

**Pairing Correlations and Particle Vibration Coupling**  
*(Including Giant Pairing Vibrations!!)*

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# Pairing Vibrations around Closed Shell Nuclei

1.D.2

*Nuclear Physics* 80 (1966) 289–313; © North-Holland Publishing Co., Amsterdam

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## PAIRING VIBRATIONS

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Received 30 September 1965

**Abstract:** We study the properties of the collective states (pairing vibrations) which are associated with fields changing the numbers of particles. In particular, we discuss which processes may be enhanced by the coherence in the pairing-vibration state.

$$H = \sum_v \varepsilon_v^{(0)} a_v^\dagger a_v - G \sum_{\mu, \nu > 0} a_\mu^\dagger a_\mu^\dagger a_\nu a_\nu$$

$$|\alpha\rangle = A_\alpha^\dagger |0\rangle = \sum_{\nu > 0} X_\nu(\alpha) a_\nu^\dagger a_\nu^\dagger |0\rangle$$

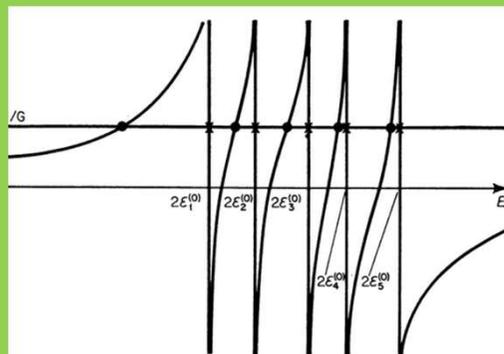
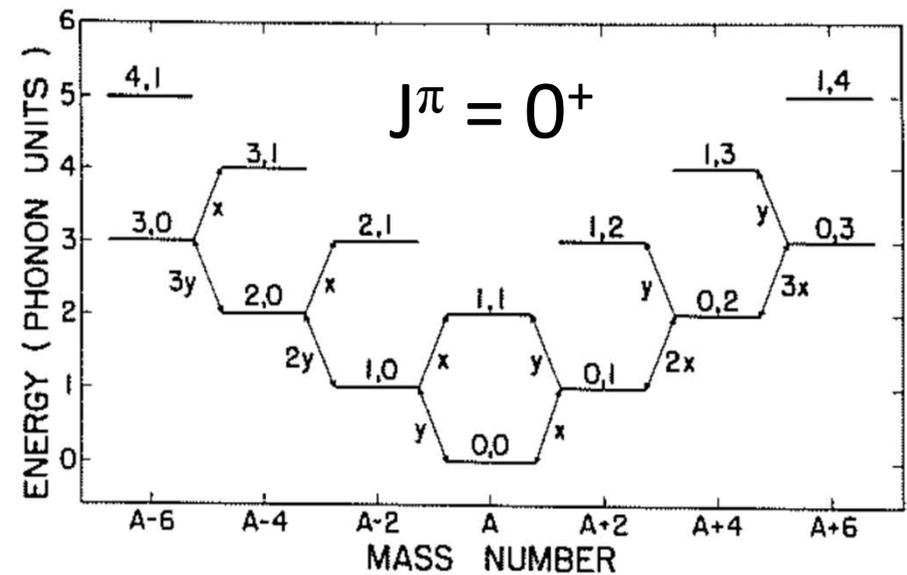
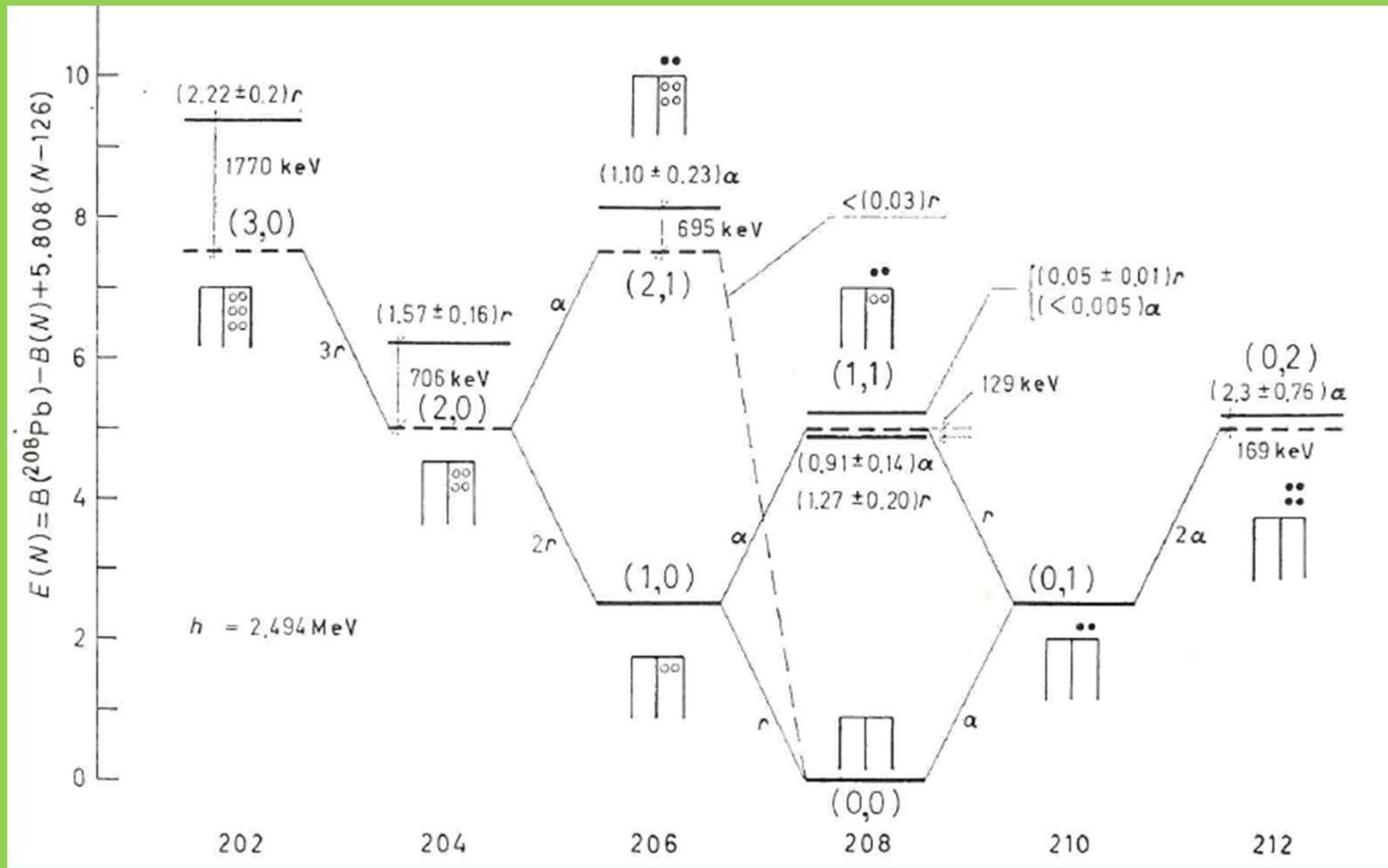


Figure 11.1. Graphical solution of the eigenvalue equation (11.31). All of



# The Pairing Vibration seminal paper



## Basic Tool: The pp-RPA equations

$$|A+2, \tau\rangle = \left( \sum_{m < n} X_{mn}^\tau a_m^+ a_n^+ - \sum_{i < j} Y_{ij}^\tau a_j^+ a_i^+ \right) |A, 0\rangle$$

Pairing Interaction  $\rightarrow$  Coherent mix  
 $\rightarrow$  Collectivity  $\rightarrow$  Cooper pair like

$$\begin{pmatrix} A & B \\ B^+ & C \end{pmatrix} \begin{pmatrix} R_p^{\tau, \lambda} \\ R_h^{\tau, \lambda} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} R_p^{\tau, \lambda} \\ R_h^{\tau, \lambda} \end{pmatrix} \cdot \hbar \Omega_{\tau, \lambda},$$

$$A_{mm'm'n'} = \delta_{mm'} \delta_{nn'} (\epsilon_m + \epsilon_n) + \bar{v}_{mnm'n'},$$

$$C_{ijj'f'} = -\delta_{ii'} \delta_{jj'} (\epsilon_i + \epsilon_j) + \bar{v}_{ijj'f'},$$

$$B_{mnij} = -\bar{v}_{mnij},$$

$$(R_p^\tau)_{mn} = X_{mn}^\tau, \quad (R_p^\lambda)_{mn} = Y_{mn}^\lambda,$$

$$(R_h^\tau)_{ij} = Y_{ij}^\tau, \quad (R_h^\lambda)_{ij} = X_{ij}^\lambda.$$

From The Nuclear Many Body Problem by Ring and Schuck

# Giant Pairing Vibrations (GPV)

Volume 69B, number 2

PHYSICS LETTERS

1 August 1977

## HIGH-LYING PAIRING RESONANCES\*

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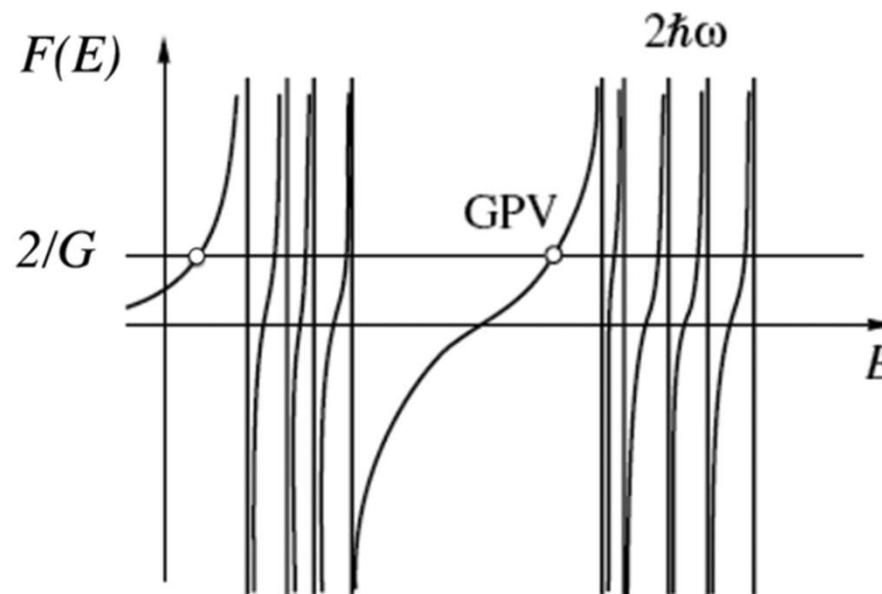
and

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Received 1 April 1977

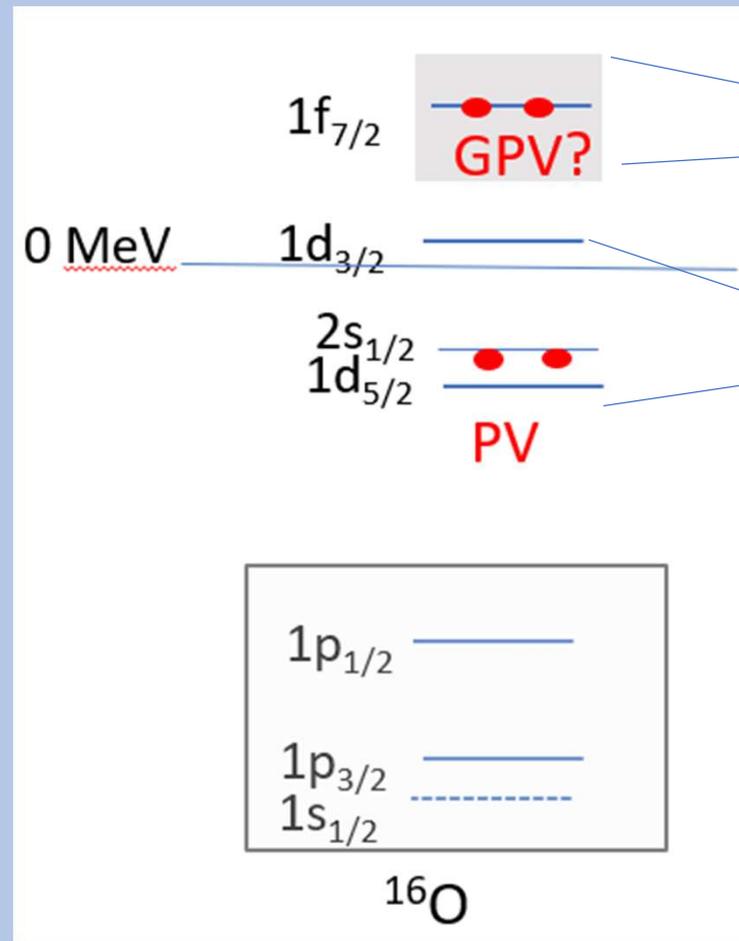
Pairing vibrations based on the excitation of pairs of particles and holes across major shells are predicted at an excitation energy of about  $70/A^{1/3}$  MeV and carrying a cross section which is 20%–100% the ground state cross section.



$$\hbar\omega_{GPV} \sim 2\hbar\omega_0 - \Omega G \sim \frac{65 \text{ MeV}}{A^{1/3}}$$

# The Pairing Vibrations; schematically

$A \rightarrow A+2$ ; for example  $^{16}\text{O} \rightarrow ^{18}\text{O}$

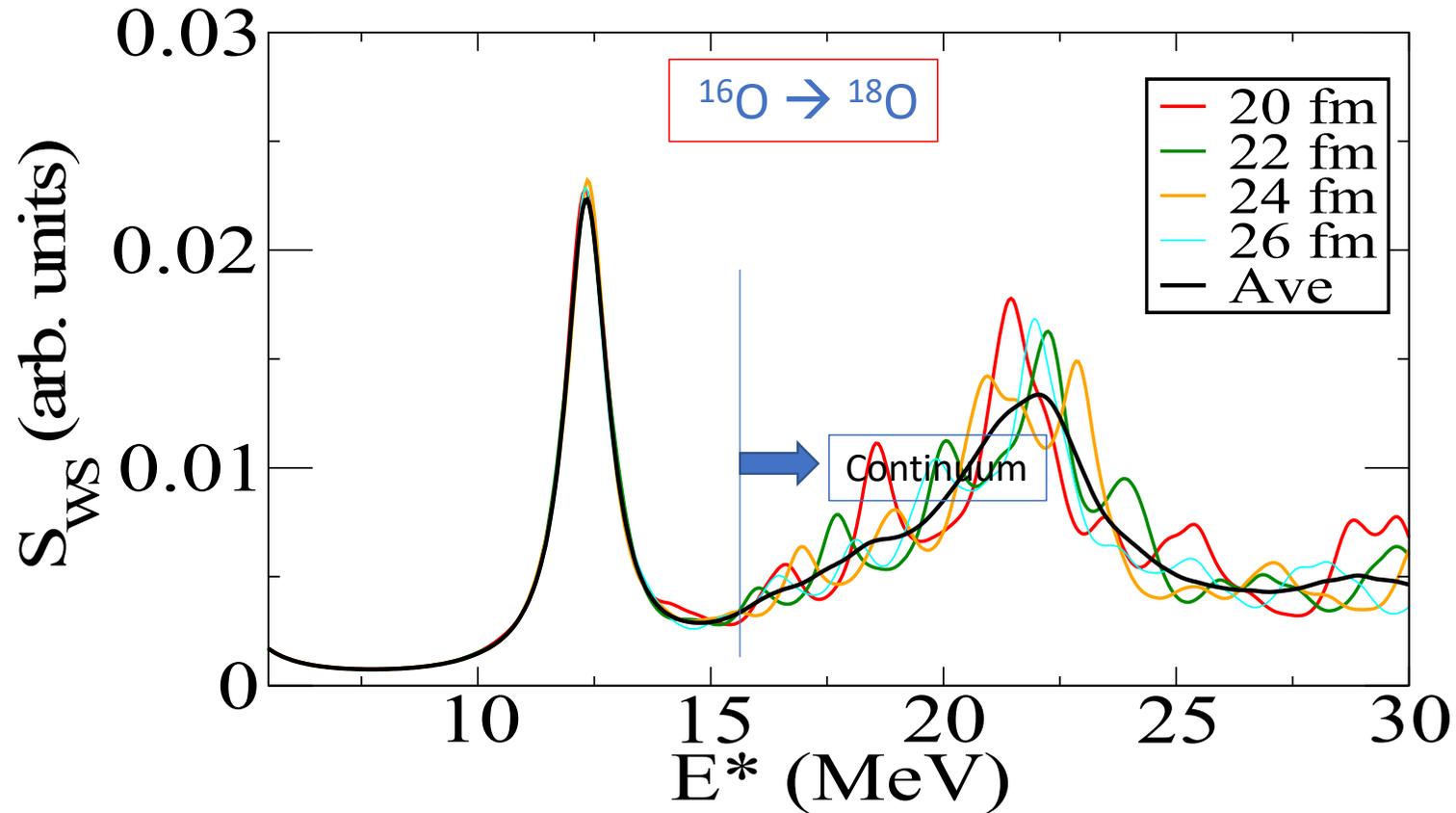


Pairing Interaction  $\rightarrow$  Coherent mix  
 $\rightarrow$  Collectivity  $\rightarrow$  Cooper pair like

Pairing Interaction  $\rightarrow$  Coherent mix  
 $\rightarrow$  Collectivity  $\rightarrow$  Cooper pair like

Pair Addition mode  
 produced in two neutron  
 transfer reactions:  
 $A(t,p)A+2$  for example

ppRPA using BOX boundary conditions



pp-RPA with the Gogny(pairing) force.

Averaging details:  
 1.Box sampling  
 2.Small Gaussian convolution.

$$S_{WS}^i = \sum_{nn'lj} [X_{nn'lj}^i + Y_{nn'lj}^i] \int dr G(r) \psi_{nlj}(r) \psi_{n'lj}(r).$$

$$G(r) \equiv (1 + \exp[(r - R_S)/a_S])$$

# The Pairing Vibrations; pure-RPA

$A \rightarrow A+2$ ; for example  $^{16}\text{O} \rightarrow ^{18}\text{O}$

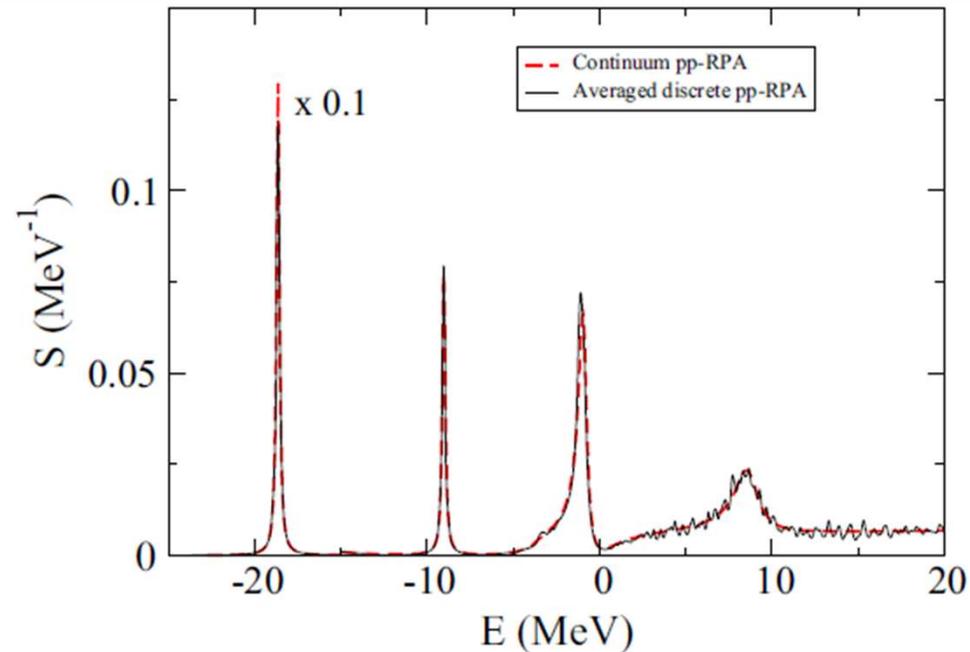
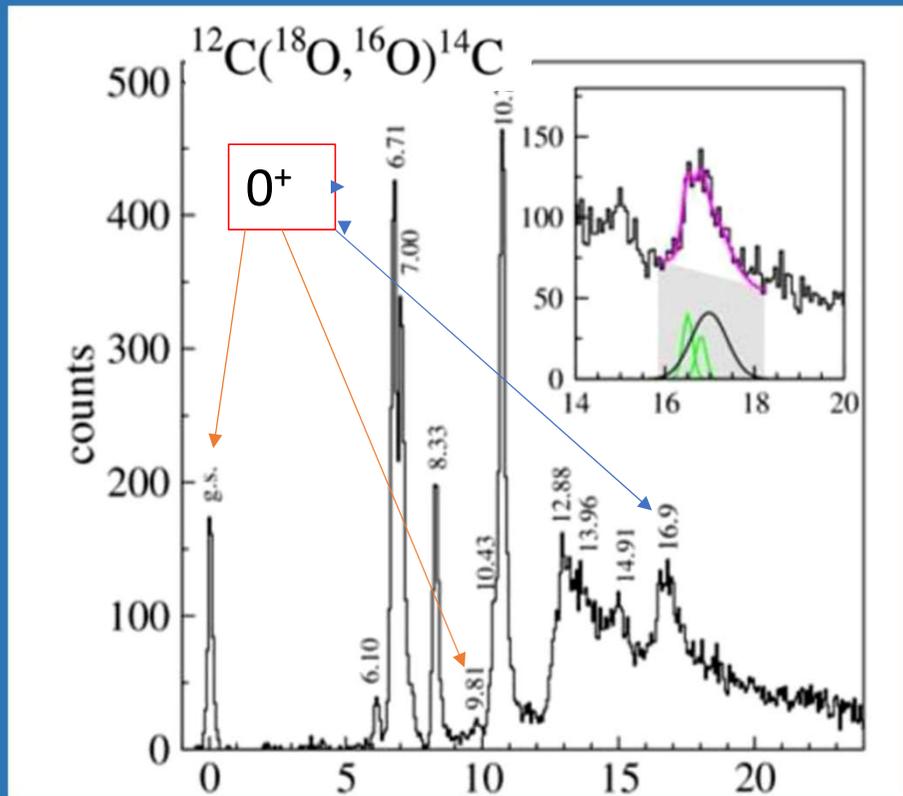
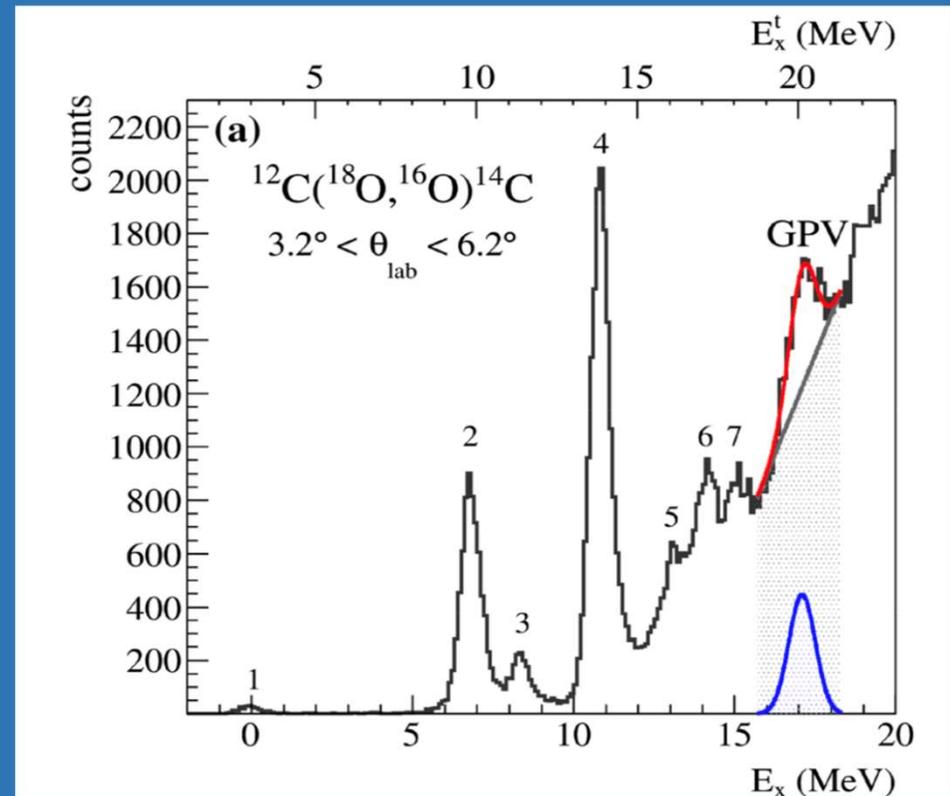


FIG. 1. The strength function calculated for  $^{18}\text{O}$  with the continuum pp-RPA (M. Matsuo, private communication) is compared to our results obtained by averaging over several boxes and using a Lorentzian with  $\text{FWHM} = 0.2 \text{ MeV}$ . In this

Several unsuccessful experimental searches have been carried out over the years , but recently a bump has been detected at  $E^* \approx 16$  MeV in the reaction  $^{12}\text{C}(^{18}\text{O},^{16}\text{O})^{14}\text{C}$  at  $E_{\text{lab}} = 84$  and 275 MeV and interpreted as a signature of GPV

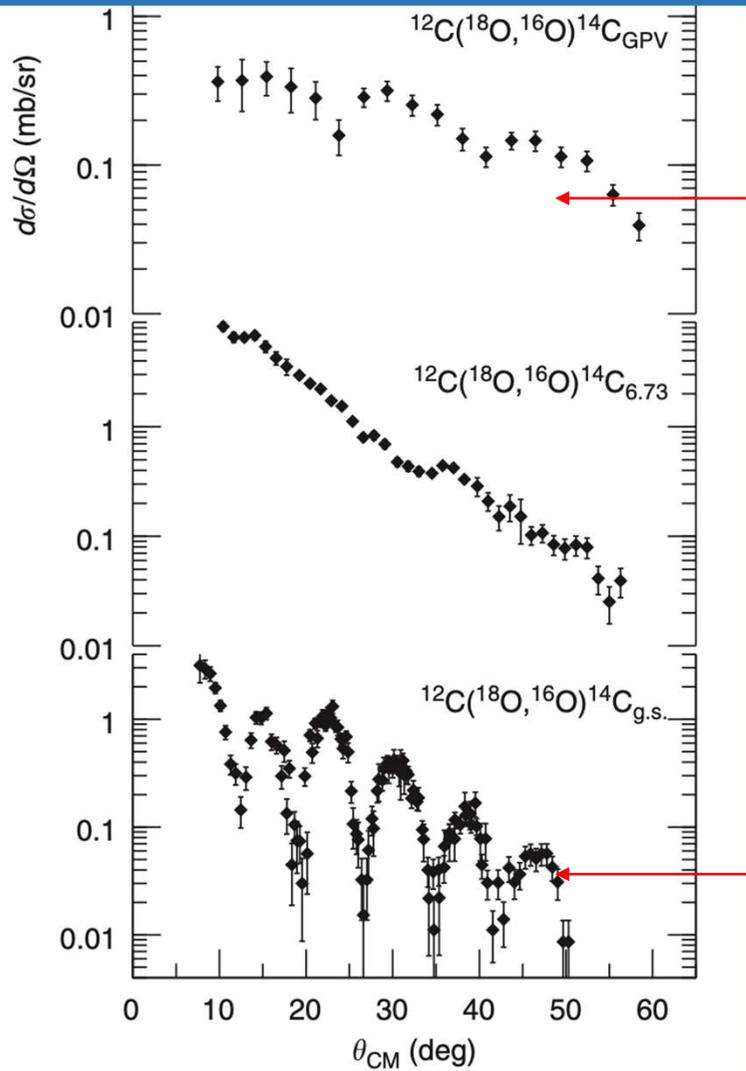


F. Cappuzzello et al., Nat. Comm. 6 (2015) 6743



F. Cappuzzello et al., Eur. Phys. J. A 57 (2021) 34

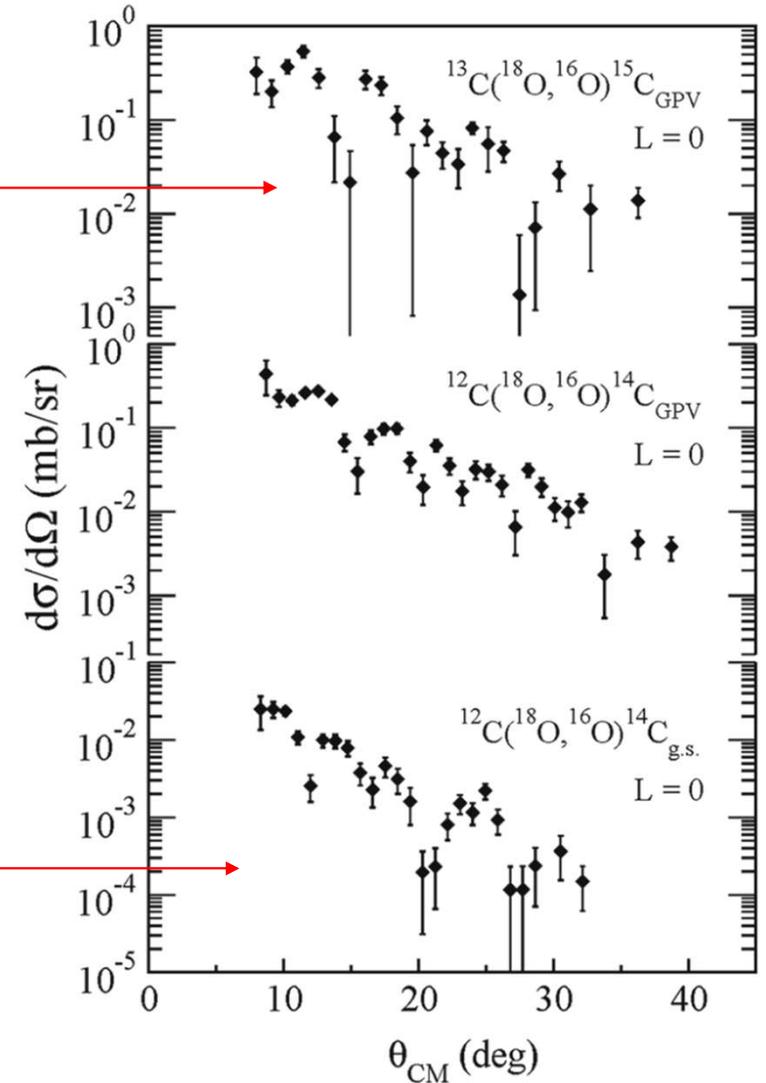
$E_{\text{lab}} = 84 \text{ MeV}$



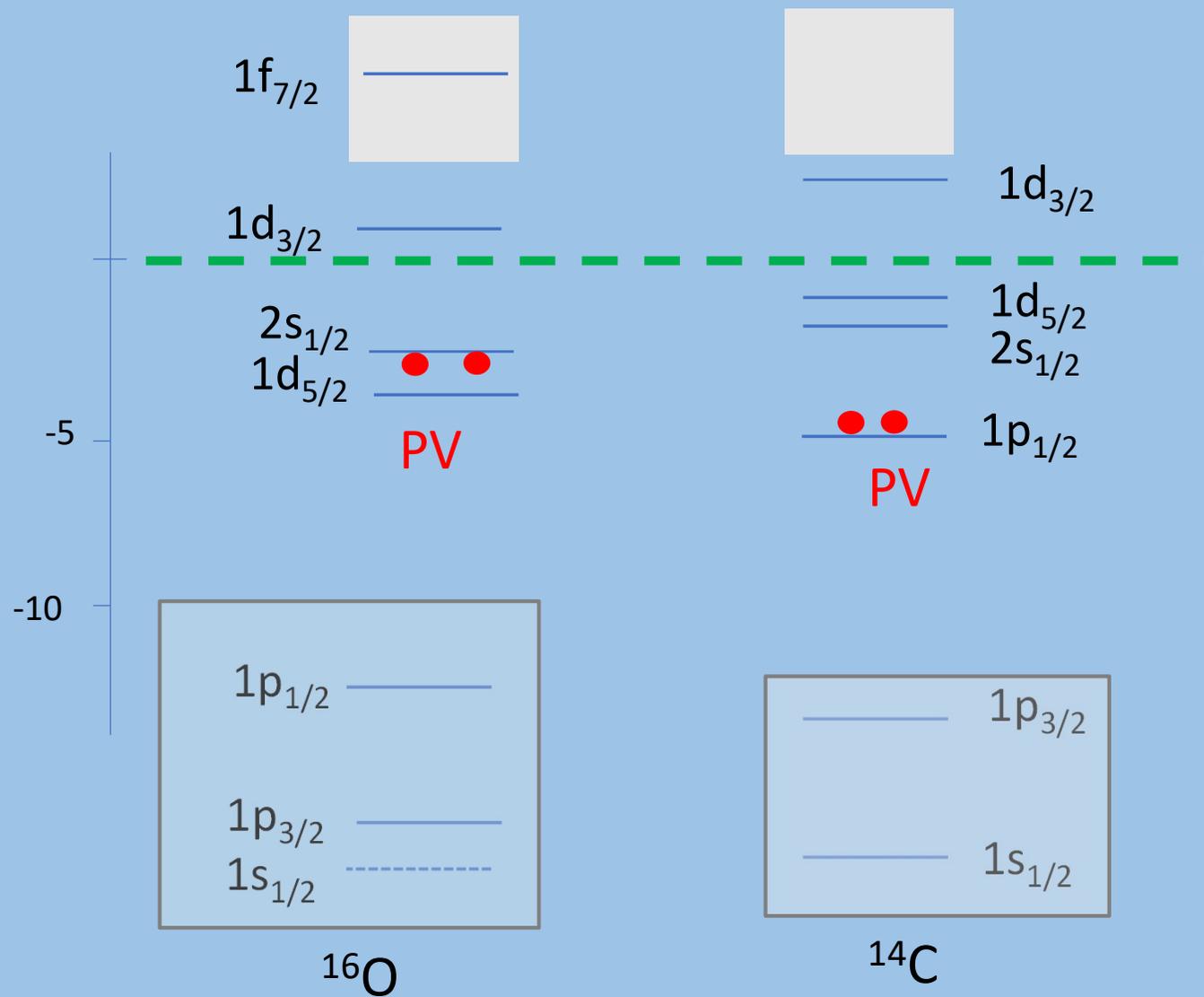
GPV?

g.s.

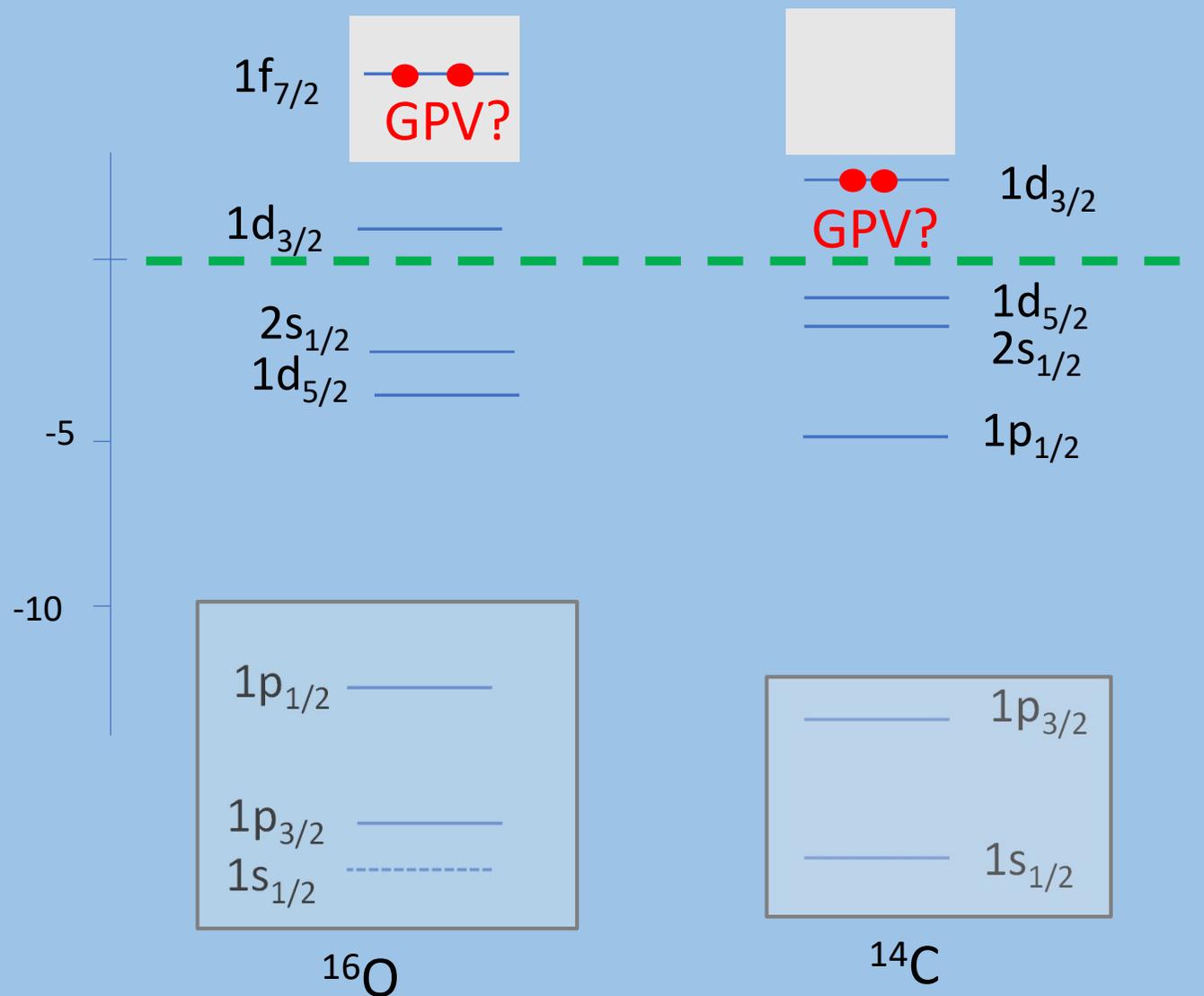
$E_{\text{lab}} = 275 \text{ MeV}$



# GPV calculation challenges



# GPV calculation challenges



In the GPV **both** neutrons lie in the continuum  $\rightarrow$  discretization method via hard wall boundary.

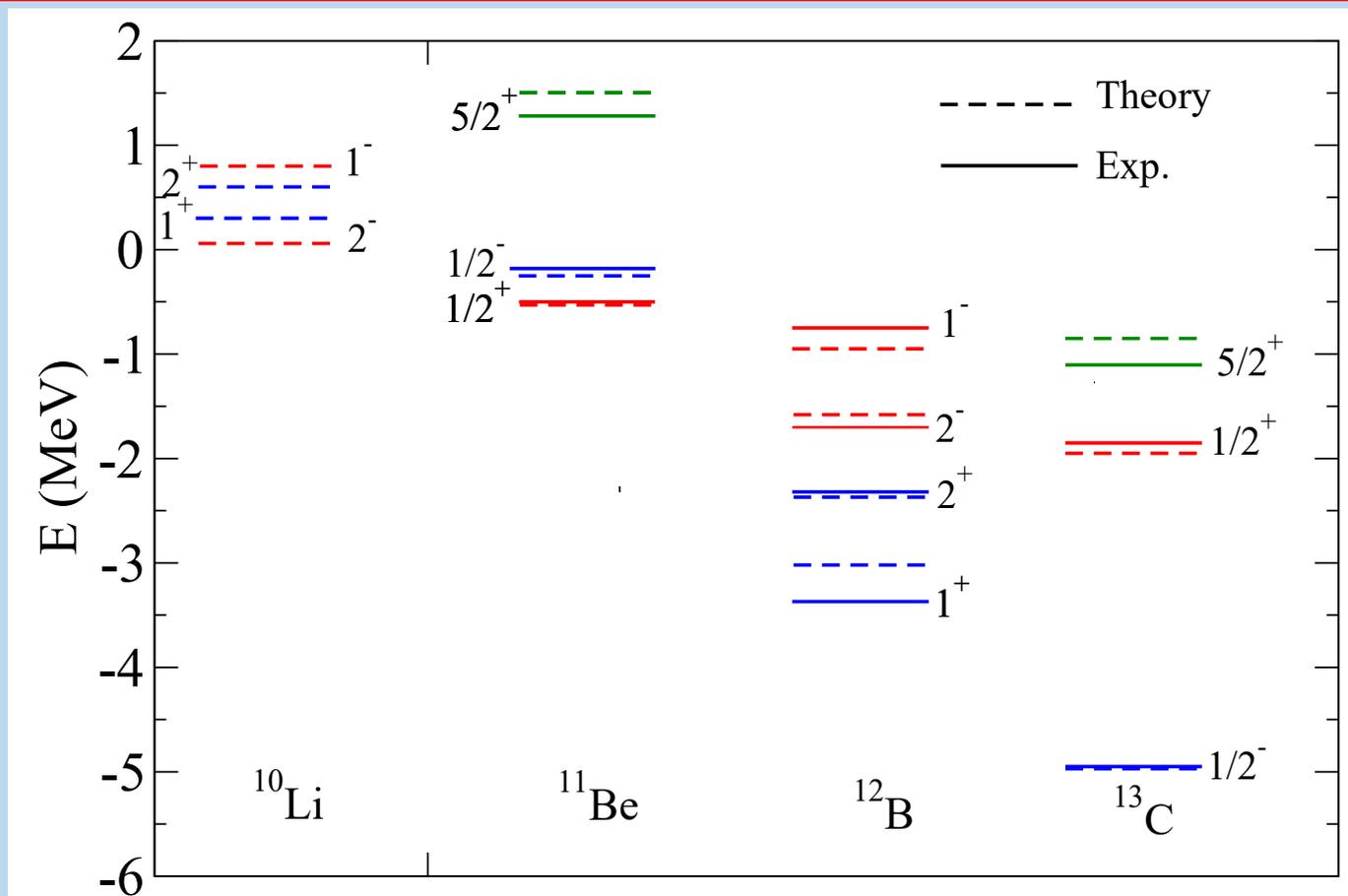
Realistic pairing interactions is needed  $\rightarrow$  Gogny, DDDI, V-14,....

An accurate description of s-p levels is crucial  $\rightarrow$  PVC

Incorporate all this in the pp-RPA eq.'s

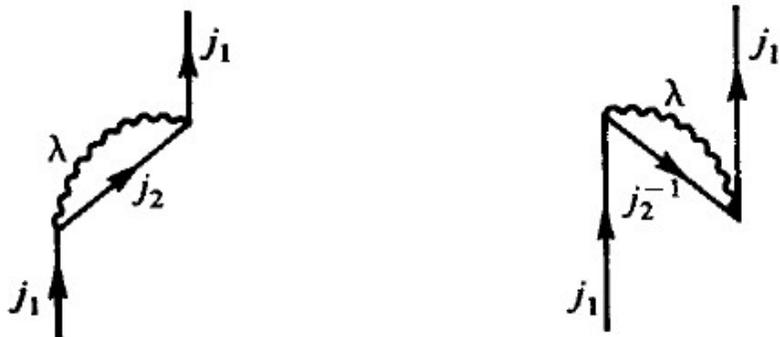
## Position of the Single Particle Levels and NFT/PVC: Self-energy

Many-body states in N=7 isotones arising from quadrupole coupling with single-particle states calculated using mean-field potentials



# Position of the Single Particle Levels and NFT/PVC: Self-energy

We will not use a "standard" mean field but a new one fitted on data after including beyond mean field



$\lambda^\pi: 2^+$  most relevant

$$\delta\varepsilon(j_1) = \begin{cases} \frac{h^2(j_1, j_2, \lambda)}{\varepsilon(j_1) - \varepsilon(j_2) - \hbar\omega_\lambda} & \varepsilon(j_2) > \varepsilon_F \\ -\frac{h^2(j_1, j_2, \lambda)}{\varepsilon(j_2) - \varepsilon(j_1) - \hbar\omega_\lambda} & \varepsilon(j_2) < \varepsilon_F \end{cases}$$

Part of the S-P strength goes to the intermediate state -> Fragmentation

PVC vertex

Experimental Collective  
Quadrupole States properties

$$h(j_1, j_2, \lambda) \equiv \langle j_2, n_\lambda = 1; I = j_1, M = m_1 | H' | j_1 m_1 \rangle$$

$$= (-1)^{j_1 + j_2} (2j_1 + 1)^{-1/2} (2\lambda + 1)^{-1/2} \langle j_2 || k_\lambda Y_\lambda || j_1 \rangle \langle n_\lambda = 1 || \alpha_\lambda || n_\lambda = 0 \rangle$$

$$\Sigma_{\gamma\delta}(\omega) = \sum_{pp'h'p''h''} V_{pp'h''\gamma} \sum_f \frac{X_{p'h'}^f X_{p''h''}^f}{\omega - \epsilon_p - \hbar\omega_f} V_{pp'h'\delta}$$

A close connection with

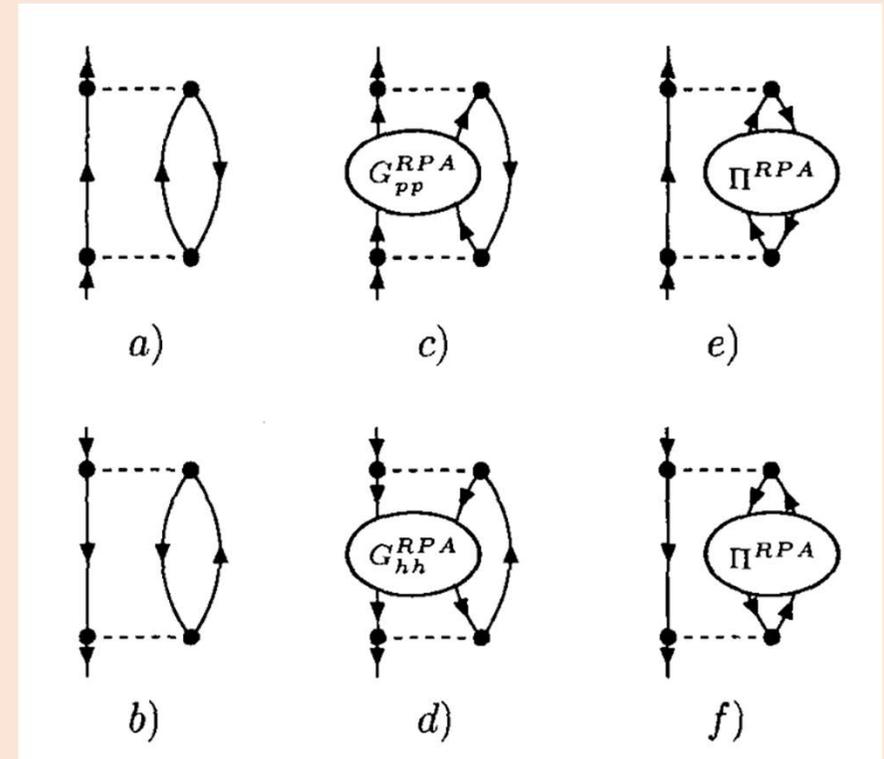
$$\Sigma^{RPA}(\alpha, \beta : E) = \frac{1}{2} \left\{ \sum_{\mu > F, n \neq 0} \frac{\Delta_{\alpha\mu}^{A+,n*} \Delta_{\beta\mu}^{A+,n}}{E - (\epsilon_\mu + (E_n^A - E_0^A)) + i\eta} + \sum_{\mu < F, m \neq 0} \frac{\Delta_{\alpha\mu}^{A-,m} \Delta_{\beta\mu}^{A-,m*}}{E - (\epsilon_\mu + (E_0^A - E_m^A)) - i\eta} \right\}$$

with

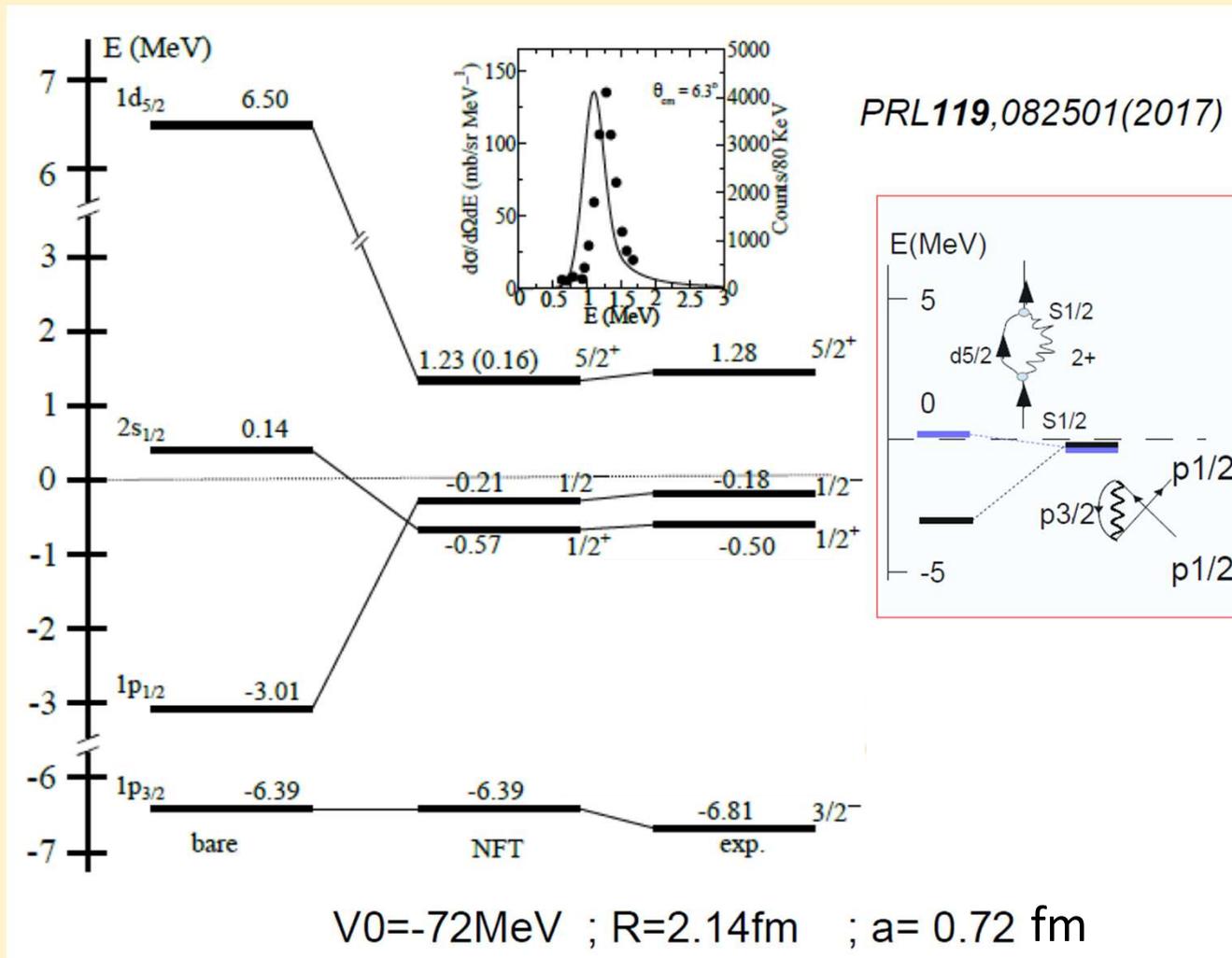
$$\Delta_{\alpha\mu}^{A+,n} = \sum_{\nu > F, \kappa < F; \nu < F, \kappa > F} \langle \alpha\kappa | G | \mu\nu \rangle R_{\nu\kappa}^{A+,n}$$

and

$$\Delta_{\alpha\mu}^{A-,m} = \sum_{\nu > F, \kappa < F; \nu < F, \kappa > F} \langle \alpha\kappa | G | \mu\nu \rangle R_{\kappa\nu}^{A-,m}$$

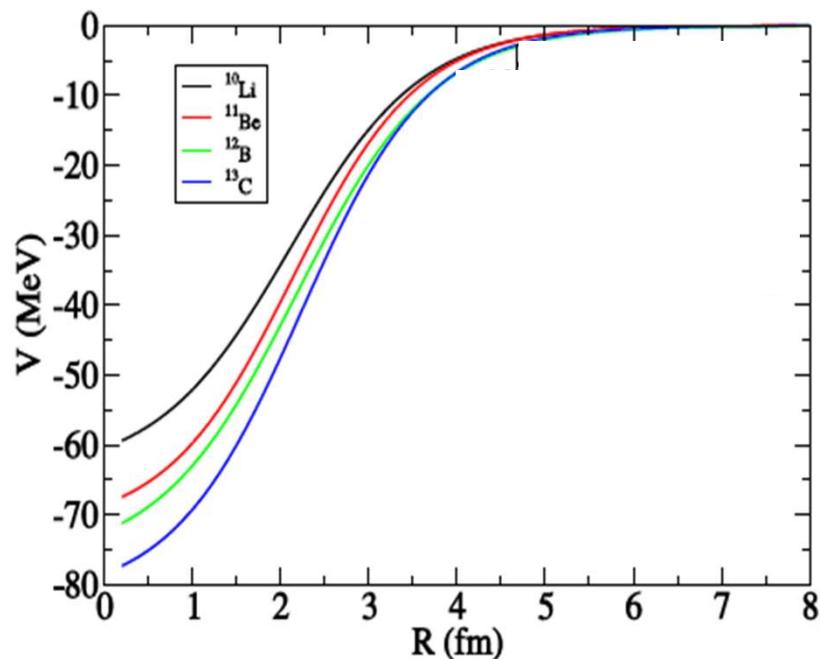


# Position of the Single Particle Levels and NFT/PVC: Self-energy



- Other observables:
- Spectroscopic factors in one-neutron transfer,
  - Population of  $2^+$  in  $11\text{Be}(p,d)10\text{Be}$  reaction.
  - Nuclear radii isotope shift

# Position of the Single Particle Levels and NFT/PVC: Self-energy



These potentials were determined by a consistent fit to data considering the PVC renormalization effects  $\rightarrow$   
No overcounting problem

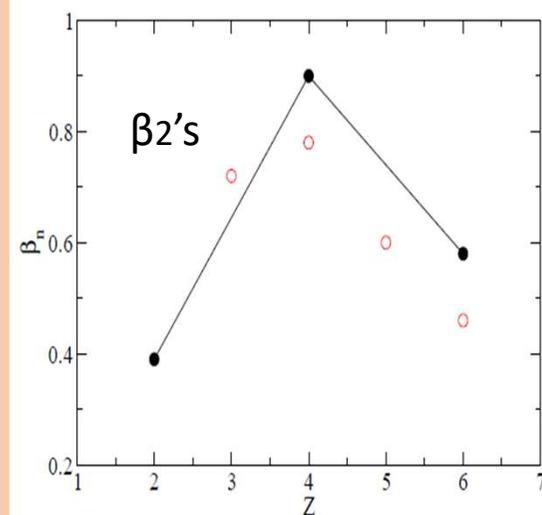
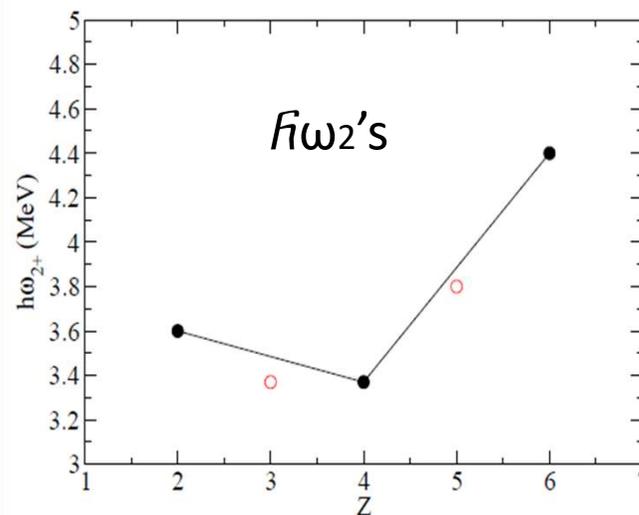
Simple parametrization:

$$V_{\text{WS}} = -82 + 54(N-Z)/A \text{ MeV}$$

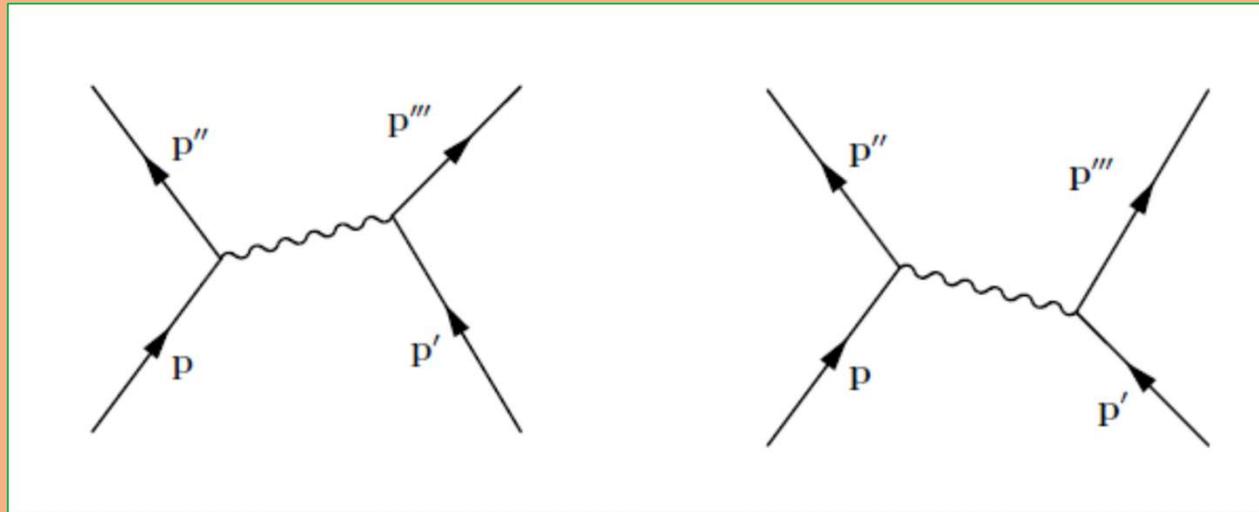
$$a = 0.75 \text{ fm}; R_{\text{WS}} = 0.99A^{1/3} \text{ fm}$$

$$V_{\text{LS}} = 0.0082V_{\text{WS}}$$

Bare mean field potential for  $N=7$  isotones



# Extended Role of Vibrations in the PV and GPV: The Phonon Exchange Induced Interaction, $V^{ind}$

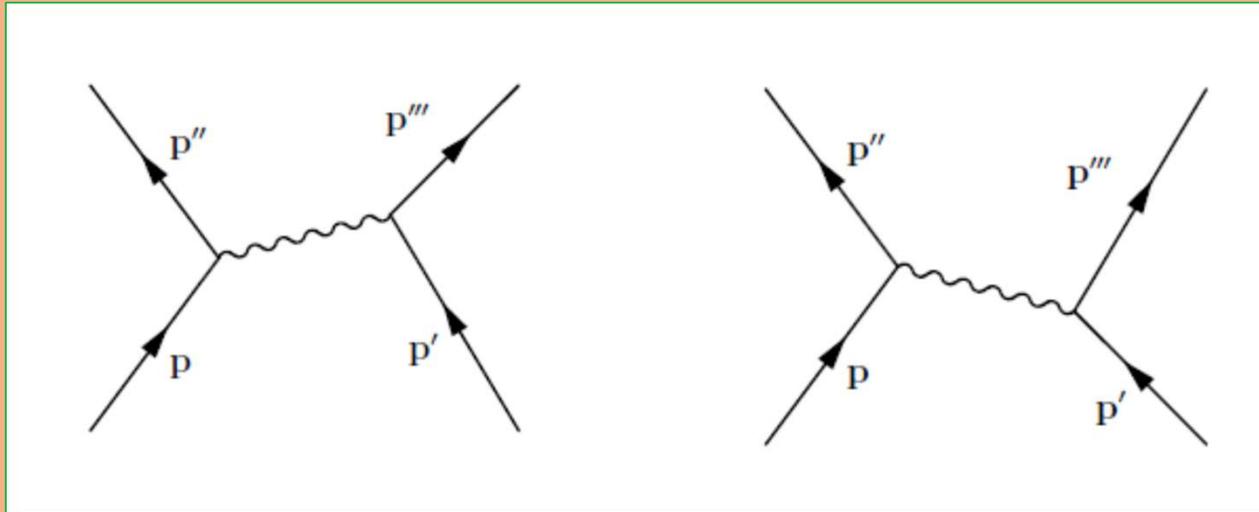


$$V_{pp'p''p'''}^{ind} = \sum_{\lambda\nu} \left[ \frac{h_{pp''\lambda\nu} h_{p''p'\lambda\nu}}{E - (\epsilon_{p''}^{emp} + \epsilon_{p'}^{emp} + \hbar\omega_{\lambda\nu})} + \frac{h_{p''p\lambda\nu} h_{p'p'''\lambda\nu}}{E - (\epsilon_p^{emp} + \epsilon_{p'''}^{emp} + \hbar\omega_{\lambda\nu})} \right]$$

Present in every nucleus!!

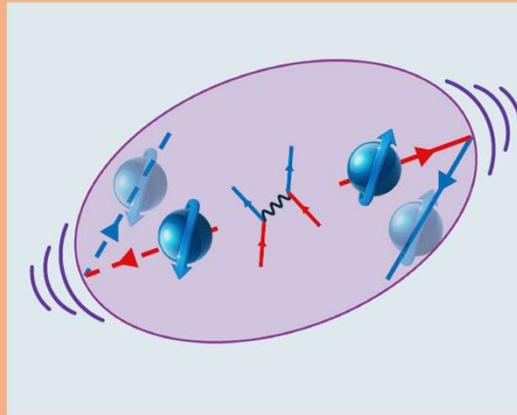
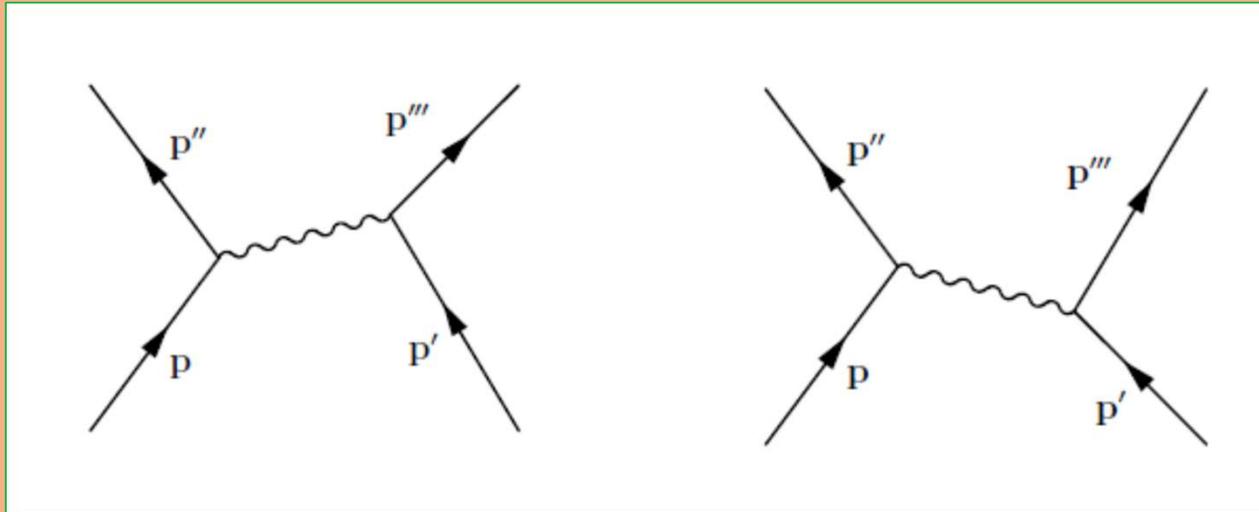
As a consequence: any pairing  $V^{eff}(p,p';p'',p''')$  must leave room to  $V^{ind}$

# Extended Role of Vibrations in the PV and GPV: The Phonon Exchange Induced Interaction, $V_{\text{ind}}$



In Spain we say:  
"An image is better than thousand words"

# Extended Role of Vibrations in the PV and GPV: The Phonon Exchange Induced Interaction, $V_{\text{ind}}$



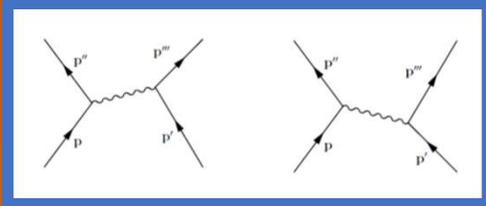
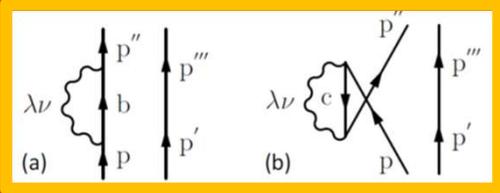
pp-RPA and PVC (E-dependent!!)

$$\begin{pmatrix} A_{pp'p''p'''} & B_{pp'h''h'''} \\ -B_{p''p''''hh'} & -A_{hh'h''h'''} \end{pmatrix} \begin{pmatrix} X_{p''p'''} \\ Y_{h''h'''} \end{pmatrix} = E \begin{pmatrix} X_{pp'} \\ Y_{hh'} \end{pmatrix}$$

Incorporating Self-energy and Induced Interaction

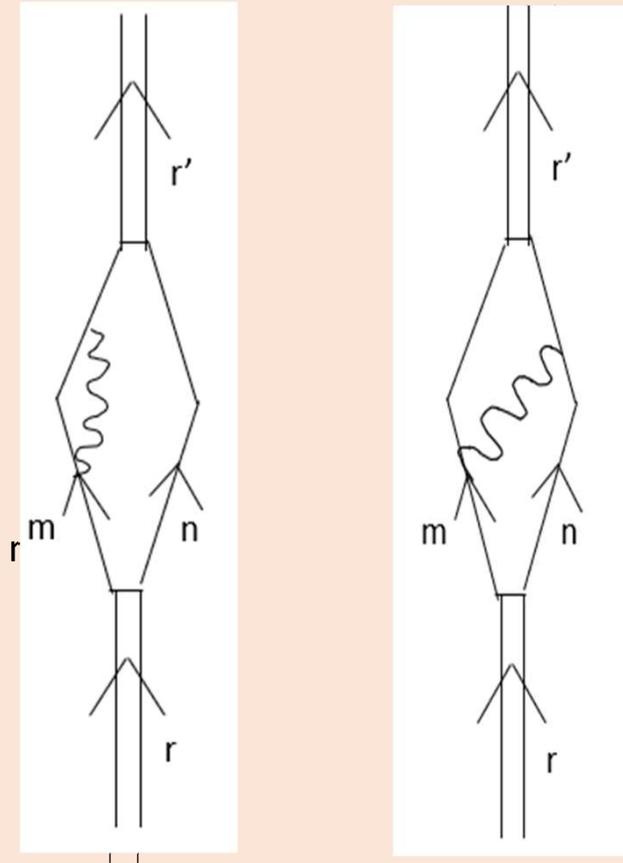
$$A_{pp'p''p'''} = [(\epsilon_p + \epsilon_{p'}) + \Sigma_{pp''(p')}(E)\delta_{p'p'''} + \Sigma_{p'p''(p)}(E)\delta_{pp''} + V_{pp'p''p'''}^{bare} + V_{pp'p''p'''}^{ind}(E) + Exch(p, p')] N_{pp'p''p'''}]$$

$$B_{pp'hh'} = [V_{pp'hh'}^{bare} + V_{pp'hh'}^{ind}(E) + Exch(p, p')] N_{pp'p''p'''}]$$



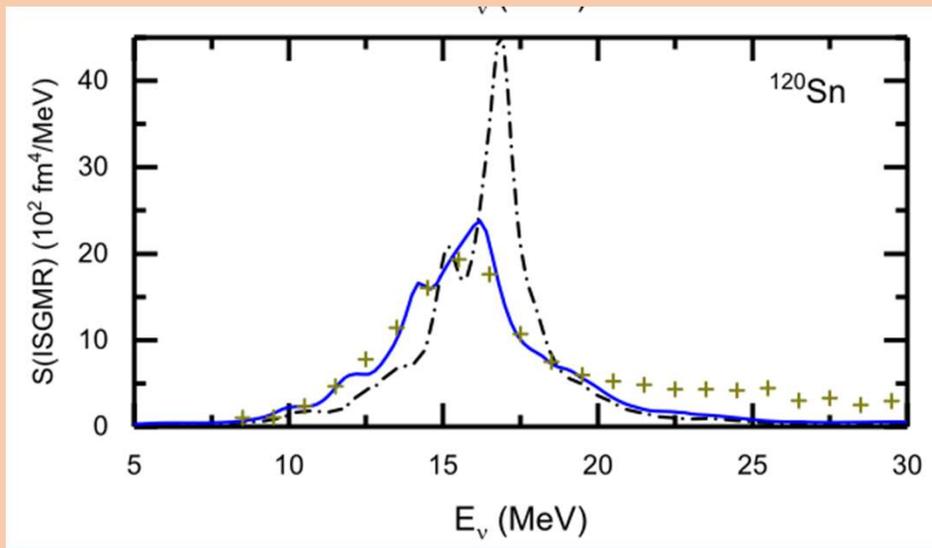
## pp-RPA and PVC (E-dependent!!)

Technical note: This extended pp-RPA is comparable to the NFT treatment: In fact, if self-energy and  $V_{\text{ind}}$  are included perturbatively in a second diagonalisation, the following “well known” NFT diagrams for the matrix elements appear:

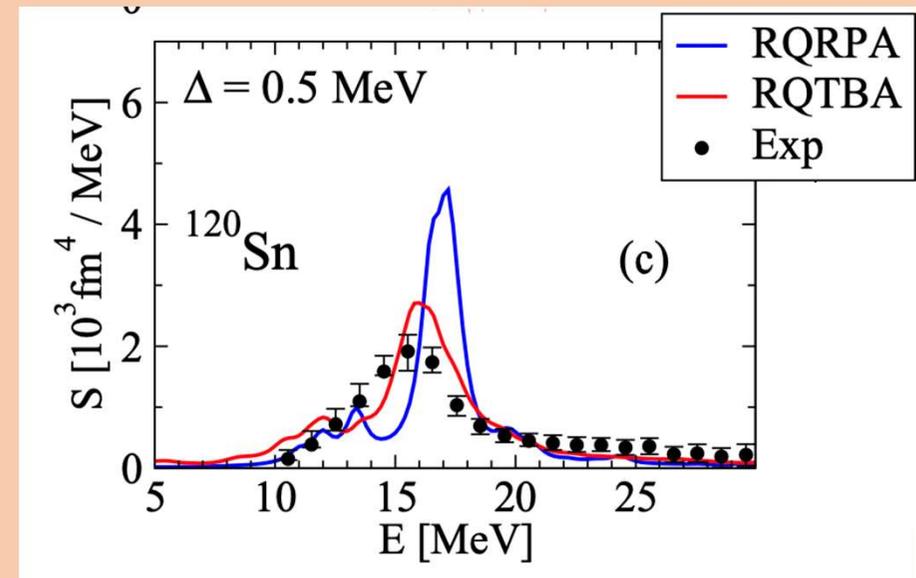




Particle-vibration coupling on top of self-consistent density functional calculations has been mostly applied to heavy nuclei near closed shells. It provides a successful reproduction of the width of giant resonance modes



Z.Z. Li, Y.F. Niu, G. Colò. PRL 131 (2023) 082501



E. Litvinova, PRC 107 (2023) L041302

Technical note II:

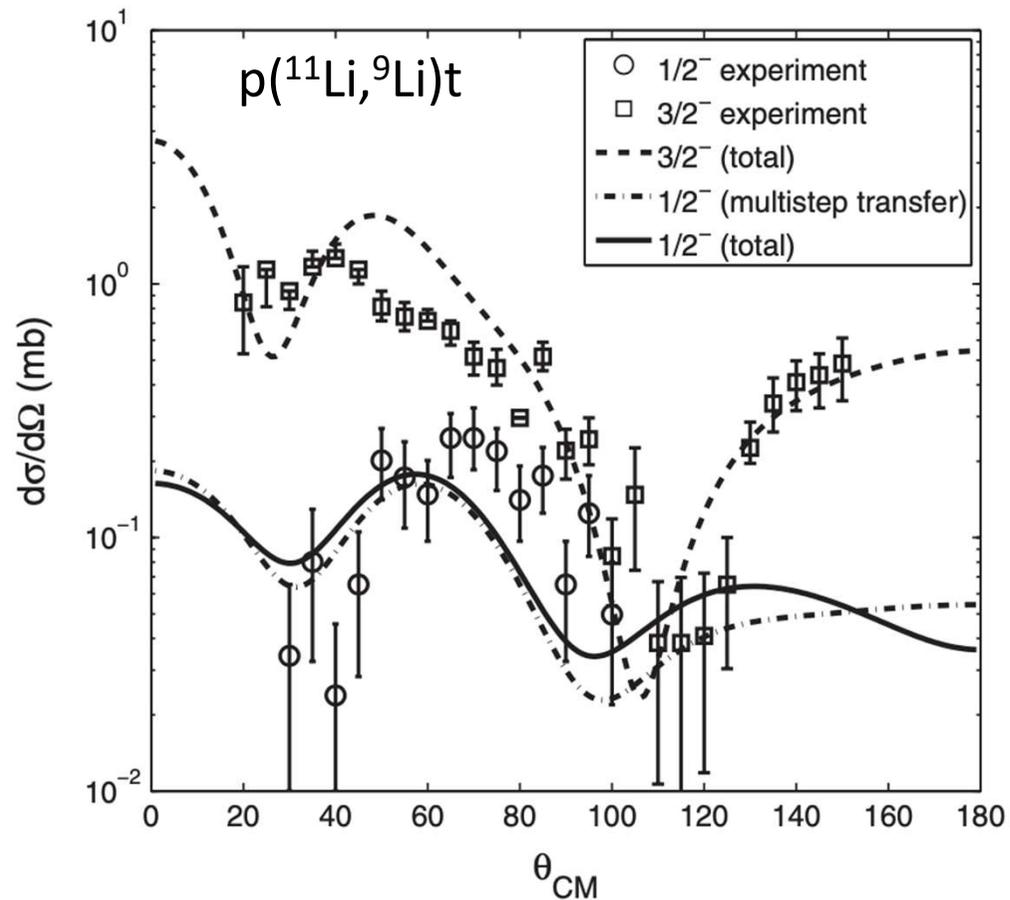
The self-energy and the induced interaction are energy-dependent, thus it is possible to reconstruct the amplitudes of the resulting  $0^+$  states on the intermediate  $2p-1$ phonon configurations, so that they can be written:

$$|0^+_n\rangle = \sum_{pp'} ( X_{pp'}(n) |pp'(0^+)\rangle + Y_{hh'}(n) |hh'(0^+)\rangle ) + \sum_{pp'\nu} R_{pp'\nu}(n) |pp'(2^+)\nu(2^+)\rangle$$

Can also be obtained by diagonalizing an energy independent matrix in the extended basis including them.

# pp-RPA and PVC (E-dependent!!)

## Role of phononic components in direct reactions

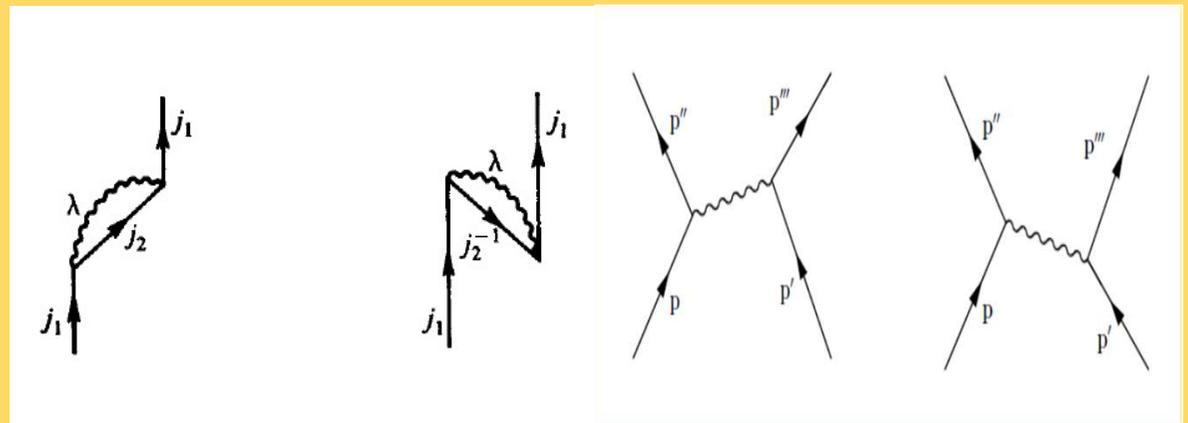
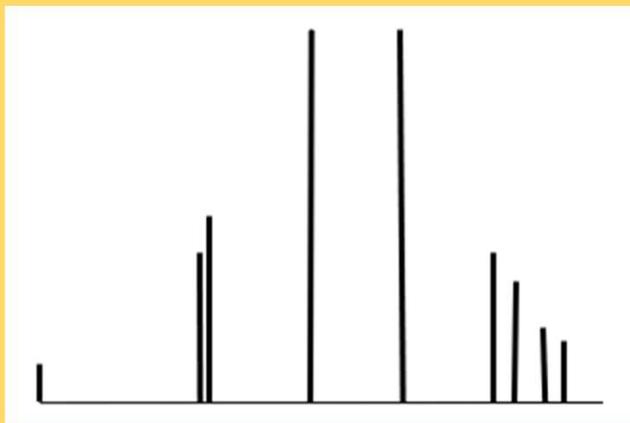
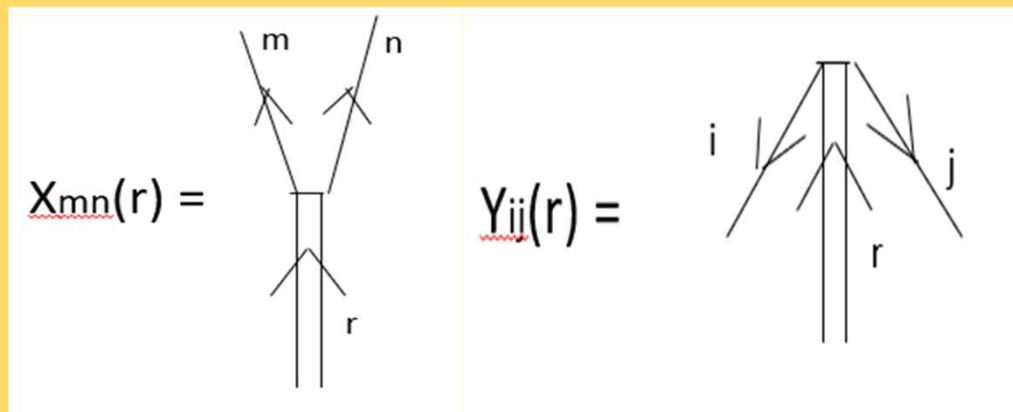


Theor.: G. Potel et al, PRL 105 (2010) 172502

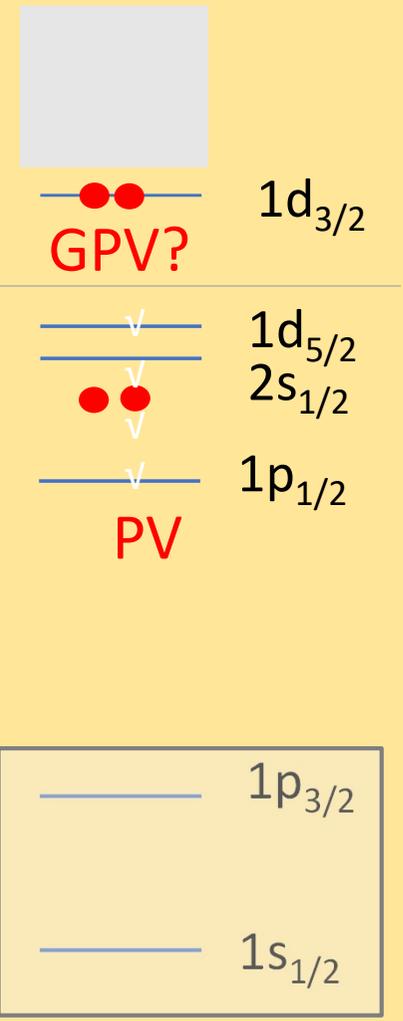
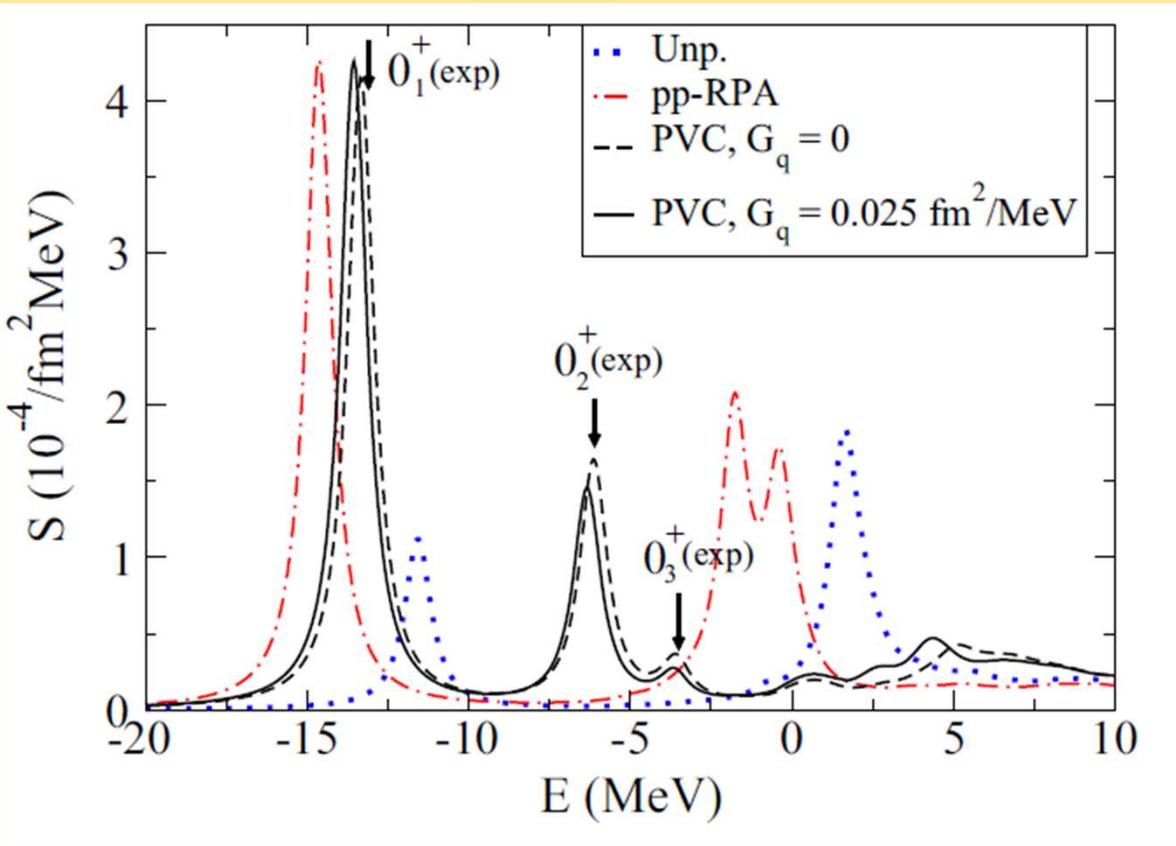
Exp.: Tanihata et al. PRL192502(2008)

# pp-RPA and PVC

Summarizing: Pairing correlations, Decay width to the continuum and PVC effects

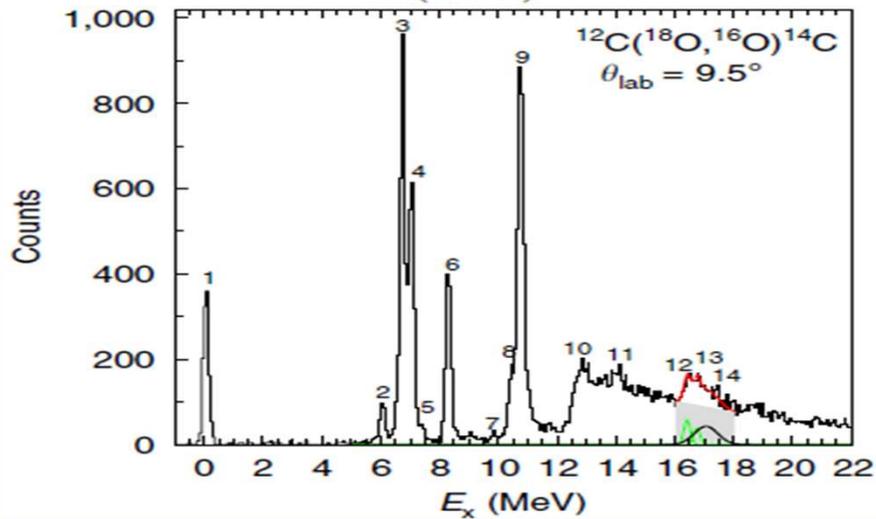
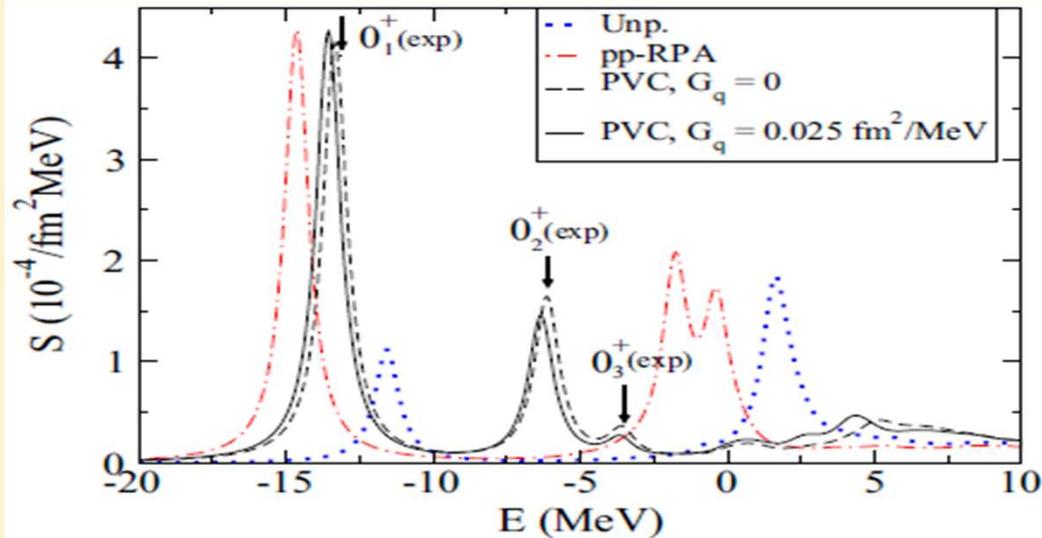


14C



14C

14C



Fragmentation of the Giant Pairing Vibration in  $^{14}\text{C}$  Induced by Many-Body Processes

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OPEN

Signatures of the Giant Pairing Vibration in the  $^{14}\text{C}$  and  $^{15}\text{C}$  atomic nuclei

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## pp-RPA and PVC

Xpp' ; Ypp' and Rpp'2+ amplitudes for 12C + 2n (0+) states: Bound states

	$E_{gs} = -13.09$ MeV $R^{2+} = 0.130$		$E_{0_2^+} = -5.96$ MeV $R^{2+} = 0.382$		$E_{0_3^+} = -3.47$ MeV $R^{2+} = 0.348$	
$l_j$	$X_{l_j}^2$	$Y_{l_j}^2$	$X_{l_j}^2$	$Y_{l_j}^2$	$X_{l_j}^2$	$Y_{l_j}^2$
$s_{1/2}$	0.006	0.003	0.283	-	0.376	-
$p_{1/2}$	0.833	-	0.050	-	0.043	-
$p_{3/2}$	-	0.002	0.001	-	-	-
$d_{3/2}$	0.003	-	0.005	-	-	-
$d_{5/2}$	0.046	-	0.327	-	0.256	-

Table 4: Main 0-phonon components of the wavefunctions of the ground state and of the two lowest excited  $0^+$  states calculated with a constant effective mass,  $m_{eff} = m_{red} = 0.92m$  ( $R_{box} = 28$  fm).

	$R_{l_j l'_j}^{2+}$						
$l_j / l'_j$	$s_{1/2}$	$p_{1/2}$	$p_{3/2}$	$d_{3/2}$	$d_{5/2}$	$f_{5/2}$	$f_{7/2}$
$s_{1/2}$	-	-	-	-	0.003	-	-
$p_{1/2}$	-	-	0.105	-	-	0.0146	-
$p_{3/2}$	-	0.105	-	0.004	-	-	-
$d_{3/2}$	-	-	0.004	-	-	-	-
$d_{5/2}$	0.003	-	-	-	0.005	-	-
$f_{5/2}$	-	0.0146	-	-	-	-	-
$f_{7/2}$	-	-	-	-	-	-	-

Table 5: Phonon components  $R_{l_j l'_j}^{2+}$  larger than 0.001, calculated in the wavefunction of the ground state of  $^{14}\text{C}$  calculated with a constant effective mass,  $m_{eff} = m_{red} = 0.92m$  ( $R_{box} = 28$  fm).

## pp-RPA and PVC

Xpp' and cumulative Rpp'2+ amplitudes for  $^{12}\text{C} + 2n$  ( $0^+; \text{GPV}$ )

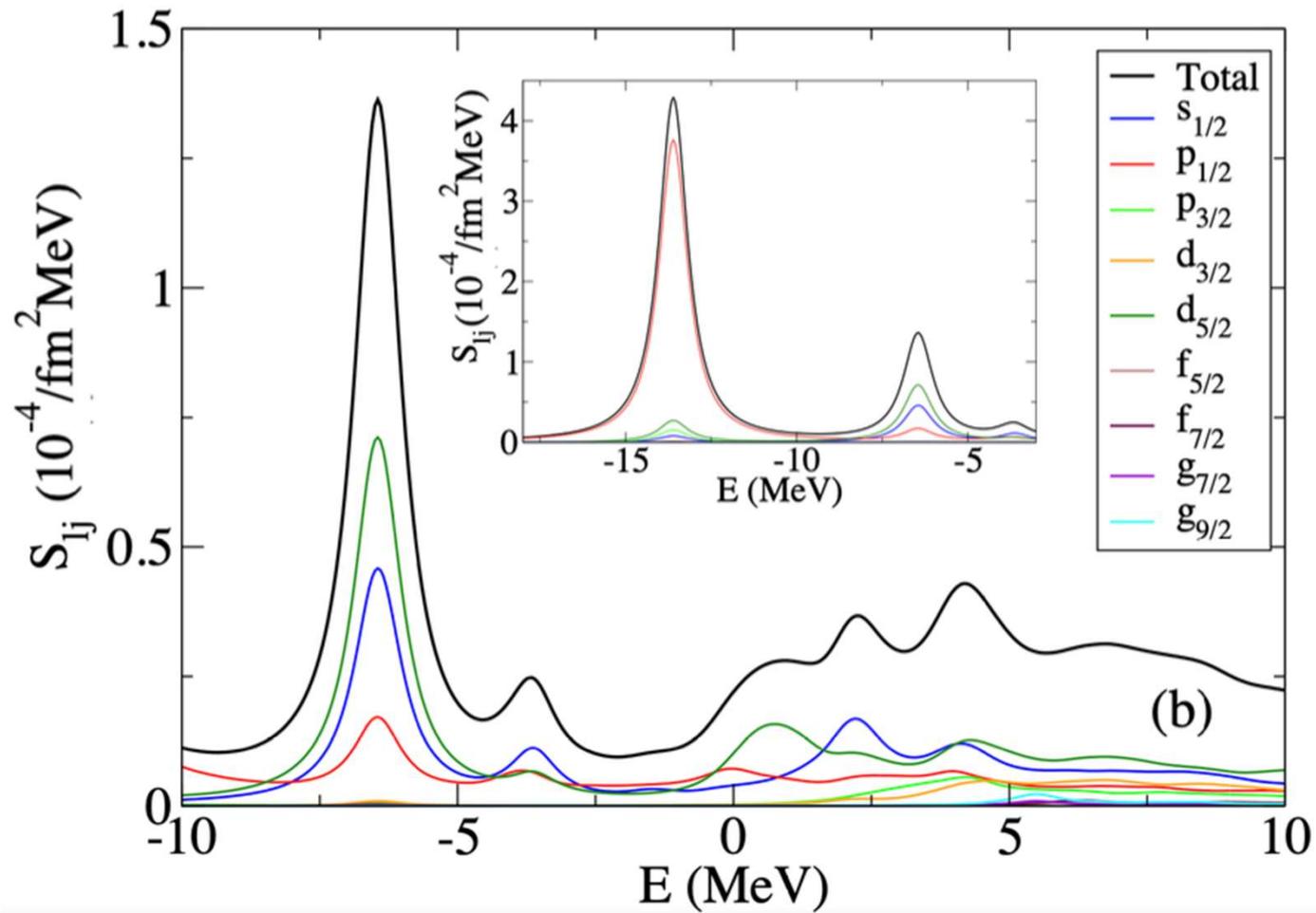
	$E=6.87$ $R_{box}=20$ $R_{i^+}^{2+} = 0.623$	$E=6.91$ $R_{box}=22$ $R_{i^+}^{2+} = 0.729$	$E=7.14$ $R_{box}=24$ $R_{i^+}^{2+} = 0.728$	$E=6.96$ $R_{box}=26$ $R_{i^+}^{2+} = 0.613$	$E=7.11$ $R_{box}=28$ $R_{i^+}^{2+} = 0.785$
$l_j$	$X_{l_j}^2$	$X_{l_j}^2$	$X_{l_j}^2$	$X_{l_j}^2$	$X_{l_j}^2$
$s_{1/2}$	0.06	0.041	0.03	0.04	0.012
$p_{1/2}$	0.112	0.004	0.001	0.005	0.012
$p_{3/2}$	0.029	0.003	0.056	0.005	0.05
$d_{3/2}$	0.006	0.019	0.007	0.003	0.007
$d_{5/2}$	0.154	0.195	0.179	0.279	0.111
$f_{5/2}$	-	-	-	-	-
$f_{7/2}$	-	-	-	-	-

Table 23: Main 0-phonon components of the wavefunctions of the excited state of  $^{14}\text{C}$  carrying the largest  $S_{dUdr}$  strength around  $E = 7$  MeV for a series of boxes ( $R_{box} = 20-28$  fm).

**Note: About 70% on the phononic side!!**

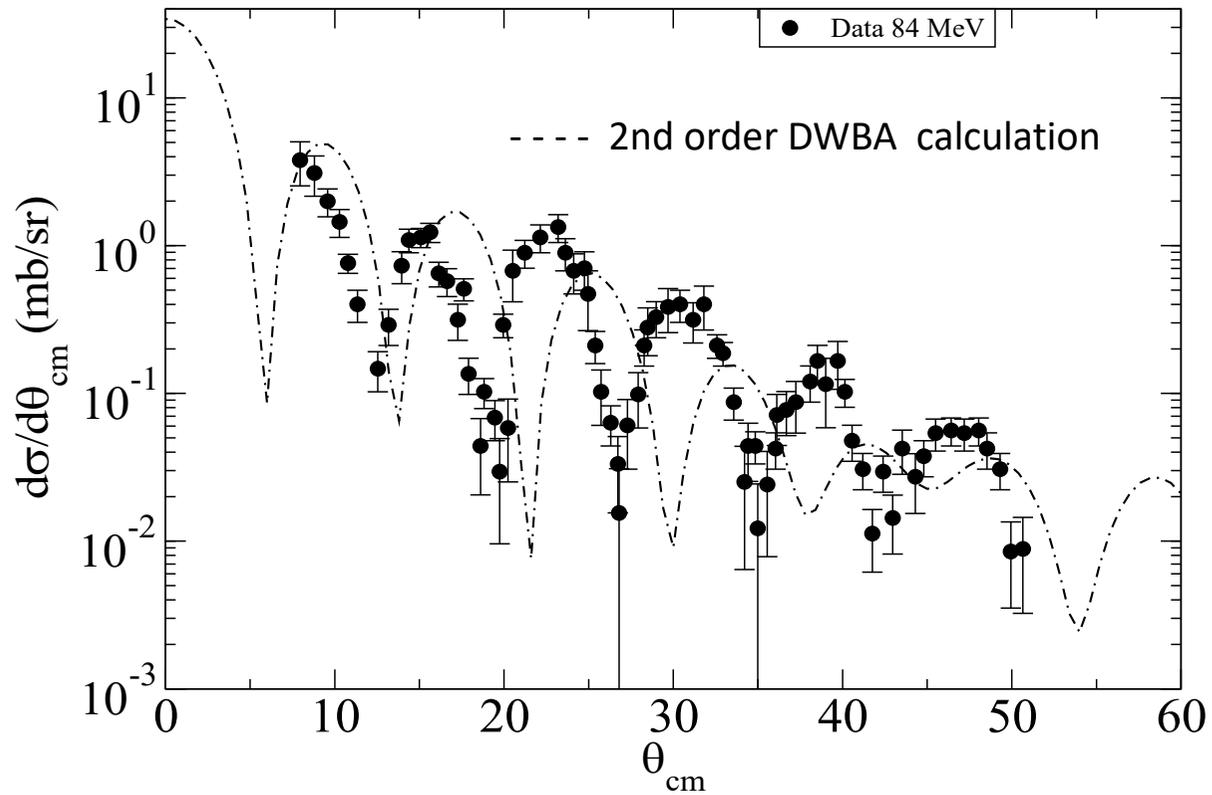
$$|0_n^+\rangle = \sum_{pp'} (X_{pp'}(n) |pp'(0^+)\rangle + Y_{hh'}(n) |hh'(0^+)\rangle) + \sum_{pp'\nu} R_{pp'\nu}(n) |pp'(2^+)\nu(2^+)\rangle$$

# Angular Momentum Decomposition



# EXTENDED pp-RPA RESULTS

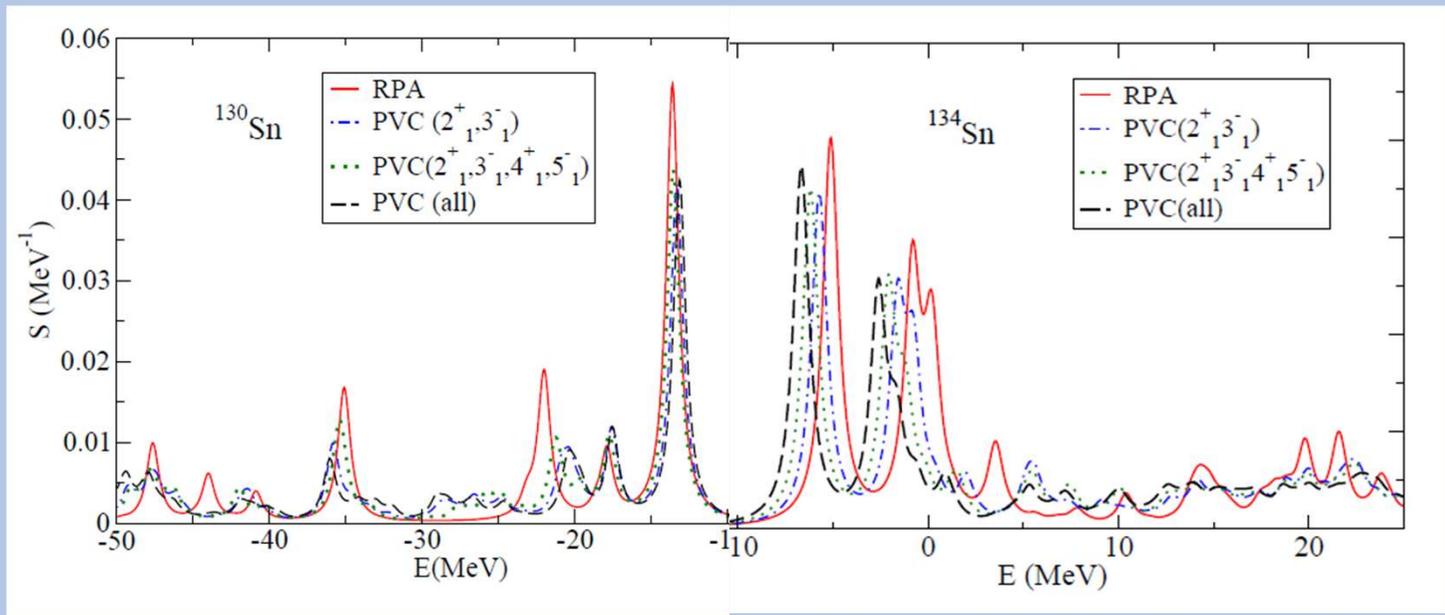
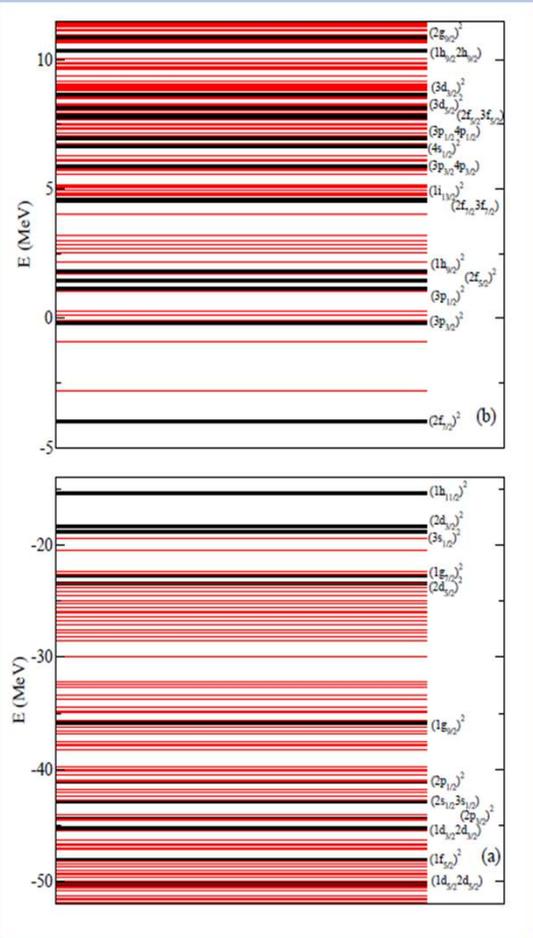
$^{12}\text{C}(^{18}\text{O}, ^{16}\text{O})^{14}\text{C}(\text{gs})$  at  $E_{\text{lab}} = 84$  MeV



# EXTENDED pp-RPA RESULTS

132Sn +- 2n

Application to médium and heavy nuclei: much more s-p levels

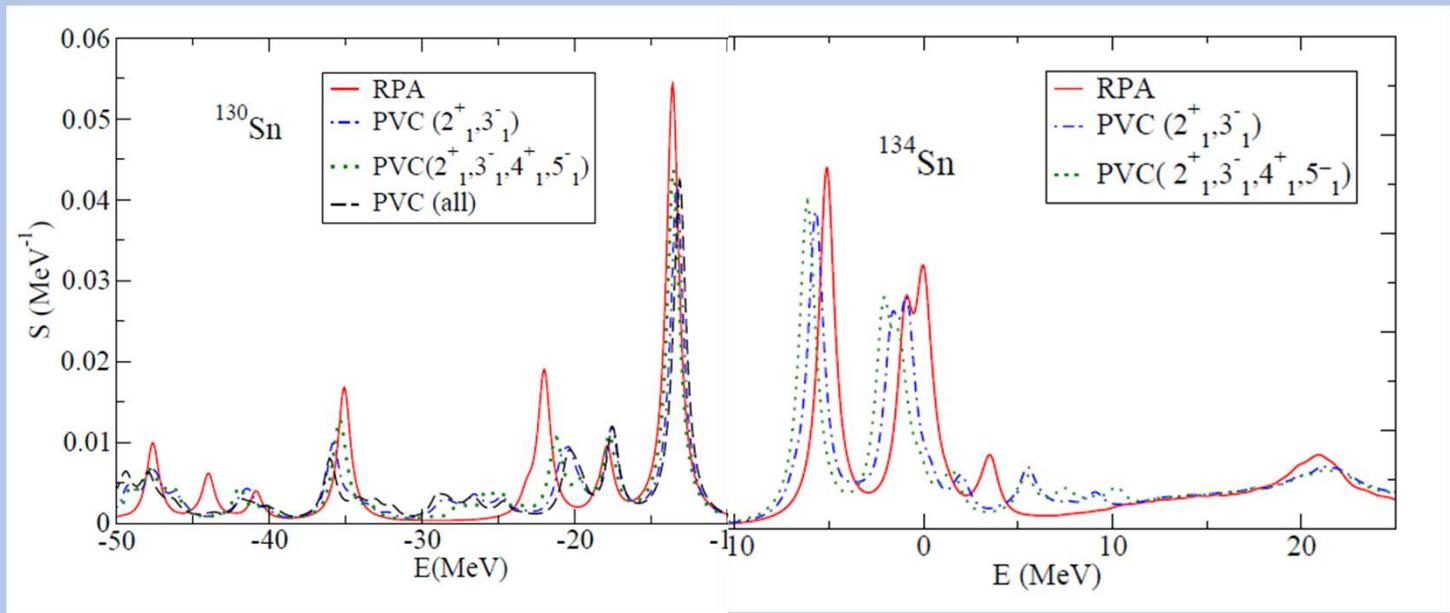
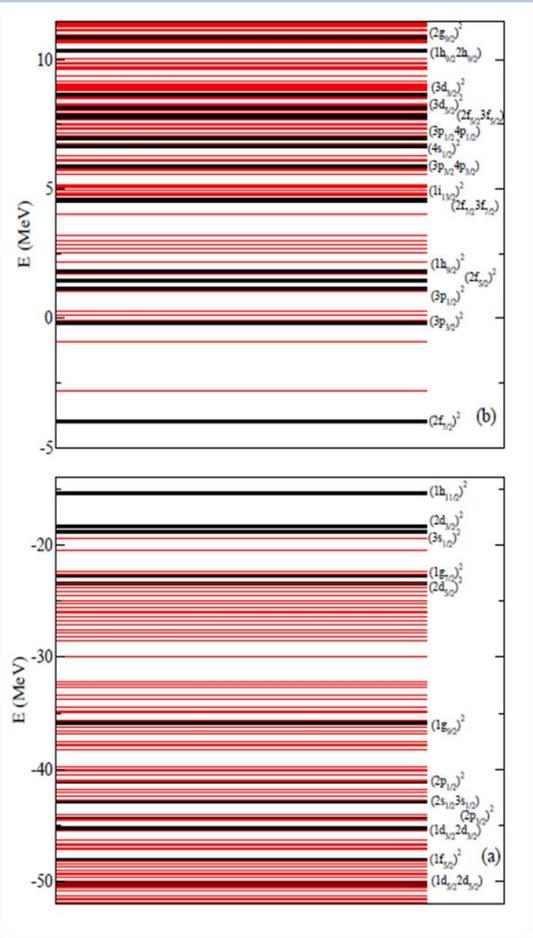


**Figure 1.** (a) The energies of the  $hh'$ -pairs in  $^{132}\text{Sn}$  up to -50 MeV are shown by black thick lines. These states are embedded among the  $hh'$   $\lambda$  states (thin red lines), arising from the coupling to the lowest  $2_1^+$ ,  $3_1^-$ ,  $4_1^+$ ,  $5_1^-$  vibrations. (b) The same, for the  $pp'$ -pairs with energies up to 11 MeV embedded among  $pp'$   $\lambda$  states, calculated in a box of radius  $R_{\text{box}} = 14$  fm.

# EXTENDED pp-RPA RESULTS

132Sn +- 2n

Application to médium and heavy nuclei: much more s-p levels

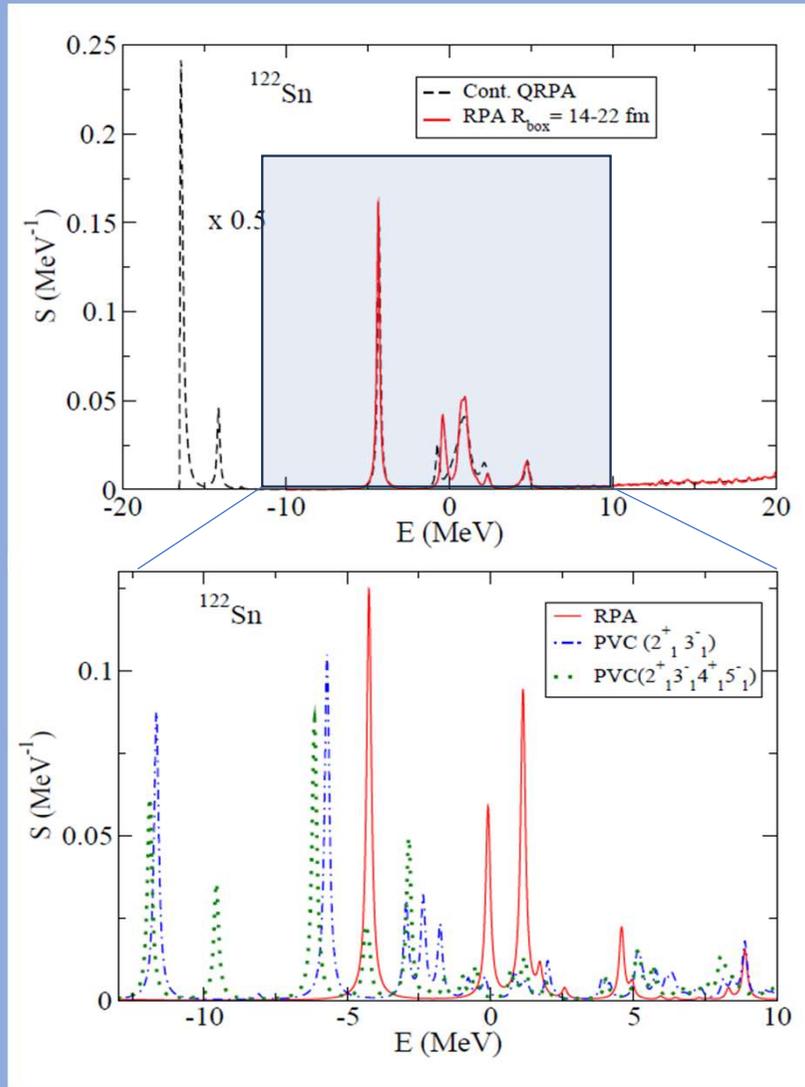
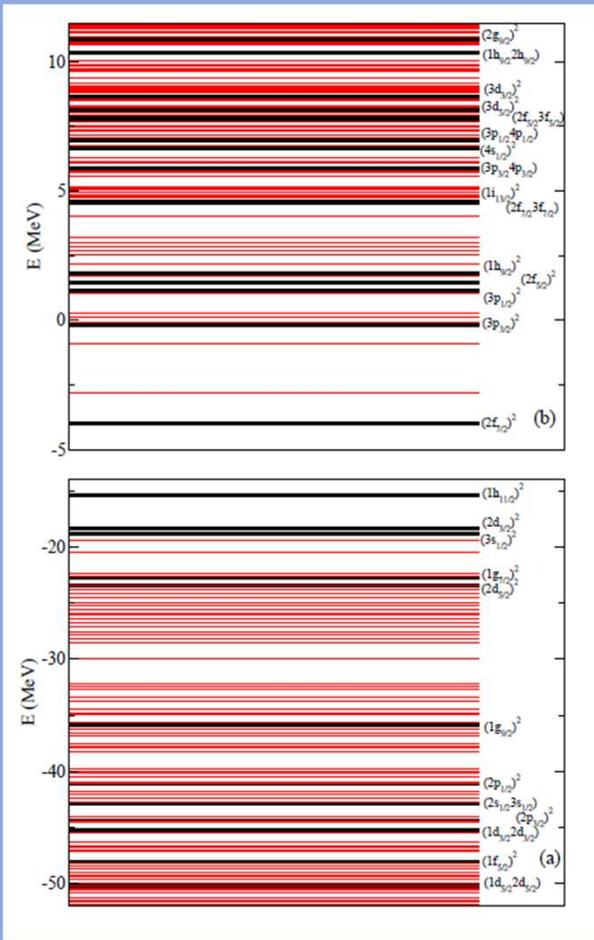


**Figure 1.** (a) The energies of the  $hh'$ -pairs in  $^{132}\text{Sn}$  up to -50 MeV are shown by black thick lines. These states are embedded among the  $hh'$   $\lambda$  states (thin red lines), arising from the coupling to the lowest  $2^+_1, 3^-_1, 4^+_1, 5^-_1$  vibrations. (b) The same, for the  $pp'$ -pairs with energies up to 11 MeV embedded among  $pp'$   $\lambda$  states, calculated in a box of radius  $R_{\text{box}} = 14$  fm.

# EXTENDED pp-RPA RESULTS

Application to médium and heavy nuclei: much more s-p levels

120Sn + 2n



## CONCLUSIONS (GPV part)

We have computed the  $2n$ -transfer strength to populate  $0+$  states in the continuum of  $^{14}\text{C}$  and made the first steps to compute the absolute cross section of the reaction  $^{12}\text{C}(^{18}\text{O}, ^{16}\text{O})^{14}\text{C}$ . The theoretical model is based on particle-particle RPA extended to include the effects of coupling to collective quadrupole vibrations, in keeping with previous calculations of weakly-bound systems.

The aim is to compare our results with the bump and the associated angular distribution revealed in the excitation spectrum and attributed to the Giant Pairing Vibration.

# Pairing Rotations (Superfluid systems) and PVC

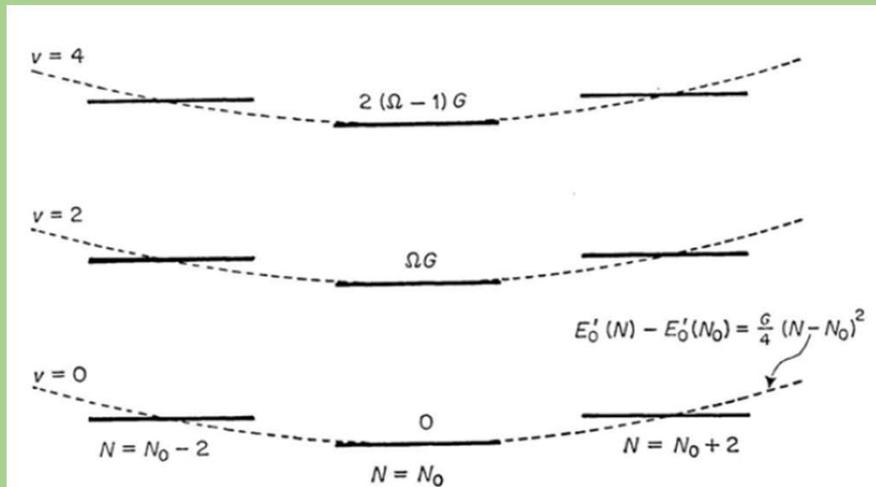


Figure 11.2. Exact energy spectrum for the Hamiltonian  $H' = H - \lambda^{(N)}n$ .

BCS theory

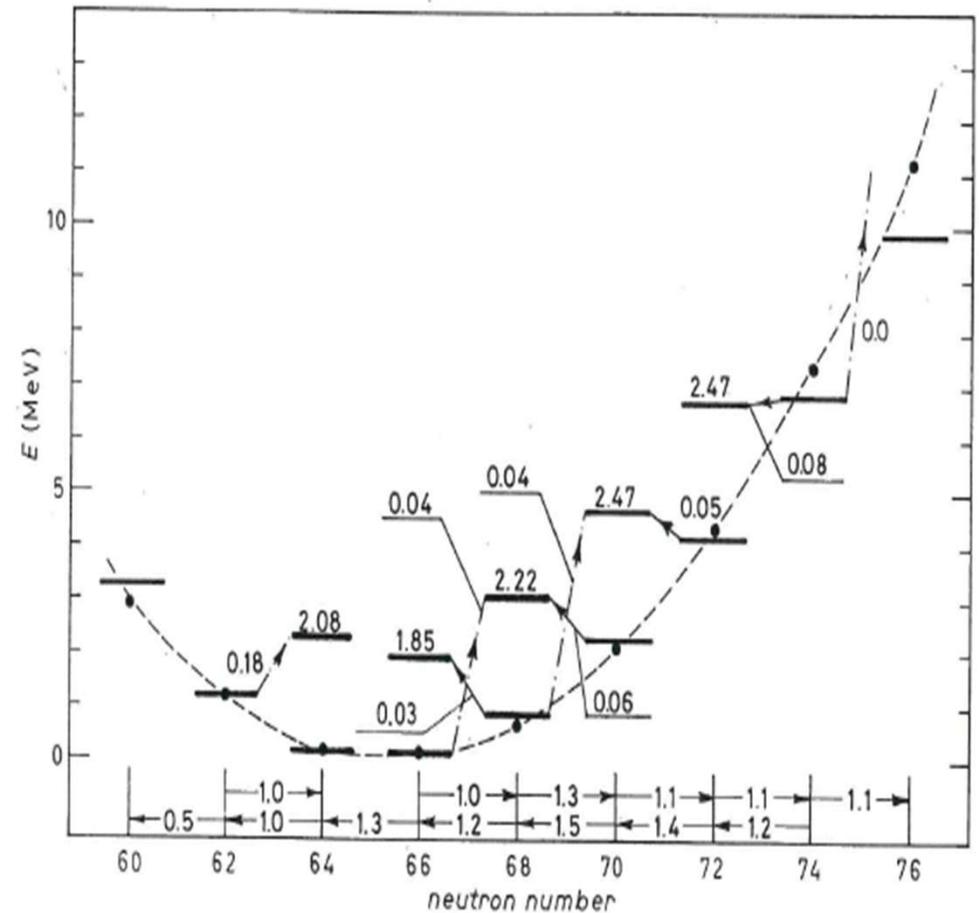
$$|> = \prod_{\nu>0} (u_{\nu} + v_{\nu}a_{\nu}^{\dagger}a_{\bar{\nu}}^{\dagger})|0>$$

$$u_{\nu}^2 = \frac{1}{2} \left\{ 1 + \frac{\epsilon_{\nu} - \lambda}{[(\epsilon_{\nu} - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\}$$

$$\frac{G}{2} \sum_{\nu>0} \frac{1}{[(\epsilon_{\nu} - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} = 1$$

$$v_{\nu}^2 = \frac{1}{2} \left\{ 1 - \frac{\epsilon_{\nu} - \lambda}{[(\epsilon_{\nu} - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\}$$

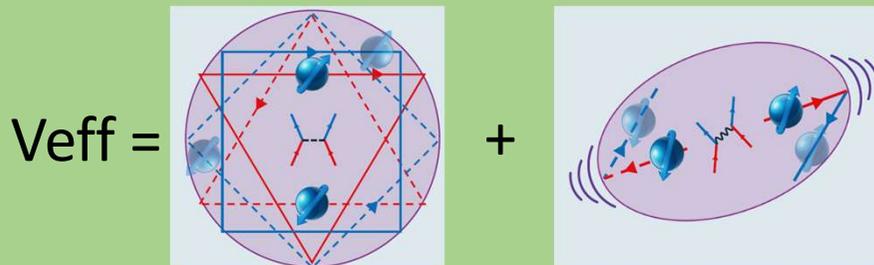
$$\sum_{\nu>0} \left\{ 1 - \frac{\epsilon_{\nu} - \lambda}{[(\epsilon_{\nu} - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\} = N$$



## Pairing Rotations and PVC

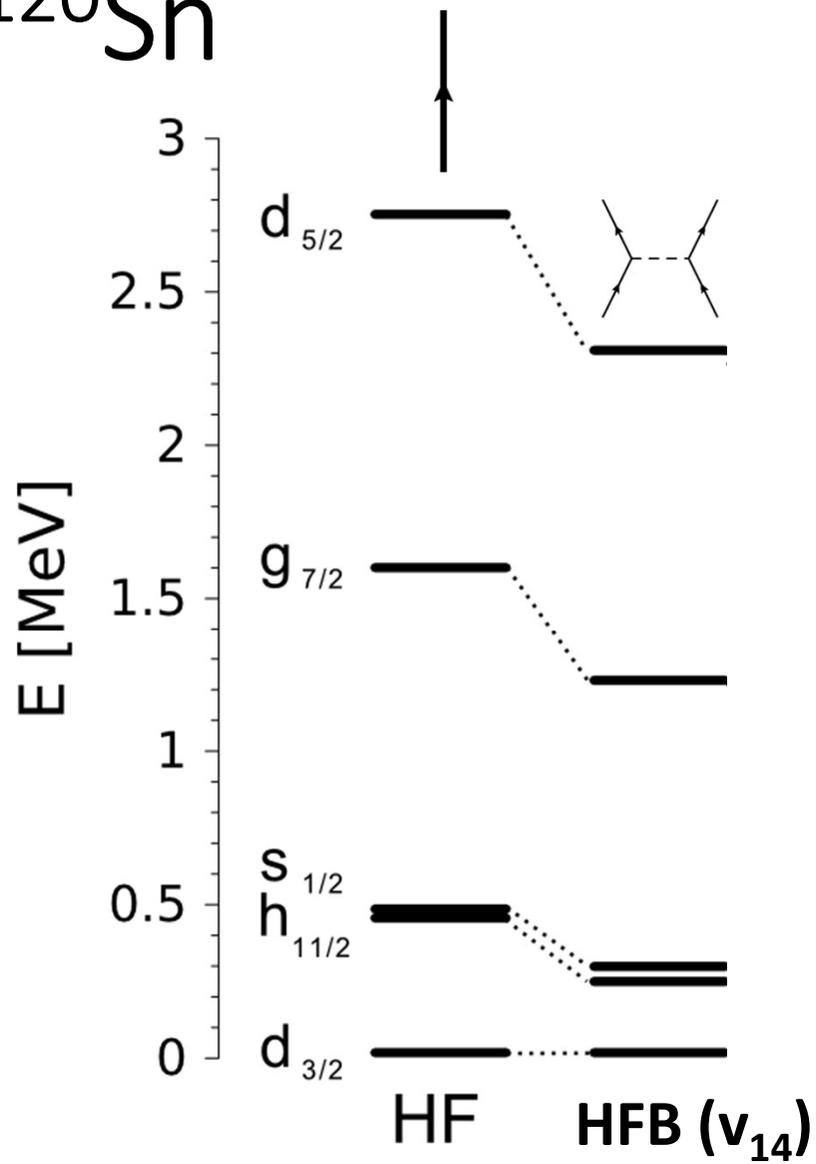
$$\tilde{\Delta}_{a(n)} = -Z_{a(n)} \sum_{b,m} V_{\text{eff}}[a(n)b(m)] N_{b(m)} \frac{\tilde{\Delta}_{b(m)}}{2\tilde{E}_{b(m)}}.$$

$$\tilde{E}_{a(n)} = \sqrt{(\tilde{\epsilon}_{a(n)} - \epsilon_F)^2 + \tilde{\Delta}_{a(n)}^2}$$



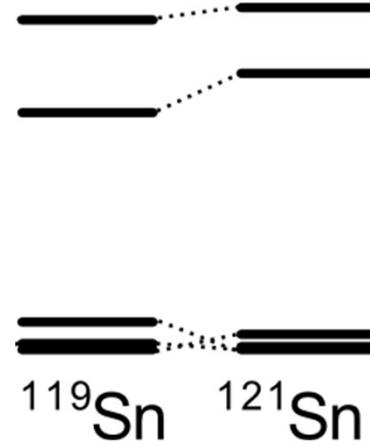
$$\Delta = \frac{G}{2} \sum_{\nu>0} \frac{\Delta}{[(\epsilon_\nu - \lambda)^2 + \Delta^2]^{\frac{1}{2}}}$$

$^{120}\text{Sn}$



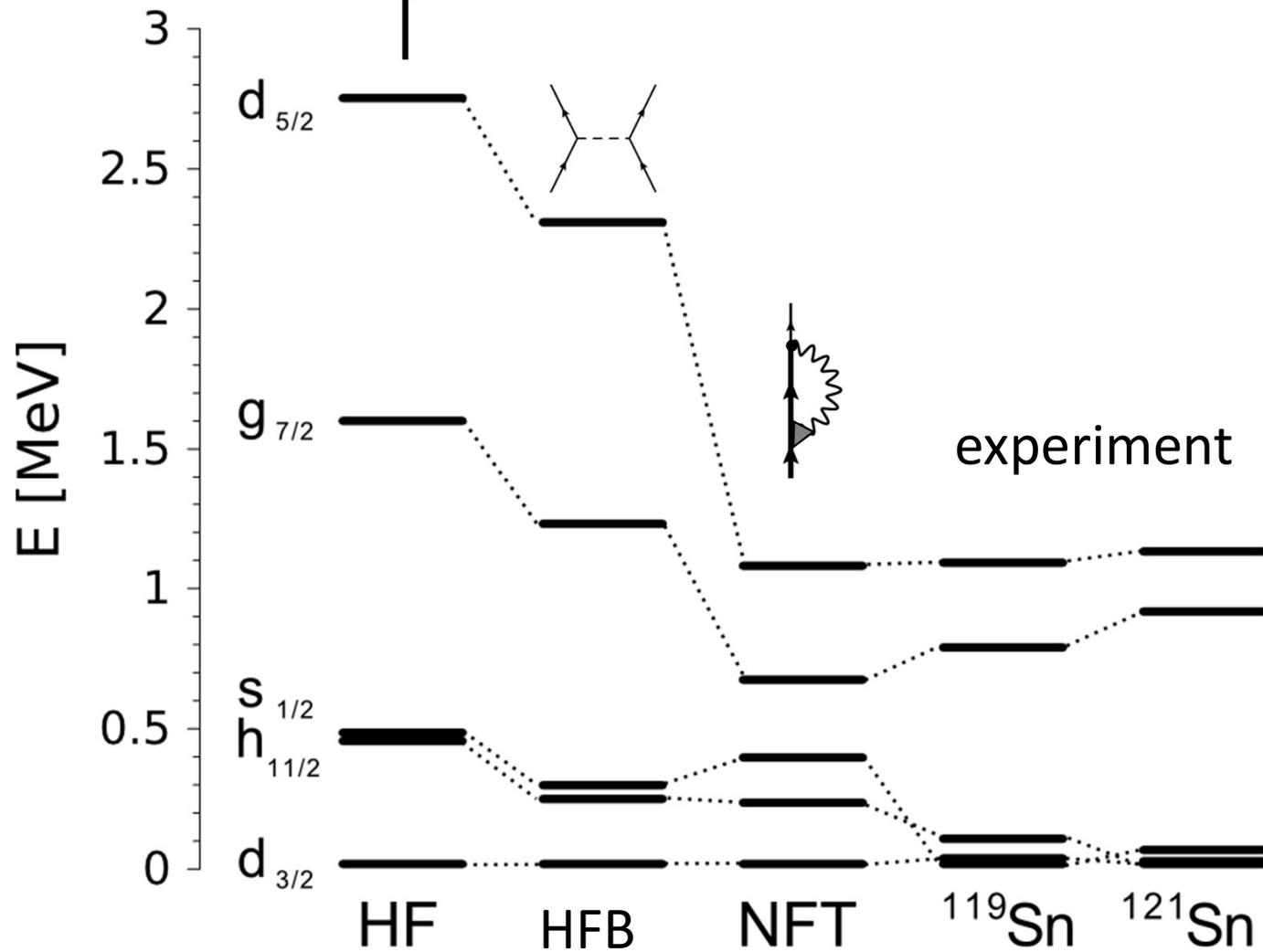
excitation spectrum

experiment

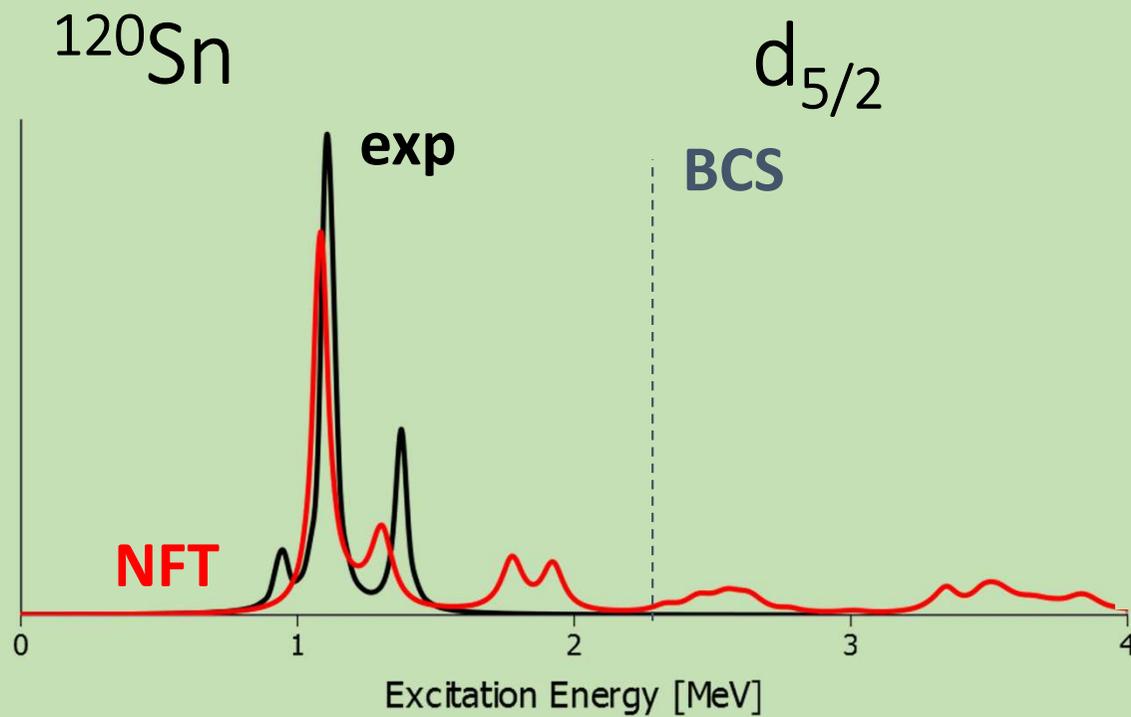


$^{120}\text{Sn}$

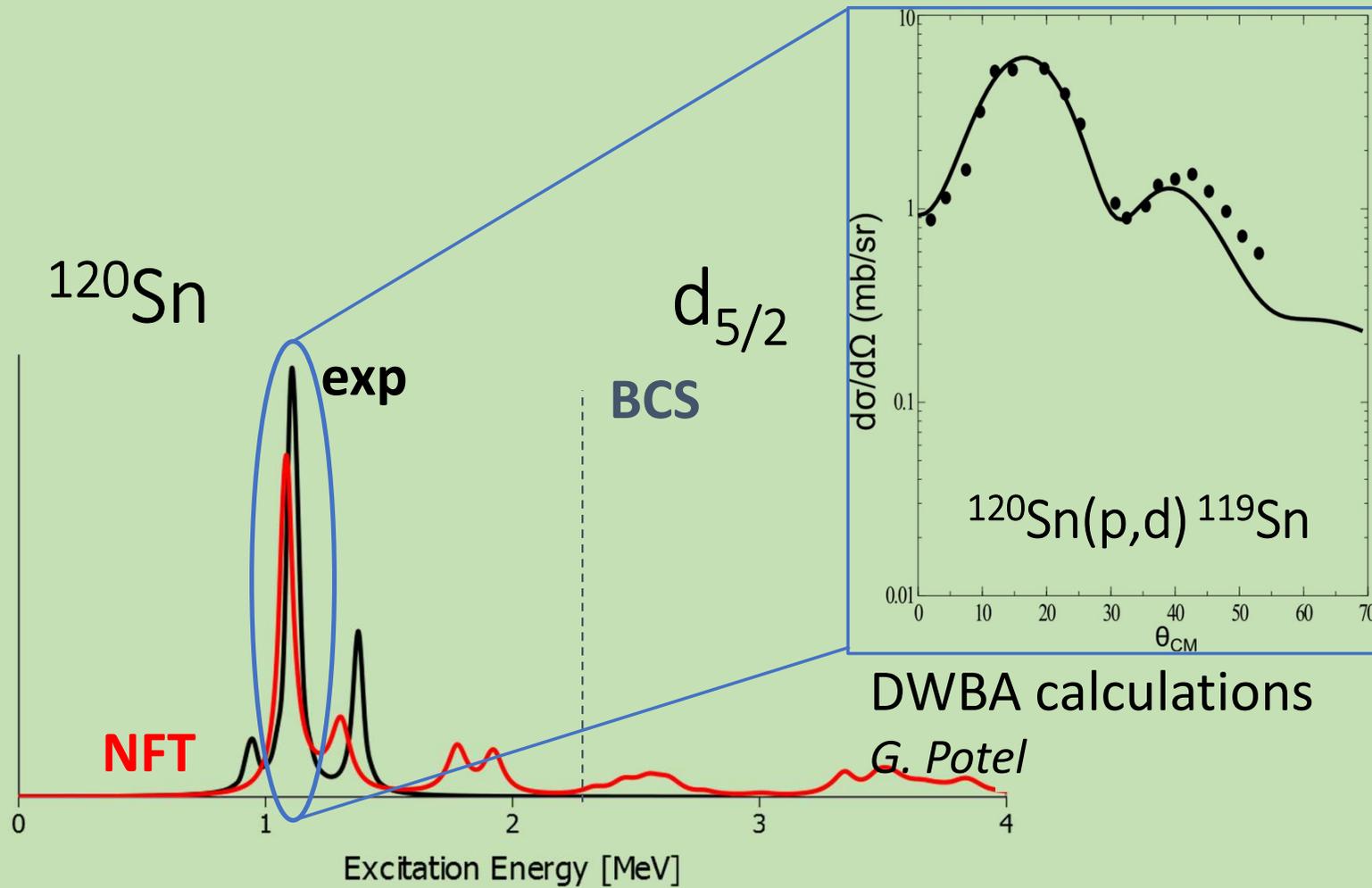
# excitation spectrum



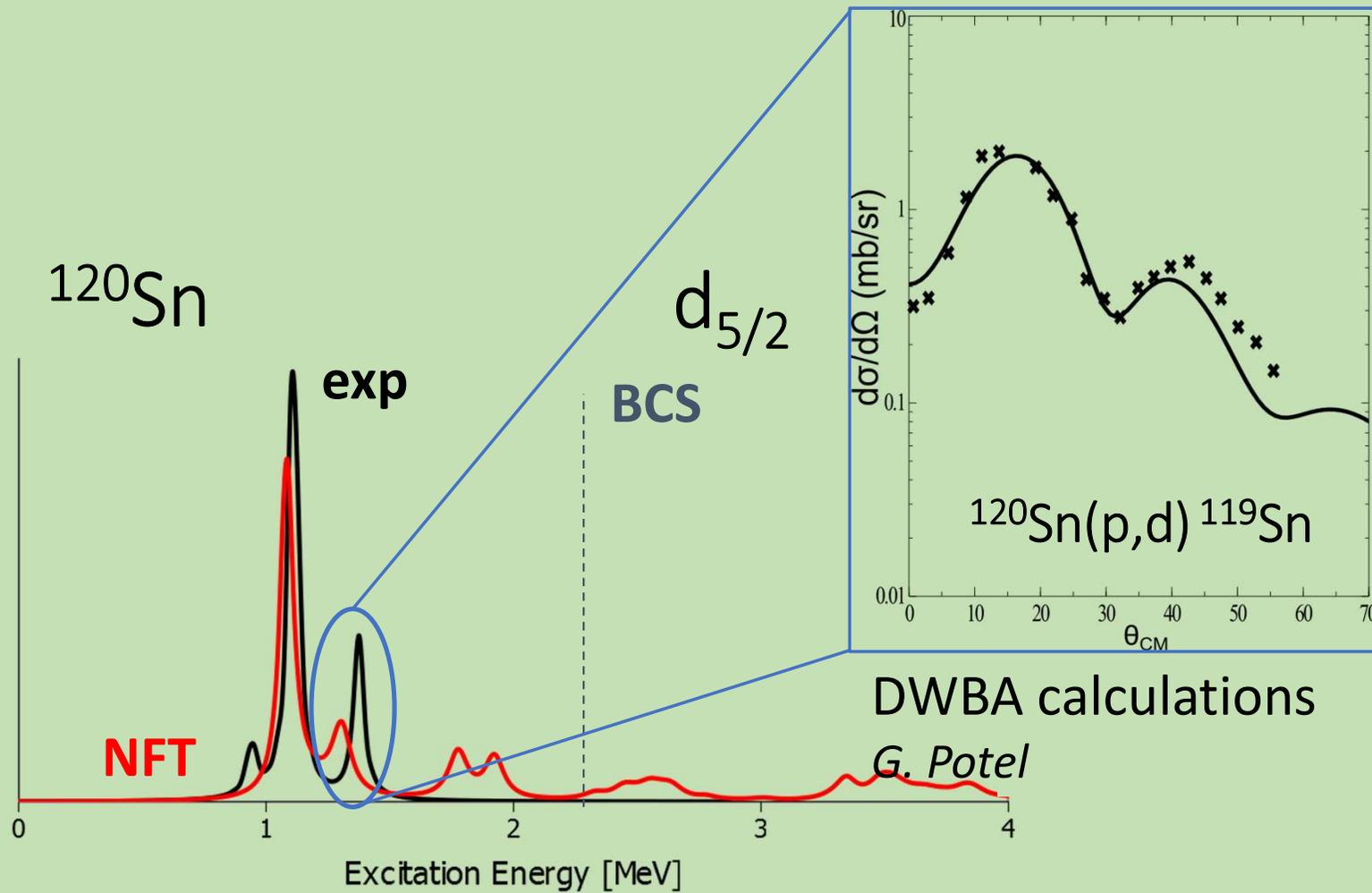
# Quasiparticle strength can now be compared with experiment

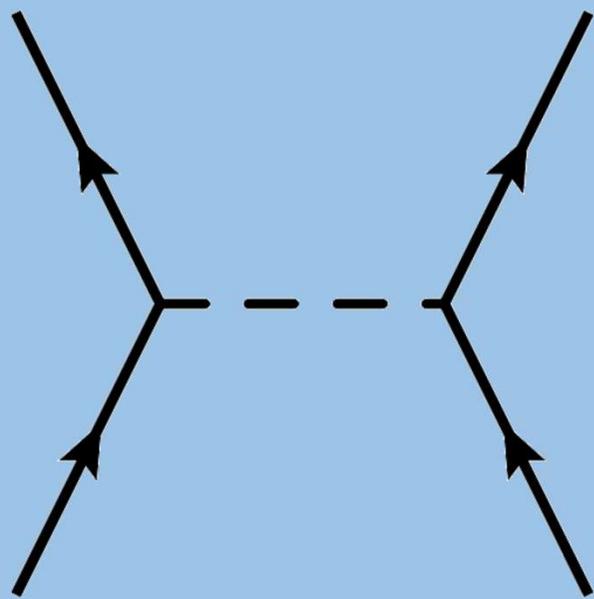


# Quasiparticle strength can now be compared with experiment

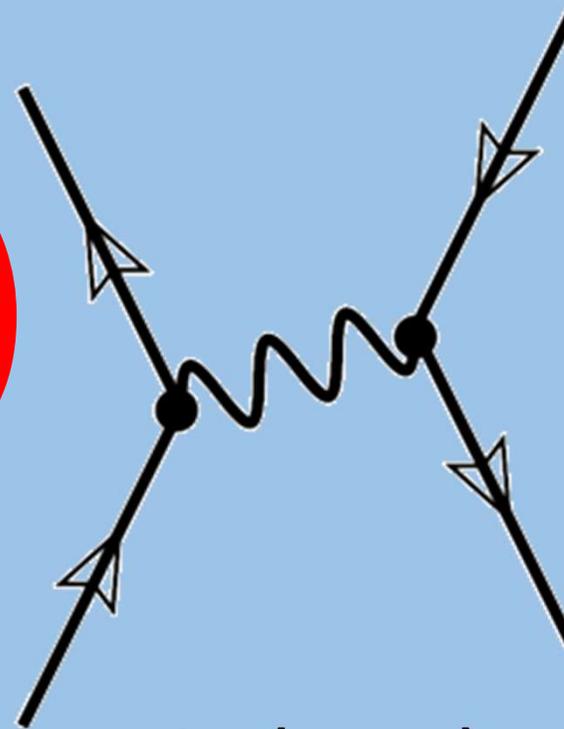
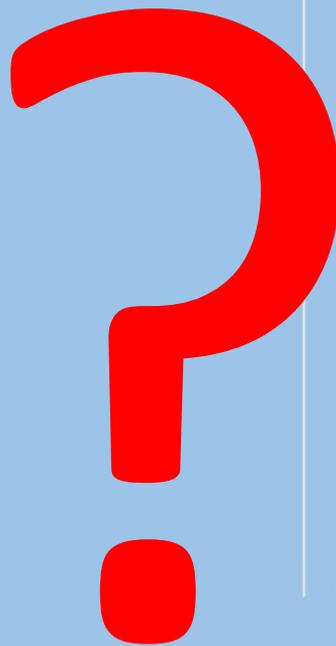


# Quasiparticle strength can now be compared with experiment





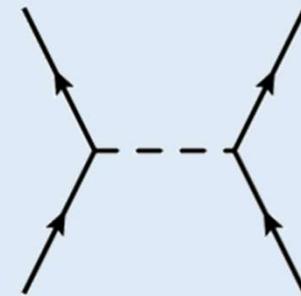
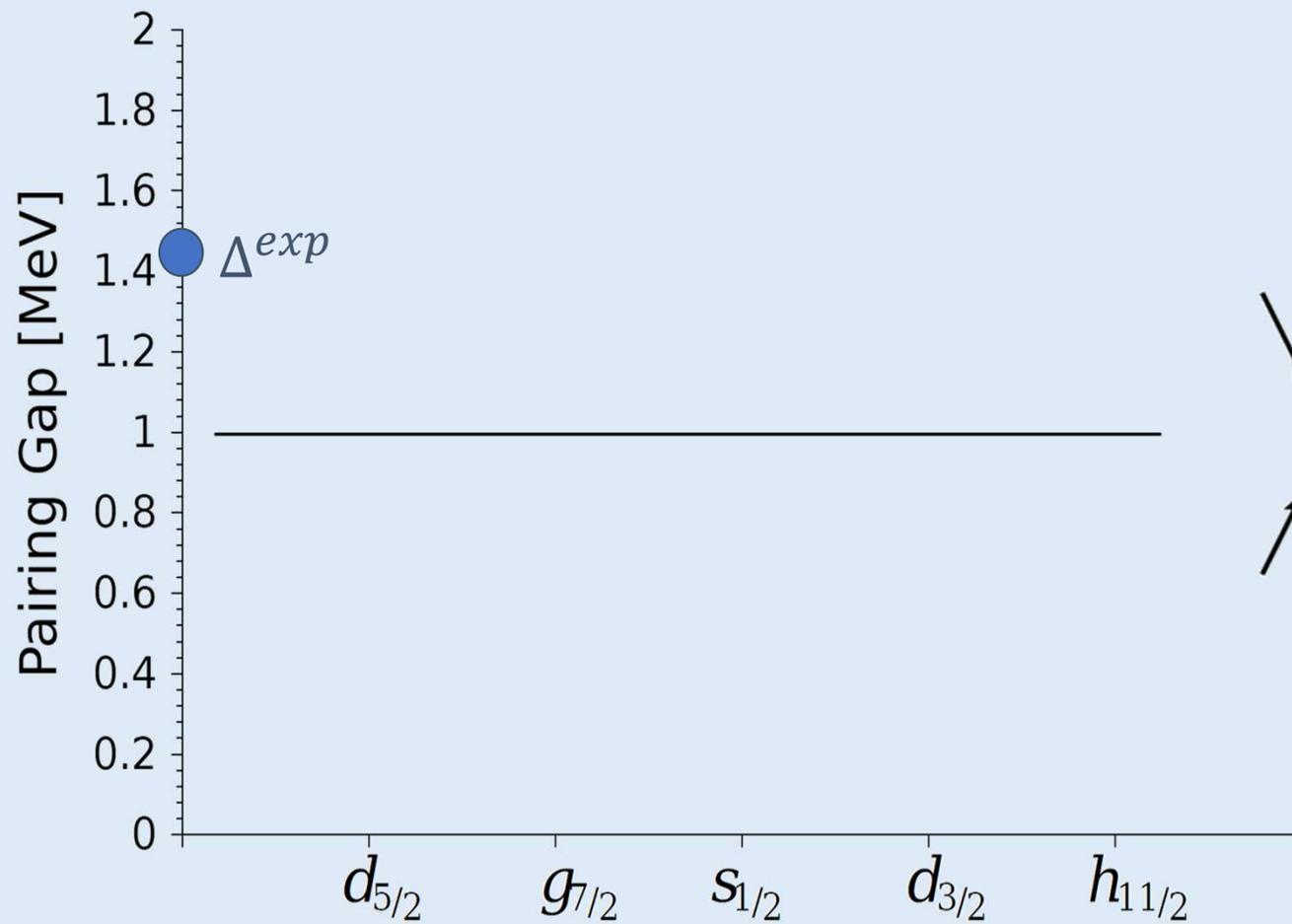
Bare  
Nucleon-Nucleon  
Interaction



Induced  
Interaction

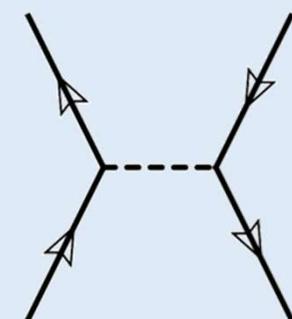
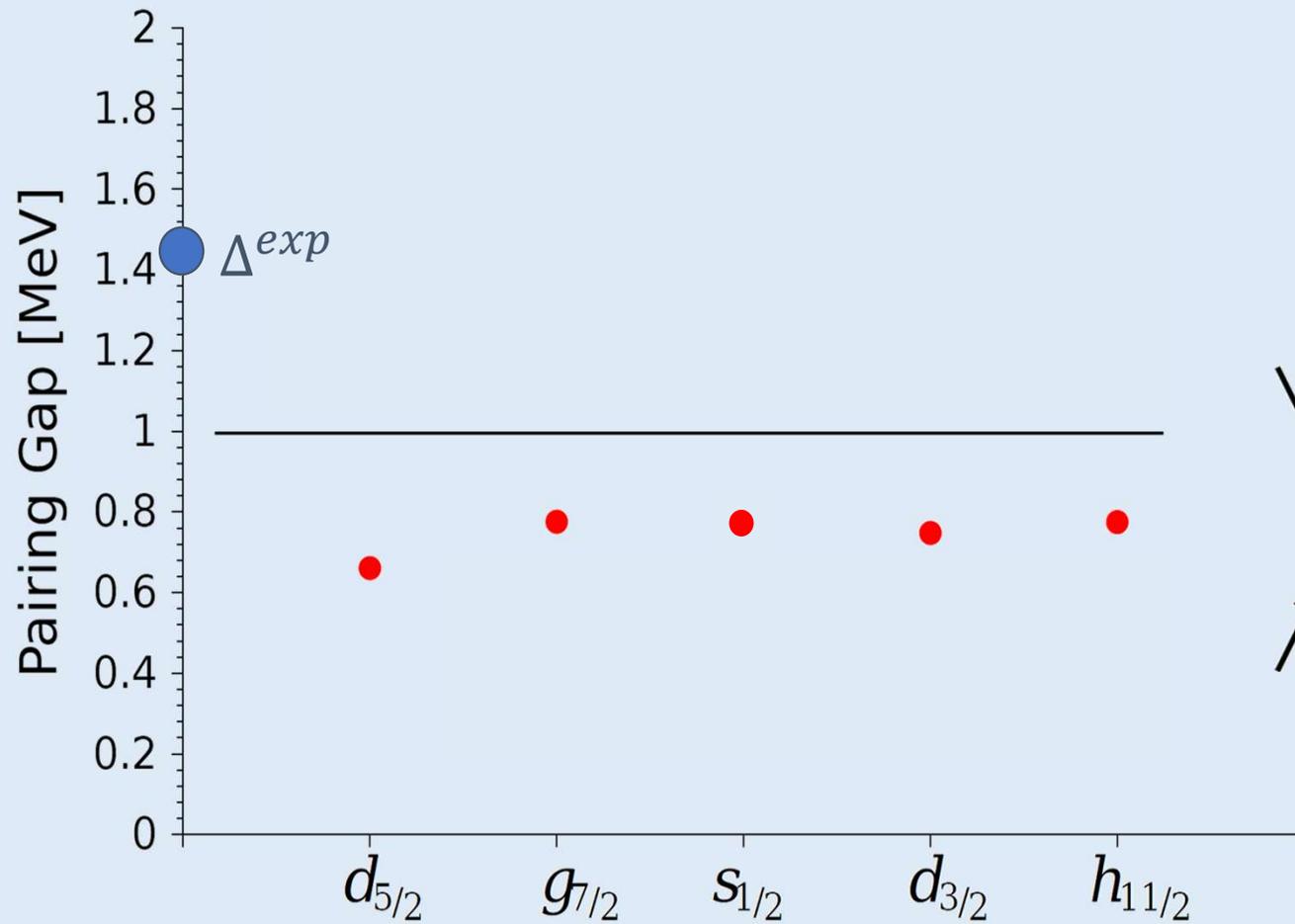
— Bare - BCS

We start from a bare  
 $v_{14}$  pairing interaction...



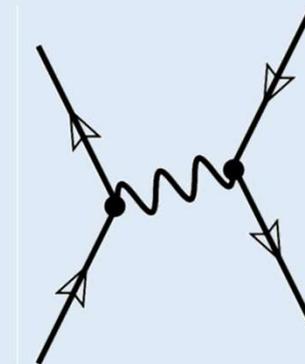
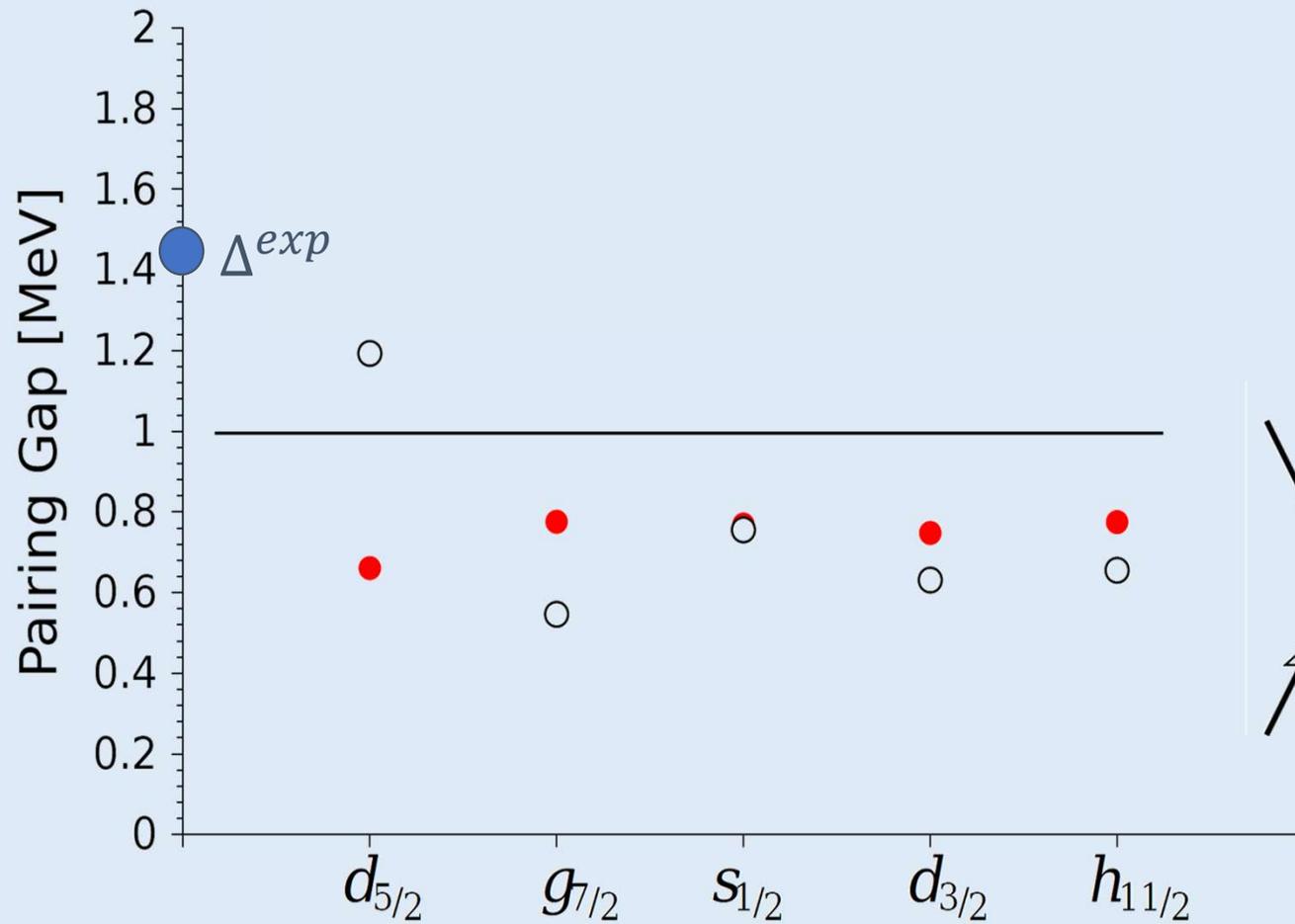
- Bare - BCS
- Bare – renorm.

...that get **renormalized**  
by Self Energy processes...



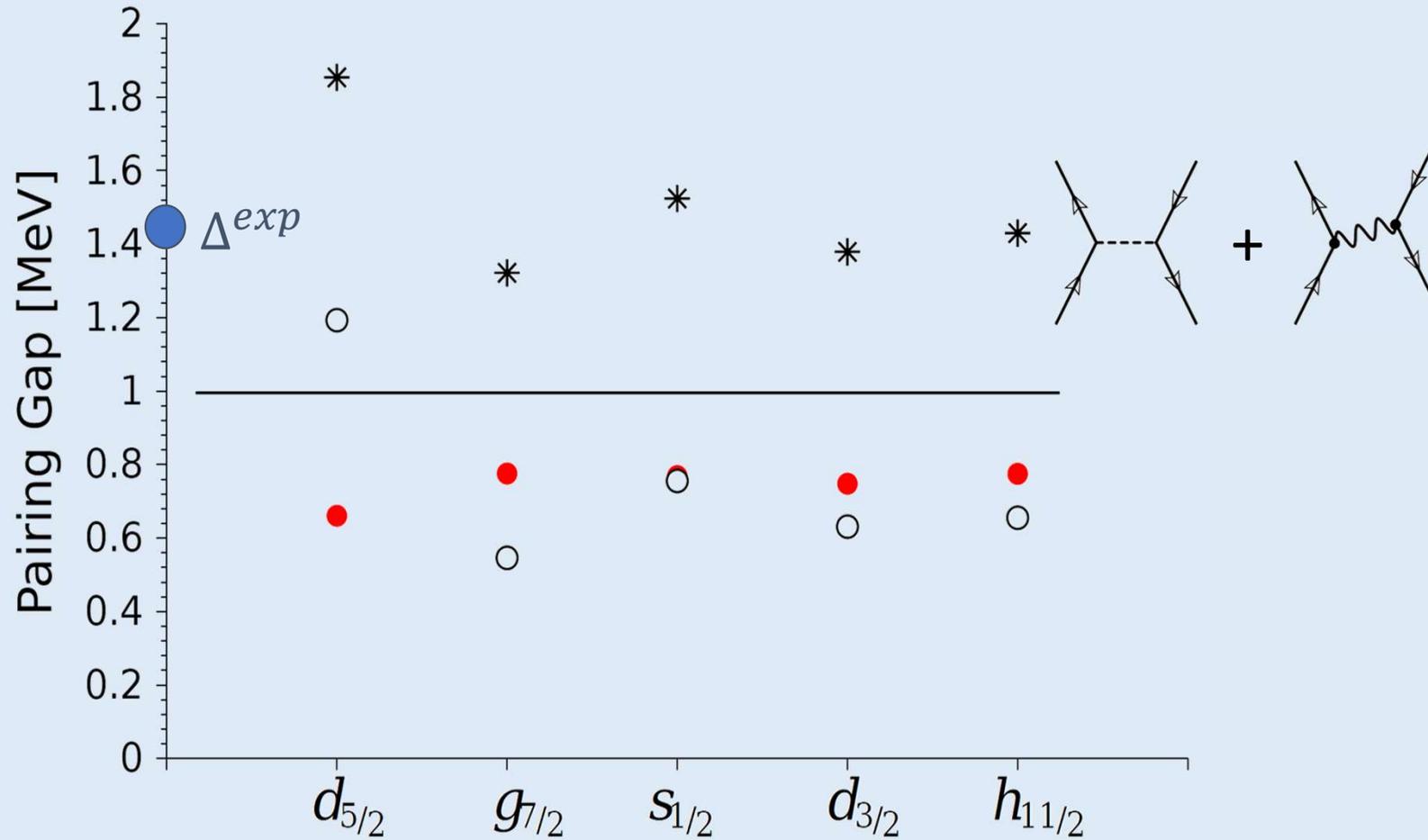
- Bare - BCS
- Bare – renorm.
- Induced

...but a novel contribution arises:  
**induced interaction (correlations)...**



- Bare - BCS
- Bare - renorm.
- Induced
- \* Total = Bare + Induced

...getting the total value close to the experiment!



We have a then consistent picture that explains

Independent particle **excitation energies**

$$\tilde{E}_j$$

Spectroscopic Factors

Pairing Gap

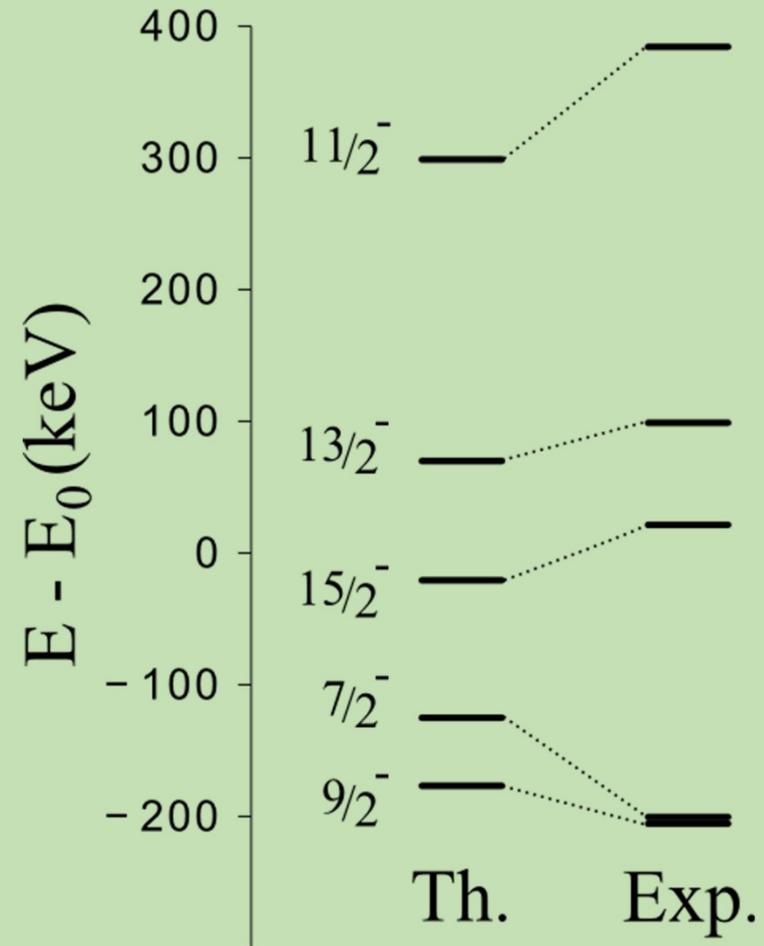
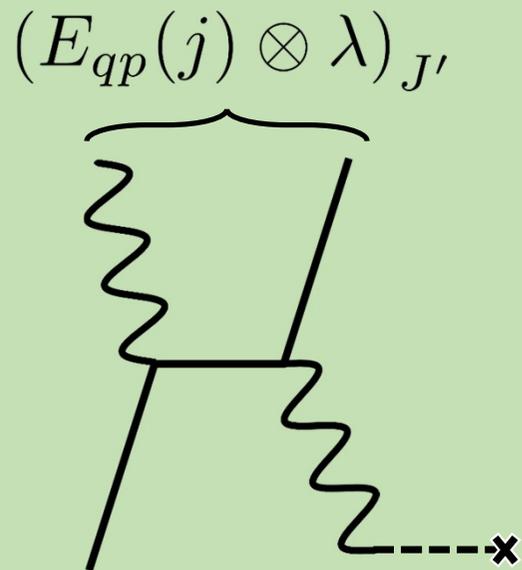
}

$$\tilde{u}_j \tilde{v}_j$$

These are inputs for...

# Multiplet

Elastic excitation of a  
quasiparticle state coupled  
to the core vibrations



Considering PVC we thus have:

- ☑ Increased Hartree-Fock excitation spectrum density.
- ☑ Introduced fragmentation of quasiparticle strength and compared with experimental 1 particle transfer cross sections.
- ☑ Increased the pairing correlations and pairing gap energy of realistic bare interaction closer to the experimental value, and reproduced the 2-particle transfer cross sections.
- ☑ Opened other reaction channels, like coupling of core excitations and quasiparticles.