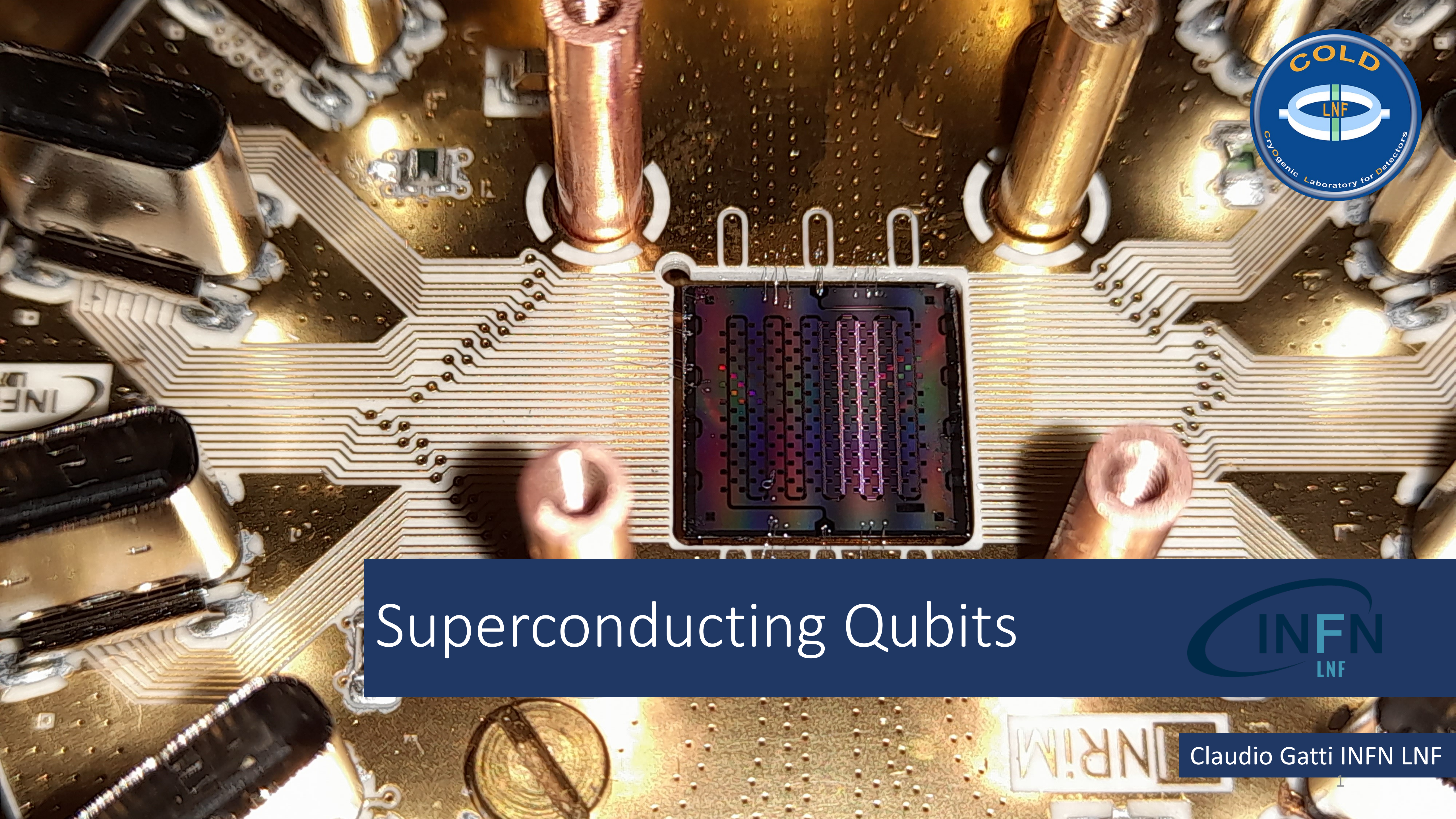
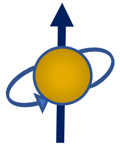
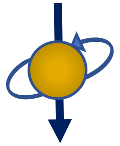


Superconducting Qubits

Claudio Gatti INFN LNF

Qubit

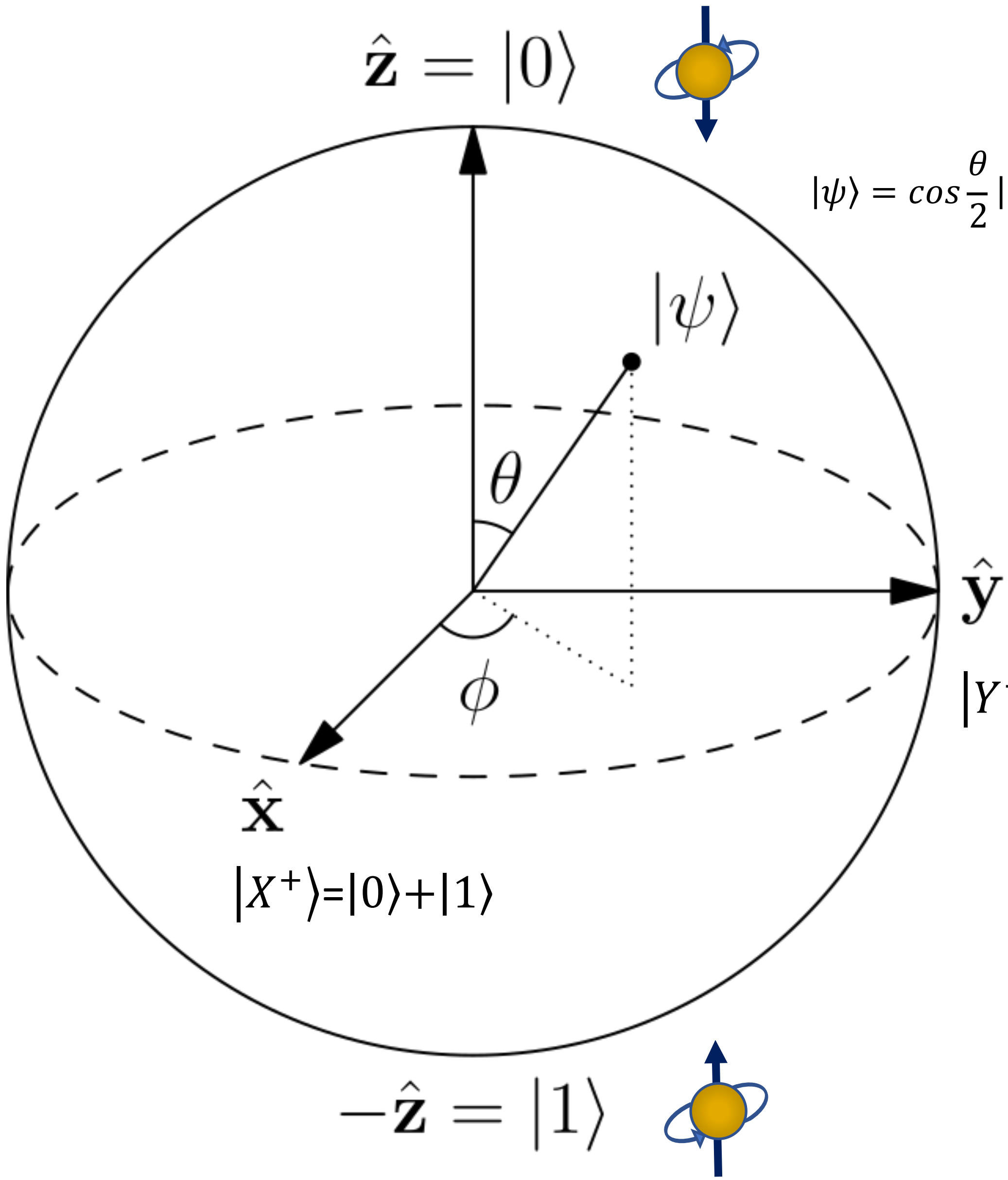
$|Z^{+}\rangle = |1\rangle$ 

$|Z^{-}\rangle = |0\rangle$ 

State superposition

$$|S\rangle = a\,|0\rangle + \mathrm{b}\,|1\rangle$$

$$Probability\ (S = 0) = |a|^2$$
$$Probability\ (S = 1) = |b|^2$$



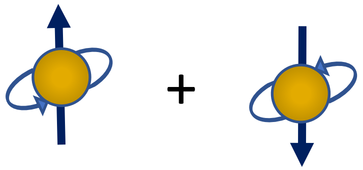
$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$

$|Y^{+}\rangle = |0\rangle + i\ |1\rangle$

Entanglement

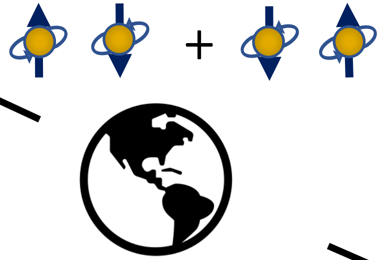
2 atoms with total spin zero

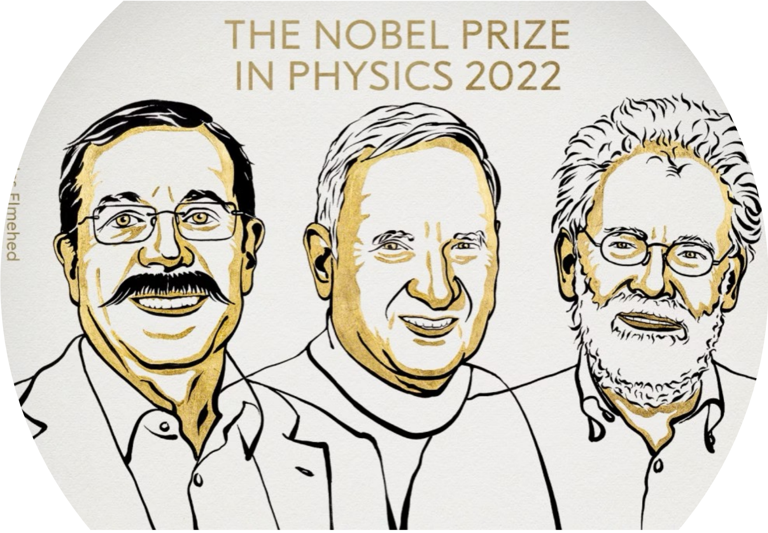
$$|S\rangle = |0,1\rangle + |1,0\rangle$$



Atom 1

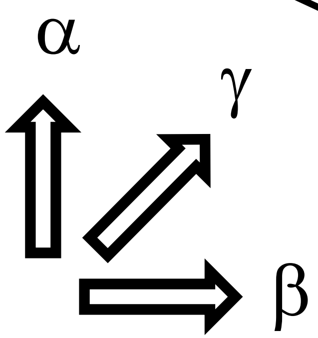
EPR

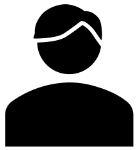




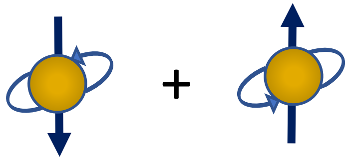
Alain Aspect, John F. Clauser e Anton Zeilinger

Alice





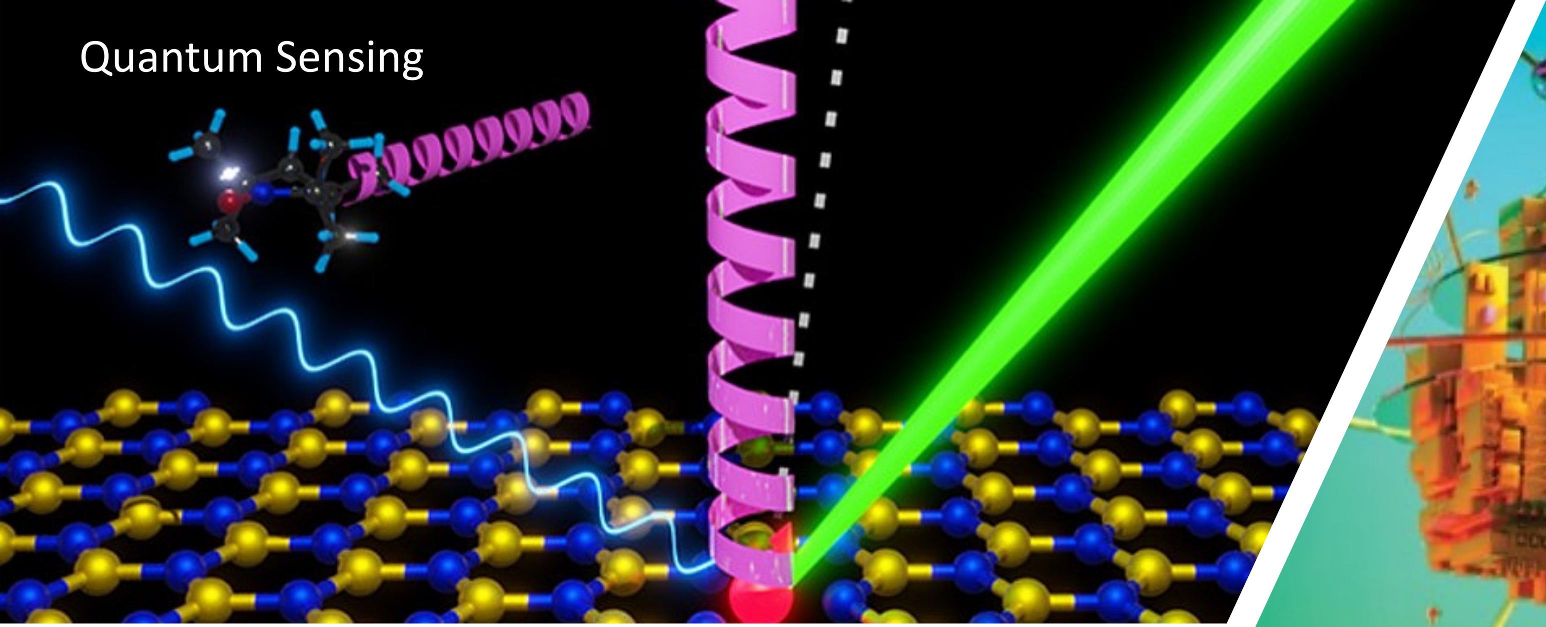
Bob

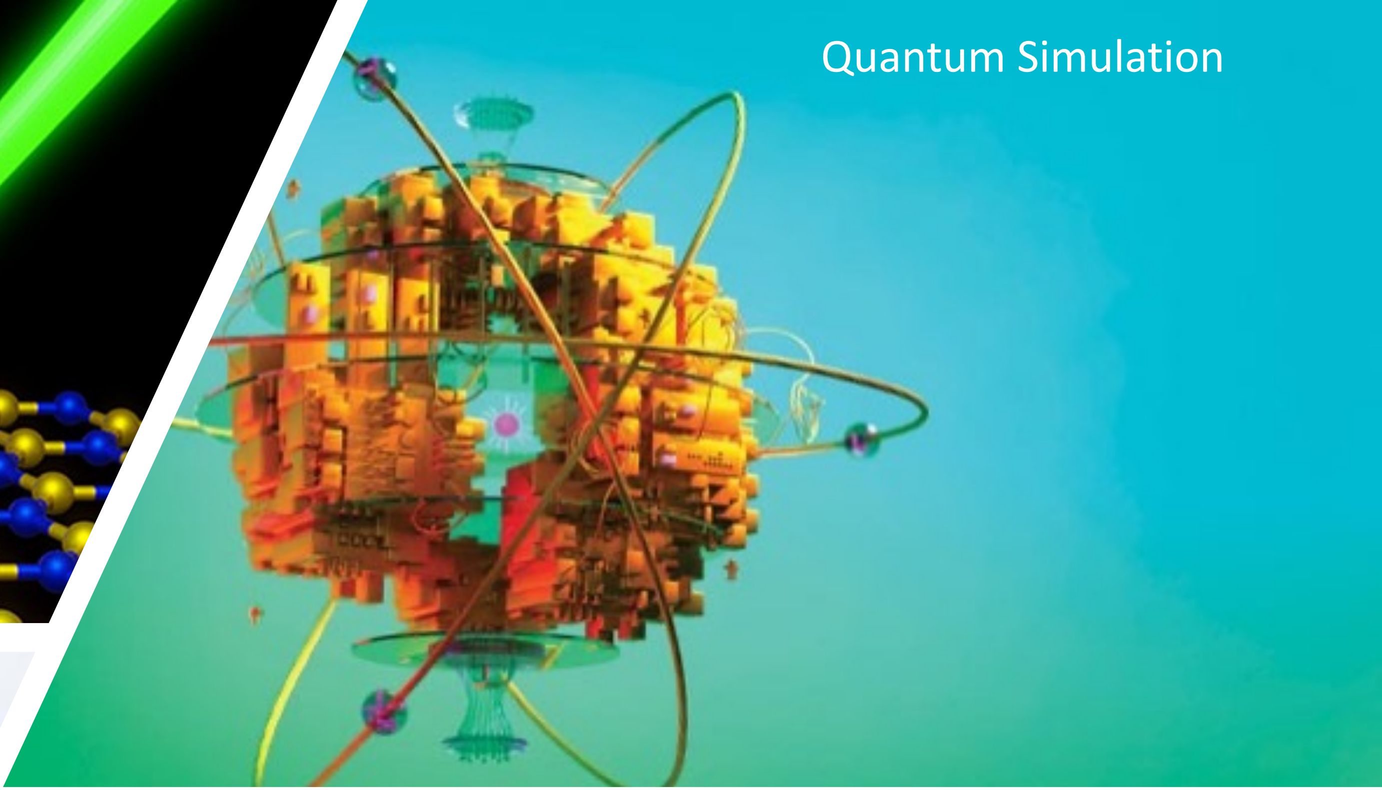


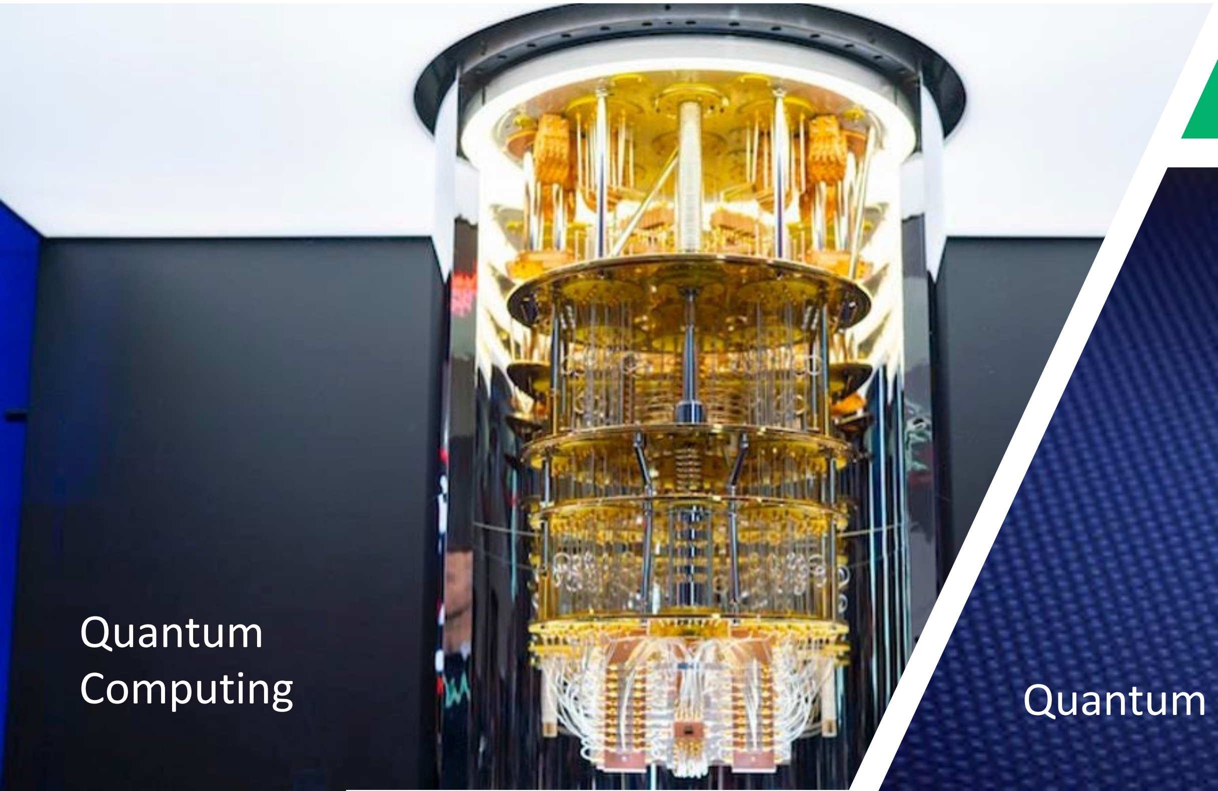
Atom 2

For a “classical state” the Bell inequality holds:

$$Prob(Alice\ \alpha = 1; Bob\ \beta = 1) \leq Prob(Alice\ \alpha = 1; Bob\ \gamma = 1) + Prob(Alice\ \gamma = 1; Bob\ \beta = 1)$$

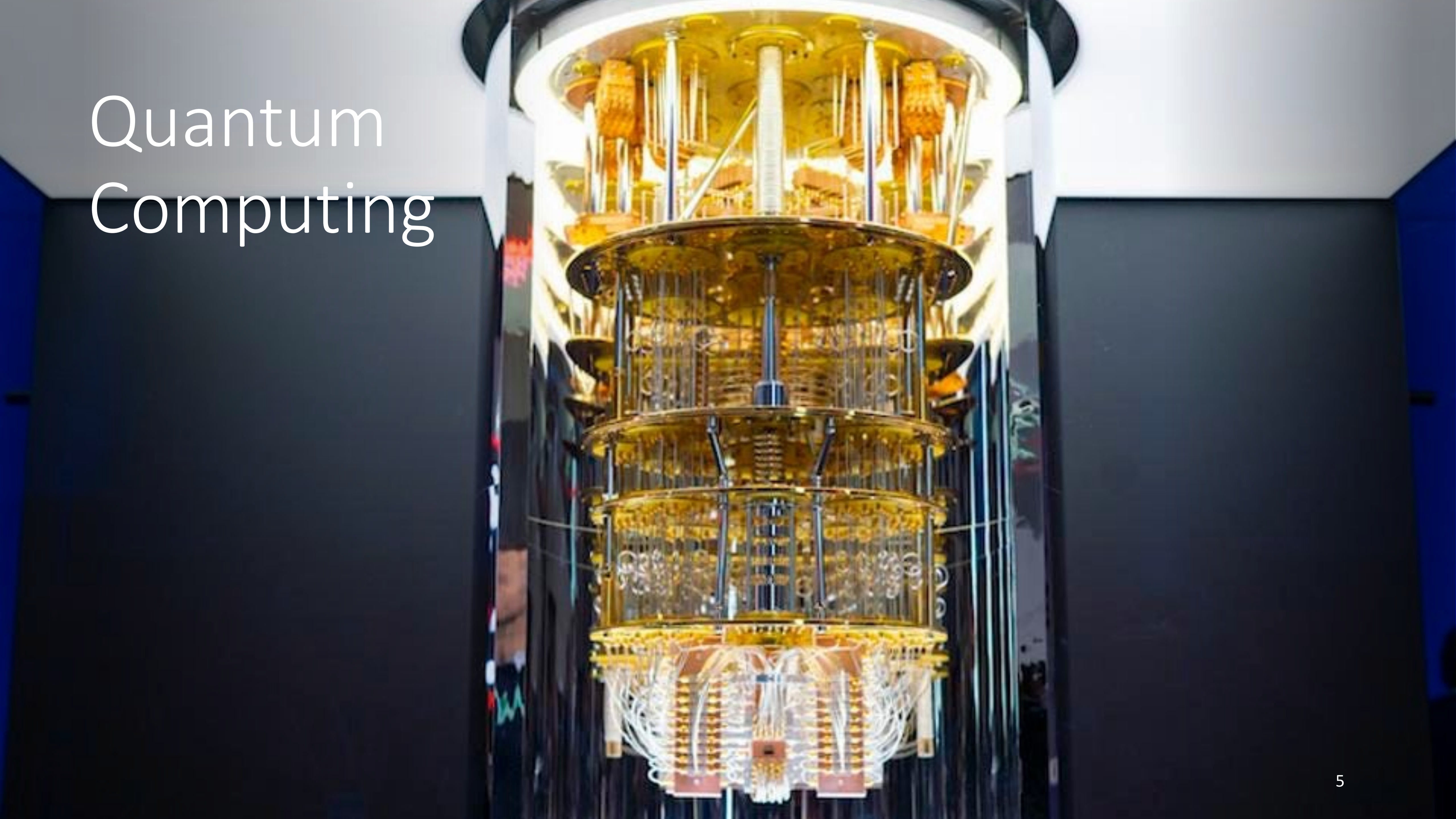

Quantum Sensing


Quantum Simulation


Quantum
Computing


Quantum Cryptography

Quantum Computing



Computational Complexity

Minimum resources needed to perform a given computation

2^n numbers	n=4 bits
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
…	…

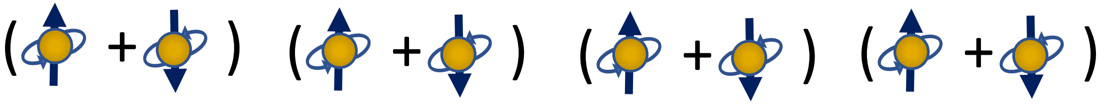
$$f(x) = 0 \; for \; x \neq x_0$$
$$f(x_0) = 1$$

Find x such that *f(x)*=1

A classical computer must compute *f(x)* for all the $N=2^n$ values of x

Grover Algorithm

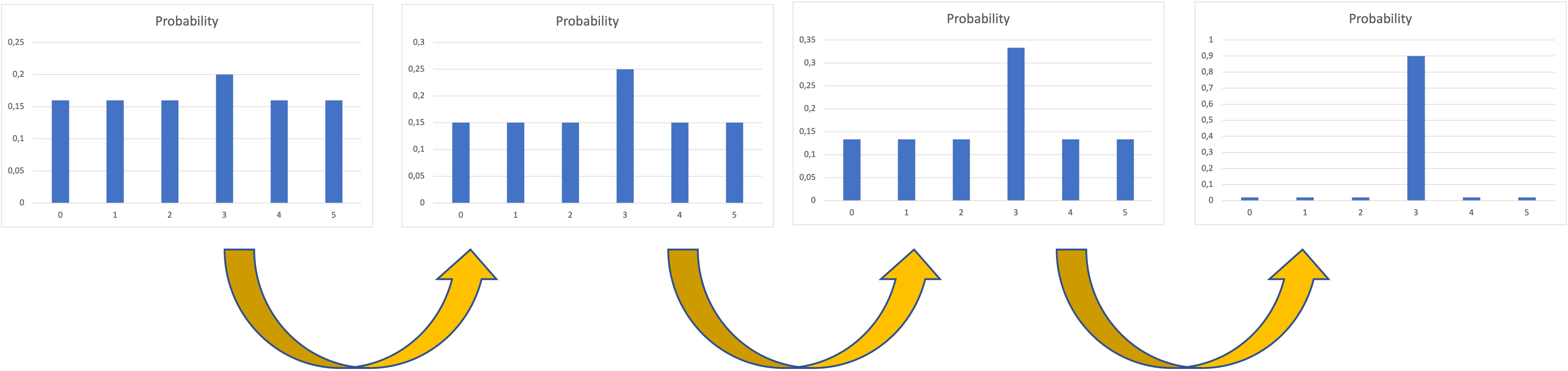
Exploits the uniform superpositions of all the states



$$|S\rangle = (\ldots + \ldots)(\ldots + \ldots)(\ldots + \ldots)(\ldots + \ldots) = |0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + \ldots$$

1. Acts in parallel on all the configurations
2. Amplifies the probability of measuring the configuration corresponding to the correct answer
3. Requires $N^{1/2}$ operations

$$a_0|0\rangle + a_1|1\rangle + a_2\,|2\rangle + a_3|3\rangle + \ \ldots$$



Grover Algorithm

$U_\omega|x\rangle = (-1)^{f(x)}|x\rangle$

$f(x=\omega) = 1$ and 0 otherwise

$U_S = 2\,|S\rangle\,\langle S| - I$

→

Grover Algorithm obtained by performing the rotation U_{GA} many times

$$U_{GA} = U_S U_\omega$$

Example of one iteration of GA with 3 qubits for ω=1:

$|S\rangle = (|0\rangle + |1\rangle + \ |2\rangle + \cdots + \ |7\rangle)/\sqrt{8}$

Grover Algorithm

$$U_\omega|x\rangle = (-1)^{f(x)}|x\rangle$$

$f(x=\omega) = 1$ and 0 otherwise

$$U_S = 2\,|S\rangle\,\langle S| - I$$

→

Grover Algorithm obtained by performing the rotation U_{GA} many times

$$U_{GA} = U_S U_\omega$$

Example of one iteration of GA with 3 qubits for ω=1:

$$|S\rangle = (|0\rangle + |1\rangle + |2\rangle + \cdots + |7\rangle)/\sqrt{8}$$

$$U_\omega|S\rangle = (|0\rangle - |1\rangle + |2\rangle + \cdots + |7\rangle)/\sqrt{8}$$

Grover Algorithm

$U_\omega|x\rangle = (-1)^{f(x)}|x\rangle$

$f(x=\omega) = 1$ and 0 otherwise

$U_S = 2\,|S\rangle\,\langle S| - I$

→

Grover Algorithm obtained by performing the rotation U_{GA} many times

$$U_{GA} = U_S U_\omega$$

Example of one iteration of GA with 3 qubits for ω=1:

$|S\rangle = (|0\rangle + |1\rangle + \ |2\rangle + \cdots + \ |7\rangle)/\sqrt{8}$

$U_\omega|S\rangle = (|0\rangle - |1\rangle + \ |2\rangle + \cdots + \ |7\rangle)/\sqrt{8}$

$$U_{GA}|S\rangle = U_S\,\frac{1}{\sqrt{8}}(|0\rangle - |1\rangle + \ |2\rangle + \cdots\ ) = \frac{2}{\sqrt{8}}(|0\rangle + |1\rangle + \cdots)\frac{6}{8} - \frac{1}{\sqrt{8}}(|0\rangle - |1\rangle + \cdots)$$

Grover Algorithm

$$U_\omega|x\rangle = (-1)^{f(x)}|x\rangle$$

$f(x=\omega) = 1$ and 0 otherwise

$$U_S = 2\,|S\rangle\,\langle S| - I$$

Grover Algorithm obtained by performing the rotation U_{GA} many times

$$U_{GA} = U_S U_\omega$$

Example of one iteration of GA with 3 qubits for ω=1:

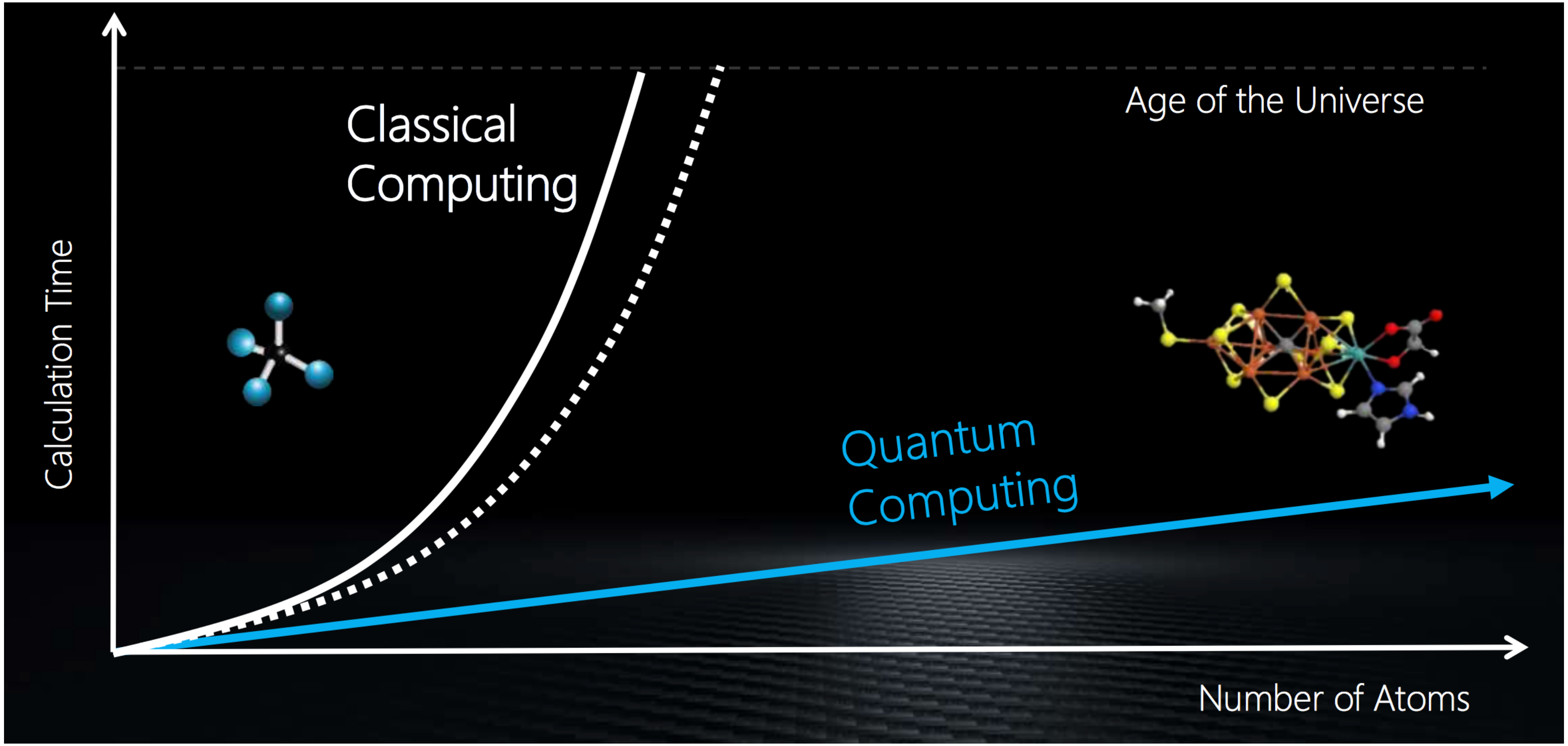
$$|S\rangle = (|0\rangle + |1\rangle + |2\rangle + \cdots + |7\rangle)/\sqrt{8}$$

$$U_\omega|S\rangle = (|0\rangle - |1\rangle + |2\rangle + \cdots + |7\rangle)/\sqrt{8}$$

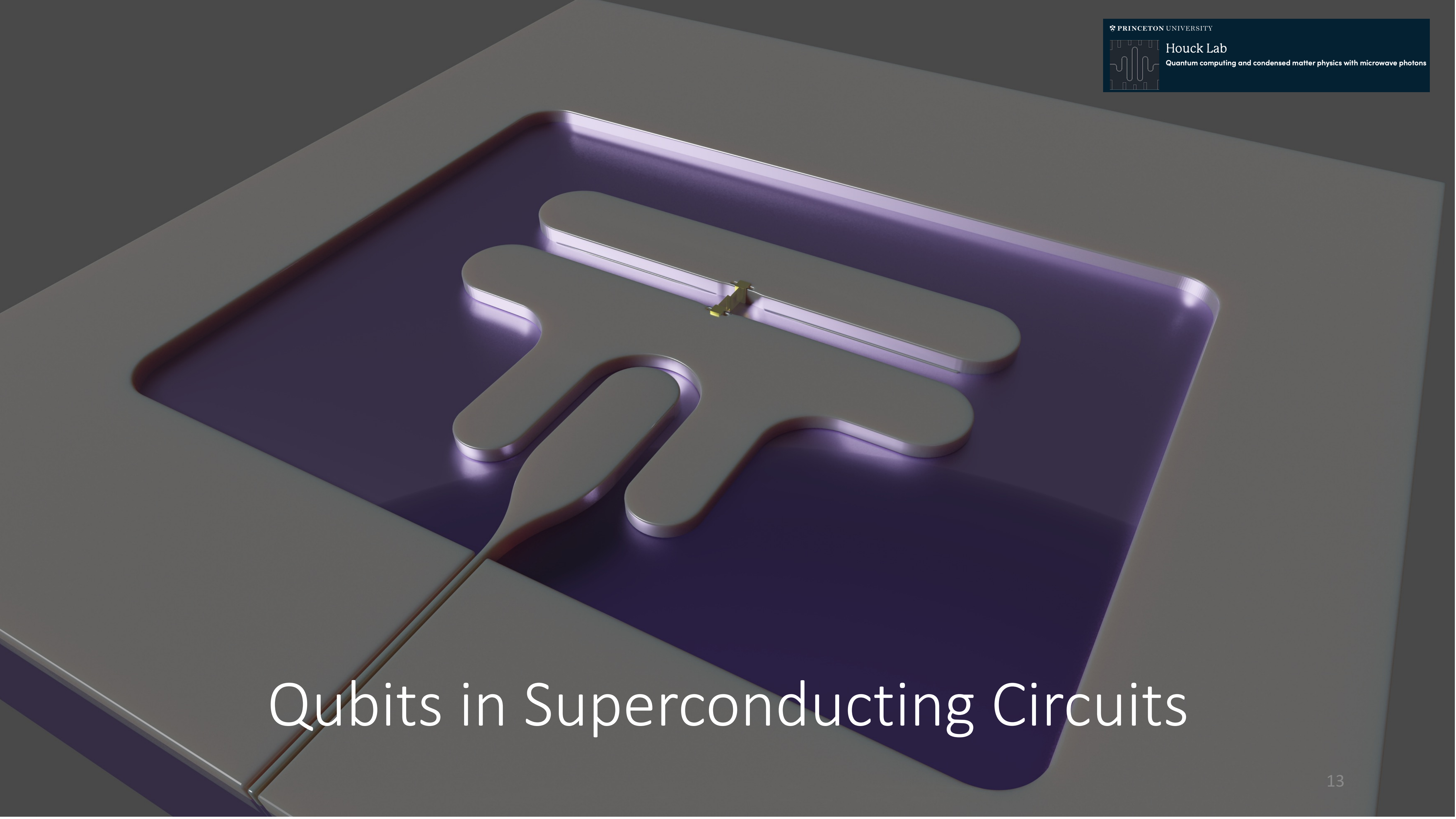
Probability = 25/32

$$U_{GA}|S\rangle = U_S \frac{1}{\sqrt{8}}(|0\rangle - |1\rangle + |2\rangle + \cdots) = \frac{2}{\sqrt{8}}(|0\rangle + |1\rangle + \cdots)\frac{6}{8} - \frac{1}{\sqrt{8}}(|0\rangle - |1\rangle + \cdots) = \frac{1}{2\sqrt{8}}(|0\rangle + 5|1\rangle + |2\rangle + \cdots)$$

Many Body Problems



PRINCETON UNIVERSITY
Houck Lab
Quantum computing and condensed matter physics with microwave photons

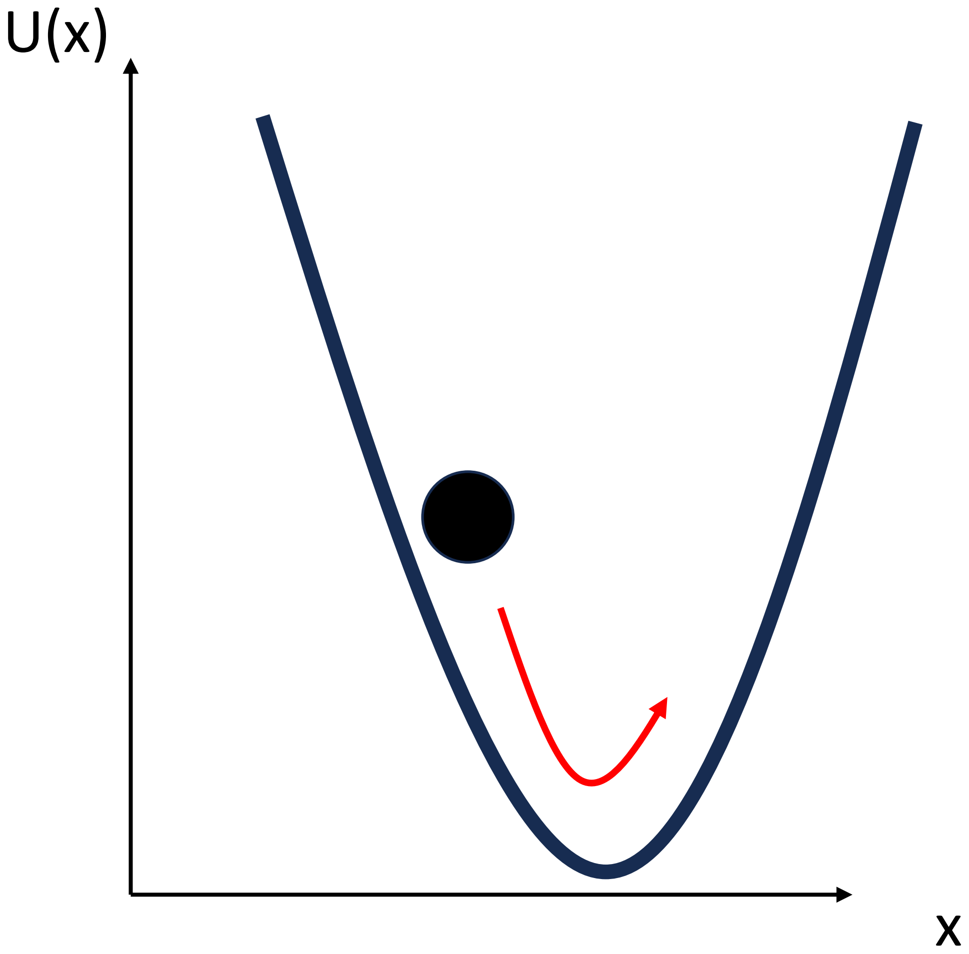


Qubits in Superconducting Circuits

Harmonic Oscillator

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$\omega = \sqrt{\frac{k}{m}}$$



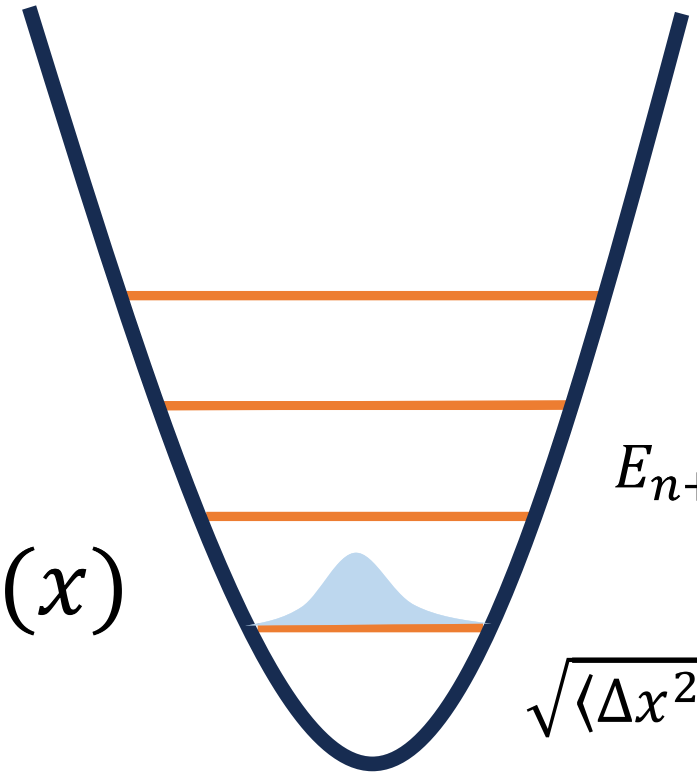
Classical Mechanics

Harmonic Oscillator

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$\psi(x)$



$$E_{n+1} - E_n = \hbar\omega$$

$$\sqrt{\langle \Delta x^2 \rangle \langle \Delta p^2 \rangle} = \frac{\hbar}{2}$$

Quantum Mechanics

Quantum Fluctuations

$$\langle \Delta x^2 \rangle = \frac{\hbar}{2m\omega}$$

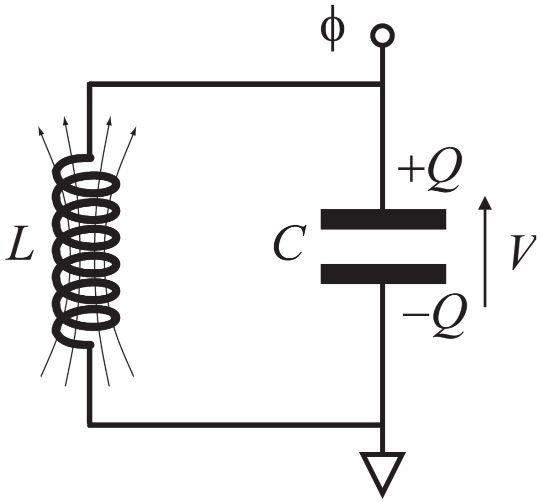
$$\langle \Delta p^2 \rangle = \frac{\hbar m\omega}{2}$$

$\hbar = 1.054572 \times 10^{-34} Js$

$k = 20\ kN$
$m = 1\ g$
$\omega = 2\pi \times 700\ Hz$
$\sqrt{\langle \Delta x^2 \rangle} = 3 \times 10^{-18} m$
$\sqrt{\langle \Delta p^2 \rangle} = 10^{-17} Kg\ m/s$

LC Oscilaltor

Q	$\leftrightarrow$	p
Φ	$\leftrightarrow$	x
C	$\leftrightarrow$	m
L	$\leftrightarrow$	$1/k$



$$E = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\Phi = LI$$

$$\omega = \frac{1}{\sqrt{LC}}$$

Quantum Fluctuations

$$\langle \Delta\Phi^2 \rangle = \frac{\hbar}{2C\omega}$$

$$\langle \Delta Q^2 \rangle = \frac{\hbar C\omega}{2}$$

$$\langle \Delta I^2 \rangle = \langle \Delta\Phi^2 \rangle / L^2 = \frac{\hbar\omega}{2L}$$

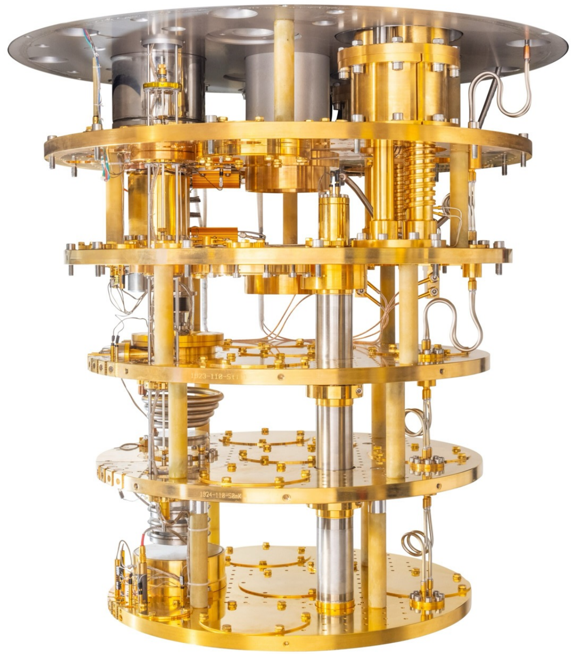
$L = 10\, nH$
$C = 100\, fF$
$\omega = 2\pi\times 5\ GHz$
$\sqrt{\langle \Delta I^2 \rangle} = 10\, nA$
$\sqrt{\langle \Delta Q^2 \rangle} \sim 2\, e$

Quantum LC Oscillator

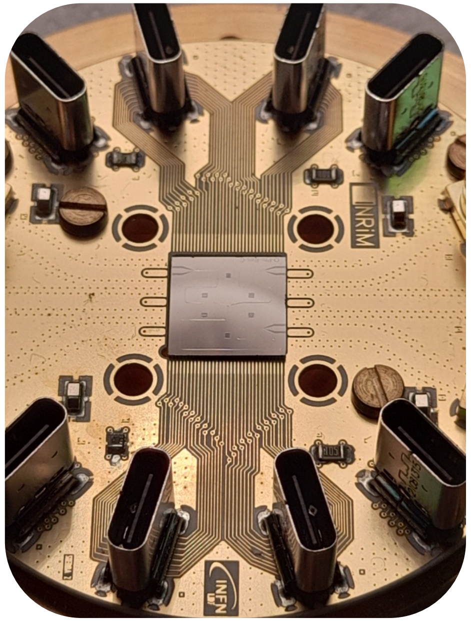
To obtain a Quantum LC Oscillator we need:

1. Negligible thermal fluctuations: $k_B T \ll \hbar\omega$
2. Negligible losses: $Q \gg 1$

Operate in a dilution refrigerator $T \ll 1K$

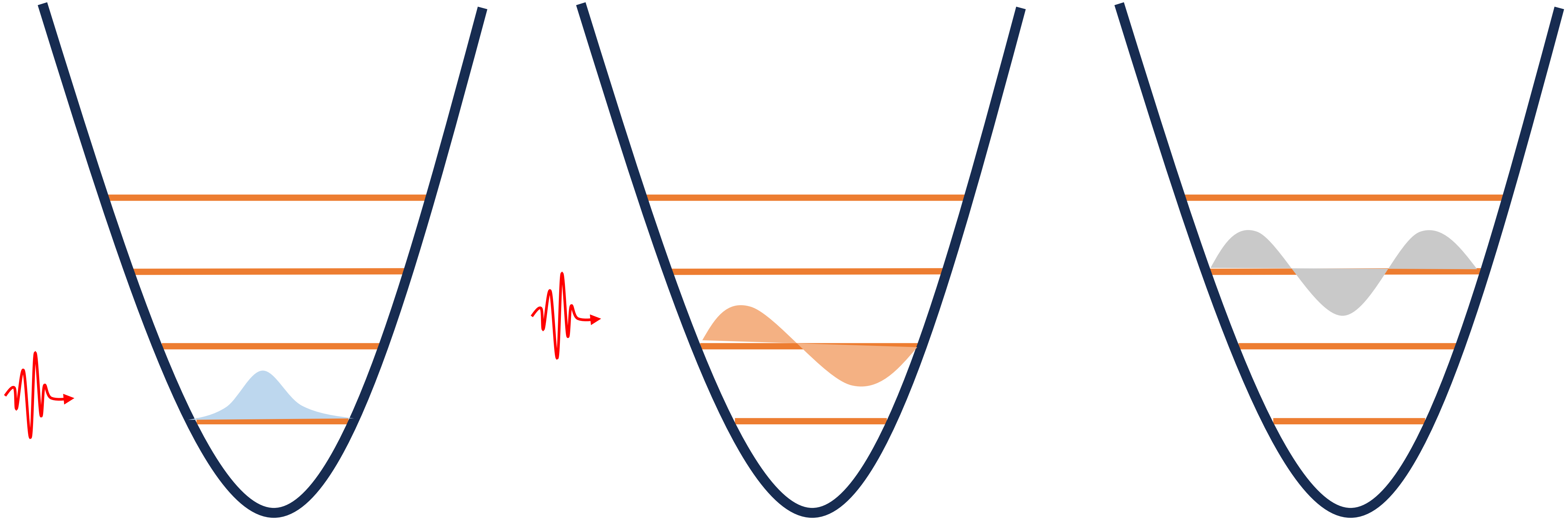


Use Superconducting Circuits R=0

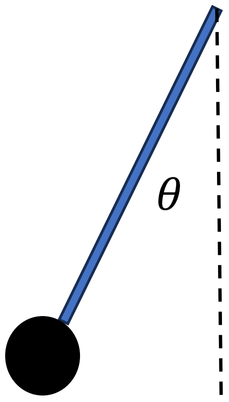


Quantum Fluctuations in Electrical Circuits M Devoret Les Houches 1995

A Quantum LC is not a Qubit



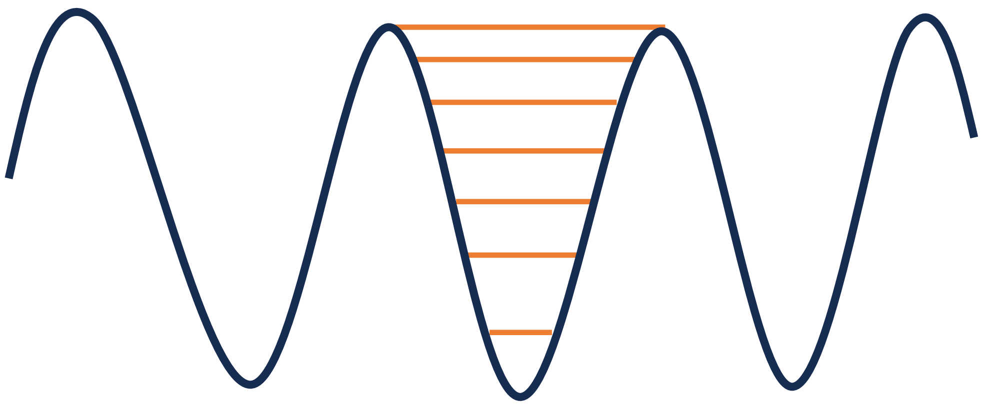
Anharmonic Oscillator



$$E = \frac{p^2}{2m} - mgl\, cos\theta$$

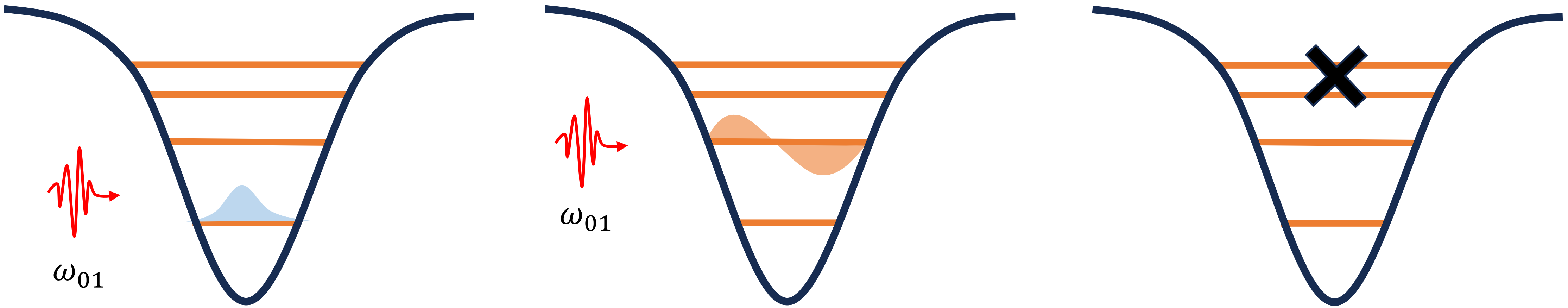
For small angles: $cos\theta \approx 1 - \frac{\theta^2}{2}$

$$E \approx \frac{p^2}{2m} + mgl\frac{\theta^2}{2}$$



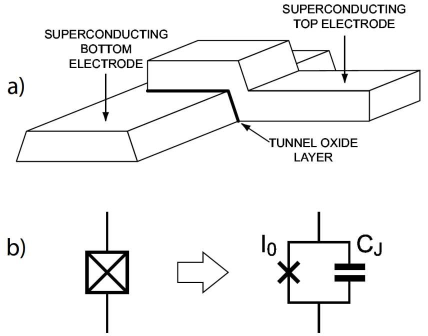
$$E_{n+1} - E_n < E_n - E_{n-1}$$

Anharmonic Oscillator

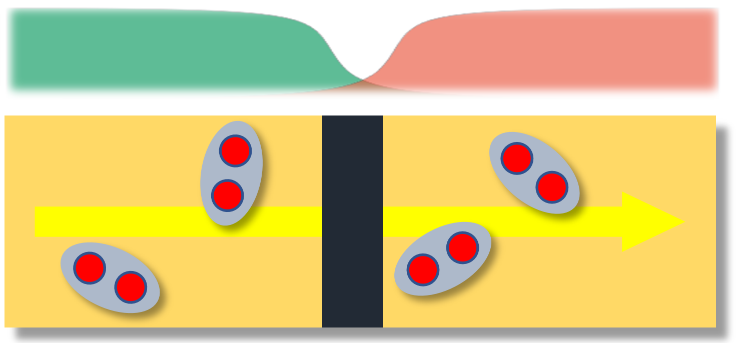


$$E_{n+1} - E_n < E_n - E_{n-1}$$

The Josephson Junction



$\psi_L = \sqrt{\rho_L}\, e^{i\varphi_L}$ $\psi_R = \sqrt{\rho_R}\, e^{i\varphi_R}$



Insulating barrier

In a SIS junction, Cooper pairs cross the insulating barrier by tunnel effect.

Tunneling current

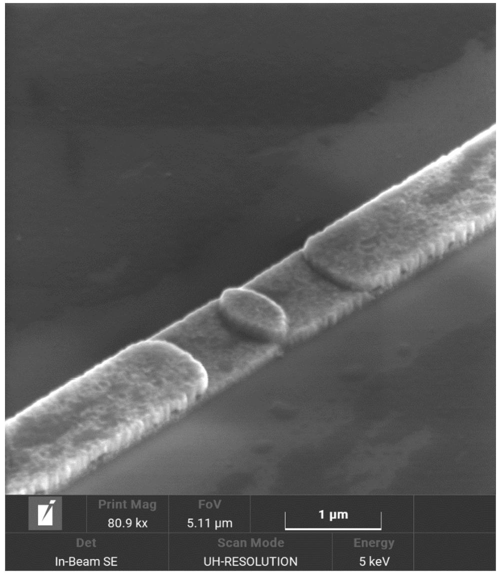
$$I = I_c \sin \varphi$$

Voltage across the junction

$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$

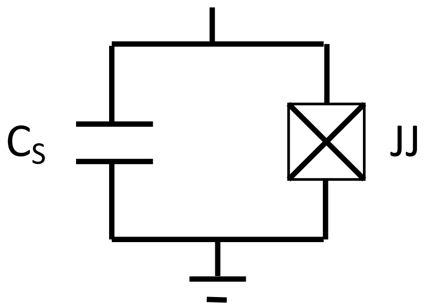
Phase difference

$$\varphi = \varphi_R - \varphi_L$$



FIB image of a JJ fabricated at FBK

The Superconducting Qubit



Charging energy

$$W_C = \frac{Q^2}{2C}$$

Inductive energy

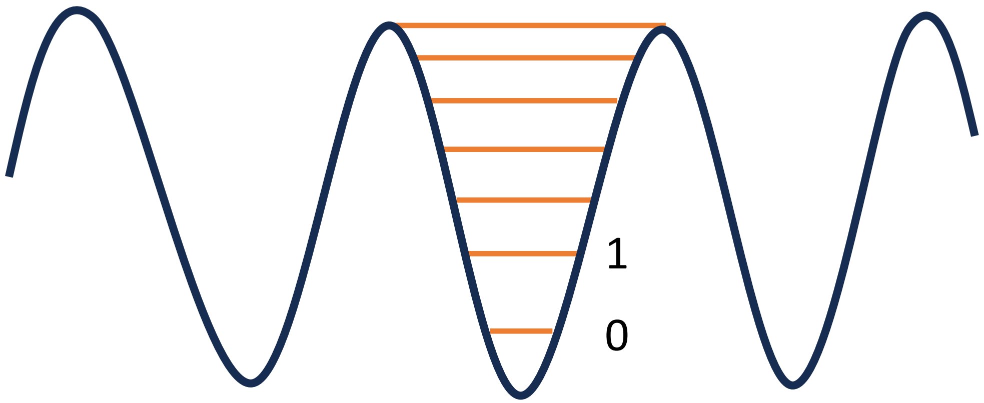
$$W_J = \int dt\ V\ I = -E_J cos2\pi\phi/\phi_0$$

Q	$\leftrightarrow$	p
$2\pi\phi/\phi_0$	$\leftrightarrow$	θ
C	$\leftrightarrow$	m
L	$\leftrightarrow$	l/mg

$$E = \frac{Q^2}{2C} - E_J cos2\pi\phi/\phi_0$$

$\phi_0 = 2.068\times10^{-15}Wb$

$$E_J = \frac{\phi_0 I_C}{2\pi} \qquad L_J = \frac{\phi_0}{2\pi I_C}$$



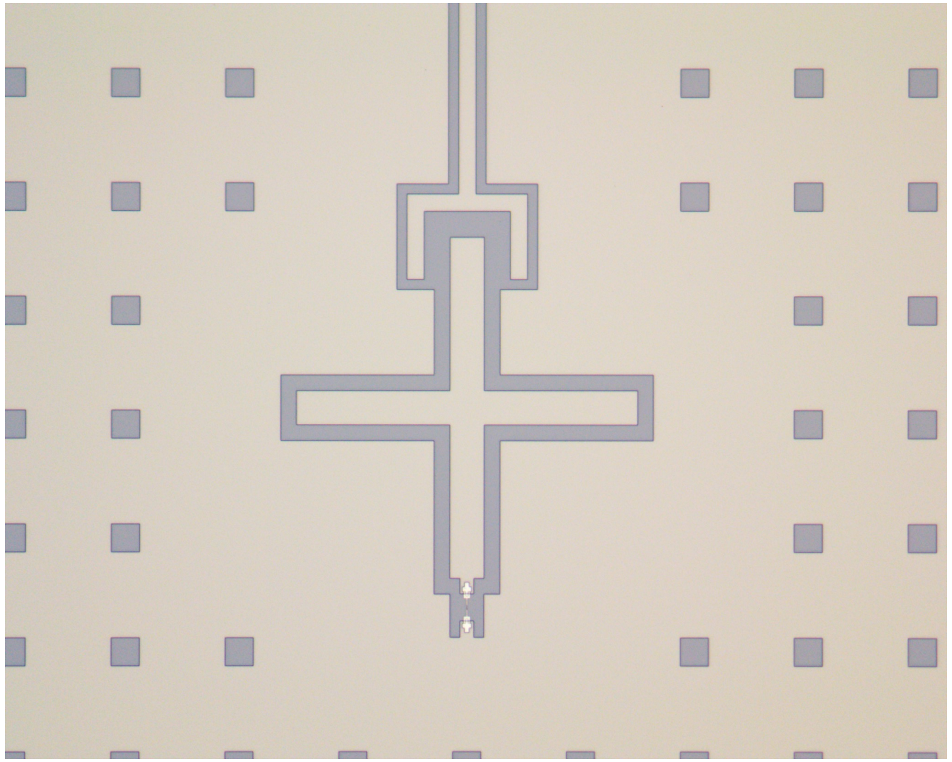
$$E_{n+1} - E_n = E_n - E_{n-1} - E_C$$

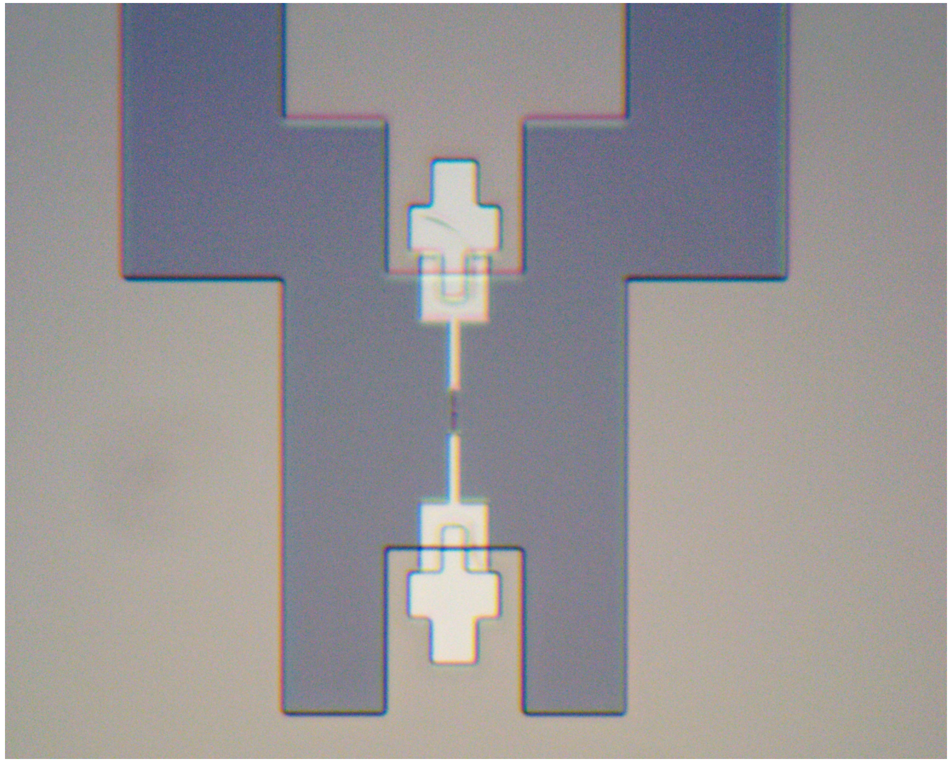
Anharmonicity

$$E_C = \frac{e^2}{2C}$$

J Koch Phys. Rev. A 76 042319 (2007)

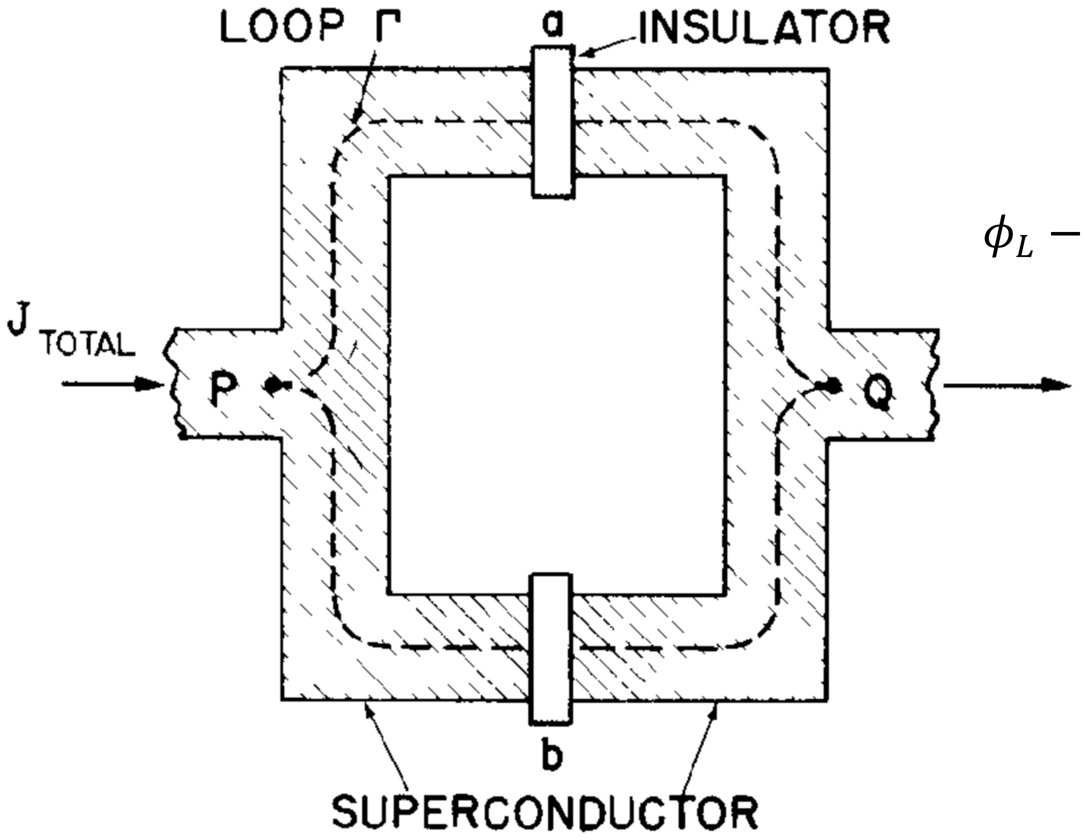
The Superconducting Qubit



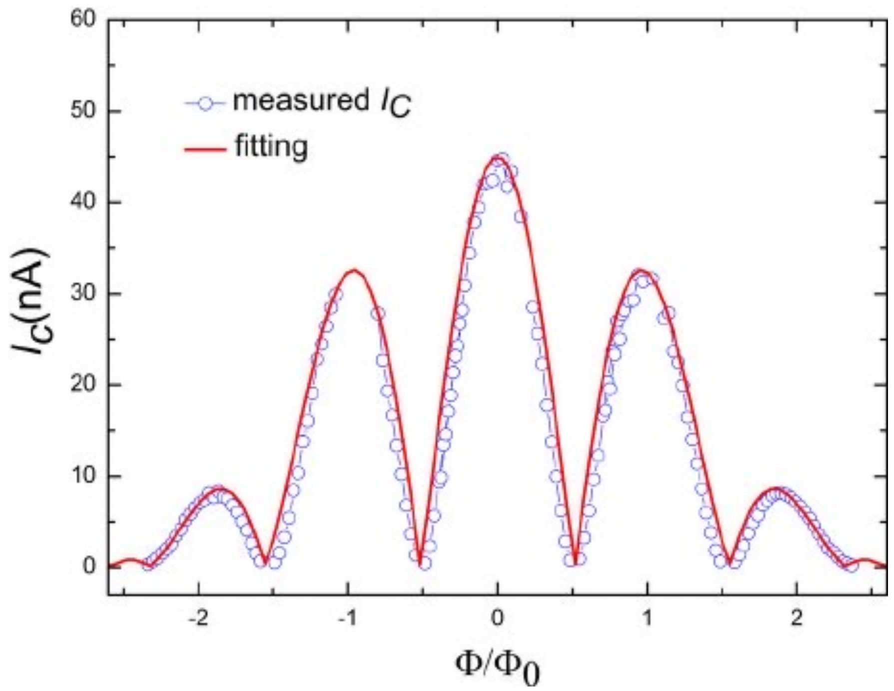


Qubit designed within the QubIT-INFN project and fabricated at NIST (thanks in particular to D. La Branca PhD Uni MiB and H. Corti PhD Uni Fi)

The Tunable Qubit

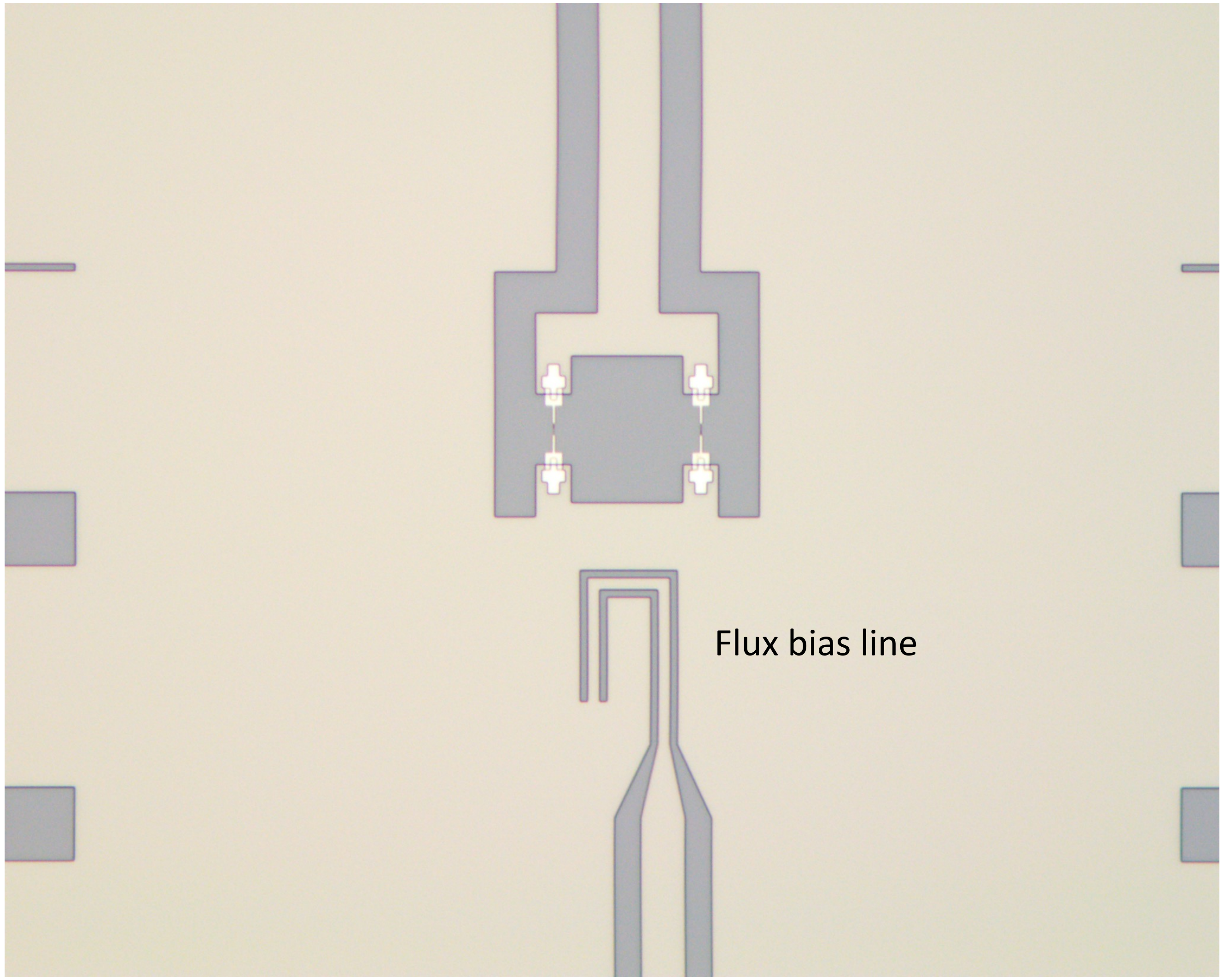


$$\phi_L - \phi_R = \Phi(B)$$



Feynman Lectures on Physics

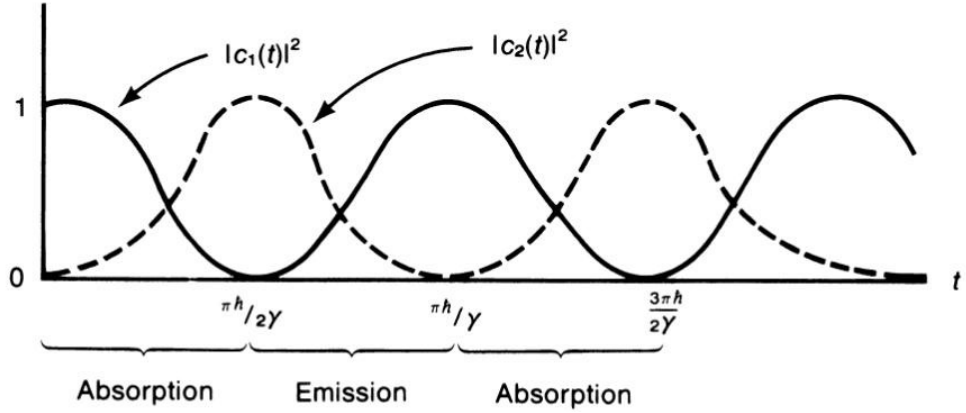
The Tunable Qubit

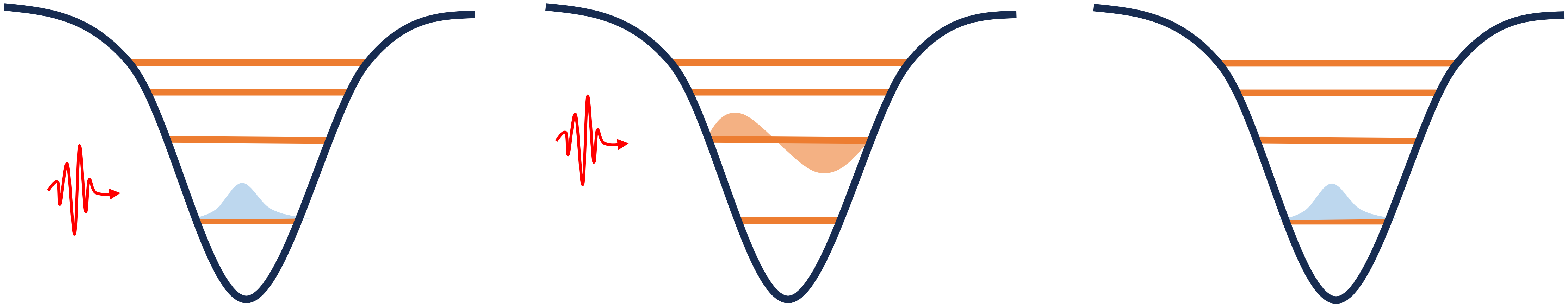


Rabi Oscillations

$$P(1) = \cos^2(\Omega_{Rabi} t/2)$$

$$\Omega_{Rabi} = 2g_{01}\sqrt{n_{photons} + 1}$$

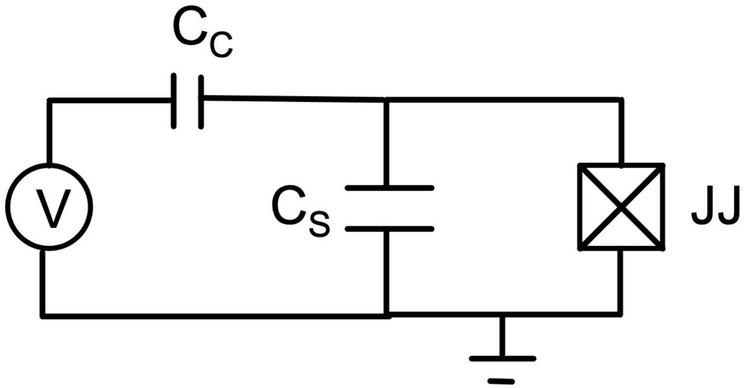


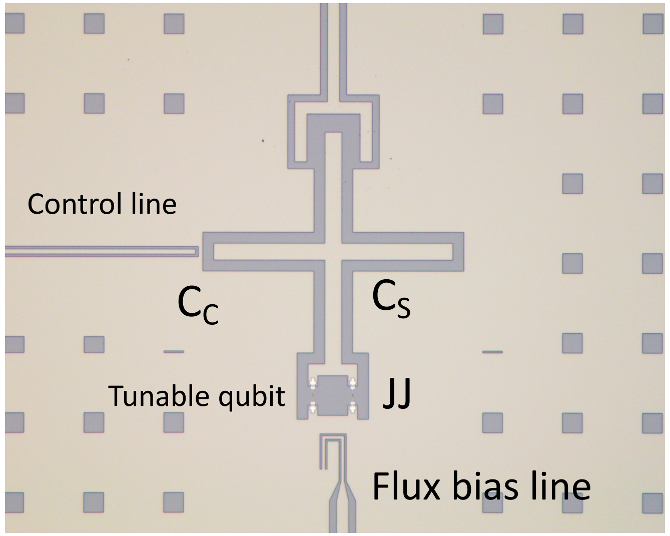


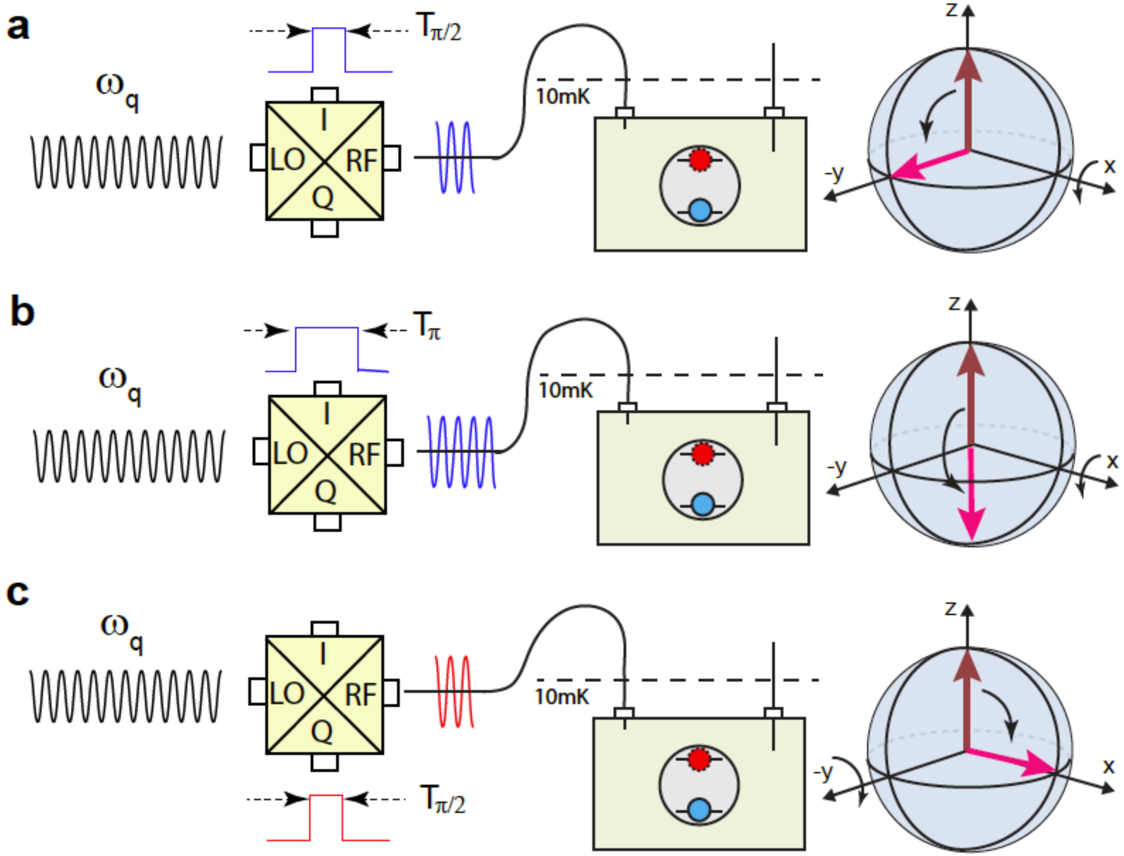
J J Sakurai “Modern Quantum Mechanics”

Qubit Control

$$\Omega_{Rabi} = 2g_{01}\sqrt{n_{photons} + 1} \qquad g_{01} \propto \frac{C_C}{C_S + C_C}$$

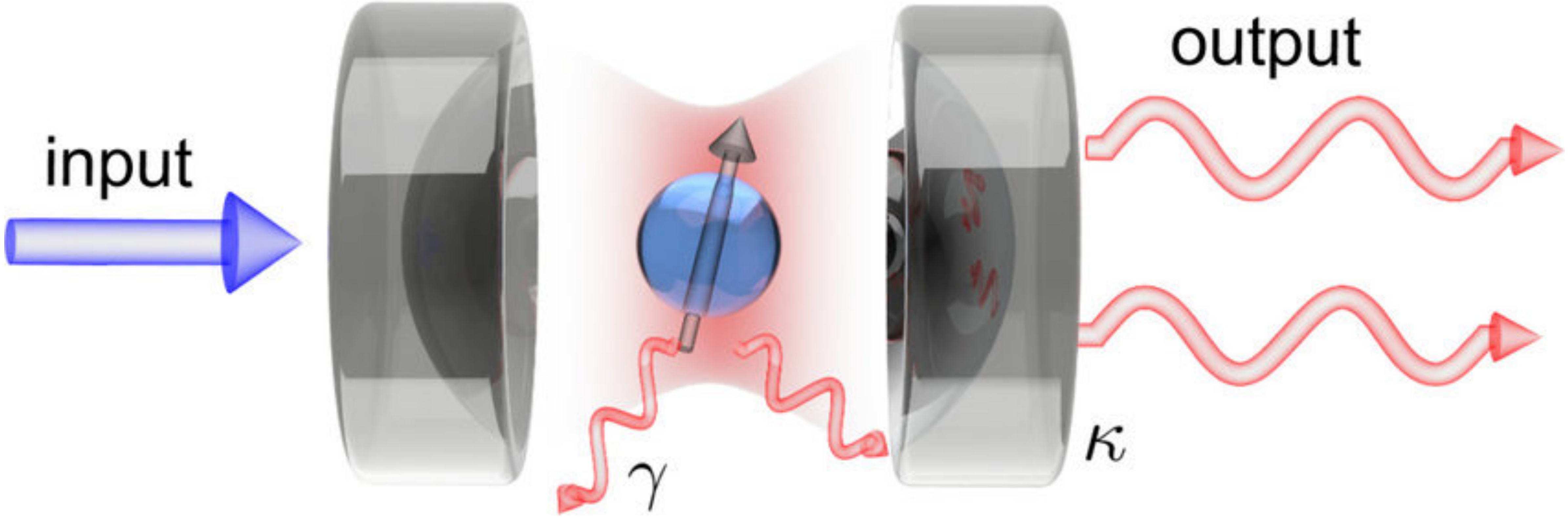




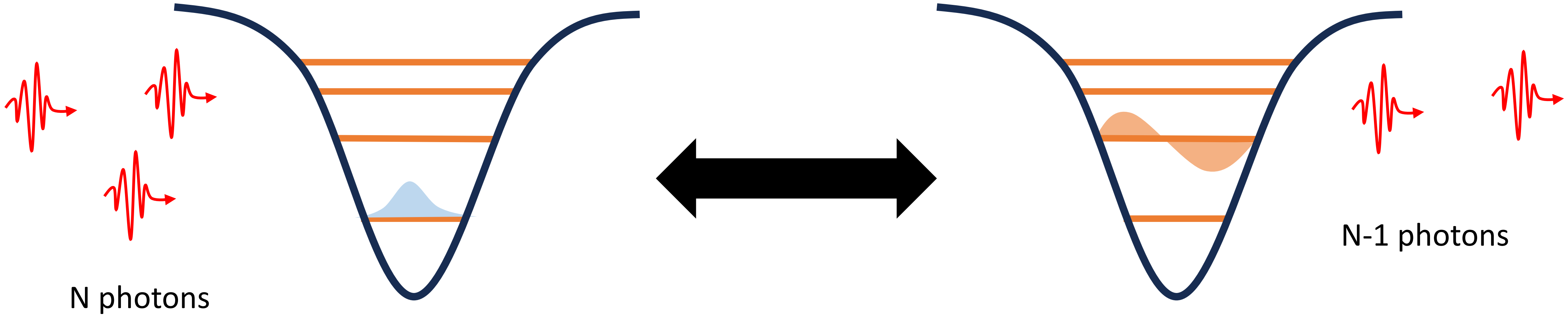


Naghiloo arxiv:1904.09291

Qubit Coupled to a Resonator



Qubit Coupled to a Resonator



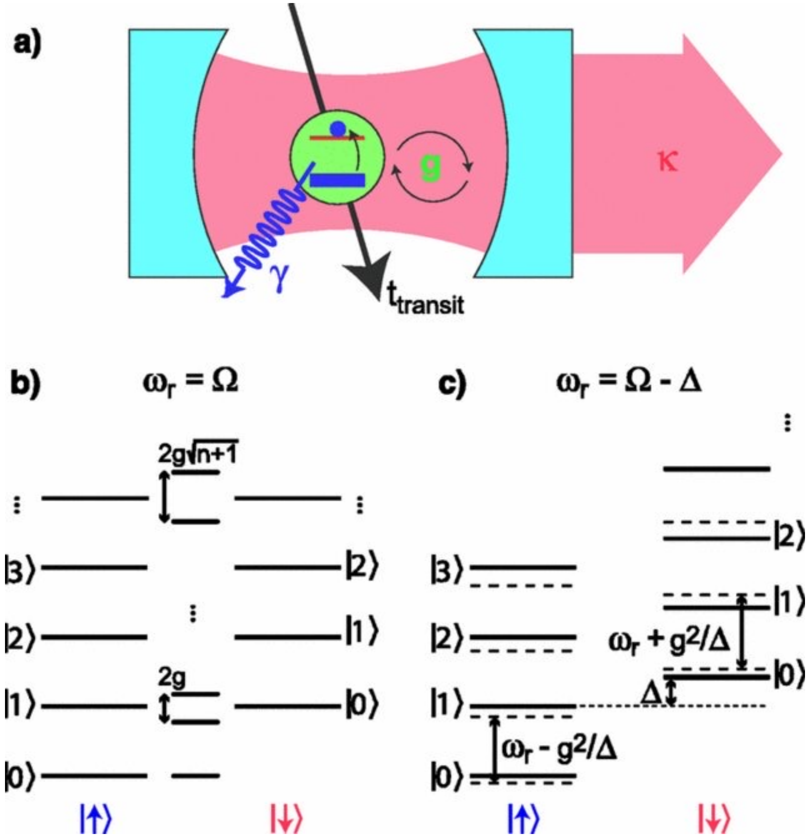
$|N, \uparrow\rangle$ $\quad$ $|N-1, \downarrow\rangle$

The number of excitations “n” is conserved

Qubit Coupled to a Resonator

The physical states are superpositions of states with equal number of excitations "n":

$$|+,n\rangle = \cos\theta_n|n,\downarrow\rangle + \sin\theta_n|n+1,\uparrow\rangle$$
$$|-,n\rangle = -\sin\theta_n|n,\downarrow\rangle + \cos\theta_n|n+1,\uparrow\rangle$$

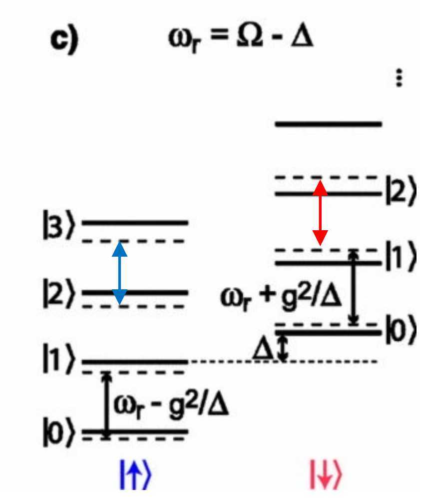


Phys. Rev. A 69, 062320

Qubit Coupled to a Resonator - Dispersive Limit

The emission spectrum of the spin-resonator system is modified by the interaction.

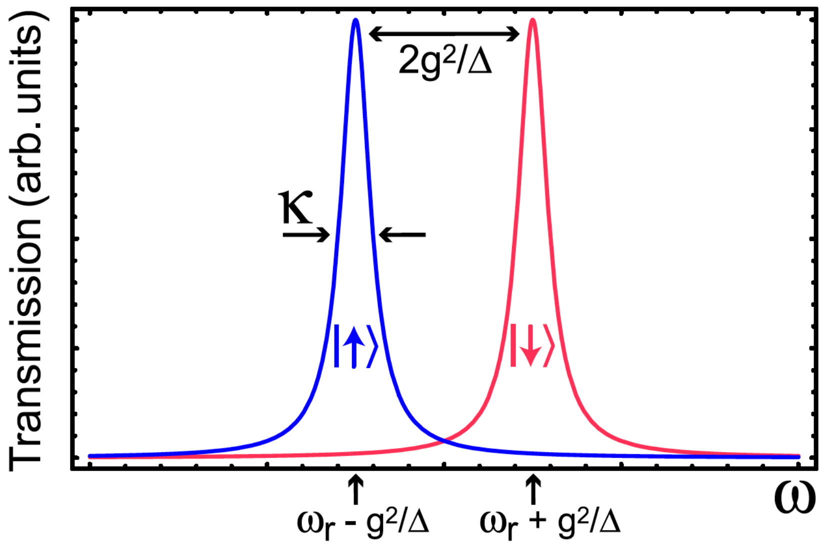
In particular, in the dispersive limit: $\left|\frac{g_{01}}{\omega_q - \omega_r}\right| \ll 1$



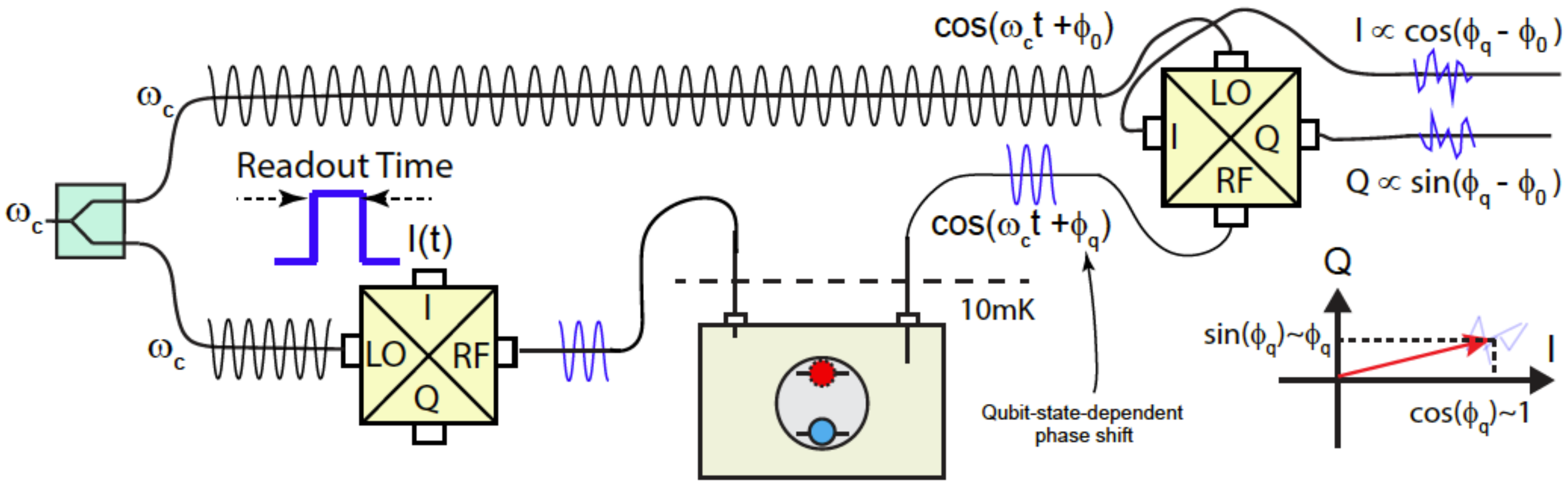
$$\hbar\omega_{r,0} = \hbar\omega_r - \frac{\hbar g_{01}^2}{\omega_q - \omega_r}$$

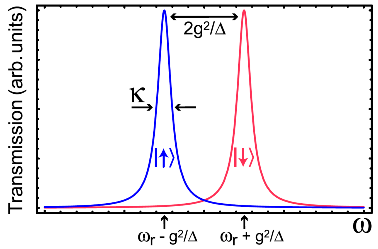
$$\hbar\omega_{r,1} = \hbar\omega_r + \frac{\hbar g_{01}^2}{\omega_q - \omega_r}$$

A. Blais et al., Phys. Rev. A 69, 062320 (2004)



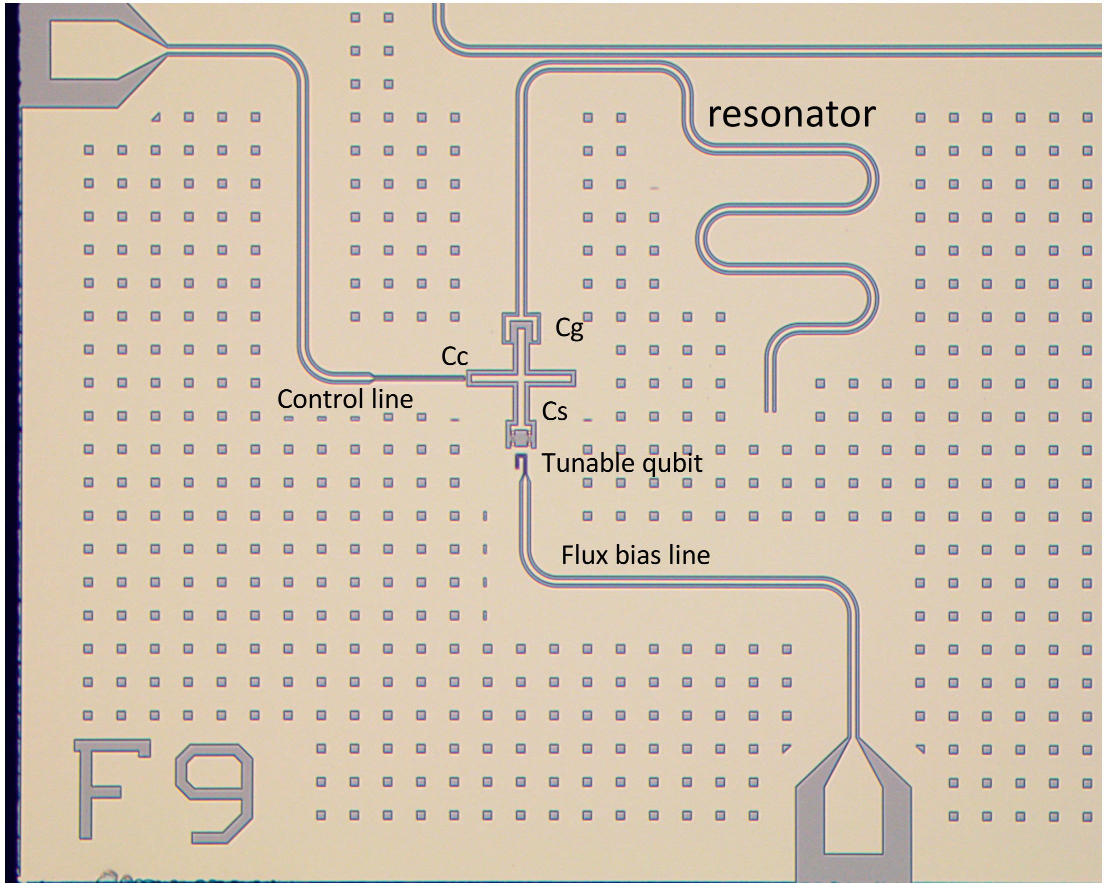
Qubit Readout





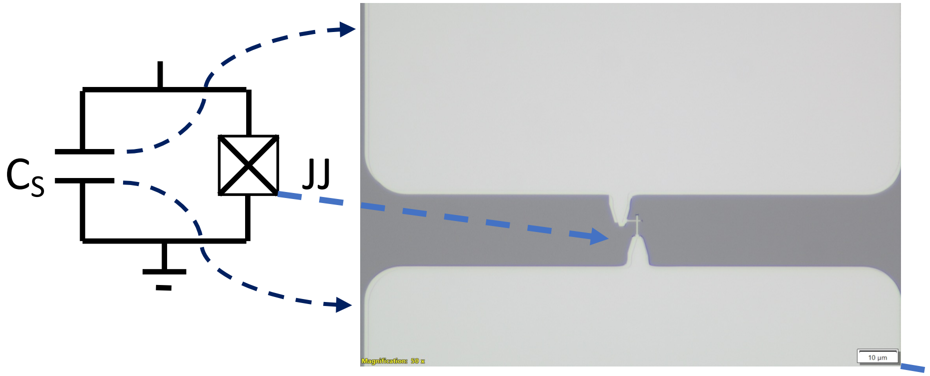
Naghiloo arxiv:1904.09291

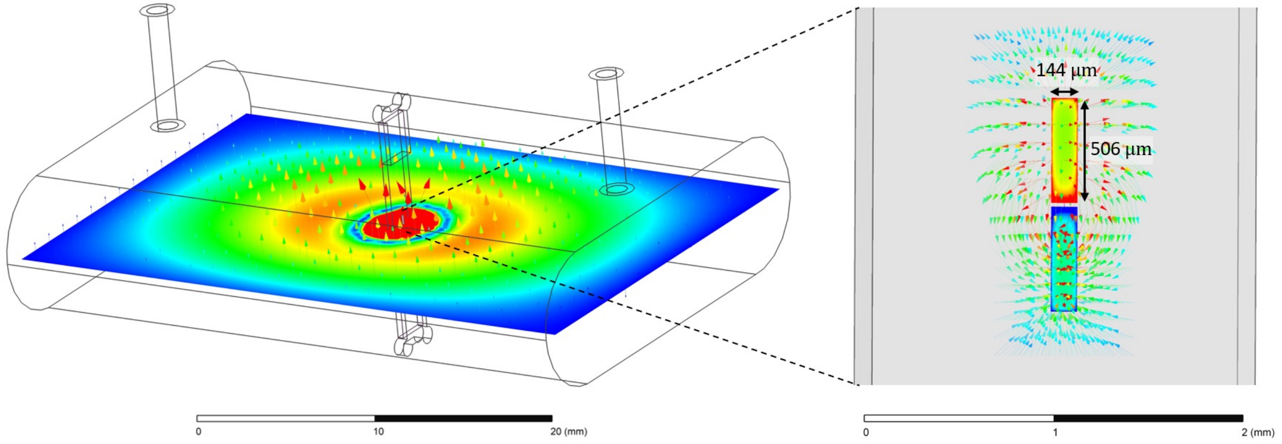
Qubit Coupled to a Resonator

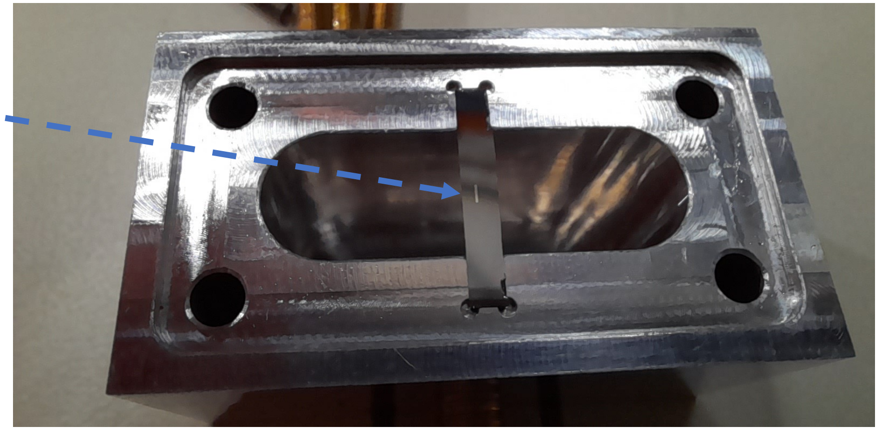


NIST

Qubit in a 3D Resonator

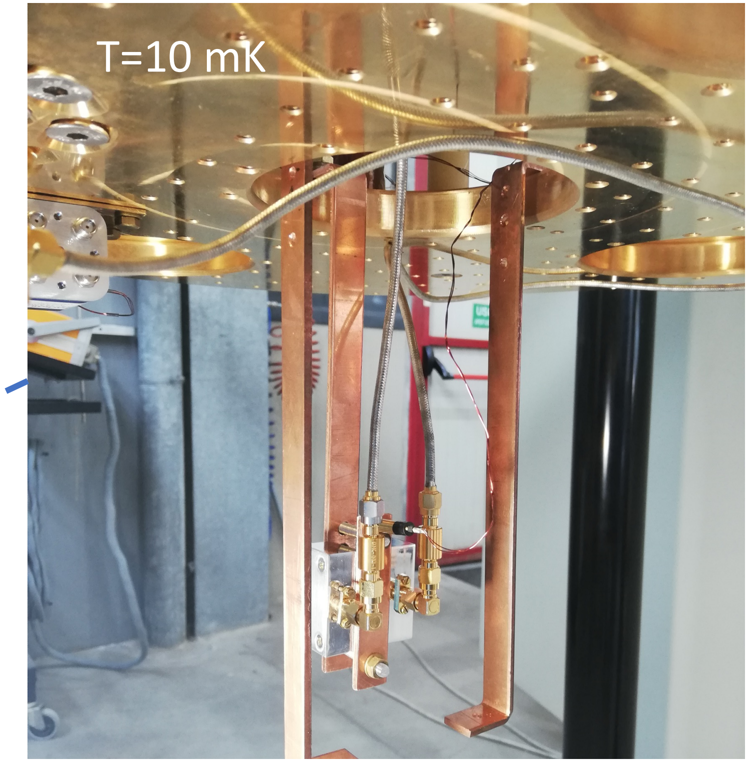




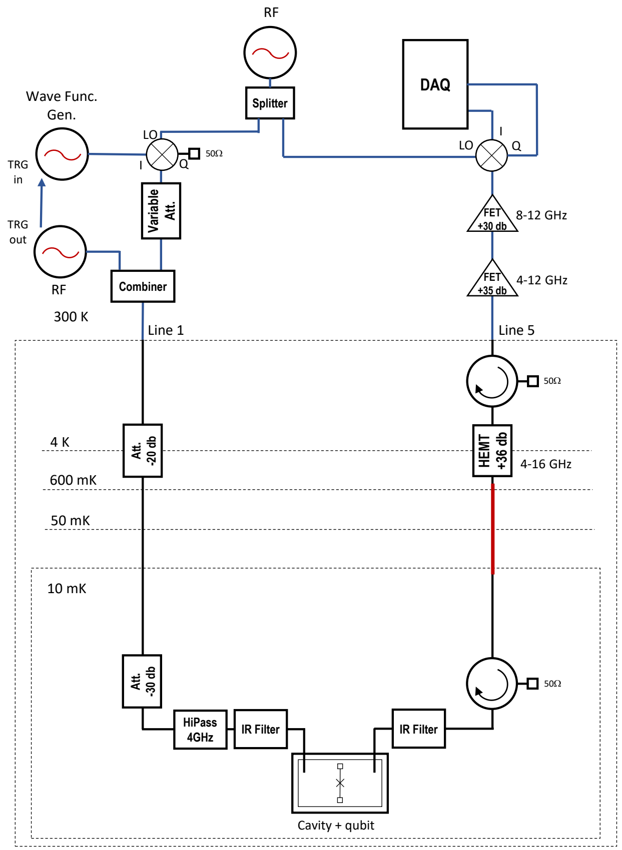


TII Technology Innovation Institute

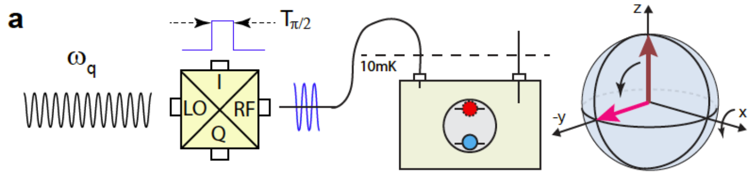
Qubit in a 3D Resonator



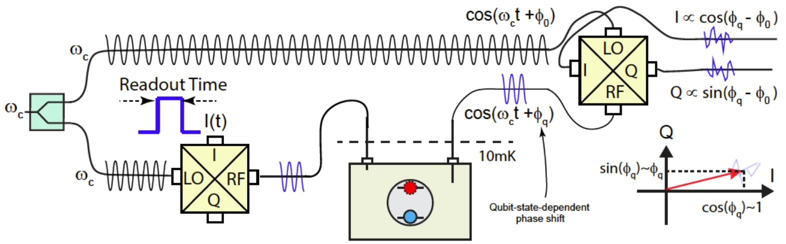
Experimental Setup

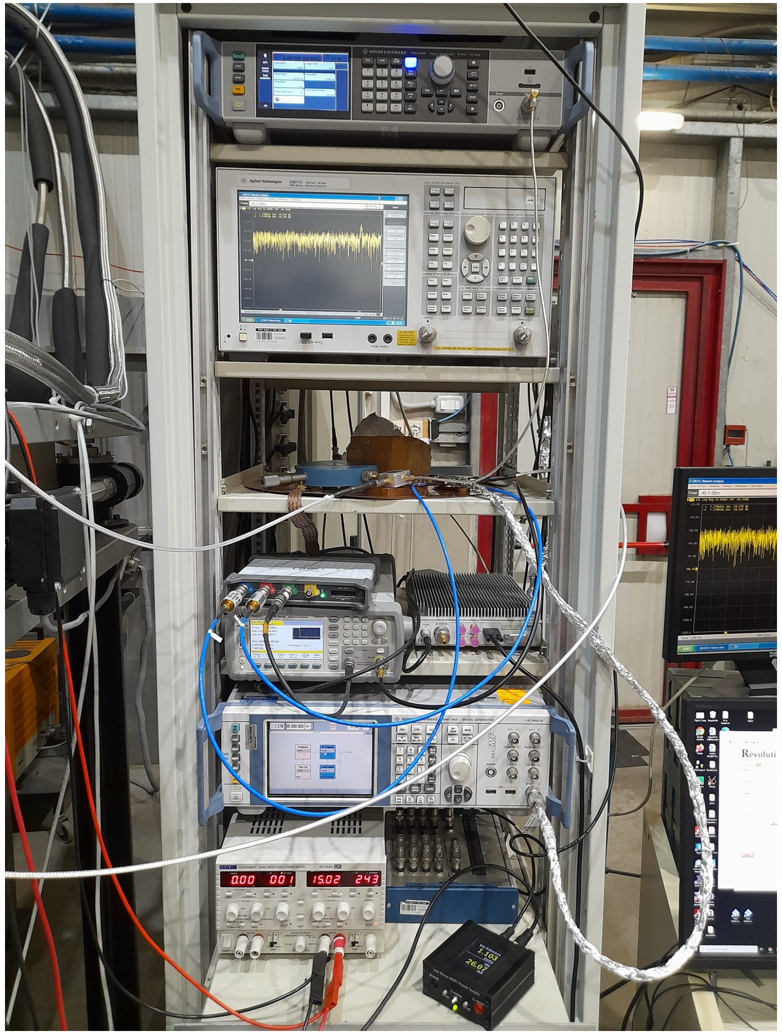


Qubit Control

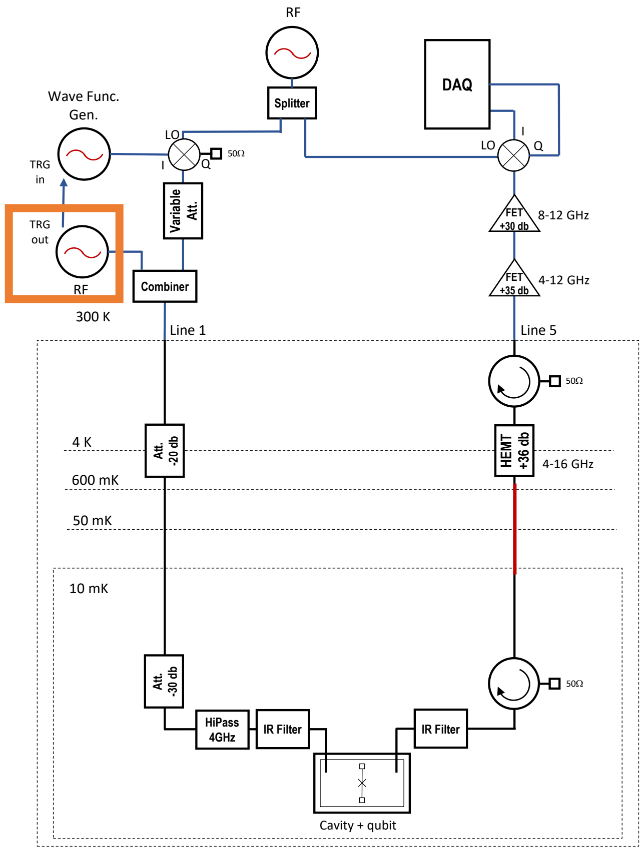


Qubit Readout

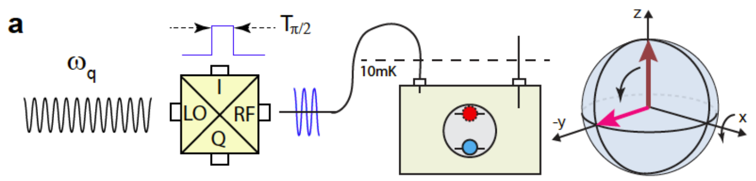




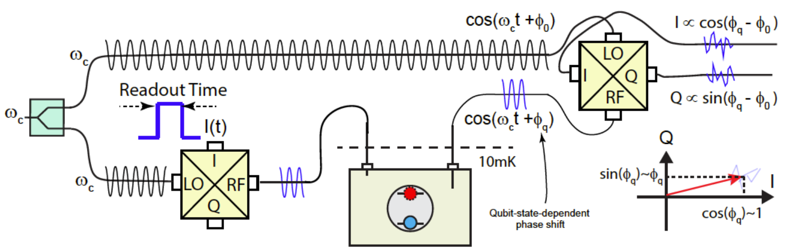
Experimental Setup

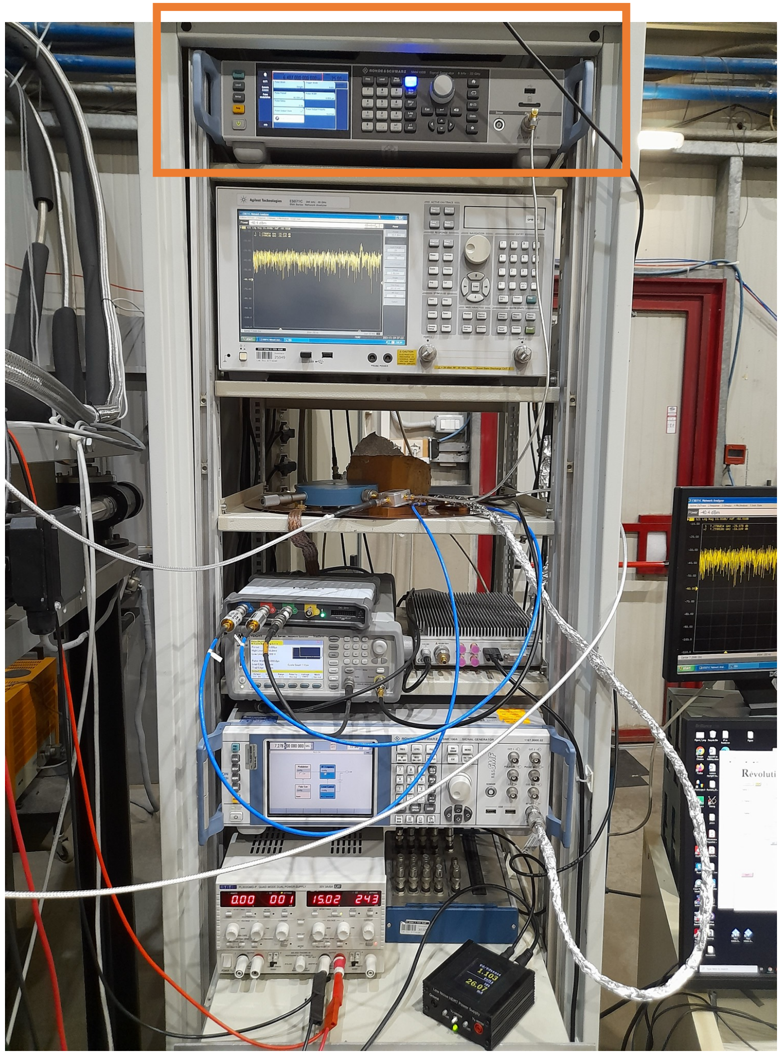


Qubit Control

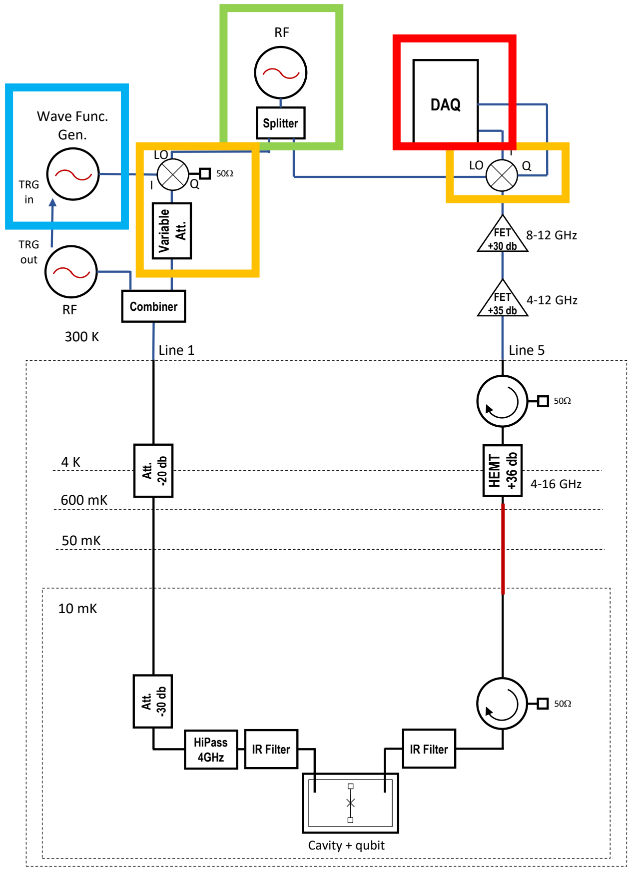


Qubit Readout

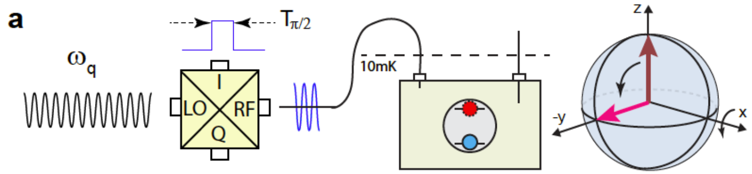




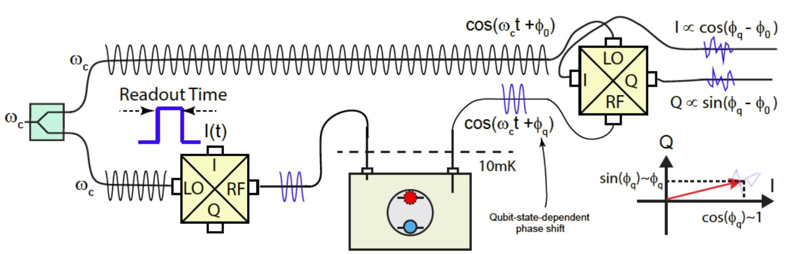
Experimental Setup

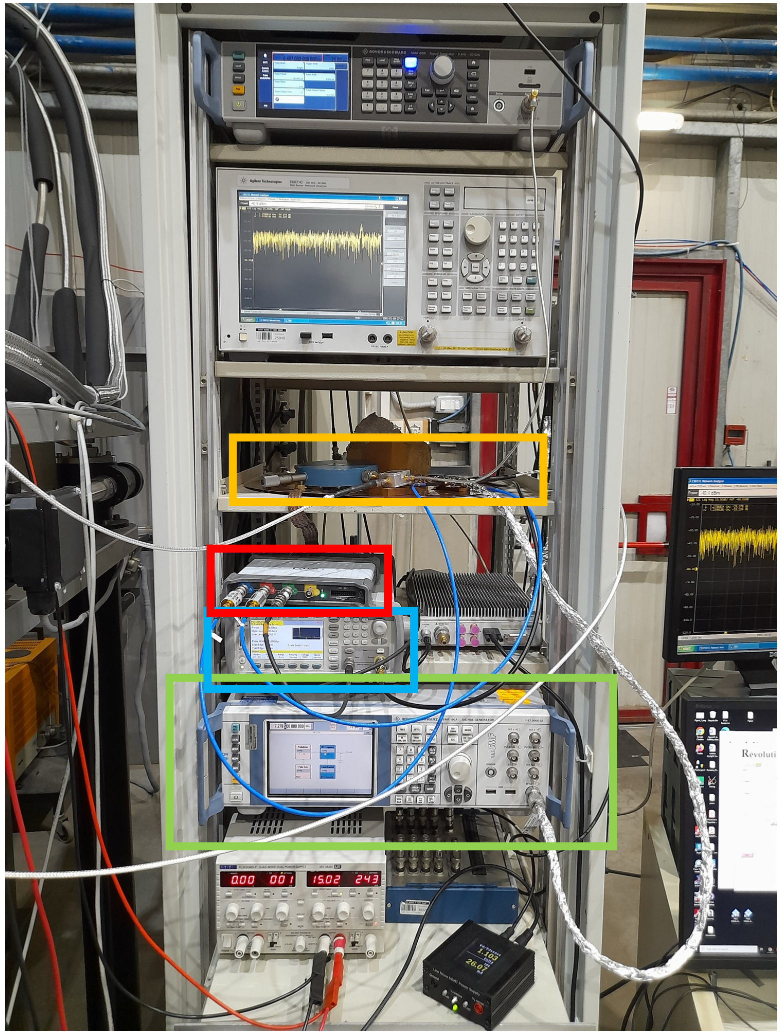


Qubit Control



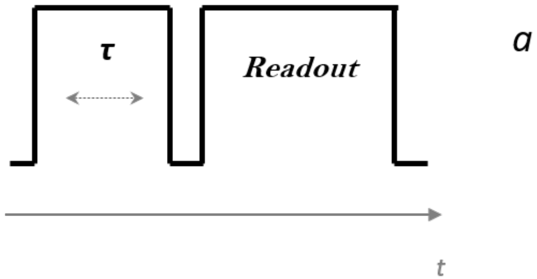
Qubit Readout

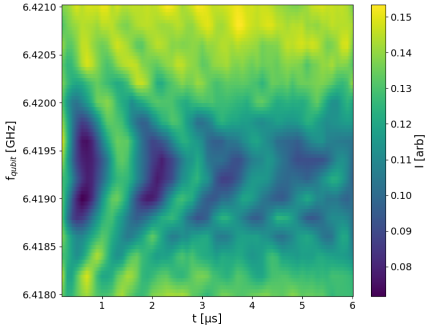


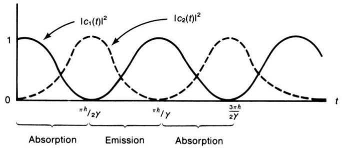


Qubit Characterization

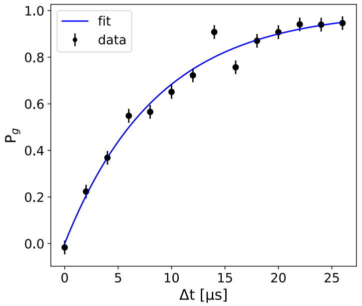
Rabi oscillations



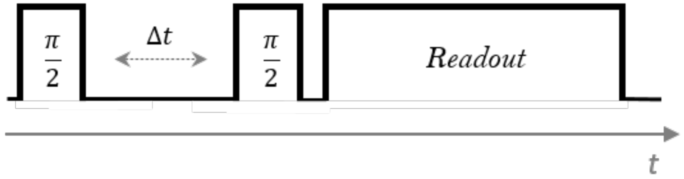


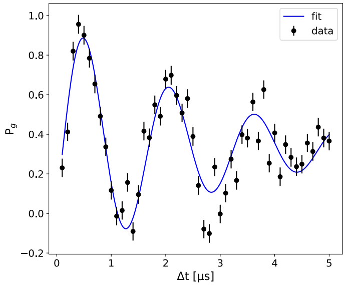


Qubit lifetime measurement

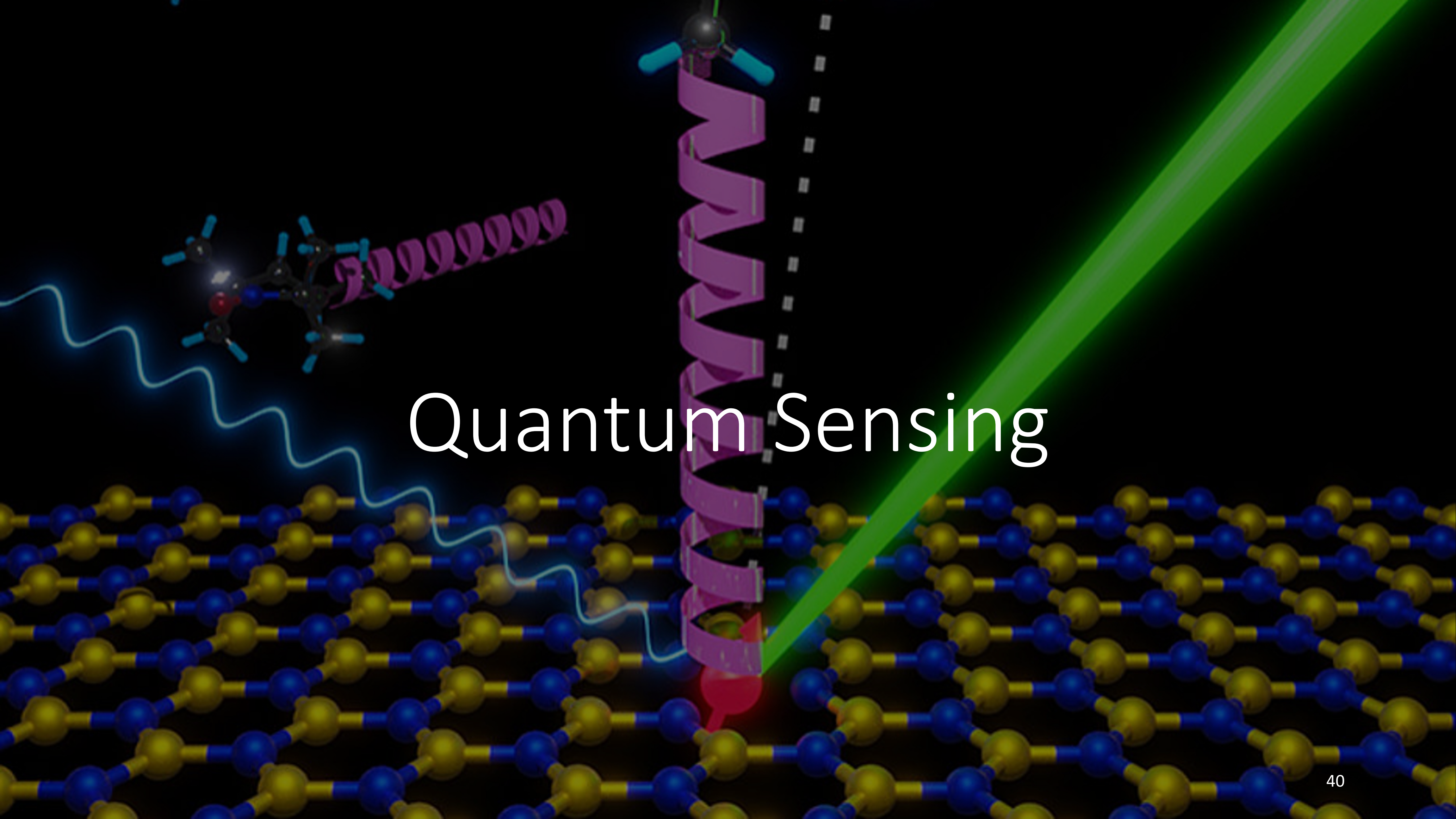


Ramsey Spectroscopy
and
T2 measurement



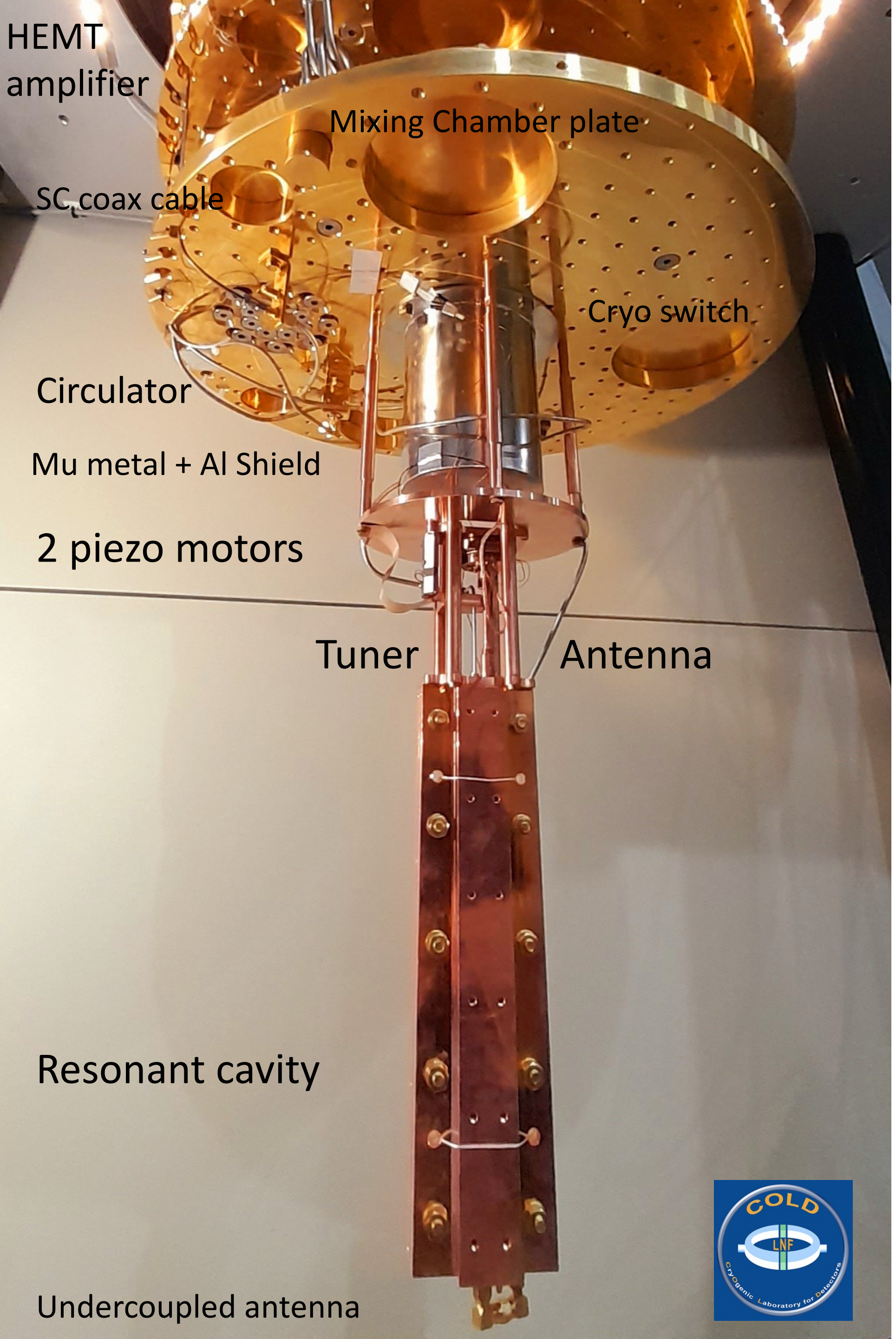


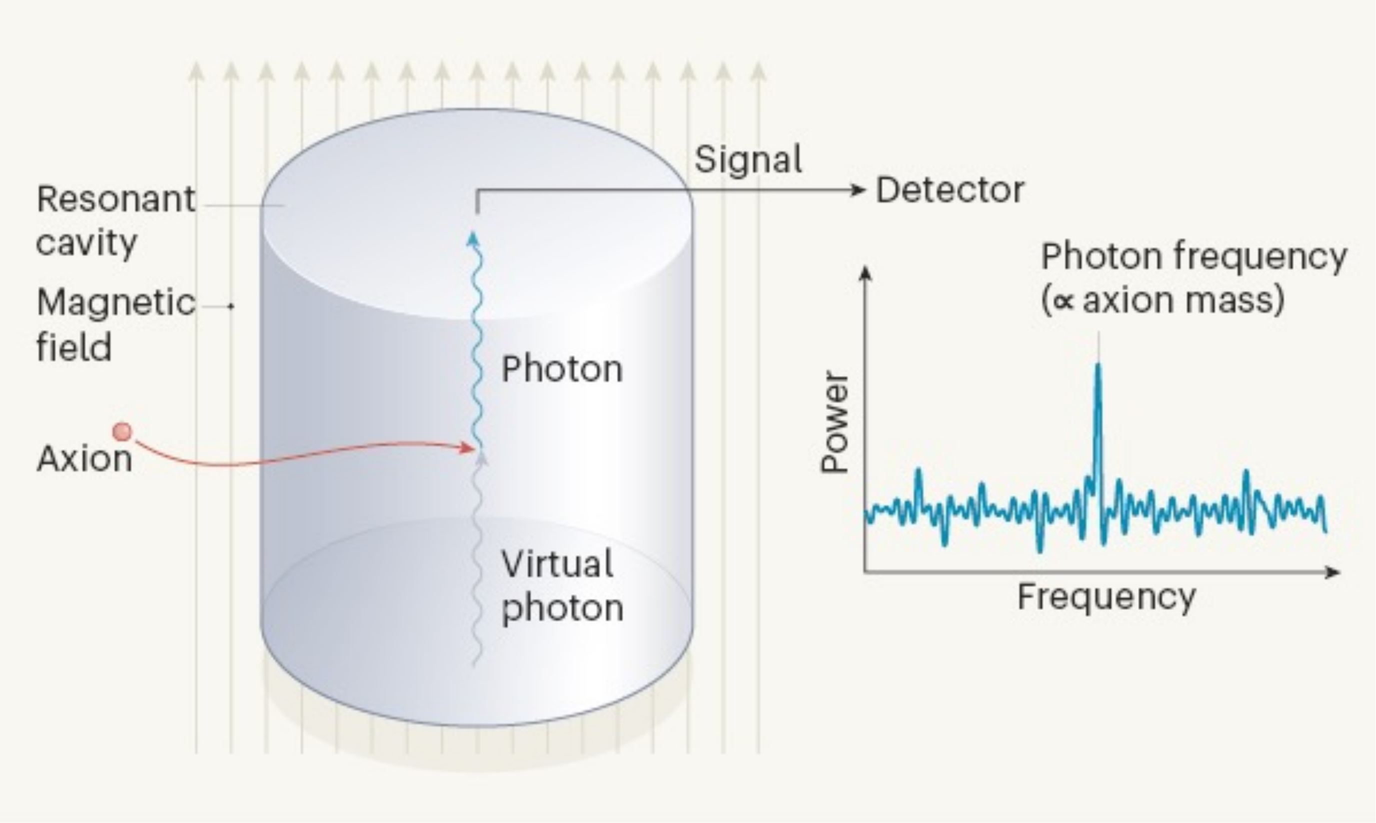
Appl. Sci. 2024, 14(4), 1478



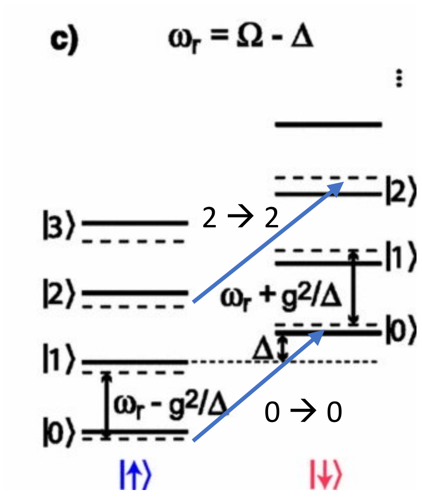
Quantum Sensing

Axion Dark Matter



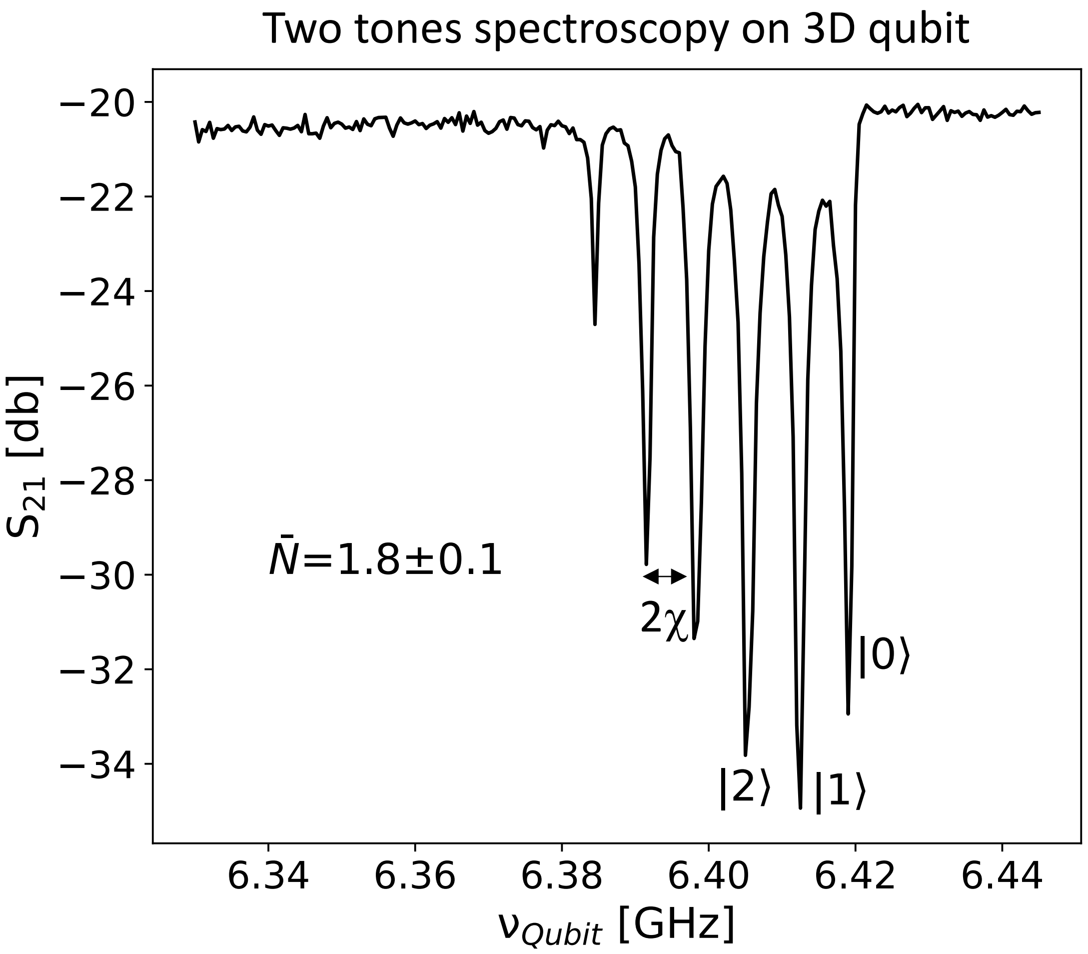


Quantum Sensing with SC Qubits



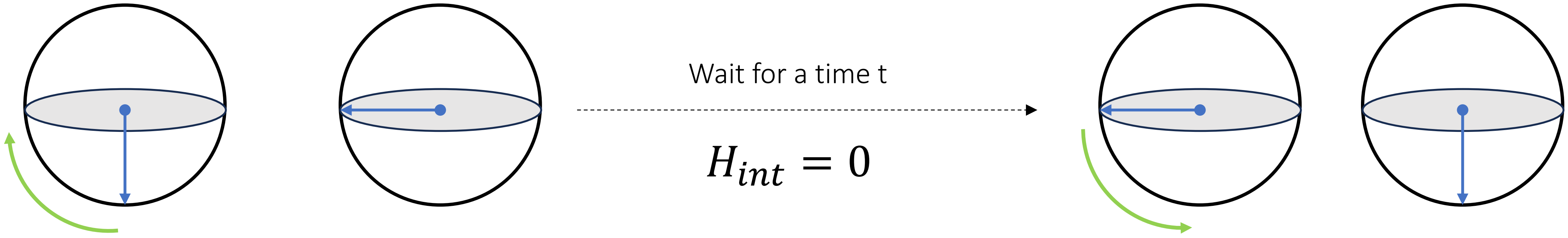
$$\omega'_q = \omega_q + 2n_\gamma\chi$$

Photon number in resonator



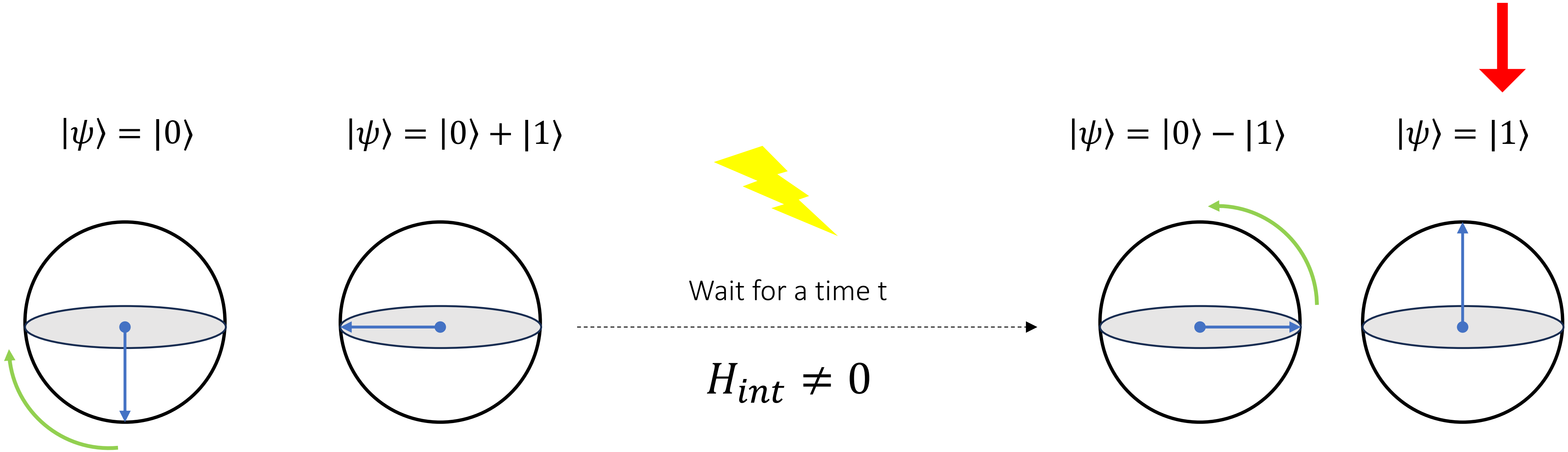
Quantum Sensing with SC Qubits

$|\psi\rangle = |0\rangle$ $|\psi\rangle = |0\rangle + |1\rangle$ $|\psi\rangle = |0\rangle + |1\rangle$ $|\psi\rangle = |0\rangle$



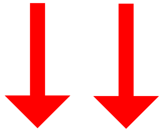
$$i\hbar \frac{\partial \psi}{\partial t} = H_{int} \psi$$

Quantum Sensing with SC Qubits

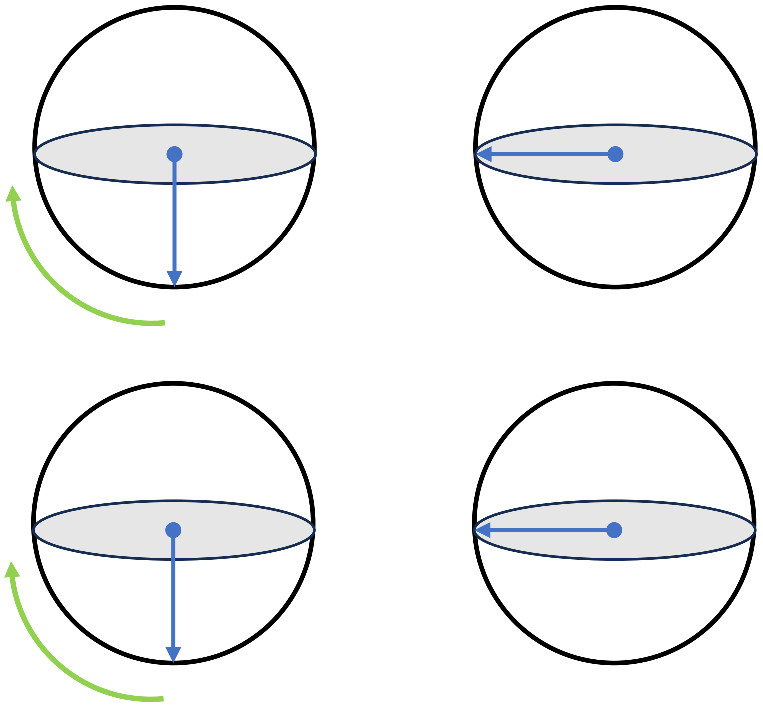


$$i\hbar \frac{\partial \psi}{\partial t} = H_{int}\psi$$

Quantum Sensing with Error Correction



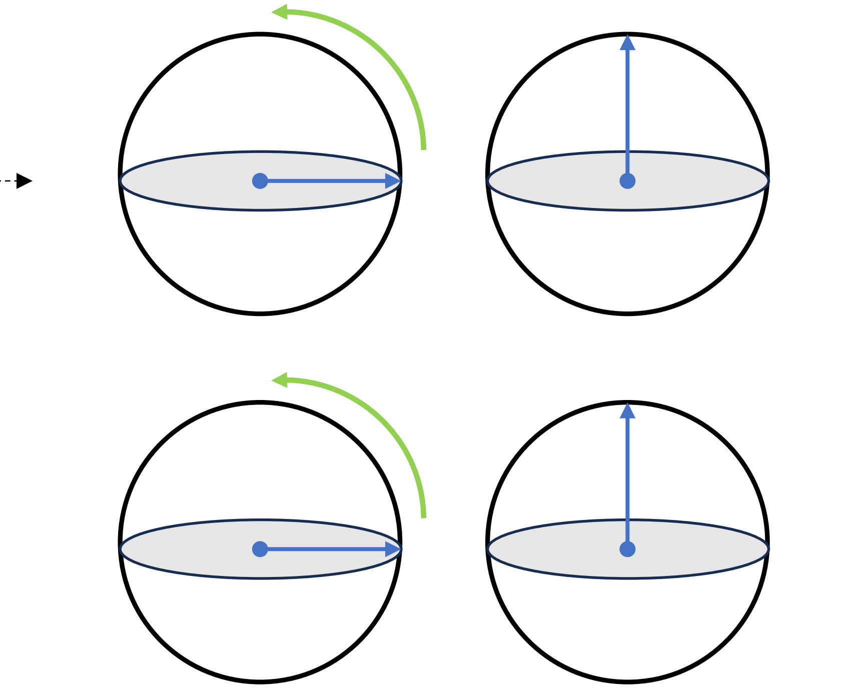
$|\psi\rangle = |0\rangle \times |0\rangle$ $\quad$ $|\psi\rangle = (|0\rangle + |1\rangle) \times (|0\rangle + |1\rangle)$ $\quad$ $|\psi\rangle = (|0\rangle - |1\rangle) \times (|0\rangle - |1\rangle)$ $\quad$ $|\psi\rangle = |1\rangle \times |1\rangle$



Wait for a time t

$$H_{int} \neq 0$$

$$i\hbar \frac{\partial \psi}{\partial t} = H_{int}\psi$$



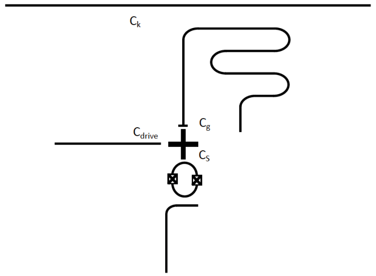
QubIT INFN CSNV Project

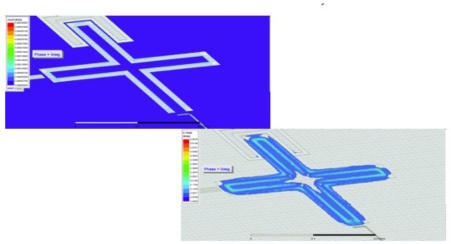
Superconducting qubits and JPA amplifiers for quantum sensing and computing

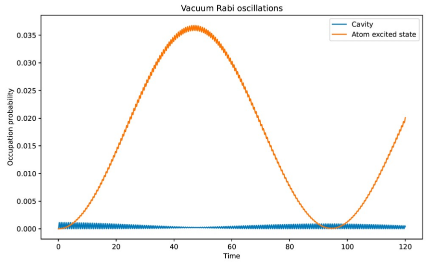
Design of 2D and 3D Superconducting Qubits

2D Qubits

$$\mathcal{L} = \frac{\dot{\vec{\Phi}} \cdot C \dot{\vec{\Phi}}}{2} - \frac{\vec{\Phi} \cdot L^{-1} \vec{\Phi}}{2} + E_j \cos\left(\frac{2\pi}{\Phi_0}\phi\right)$$







Circuit Modeling → Circuit Design → Electromagnetic Simulations → Quantum Simulation

1) Connect physical elements (C, L, Ic, Z0) to quantum-circuit properties (lifetime, frequency, couplings)

2) Design of circuit with first estimate of circuit element values

3) Layout realization, E.M. simulation and design optimization.

4) Evolution of quantum Hamiltonian based on circuit parameters

3D Qubits

From HFSS simulation

$$g_{01} = \frac{2e \cdot d_{eff}}{\hbar} E_0 \frac{1}{\sqrt{2}} \left(\frac{Ej}{8Ec}\right)^{1/4}$$

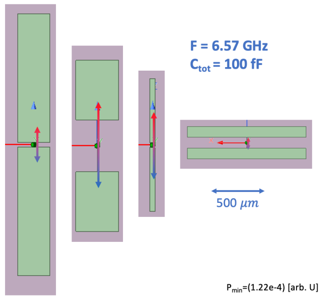
$$\hbar\, w_q = \sqrt{8\, Ec\, Ej} - Ec$$

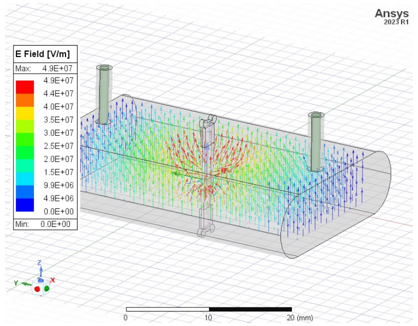
$$Ec = \frac{e^2}{2C}$$

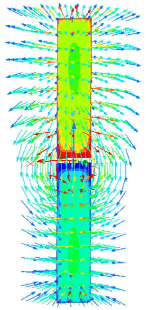
$$E_0 = \sqrt{\frac{\hbar w_r}{2\varepsilon_0 V}}$$

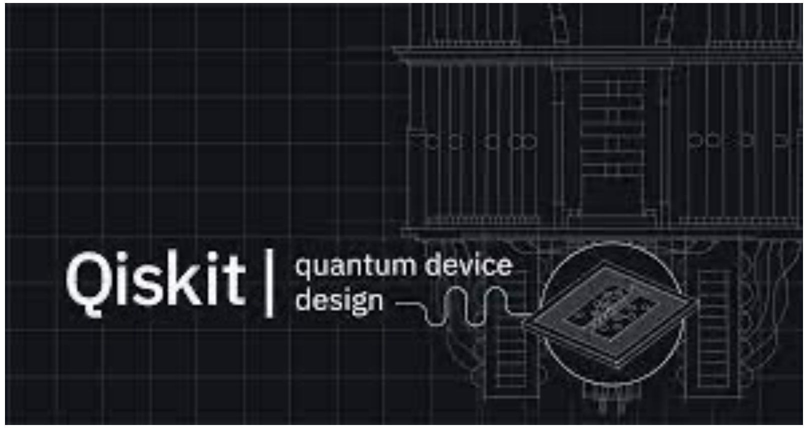
$$V = \frac{\int \varepsilon_r(\vec{r}) \| \vec{E}(\vec{r}) \|^2 d\vec{r}}{\max(\| \vec{E}(\vec{r}) \|^2)} \approx \frac{1}{4} Vcavity$$

For TE_{110} mode

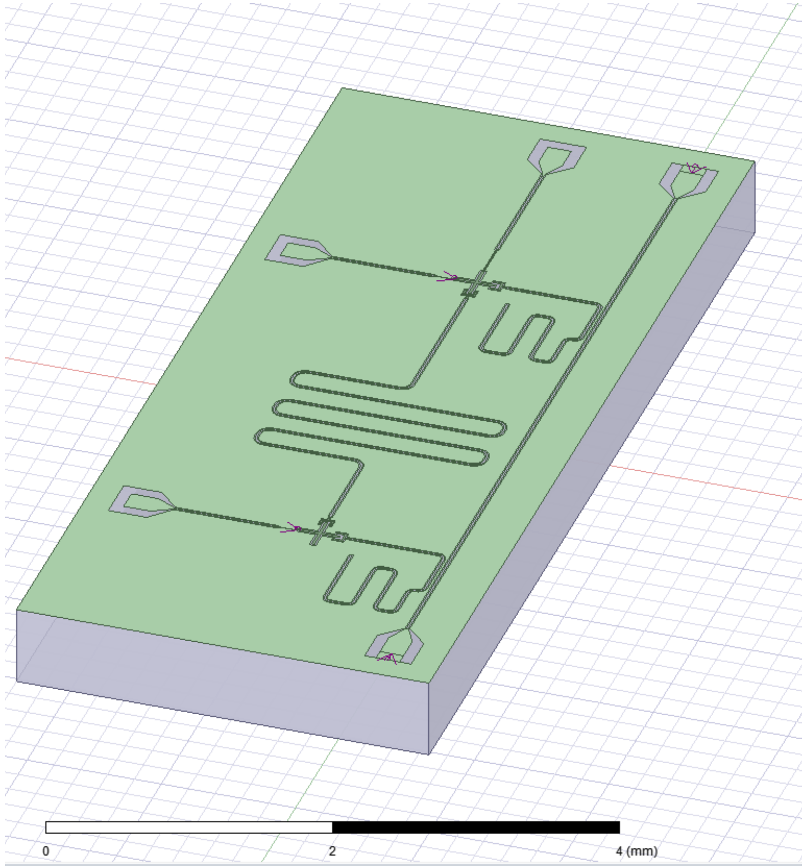


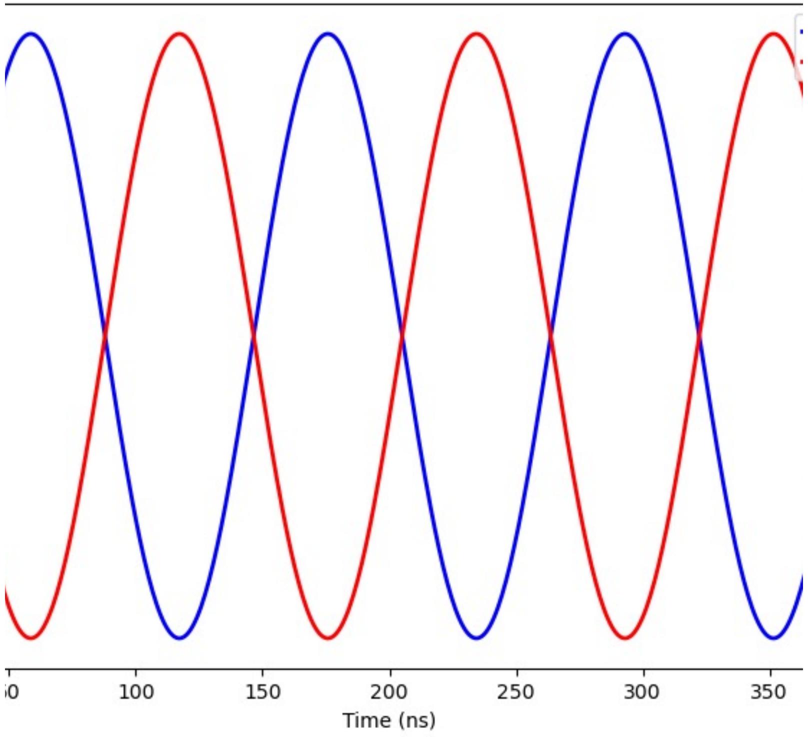


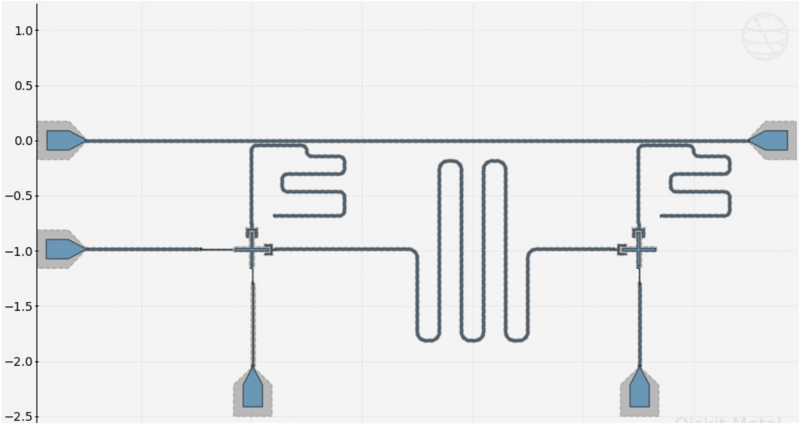




Ansys / HFSS



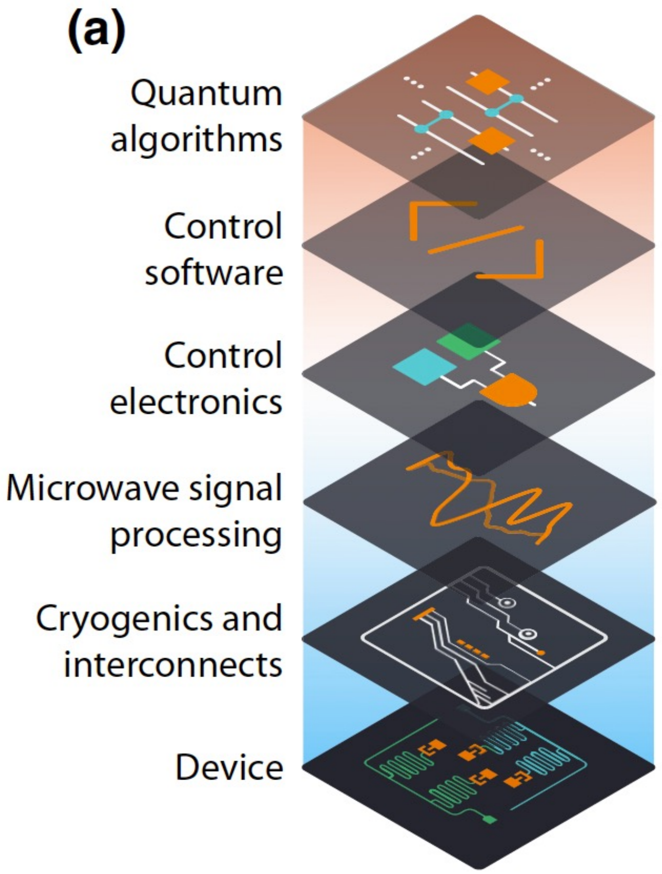


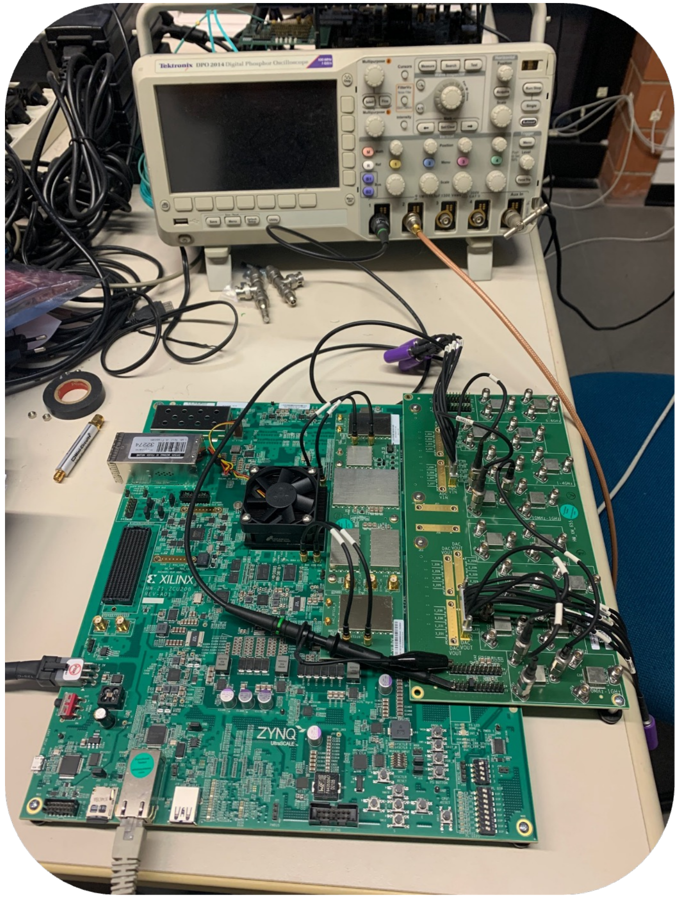


iSWAP Gate

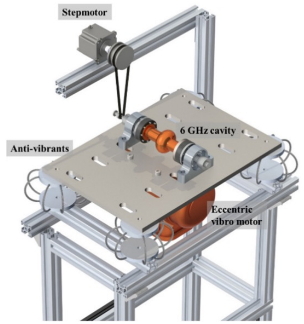
Thanks to Alex Piedjou PostDoc at LNF

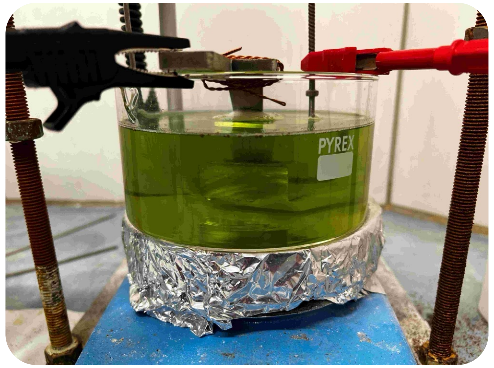
Qubit Control with RFSoC





3D Cavity Fabrication

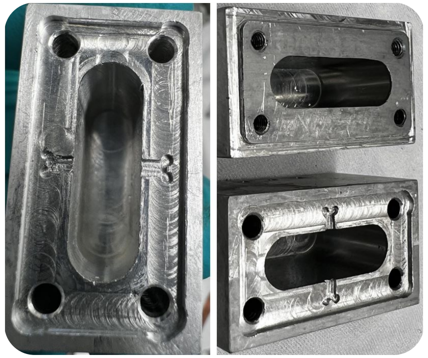


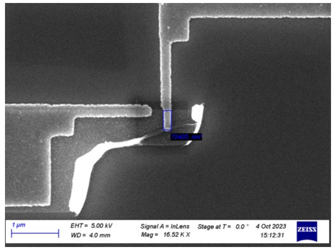
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Legnaro

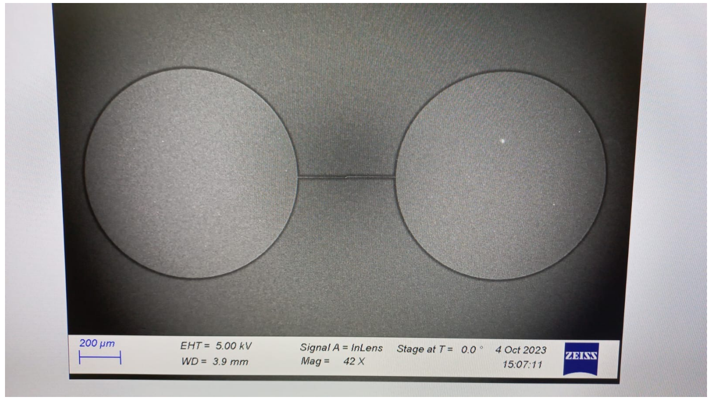
Mechanical machining

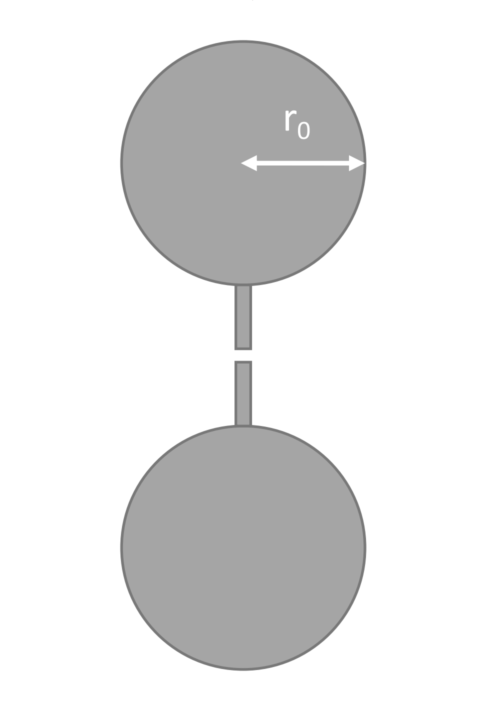
Vibro-tumbling

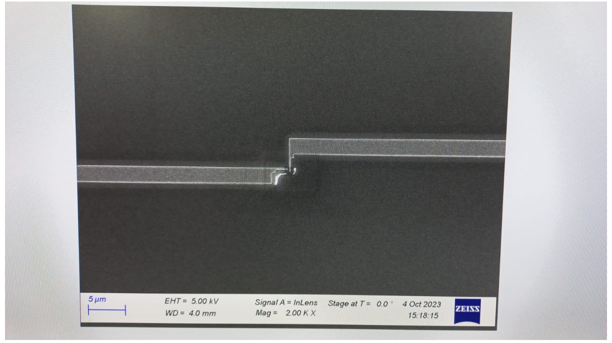
Electropolishing





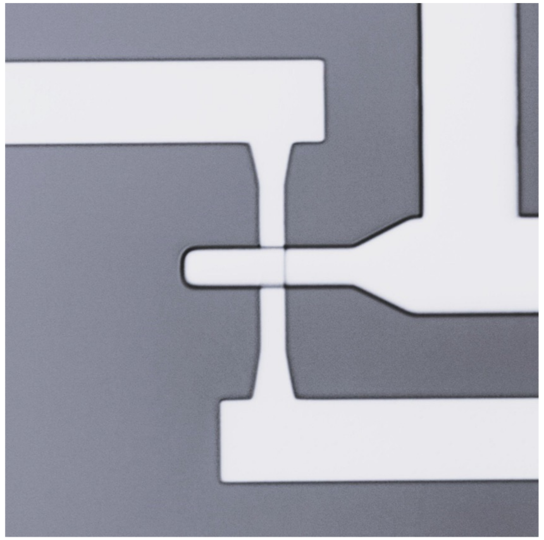


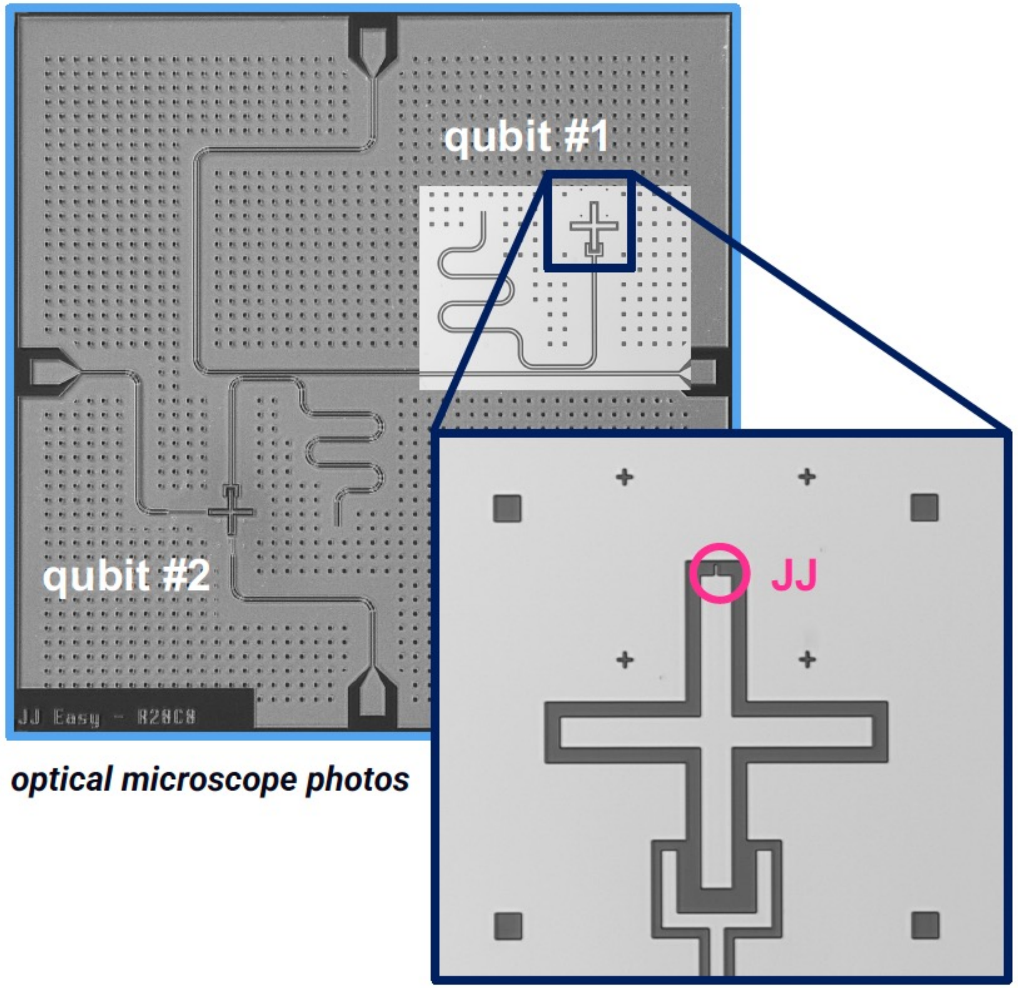




- Aluminum JJ with area approx. 200 x 350 nm

Manufactoring of 3D qubits with circular pads at CNR





optical microscope photos

FONDAZIONE
BRUNO KESSLER

The End