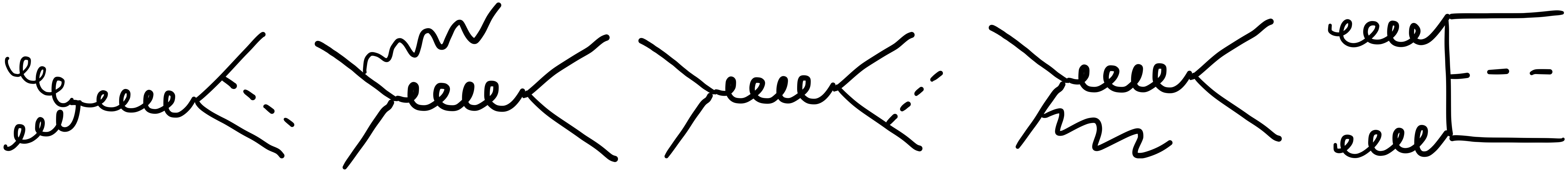


PRECISION QCD FOR ASSOCIATED TOP PRODUCTION

Genova, 03/04/2024

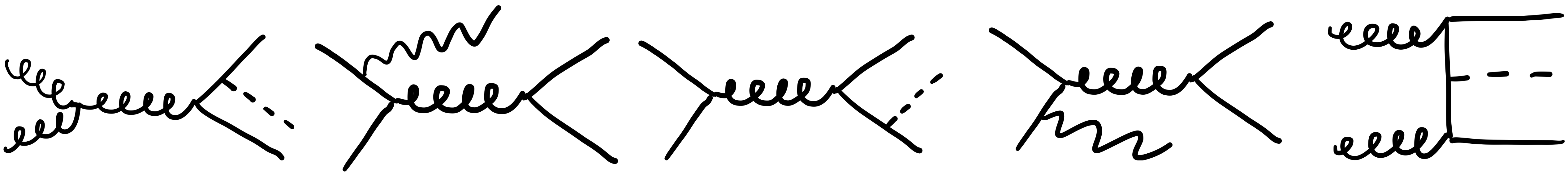
Simone Devoto

GHENT UNIVERSITY

FACULTY OF SCIENCES

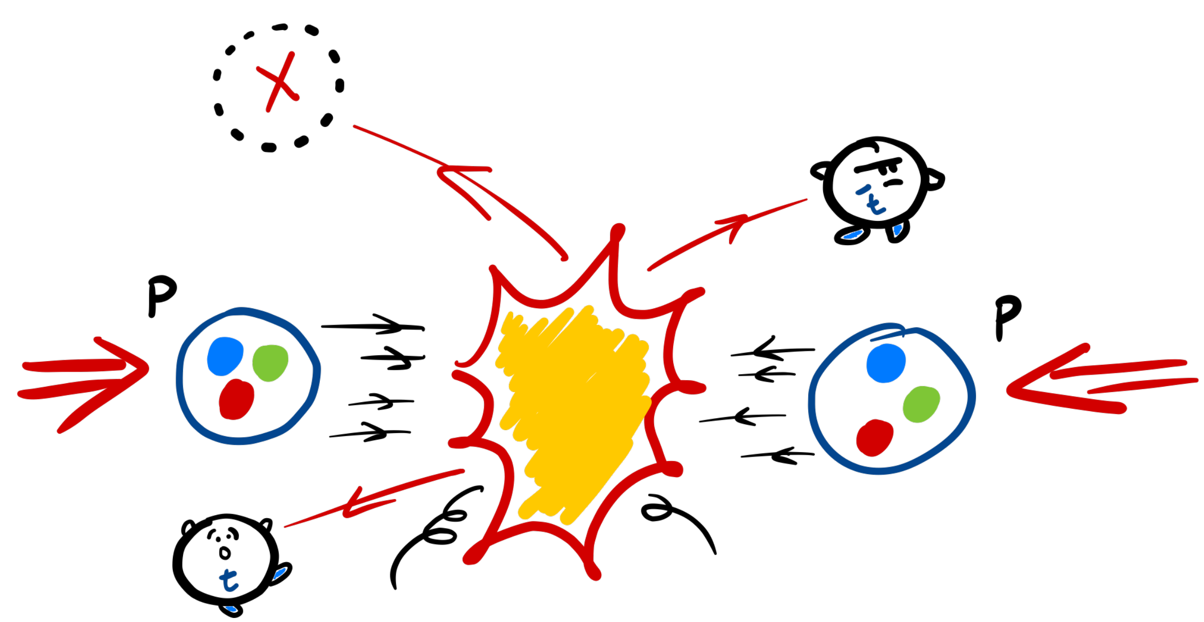
European Research Council
Established by the European Commission

In collaboration with:
L. Buonocore, S. Catani, M. Grazzini,
S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini

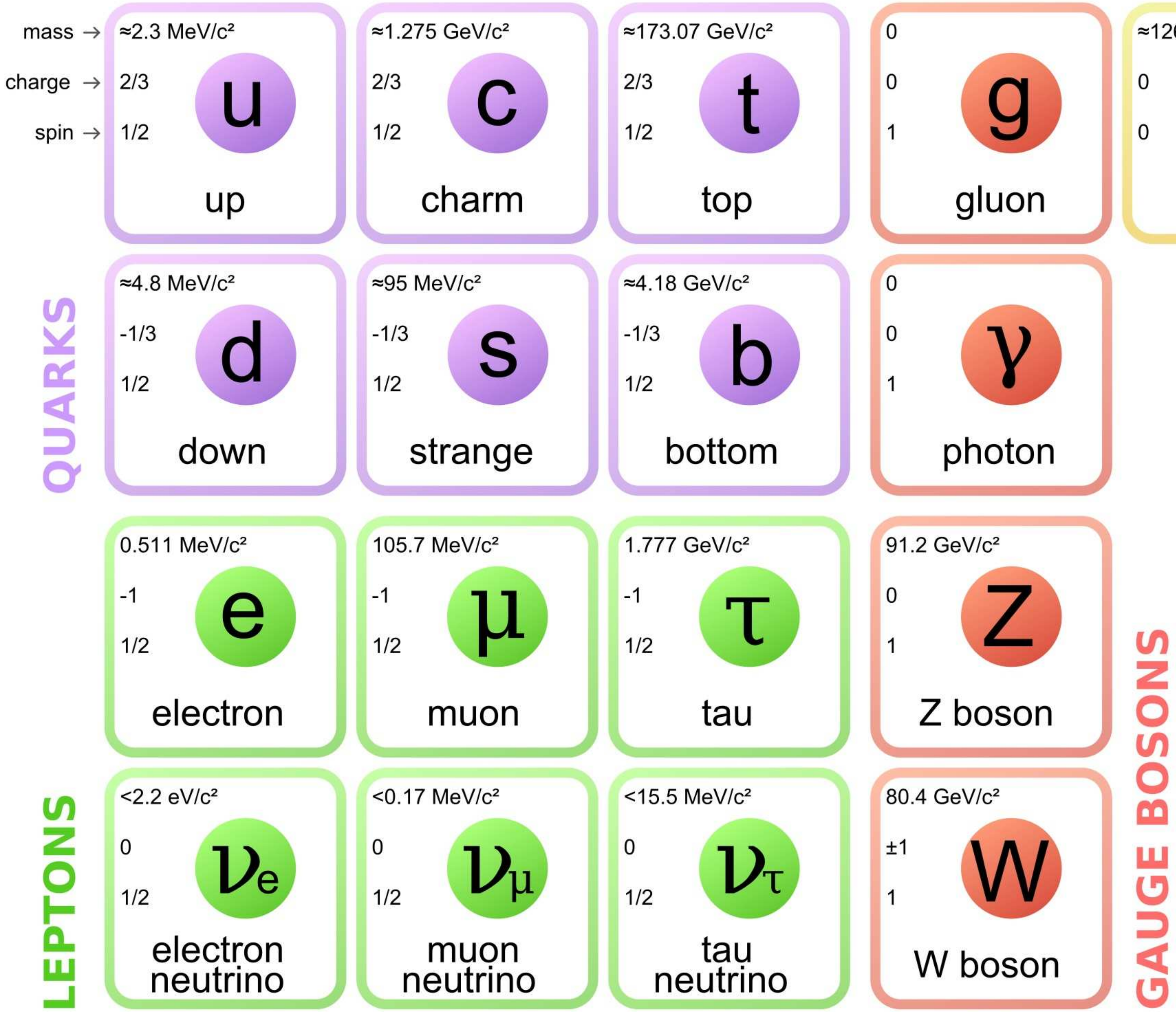


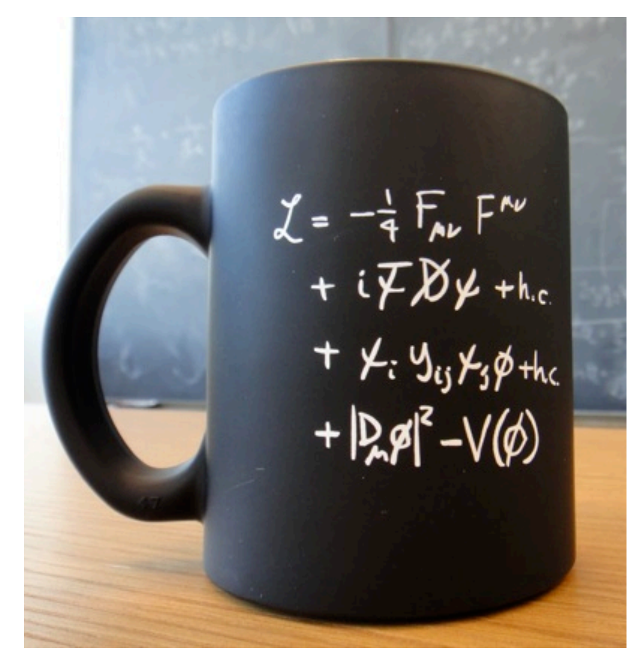
CONTENTS

- **Motivations**;
- Theory bottlenecks:
  - subtraction;
  - two-loop amplitudes;
- $t\bar{t}H$ @ **NNLO**;
- $t\bar{t}W$ @ **NNLO**;
- **Conclusions**.



THE STANDARD MODEL

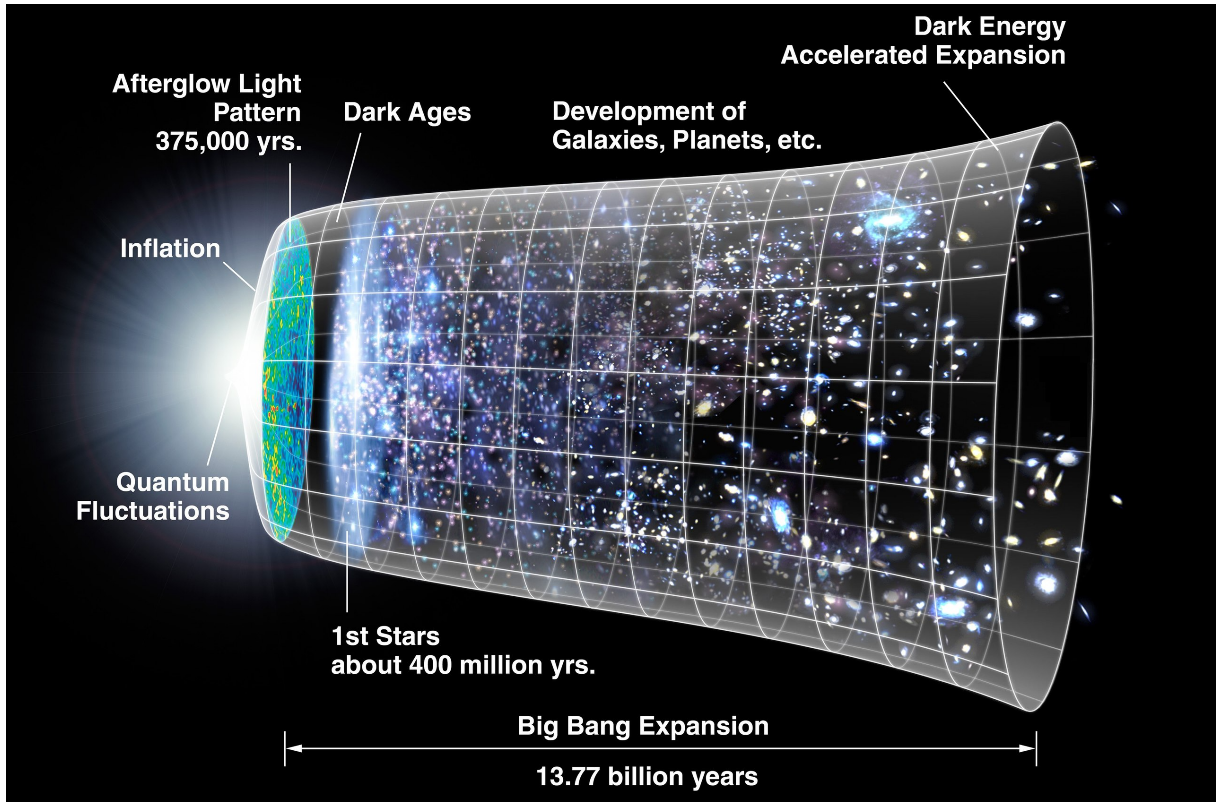




OPEN QUESTIONS

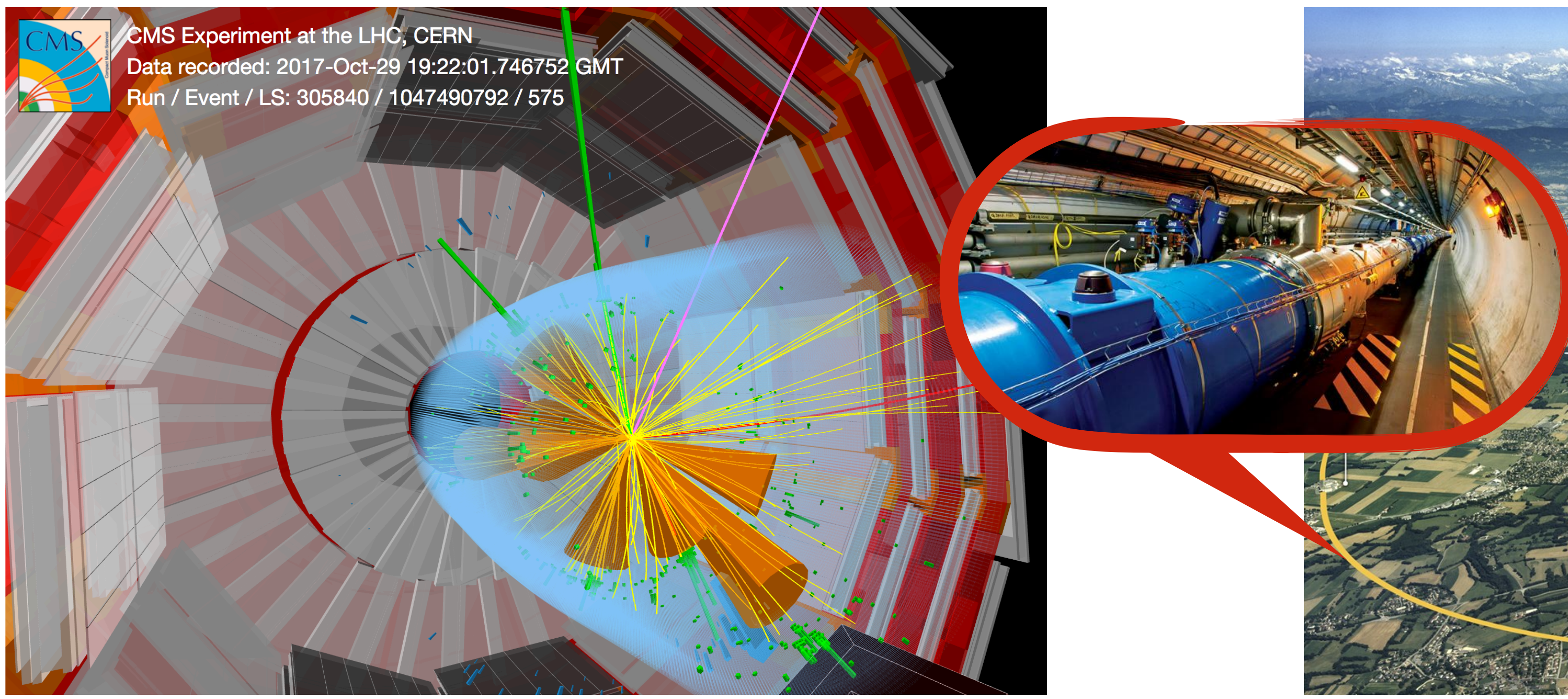
The Standard Model has still **open questions**:

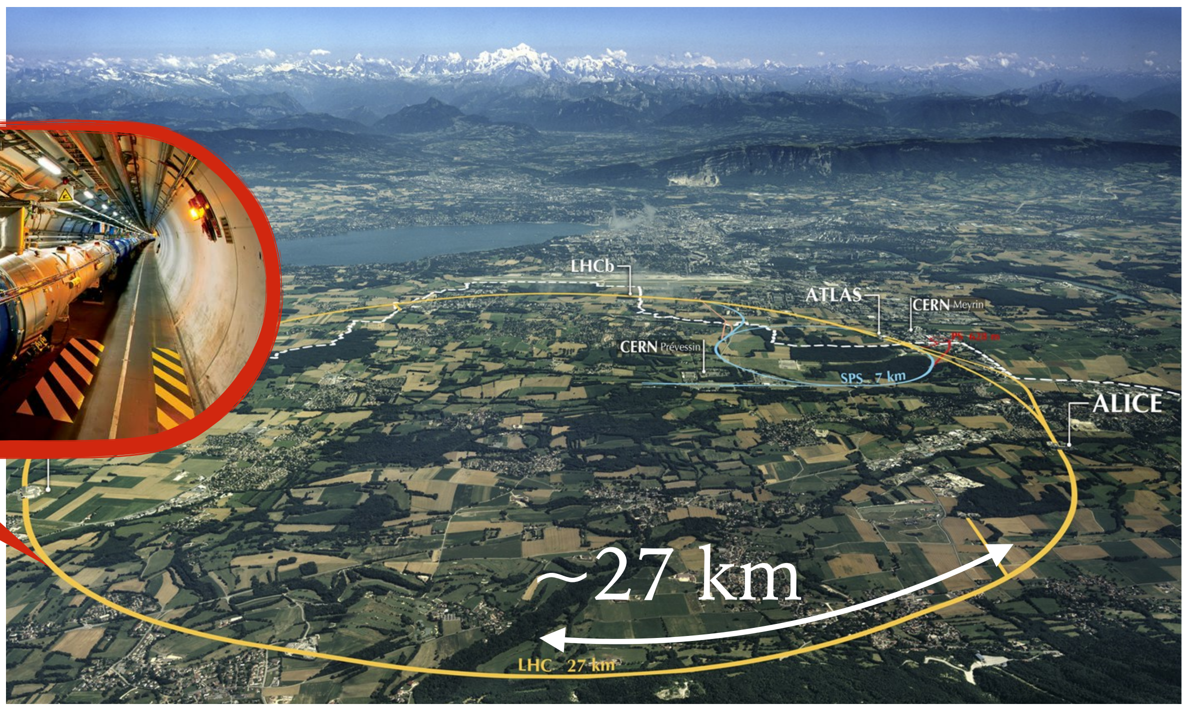
- How can we accommodate **Gravity**?
- How can we accomodate **Neutrinos masses**?
- What is the **Dark Matter**?
- What is the **Dark Energy**?
- Where is all the **antimatter** (origin of the matter-antimatter asymmetry)?
- …

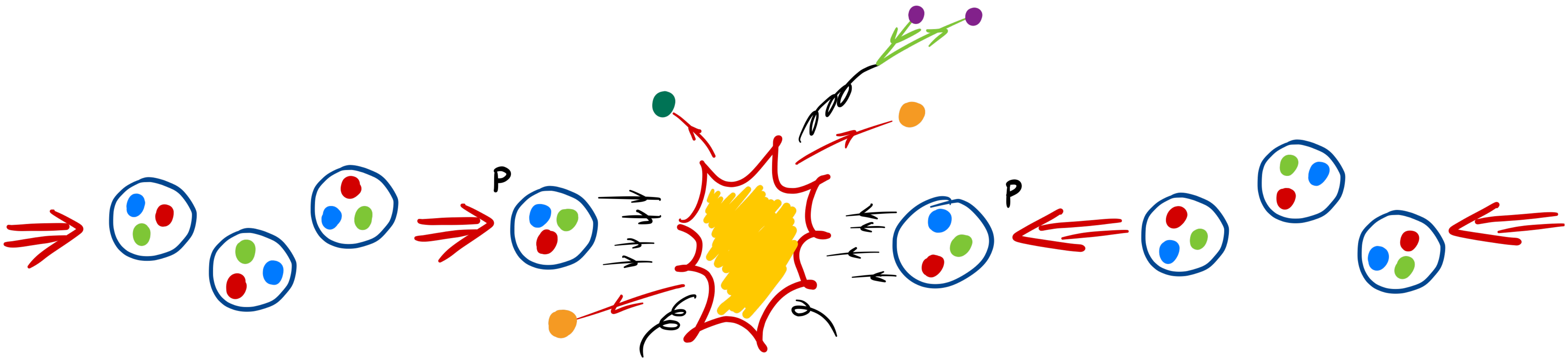


THE STANDARD MODEL AT LHC

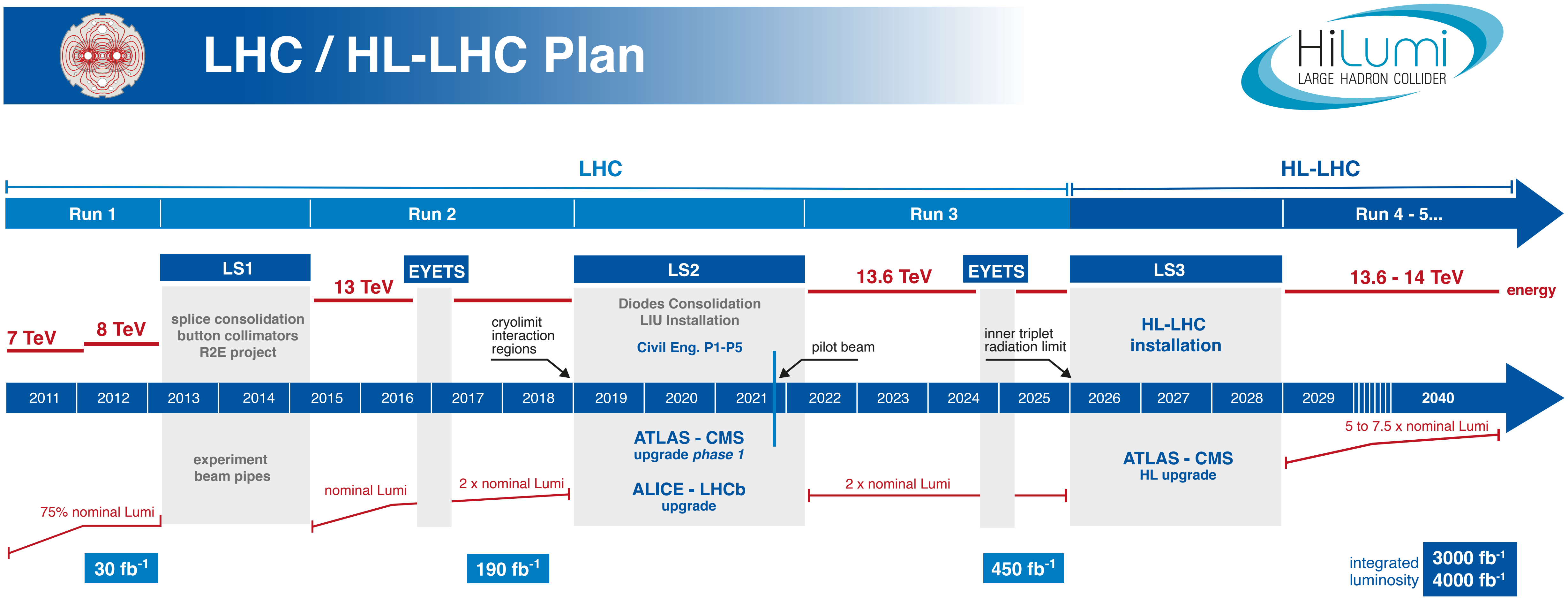
LHC: the biggest particle accelerator in the world!

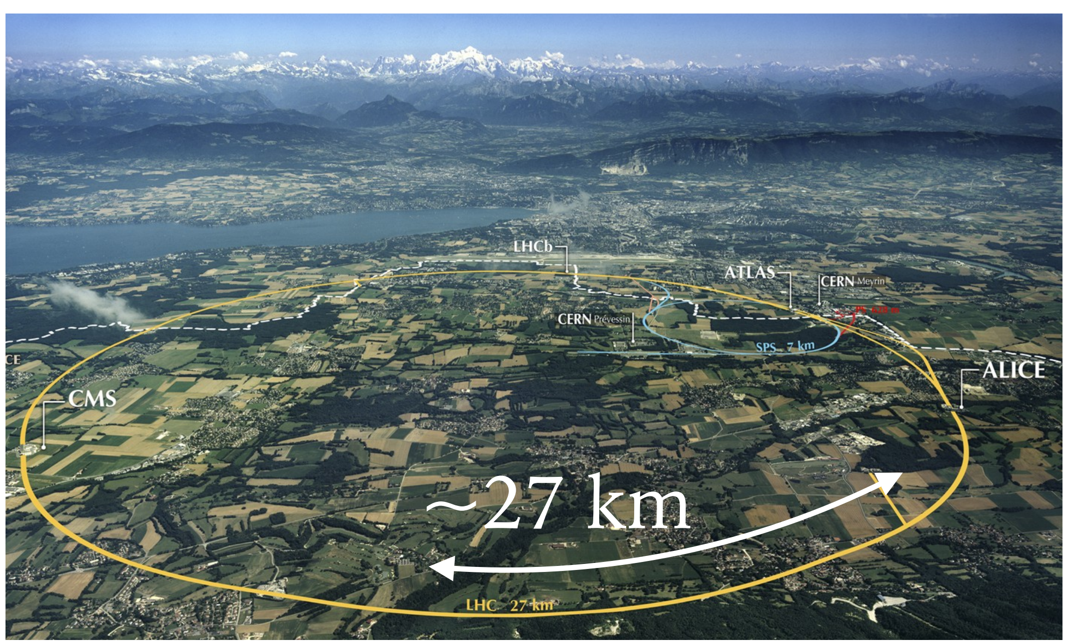




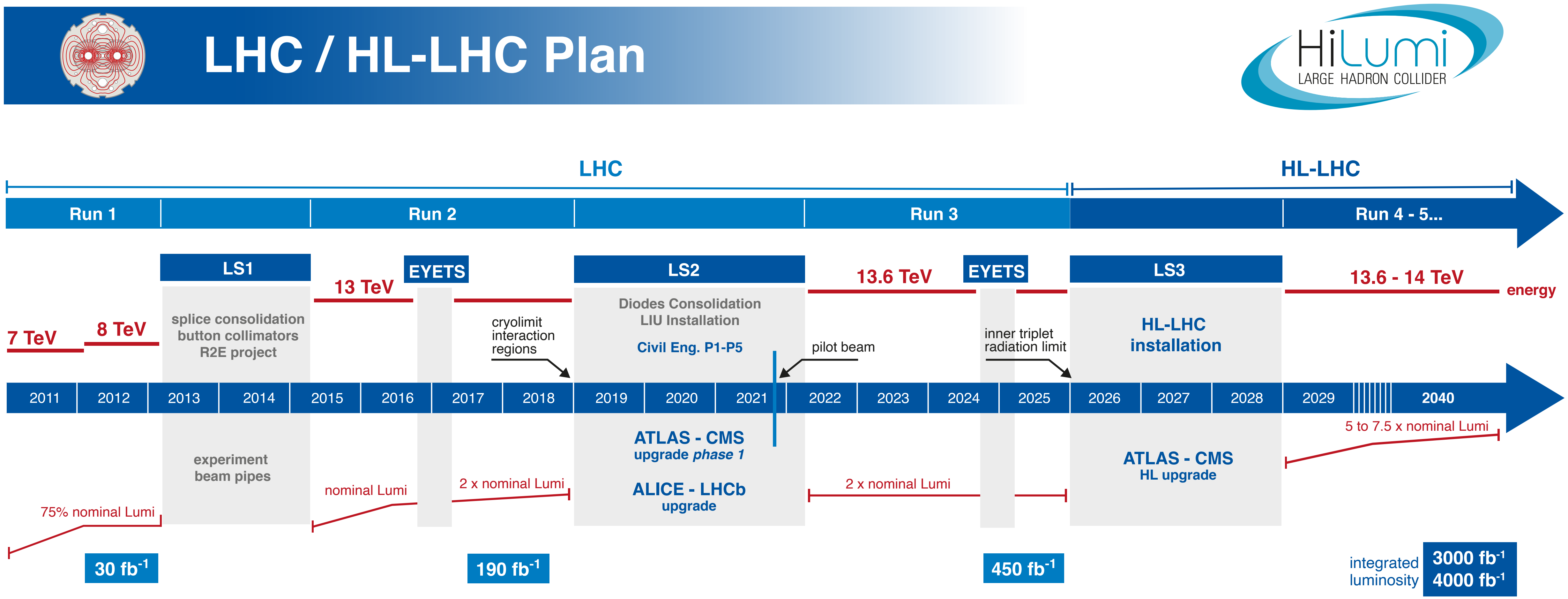


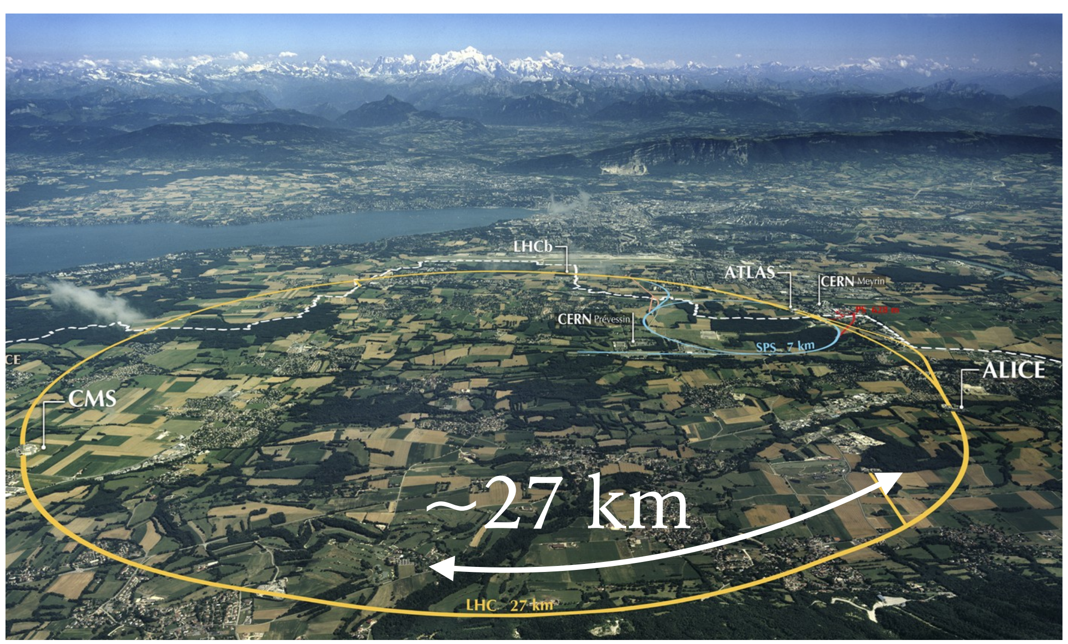
FUTURE OF LHC

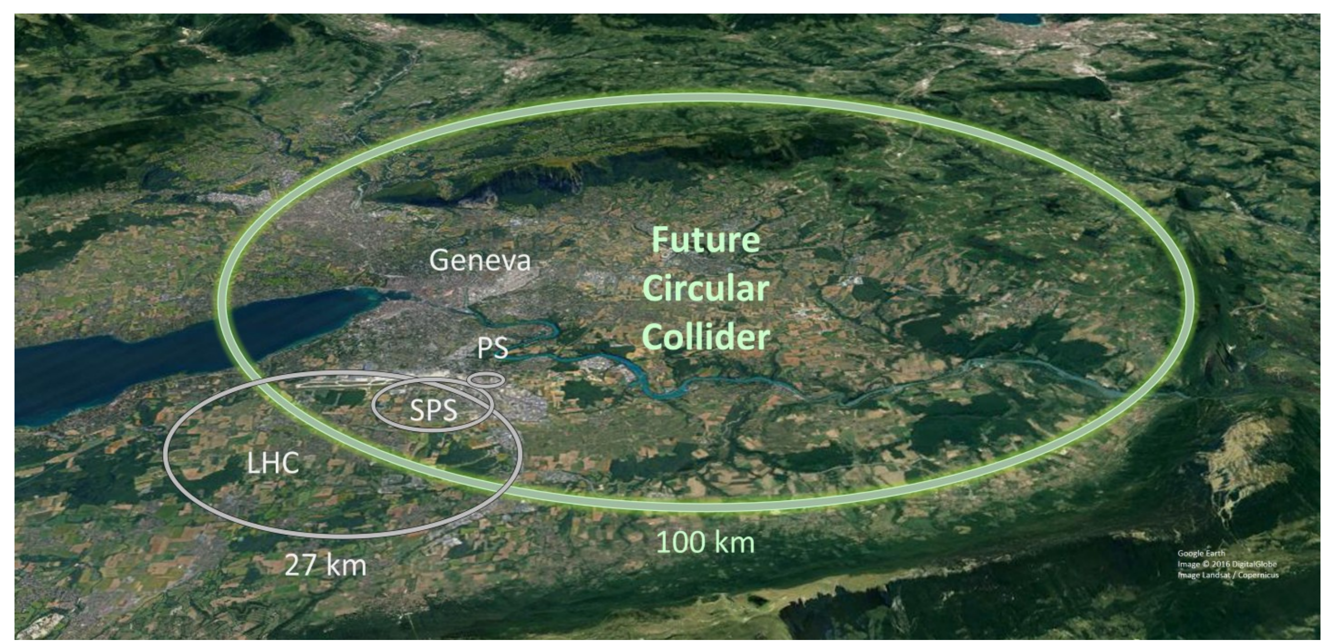




FUTURE OF LHC

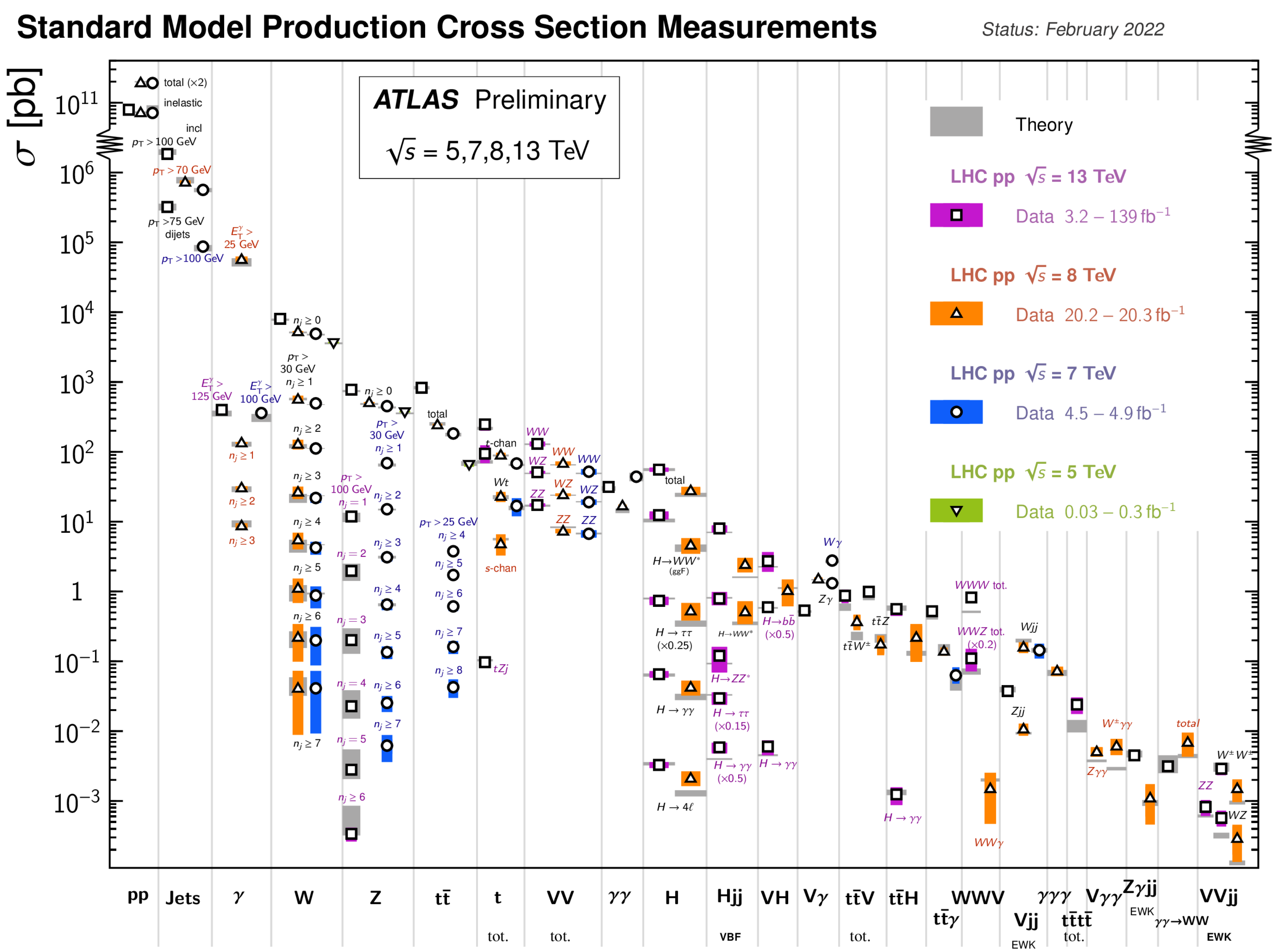






WHAT ARE WE LEARNING FROM LHC?

- LHC has proven itself to be:
  - A **discovery machine**: **Higgs boson** discovery;
  - A **precision machine**: **precise measurement** of SM parameters.



Model Building (BSM)

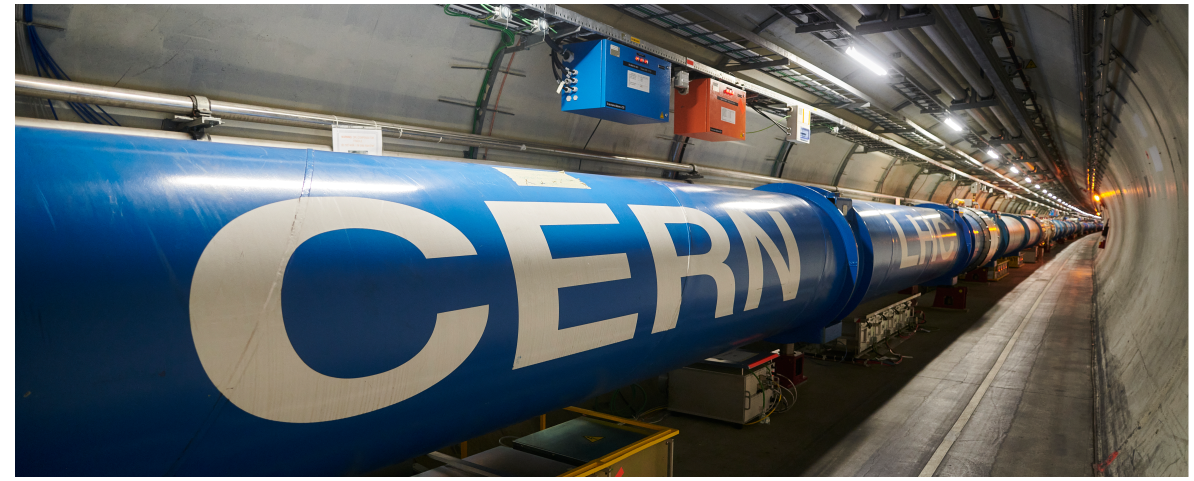
Create new models that go Beyond the Standard Model (BSM) trying to explain the open questions.

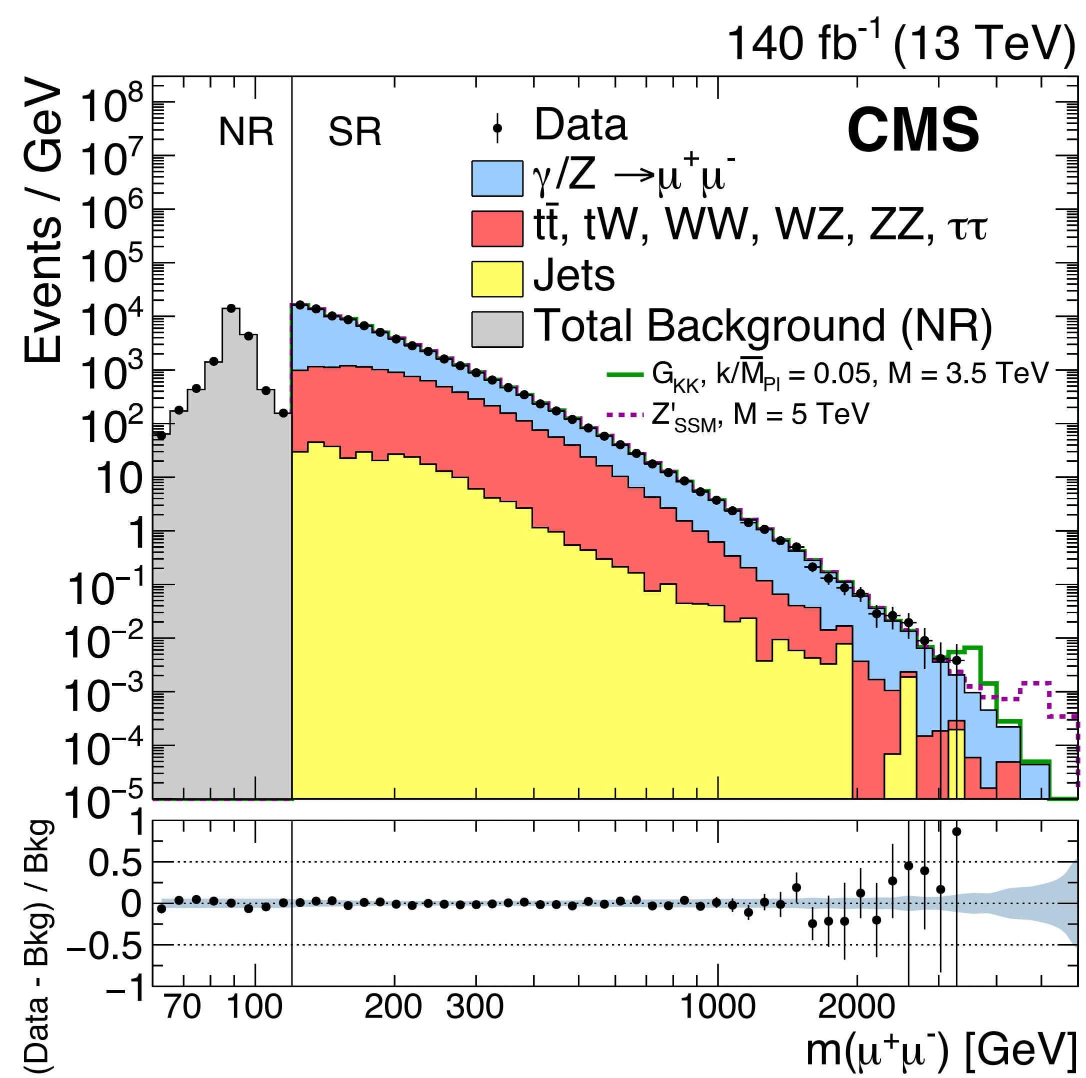
Precision Studies

Search for small deviations from the predictions of the Standard Model by systematically comparing the best theoretical predictions with data.

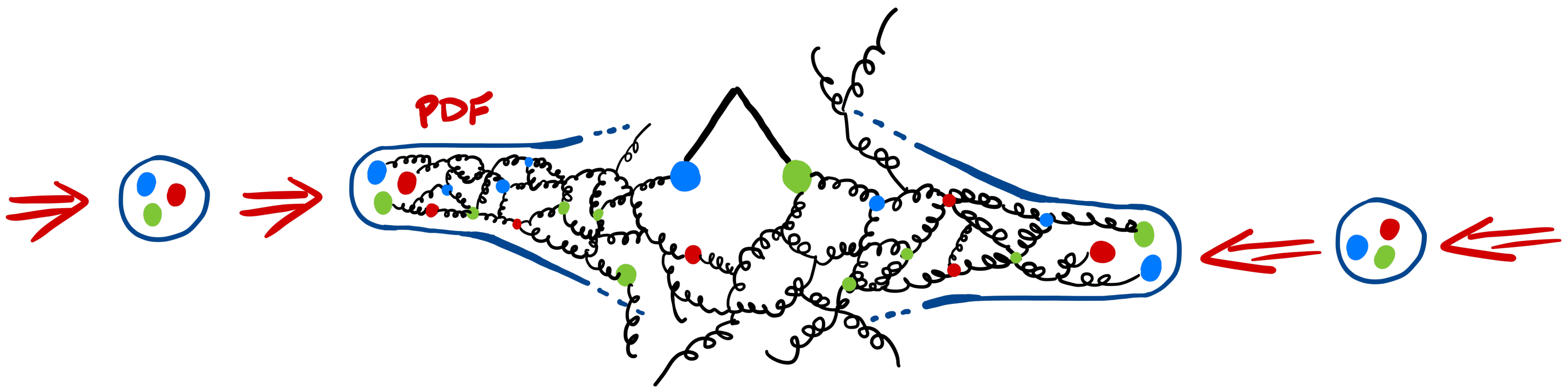
WHY PRECISION?

- **Run3** at LHC: factor of **2** increase of the data set;
- High Luminosity program (**HL-LHC**): factor of **10** increase of the data set;
- Dramatic **improvement in the experimental precision**, with an expected goal of **1% accuracy or better** in a key set of observables (**1‰ at FCC**!)
- **Theoretical predictions need to match experimental accuracy**.
- Need for the computation of **N3LO QCD**, **NNLO EW** and mixed **NNLO QCDxEW** correction.

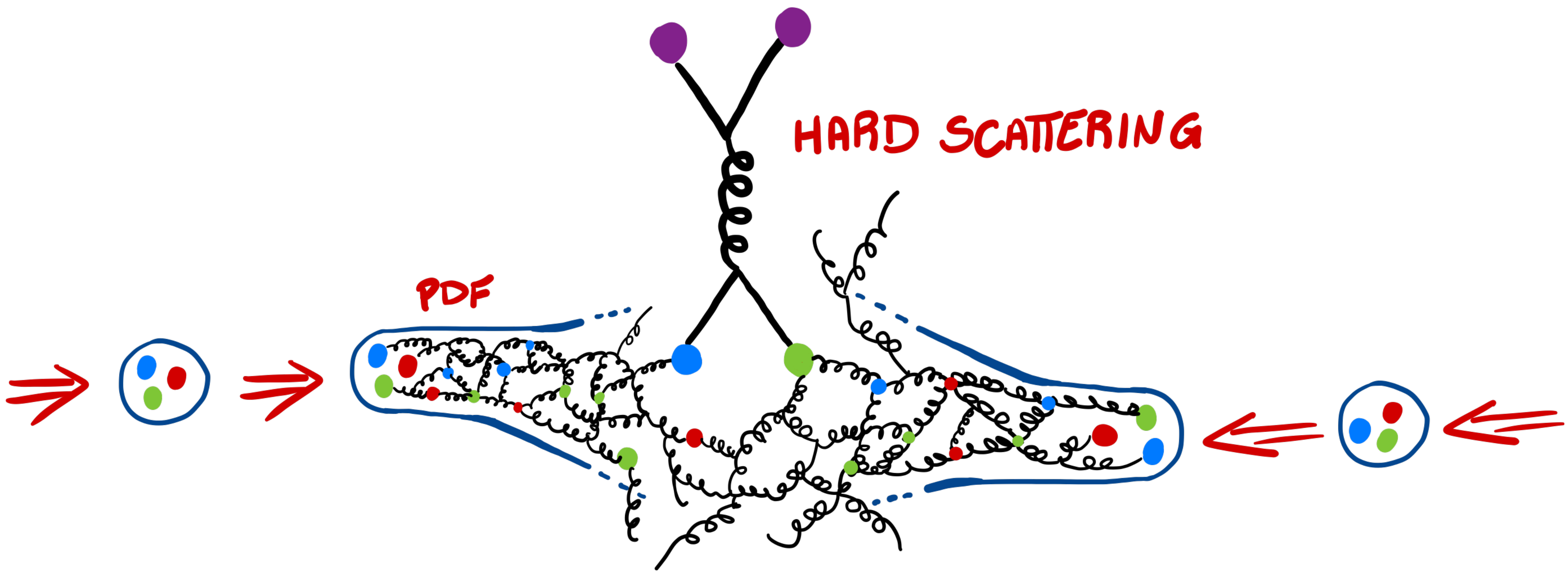




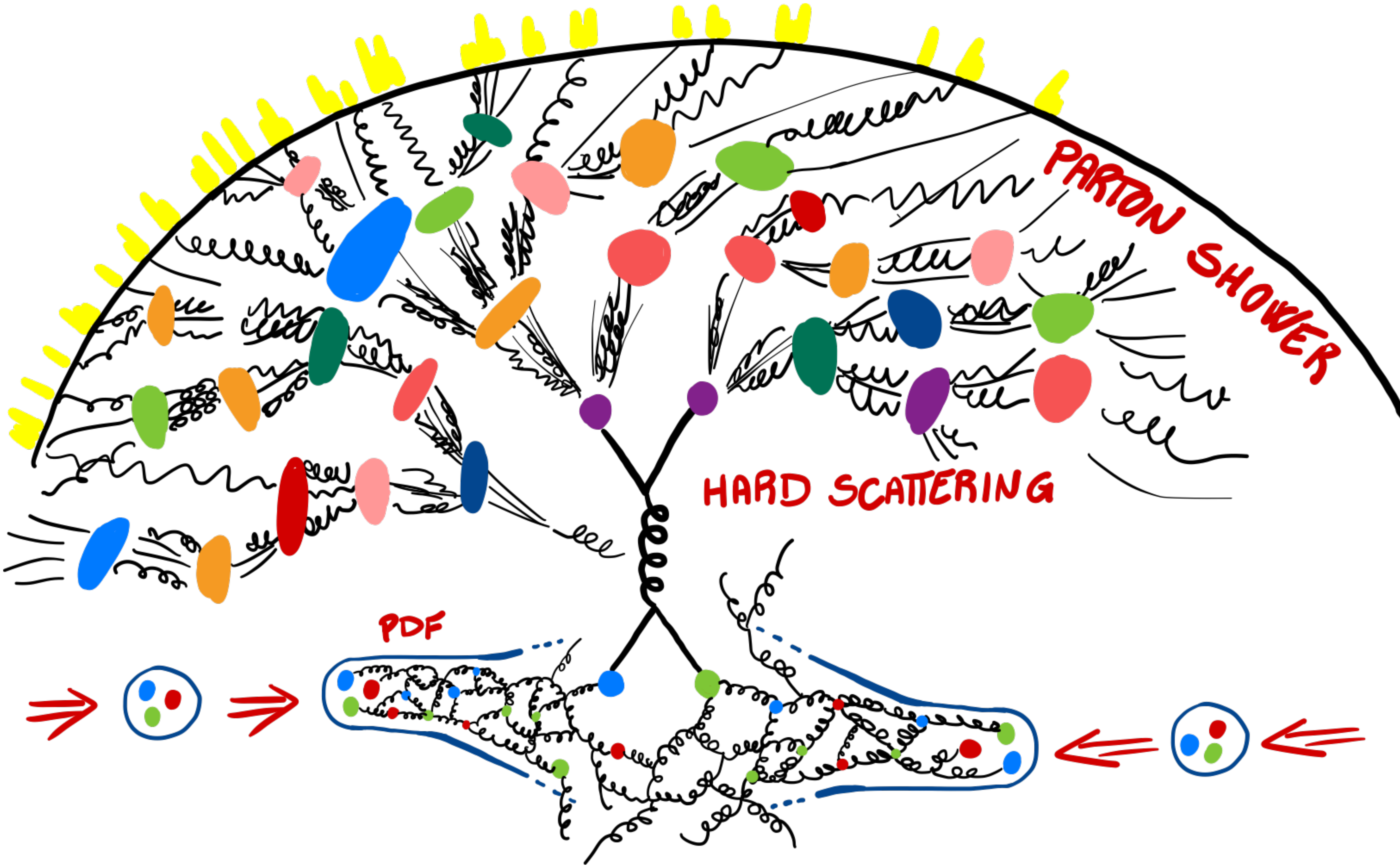
THEORETICAL INGREDIENTS



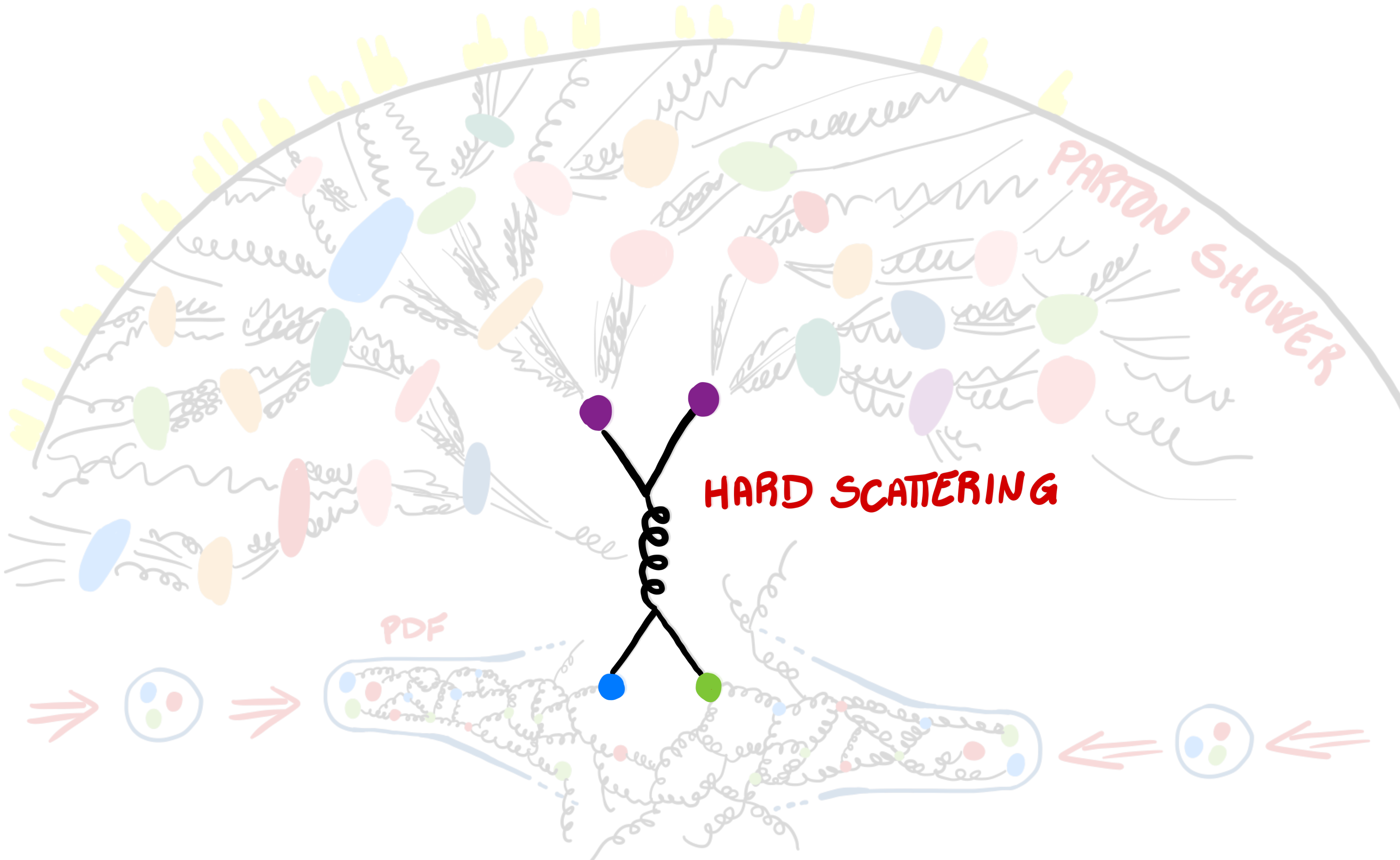
THEORETICAL INGREDIENTS



THEORETICAL INGREDIENTS

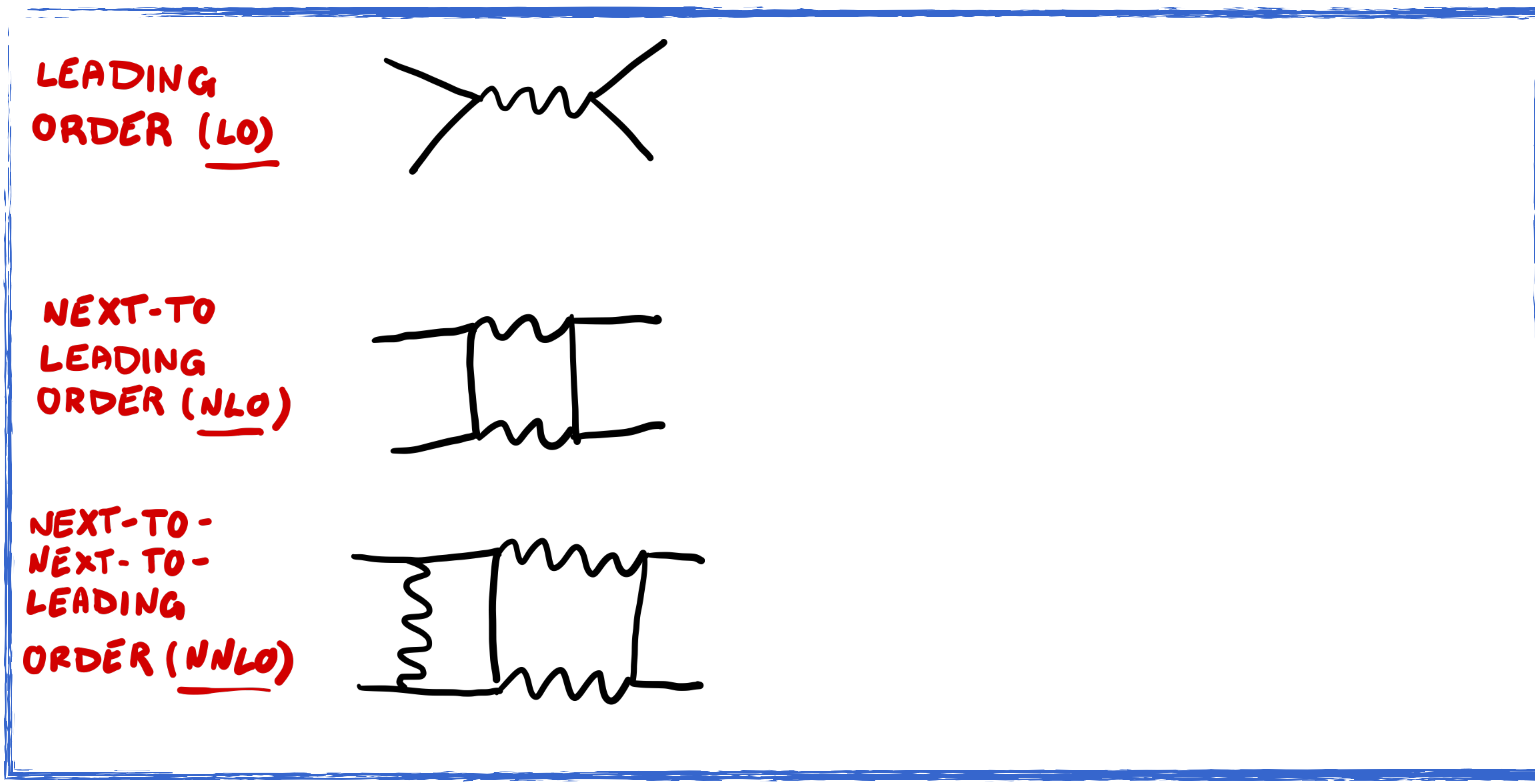


THEORETICAL INGREDIENTS



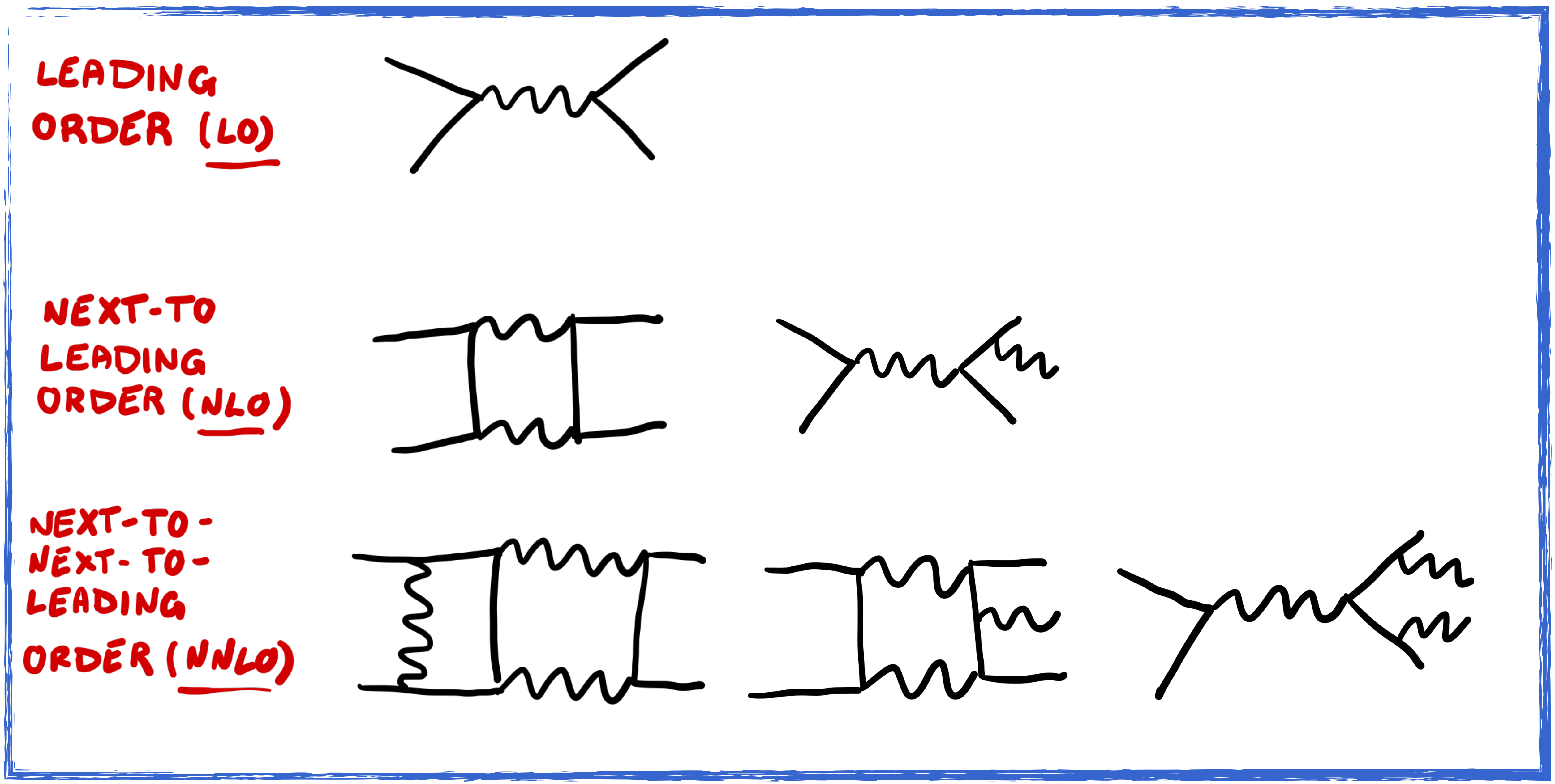
PRECISION TESTS OF THE STANDARD MODEL

- High precision in the theoretical prediction requires the computation of **higher order corrections**.

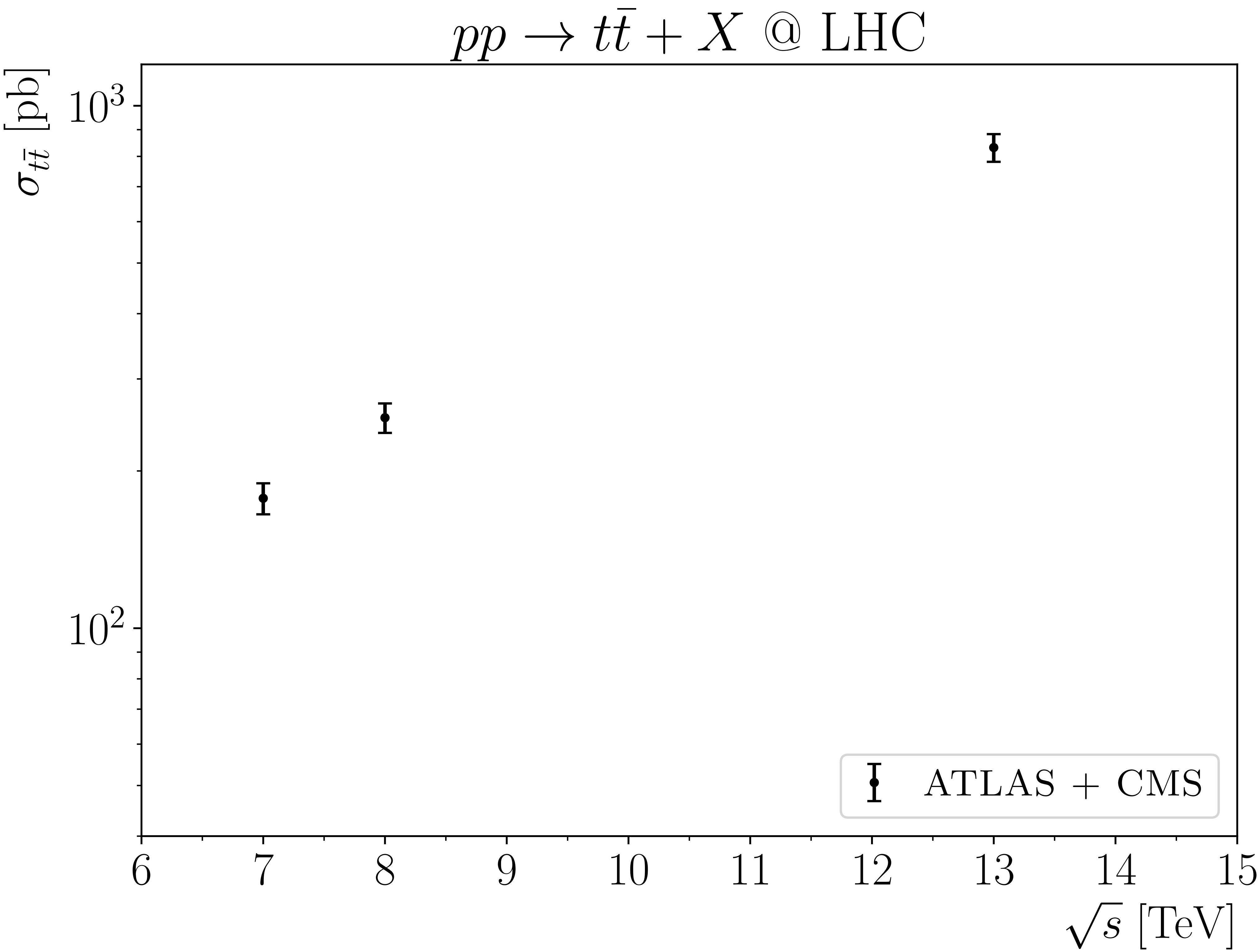


PRECISION TESTS OF THE STANDARD MODEL

- High precision in the theoretical prediction requires the computation of **higher order corrections**.

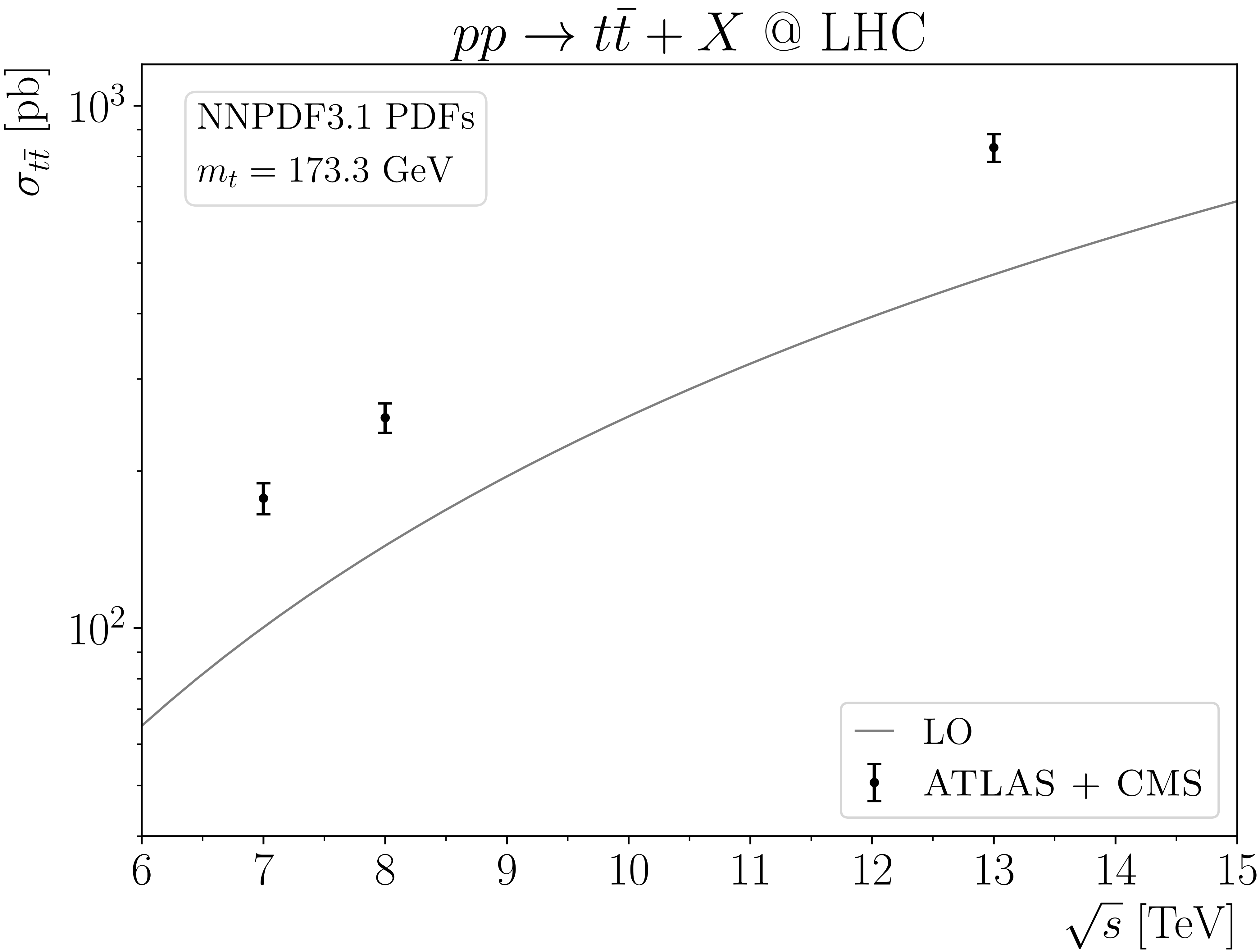


THE IMPORTANCE OF PRECISION



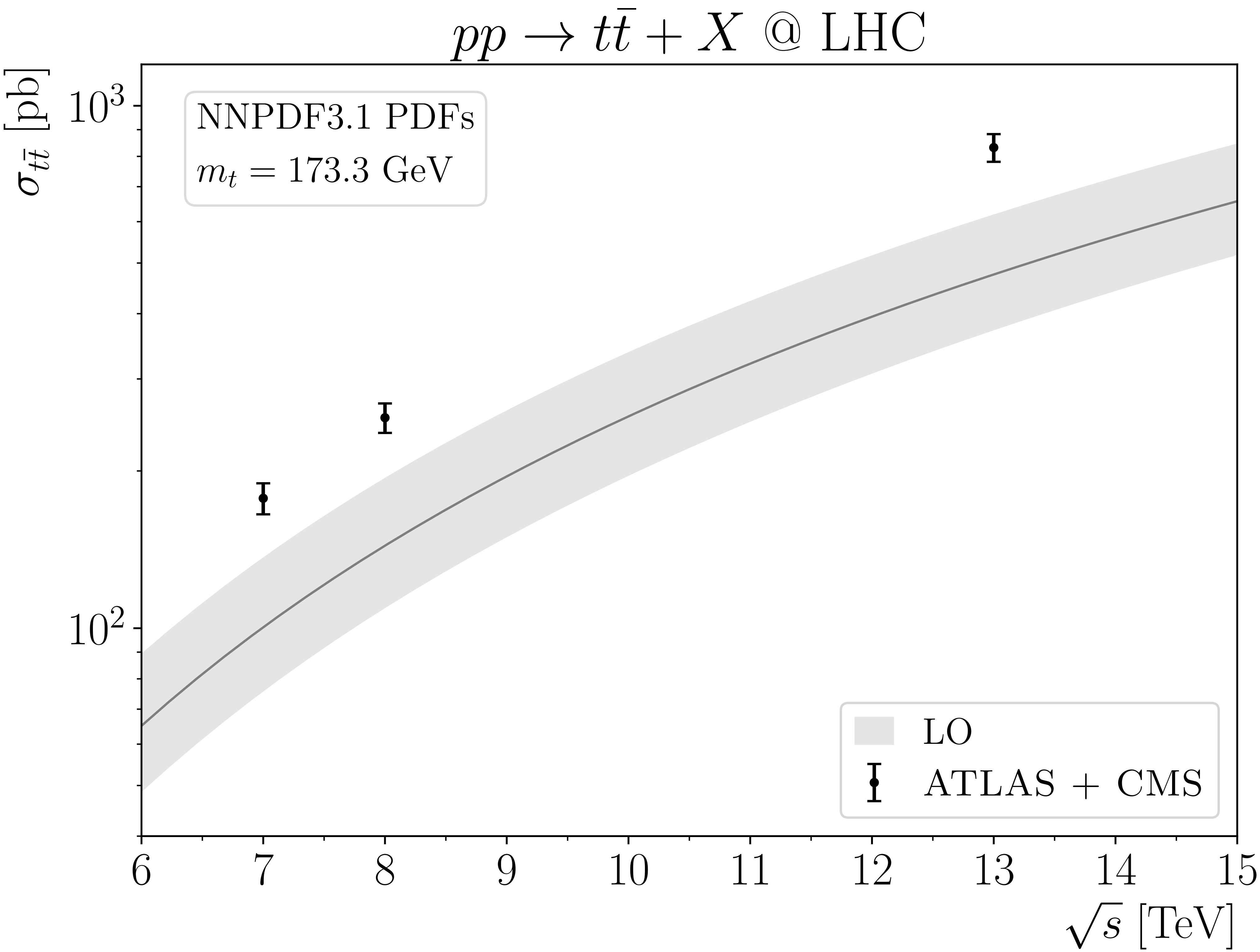
[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019]

THE IMPORTANCE OF PRECISION



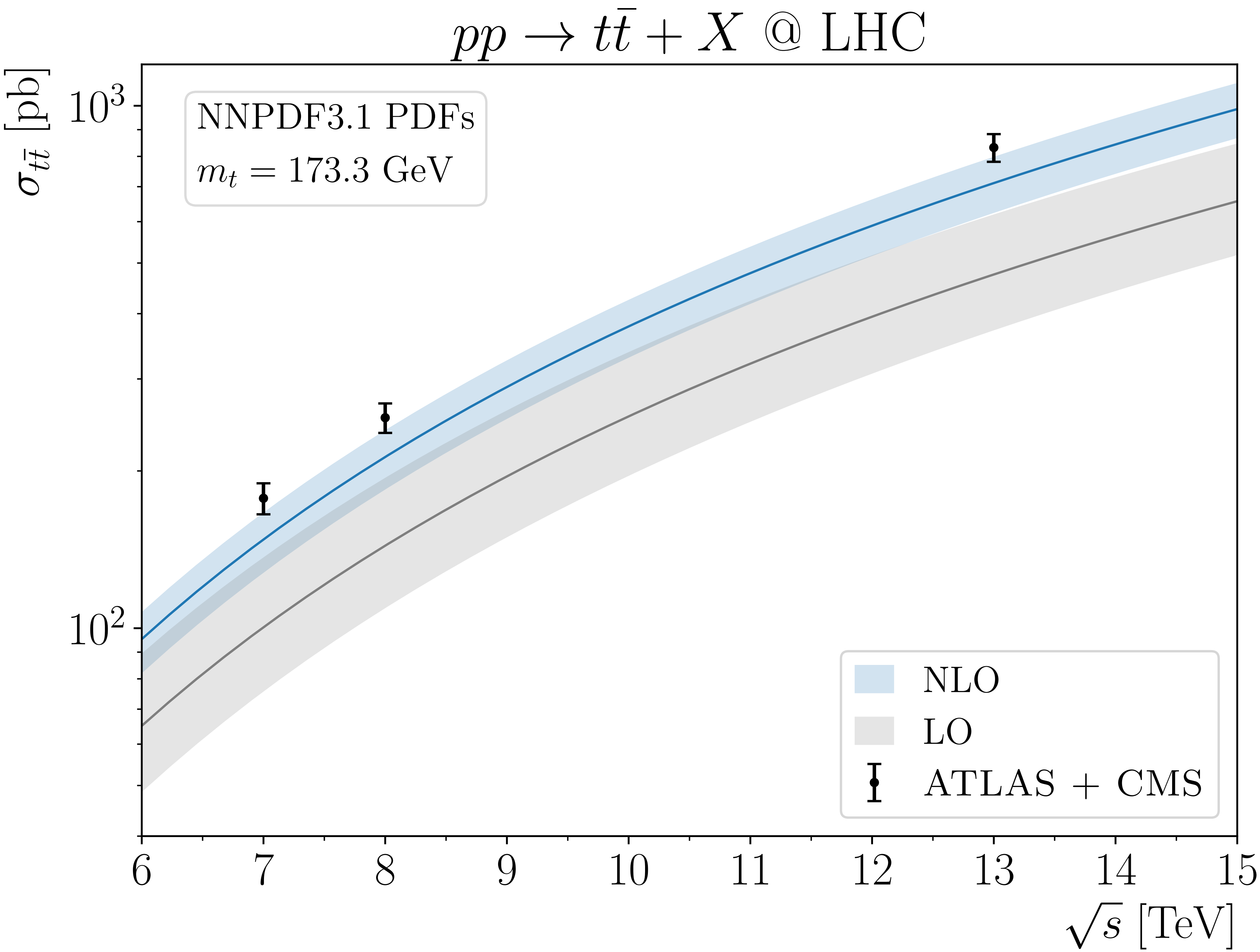
[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019]

THE IMPORTANCE OF PRECISION



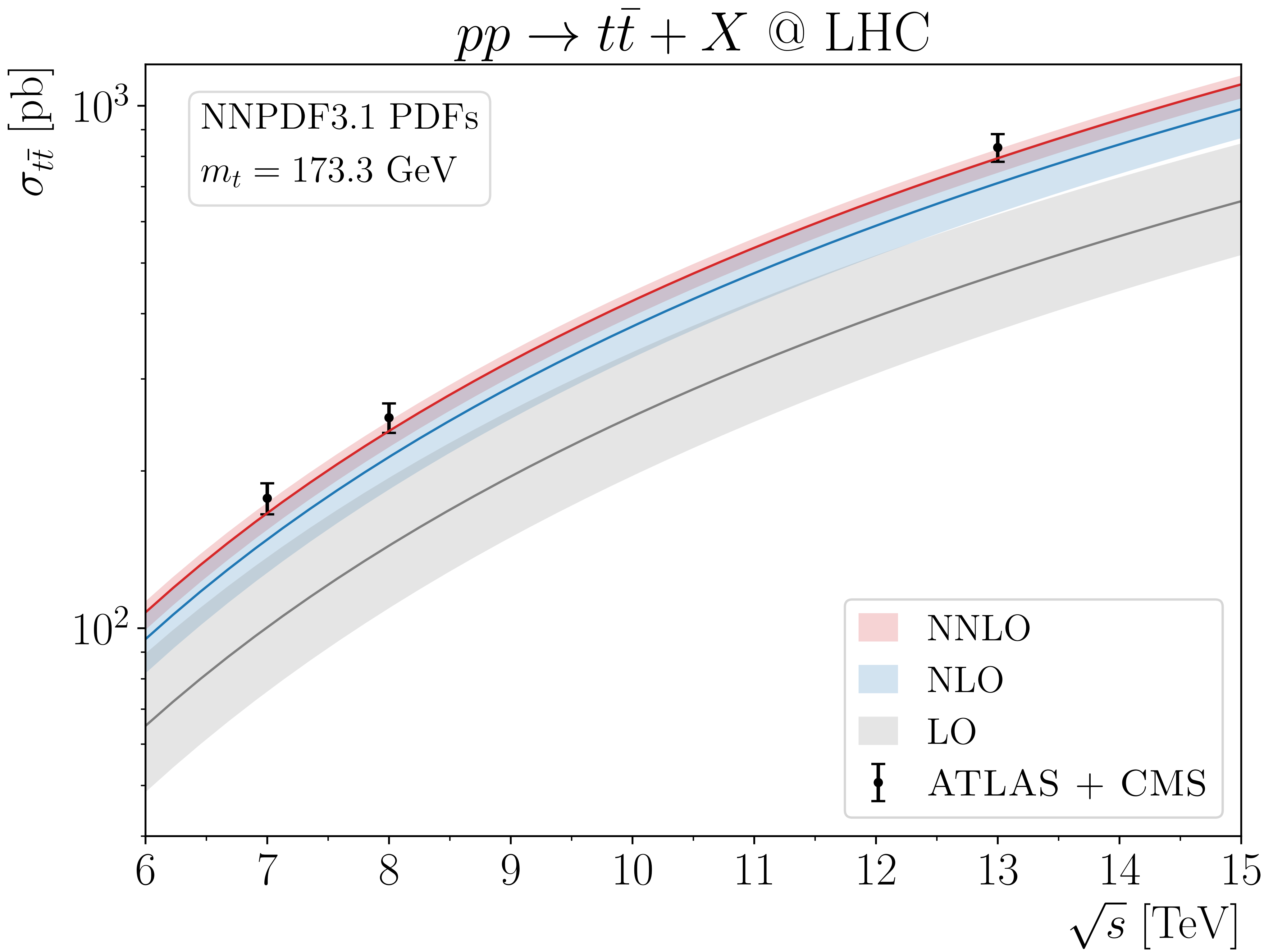
[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019]

THE IMPORTANCE OF PRECISION



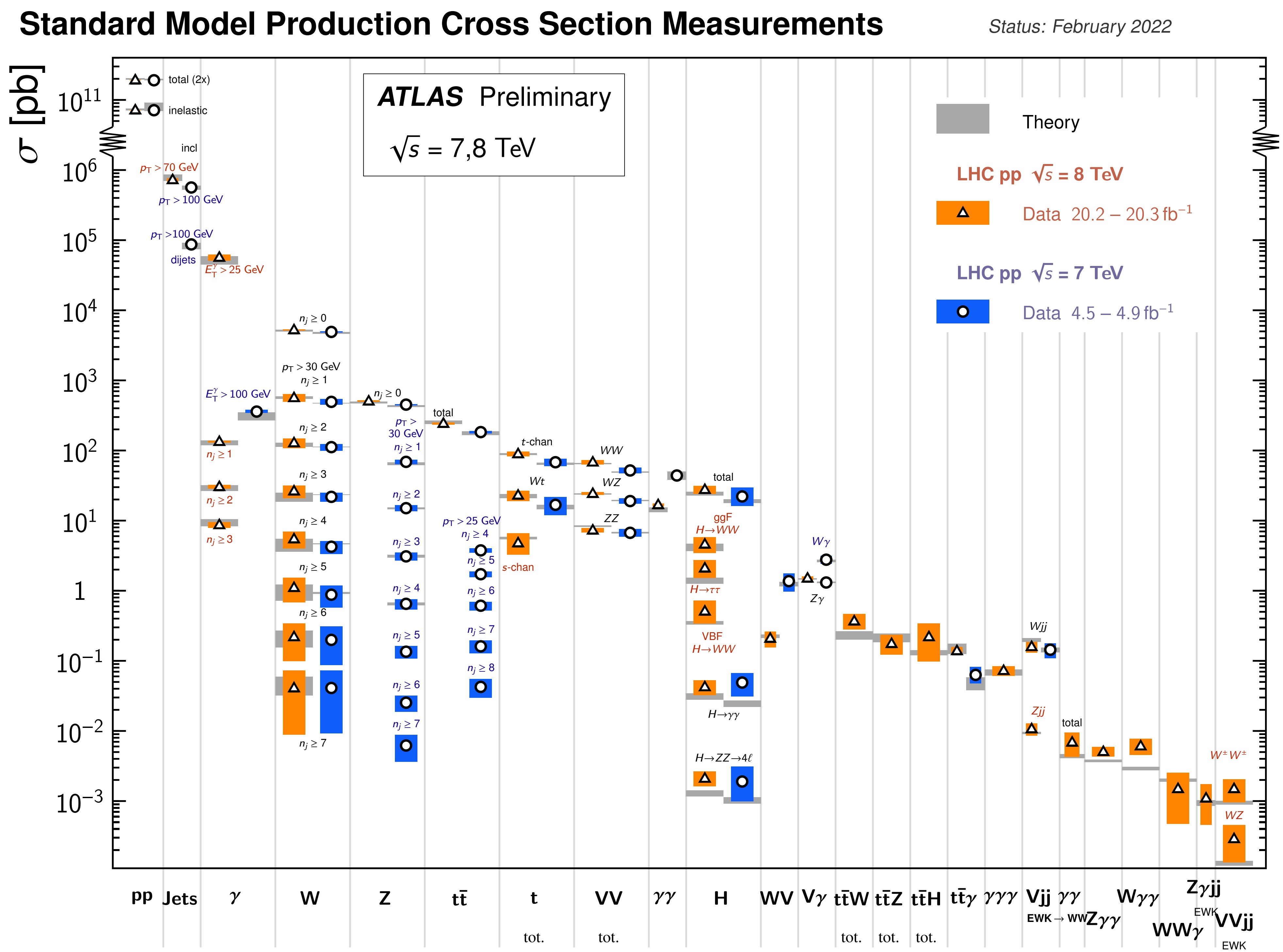
[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019]

THE IMPORTANCE OF PRECISION

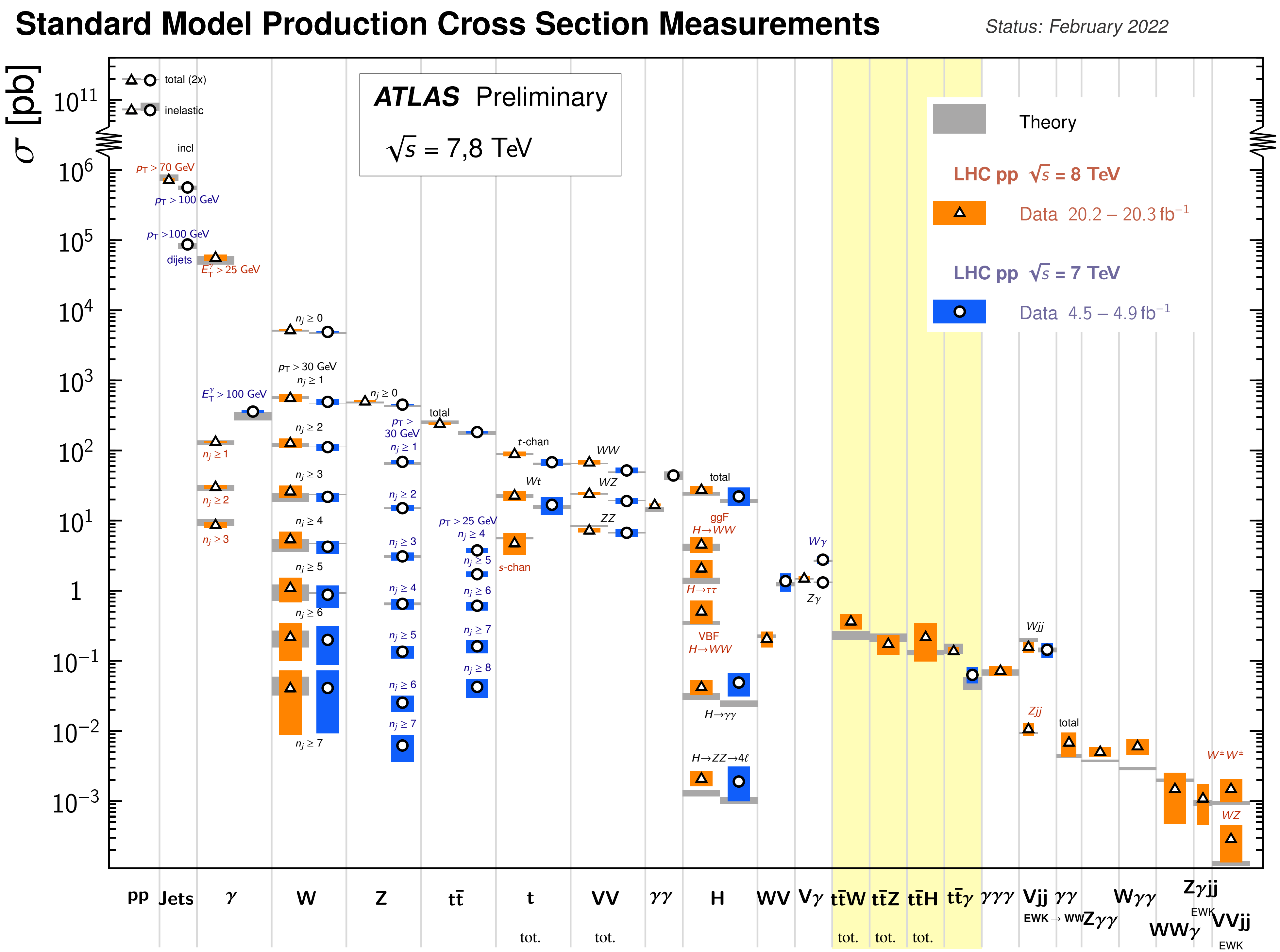


[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019]

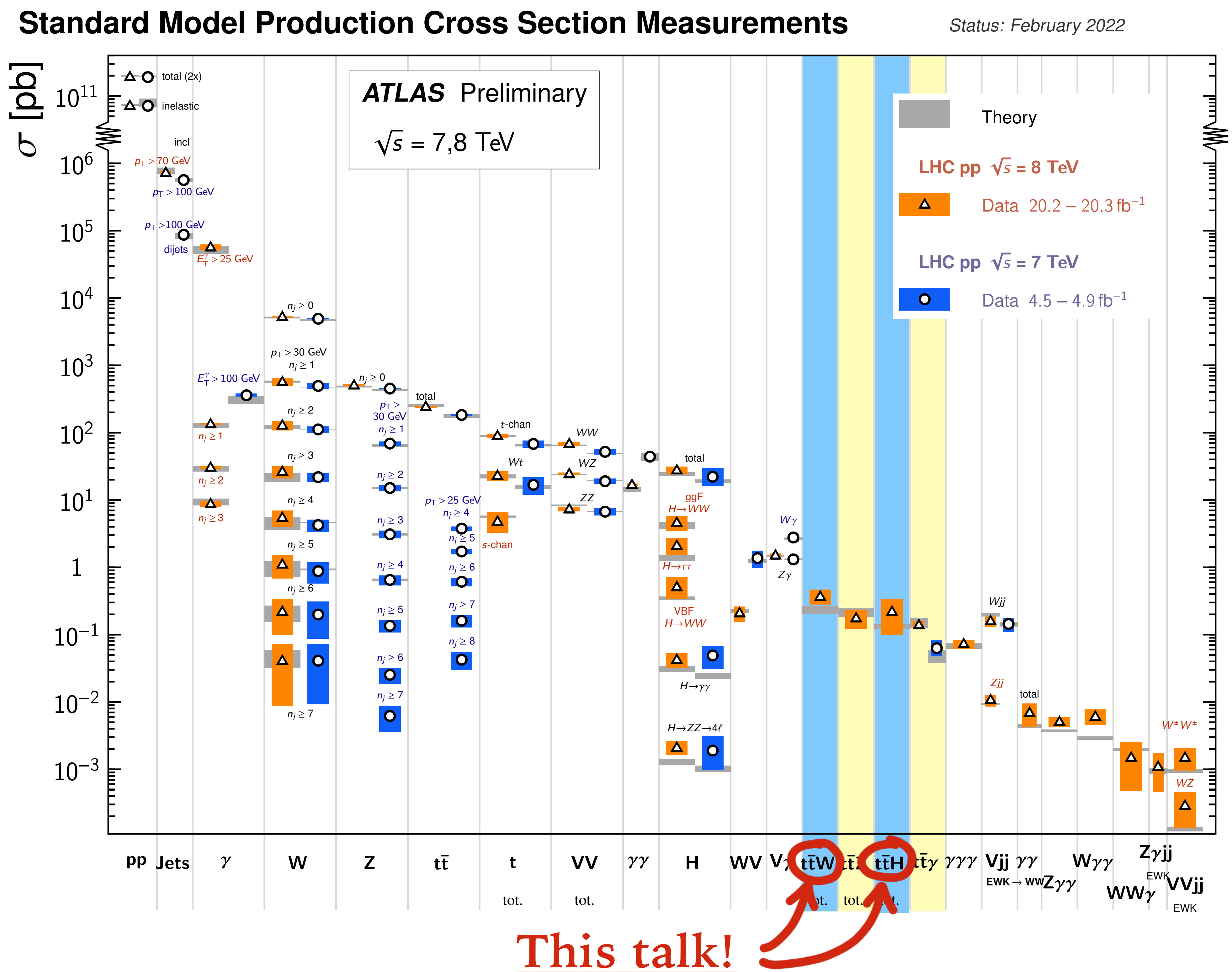
INTRODUCTION



INTRODUCTION

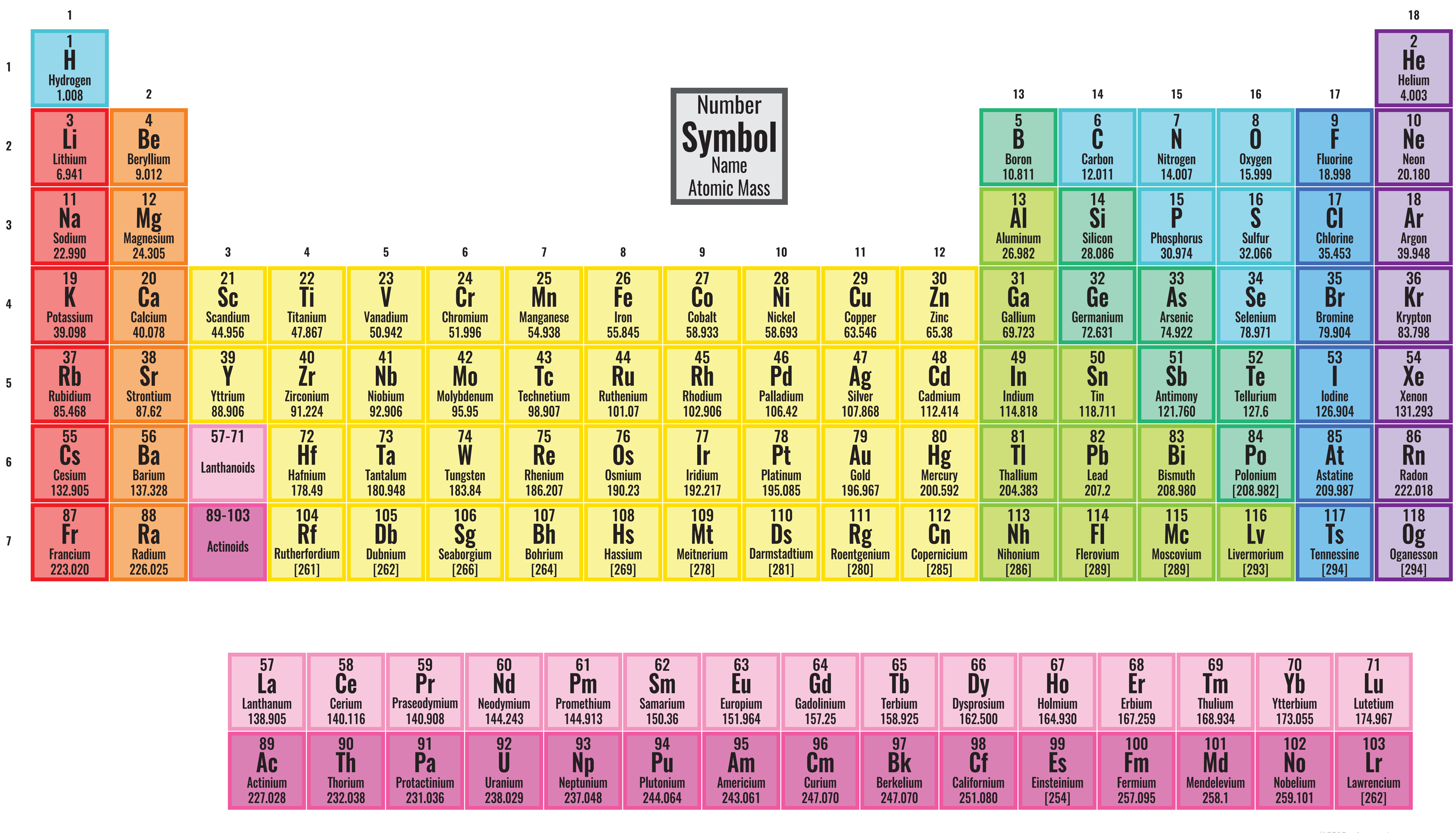


INTRODUCTION



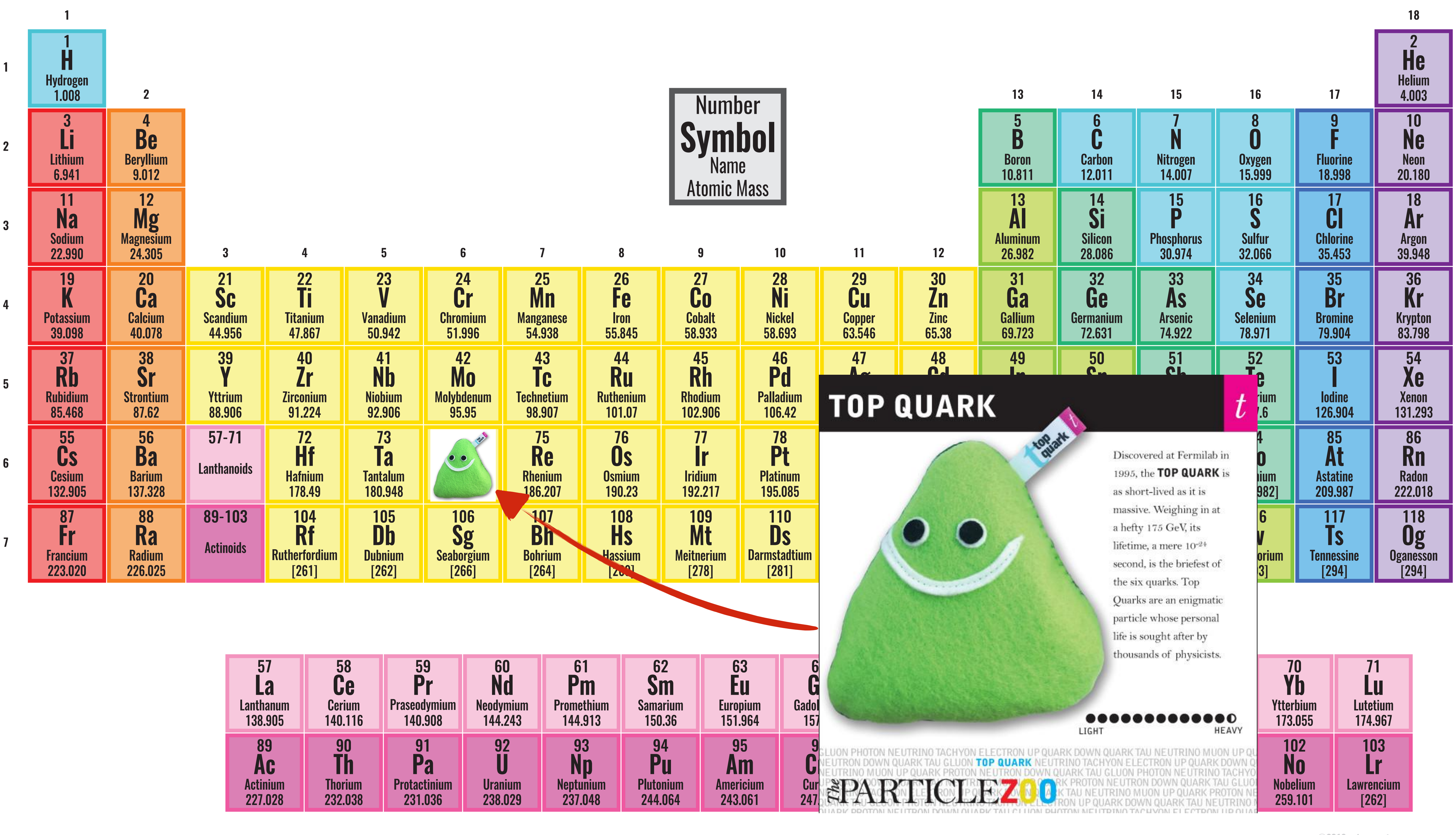
THE TOP QUARK

The **top quark** is the **heaviest** particle in the Standard Model: $m_t \simeq 173$ GeV.



THE TOP QUARK

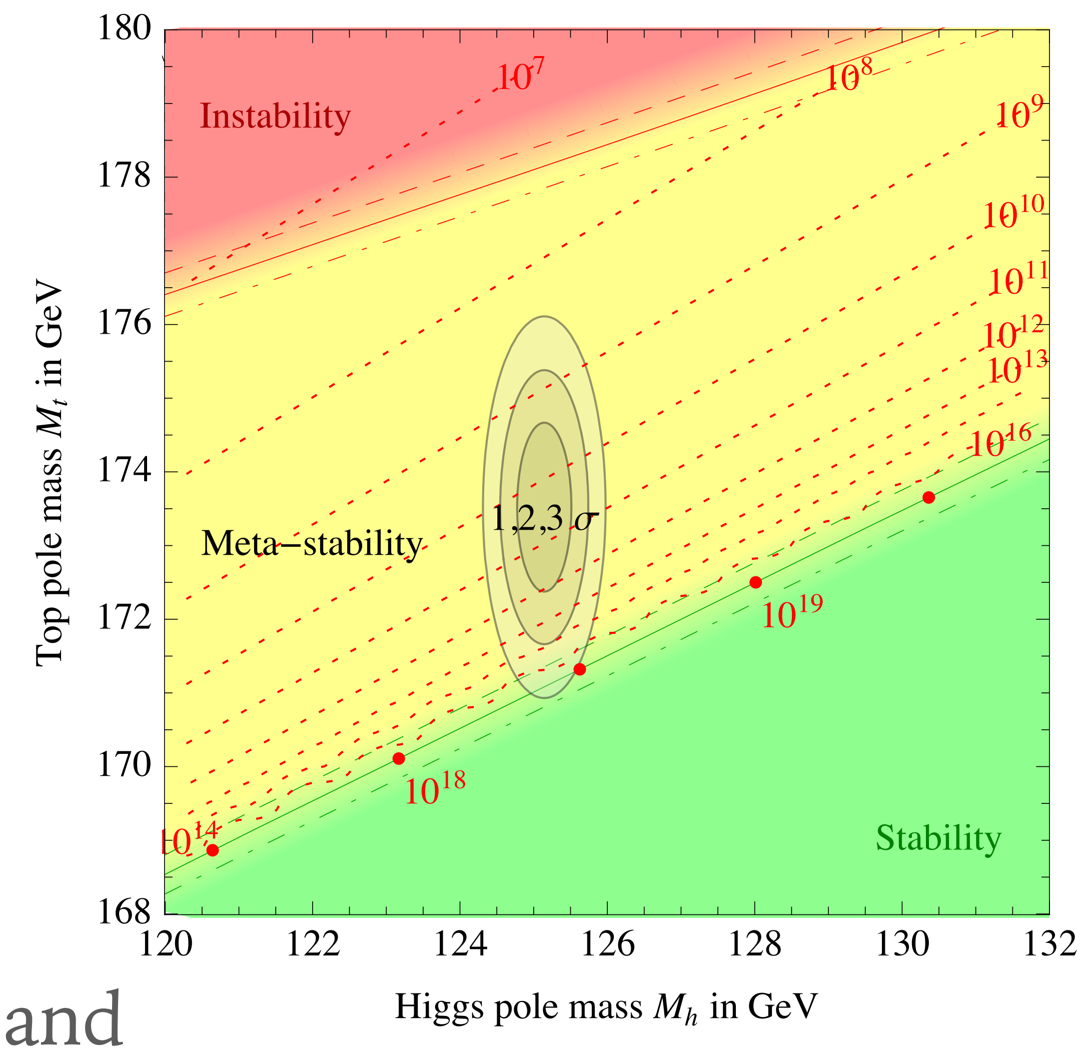
The **top quark** is the **heaviest** particle in the Standard Model: $m_t \simeq 173$ GeV.



THE TOP QUARK

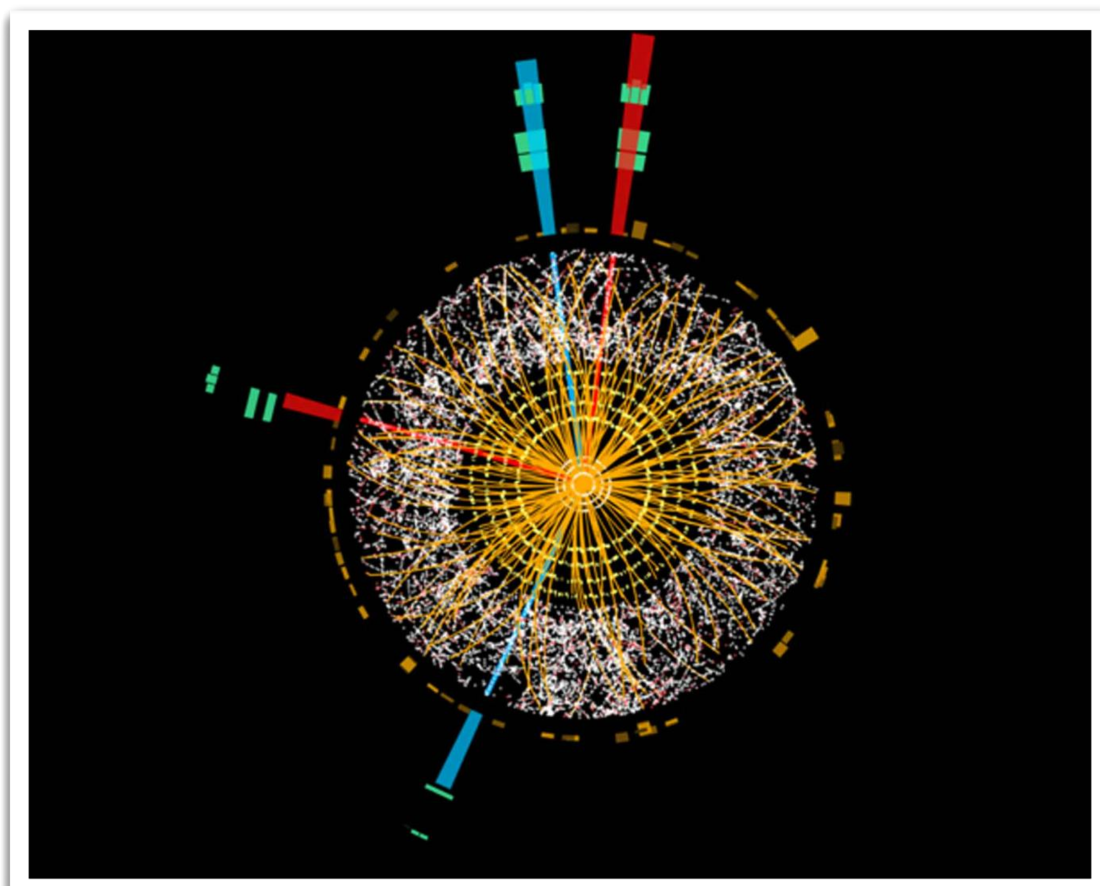
The Top Quark plays a major role both in Standard Model studies and Beyond the Standards Model searches!

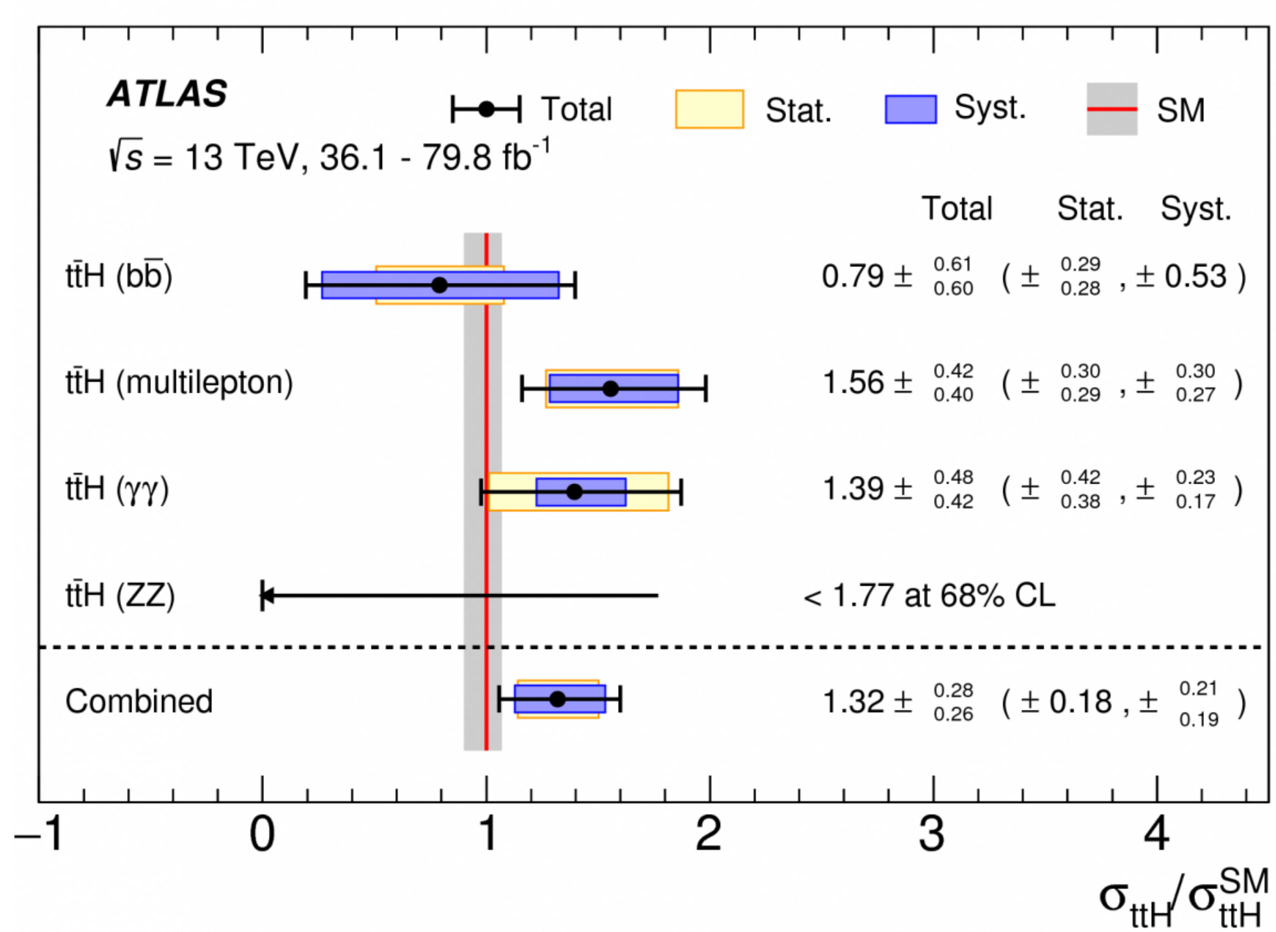
- Has a strong coupling with the **Higgs boson**;
- The top mass is a **fundamental parameter** of the Standard Model;
- The value of the top mass plays a key role in **vacuum stability**;
- The top mass is a **standard candle** at LHC;
- possible window on **New Physics**;
- **Important background** both for SM and BSM studies.



INTRODUCTION $(t\bar{t}H)$

- The **discovery of the Higgs boson** in 2012 confirmed one of the most glaring predictions of the Standard Model.
- The **study of the Higgs boson** proprieties is one of the priorities of LHC.
- Special role played by the **top quark**: strong coupling because of the top mass!
- $t\bar{t}H$ production allows direct measurement of the **top-quark Yukawa coupling**! (possible window on new physics scenarios…)

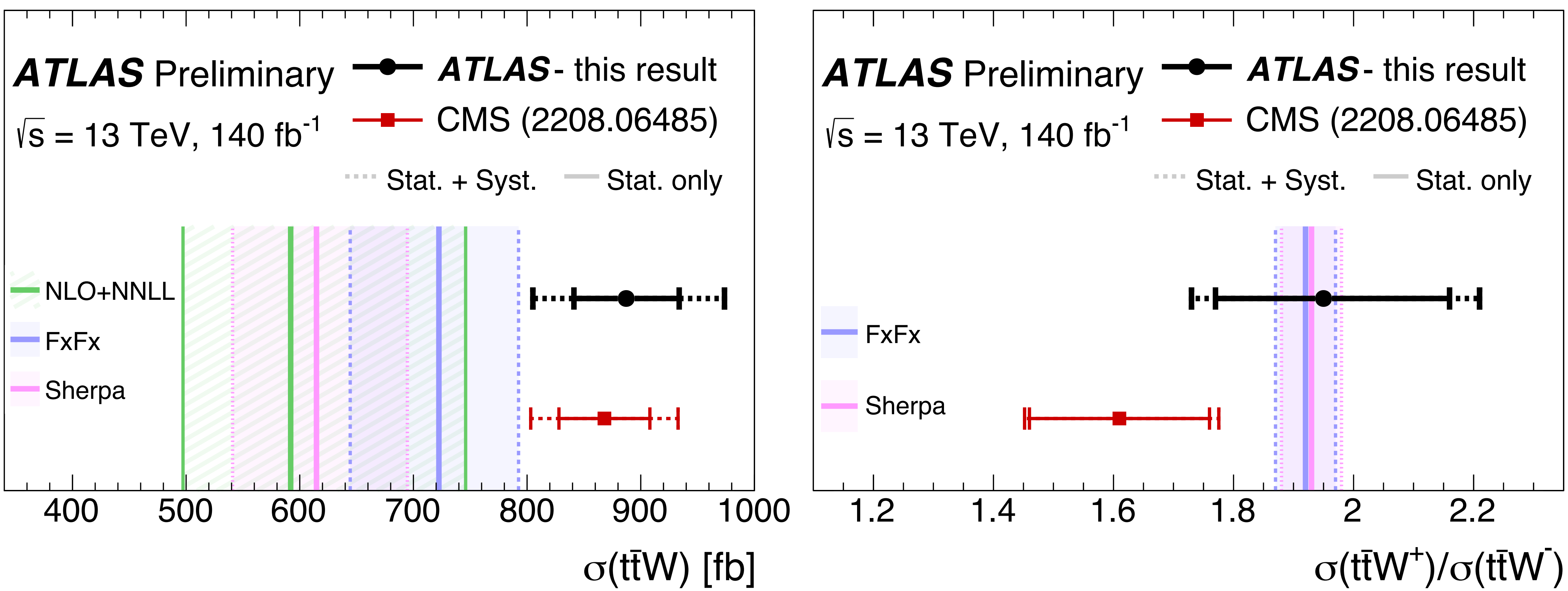




[M.Cepeda et al.: arXiv 1902.00134]

INTRODUCTION $(t\bar{t}W)$

- Together with $t\bar{t}H$ production, one of the **most massive** Standard Model (SM) signatures accessible at the LHC;
- Relevant as a $t\bar{t}H$ **background**;
- Measurements carried out by the ATLAS and CMS collaborations lead to rates consistently **higher than the SM predictions**;
- Most recent measurements confirm excess at the 2σ **level**.



[ATLAS-CONF-2023-019]

STATUS OVERVIEW $(t\bar{t}H)$

THEORY

- NLO QCD:

  [W. Beenakker, S. Dittmaier, M. Krämer, B. Plumper, M. Spira, and P. Zerwas; 0107081, 0211352], [L. Reina and S. Dawson; 0107101], [L. Reina, S. Dawson, and D. Wackeroth; 0109066], [S. Dawson, L. Orr, L. Reina, and D. Wackeroth; 0211438], [S. Dawson, C. Jackson, L. Orr, L. Reina, and D. Wackeroth; 0305087], [A. Denner and R. Feger, 1506.07448];

- NLO EW:
  [S. Frixione, V. Hirschi, D. Pagani, H. Shao, and M. Zaro; 1407.0823, 1504.03446], [Y. Zhang, W.-G. Ma, R.-Y. Zhang, C. Chen, and L. Guo; 1407.1110];

- NLO QCD + EW:
  [A. Denner, JN. Lang, M. Pellen, and S. Uccirati; 1612.07138];

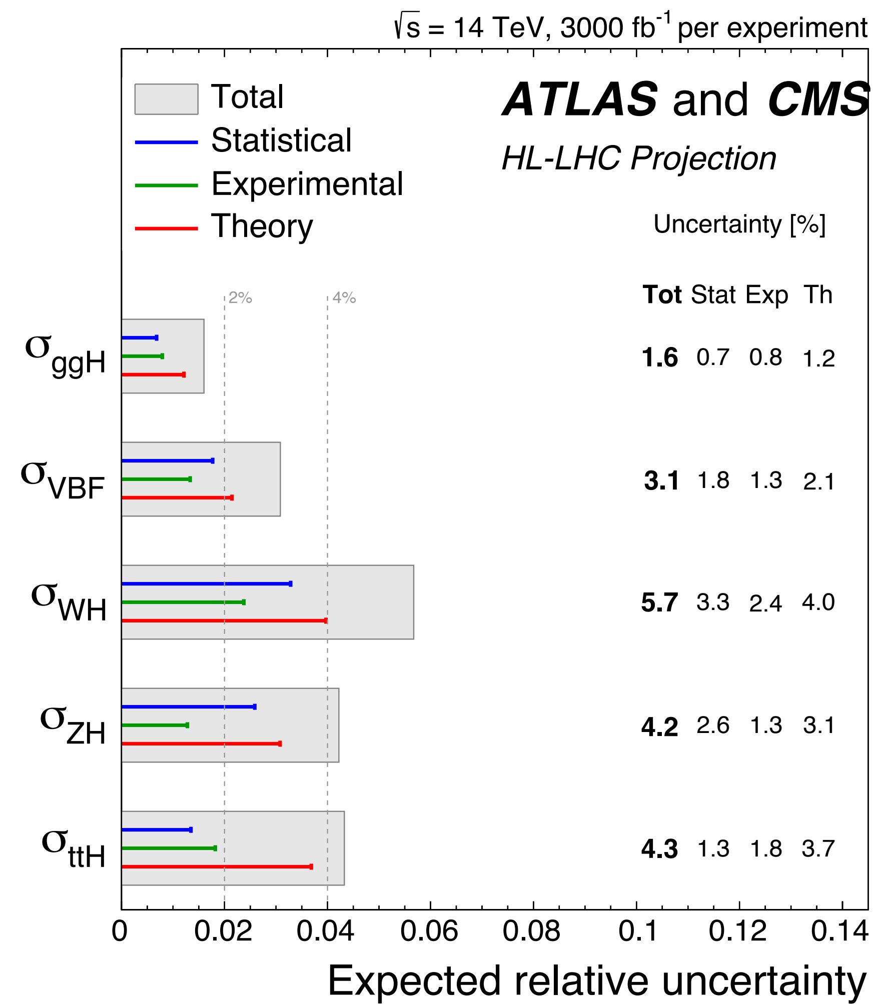
- Resummation of soft gluons:
  [A. Kulesza, L. Motyka, T. Stebel, and V. Theeuwes; 1509.02780, 1704.03363], [A. Broggio, A. Ferroglia, B. D. Pecjak, A. Signer, and L. L. Yang; 1510.01914], [A. Broggio, A. Ferroglia, B. D. Pecjak, and L. L. Yang; 1611.00049], [A. Broggio, A. Ferroglia, R. Frederix, D. Pagani, B. D. Pecjak, and I. Tsinikos; 1907.04343], [W.-L. Ju and L. L. Yang; 1904.08744], [A. Kulesza, L. Motyka, D. Schwartländer, T. Stebel, and V. Theeuwes; 2001.03031]

- First steps to NNLO: *off-diagonal channels [S. Catani, I. Fabre, M. Grazzini, S. Kallweit; 2102.03256]*

Current theoretical uncertainties $\mathcal{O}(10\%)$

EXPERIMENTS

- ATLAS collaboration: *[1806.00425];*
- CMS collaboration: *[1804.02610].*



[M. Cepeda et al.: arXiv 1902.00134]

Current experimental uncertainties $\mathcal{O}(20\%)$
Expected at the end of HL-LHC $\mathcal{O}(2\%)$

STATUS OVERVIEW $(t\bar{t}W)$

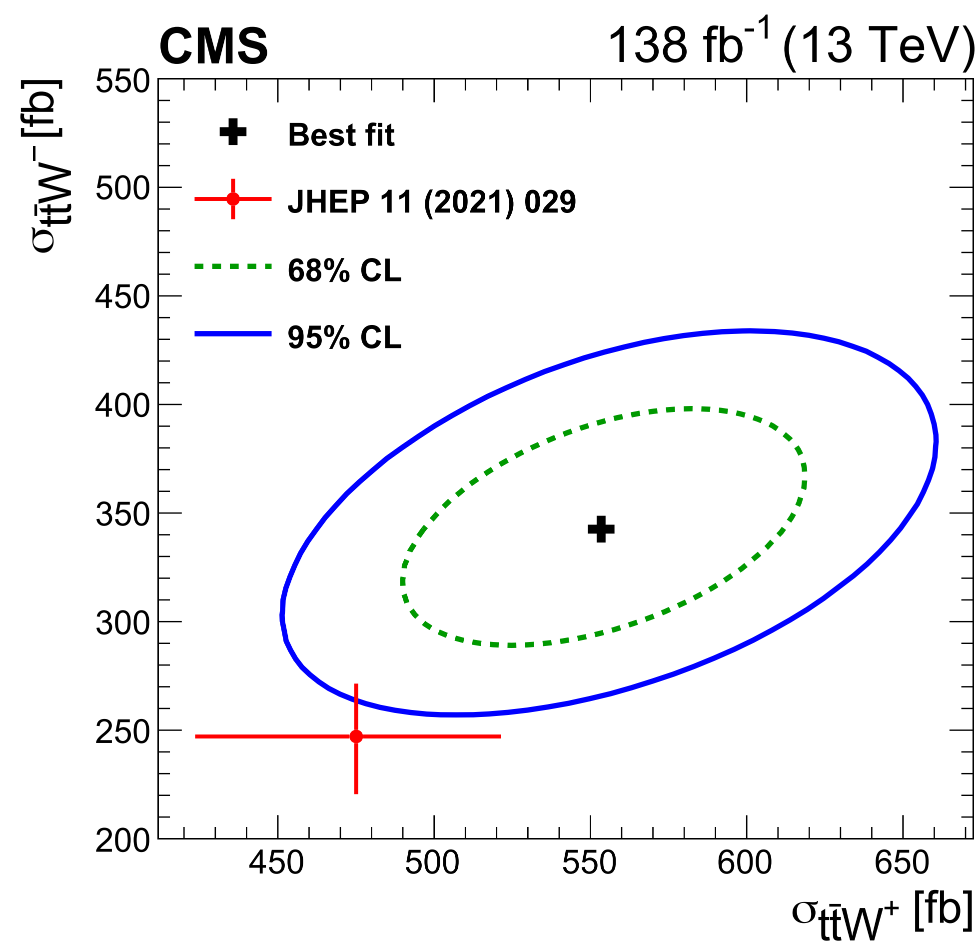
THEORY

- NLO QCD:
  [S. Badger, J. M. Campbell, R. K. Ellis, 1011.6647], [J. M. Campbell, R. K. Ellis, 1204.5678], [A. Denner, G. Pelliccioli, 2102.03264];
- NLO QCD with light jet:
  [G. Bevilacqua, H. Y. Bi, F. Febres Cordero, H. B. Hartanto, M. Kraus, J. Nasufi, L. Reina, and M. Worek , 2109.1581, 2305.03802]
- NLO QCD + EW:
  [S. Frixione, V. Hirschi, D. Pagani, H. S. Shao, M. Zaro, 1504.03446], [R. Frederix, D. Pagani, M. Zaro, 1711.02116], [Denner, Pelliccioli, 2020]
- Resummation of soft gluons:
  [H. T. Li, C. S. Li, S. A. Li, 1409.1460] [A. Broggio, G. Ferroglia, G. Ossola, B. D. Pecjak, 1607.05303], [A. Kulesza, L. Motyka, D. Schwartlaender, T. Stebel, V. Theeuwes, 1812.08662]
- NLO QCD + EW (on-shell) predictions supplemented with multi-jet merging as la FxFx: *[R. Frederix, S. Frixione, 1209.6215] [R. Frederix, I. Tsinikos, 2108.07862]*

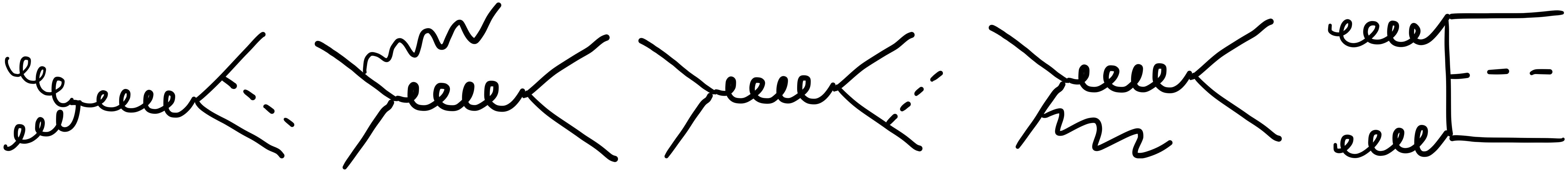
Current theoretical uncertainties $\mathcal{O}(10\%)$

EXPERIMENTS

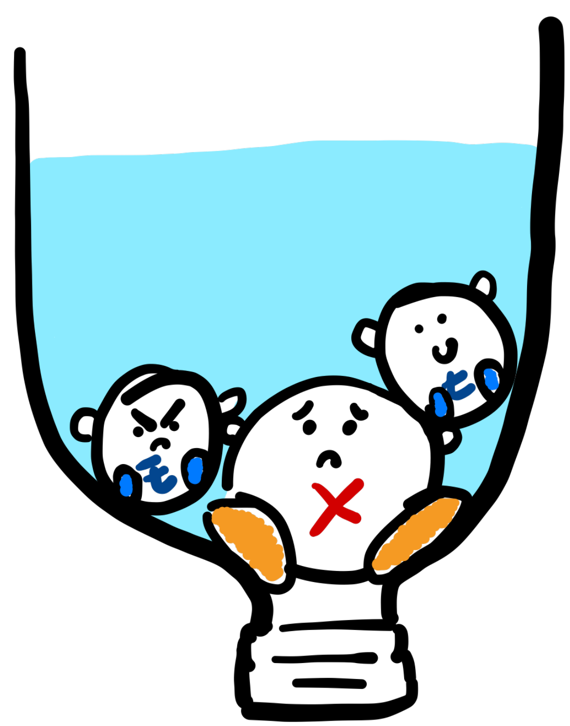
- ATLAS collaboration: *[ATLAS-CONF-2023-019];*
- CMS collaboration: *[2208.06485].*



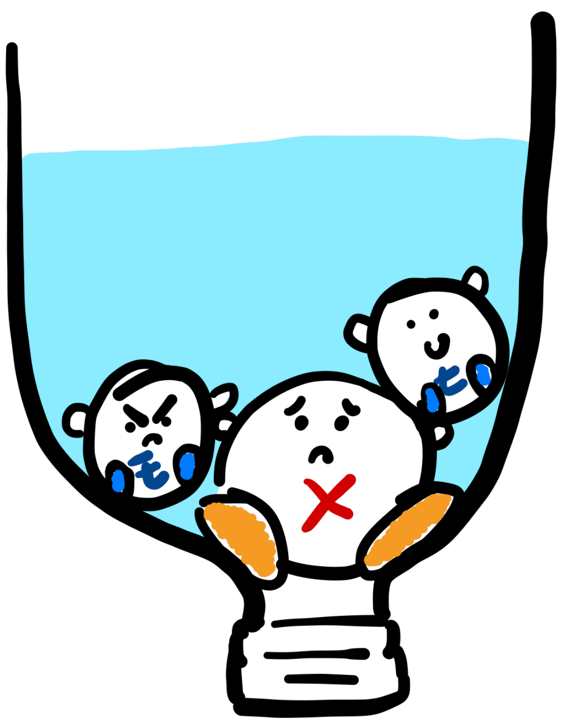
Theory-experiment tension at 2σ level:
Explained by higher order corrections?

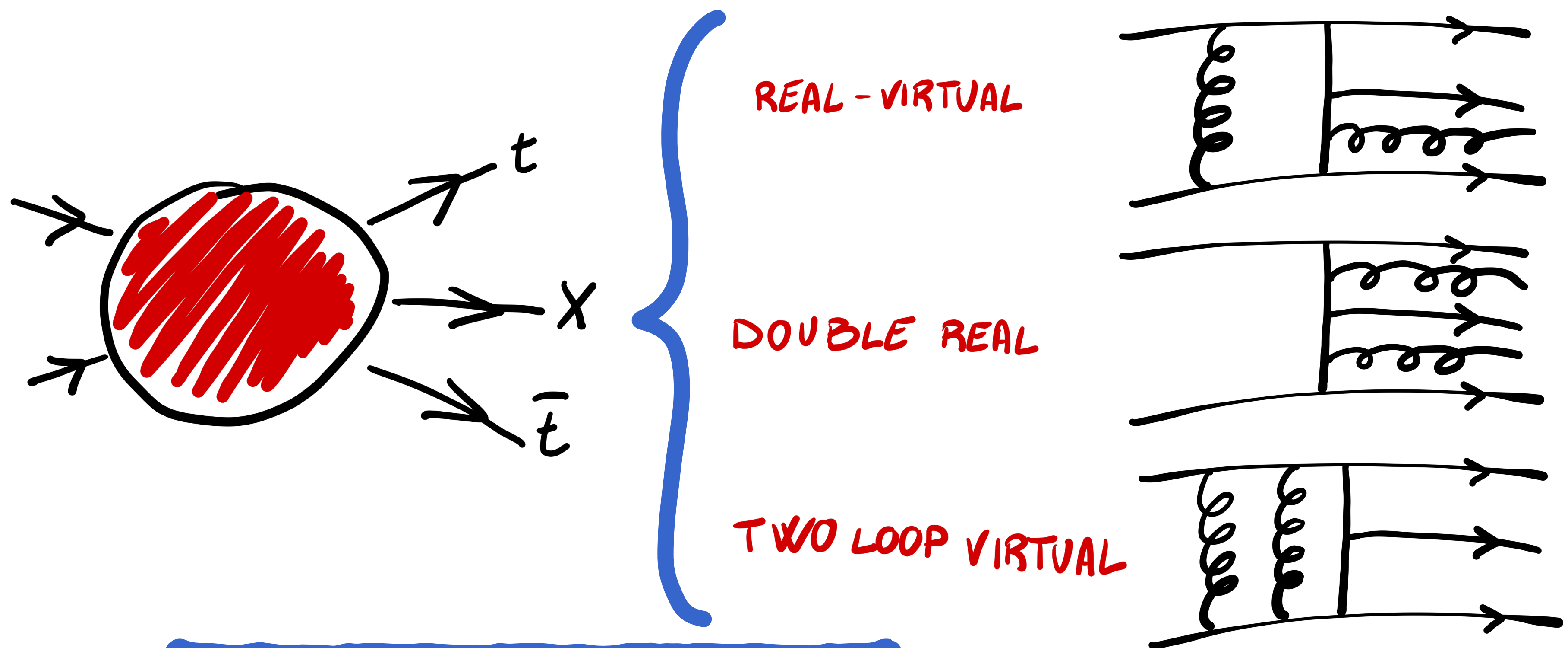


THEORETICAL CHALLENGES



THEORY BOTTLENECKS





What makes NNLO challenging?

Subtraction procedure

- We use **q_T-subtraction**;
- We **generalised** the method to this class of processes.

Two loop amplitudes

- Not known: current frontier!

[F. Febres Cordero, G. Figueiredo, M. Kraus, B. Page, L. Reina, 2312.08131],[B. Agarwal, G. Heinrich, S. P. Jones, M. Kerner, S. Y. Klein, J. Lang, V. Magerya, A. Olsson, 2402.03301]

- We developed **approximations**.

q_T SUBTRACTION FORMALISM

[S. Catani, M. Grazzini Phys.Rev.Lett. 98 (2007)]

$$d\sigma^F_{NNLO} = d\sigma^F_{NNLO}\Big|_{q_T=0} + d\sigma^F_{NNLO}\Big|_{q_T\neq 0}$$

$$d\sigma^{F+jets}_{NLO}$$

$$d\sigma^F_{NNLO} = \mathscr{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{NLO} - d\sigma^{CT}_{NLO}\right]$$

HARD COLLINEAR COEFFICIENT

Contains information on virtual corrections to the process.

$$\mathscr{H}^F_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathscr{H}^{(2)}$$

Contains the genuine 2-loop contribution:

$$H^{(2)} = \frac{2\,Re\left(\mathscr{M}^{(2)}(\mu_{IR},\mu_R)\mathscr{M}^{(0)}\right)}{|\mathscr{M}^{(0)}|^2}$$

- APPROXIMATED -

Includes:

- one-loop squared contribution;
- soft parton contribution.

- EXACT -

SOFT PARTON CONTRIBUTION

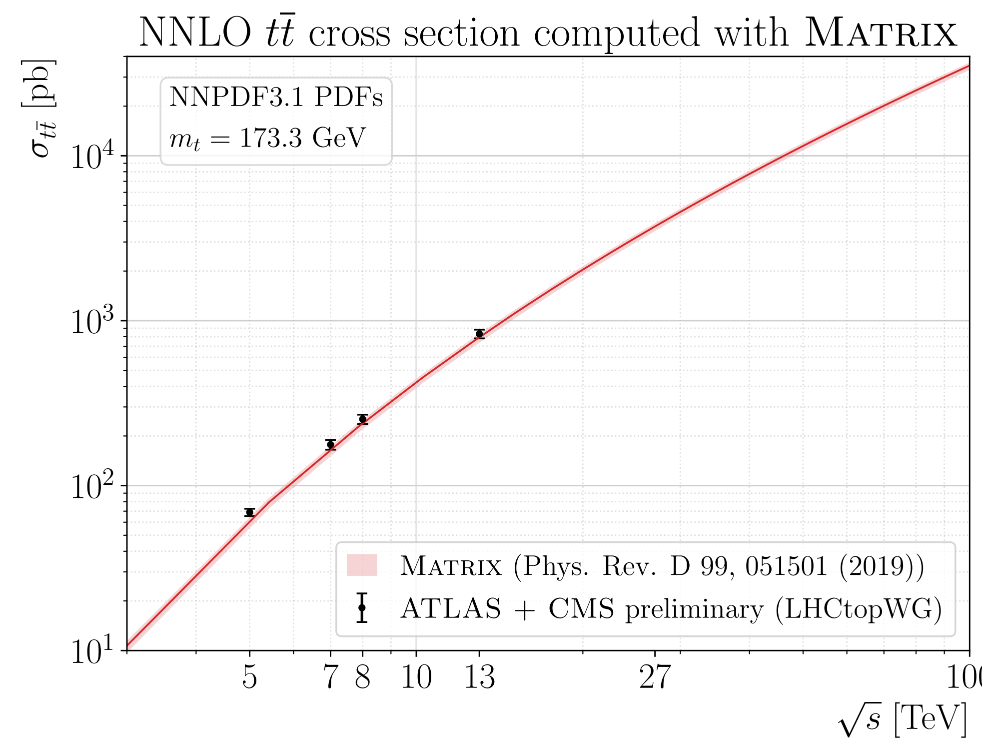
[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786
SD, J. Mazzitelli, In preparation]

The soft contribution from a massive final state was a key ingredient to extend q_T subtraction to a massive coloured final state.

Soft contributions to heavy-quark (Q) production

[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786]

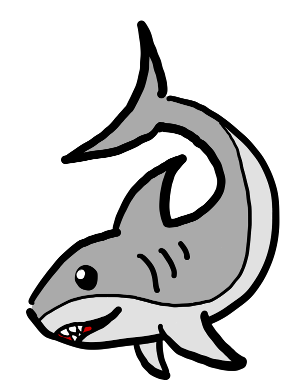
- Applied to top pair and bottom pair production: *[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019, 2020]*;
- Mostly **analytic** expressions;
- Assumption of $Q\bar{Q}$ **back-to-back** at LO.



NEW: generalisation to $Q\bar{Q}F$ kinematics

[SD, J. Mazzitelli, IN PREPARATION]

- removed the back-to-back assumption;
- Extra contribution computed **numerically**;
- On-the-fly numerical integration implemented in a **library**: **SHARK**
  Soft function for **H**eavy quark production in **AR**bitrary **K**inematics



2-LOOP CONTRIBUTION

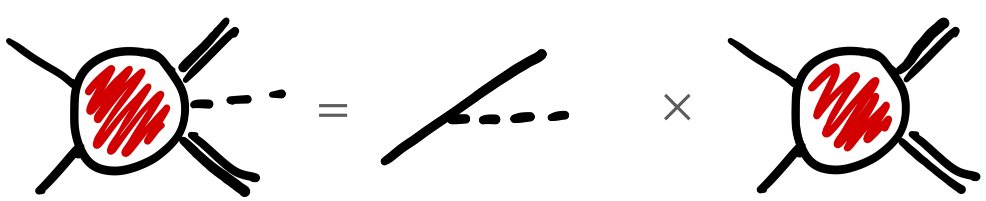
$$H^{(2)}_{t\bar{t}X} = \frac{2\,Re\left(\mathcal{M}^{(2)}_{t\bar{t}X}(\mu_{IR},\mu_R)\mathcal{M}^{(0)}_{t\bar{t}X}\right)_{appr.}}{|\mathcal{M}^{(0)}_{t\bar{t}X}|^2_{appr.}}$$

subtraction scale μ_{IR} (we use $\mu_{IR} = Q_{t\bar{t}H}$)

- We need to find an approximation of the virtual amplitude;
- We apply the approximation both on the numerator and denominator of $H^{(2)}_{t\bar{t}X}$: effectively a **reweighting**.

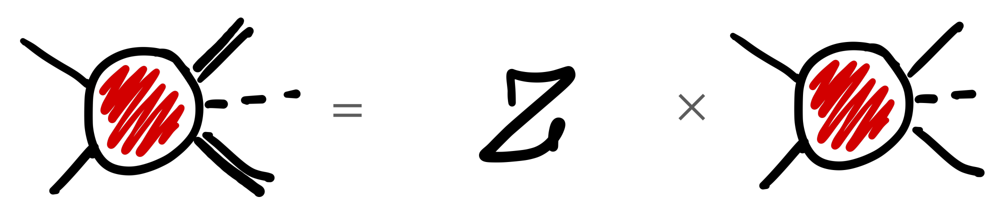
Two independent approximations

Soft approximation



- Captures the leading behaviour when the energy and **mass of the associated boson** are smaller than the other relevant scales

Massification procedure



- Captures the leading behaviour when the **mass of the top pair** are smaller than the other relevant scales

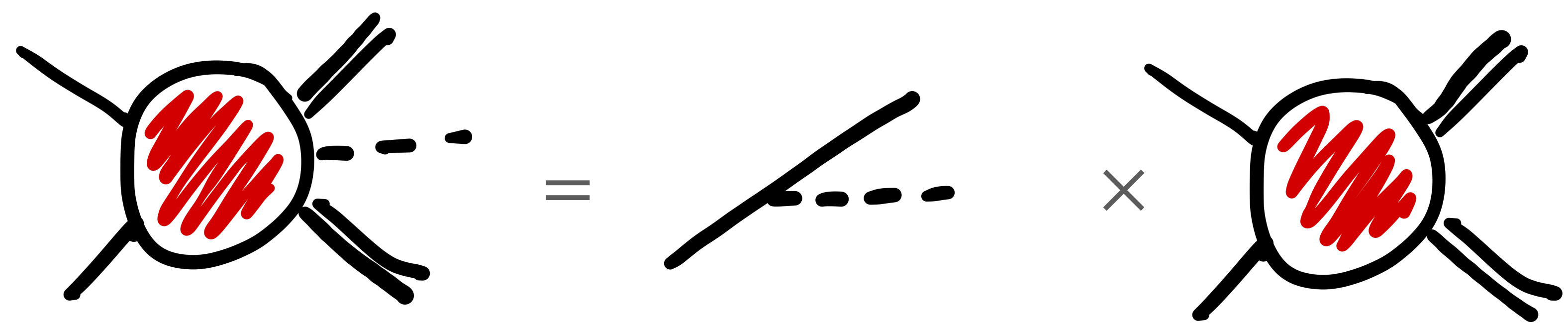
SOFT APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

Process: $c(p_1) + \bar{c}(p_2) \to t(p_3) + \bar{t}(p_4) + X(k)$

Soft approximation:

$k \to 0, \quad m_X \ll m_t$



$$\mathcal{M}_{q\bar{q}'\to t\bar{t}H}(\{p_i\},k) \simeq F(\alpha_S(\mu_R); m_t/\mu_R)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\mathcal{M}_{q\bar{q}'\to t\bar{t}}(\{p_i\})$$

$$\mathcal{M}_{q\bar{q}'\to t\bar{t}W}(\{p_i\},k) \simeq \frac{g}{\sqrt{2}}\left(\frac{p_2\cdot\varepsilon^*(k)}{p_2\cdot k} - \frac{p_1\cdot\varepsilon^*(k)}{p_1\cdot k}\right)\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}(\{p_i\})$$

- The formula captures the leading behaviour in the **soft limit** $k \to 0$: the emission from highly **off-shell top propagators** is **not captured**.
- The formula can be obtained both from the **eikonal approximation** and the **low energy theorems**;
- To use the approximation, we need a **recoil prescription** to map the $t\bar{t}X$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}X} \to Q_{t\bar{t}}$);

MASSIFICATION PROCEDURE

[A. A. Penin: 0508127]
[A. Mitov, S. Moch: 0612149]
[T. Becher and K. Melnikov: 0704:3582]

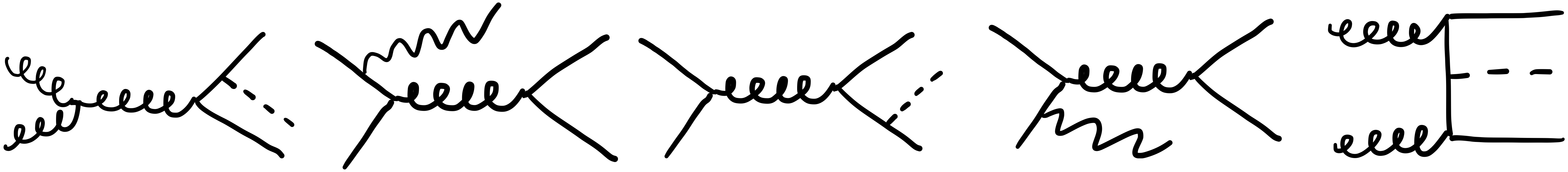
Process: $c(p_1) + \bar{c}(p_2) \to t(p_3) + \bar{t}(p_4) + X(k)$

Massification procedure:

$m_t \ll Q$

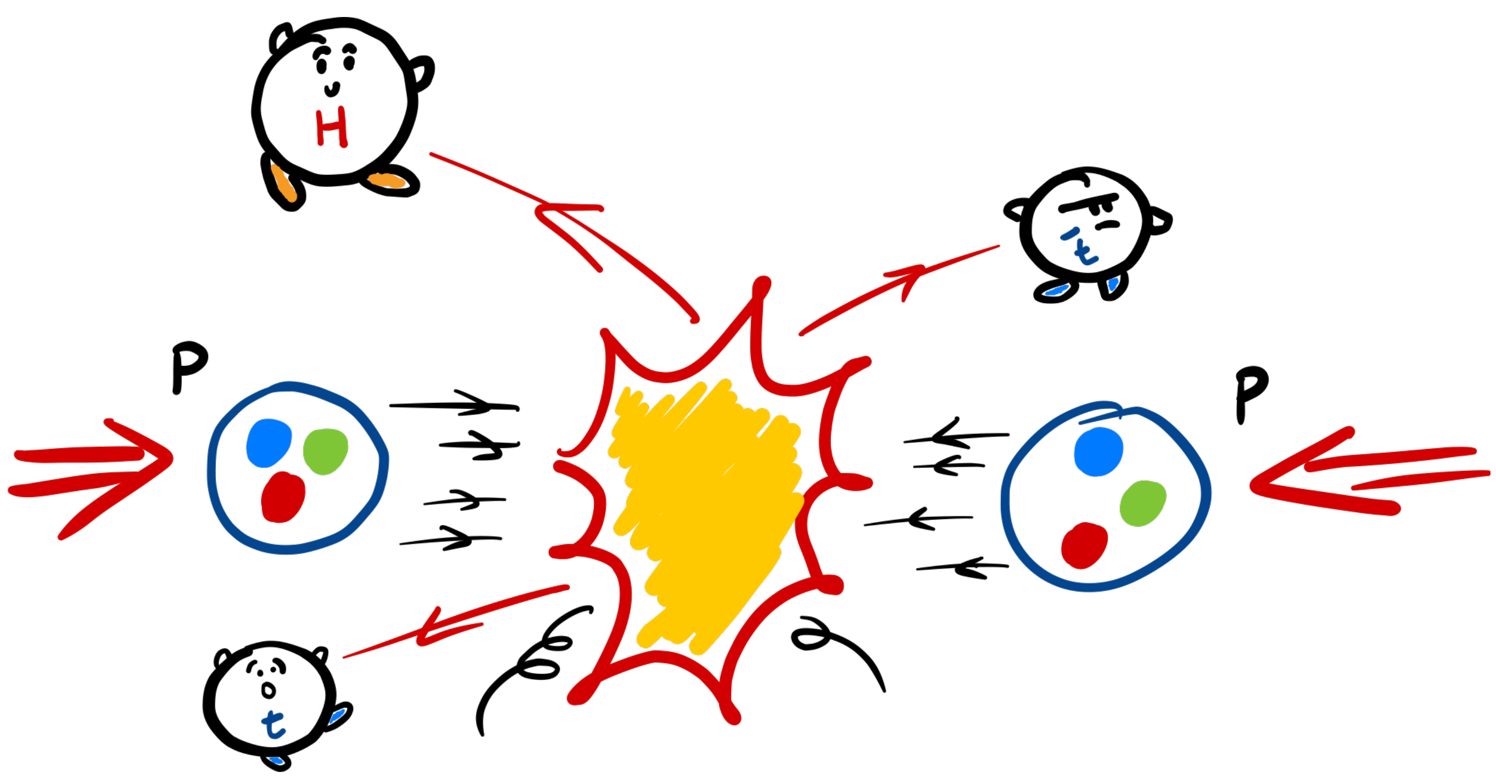
$$\mathcal{M}(\{p_i\}, k; \mu_R, \epsilon) \sim Z_{[q]}^{(m_t|0)}\left(\alpha_S(\mu_R), \frac{m_t}{\mu_R}, \epsilon\right)\mathcal{M}^{m_t=0}(\{p_i\}, k; \mu_R, \epsilon)$$

- The perturbative factor $Z_{[q]}^{(m_t|0)}$ was computed in *[A. Mitov, S. Moch: 0612149]*;
- The procedure retrieves the correct **mass logarithms**;
- The contribution from **massive top loops** is **not captured**;
- Successfully employed to derive and cross check results for $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$ amplitudes *[M. Czakon, A. Mitov, S. Moch: 0705.1975]*;
- Successfully applied to $b\bar{b}W$ production *[L. Buonocore, SD, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2212.04954]*.



$t\bar{t}H$ PRODUCTION

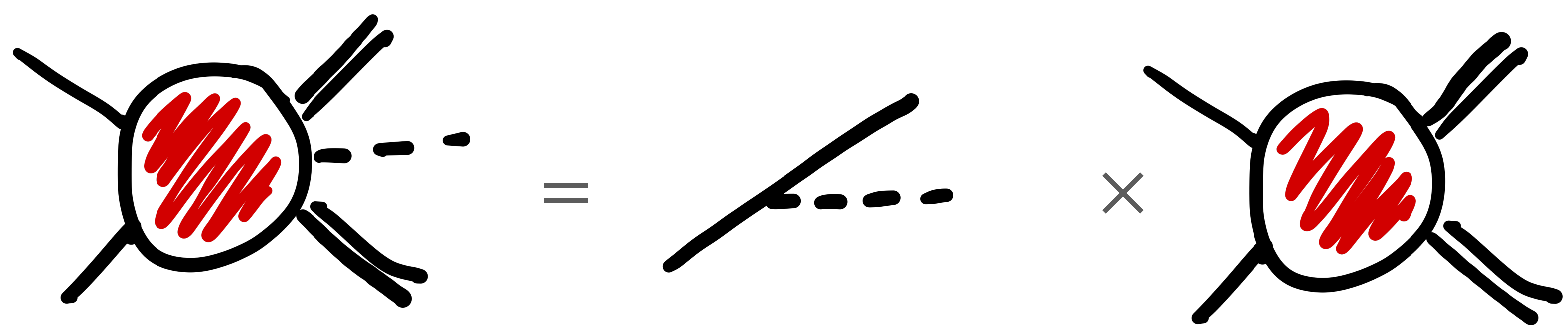
[ArXiv:2210.07846]



CHOICE OF THE APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

- Amplitudes for the process $c\bar{c} \to t\bar{t}$ available *[Czakon (2008); Barnreuther et al.(2013)]*: **we can use the soft approximation**.



$$\mathscr{M}_{q\bar{q}' \to t\bar{t}H}\left(\{p_i\}, k\right) \simeq F\left(\alpha_S(\mu_R); m_t/\mu_R\right) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathscr{M}_{q\bar{q}' \to t\bar{t}}\left(\{p_i\}\right)$$

- The perturbative function $F\left(\alpha_S(\mu_R); m_t/\mu_R\right)$ is an **effective coupling** which also takes into account the **renormalisation** of the mass and of the wave function;
- To map the $t\bar{t}H$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}H} \to Q_{t\bar{t}}$), we use the **$q_T$ recoil prescription**:
  - We reabsorb the Higgs momentum equally in the initial-state parton momenta;
  - We leave unchanged the top and anti-top momenta.

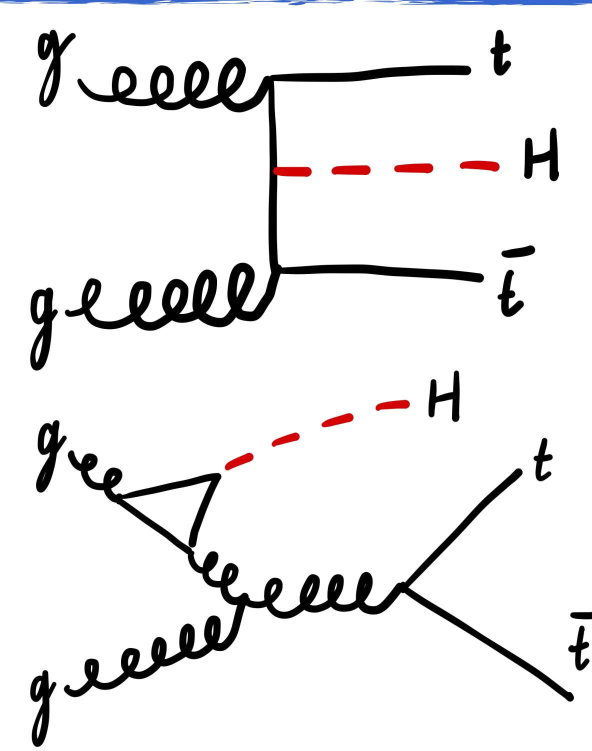
TESTING THE APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

To **validate** our procedure: test the approximation at NLO!

$\Delta\sigma_{\mathrm{NLO,H}}$[fb]	13 TeV		100 TeV	
	gg	$q\bar{q}$	gg	$q\bar{q}$
Exact	88.62	7.826	8205	217.0
Soft Approximation	61.92	7.413	5612	206.0
Difference	**30.1%**	**5.27%**	**31.6%**	**5.06 %**

- Deviation w.r.t. exact computation is about **30%** for the gg **channel** and **5%** for the $q\bar{q}$ **channel**;
- Deviation **independent** of kinematic variables;
- **Better agreement** for $q\bar{q}$ channel can be explained by the presence, both at LO and NLO, of diagrams where a **Higgs boson is radiated from a virtual top** only present in the gg channel.



UNCERTAINTIES ESTIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

How to estimate the NNLO uncertainties?

- We use the **deviation from the exact results at NLO** as a **lower bound** on the NNLO uncertainty;
- We multiply by a **tolerance factor** of **3**;
- We combined **linearly** the uncertainty for the gg and $q\bar{q}$ channel;

How to test the NNLO uncertainties?

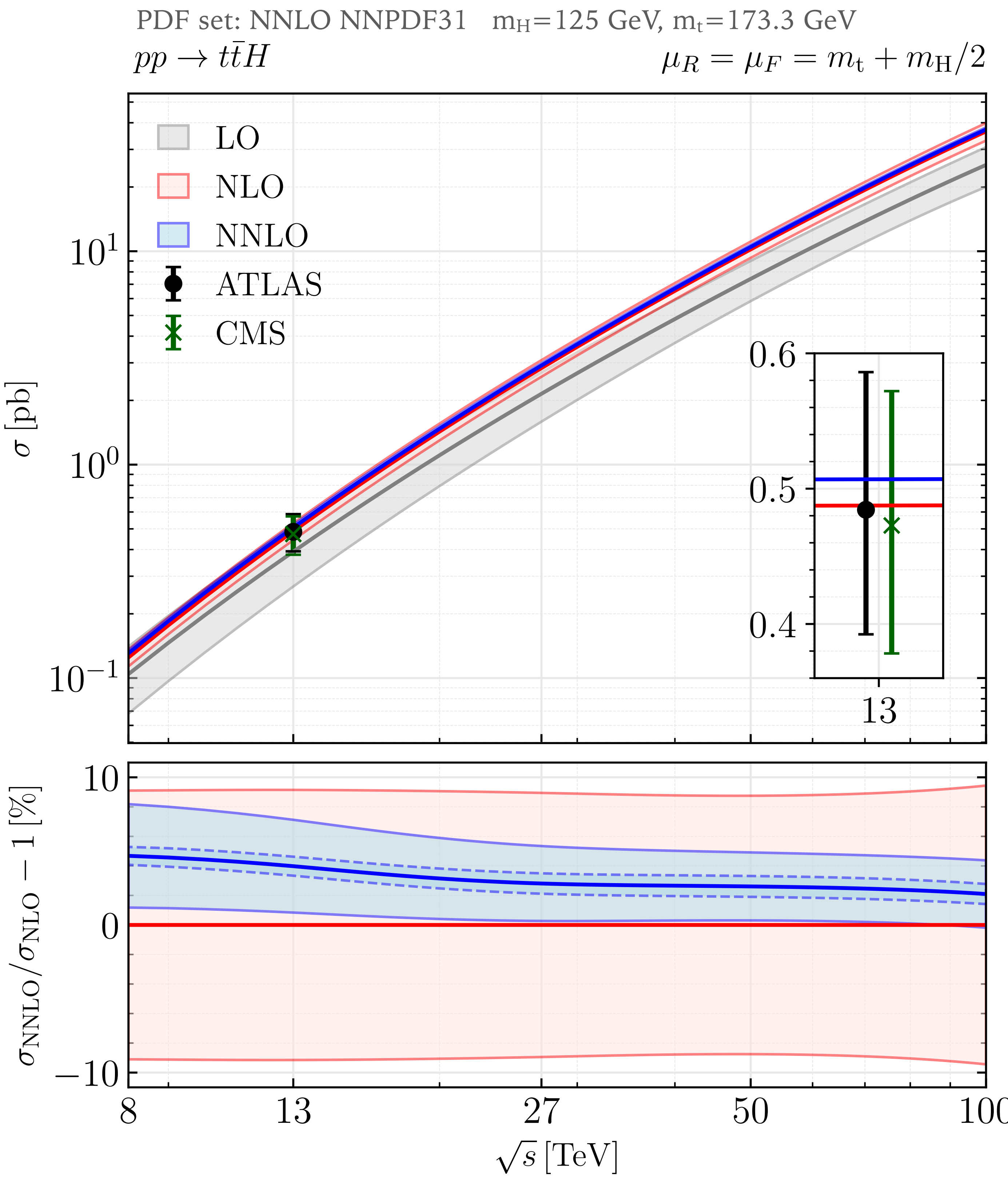
- Check the effect of using **different recoil prescription**;
- Check the effect of using a **different subtraction scales** $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$.

Final uncertainty:
- **$\pm 15\,\%$ on $\Delta\sigma_{\rm NNLO}$**
- **$\pm 0.6\,\%$ on $\sigma_{\rm NNLO}$**

*Effect on the total cross section modulated by the (small) contribution of the hard factor: about **1%** of the LO cross section in the gg and **2-3%** in the $q\bar{q}$ channel.*

RESULTS

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

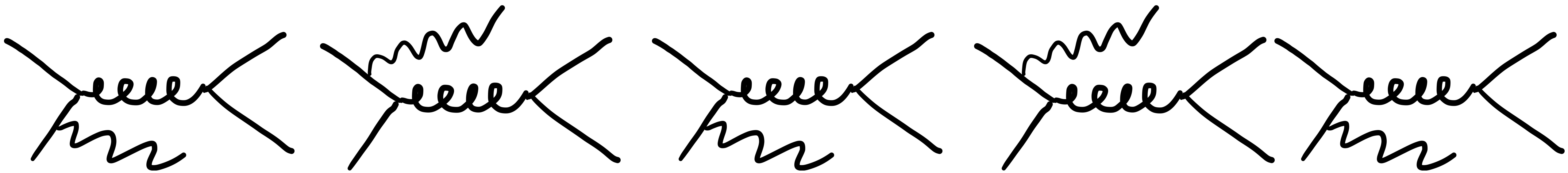


σ [pb]	13 TeV	100 TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

Numerical + soft Higgs uncertainties

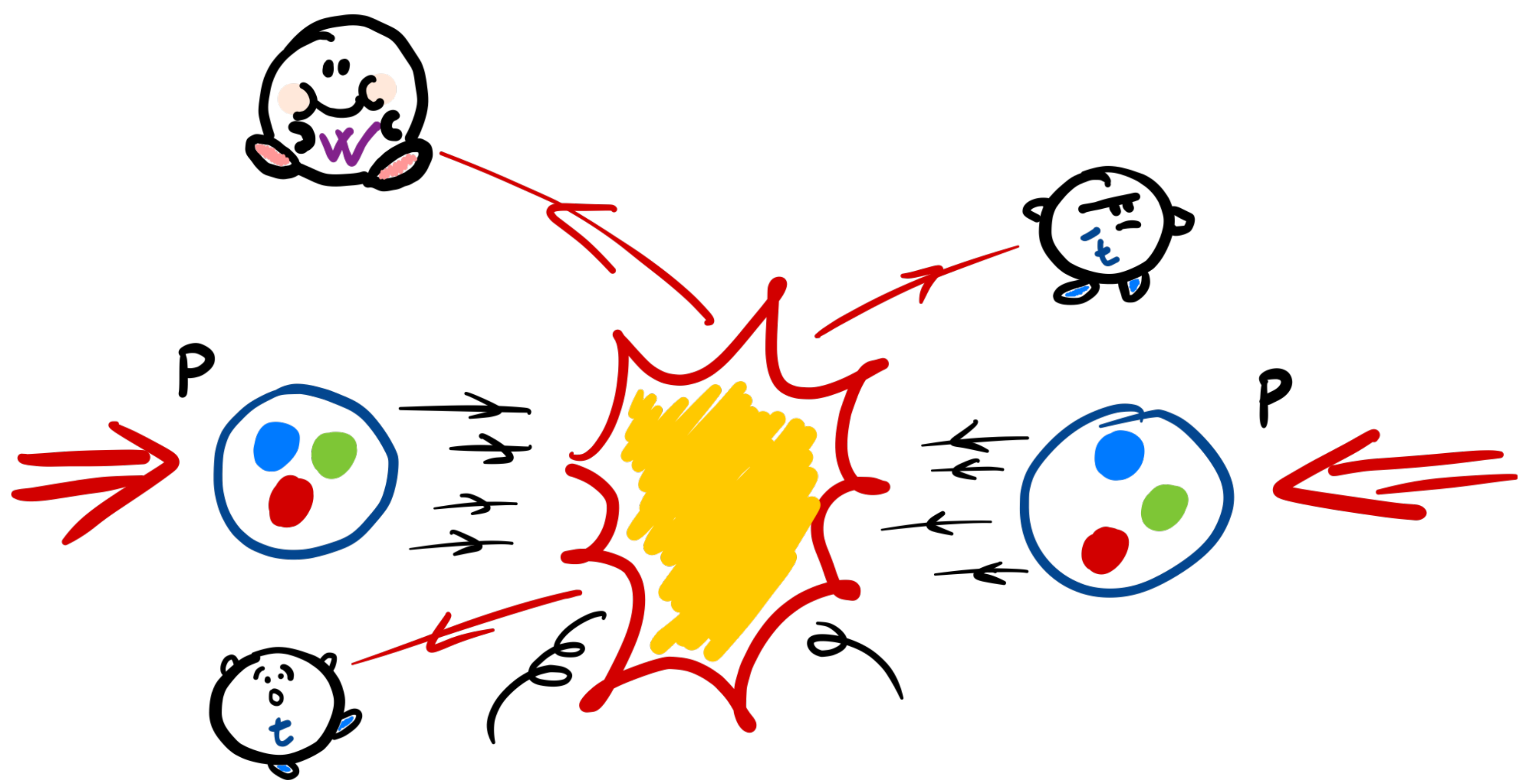
Scale uncertainties

- **NNLO corrections**: **+4%** (13 TeV), **+2%** (100 TeV);
- Reduction of **scale uncertainties**;
- Soft approximation uncertainty significantly **smaller** than remaining perturbative uncertainties.



$t\bar{t}W$ PRODUCTION

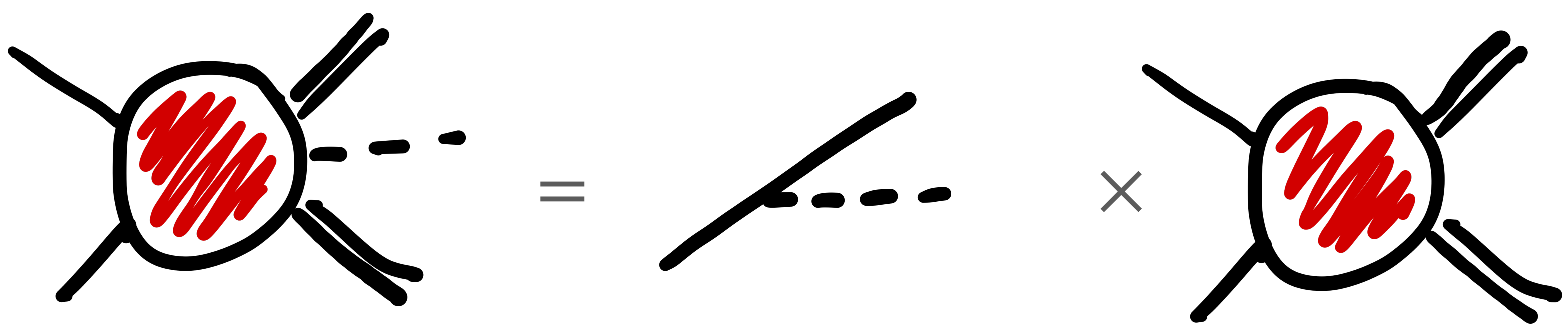
[ArXiv:2306.16311]



CHOICE OF THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

- Amplitudes for the process $c\bar{c} \to t\bar{t}$ available [P. Bärnreuther, M. Czakon, P. Fiedler: 1312.6279]: **we can use the soft approximation**.



$$\mathcal{M}_{q\bar{q}'\to t\bar{t}W}\left(\{p_i\},k\right) \simeq \frac{g}{\sqrt{2}}\left(\frac{p_2\cdot\varepsilon^*(k)}{p_2\cdot k}-\frac{p_1\cdot\varepsilon^*(k)}{p_1\cdot k}\right)\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}\left(\{p_i\}\right)$$

- The soft emission of a W selects the **helicity configuration** $\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}$;
- In contrast with the $t\bar{t}H$ case, the soft W is emitted by the **initial-state partons**;
- To map the $t\bar{t}W$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}W} \to Q_{t\bar{t}}$), we use use a **prescription symmetrised** with respect to the one employed for $t\bar{t}H$ case:
  - We reabsorb the W momentum equally in the top-quark momenta;
  - We leave unchanged the initial-state parton momenta.

CHOICE OF THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

- Amplitudes for the massless process $c\bar{c} \to q\bar{q}W$ available *[S. Abreu, F. Febres Cordero, H. Ita, M. Klinkert, B. Page, V. Sotnikov: 2110.07541]*: **we can use the massification procedure**;

$$\mathcal{M}(\{p_i\}, k; \mu_R, \epsilon) \sim Z_{[q]}^{(m_t|0)}\left(\alpha_S(\mu_R), \frac{m_t}{\mu_R}, \epsilon\right) \mathcal{M}^{m_t=0}(\{p_i\}, k; \mu_R, \epsilon)$$

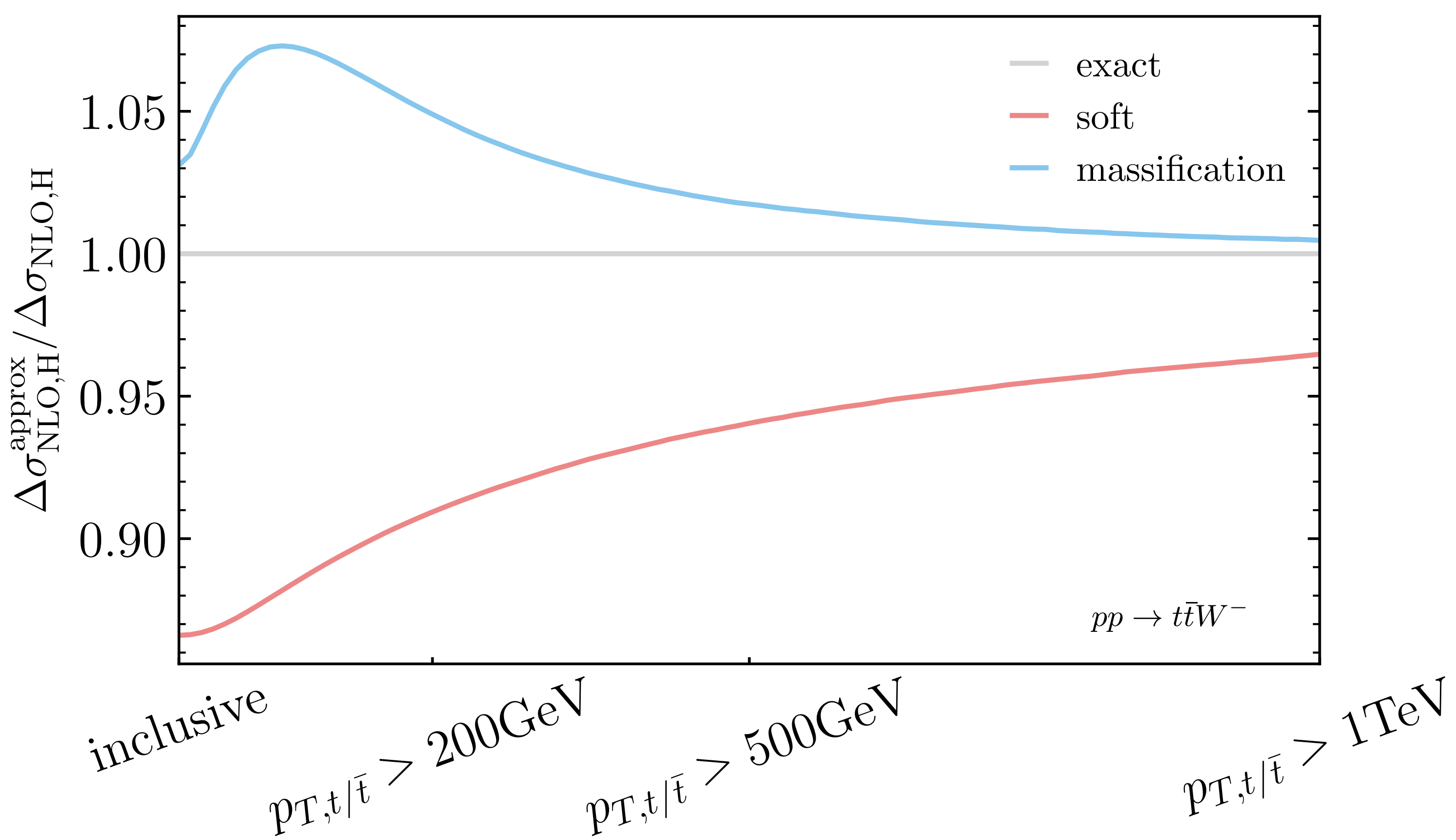
- Massification of the amplitudes implemented in a **C++ library**, **WQQAmp** *[L. Buonocore, L. Rottoli, C. Savoini, https://gitlab.com/lrottoli/WQQAmp]*;
- We need to map the massless kinematics into a massive one: we do it by preserving the momentum of the $t\bar{t}$ pair.

TESTING THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

To validate our procedure: test the approximations at NLO!

- Both approximations provide a **good estimation** also at the inclusive level;
- We observe a **pattern**: **soft approximation undershoots** the exact result, while the **massification procedure overshoots**;
- As expected, both approximations get closer to the exact result when a **harder cut** is imposed

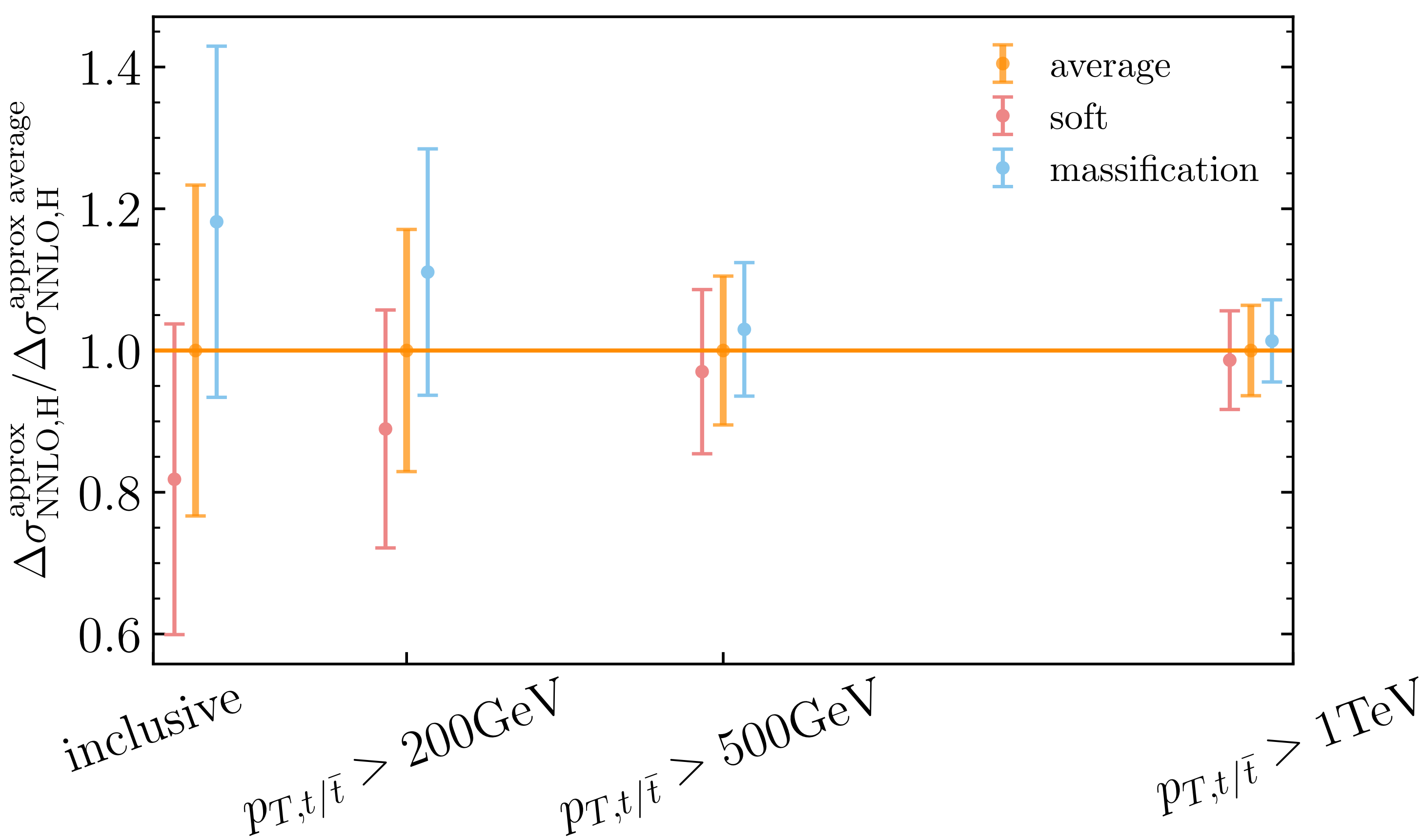


UNCERTAINTIES ESTIMATION

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

How to estimate the NNLO uncertainties of each approximation?

- **Method 1**: we take the difference between exact and approximated result at NLO and we multiply by a **tolerance factor** of **2**;
- **Method 2**: we consider the effect of using a **different subtraction scales** $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$;
- The uncertainty is defined as the **maximum between these two estimates**.



- The two approximations are **fully consistent**;
- Our best prediction is obtained by taking their **average** and **linearly combing** the uncertainties.

Final uncertainty:

- **$\pm 25\,\%$ on $\Delta\sigma_{\rm NNLO,H}$**
- **$\pm 2\,\%$ on $\sigma_{\rm NNLO}$**

RESULTS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

LHC@13TeV	$\sigma_{t\bar{t}W^+}$ $[fb]$	$\sigma_{t\bar{t}W^-}$ $[fb]$	$\sigma_{t\bar{t}W}$ $[fb]$	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
LO_{QCD}	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.0^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
NLO_{QCD}	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
NNLO_{QCD}	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\,\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\,\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\,\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\text{NNLO}_{\text{QCD}}+\text{NLO}_{\text{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\,\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\,\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\,\%$	$2.007^{+2.1\%}_{-2.1\%}$

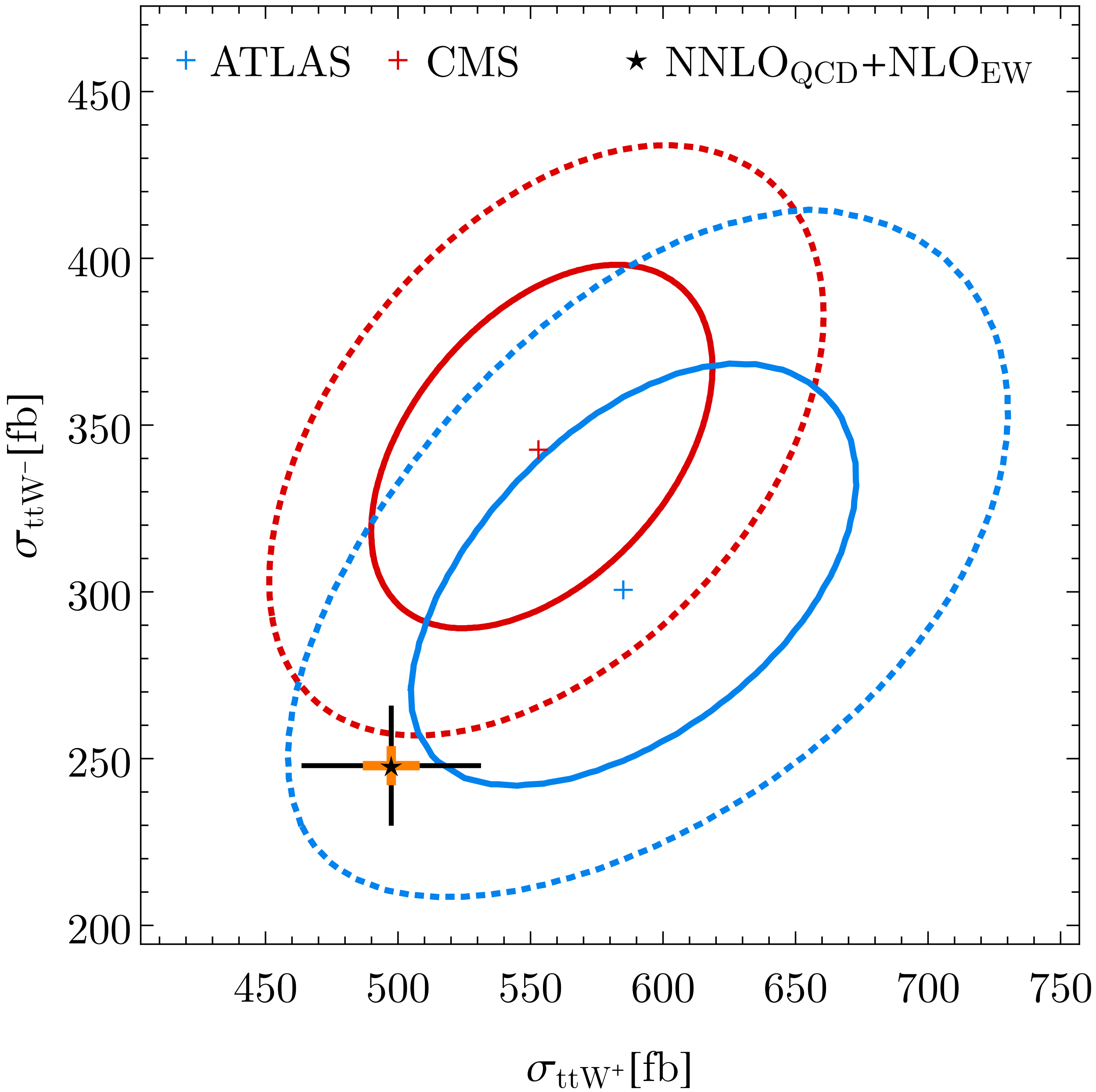
Scale uncertainties

Uncertainties from 2 loop amplitudes

- We choose $\mu_0 = M/2$;
- NNLO predictions show first sign of **perturbative convergence**;
- ratio $\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$ have a **very stable** perturbative behaviour;
- **PDF uncertainties** $\pm 1.8\,\%$ (computed with MATRIX + PINEAPPL interface *[SD, T. Ježo, S. Kallweit, C. Schwan, in preparation]*)
- α_S **uncertainties** $\pm 1.8\,\%$;
- by combining with EW corrections, we get our **best prediction**;
- to be conservative, scale uncertainties for $\text{NNLO}_{\text{QCD}}+\text{NLO}_{\text{EW}}$ are **symmetrised**.

RESULTS

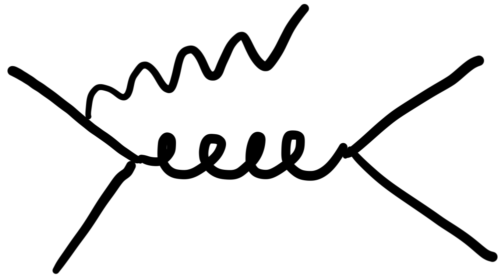
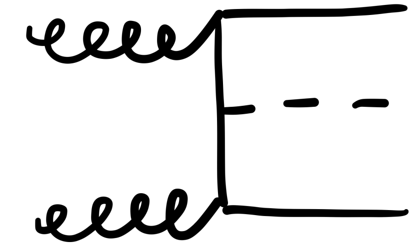
[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]



- We compare our best prediction to **ATLAS and CMS measurements**;
- With respect to the **FxFx prediction**, the current theory reference, higher rate and smaller uncertainties;

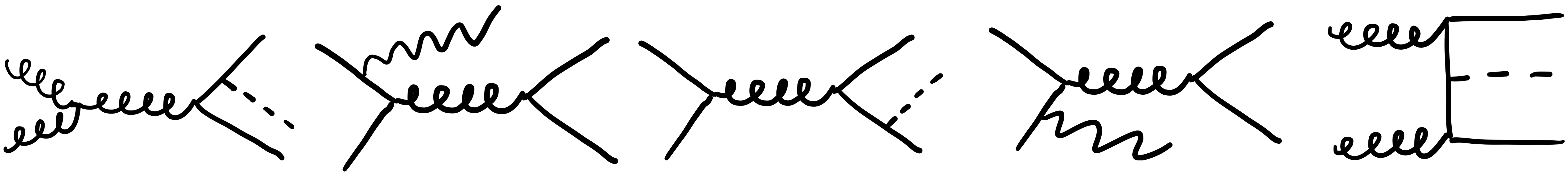
$$\sigma_{t\bar{t}W}^{NNLO_{QCD}+NLO_{EW}} = 745.3^{+6.7\%}_{-6.7\%}$$
$$\sigma_{t\bar{t}W}^{FxFx} = 722.3^{+9.7\%}_{-10.8\%}$$

- Tension remains at the $1\sigma - 2\sigma$ **level**.

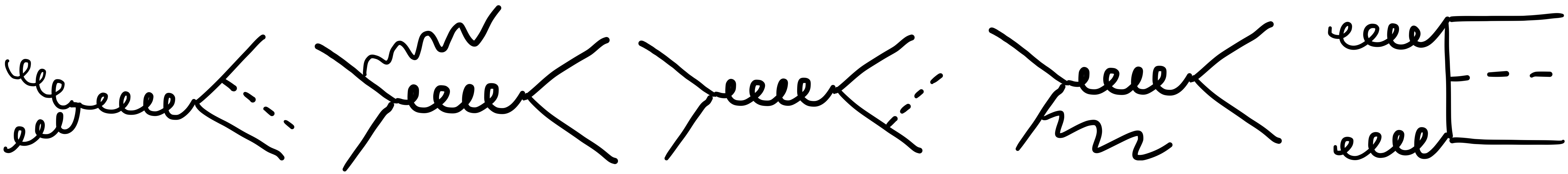
SUMMARY & OUTLOOK

- We computed within q_T subtraction formalism the **NNLO QCD corrections** to $t\bar{t}H$ production and $t\bar{t}W$ production;
- The **missing ingredients** we needed for the computation are:
  - **NNLO soft contribution** in arbitrary kinematics;
  - **two-loop amplitudes** (**massification** and/or **soft approximation**);
- **First** (almost) exact computations at NNLO QCD for a **$2 \rightarrow 3$ process** with massive coloured particles.



SUMMARY & OUTLOOK

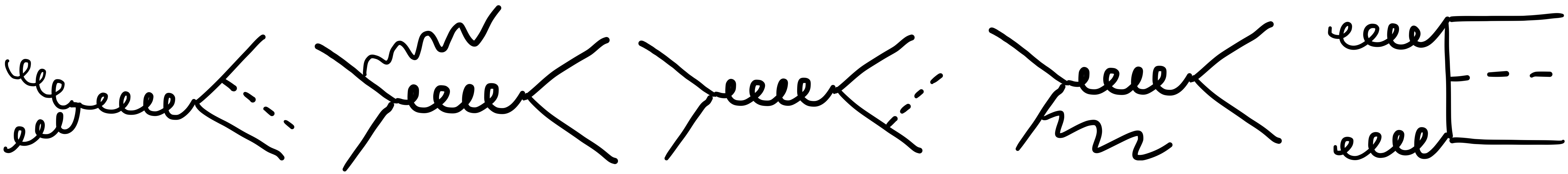
- **Differential distributions**;
- Further phenomenological studies.



SUMMARY & OUTLOOK

- **Differential distributions**;
- Further phenomenological studies.

THANKS!



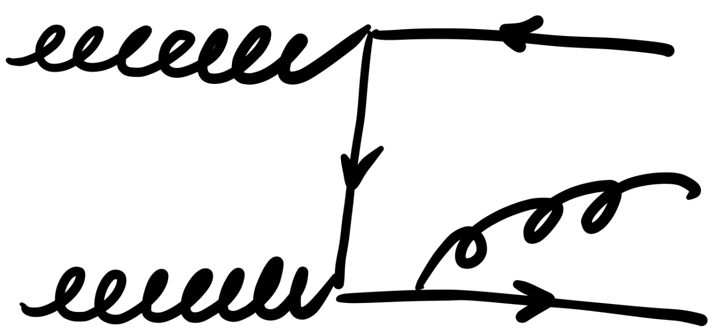
BACKUP SLIDES

IR SUBTRACTION METHODS (NLO)

$$\Delta\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

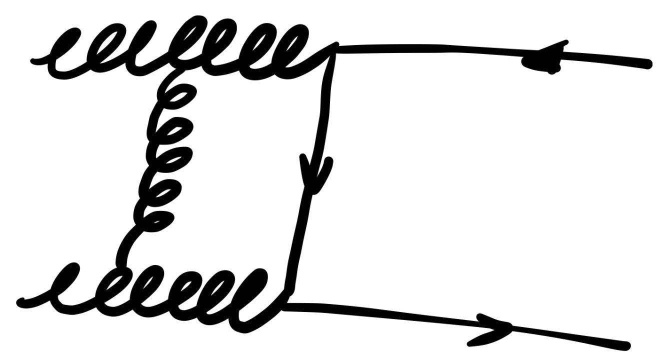
Divergent   Divergent

REAL



- No explicit ϵ poles;
- Singular in unresolved limit.

VIRTUAL



- Explicit poles up to order $1/\epsilon^2$;
- No additional PS singularity

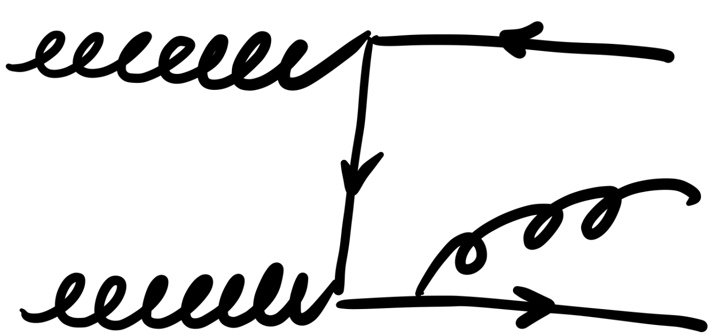
IR SUBTRACTION METHODS (NLO)

$$\Delta\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1} \left[d\sigma^{R} - d\sigma^{CT} \right] + \left[\int_{m} d\sigma^{V} + \int_{1} d\sigma^{CT} \right]$$

~~Divergent~~ CONVERGENT!

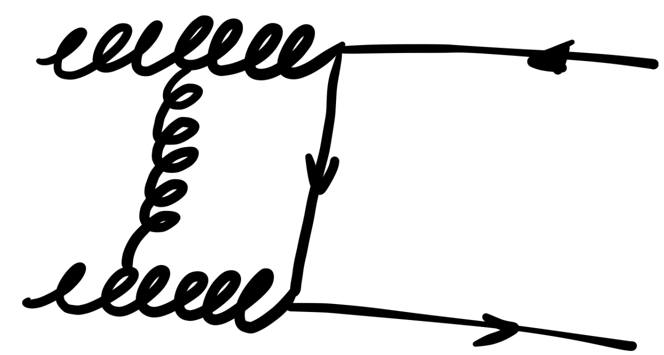
~~Divergent~~ CONVERGENT!

REAL



- No explicit ϵ poles;
- Singular in unresolved limit.

VIRTUAL



- Explicit poles up to order $1/\epsilon^2$;
- No additional PS singularity

TOTAL CROSS SECTION

	$\sqrt{s} = 13\,\text{TeV}$		$\sqrt{s} = 100\,\text{TeV}$	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}}\vert_{\text{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\text{NNLO,H}}\vert_{\text{soft}}$	−2.980(3)	2.622(0)	−239.4(4)	65.45(1)

- Soft Higgs approximation at LO:
  - gg channel: factor 2.3 ($\sqrt{s}$ = 13 TeV)/factor 2.0 ($\sqrt{s}$ = 100 TeV)
  - $q\bar{q}$ channel: factor 1.11 ($\sqrt{s}$ = 13 TeV)/factor 1.06 ($\sqrt{s}$ = 100 TeV)
- At LO there is no reweighting!

CHANGING THE SUBTRACTION SCALE

$$H^{(2)}_{t\bar{t}H} = \frac{2\,Re\left(\mathcal{M}^{(2)}_{t\bar{t}H}(\mu_{IR},\mu_R)\mathcal{M}^{(0)}_{t\bar{t}H}\right)_{soft}}{|\,\mathcal{M}^{(0)}_{t\bar{t}H}|^2_{soft}}$$

- The subtraction scale μ_{IR} is the scale at which the IR poles are subtracted (equivalently, at which the soft approximation is applied);
- Effect of using a different subtraction scales $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$.
  - gg channel +164%/-25% ($\sqrt{s} = 13$ TeV)
    +142%/-20% ($\sqrt{s} = 100$ TeV)
  - $q\bar{q}$ channel +4%/-0% ($\sqrt{s} = 13$ TeV)
    +3%/-0% ($\sqrt{s} = 100$ TeV)

SOFT HIGGS APPROXIMATION

Eikonal approximation

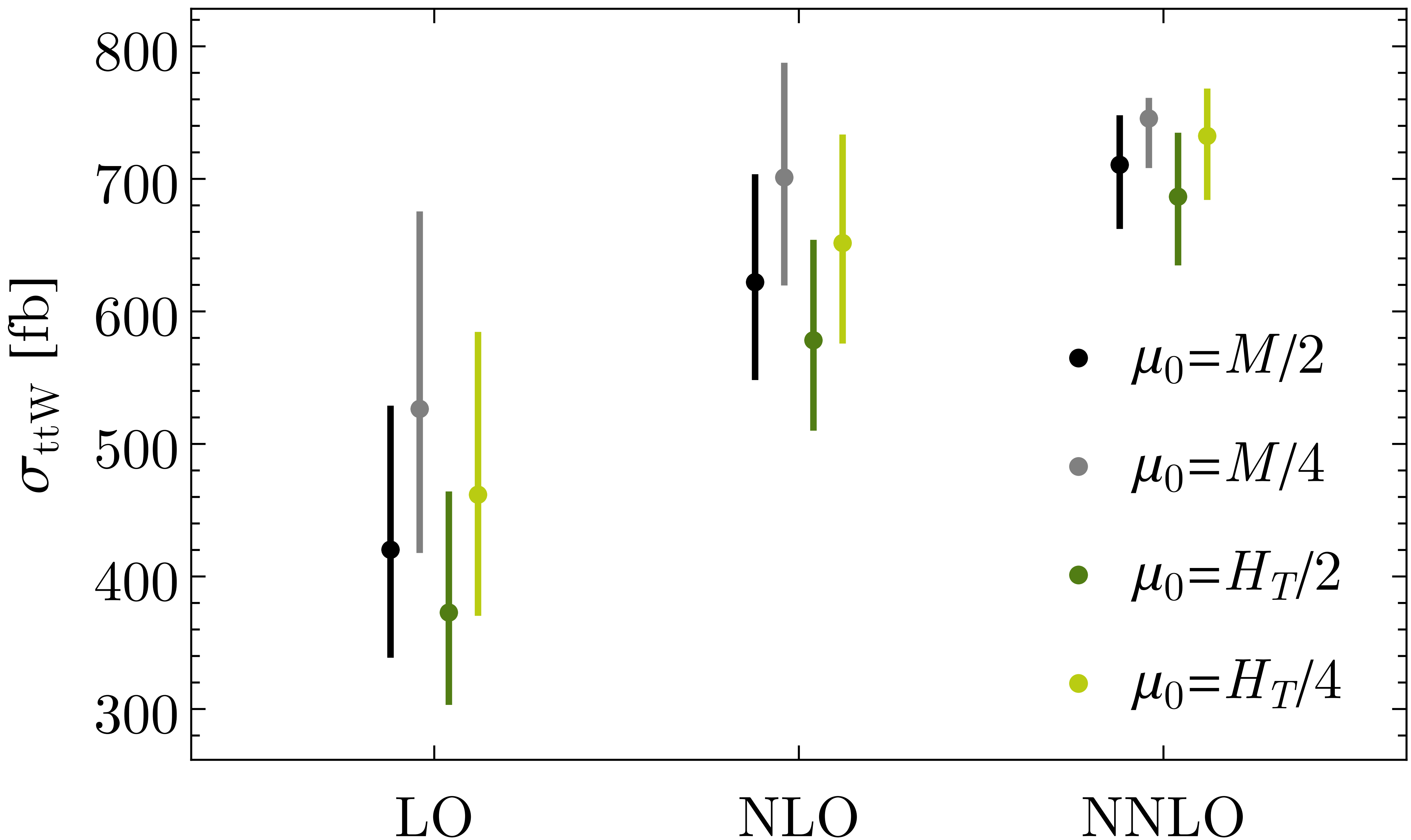
$$\lim_{k\to 0} \mathscr{M}_{t\bar{t}H}\big(\{p_i\},k\big) = F\big(\alpha_S(\mu_R); m_t/\mu_R\big)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\mathscr{M}_{t\bar{t}}\big(\{p_i\}\big)$$

Low Energy Theorem

$$\lim_{q\to 0} \mathscr{M}^{\text{bare}}(p\to p+q) = \frac{1}{v}m_0\frac{\partial}{\partial m_0}\mathscr{M}^{\text{bare}}(p\to p)\Big|_{p^2=m^2}$$

$$F(\alpha_S(\mu_R); m_t/\mu_R) = 1 + \frac{\alpha_S(\mu_R)}{2\pi}\left(-3\,C_F\right)$$
$$+\left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2\left(\frac{33}{4}C_F^2 - \frac{185}{12}C_FC_A + \frac{13}{6}C_F(n_L+1) - 6\,C_F\beta_0\ln\frac{\mu_R^2}{m_t^2}\right) + \mathcal{O}(\alpha_S^3)$$

$t\bar{t}W$: DIFFERENT SCALE CHOICES



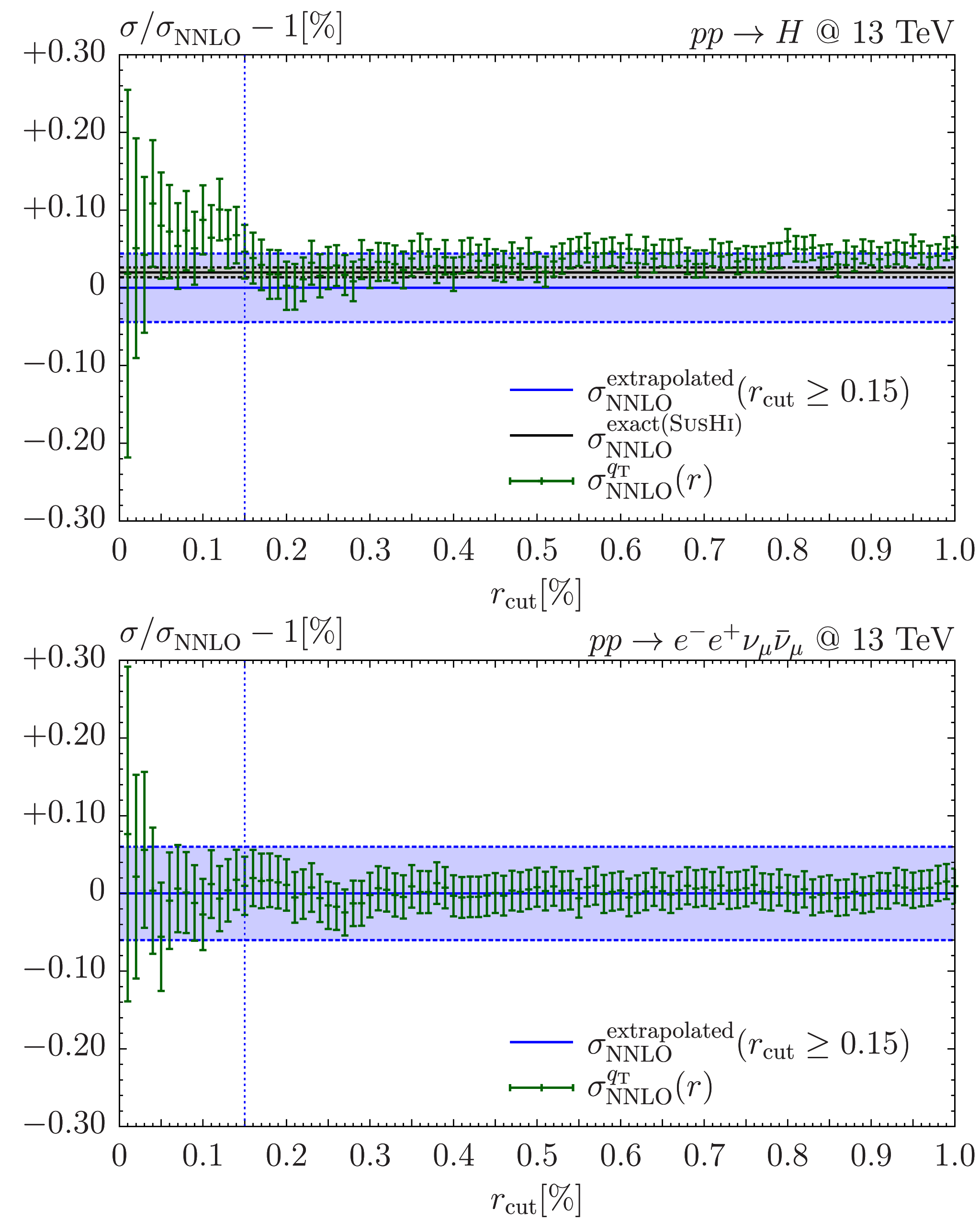
THE SLICING

$$d\sigma^{F}_{(N)NLO} = \mathscr{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

$d\sigma^{F+jets}_{(N)LO}$ and $d\sigma^{CT}_{(N)LO}$ are separately divergent. In practice, q_T subtraction is implemented as a slicing method:

- introducing a cutoff $r_{cut} = Q/M$;
- performing the limit $r_{cut} \to 0$.

Quality of the $q_T \to 0$ extrapolation can be understood looking at the r_{cut} dependence



[M.Grazzini, S. Kallweit, M. Wiesemann: arXiv 1711.06631]

r_{CUT} DEPENDENCE