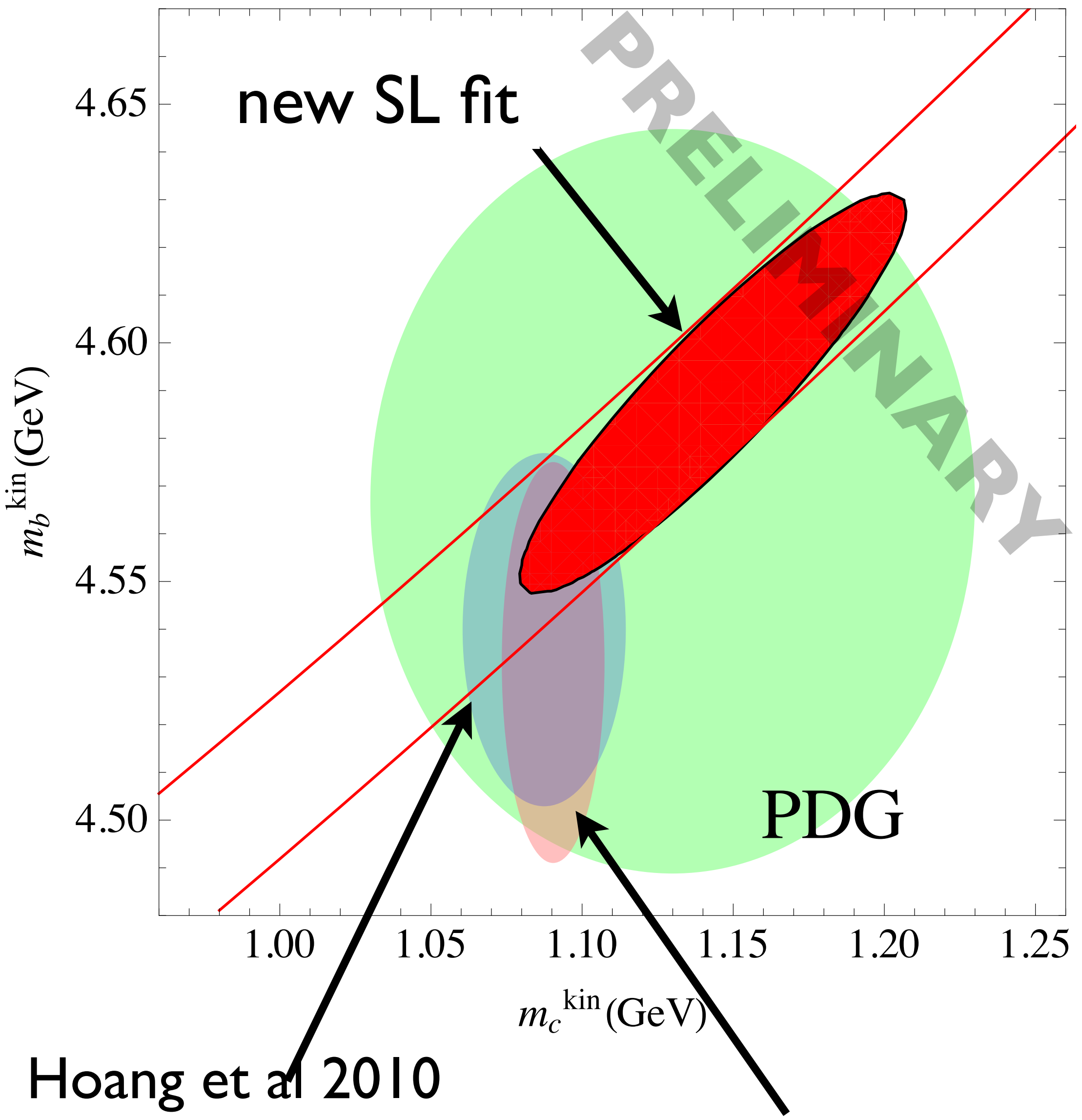
1. 100% correlations
2. corr. computed from low-order expressions
3. experimental correlations (very similar to no correlation)

always assume different central moments uncorrelated

Using mass determinations



Recent sum rules determinations converted to kin scheme



Hoang et al 2010
Hoang (m_b)

Kuhn et al 2009

Comparisons and combinations for $m_{b,c}$ penalized by changes of scheme. Better to avoid $m_c(m_c)$.

Preliminary C, F determination

Schwanda, PG

- *Direct fit to $m_c(3\mathrm{GeV})$ with Karlsruhe constraint on m_c leads to*

  m_b^{kin}=4.535(21)GeV ➨ $m_b(m_b)$=4.165(36)GeV (MSbar)

  consistent with Karlsruhe's group m_b determination

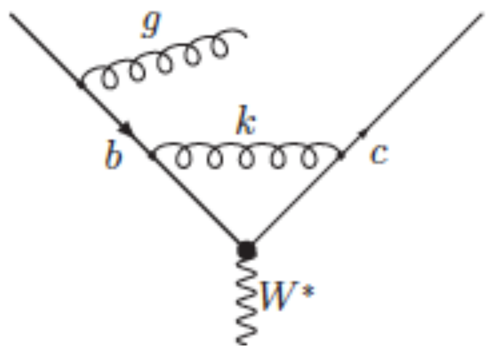
- *changing th correlations results vary widely:*
  C= 0.569(26) 0.553(22) 0.548(17) ...

- *including $m_{c(3\mathrm{GeV})}$=0.986(13)GeV (Kuhn et al) in the fit*

  C=0.571(7) F=1.794(23)

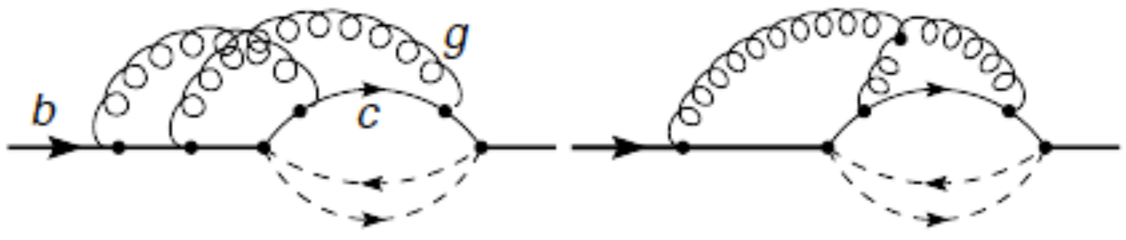
  where only the experimental uncertainty is shown

Fits at NNLO



- Complete 2loop corrections to width and moments with cuts are now known, either in expansion m_c/m_b or numerically Biswas-Melnikov

Pak-Czarnecki, PG



$$d\Gamma = \Gamma_0 \left[dF_0 + \frac{\alpha_s(m_b)}{\pi} dF_1 + (\frac{\alpha_s}{\pi})^2 (\beta_0\, dF_{\rm BLM} + dF_2) + ... \right]$$

- Non-BLM minor corrections to BLM, residual th error on V_{cb} O(0.5%).

- Strong cancellations between different contributions make NNLO to moments: non-accidental, numerical accuracy crucial

$$\langle E_l \rangle_{E_l>1{\rm GeV}} = 1.54\,{\rm GeV} \left[1 + (0.96_{den} - 0.93)\,\frac{\alpha_s}{\pi} + (0.48_{den} - 0.46)\,\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 \right.$$
$$\left. + [1.69(7) - 1.75(9)_{den}] \left(\frac{\alpha_s}{\pi}\right)^2 + O(1/m_b^2, \alpha_s^3) \right]$$

$$\ell_2 = \langle E_\ell^2 \rangle - \langle E_\ell \rangle^2 = (2.479 - 2.393)\,{\rm GeV}^2 = 0.087\,{\rm GeV}^2.$$

NNLO code is ready

PG, JHEP 9(2011)055

	ℓ_1	ℓ_2	ℓ_3	R^*
		$\mu = 0$		
tree	1.5674	0.0864	-0.0027	0.8148
$1/m_b^3$	1.5426	0.0848	-0.0010	0.8003
$O(\alpha_s)$	1.5398	0.0835	-0.0010	0.8009
$O(\beta_0\alpha_s^2)$	1.5343	0.0818	-0.0009	0.7992
$O(\alpha_s^2)$	1.5357(2)	0.0821(6)	-0.0011(16)	0.7992(1)
		$\mu = 1\text{GeV}$		
$O(\alpha_s)$	1.5455	0.0858	-0.0003	0.8029
$O(\beta_0\alpha_s^2)$	1.5468	0.0868	0.0005	0.8035
$O(\alpha_s^2)$	1.5466(2)	0.0866(6)	0.0002(16)	0.8028(1)
$O(\alpha_s^2)^*$	–	0.0865	0.0004	–
tot error [6]	0.0113	0.0051	0.0022	

E_{cut}=1GeV, m_c/m_b=0.25

		$\mu = 1\text{GeV},\ m_c^{\overline{\text{MS}}}(3\text{GeV})$		
	ℓ_1	ℓ_2	ℓ_3	R^*
tree	1.6021	0.0940	-0.0043	0.8296
$1/m_b^3$	1.5748	0.0922	-0.0020	0.8159
$O(\alpha_s)$	1.5613	0.0894	-0.0004	0.8118
$O(\beta_0\alpha_s^2)$	1.5629	0.0904	0.0004	0.8125
$O(\alpha_s^2)$	1.5571(4)	0.0890(9)	-0.0008(25)	0.8090(2)
$O(\alpha_s^2)^*$	–	0.0889	0.0006	–

Small corrections. Cancellations may be partially spoiled by choice of scheme

		$\mu = 0$			$\mu = 1\text{GeV}$	
	h_1	h_2	h_3	h_1	h_2	h_3
LO	4.345	0.198	-0.02	4.345	0.198	-0.02
$1/m_b^3$	4.452	0.515	4.90	4.452	0.515	4.90
$O(\alpha_s)$	4.563	0.814	5.96	4.426	0.723	4.50
$O(\beta_0\alpha_s^2)$	4.701	1.105	6.85	4.404	0.894	4.08
$O(\alpha_s^2)$	4.682(1)	1.066(3)	6.69(4)	4.411(1)	0.832(4)	4.08(4)
tot error [6]				0.149	0.501	1.20

Higher power corrections

Mannel,Turczyk,Uraltsev **1009.4622**

Proliferation of non-pert parameters: for ex at $1/m_b^4$

$$
\begin{aligned}
2M_B m_1 &= \langle ((\vec{p})^2)^2 \rangle \\
2M_B m_2 &= g^2 \langle \vec{E}^2 \rangle \\
2M_B m_3 &= g^2 \langle \vec{B}^2 \rangle \\
2M_B m_4 &= g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle
\end{aligned}
$$

$$
\begin{aligned}
2M_B m_5 &= g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle \\
2M_B m_6 &= g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle \\
2M_B m_7 &= g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle \\
2M_B m_8 &= g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle \\
2M_B m_9 &= g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle
\end{aligned}
$$

can be estimated by Ground State Saturation

$$\langle \Omega_0 | \bar{Q}\, iD_j iD_k iD_l iD_m\, Q | \Omega_0 \rangle = \langle \Omega_0 | \bar{Q} iD_j iD_k Q | \Omega_0 \rangle \, \langle \Omega_0 | \bar{Q}\, iD_l iD_m Q | \Omega_0 \rangle$$

$$\frac{\delta\Gamma_{1/m^4} + \delta\Gamma_{1/m^5}}{\Gamma} \approx 0.013 \qquad \frac{\delta V_{cb}}{V_{cb}} \approx +0.4\%$$

after inclusion of the corrections in the moments. While this might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values.*

$O(\alpha_s/m_b^2)$ effects in $B\to X_s\gamma$

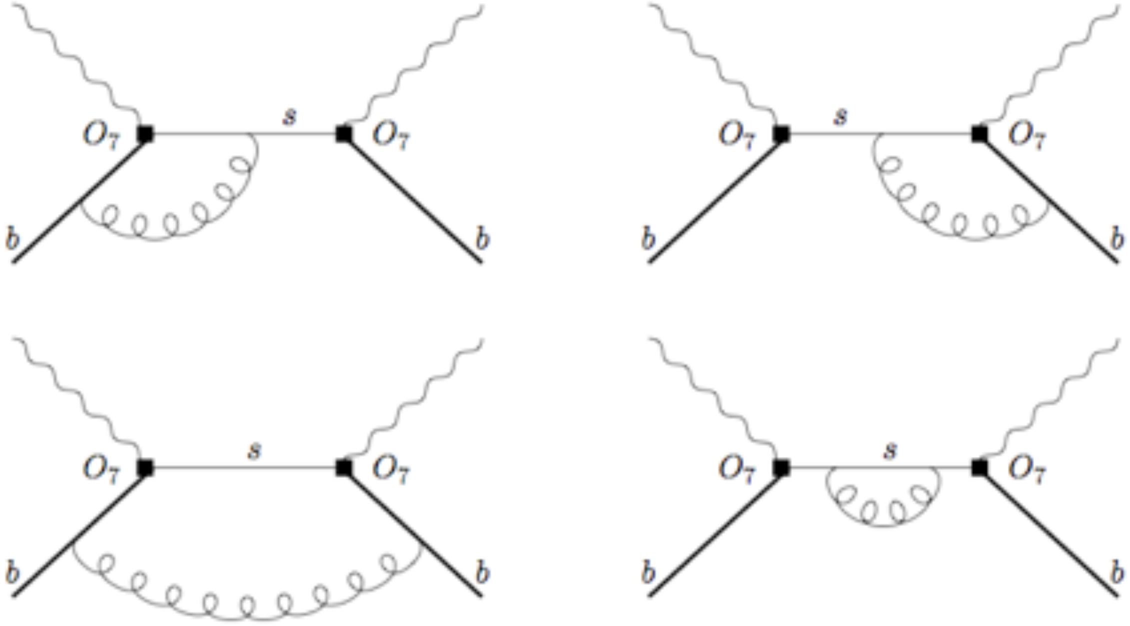
Ewerth,Nandi,PG arXiv:0911.2175

$$T\{\bar b(x)\sigma_{\mu\nu}P_L s(x)\bar s(0)\sigma_{\alpha\beta}P_R b(0)\} =$$

$$c_{\dim 3}O_{\dim 3} + \frac{1}{m_b}c_{\dim 4}O_{\dim 4} + \frac{1}{m_b^2}c_{\dim 5}O_{\dim 5} + \ldots$$

$$O_b^\mu = \bar b\gamma^\mu b, \qquad O_2^{\mu\nu} = \bar b_v \frac{1}{2}\{iD^\mu, iD^\nu\}b_v\,,$$

$$O_1^\mu = \bar b_v iD^\mu b_v, \qquad O_3^{\mu\nu} = \bar b_v \frac{g_s}{2}G^{a\mu}{}_\alpha\sigma^{\alpha\nu}T^a b_v\,,$$



One-loop matching onto local operators with HQET fields in dim reg

$$\frac{d\Gamma_{77}}{dz} = \Gamma_{77}^{(0)}\left[c_0^{(0)} + c_{\lambda_1}^{(0)}\frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(0)}\frac{\lambda_2(\mu)}{2m_b^2} + \frac{\alpha_s(\mu)}{4\pi}\left(c_0^{(1)}\boxed{+c_{\lambda_1}^{(1)}}\frac{\lambda_1}{2m_b^2}\boxed{+c_{\lambda_2}^{(1)}}\frac{\lambda_2(\mu)}{2m_b^2}\right)\right]$$

$\lambda_{1,2}$ are HQET analogues of $\mu^2_{\pi,G}$

The NLO effect 10-20% in coefficients of first few moments, leading to δm_b~10MeV, $\delta\mu_\pi^2$ ~ 0.04GeV^2

Extension to semileptonic case almost complete: these corrections likely more important than non-BLM ones.

$O(\alpha_s\mu^2_\pi/m_b^2)$ to moments known numerically Becher,Boos,Lunghi

Summary

- *Dominant parametric uncertainties in BR_γ due to b,c masses, V_{cb} and local OPE power corrections. Strong correlations, semileptonic moments provide crucial information.*
- *Global fits including precise constraints on m_c and possibly m_b are the way to go. Preliminary results for C, F have >50% smaller experimental uncertainty.*
- *Inclusion of higher order effects in the fits under way, improvements in parametric uncertainty look possible.*