INFLATIONARY SCENARIO AND DE SITTER CONJECTURE

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I will show you the relationship between the swampland criteria in the field theory and slow-roll condition in inflationary scenario.

☆ Plan of my talk

[1] Introduction
[2] de Sitter conjecture
[3] No Go theorem & Swampland
[4] Discussions and remarks

Kwansei Gakuin University

Region name

Kansai Region (including Kyoto, Osaka, …)

In 1880's, Japanese called it "Kwansei"

[1] Introduction

☆ What is your research interest in physics?

-> My research interest is general relativity and cosmology which is motivated by string theory.

☆ What do you want to do in string theory?

I would like to investigate

~ how to produce the inflation, moduli stabilization and late-time acceleration of our universe.

☆ Why is that?

- There is the practical confirmation of the inflationary universe scenario of the early universe.

- It is now the most challenging problem to construct a consistent cosmological model that explains these observational facts, on the basis of unified theory.

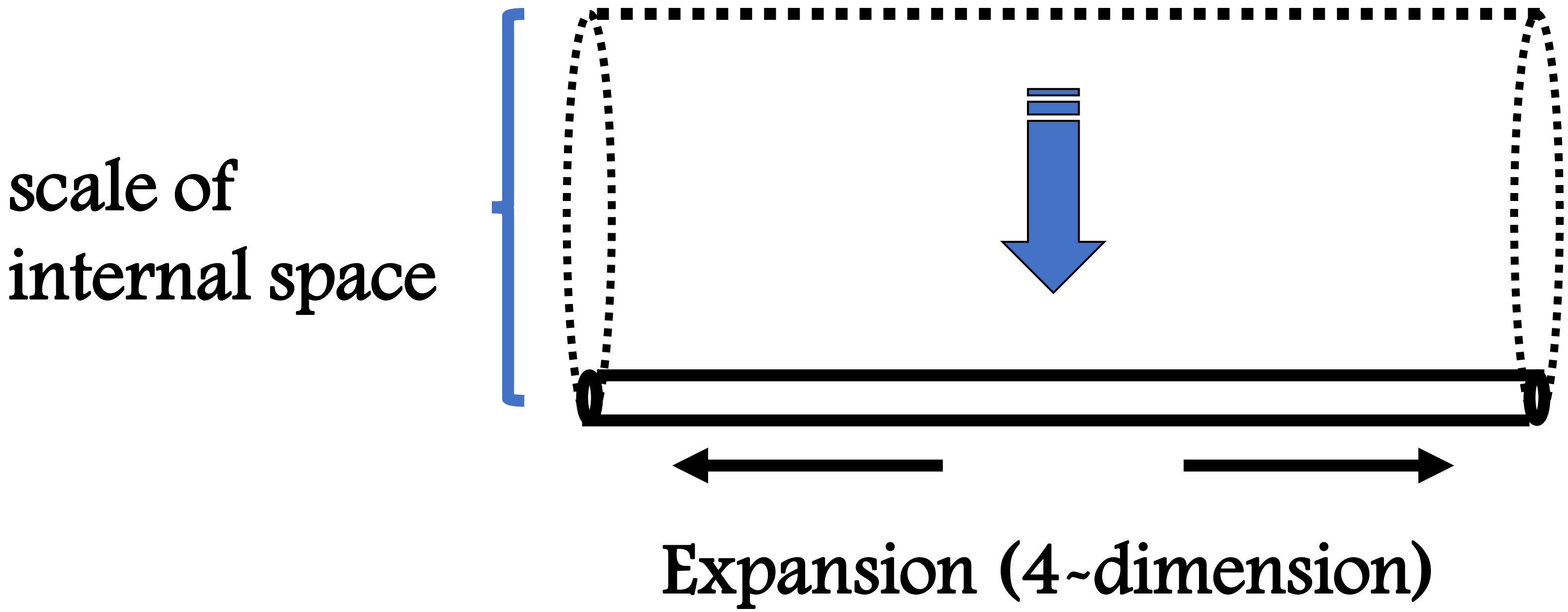
- String theory is one of candidate for viable unified fundamental theories in particle physics.

Then, it is important to study the dynamics of inflation in the string theory!

☆ What is problem?

- These theories require the spacetime to be higher dimensional (more than 4~dimensional spacetime).
- In order to obtain a 4~dimensional universe at low energies, we have to find a natural way to conceal "extra" dimensions, which is usually called a compactification.
- This compactification gives rise to various new problems. One of the most serious problems is the "moduli stabilization".

◆ Compactification



Moduli:
volume of internal space, shape of internal space
⇒ It is necessary to fix the moduli in order to predict a realistic cosmology.

☆Kaluza-Klein spherical compactification:

D-dim metric :

Extra d-dimensions (S^d)

$$ds^2 = g_{MN}\, dx^M dx^N = \left[\frac{b(x)}{b_0}\right]^{-d} q_{\mu\nu}(\mathrm{X})\, dx^\mu\, dx^\nu + b^2(x) u_{ij}\, dy^i dy^j$$

4D effective action

4-dim universe

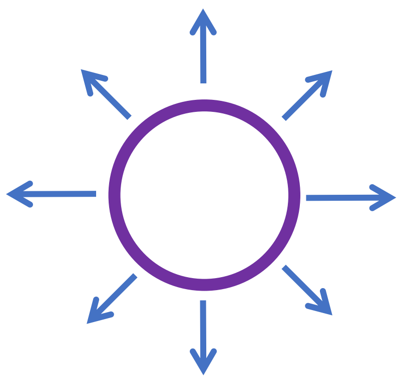
D-dim action

$$\begin{aligned} S &= \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\,(R - \mathcal{L}_m) \\ &= \int d^4 x\sqrt{-q}\left[\frac{1}{2\bar{\kappa}^2}R(\mathrm{X}) - \frac{1}{2}q^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right], \\ \phi &= \phi_0 \ln\left[\frac{b(x)}{b_0}\right], \qquad \phi_0 = \sqrt{\frac{d(d+2)}{2\kappa^2}}, \qquad \bar{\kappa}^2 = \frac{\kappa^2}{V_{\mathrm{int}}}, \\ U(\phi) &= \frac{\mathcal{L}_m}{\kappa^2}\,\mathrm{e}^{-d\frac{\phi}{\phi_0}} - \frac{d(d-1)}{2\kappa^2\phi_0^2}\,\mathrm{e}^{-(d+2)\frac{\phi}{\phi_0}} \end{aligned}$$

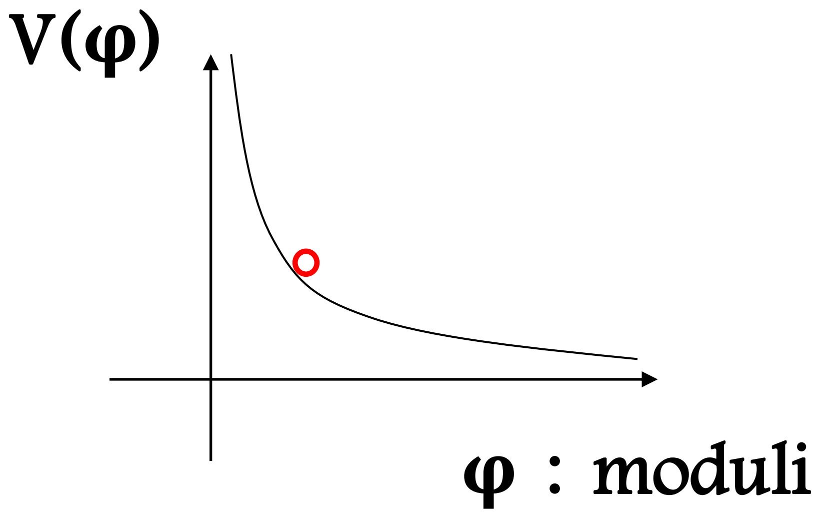
- Dynamics of extra dimension and moduli stabilization

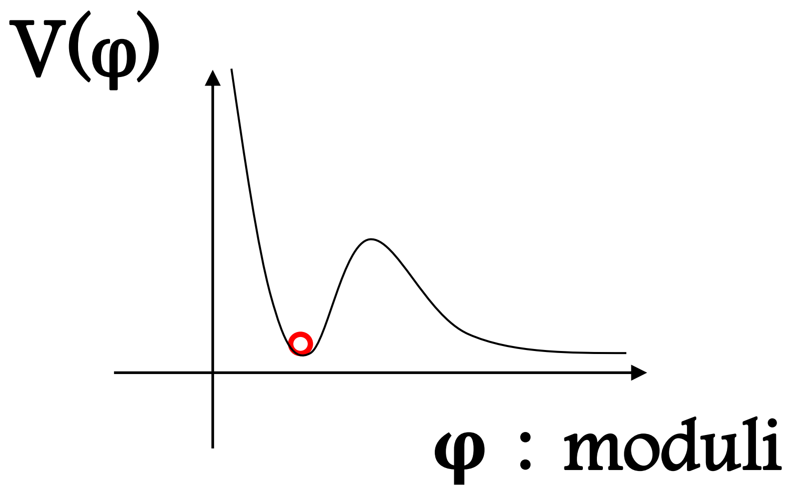
(Alan Chodos, Steven Detweiler, Phys.Rev.D21:2167,1980.)
(Philip Candelas, Steven Weinberg, Nucl.Phys.B237:397,1984.)
(S. Kachru, R. Kallosh, A. Linde, S. P. Trivedi; Phys.Rev.D68:046005,2003)

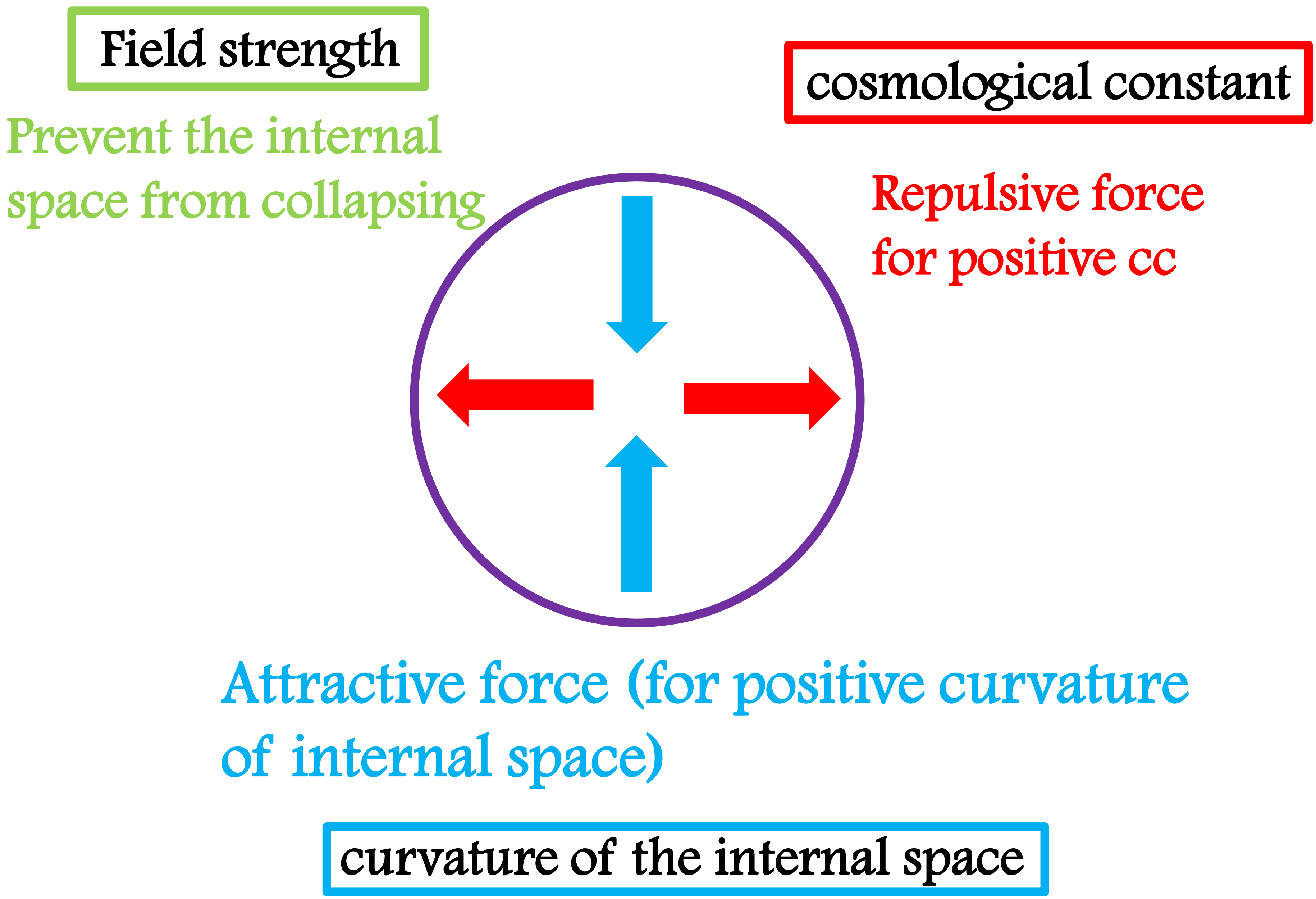
Run away potential



Stabilization of moduli
Field strength, quantum corrections






Field strength
Prevent the internal space from collapsing
cosmological constant
Repulsive force for positive cc
Attractive force (for positive curvature of internal space)
curvature of the internal space

- Another is the no-go theorem against accelerating expansion of the universe in simple Kaluza-Klein type or stationary warped compactification with a smooth compact internal space.
(J. Maldacena, C. Nunez, hep-th/0007018)

$\Rightarrow$ It is not so easy to realize the accelerating universe in the string theory.

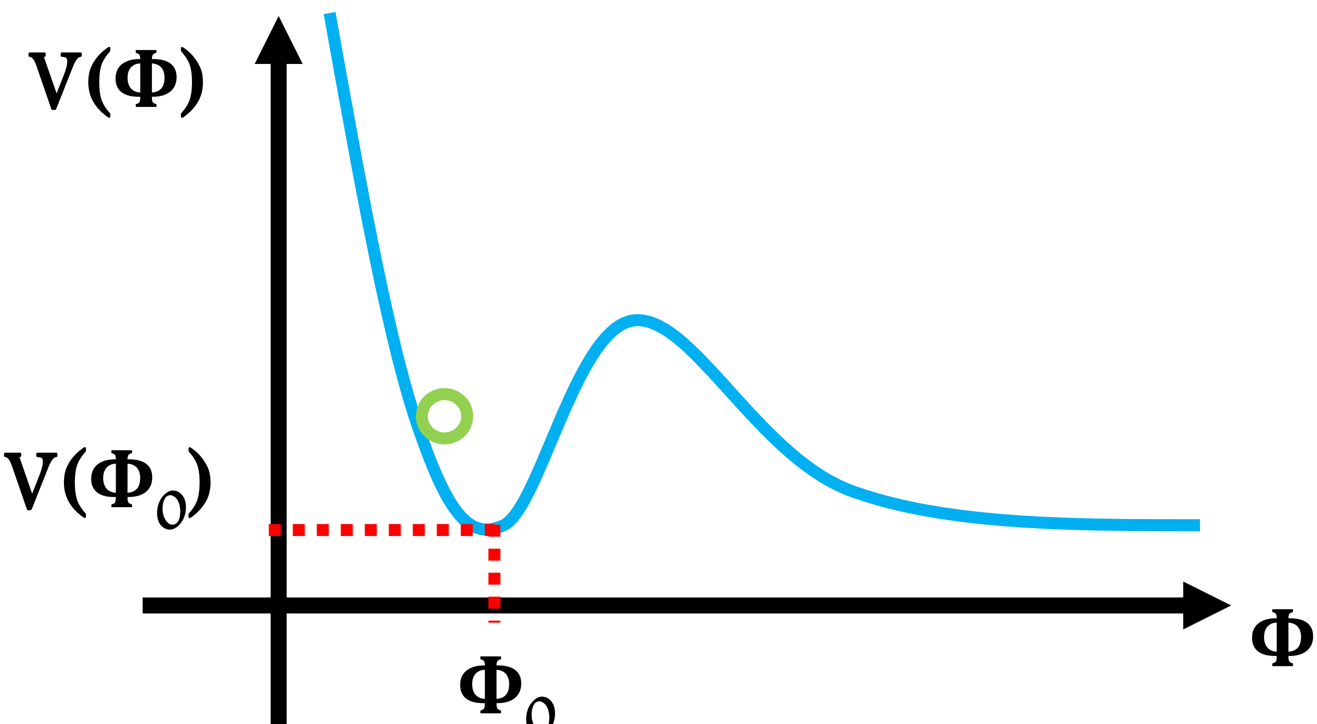
- The potential energy of moduli is the effective cosmological constant in four dimensions.

- The potential energy in general is negative at its minimum even if the potential has the minimum.

Einstein equation with cosmological constant:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu} \ ,$$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \left[g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi + V(\phi) \right]$$



Anything that contributes to the energy density of the vacuum acts just like a cosmological constant.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{1}{2}\kappa^2 V(\phi_0) g_{\mu\nu}\,,$$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \underbrace{\left[\Lambda + \frac{1}{2}\kappa^2 V(\phi_0)\right]}_{\Lambda_{\mathrm{eff}}} g_{\mu\nu} = 0\,,$$

Λ_{eff}

This has the same effect as adding a term Φ_0 to the effective cosmological constant.

- In the gravity theory, an initial clue of the de Sitter compactification was that the hyperbolic space associated to a higher-dimensional theory can be regarded as internal space.

$$R = R_{(4)} + R_{(d)} = 0$$

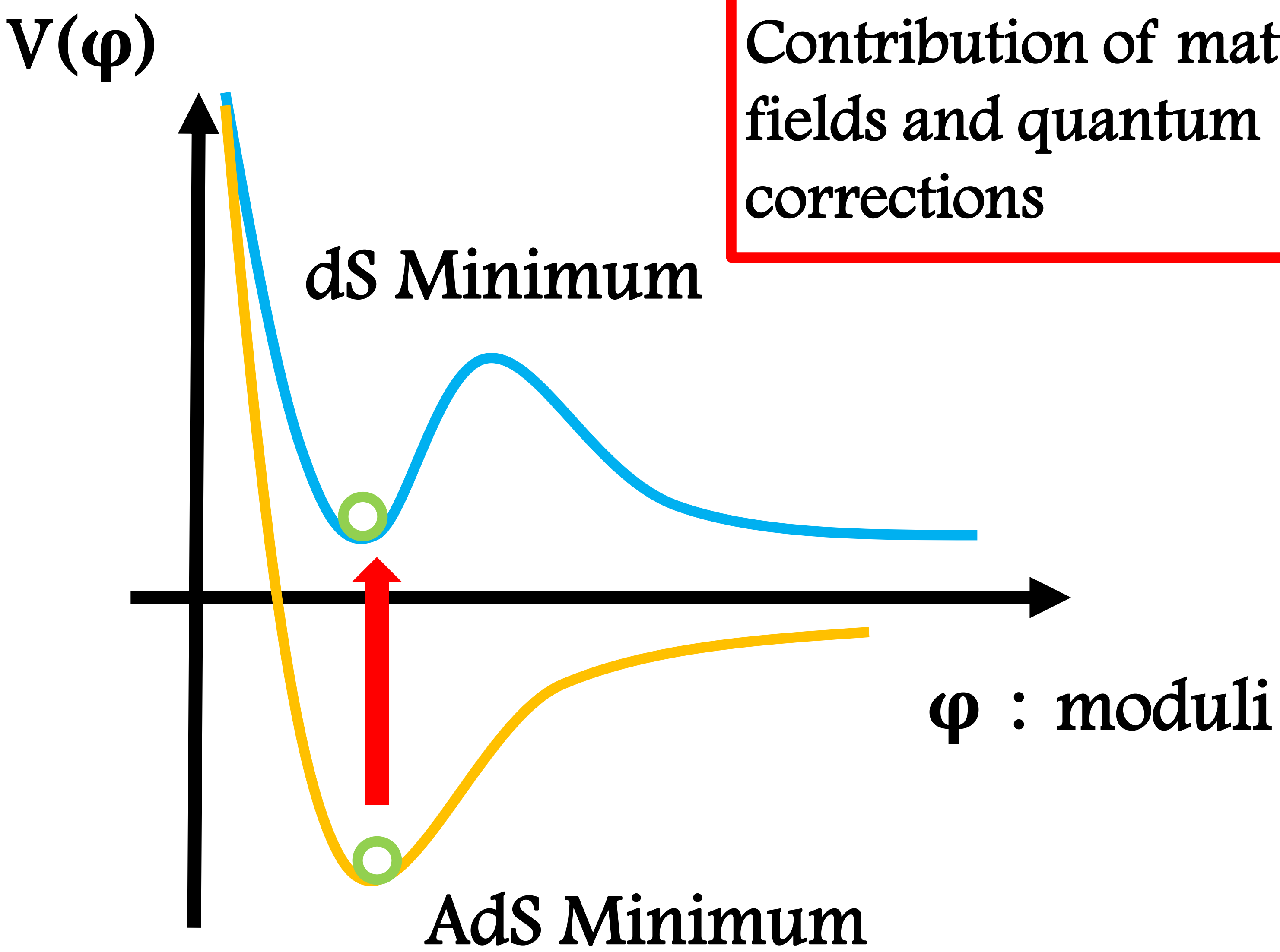
4D universe	Extra dimension
dS_4	H^d

(P. K. Townsend and M. N. R. Wohlfarth, arXiv:hep-th/0303097)

4D universe	Extra dimension
AdS_4	S^d

(Peter G.O. Freund, Mark A. Rubin, Phys.Lett.B97:233-235,1980)

(S. Kachru, R. Kallosh, A. Linde, S. P. Trivedi; Phys.Rev.D68:046005,2003)



Contribution of matter fields and quantum corrections

[2] de Sitter conjecture

★ Swampland

It originated in string Landscape.
(S. Kachru, R. Kallosh, A. D. Linde, S. P. Trivedi, hep-th/0301240)
(L. Susskind, hep-th/0302219)

The number of possible consistent flux compactifications of F-theory to 4D is at least 10^{272000} !!
(W. Taylor and Y.-N. Wang, arXiv:1511.03209 [hep-th])

There is huge number of choices to make when using string theory for model building.

☆ String landscape and swampland

★String Landscape ~ large space of inequivalent string backgrounds

- It is difficult for us to construct string vacua from a very large number of possible choices
  ⇒ Is there no predictability in string theory?

- The attitude towards identifying the correct string vacuum has shifted from using a top-down approach to a bottom-up one over the past decade.
  (T. D. Brennan, F. Carta and C. Vafa, arXiv:1711.00864 [hep-th])

Top-down : $10d$ string ⇒ $4d$ EFT

Bottom-up : $4d$ QFT ⇒ 4d QFT + gravity

- Not all consistent looking effective field theories can be coupled consistently to gravity with a UV completion.

- We call the set of all effective field theory which is ultimately inconsistent with a string theory as the swampland.
  (C. Vafa, hep-th/0509212)
  (H. Ooguri, C. Vafa, hep-th/0605264)

- There are some conjectures, minimal criteria that allows us to exclude a theory from the string landscape.

☆ Swampland conjecture

- Theory of quantum gravity coupled to two scalar fields (in reduced Planck units) :

$$\frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^1 \partial_\nu \phi^1 - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^2 \partial_\nu \phi^2 - V(\phi^1 \, , \, \phi^2) + \cdots$$

☆ Criterion 1 :

- The range of the scalar field has an upper bound.

$$\triangle \phi^i \sim \mathcal{O}(1)$$

(C. Vafa, hep-th/0509212)
(H. Ooguri and C. Vafa, hep-th/0605264)

- There is a finite radius in field space where the effective Lagrangian above is valid.

☆ de Sitter conjecture (Criterion 2) :

(G. Obied, *et. al.*, arXiv:1806.08362 [hep-th])

- For V > 0, there is a lower bound on the relative variation of the potential in field space.

$$\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$$

- The parameter *c* can depend on the macroscopic dimension of spacetime *d*.

- This is motivated by the observation that it appears to be difficult to construct any reliable de Sitter vacuum in string theory.

$$\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$$

- If the quantum theory of gravity with V > 0 does not satisfy this criteria, it belongs to swampland.
- If the quantum theory of gravity with V > 0 satisfy this criteria, it belongs to landscape.

$$\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$$

- If the theory does not satisfy this criteria, what is problem?
- The theory is not consistent with equation of motion (higher~ or lower~dimensional), or energy condition (strong, null).

$$\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$$

- If the potential in quantum gravity has a local or global minimum and the potential energy at minimum is positive, $|\nabla V|/V=0$!

  ⇒ The theory will be the swampland.

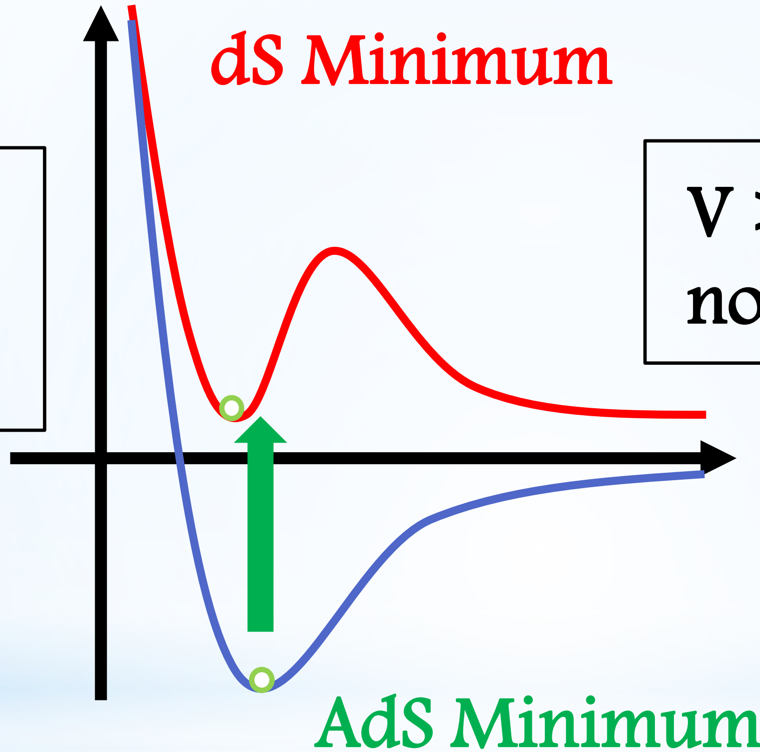
- The stable vacua in string theory will give AdS!

★ Is there no de Sitter vacuum in string theory?

(Y. Akrami, R. Kallosh, A. Linde, V. Vardanyan, arXiv:1808.09440 [hep-th])

(S. Kachru, R. Kallosh, A. Linde, S. P. Trivedi; Phys.Rev.D68:046005,2003)

Conjecture:
- $\nabla V > c\, V$
- $c > 0$



V > 0 and ∇V=0
not allowed !

- Is KKLT counterexample to conjecture or swampland?

  (H. Murayama, *et. al.*, arXiv:1809.00478 [hep-th])

  (R. Blumenhagen, *et. al.*, arXiv:1902.07724 [hep-th])

I do not think so, because…

[3] No Go theorem & Swampland

(J. M. Maldacena and C. Nunez, hep-th/0007018)
(P. J. Steinhardt and D. Wesley, arXiv:0811.1614 [hep-th])
(G. Obied, *et. al.*, arXiv:1806.08362 [hep-th])

♦ Assumptions :

- no α', g_s-corrections
- no D-branes/O-planes
- no singularities
- all matter satisfies strong and null energy condition.
- speed of all fields is set to zero

In KKLT model, there are D-branes and O-planes.

$\Rightarrow$ The KKLT construction contained novel ingredients invalidating the no-go theorem.

☆ de Sitter conjecture :

(G. Obied, *et. al.*, arXiv:1806.08362 [hep-th])

$$\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$$

- The extension of Maldacena-Nunez (No-Go theorem) argument.
  (J. Maldacena, C. Nunez, hep-th/0007018)

★ Assumption:

- Existence a stationary point $V'(\Phi_0) = 0$ with $V(\Phi) > 0$.

⇒ stationary de Sitter solution of 4D SUGRA.

⇒ But, 10d Einstein eq. shows that this solution is incompatible with the strong energy condition!

- Swampland conjecture : making one step further
- Placing our system at any point of the potential with positive energy $V(\Phi) > 0$ and choosing the condition to be $\partial_t \Phi = 0$.
- Using SEC & higher-dimensional EOM
  ⇒ Hubble is related to the acceleration of scalar field.
- Solving the lower-dimensional EOM
  ⇒ finding the relation between the value of the scalar field potential and the gradient of the potential energy.

☆ Motivation

- The velocity of all scalar fields (moduli) are zero.

  Why ?

  ~ We can easily solve the effective *d*-dimensional equations of motion.

- However, the moduli will be stabilized when the velocity of the moduli is always zero.

  ~ This is not consistent with the assumption (no stationary point of the potential).

- Moreover, there is no dynamical fields such as inflaton in the background.

~ We cannot discuss the dynamics of the inflation in terms of de Sitter criteria.

- If we set non-zero velocities of the moduli, these couple to the scale factor of the universe in EOM.

~ This affects the evolution of our universe.

- Are there the relationship between the conjecture and slow-roll condition?

(Murayama’s talk in ”Accelerating Universe in the Dark”)

- Background: gravity & scalar fields

d~dim

$$ds^2 = e^{-2\sqrt{\frac{D-d}{(D-2)(d-2)}}\tau}\, \tilde{\Omega}^2(\Phi, y)\left[-dt^2 + a^2(t)\delta_{\alpha\beta}dx^\alpha dx^\beta\right]$$

D~dim

$$+e^{2\sqrt{\frac{d-2}{(D-d)(D-2)}}\tau}\, \tilde{g}_{mn}(\Phi, y)\, dy^m dy^n$$

- Strong energy condition ($R_{tt} \geqq 0$)

- D~dim Einstein equation & d~dim EOM

- d~dimensional gravitational constant κ^2 to be equal to D~dimensional one

- Combining D-dim & d-dim equations with the SEC bound, we obtain the following bound on the gradient of the potential

$$\frac{|\partial_\tau V|}{V} \geq \left| 2\sqrt{\frac{D-2}{(D-d)(d-2)}} - \alpha \frac{\dot{\tau}^2}{V} \right| ,$$

$$\alpha = \frac{1}{4}\sqrt{\frac{D-d}{(D-2)(d-2)}} \left[-5d^2 + d\,(5D+12) - 11D - 2 \right] ,$$

- α > 0 for D=10, 4≦d≦9 or D=11, 4≦d≦10

- Slow roll inflation model:

$$|\ddot{\tau}/\dot{\tau}| \ll |H| \,, \quad V \gg \dot{\tau}^2$$

$\Rightarrow$ Swampland criterion

(G. Obied, *et. al.*, arXiv:1806.08362 [hep-th])

$$\frac{|\partial_\tau V|}{V} \geq 2\sqrt{\frac{D-2}{(D-d)(d-2)}}$$

[4] Discussion and remarks

(1) Swampland criteria :

- The consequences of the strong and null energy condition support the criteria.
- However, we must assume that the velocity of scalar fields is zero.

(2) Modification of de Sitter conjecture :

- If we consider the contribution of the velocity, we have a new tern in the criterion.

- Upon setting the slow-roll condition, the criterion has the same form as the case of de Sitter conjecture.

- If $(\partial_t \tau)^2/V \sim O(10^{-1})$, we can obtain $|\nabla V|/V \geqq 0$.

~ *Epilogue* ~

Is de Sitter conjecture physically plausible?

In my opinion, I have to modify the criteria.

Because there are many assumptions for the conjecture and the no-go theorem.

For instance, they assume a nonsingular compactification manifold.

- M-theory compactification on a manifold of G_2 holonomy can give chiral fermions in four dimensions only if the compactification manifold is singular.

  (B. S. Acharya, E. Witten, hep-th/0109152)

- Thus, the conjecture may hardly be used as a general argument against dS vacua in string theory.