Precision SM tests with hadronic τ decays

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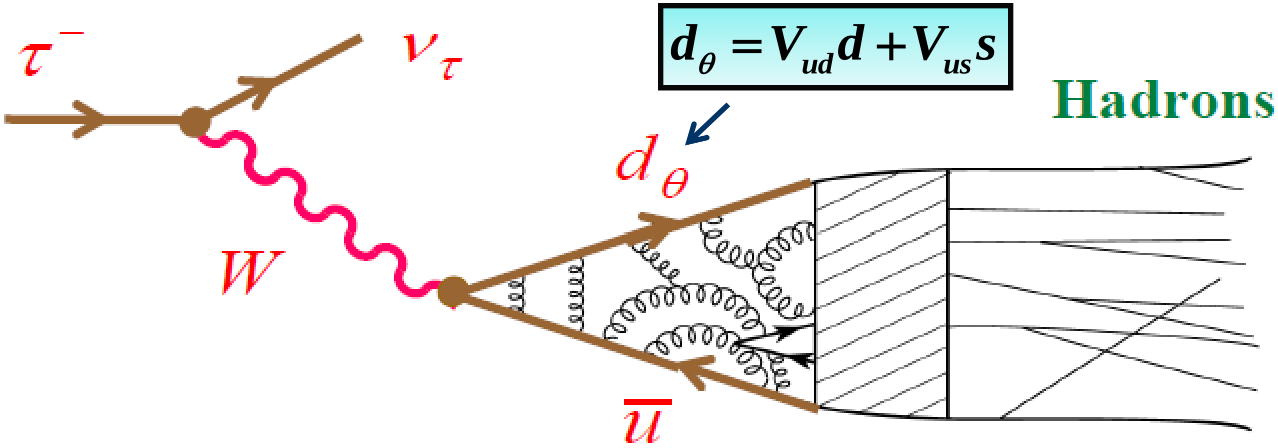
Outline :

1. Introduction and Motivation
2. Hadronic τ-decays as a QCD laboratory
3. Hadronic τ-decays as a probe of electroweak interactions
4. Conclusion and outlook

1. Introduction and Motivation

1.1 Hadronic τ-decays

- τ the only lepton heavy enough $\boxed{m_\tau = 1.777\ \mathrm{GeV}}$ to decay into hadrons

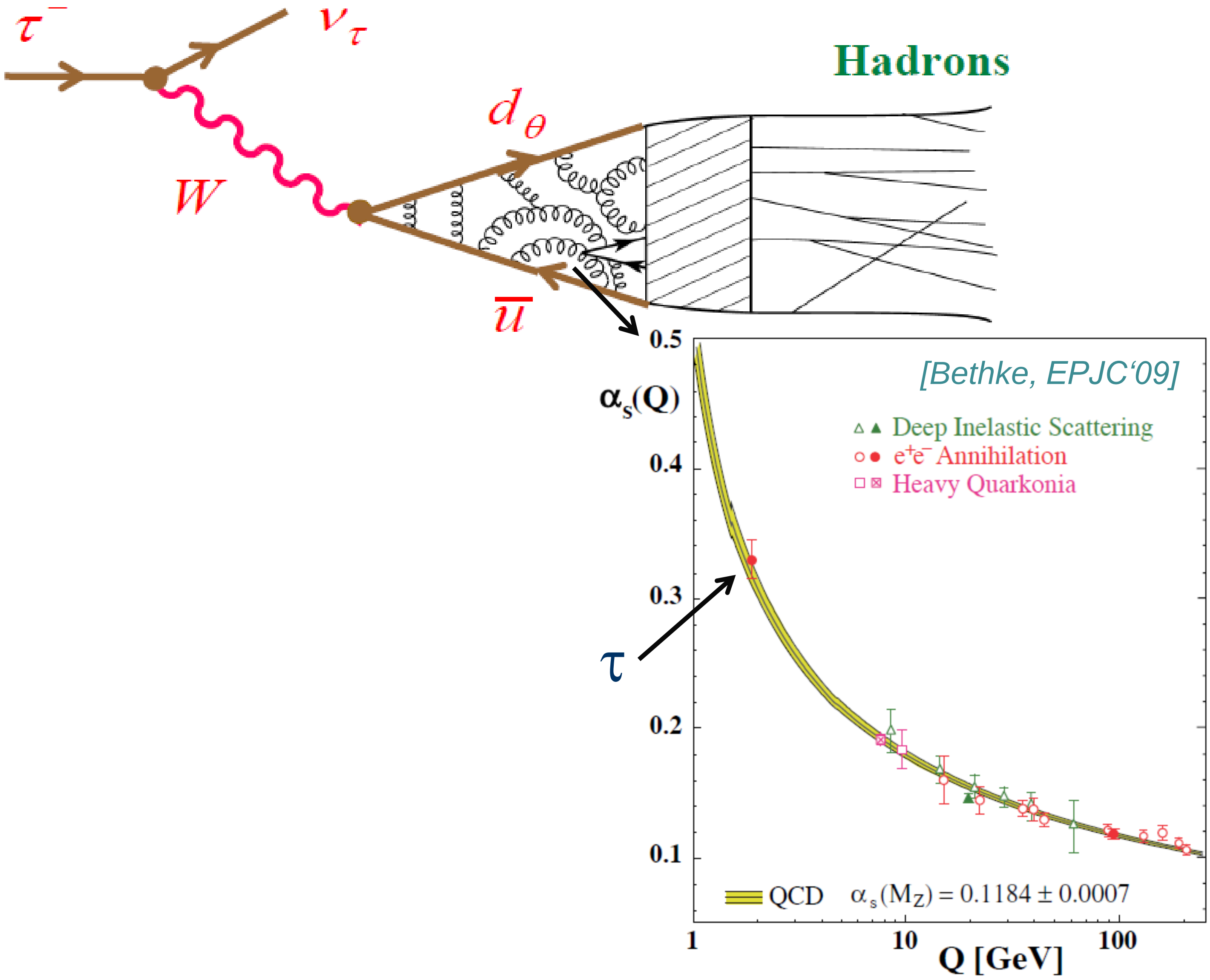


$$\boxed{d_\theta = V_{ud}\, d + V_{us}\, s}$$

- Very rich phenomenology but
  - Precise measurements needed
  - Have the hadronic uncertainties under control

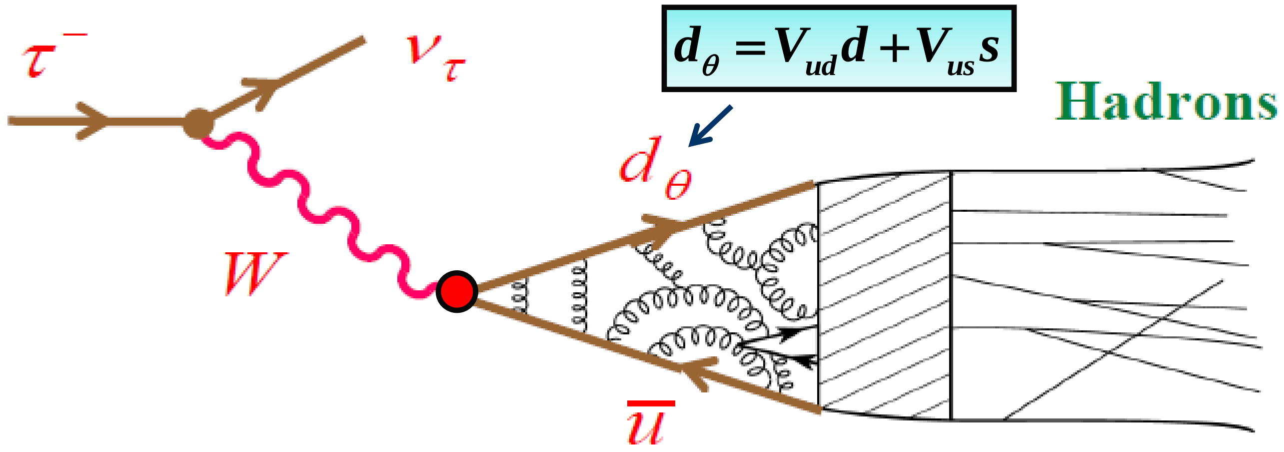
⇨ Tests of QCD and EW interactions

1.2 Hadronic τ-decays as a QCD laboratory



[Bethke, EPJC‘09]

1.3 Hadronic τ-decays to probe new physics



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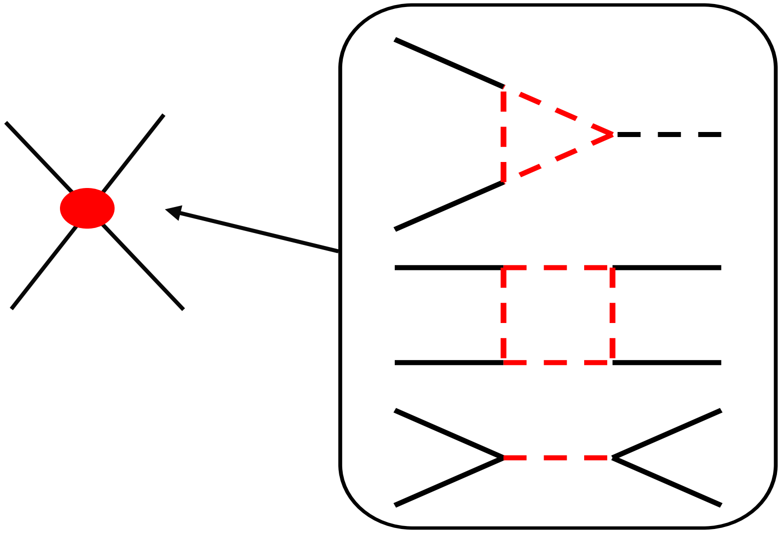


SUSY loops
Z’, Charged Higgs,
Right-Handed
Currents,….

2. Hadronic τ-decays as a QCD laboratory

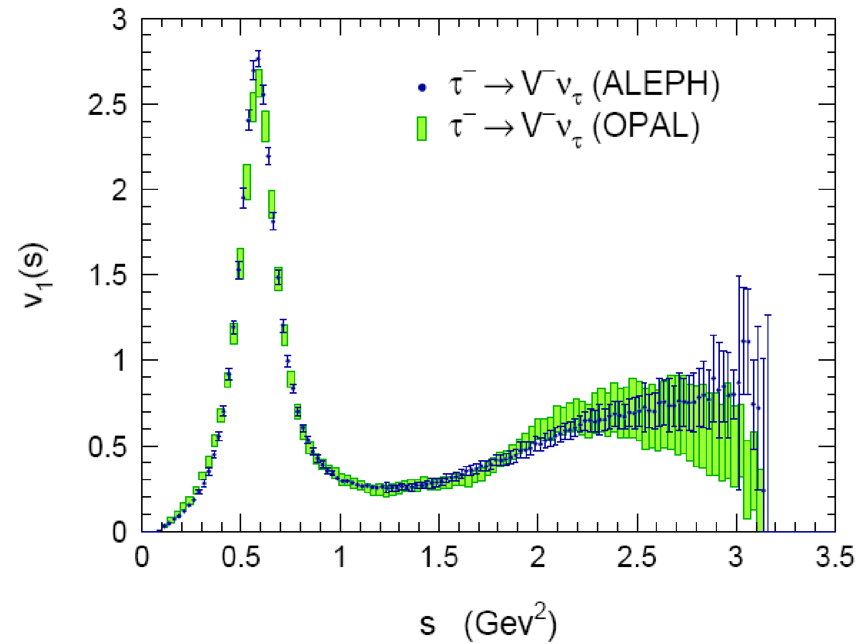
2.1 Introduction

- $\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \mathbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$ naïve QCD prediction

Experimentally $R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$

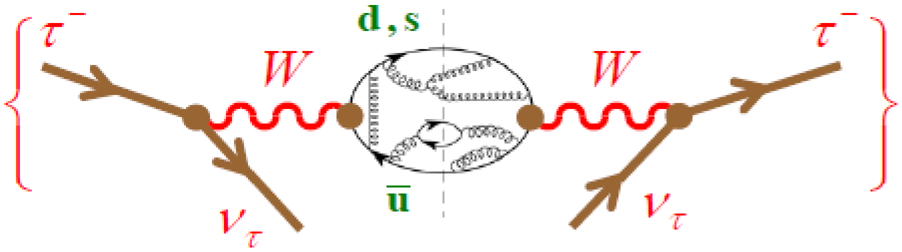
- In tau decays mixing between
  - Perturbative QCD
  - Non-perturbative QCD: resonance structure
- Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$



2.2 Theoretical Method

- Optical theorem: $\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}\left\{ \quad \right\}$



$$\boxed{\Gamma \alpha \, \mathrm{Im}\Pi^{\mu\nu}(q)} \Rightarrow \Pi^{\mu\nu}(q) = i\int d^4x \, e^{iqx} \left\langle 0 \middle| T\left\{ J^{\mu}(x) J^{\nu\dagger}(0) \right\} \middle| 0 \right\rangle$$

- Lorentz decomposition: $\Pi^{\mu\nu}(q) = \left(-g_{\mu\nu}\, q^2 + q^{\mu}q^{\nu} \right) \Pi^{1}(q^2) + q^{\mu}q^{\nu}\, \Pi^{0}(q^2)$

$$\Rightarrow \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2} \right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2} \right) \mathrm{Im}\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\Pi^{(0)}(s+i\varepsilon) \right]}$$

$$\boxed{\Pi^{(J)}(s) = \left| V_{ud} \right|^2 \left(\Pi^{(J)}_{ud,VV}(s) + \Pi^{(J)}_{ud,AA}(s) \right) + \left| V_{us} \right|^2 \left(\Pi^{(J)}_{us,VV}(s) + \Pi^{(J)}_{us,AA}(s) \right)}$$

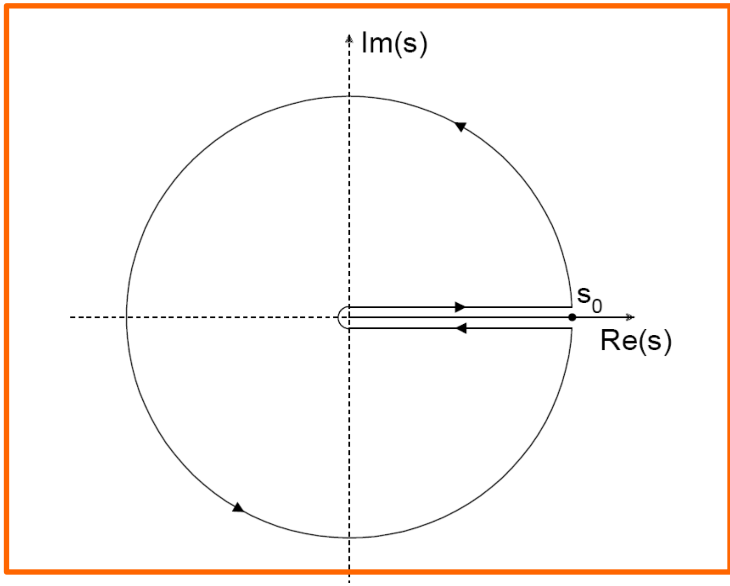
2.3 Correlators

Braaten, Narison, Pich'92

- Analyticity: Π analytic in the entire complex plane except for s real positive

⇨ Cauchy theorem:

$$\frac{1}{\pi}\int_0^{s_0} ds\; g(s)\;\mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\; g(s)\;\Pi(s)$$



⇨ $$R_\tau(m_\tau^2) = 6i\pi S_{EW}\oint_{|s|=m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s)+\Pi^{(0)}(s)\right]$$

- Sufficient high energy for *Operator Product Expansion*
  Kinematic factor ⇨ decreases the weight close to the real axis where Π has a cut

2.4 Operator Product Expansion

Braaten, Narison, Pich'92

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\mathrm{dim}O=D} \mathcal{C}^{(J)}(s,\mu)\ \langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ separation scale between short and long distances

- D=0: Perturbative contributions
- D=2: Quark mass corrections
- D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\langle m_j \bar{q}_i q_i \rangle$
- D=6: 4 quarks operators, $\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \rangle$
- D≥8: Neglected terms, supposed to be small…

⇨ $$R_{\tau,V}(s_0) = \frac{3}{2} \left|V^{ud}\right|^2 S_{EW} \left(1 + \delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V}\right)$$

similar for $R_{\tau,A}(s_0)$ and $R_{\tau,S}(s_0)$

2.4 Operator Product Expansion

Braaten, Narison, Pich'92

- $$\boxed{R_{\tau,V+A}(s_0) = N_C\ S_{EW}\left|V_{ud}\right|^2\left(1+\delta_P+\delta_{NP}\right)}$$

$$S_{EW} = 1.0201(3)$$

Marciano &Sirlin'88, Braaten & Li'90, Erler'04

2.4 Operator Product Expansion

Braaten, Narison, Pich'92

- $R_{\tau,V+A}(s_0) = N_C\, S_{EW}\, |V_{ud}|^2 \left(1+\delta_P+\delta_{NP}\right)$

  $\nearrow$

  $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (m_q=0)

$$\boxed{-s\frac{d}{ds}\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_S(\text{-}s)}{\pi}\right)^n}$$

$K_0 = K_1 = 1,\ K_2 = 1.63982,\ K_3 = 6.37101$

$K_4 = 49.07570$ *Baikov, Chetyrkin, Kühn'08*

⇨ $$\boxed{\delta_P = \sum_{n=1} K_n A^n(\alpha_S) = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots}$$

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

with $A^n(\alpha_S) = \dfrac{1}{2\pi i}\oint_{|x|=1}\dfrac{dx}{x}\left(1-2x+2x^3-x^4\right)\left(\dfrac{\alpha_S(-s)}{\pi}\right)^n$

⇨ $$\boxed{\delta_P \approx 20\%}$$

2.4 Operator Product Expansion

Braaten, Narison, Pich'92

- $R_{\tau,V+A}(s_0) = N_C\, S_{EW}\, \left|V_{ud}\right|^2 \left(1+\delta_P+\delta_{NP}\right)$

  $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Non perturbative part

$$\boxed{\Pi^{(0+1)}_{OPE}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\left\langle O_{2n}\right\rangle}{(-s)^n}}$$

$$C_4\left\langle O_4\right\rangle \approx \frac{2\pi}{3}\left\langle 0\right|\alpha_S G^{\mu\nu}G_{\mu\nu}\left|0\right\rangle$$

⇒

$$\boxed{\delta_{NP} = \frac{-1}{2\pi i}\oint_{|x|=1} dx\left(1-3x^2+2x^3\right)\sum_{n\geq 2}\frac{C_{2n}\left\langle O_{2n}\right\rangle}{\left(-xm_\tau^2\right)^n} = -3\frac{C_6\left\langle O_6\right\rangle}{m_\tau^6} - 2\frac{C_8\left\langle O_8\right\rangle}{m_\tau^8} + \ldots}$$

Suppression by m_τ^{2n} and additionnal chiral suppression for $\boxed{C_6\left\langle O_6\right\rangle^{V+A}}$

⇒ $\boxed{\delta_{NP} = -0.0059 \pm 0.0014}$ fitted from data *Davier et al'08*

2.5 Extraction of α_S

- Use moments: weighted distribution to estimate the non-perturbative contributions

$$R^{kl}_{\tau,V/A} = \int_0^{m_\tau^2} ds \left(1-\frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_{\tau,V/A}}{ds}$$

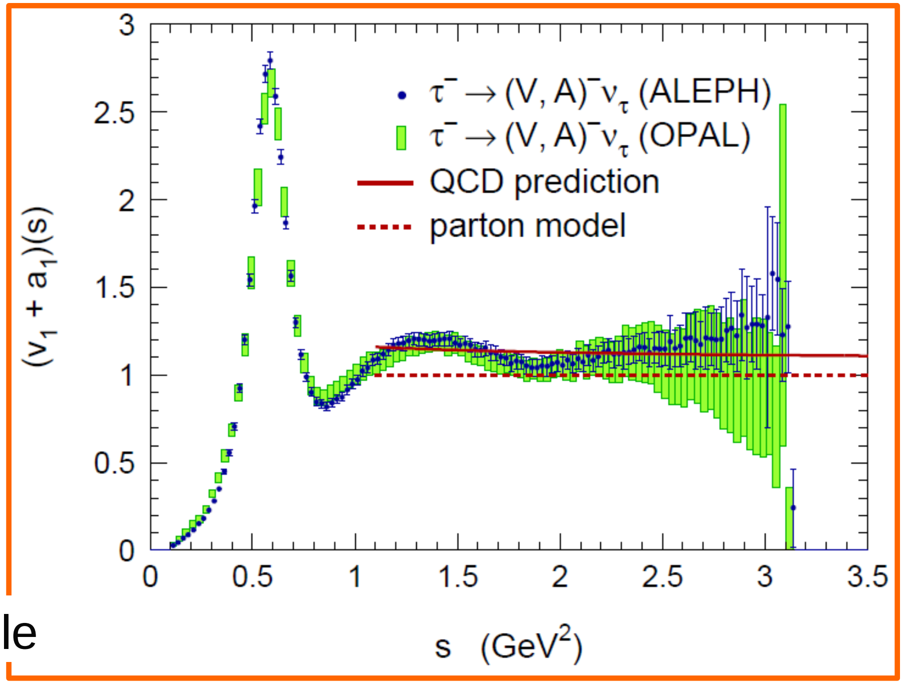
Strong correlations!

⇨ Use V+A where the non-perturbative contribution is supposed to be small

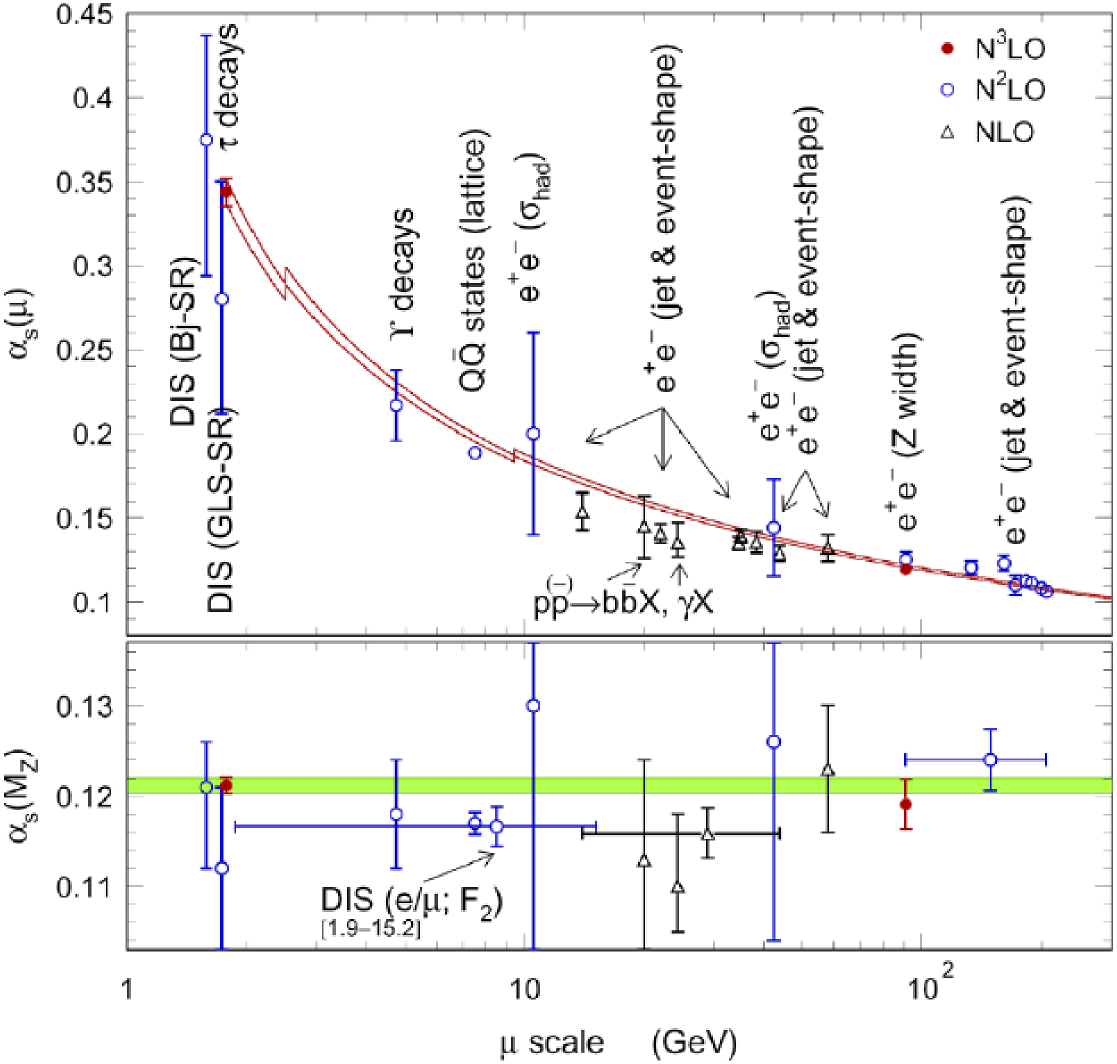
- Data from ALEPH and OPAL

- Analysis from *Davier et al'08*
  - ALEPH data
  - Improved BRs from BaBar and Belle

Moments (k,l)= (0,0), (1,0), (1,1), (1,2), (1,3)



2.6 Test of QCD



Davier et al’08

$$\boxed{\alpha_s\left(m_\tau^2\right) = 0.344 \pm 0.009}$$

$$\boxed{\alpha_s\left(M_Z^2\right) = 0.1212 \pm 0.0011}$$

to be compared to

$$\boxed{\alpha_s\left(M_Z^2\right)_{Z\ \text{width}} = 0.1190 \pm 0.0027}$$

The most precise test of *asymptotic freedom*!

2.7 Extraction of α_S: Other analyses

Pich Tau'10

Reference	Method	δ_P	$\alpha_s(m_\tau)$	$\alpha_s(m_Z)$
Baikov et al	CIPT, FOPT	0.1998 (43)	**0.332 (16)**	**0.1202 (19)**
Davier et al	CIPT	0.2066 (70)	**0.344 (09)**	**0.1212 (11)**
Beneke-Jamin	BSR + FOPT	0.2042 (50)	**0.316 (06)**	**0.1180 (08)**
Maltman-Yavin	PWM + CIPT		**0.321 (13)**	**0.1187 (16)**
Menke	CIPT, FOPT	0.2042 (50)	**0.342 (11)**	**0.1213 (12)**
Narison	CIPT, FOPT		**0.324 (08)**	**0.1192 (10)**
Caprini-Fischer	BSR + CIPTm	0.2042 (50)	**0.321 (10)**	
Cvetič et al	βexp + CIPT	0.2040 (40)	**0.341 (08)**	**0.1211 (10)**
Pich	CIPT	0.2038 (40)	**0.342 (12)**	**0.1213 (14)**
Boito et al	CIPT+DV		0.322 (19)	0.1187 (32)
Boito et al	FOPT+DV		0.307 (19)	0.1169 (25)

➡ New analyses going on *Boito et al'11, Gonzalez-Alonso, Pich, E.P.*

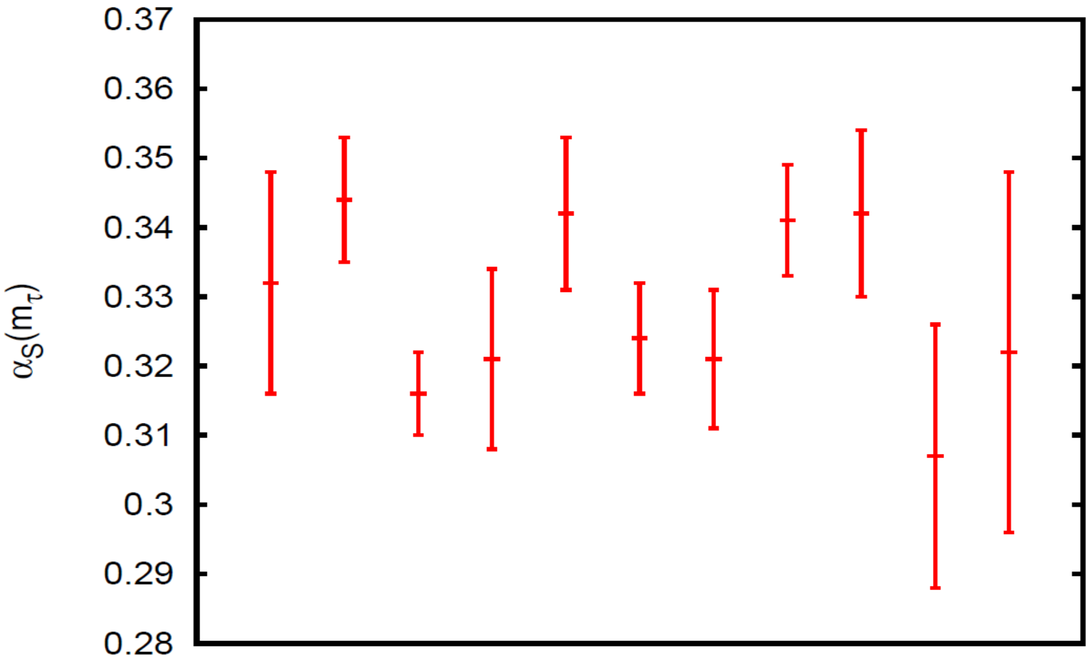
2.7 Extraction of α_S: Other analyses

- Theoretically:
  - Estimate of the perturbative part: CIPT vs. FOPT
  - Choice of the moments ⇨ correlations!
  - Inputs for gluon condensate, higher order operators…
  - Duality violations

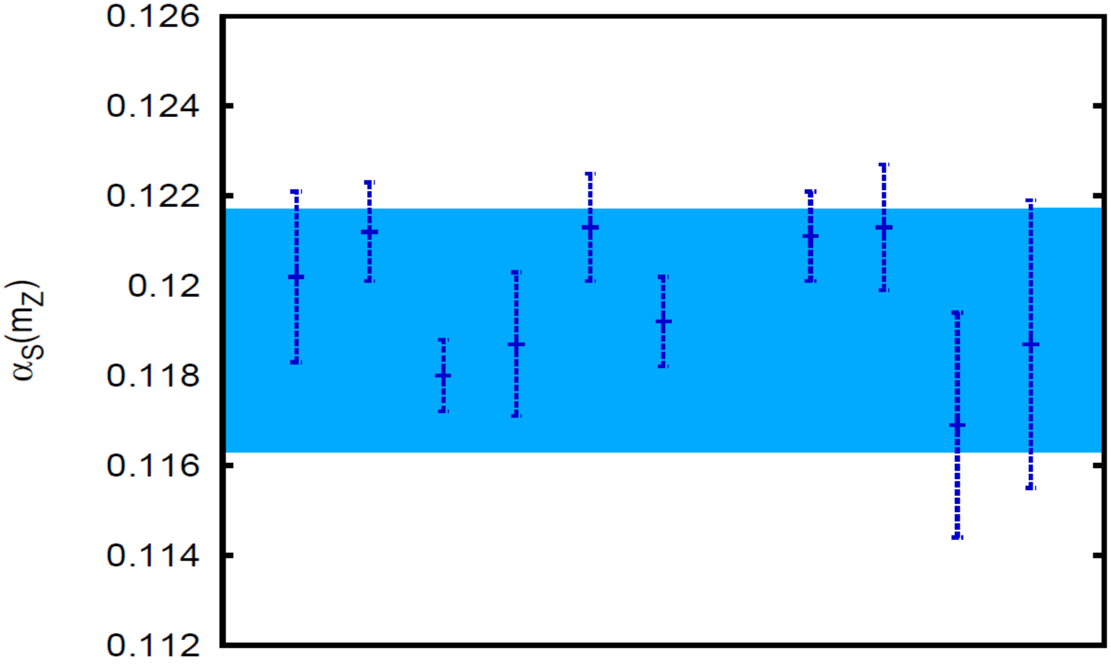
- Experimentally:
  - Data from ALEPH and OPAL: unfolding? Correlations point by point?
  - Difficulty to separate V and A ⇨ Use BaBar and Belle
  - Improved BRs from BaBar and Belle

⇨ New data from BaBar, Belle, SuperB

2.7 Extraction of α_S: Other analyses



2.7 Extraction of α_S: Other analyses



- *Extraction of α_S* from hadronic τ decays very *competitive*!
- If new data room for *improvement*!
  - Study of duality violation effects
  - Higher order condensates
  - New physics?

3. Hadronic τ-decays as a probe of electroweak interactions

3.1 New Physics in R_τ

- Models with modifications of the couplings:
  - Right-handed currents

Bernard, Oertel, E.P., Stern'07

$$\Pi^{(J)}(s)=\left|V_{ud}\right|^2\left(\Pi^{(J)}_{ud,VV}(s)+\Pi^{(J)}_{ud,AA}(s)\right)+\left|V_{us}\right|^2\left(\Pi^{(J)}_{us,VV}(s)+\Pi^{(J)}_{us,AA}(s)\right)$$

$$\Downarrow$$

$$\Pi^{(J)}(s)=\left|V^{eff}_{ud}\right|^2\Pi^{(J)}_{ud,VV}(s)+\left|A^{eff}_{ud}\right|^2\Pi^{(J)}_{ud,AA}(s)+\left|V^{eff}_{us}\right|^2\Pi^{(J)}_{us,VV}(s)+\left|A^{eff}_{us}\right|^2\Pi^{(J)}_{us,AA}(s)$$

$$\Rightarrow \frac{R_A}{R_V}=\frac{\left|A^{ud}_{eff}\right|^2 S_{EW}\left(1+\delta^{(0)}+\sum\limits_{D=2,4..}\delta^{(D)}_{ud,A}\right)}{\left|V^{ud}_{eff}\right|^2 S_{EW}\left(1+\delta^{(0)}+\sum\limits_{D=2,4..}\delta^{(D)}_{ud,V}\right)}=(1-4\varepsilon_{ns})\frac{\left(1+\delta^{(0)}+\sum\limits_{D=2,4..}\delta^{(D)}_{ud,A}\right)}{\left(1+\delta^{(0)}+\sum\limits_{D=2,4..}\delta^{(D)}_{ud,V}\right)}$$

3.1 New Physics in R_τ

- Models with modifications of the couplings:
  - Tensor & scalar interactions ex: leptoquarks

Cirigliano, Filipuzzi, Gonzalez-Alonso, E.P. in progress

$$\begin{aligned} R_\tau^{NS}(s_0) = 6\pi i \left|V^{ud}\right|^2 \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \Bigg\{ & \left|\kappa_V\right|^2 \left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,VV}(s)+\Pi^{(0)}_{ud,VV}(s)\right] \\ & + \left|\kappa_A\right|^2 \left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,AA}(s)+\Pi^{(0)}_{ud,VV}(s)\right] \\ & + 2\,\mathrm{Re}\left(\kappa_V\kappa_S^*\right)\frac{\Pi_{ud,VS}(s)}{m_\tau} + 2\,\mathrm{Re}\left(\kappa_A\kappa_P^*\right)\frac{\Pi_{ud,AP}(s)}{m_\tau} \\ & + 12\,\mathrm{Re}\left(\kappa_V\kappa_T^*\right)\frac{\Pi_{ud,VT}(s)}{m_\tau}\Bigg\}\left[1-2\tilde{v}_L\right] \end{aligned}$$

- But also charged Higgs, little Higgs, SUSY…

3.1 New Physics in R_τ

- Disentangle New Physics from QCD effects:
  - Take QCD observables from other sources or more data

    Inputs for $\alpha_s(m_\tau),\ \left\langle \frac{\alpha_s}{\pi} GG \right\rangle,\ m_{u,d,s},\ \left\langle \bar{q}_i q_i \right\rangle,\ \left\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \right\rangle$

    Lattice QCD, SCET, moments…

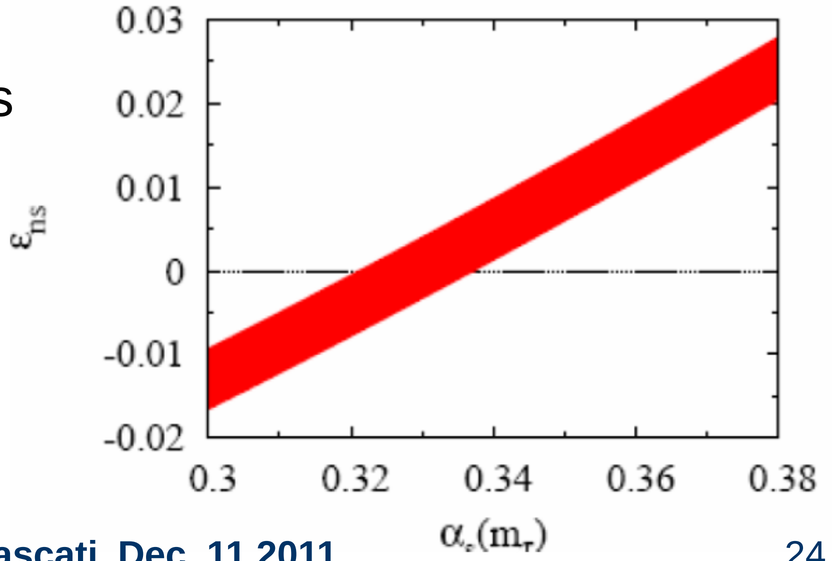
  - Experimental separation V/A very important

    ➡ only data from OPAL, need more data

- Possible constraint on NP parameters

  Ex: RHCs

  Bernard, Oertel, E.P., Stern'07



3.2 Extraction of V_{us}

Experimental processes	Parameters extracted	Low Energy/QCD inputs
K_{l3}: $K \to \pi \ell \nu_\tau$	$f_+(0)\ V_{us}$	Form factors $f_+(q^2),\ f_0(q^2)$
K_{l2}/π_{l2}:$K \to \ell\ \nu_\tau/\pi \ell \nu_\tau$	$F_K/F_\pi\ V_{us}/V_{ud}$	F_K/F_π, V_{ud}
$\tau \to s$ inclusive	V_{us} or m_s	SU(3) breaking
$\tau \to K\nu$ absolute	$F_K\ V_{us}$	F_K
$\tau \to K\nu_\tau/\ \tau \to \pi\nu_\tau$	$F_K/F_\pi\ V_{us}/V_{ud}$	F_K/F_π, V_{ud}
$\tau \to K\pi\nu_\tau$	$f_+(0)\ V_{us}$	Form factors $f_+(q^2),\ f_0(q^2)$

Inclusive τ decays

- $$\boxed{R_\tau^{kl} = N_C S_{EW}\left\{\left(|V_{us}|^2+|V_{ud}|^2\right)\left[1+\delta^{kl(0)}\right]+\sum_{D\geq 2}\left[|V_{ud}|^2\,\delta_{ud}^{kl(D)}+|V_{us}|^2\,\delta_{us}^{kl(D)}\right]\right\}}$$

⇒ Use instead

$$\boxed{\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2}-\frac{R_{\tau,S}^{kl}}{|V_{us}|^2}} \approx N_C S_{EW}\sum_{D\geq 2}\left[\delta_{ud}^{kl(D)}-\delta_{us}^{kl(D)}\right]$$

- $\boxed{\delta R_\tau^{kl} \approx 24\frac{m_s^2(m_\tau^2)}{m_\tau^2}\Delta_{kl}\left(\alpha_S\right)}$ ⇒ m_s and/or $\boxed{V_{us}}$

- Δ_{kl} known to order $O\left(\alpha_S^3\right)$: *Gámiz, Jamin, Pich, Prades,Schwab'03,'05*
  - transverse contribution (J=0+1) computed from theory
  - longitudinal contribution (J=0) divergent ⇒ determined from data
    Phenomenological model (pion, kaon poles + Kπ scattering)

Inclusive τ decays

- $$\boxed{|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,th}}}$$

τ data: $R^{00}_{\tau,S} = 0.1615(40)$ and $R^{00}_{\tau,V+A} = 3.479(11)$

PDG 10: $|V_{ud}| = 0.97425(22)$

$$\boxed{\delta R^{00}_{\tau,th} = 0} \Rightarrow \boxed{|V_{us}| = 0.210(3)}$$

.

- $\delta R^{00}_{\tau,th}$ computed from phenomenology *Gámiz, Jamin, Pich, Prades,Schwab'02,'03*

$$\delta R^{00}_{\tau,th} = 0.216(16) \Rightarrow \boxed{|V_{us}| = 0.2164 \pm 0.0027_{\mathrm{exp}} \pm 0.0005_{\mathrm{th}}}$$

dominated by experimental uncertainties

- V_{us} from K_{l3}: $\boxed{|V_{us}| = 0.2254 \pm 0.0013}$ dominated by uncertainty on $f_+(0)$

$\boxed{f_+(0)|V_{us}| = 0.2163 \pm 0.0005}$ and $f_+(0) = 0.959 \pm 0.005$

Flavianet Kaon WG'10 *RBC-UKQCD'09*

Inclusive τ decays

- $\boxed{\left|V_{us}\right|^2 = \dfrac{R^{00}_{\tau,S}}{\dfrac{R^{00}_{\tau,V+A}}{\left|V_{ud}\right|^2} - \delta R^{00}_{\tau,th}}}$

τ data: $R^{00}_{\tau,S} = 0.1615(40)$ and $R^{00}_{\tau,V+A} = 3.479(11)$

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$\delta R^{00}_{\tau,th} = 0.216(16) \Rightarrow \boxed{\left|V_{us}\right| = 0.2164 \pm 0.0027_{\text{exp}} \pm 0.0005_{\text{th}}}$

dominated by experimental uncertainties

- V_{us} from unitarity: $\boxed{\left|V_{us}\right| = 0.2255 \pm 0.0010}$ 3σ away!

Inclusive τ decays

- Possible normalization problem:

Smaller τ → K branching ratios ⇒ smaller $R_{\tau,S}$ ⇒ smaller $\boxed{V_{us}}$

$$\boxed{R^{00}_{\tau,S}\Big|_{\text{old}} = 0.1686(47)} \quad \Rightarrow \quad \boxed{R^{00}_{\tau,S}\Big|_{\text{new}} = 0.1615(40)}$$

$$\boxed{\left|V_{us}\right|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0005_{\text{th}}} \quad \Rightarrow \quad \boxed{\left|V_{us}\right|_{\text{new}} = 0.2164 \pm 0.0027_{\text{exp}} \pm 0.0005_{\text{th}}}$$

Missing modes at B factories?

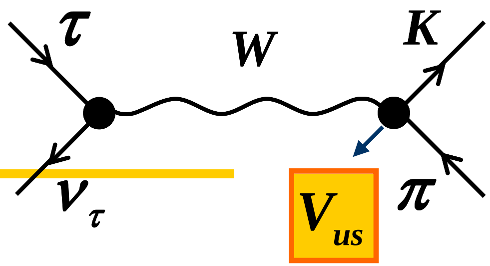
- Spectral moment analysis

Maltman et al'09

$$\boxed{|V_{us}| = \begin{cases} 0.2180(32)(15) & (\hat{w}_{10}) \\ 0.2188(29)(22) & (w_{20}) \\ 0.2172(34)(11) & (w_{10}) \\ 0.2160(26)(8) & (w_{(00)}) \end{cases}}$$

τ + electroproduction data ⇒ $\boxed{|V_{us}| = 0.2208(27)(28)(5)(2)}$

Exclusive decays: $\tau \to K\pi\nu_\tau$ decays



Bernard, Boito, E.P., in progress

- $$\Gamma_{\tau\to K\pi\nu_\tau} \equiv \Gamma_{K\pi} = \frac{G_F^2 m_\tau^3}{32\pi^3} C_K S_{EW} \left| f_+(0) V_{us} \right|^2 I_K^\tau \left(1+\delta_{EM}\right)^2$$

$$I_K^\tau = \int_{(m_K+m_\pi)^2}^{m_\tau^2} ds\, \lambda^{3/2} F\left(s, \bar{f}_+(s), \bar{f}_0(s)\right)$$

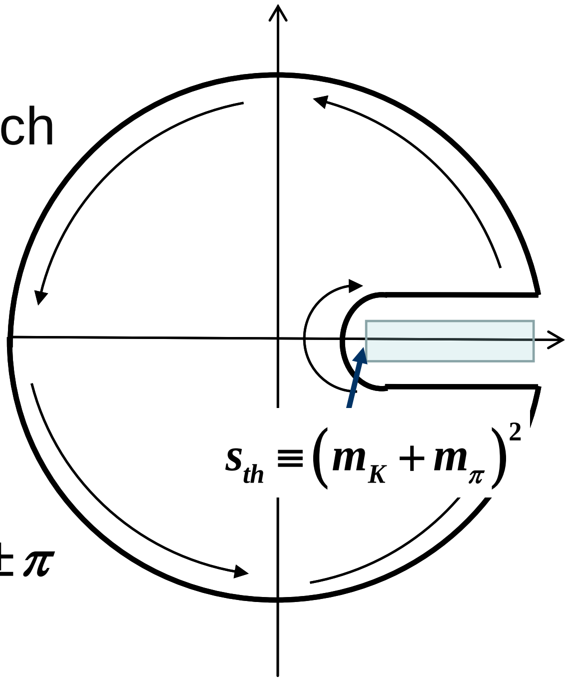
~0.1% level expected
Neglected at this stage
Work in progress by F. Flores

- Form factors described using a dispersive approach

$$\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{(s'-\bar{s})^n} \frac{\phi_{+,0}(s')}{s'-s-i\varepsilon} \right]$$

$\phi_{+,0}$ (s): phase of the form factor

- – $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$
- – $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown ⇨ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi$



Exclusive decay: $\tau \to K\pi\nu_\tau$ decays

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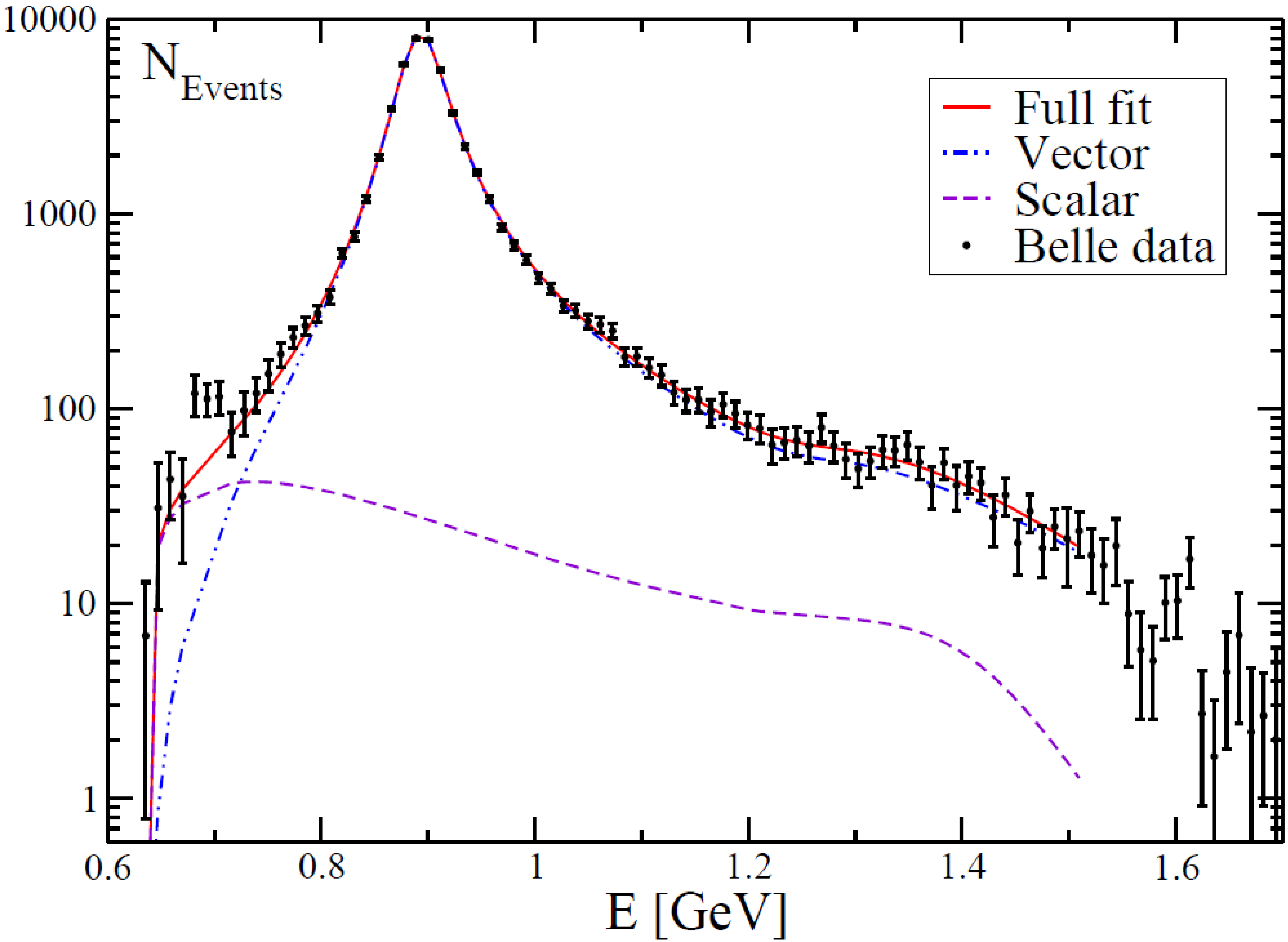
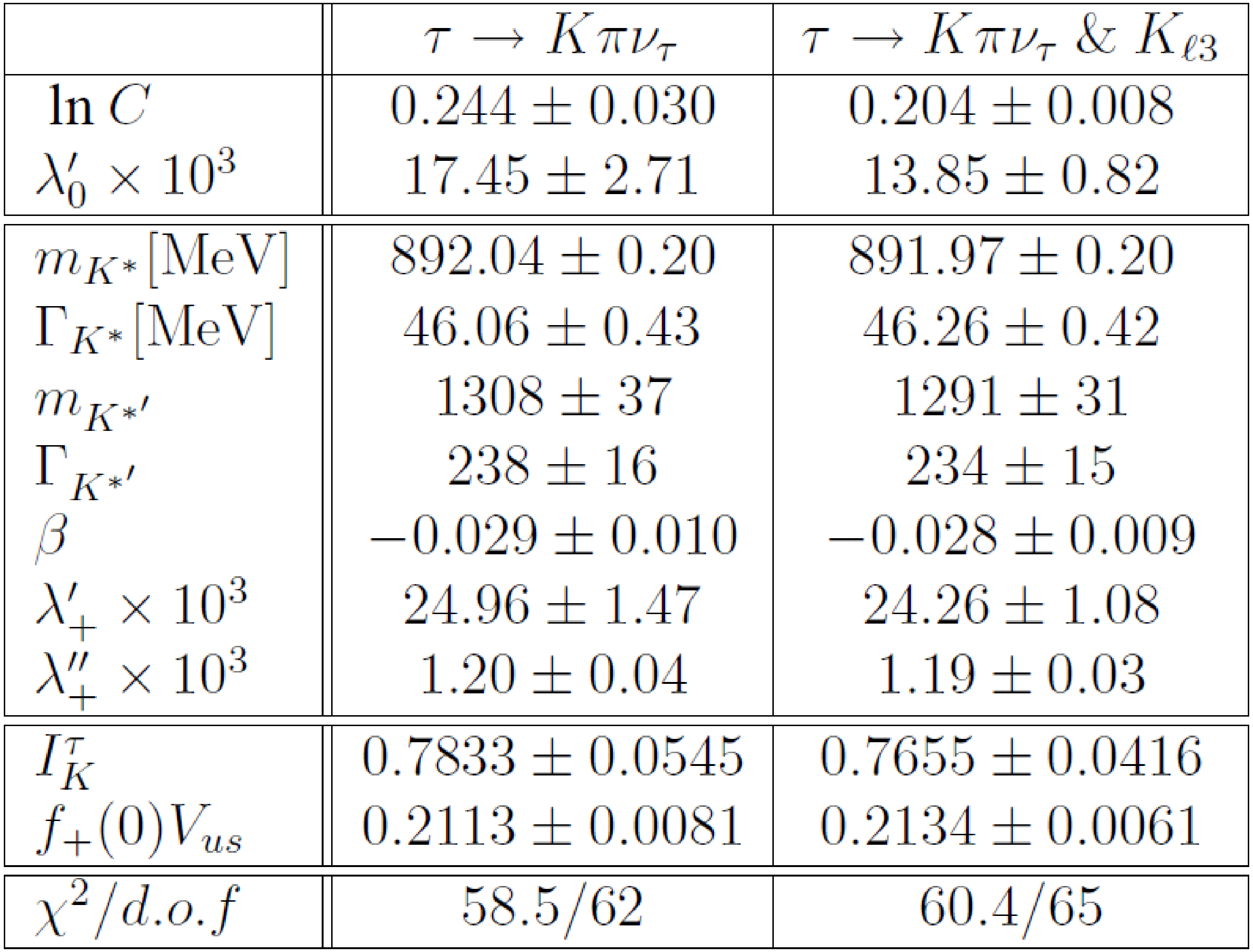


Exclusive decay: $\tau \to K\pi\nu_\tau$ decays

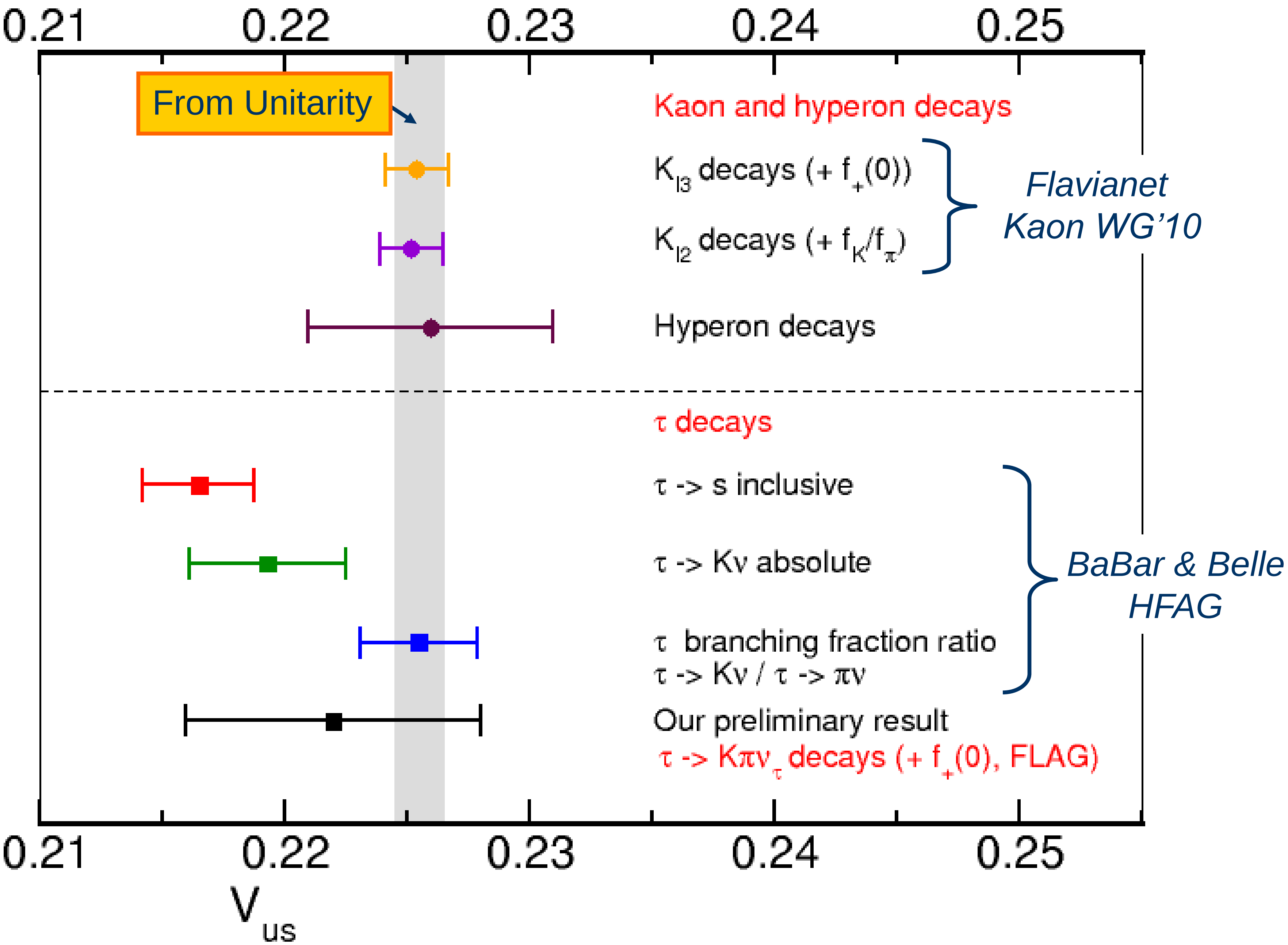
Bernard, Boito, E.P., in progress

- Use of the vector parametrization of *[Boito, Escribano & Jamin'09,'10]* but fit now the scalar form factor



	$\tau \to K\pi\nu_\tau$	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$
$\ln C$	0.244 ± 0.030	0.204 ± 0.008
$\lambda'_0 \times 10^3$	17.45 ± 2.71	13.85 ± 0.82
m_{K^*} [MeV]	892.04 ± 0.20	891.97 ± 0.20
Γ_{K^*} [MeV]	46.06 ± 0.43	46.26 ± 0.42
$m_{K^{*\prime}}$	1308 ± 37	1291 ± 31
$\Gamma_{K^{*\prime}}$	238 ± 16	234 ± 15
β	-0.029 ± 0.010	-0.028 ± 0.009
$\lambda'_+ \times 10^3$	24.96 ± 1.47	24.26 ± 1.08
$\lambda''_+ \times 10^3$	1.20 ± 0.04	1.19 ± 0.03
I_K^τ	0.7833 ± 0.0545	0.7655 ± 0.0416
$f_+(0)V_{us}$	0.2113 ± 0.0081	0.2134 ± 0.0061
$\chi^2/d.o.f$	$58.5/62$	$60.4/65$

Only statistical errors but the dominant ones !


0.21
0.22
0.23
0.24
0.25
From Unitarity
Kaon and hyperon decays
K_{l3} decays (+ $f_+(0)$)
K_{l2} decays (+ f_K/f_π)
Flavianet Kaon WG'10
Hyperon decays
τ decays
τ -> s inclusive
τ -> Kν absolute
τ branching fraction ratio τ -> Kν / τ -> πν
BaBar & Belle HFAG
Our preliminary result
τ -> Kπν_τ decays (+ $f_+(0)$, FLAG)
V_{us}

V_{us} from Tau decays

- Problem inclusive versus exclusive:
  - Take the K_{l3} Br and predict the $\tau \to K\pi\nu_\tau$ Br

$$\frac{\Gamma_{\tau\to K\pi\nu_\tau}}{\Gamma_{K\to\pi\ell\nu_\ell}} = \frac{\frac{G_F^2 m_\tau^3}{48\pi^3} C_K^{\tau 2} S_{\mathrm{EW},\tau} \left(|V_{us}| f_+^{K^0\pi^-}(0)\right)^2 I_K^\tau \left(1+\delta_{\mathrm{EM}}^\tau+\delta_{\mathrm{SU(2)}}^\tau\right)^2,}{\frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\mathrm{EW,K}} \left(|V_{us}| f_+^{K^0\pi^-}(0)\right)^2 I_K^\ell \left(1+\delta_{\mathrm{EM}}^{K\ell}+\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

⇨ $$\mathrm{BR}(\tau\to K\pi\nu_\tau) = \frac{8m_\tau^3}{m_K^5}\,\frac{C_K^{\tau 2}}{C_K^2}\,\frac{S_{\mathrm{EW},\tau}}{S_{\mathrm{EW,K}}}\,\frac{I_K^\tau}{I_K^\ell}\,\frac{\left(1+\delta_{\mathrm{EM}}^\tau+\delta_{\mathrm{SU(2)}}^\tau\right)^2}{\left(1+\delta_{\mathrm{EM}}^{K\ell}+\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}\,\frac{\tau_\tau}{\tau_K}\,\mathrm{BR}(K_{e3})$$

$$\boxed{\mathrm{BR}(\tau^-\to K_S\pi^-\nu_\tau) = \mathrm{BR}(\tau^-\to K^-\pi^0\nu_\tau) = (0.43814\pm 0.01348)\%}$$

To be compared with $\boxed{\mathrm{BR}(\tau^-\to K_S\pi^-\nu_\tau) = \left(0.404\pm 0.002_{\mathrm{stat}}\pm 0.013_{\mathrm{syst}}\right)\%}$ *Belle'07*

$$\left(\mathrm{BR}(\tau\to K\pi\nu_\tau)_{\exp} = \left(0.427\pm 0.024\right)\%\right)$$

  - New Physics?

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- CP violating asymmetry

$$\boxed{A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \overline{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \overline{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}}$$

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle$$

$$\left|K_L^0\right\rangle = p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle$$

$$\boxed{\left\langle K_L \middle| K_S \right\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}\left(\varepsilon_K\right)}$$

$$= |p|^2 - |q|^2 \quad \boxed{\approx \left(0.33 \pm 0.01\right)\%}$$

in the Standard Model *Bigi & Sanda'05*

- Experimental measurement: $\boxed{A_{Q\,\mathrm{exp}} = \left(-0.45 \pm 0.24_{\mathrm{stat}} \pm 0.11_{\mathrm{syst}}\right)\%}$

BaBar'11

⇨ $\boxed{\sim 3\sigma}$ from the SM!

- New physics: Charged Higgs, leptoquarks or others?

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- Analysis of the angular CP violating asymmetry

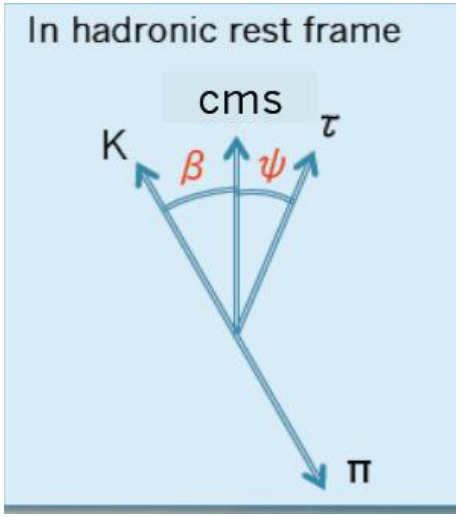
$$\frac{d\Gamma\left(\tau^- \to K\pi\nu_\tau\right)}{dq^2 d\cos\theta\; d\cos\beta} = \left[A(q^2) - B(q^2)\;\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2 + m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\,\tilde{f}_0^*(s)\right)$$

CP violating term
S-P interference

- $A(Q^2)$, $B(Q^2)$, $C(Q^2)$ kinematic functions
- Angles:

  in Kπ rest frame
  - β: angle between kaon and e^+e^- CMS frame
  - Ψ: angle between τ and CMS frame

  in τ rest frame
  - θ: angle between τ direction in CMS and direction of Kπ system (dependence with Ψ)



3.3 $\tau \rightarrow K\pi\nu_\tau$ CP violating asymmetry

- Analysis in the framework of multiple Higgs models

$$\frac{d\Gamma\left(\tau^- \rightarrow K\pi\nu_\tau\right)}{dq^2 d\cos\theta\; d\cos\beta} = \left[A(q^2) - B(q^2)\;\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2$$

$$+\, m_\tau^2 \left|\tilde{f}_0(s)\right|^2 - C(q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\,\tilde{f}_0^*(s)\right)$$

CP violating term
S-P interference

  - $A(Q^2)$, $B(Q^2)$, $C(Q^2)$ kinematic functions
  - form factors

$$\langle K\pi|\;\bar{s}\gamma_\mu u\;|0\rangle = \left[\left(p_K - p_\pi\right)_\mu + \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu\right] f_+(s) - \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu f_0(s)$$

with $s = q^2 = (p_K + p_\pi)^2$

vector

scalar

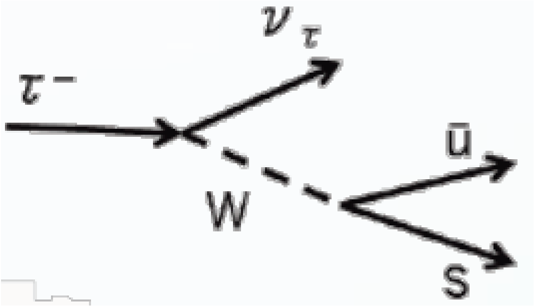
3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- Analysis in the framework of multiple Higgs models

$$\frac{d\Gamma\left(\tau^- \to K\pi\nu_\tau\right)}{dq^2 d\cos\theta\, d\cos\beta} = \left[A(q^2) - B(q^2)\,\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2 + m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\,\tilde{f}_0^*(s)\right)$$

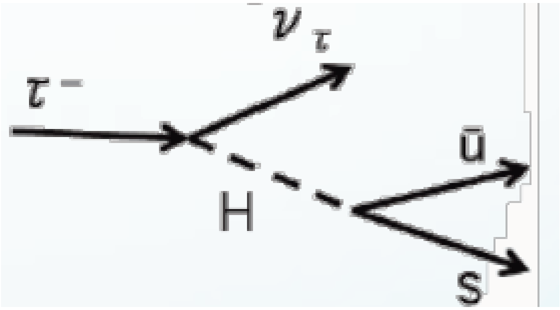
CP violating term
S-P interference

  - $A(Q^2)$, $B(Q^2)$, $C(Q^2)$ kinematic functions
  - form factors:



+

Charged Higgs contribution



Khün & Mirkes'05

$$\tilde{f}_0(s) = f_0(s) + \frac{\eta^2}{m_\tau^2}\, f_H(s)$$

with

$$f_H(s) = \frac{s}{m_u - m_s}\, f_0(s)$$

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- Analysis in the framework of multiple Higgs models

$$\boxed{\begin{aligned}\frac{d\Gamma\left(\tau^- \to K\pi\nu_\tau\right)}{dq^2 d\cos\theta\; d\cos\beta} = &\Big[A(q^2) - B(q^2)\,\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\Big]\left|f_+(s)\right|^2 \\ &+ m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\,\tilde{f}_0^*(s)\right)\end{aligned}}$$

CP violating term
S-P interference

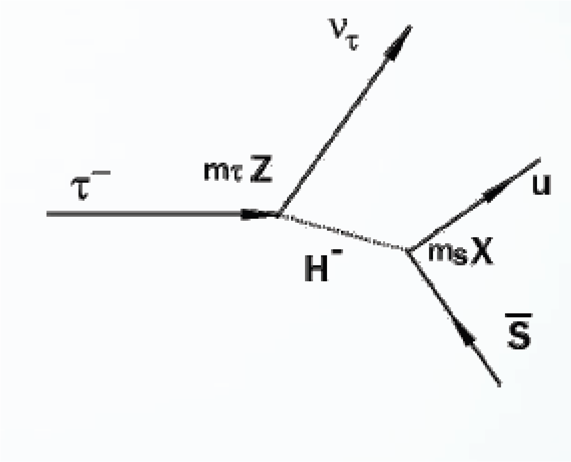
  - $A(Q^2), B(Q^2), C(Q^2)$ kinematic functions
  - Experimental measurement requires precise hadronic parametrization of the form factors $f_+(s), f_0(s)$
    - ➢ Use integrated $\tau \to K\pi\nu_\tau$ invariant mass $\Gamma_{\tau\to K\pi\nu_\tau}$ (*dispersive method*!)
    - ➢ FB asymmetries ⇨ disentangle vector and scalar form factors

$$\boxed{A_{FB} = \frac{d\Gamma\left(\cos\theta\right) - d\Gamma\left(-\cos\theta\right)}{d\Gamma\left(\cos\theta\right) + d\Gamma\left(-\cos\theta\right)}}$$

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- Measured CP violating parameter

$$\boxed{\Delta \equiv \frac{d\Gamma\left(\tau^+ \to K_S^0\pi^+\nu_\tau\right)}{dsd\cos\theta d\cos\beta} - \frac{d\Gamma\left(\tau^- \to K_S^0\pi^-\nu_\tau\right)}{dsd\cos\theta d\cos\beta}}$$



- In NP scenarios with charged Higgs

$$\boxed{\Delta = C^{'}(s)\,\mathrm{Im}(\eta_S)\,\frac{\mathrm{Im}\left(f_+(s)\,f_H^*(s)\right)}{m_\tau}\cos\beta\cos\Psi}$$

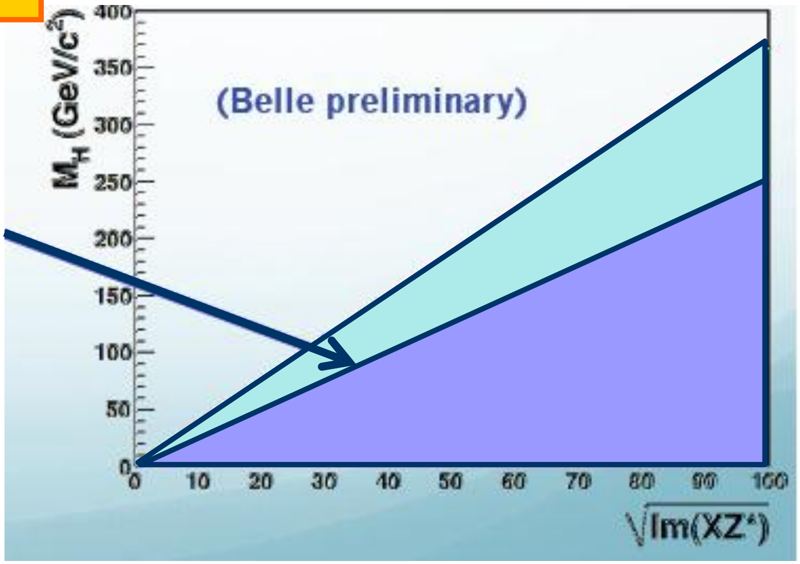
- Measurement:

$$\boxed{\left|\mathrm{Im}\,\eta_S\right| < 0.19} \quad \Rightarrow \quad \boxed{\left|\mathrm{Im}\,\eta_S\right| < 0.026}$$

CLEO'02 *Belle'11*

Constraints on the couplings and M_H

Bishchofberger Tau2010



4. Conclusion and outlook

4.1 Conclusion and Outlook

Hadronic τ-decays very interesting to study

- Very precise determination of α_S
  But error assignment and treatment of the NP part and new data needed

- Test of electroweak couplings very promising
  - New physics in R_τ: analyses in progress but it would be nice to have more data and a precise separation between V and A. Hadronic uncertainties have to be under control

  - Extraction of V_{us} : the τ could give a very precise determination of V_{us} but difference between inclusive/exclusive modes:
    Data normalization, unmeasured modes? New Physics?

4.2 Conclusion and Outlook

Hadronic τ-decays very interesting to study

- Very precise determination of α_S
  But error assignment and treatment of the NP part and new data needed

- Test of electroweak couplings very promising
  - CP violating asymmetry: very interesting measurements to constrain new physics:
    Experimentally: BaBar & Belle agreement?
    Theoretically: Hadronic form factors precisely described
    ⇨ measurement of A_{FB} would help!
    Model of new physics to investigate

⇨ With SuperB and B & Tau-Charm factories very interesting prospects!

- g-2 not covered, sorry! ⇨ SuperB collaboration meeting

5. Back-up

Exclusive decays: $\tau \rightarrow K\nu_\tau$ decays vs. $\tau \rightarrow K\nu_\tau/\tau \rightarrow \pi\nu_\tau$

- $$\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi\hbar}\left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}$$

- $$\frac{\mathcal{B}(\tau^- \to K^-\nu_\tau)}{\mathcal{B}(\tau^- \to \pi^-\nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K^2}{f_\pi^2}\frac{(1 - m_K^2/m_\tau^2)^2}{(1 - m_\pi^2/m_\tau^2)^2}(1 - \delta_{LD})$$

2.3 Measurements

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau\, h^-(\gamma)\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e(\gamma)\right)}}$$

Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

2.3 Measurements

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau\, h^-(\gamma)\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)\right)}}$$

Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

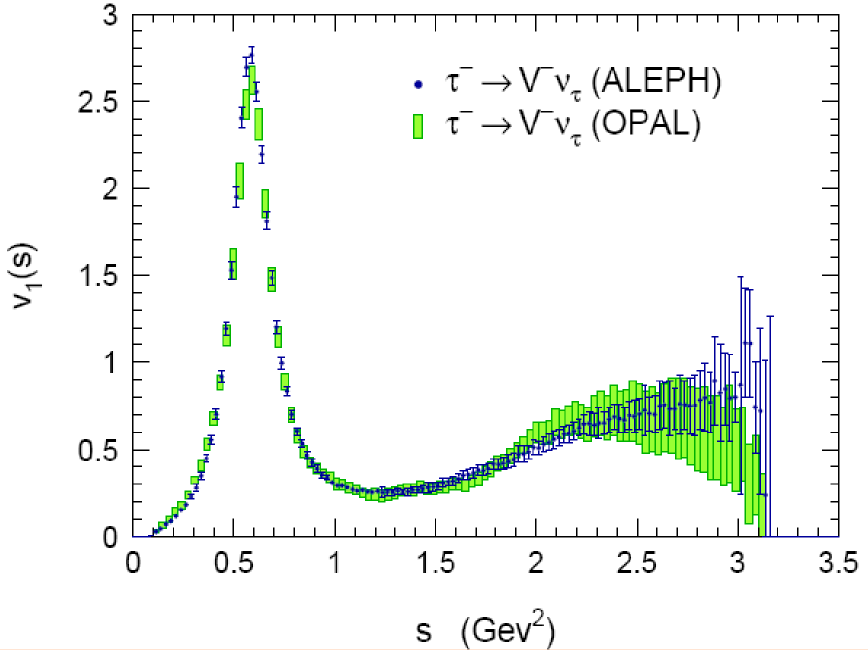
$$R_{\tau,V} \Rightarrow \boxed{\tau^- \to \nu_\tau + h_{V,s=0}}$$

(even number of pions)

$$R_{\tau,A} \Rightarrow \tau^- \to \nu_\tau + h_{A,s=0}$$

(odd number of pions)

$$R_{\tau,S} \Rightarrow \tau^- \to \nu_\tau + h_{V+A,s=1}$$



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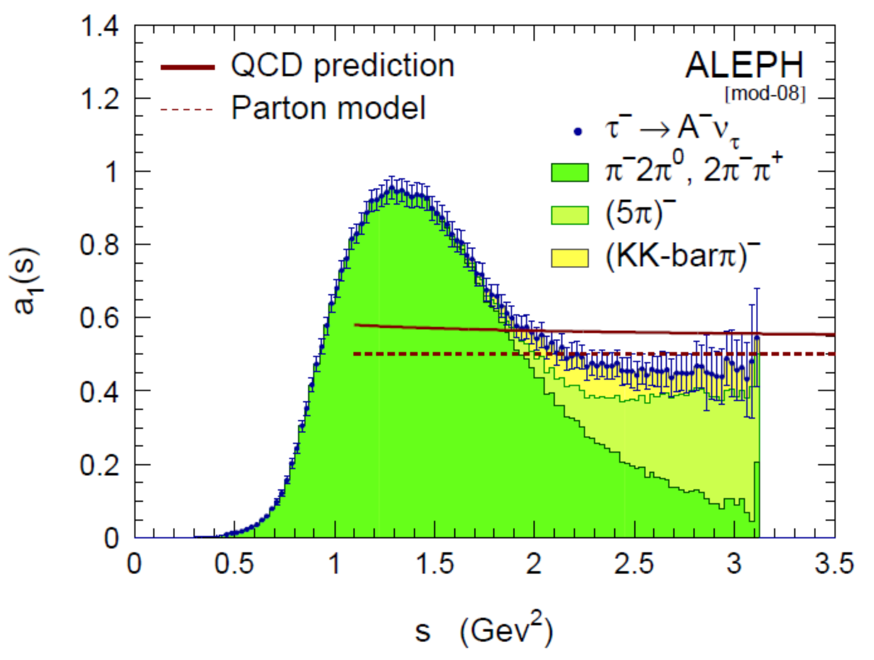
$R_{\tau,V}$ ⇨ $\tau^- \to \nu_\tau + h_{V,s=0}$

(even number of pions)

$R_{\tau,A}$ ⇨ $\boxed{\tau^- \to \nu_\tau + h_{A,s=0}}$

(odd number of pions)

$R_{\tau,S}$ ⇨ $\tau^- \to \nu_\tau + h_{V+A,s=1}$



2.3 Measurements

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau\, h^-(\gamma)\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)\right)}}$$

Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + \color{red}{R_{\tau,S}}$$

$$R_{\tau,V} \Rightarrow \tau^- \to \nu_\tau + h_{V,s=0}$$

(even number of pions)

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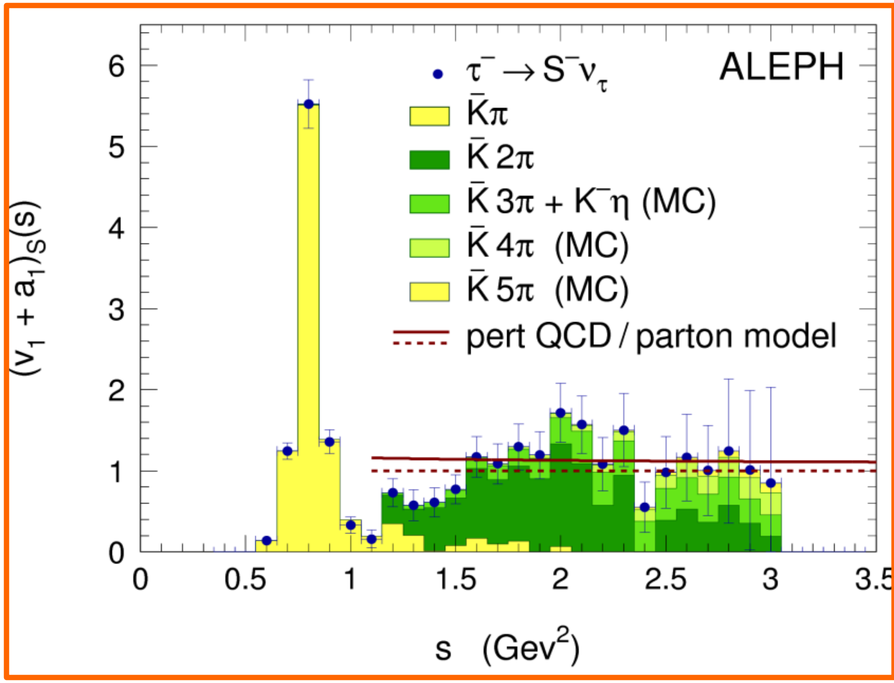
(odd number of pions)

$$\color{red}{R_{\tau,S}} \Rightarrow \boxed{\tau^- \to \nu_\tau + h_{V+A,s=1}}$$



2.3 Measurements

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau\, h^-(\gamma)\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e (\gamma)\right)}}$$

Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

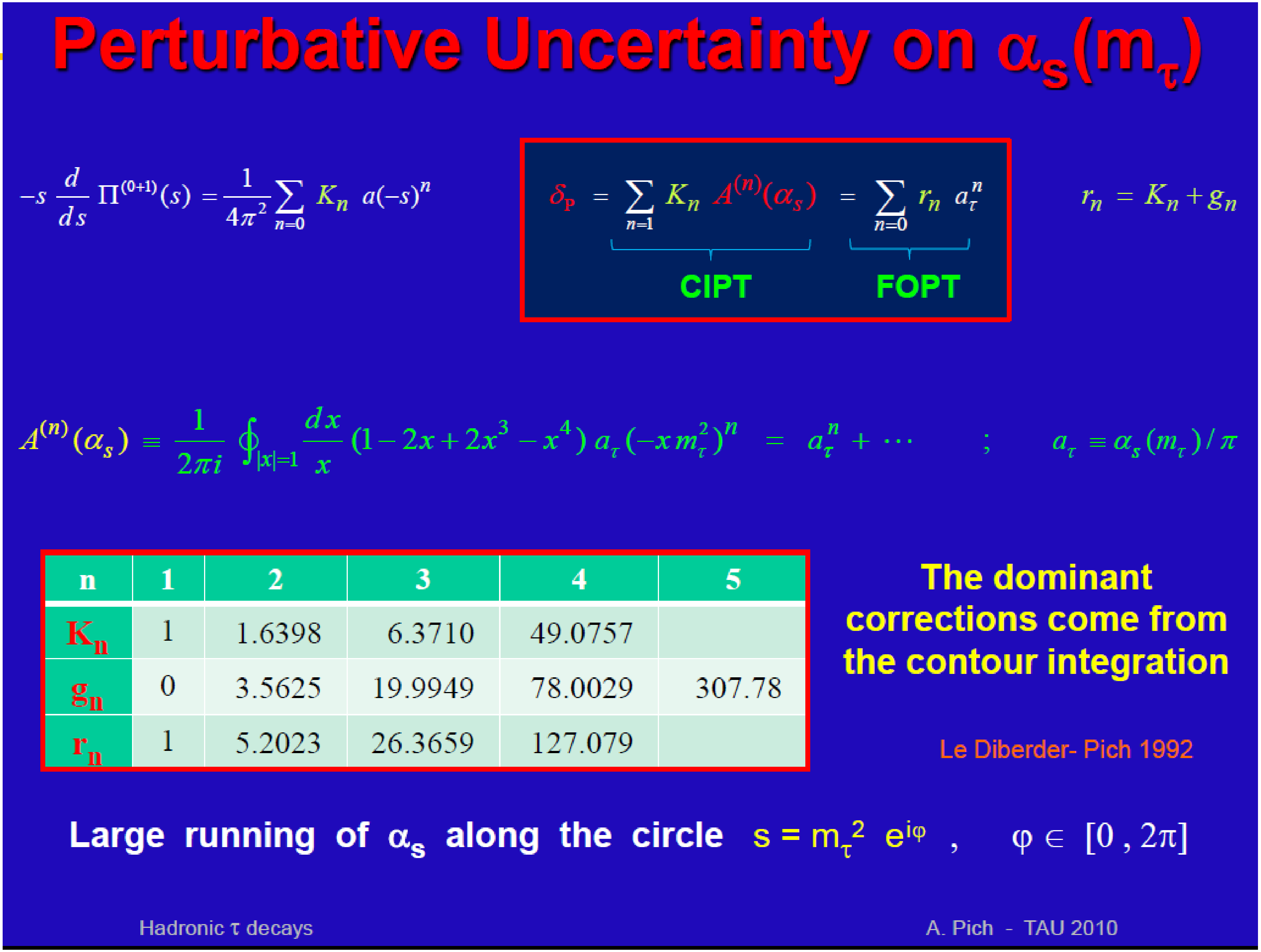
$R_{\tau,V}$ ⇒ $\tau^- \to \nu_\tau + h_{V,s=0}$ (even number of pions)

$R_{\tau,A}$ ⇒ $\tau^- \to \nu_\tau + h_{A,s=0}$ (odd number of pions)

$R_{\tau,S}$ ⇒ $\tau^- \to \nu_\tau + h_{V+A,s=1}$

- Theoretically $$\boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

⇒ Compute the correlators: not known, non perturbative QCD



Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \; a(-s)^n$$

$$\boxed{\delta_P = \underbrace{\sum_{n=1} K_n \; A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n \; a_\tau^n}_{\text{FOPT}}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\;a_\tau(-x m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

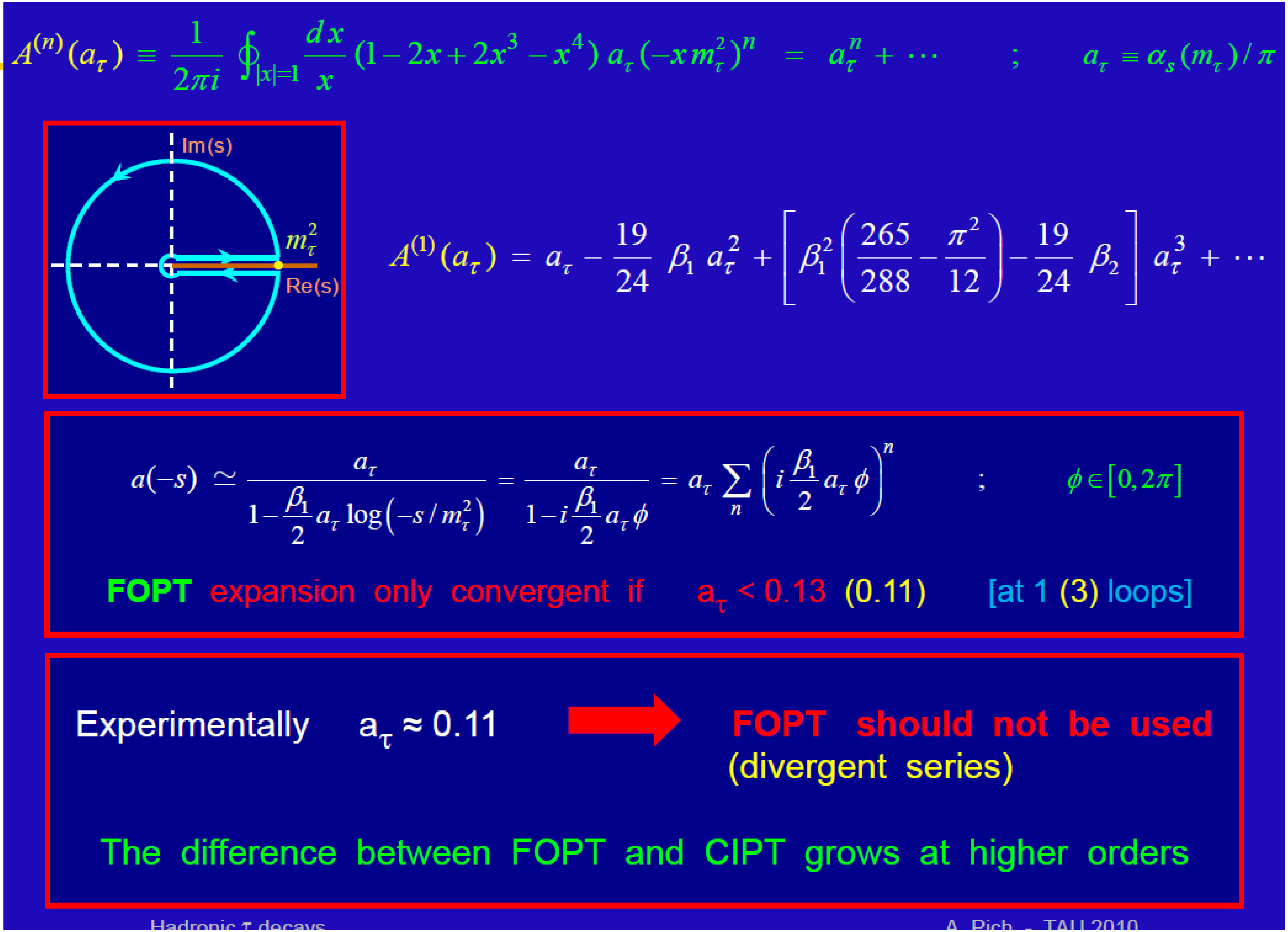
The dominant corrections come from the contour integration

Le Diberder- Pich 1992

Large running of α_s along the circle $s = m_\tau^2\ e^{i\varphi}$, $\varphi \in [0\,, 2\pi]$

Hadronic τ decays

A. Pich - TAU 2010



$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\;a_\tau(-x m_\tau^2)^n \;=\; a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

$$A^{(1)}(a_\tau) = a_\tau - \frac{19}{24}\,\beta_1\,a_\tau^2 + \left[\beta_1^2\left(\frac{265}{288}-\frac{\pi^2}{12}\right)-\frac{19}{24}\,\beta_2\right]a_\tau^3 + \cdots$$

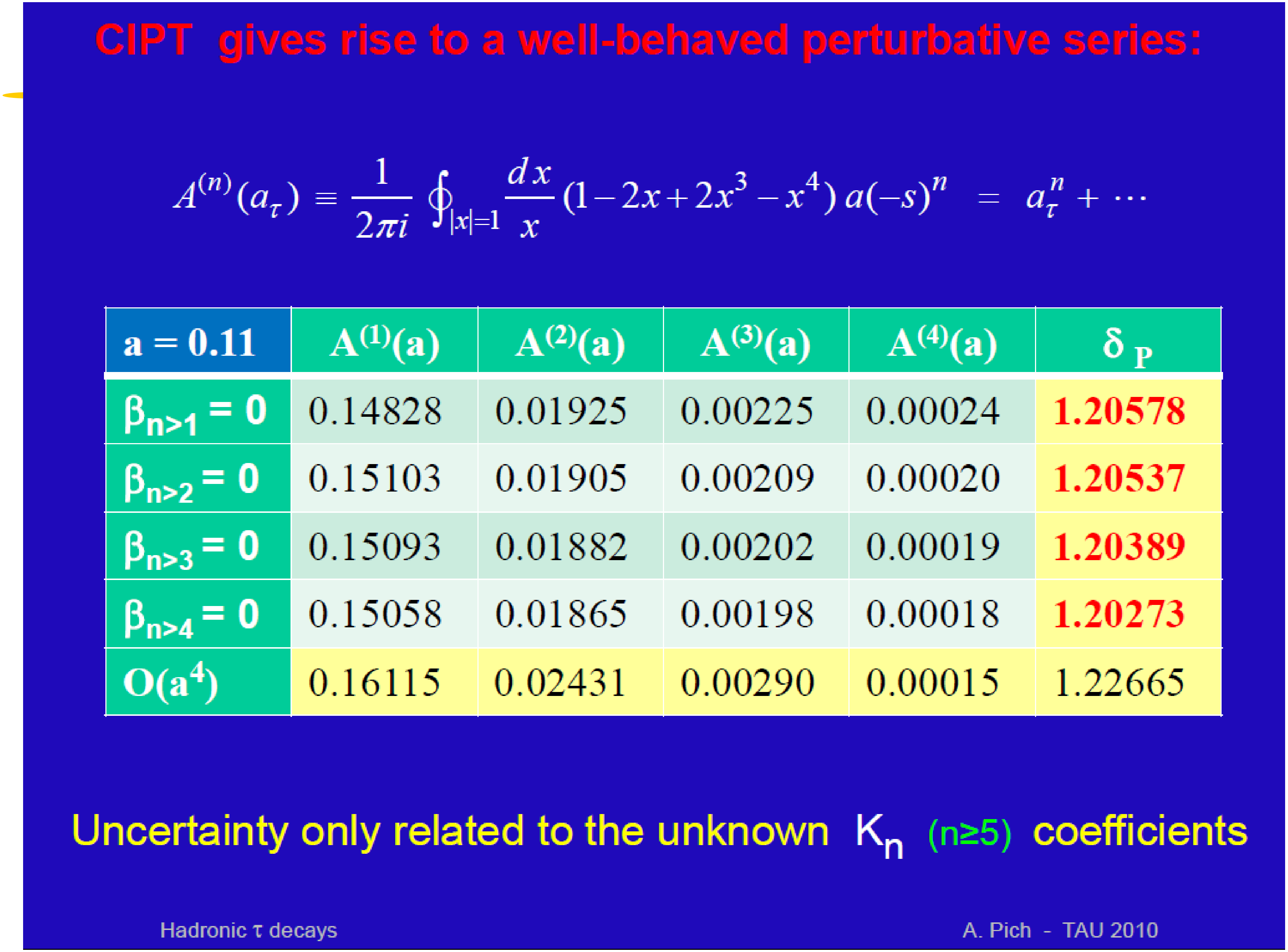
$$a(-s) \simeq \frac{a_\tau}{1-\frac{\beta_1}{2}a_\tau\log\left(-s/m_\tau^2\right)} = \frac{a_\tau}{1-i\frac{\beta_1}{2}a_\tau\phi} = a_\tau\sum_n\left(i\frac{\beta_1}{2}a_\tau\phi\right)^n \qquad ; \qquad \phi\in[0,2\pi]$$

FOPT expansion only convergent if $a_\tau < 0.13$ (0.11) [at 1 (3) loops]

Experimentally $a_\tau \approx 0.11$ → **FOPT should not be used**
(divergent series)

The difference between FOPT and CIPT grows at higher orders

Hadronic τ decays A. Pich - TAU 2010



CIPT gives rise to a well-behaved perturbative series:

$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a(-s)^n \;=\; a_\tau^n + \cdots$$

$a = 0.11$	$A^{(1)}(a)$	$A^{(2)}(a)$	$A^{(3)}(a)$	$A^{(4)}(a)$	δ_P
$\beta_{n>1} = 0$	0.14828	0.01925	0.00225	0.00024	1.20578
$\beta_{n>2} = 0$	0.15103	0.01905	0.00209	0.00020	1.20537
$\beta_{n>3} = 0$	0.15093	0.01882	0.00202	0.00019	1.20389
$\beta_{n>4} = 0$	0.15058	0.01865	0.00198	0.00018	1.20273
$O(a^4)$	0.16115	0.02431	0.00290	0.00015	1.22665

Uncertainty only related to the unknown K_n $(n \geq 5)$ coefficients

Hadronic τ decays

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Modelling a better behaved FOPT

(Beneke – Jamin)

- **Large higher-order K_n corrections could cancel the g_n ones**

  Happens in the "large-β_0" approximation (UV renormalon chain)

- D = 4 corrections very suppressed in R_τ

  → **n = 2 IR renormalons can do the job** **($K_n \approx - g_n$)**

- No sign of renormalon behaviour in known coefficients

  → **n = -1,2,3 renormalons + linear polynomial**

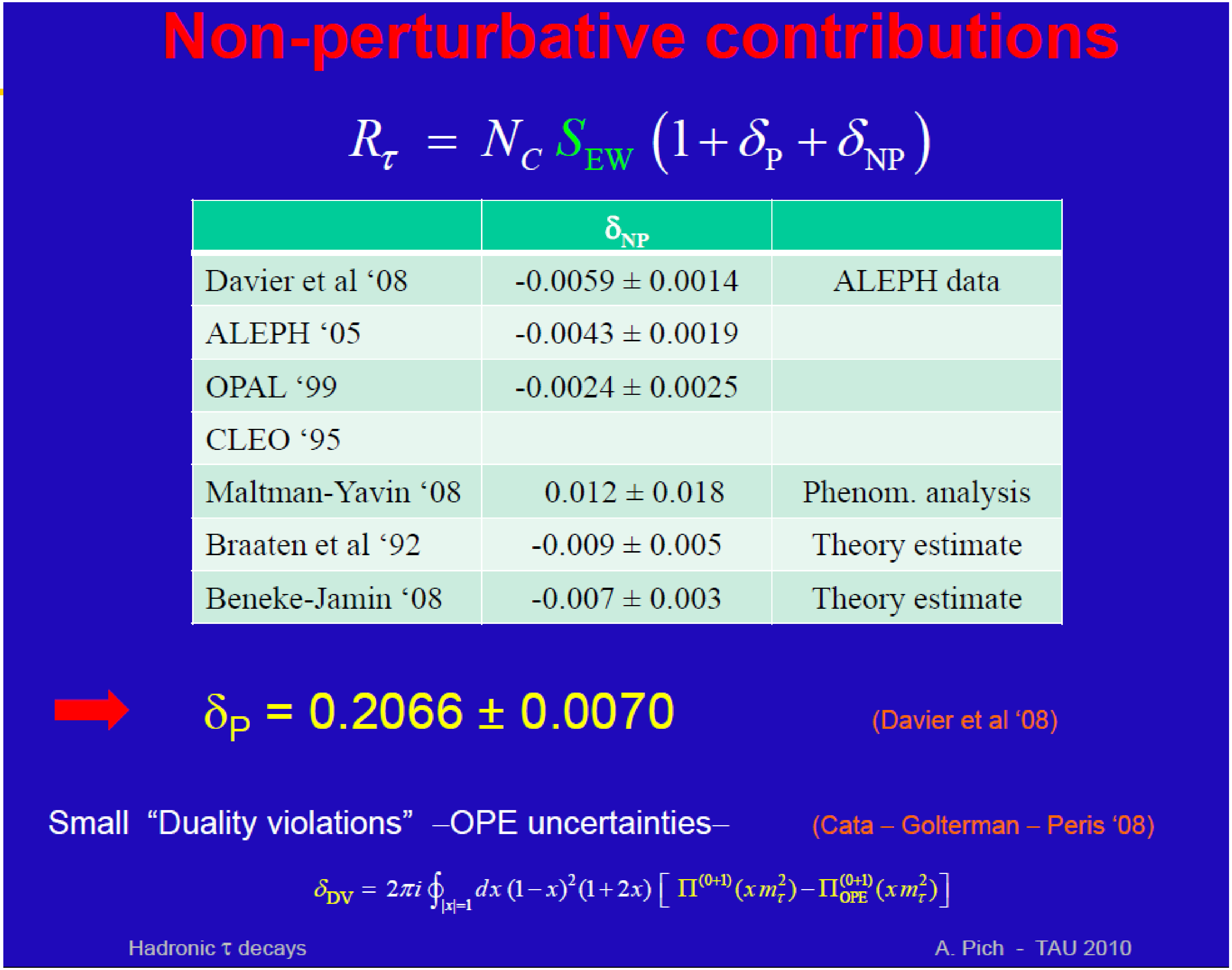
  5 unknown constants fitted to K_n $(2 \le n \le 5)$. $K_5 = 283$ assumed

- **Borel summation:** large renormalon contributions. Smaller α_s

  Same result with Modified (conformal mapping) CIPT (Fischer – Caprini)

Nice model of higher orders. But too many different possibilities …

(Descotes-Genon – Malaescu)



Non-perturbative contributions

$$R_\tau = N_C \, S_{EW} \left(1 + \delta_P + \delta_{NP}\right)$$

	δ_{NP}	
Davier et al ‘08	-0.0059 ± 0.0014	ALEPH data
ALEPH ‘05	-0.0043 ± 0.0019	
OPAL ‘99	-0.0024 ± 0.0025	
CLEO ‘95		
Maltman-Yavin ‘08	0.012 ± 0.018	Phenom. analysis
Braaten et al ‘92	-0.009 ± 0.005	Theory estimate
Beneke-Jamin ‘08	-0.007 ± 0.003	Theory estimate

➡ $\delta_P = 0.2066 \pm 0.0070$ (Davier et al ‘08)

Small “Duality violations” –OPE uncertainties– (Cata – Golterman – Peris ‘08)

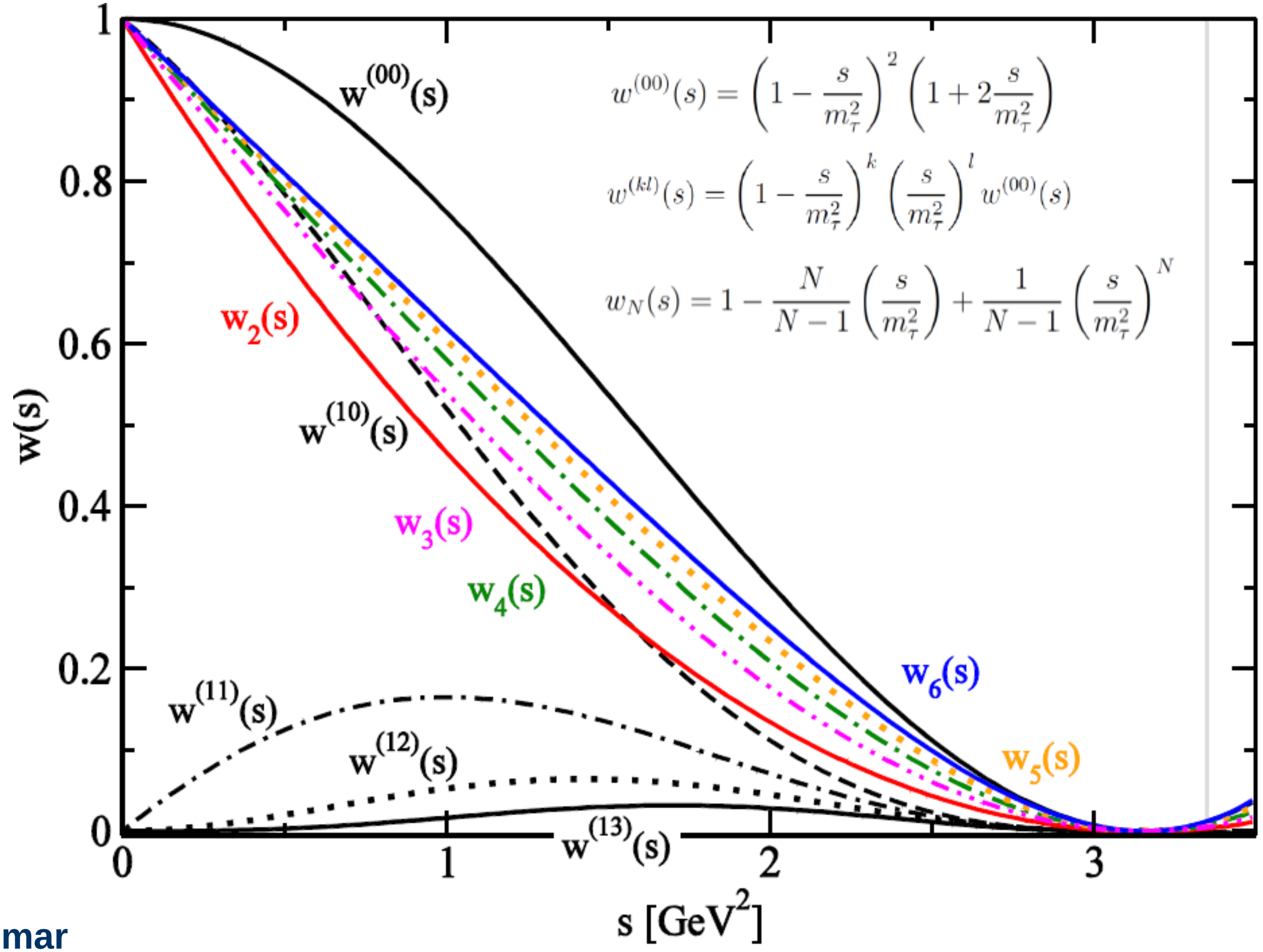
$$\delta_{DV} = 2\pi i \oint_{|x|=1} dx\,(1-x)^2(1+2x)\left[\,\Pi^{(0+1)}(x m_\tau^2) - \Pi^{(0+1)}_{OPE}(x m_\tau^2)\right]$$

Hadronic τ decays

A. Pich - TAU 2010

2.6 Extraction of α_s

- Data from ALEPH and OPAL
- Analysis from *Davier et al'08* ALEPH data + Improved BRs from BaBar and Belle and moments (k,l)= (0,0), (1,0), (1,1), (1,2), (1,3)



3.3 $\tau \to K\pi\nu_\tau$ CP asymmetries

- CP violating asymmetry

$$\boxed{A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}}$$

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle$$

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle$$

$= |p|^2 - |q|^2$ $\boxed{\approx (0.33 \pm 0.01)\,\%}$ in the Standard Model *Bigi & Sanda'05*

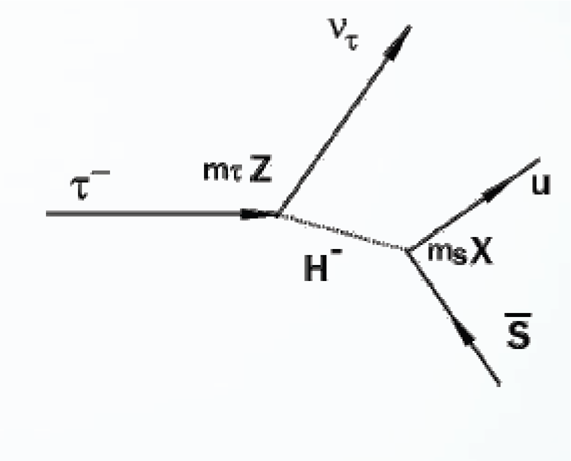
- Experimental measurement requires precise hadronic parametrization of the form factors $\bar{f}_+(s)$, $\bar{f}_0(s)$
  - Use integrated $\tau \to K\pi\nu_\tau$ invariant mass $\Gamma_{\tau\to K\pi\nu_\tau}$ (*dispersive method*!)
  - FB asymmetries ⇨ disentangle vector and scalar form factors

$$\boxed{A_{\mathrm{FB}} = \frac{d\Gamma\left(\cos\theta\right) - d\Gamma\left(-\cos\theta\right)}{d\Gamma\left(\cos\theta\right) + d\Gamma\left(-\cos\theta\right)}}$$

3.2 $\tau \to K\pi\nu_\tau$ CP asymmetries

- Constraint on models with 3 charged Higgs

$$\eta_S \simeq \frac{m_\tau m_s}{M^2_{H^\pm}} X^* Z$$



$$\boxed{\left|\mathrm{Im}\,\eta_s\right| < 0.026} \Rightarrow |\Im(XZ^*)| < 0.15 \frac{M^2_{H^\pm}}{1\,\mathrm{GeV}^2/c^4}$$