Fitting the distribution of angular separation of He tracks in absence of correlations

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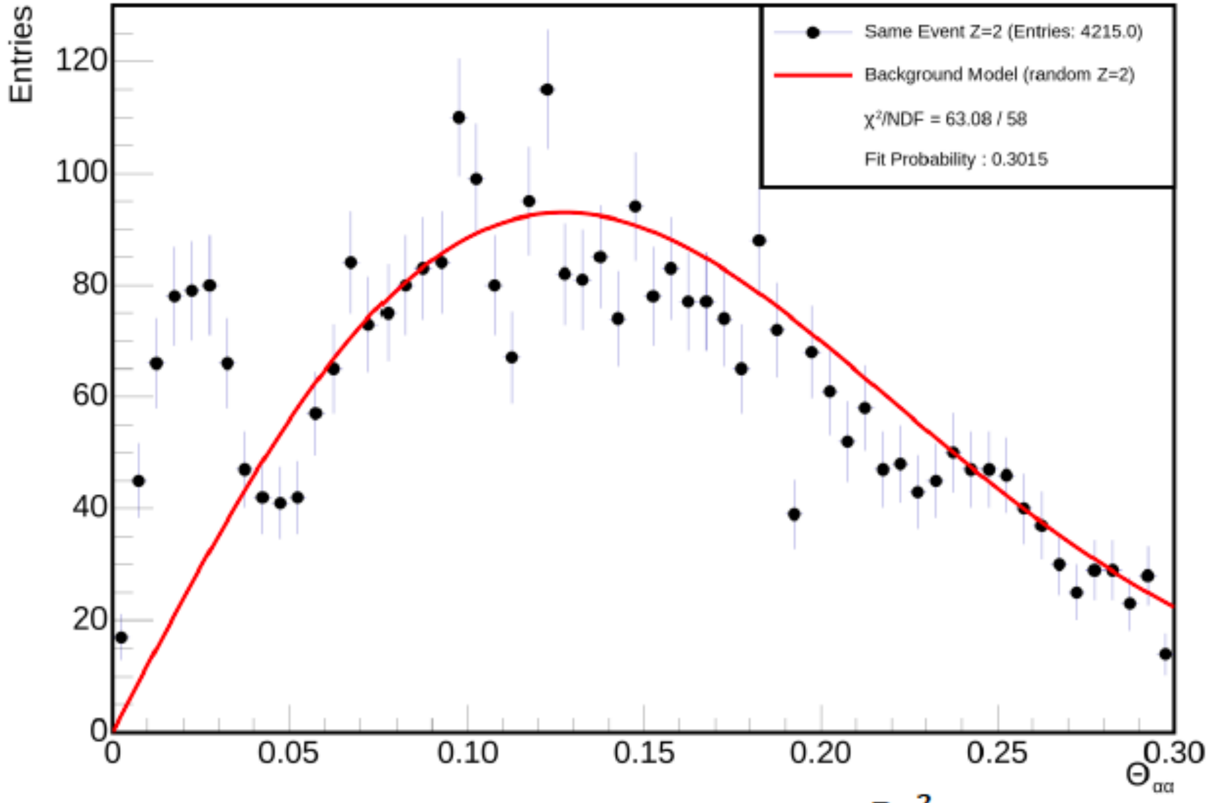
Introduction

At the last Physics Meeting, V. Boccia attempted the fit of background in the distribution of angular separation of Z=2 tracks using a function ~ x exp(-bx²), which was determined empirically as a reasonable behaviour.

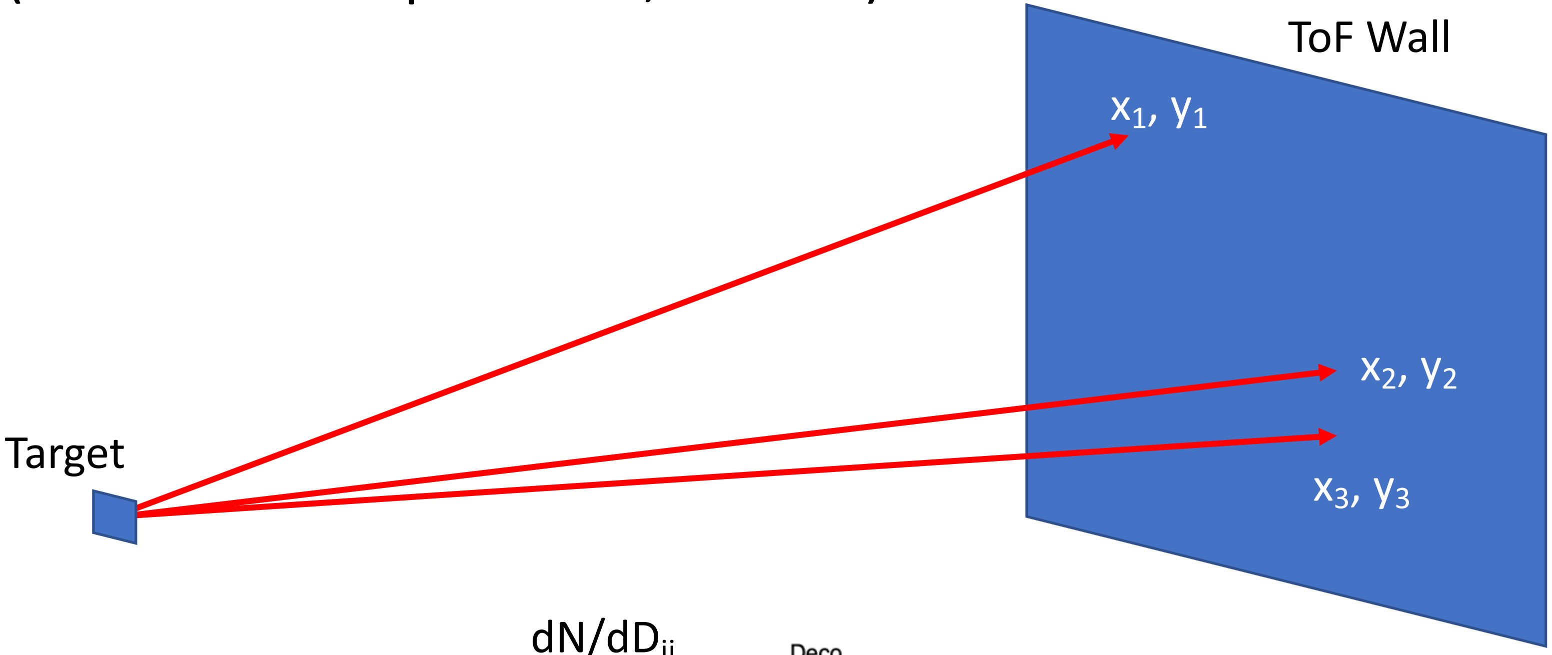


$$Fit\ Function: f(x) = Axe^{-Bx^2}$$

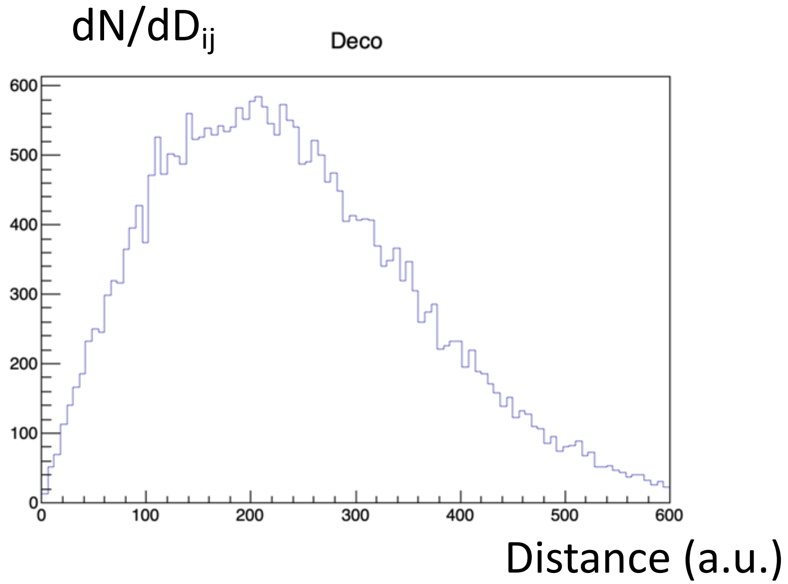
It was requested to give a motivation of the choice for this function.

Here we shall give a demonstration why this choice is the best one (in case of absence of correlations)

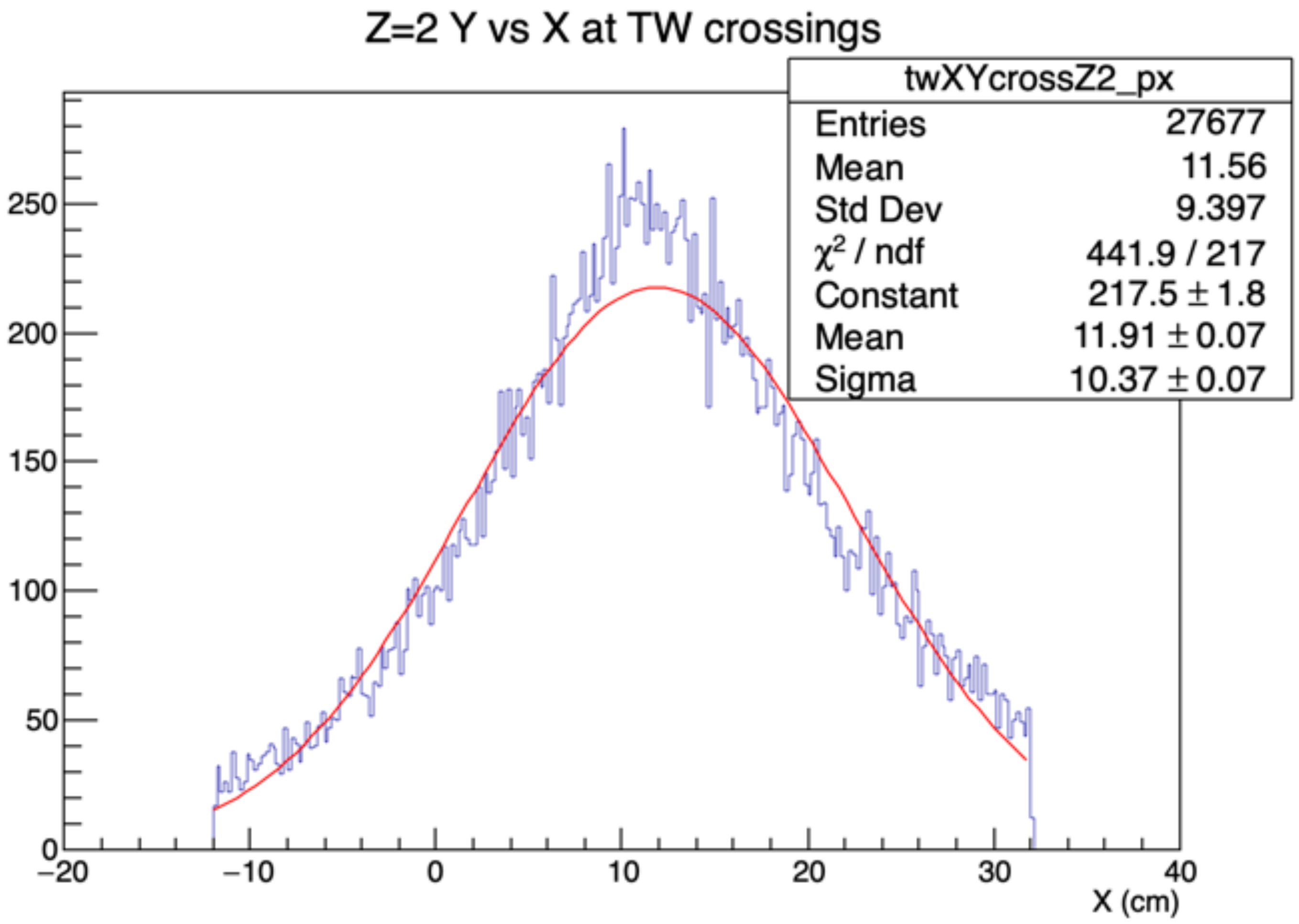
Let us first consider the spatial separation of points in a plane (in the FOOT experiment, the TW)



$$Distance_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$



Helium tracks at the TW have a lateral spread which can be roughly approximated by a gaussian



Under the gaussian hypothesis, *in absence of correlations*, x_1, y_1, x_2, y_2 are all independently distributed by the same gaussian with rms σ:

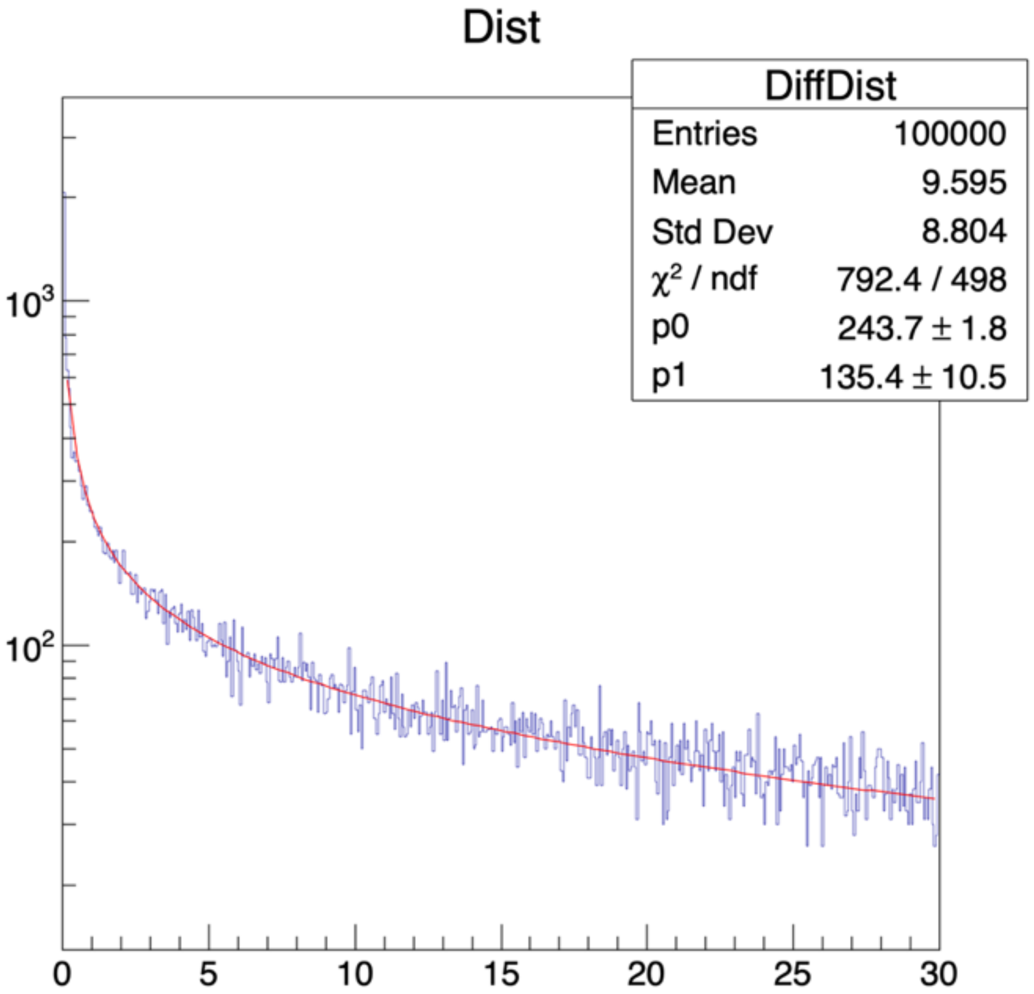
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

Both $(x_1 - x_2)$ and $(y_1 - y_2)$ are again normally distributed with $\sigma' = \sqrt{2}\sigma$.

As a first step we shall first look for the probability distribution of Δx^2 and Δy^2, i.e. the distribution of the square of a gaussian random number.

It can be shown that the square of a gaussian random number is distributed according to the following distribution:

$$p(y) = \frac{1}{\sqrt{2\pi}\sigma'\sqrt{y}} e^{-\frac{y}{2\sigma'^2}}$$

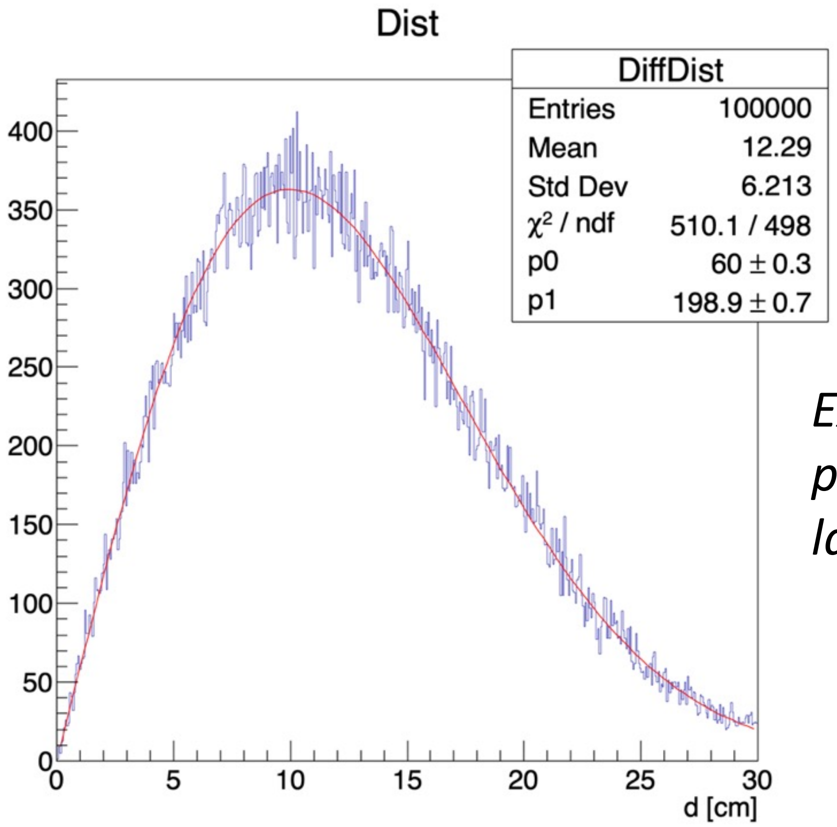


Example of the distribution of the square of a gaussian random coordinate with σ=10 cm

Now we can find the distribution for the variable $z = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{u+v}$

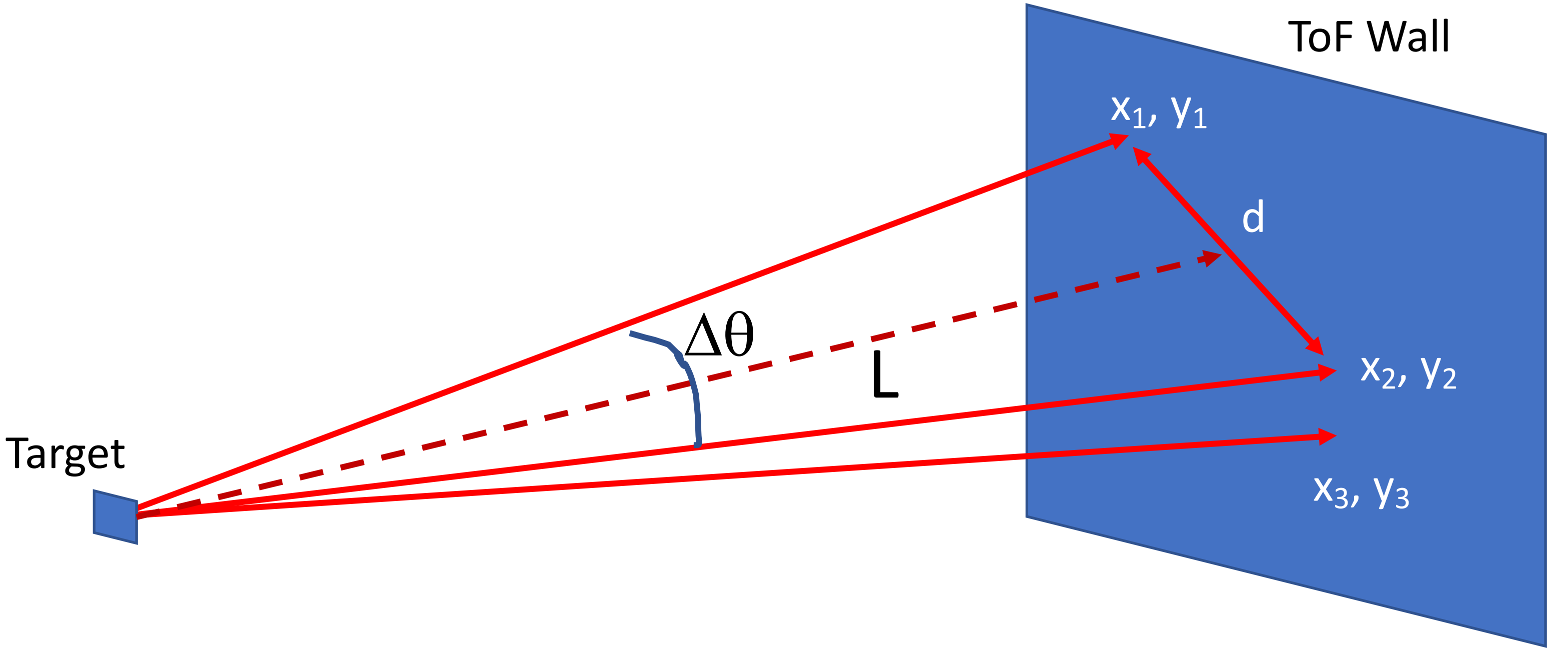
where u and v are square of gaussian random numbers. It can be shown, *("after a few simple passages left to the reader")* that the resulting p.d.f. for z is:

$$p(z) = \frac{z}{\sigma'^2} e^{-\frac{z^2}{2\sigma'^2}}$$



Example of the distribution of the distance between 2 points in a plane (unlimited) sampled from a gaussian lateral distribution with σ=10 cm

We can now move to the angular separation $\Delta\theta$.
If the separation between 2 points is d and the TW is at a distance L>>d from the event vertex (target) then $Tan(\Delta\theta) \sim \frac{d}{L}$ or $\Delta\theta \sim ArcTan(\frac{d}{L})$



By applying a change of variable for the p.d.f.:

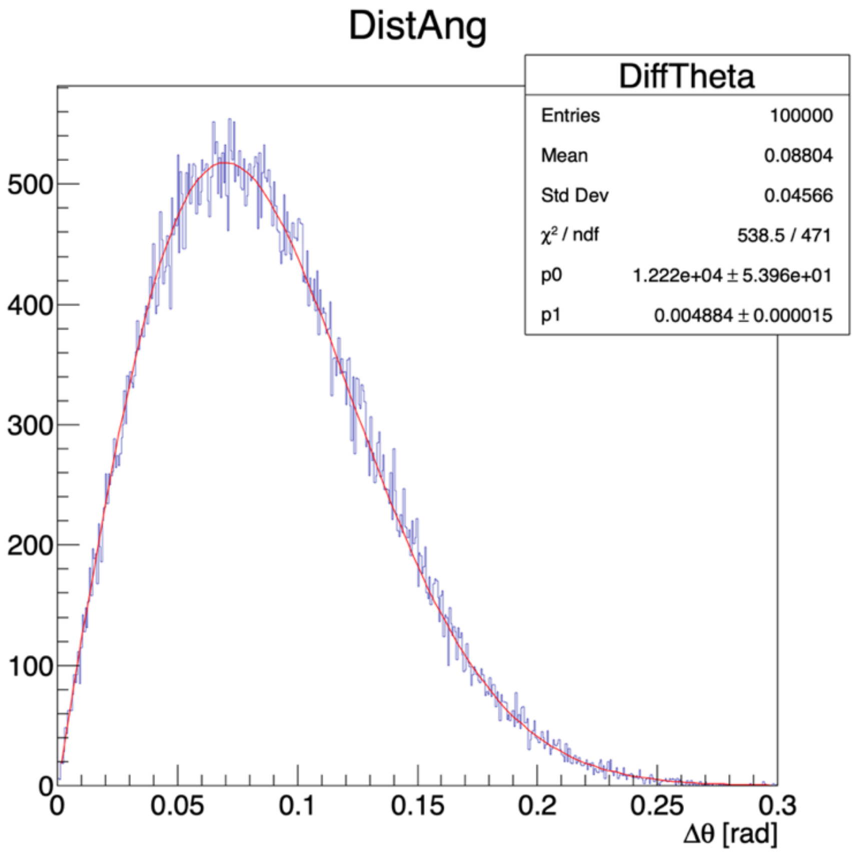
$$p[Tan(\Delta\theta)] = \frac{Tan(\Delta\theta)}{\sigma_{\Delta\theta}^2[1 + \mathrm{Tan}(\Delta\theta)^2]} e^{-\frac{Tan(\Delta\theta)^2}{2\sigma_{\Delta\theta}^2}} \qquad \text{where } \sigma_{\Delta\theta} = \frac{\sigma'}{L}$$

For $\Delta\theta$ small, we can introduce the further approximation $Tan(\Delta\theta) \sim \theta$:

$$p(\Delta\theta) \sim \frac{\Delta\theta}{\sigma^2_{\Delta\theta}(1+\Delta\theta^2)} e^{-\frac{\Delta\theta^2}{2\sigma^2_{\Delta\theta}}}$$

At first order, the term $1+\Delta\theta^2$ in the denominator can be neglected, so the function can be further simplified to:

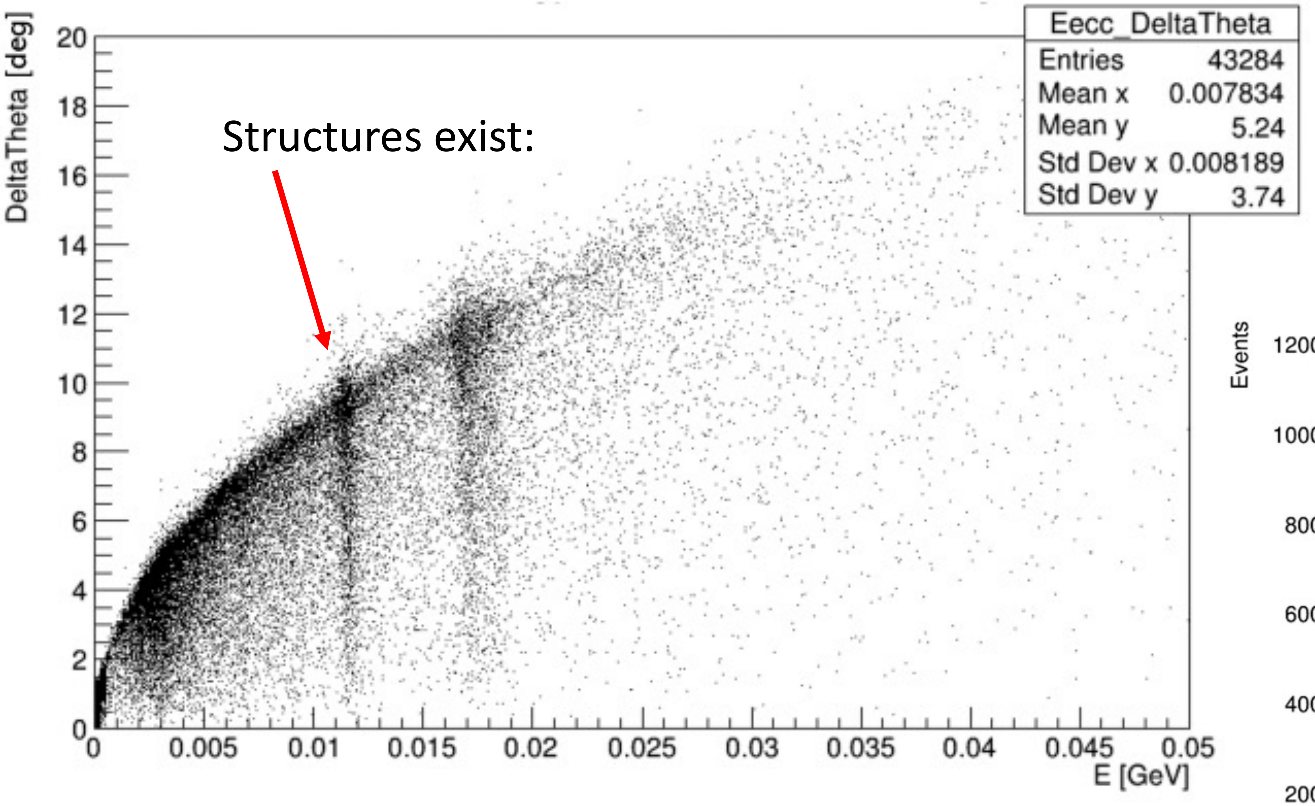
$$p(\Delta\theta) \sim \frac{\Delta\theta}{\sigma^2_{\Delta\theta}} e^{-\frac{\Delta\theta^2}{2\sigma^2_{\Delta\theta}}}$$



Example of the distribution of the angular distance between 2 points in the TW plane (x,y sampled from a gaussian distribution with σ=10 cm) L=200 cm.

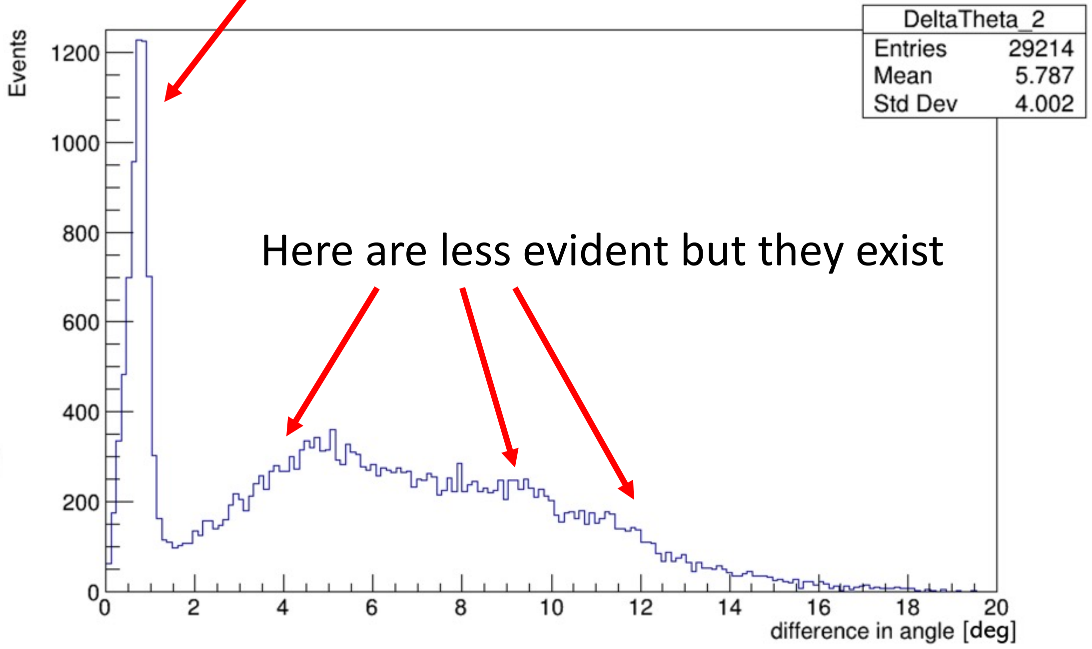
Warning: deviations from this behaviour <u>must</u> appear when correlations exist:

Structures exist:



Obvious

Here are less evident but they exist



Opening angular distribution between α particles vs. excitation energy spectrum of the breakup of ^{8}Be intermediate stage of ^{12}C into 2 α particles

(A. Caglioni's talk at Trento General Meeting)

Conclusions

- The choice of $f(x) = x \exp(-bx^2)$, which was initally chosen on an empirical basis, is actually the correct function to be used to fit angular- or space-separation, if correlation are not present, when the lateral distribution of He tracks can be approximated by a gaussian. This seems the case at FOOT energies.
- Actually, correlations even beyond the peak at very small angles, due to ^{8}Be ground state, should exist, manifesting themselves as less visibile structures at higher angular separation.
- In the case of emulsion data, statistical fluctuations are probably masking these correlation structures
- Data from electronic setup should put more in evidence such structures

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