Line shapes of near-threshold resonances

A. Nefediev (ITEP, Moscow)

Original part based on

- Yu.Kalashnikova, A.N., Phys. Rev. **D80**, 074004 (2009)
- V.Baru, C.Hanhart, Yu.Kalashnikova, A.Kudryavtsev, A.N. Eur.Phys.J. **A44**, 93 (2010)
- C.Hanhart, Yu.Kalashnikova, A.N., Phys. Rev. **D81**, 094028 (2010)
- C.Hanhart, Yu.Kalashnikova, A.N., Eur.Phys.J. **A47**, 101 (2011);

Charmonium spectrum before and after B factories

Before B factories:

- most of known charmonia lied below the open-charm threshold
- well-developed phenomenology based on potential quark models
- threshold effects reduced to constant mass shifts

After B factories:

- lots of new states observed, mainly above the open-charm threshold, including enigmatic states like $X(3872)$, $Y(4260)$, charged Z's, etc
- lots of new approaches involved: hybrids, molecules, tetraquarks
- threshold effects and D-meson loops are of paramount importance!
- drastic departure from the Breit–Wigner form in line shapes

Bottomonia: $\Upsilon(5S)$ — new thresholds, new surprises

- $\Upsilon(5S)$ versus $\Upsilon(4S)$: new threshold involved — $B_s\bar{B}_s$
- anomolous two-pion decays $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ and $\Upsilon(5S) \to h_b(nP)\pi^+\pi^-$
- anomalous three-body decays $\Upsilon(5S) \to B\bar{B}^*\pi$
- enigmatic charged Z_b states residing close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds
- to be continued?

Bottomonia: $\Upsilon(5S)$ — new thresholds, new surprises

- $\Upsilon(5S)$ versus $\Upsilon(4S)$: new threshold involved — $B_s\bar{B}_s$
- anomolous two-pion decays $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ and $\Upsilon(5S) \to h_b(nP)\pi^+\pi^-$
- anomalous three-body decays $\Upsilon(5S) \to B\bar{B}^*\pi$
- enigmatic charged Z_b states residing close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds
- to be continued?

To understand physics of $\Upsilon(5S)$

we need to understand relevant thresholds!

Threshold effects: from Breit–Wigner to Flatté

- Elementary state plus remote thresholds (Breit–Wigner form):

$$|X\rangle = c|\psi_0\rangle + \ldots \Longrightarrow f(E) \propto \frac{1}{E - E_f + \frac{i}{2}\Gamma_0}$$

- Elementary state attracted to an S wave threshold, plus remote thresholds (Flattè form):

$$|X\rangle = c|\psi_0\rangle + \chi(k)|M_1 M_2\rangle + \ldots \Longrightarrow f(E) \propto \frac{1}{E - E_f + \frac{i}{2}(\Gamma_0 + gk)}$$

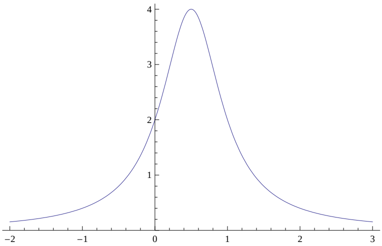
relative momentum in the $\{M_1 M_2\}$ system $k = \sqrt{2\mu E}$ (analytically continued below threshold, for $E < 0$) is a rapid function around the threshold at $E = 0$

Example of the threshold effects

$$\frac{dBr_{in}(E)}{dE} = \text{const} \times \frac{\Gamma_0}{\left(E - E_f - \frac{1}{2} g\kappa(E)\right)^2 + \frac{1}{4}\left(\Gamma_0 + gk(E)\right)^2}$$

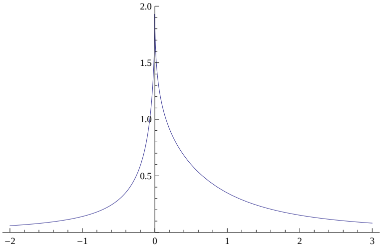
$k(E)$ — relative momentum in hadronic channel above threshold
$\kappa(E)$ — analytical continuation of $k(E)$ below threshold

Breit–Wigner ($g = 0$)



$E_{\rm peak} = E_f \quad \Gamma_{\rm vis} = \Gamma_0$

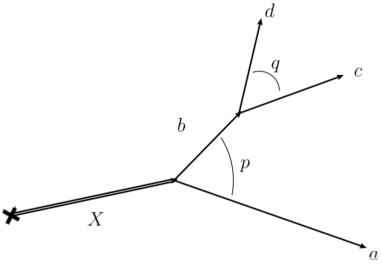
Flatté ($E_f > 0$, $g \neq 0$)



$E_{\rm peak} = E_{\rm thresh} \quad \Gamma_{\rm vis} \ll \Gamma_0$

Unstable constituent

$$X(3872) \to D\bar{D}^* \to D[\bar{D}\pi], \quad Z_b(10610) \to B\bar{B}^* \to B[\bar{B}\gamma], \quad \ldots$$



$$E_R = m_b - (m_c + m_d) > 0$$

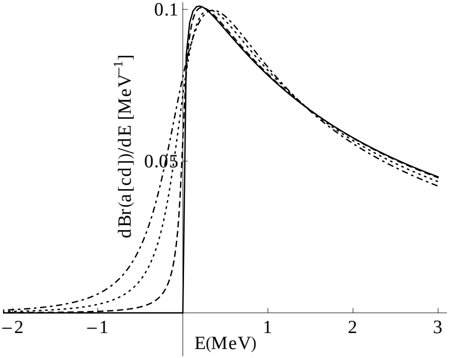
$$\Gamma_R = \Gamma(b \to c + d)$$

$$\frac{d\mathrm{Br}(a[cd])}{dE} = \mathrm{const} \times \frac{gk(E)}{\left(E - E_X - \frac{1}{2}g\kappa(E) + \frac{1}{2}g\kappa(E_X)\right)^2 + \frac{1}{4}\left(\Gamma_0 + gk(E)\right)^2}$$

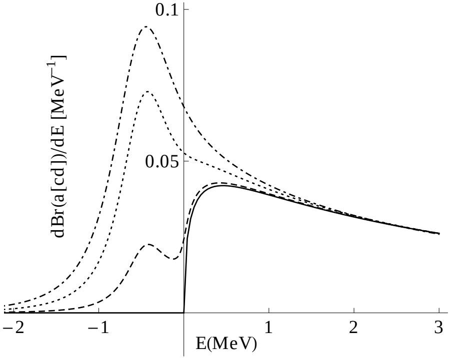
- if $E_X < 0$ then X is a bound state
- if $E_X > 0$ then X is a virtual state

Unstable constituent: Examples of the line shapes

Virtual state ($E_X > 0$)



Bound state ($E_X < 0$)



Solid line:	$\Gamma_R = 0$
Dashed line:	$\Gamma_R/E_R = 0.01$
Dotted line:	$\Gamma_R/E_R = 0.07$
Dash-dotted line:	$\Gamma_R/E_R = 0.1$

Direct interactions in the mesonic channel

Let X be a compact state ψ_0 (quarkonium, tetraquark,...) attracted to an S wave threshold $M_1 M_2$:

$$|X\rangle = c|\psi_0\rangle + \chi(k)|M_1 M_2\rangle$$

and let us allow for the direct interaction between mesons M_1 and M_2

Direct interactions in the mesonic channel

Let X be a compact state ψ_0 (quarkonium, tetraquark,...) attracted to an S wave threshold $M_1 M_2$:

$$|X\rangle = c|\psi_0\rangle + \chi(k)|M_1 M_2\rangle$$

and let us allow for the direct interaction between mesons M_1 and M_2

Near threshold the meson–meson scattering t matix takes the form:

$$t(E) \approx \frac{1}{-\gamma_V - ik}$$

Line shapes depend therefore on a new parameter γ_V (the inverse scattering length in the mesonic channel)

Modified Flatté distribution

$$f(E) \propto \frac{1}{E - E_f + \frac{i}{2} g k - \frac{(E - E_f)^2}{E - E_C}}$$

$$E_C = E_f - \frac{1}{2} g \gamma_V$$

Modified Flatté distribution

$$f(E) \propto \frac{1}{E - E_f + \frac{i}{2}gk - \frac{(E-E_f)^2}{E - E_C}}$$

$$E_C = E_f - \frac{1}{2}g\gamma_V$$

- $|E_C| \gg |E_f|$ (large $|\gamma_V|$)

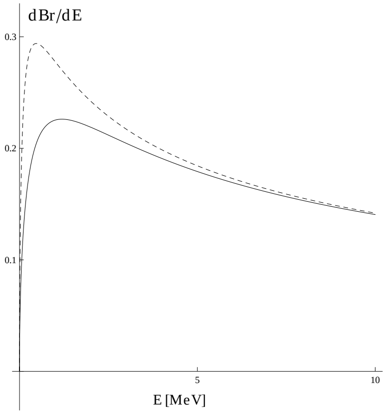
$$f(E) \propto \frac{1}{E - E_f + \frac{i}{2}gk}$$

- $|E_C| \sim |E_f|$ (small $|\gamma_V|$)

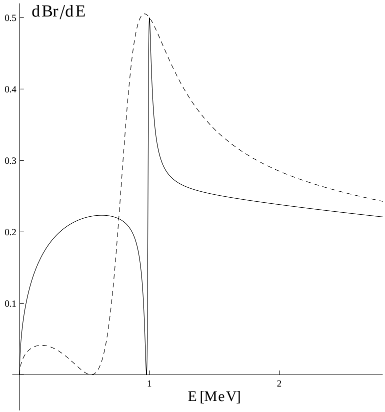
$$f(E) \propto (E - E_C)$$

Production through the hadronic component

$|E_C| \gg |E_f|$



$|E_C| \sim |E_f|$



Multi-channel Flatté distribution

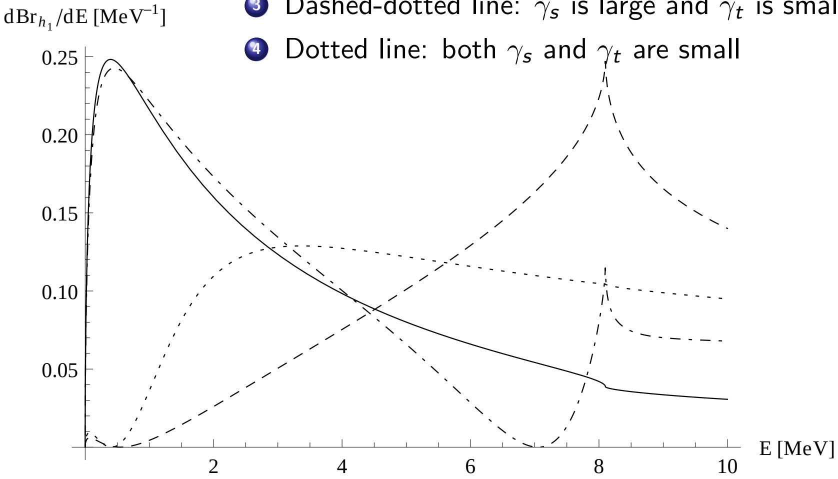
Let X be a compact state residing in the vicinity of two S wave thresholds (example: $X(3872)$ located close to both neutral and charged $D\bar{D}^*$ thresholds):

$$|X\rangle = c|\psi_0\rangle + \chi_1(k_1)|M_{11}M_{12}\rangle + \chi_2(k_2)|M_{21}M_{22}\rangle$$

Direct interactions between mesons in both channels can be parametrised via two (inverse) scattering lengths: singlet γ_s and triplet γ_t

- for small γ_s the zero E_C appears in the near-threshold region
- for small γ_t the nonlinear term $k_1 k_2$ has to be kept

1. Solid line: both γ_s and γ_t are large
2. Dashed line: γ_s is small and γ_t is large
3. Dashed-dotted line: γ_s is large and γ_t is small
4. Dotted line: both γ_s and γ_t are small



Practical guidelines

- Below-threshold region provides information about the bound-state peak. Unbiased analysis is required (no artificial events moving from the below-threshold region to the above-threshold region)!

Practical guidelines

- Below-threshold region provides information about the bound-state peak. Unbiased analysis is required (no artificial events moving from the below-threshold region to the above-threshold region)!
- If off-peak data contain irregularities, these can signal a nontrivial interplay of various degrees of freedom in the near-threshold resonance

Practical guidelines

- Below-threshold region provides information about the bound-state peak. Unbiased analysis is required (no artificial events moving from the below-threshold region to the above-threshold region)!
- If off-peak data contain irregularities, these can signal a nontrivial interplay of various degrees of freedom in the near-threshold resonance
- If off-peak data in one channel do not contain irregularities, this can be a result of the interference of different production mechanisms which tames the resulting line shape. Other channels should be checked and a **combined** analysis of all channels is to be performed

Practical guidelines (data analysis)

- Start from the simple Flatté. If the data are described satisfactory, then stop at this point

Practical guidelines (data analysis)

- Start from the simple Flatté. If the data are described satisfactory, then stop at this point
- Parameters found can be used to make conclusions on the nature of the resonance (bound versus virtual state, admixture of the molecule/compact components in the resonance w.f.)

Practical guidelines (data analysis)

- Start from the simple Flatté. If the data are described satisfactory, then stop at this point
- Parameters found can be used to make conclusions on the nature of the resonance (bound versus virtual state, admixture of the molecule/compact components in the resonance w.f.)
- If the simple Flatté distribution fails to describe the data, add (one by one) extra ingredients:
  - widths of the constituents
  - direct interaction in the mesonic channels
  - fine tuning between different production mechanisms

  Then additional information can be extracted concerning possible exchanges and binding forces in system

Application to the $X(3872)$

- For the existing data, the simple Flatté formulae work fine.
- The conclusion is that the X is a bound state with the approximately equal weights ($1/2$ and $1/2$) of the molecule and charmonium component in its w.f.
- The finite D^* width should be taken into account to consider the below-threshold peak (to search for the bound state peak in the data, if/when statistics allows).
- No hint for extra structures in the X line shapes exists so far, so there is no need to include extra ingredients.
- Binding mechanisms for the $X(3872)$ are still obscure

Outlook for the B physics

A similar analysis has to be done for the recently observed charged Z_b states, which requires large statistics at $\Upsilon(5S)$

Outlook for the B physics

A similar analysis has to be done for the recently observed charged Z_b states, which requires large statistics at $\Upsilon(5S)$

A lot of work for SuperB!

A similar analysis has to be done for the recently observed charged Z_b states, which requires large statistics at $\Upsilon(5S)$

A lot of work for SuperB!

Thank you!