The V_{cb} puzzle: status and progress

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Outline:

1. Introduction: the CKM matrix and the V_{cb} puzzle
2. Inclusive $B \to X_c \ell \bar{\nu}$ decays
3. Exclusive $B \to D^{(*)} \ell \bar{\nu}$ decays
4. Open problems and conclusions

Introduction

Interaction basis

⇒ gauge interactions are diagonal

⇒ mass terms are not diagonal

$$-\mathcal{L}_{\mathrm{Y}} = \underbrace{Y_d^{ij}}_{\text{Non-diagonal Yukawa}} \bar{Q}_L^i H d_R^j + \underbrace{Y_u^{ij}}_{\text{Non-diagonal Yukawa}} \bar{Q}_L^i \tilde{H} u_R^j + \mathrm{h.c.}$$

Non-diagonal Yukawa

Mass basis

⇒ Yukawa couplings are diagonal

⇒ The CKM matrix is the remnant of the diagonalisation

$$\mathcal{L}_{cc} \propto \bar{u}_L^i \gamma^\mu d_L^j W_\mu^+ V_{ij}$$

CKM matrix

The CKM matrix

nuclear β decays

$K \to \pi \ell \bar{\nu}$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$b \to u \ell \nu$

$b \to c \ell \nu$

$\nu + d \to c + \ell$

- The CKM is a unitary matrix
- The elements associated with the first and second families are better determined
- How do we determine the other elements?

The unitarity triangles

- The CKM is parametrised by three mixing angles and one CP-violating phase

  $\Rightarrow$ We can determine the not-so-precise elements from the most precise ones

- Unitarity is essential to assess whether there are any deviations that could hint at BSM physics

- Useful to use the Wolfenstein parametrisation

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Unitarity: $V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$

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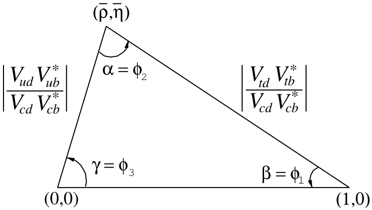
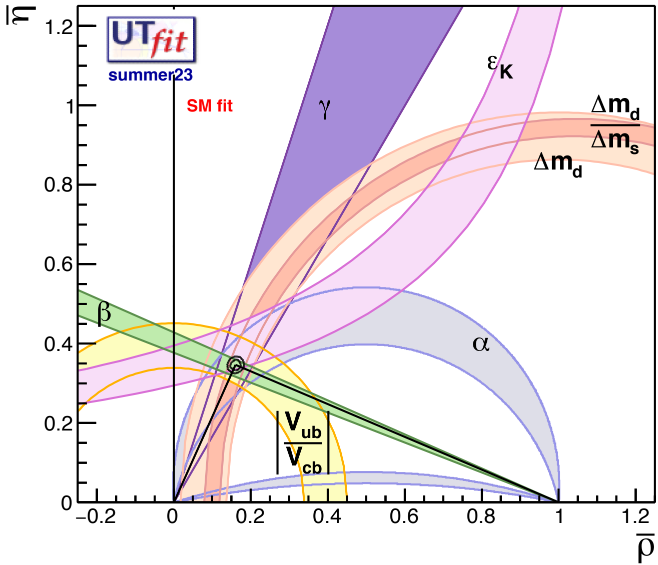


The Unitarity triangles



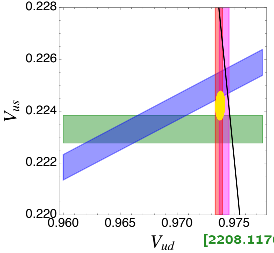
The CKM status in 2024

"...there is a general consistency, at the percent level, between the SM predictions and the experimental measurements. Thus in order to discover new physics effects a further effort in theoretical and experimental accuracy is required."

[2212.03894]

However, there are tensions which (in my opinion) require even more urgent attention

- Violation of unitarity in the first row
- The V_{cb}/V_{ub} puzzle



[2208.11707]

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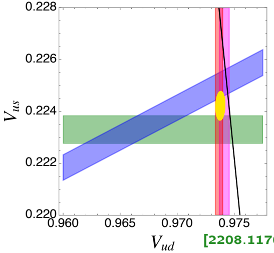


[2208.11707]

The V_{cb} saga

How do we extract V_{cb}?

Inclusive processes:

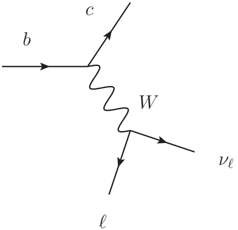
- We start from a well determined initial hadronic state and we sum over all possible hadronic final states

Exclusive processes

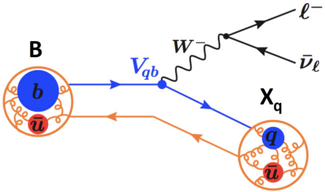
- We resolve the hadronic final state
- More data available

Requires predictions for observables related to hadronic decays

The theory drawback



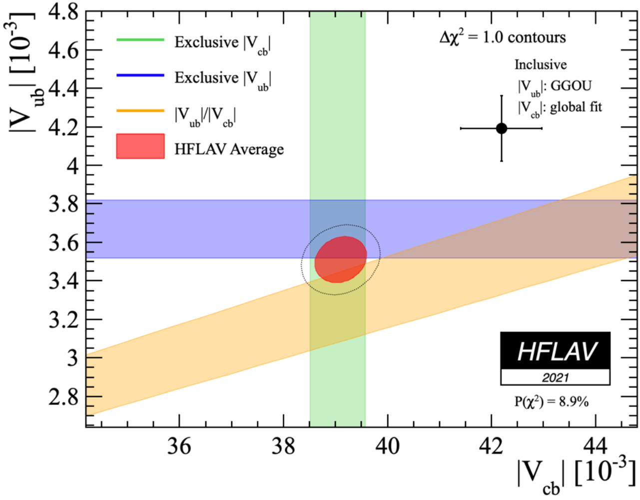
$\mu_{\text{partonic}} = m_b$



$\mu_{\text{hadronic}} = \Lambda_{\text{QCD}}$

Fundamental challenge to match partonic and hadronic descriptions

The long-standing V_{cb} puzzle



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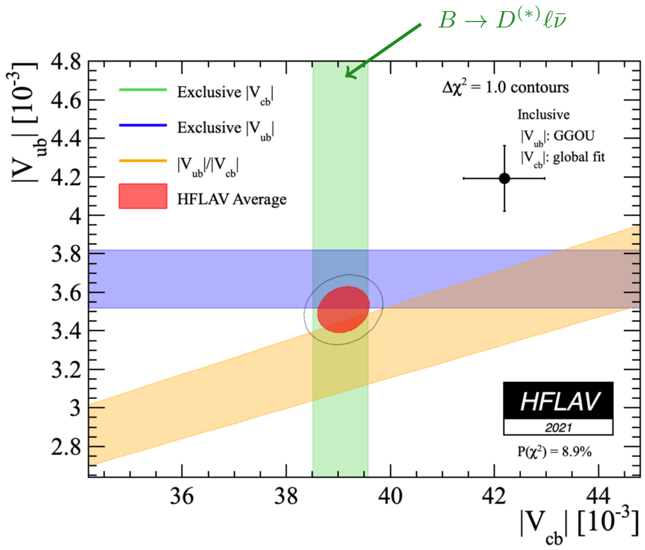
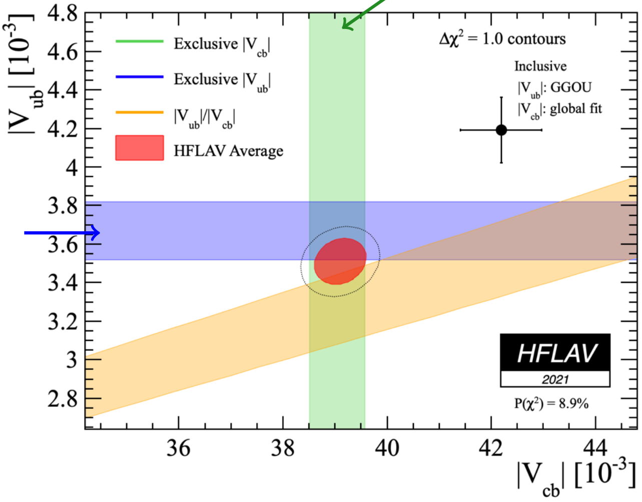


The long-standing V_{cb} puzzle

$B \to D^{(*)}\ell\bar{\nu}$

$B \to \pi\ell\bar{\nu}$



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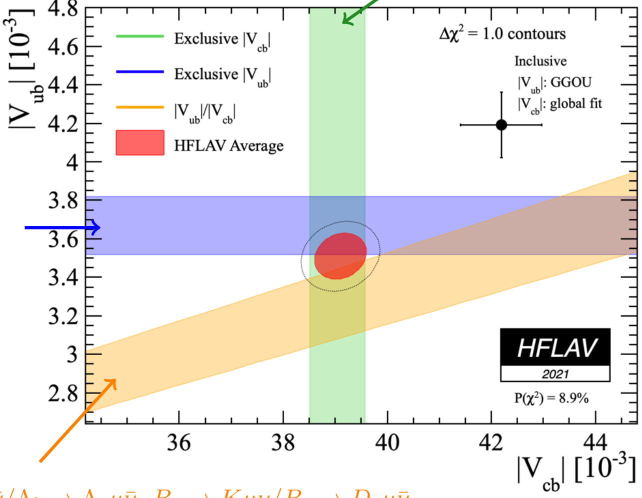
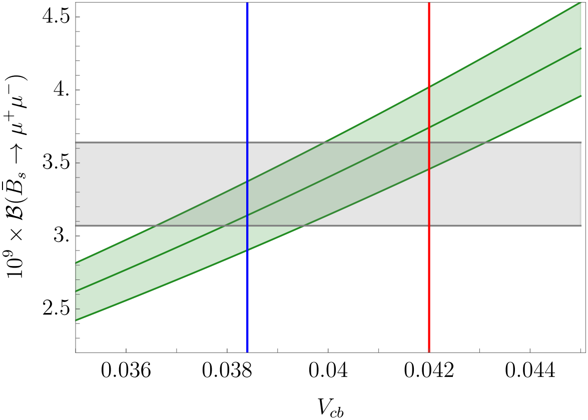


$\Lambda_b \to p\mu\bar{\nu}/\Lambda_b \to \Lambda_c\mu\bar{\nu}$, $B_s \to K\mu\nu/B_s \to D_s\mu\bar{\nu}$

Why we need a better determination of V_{cb}?



- The value of V_{cb} has a major impact on flavour observables like $\mathcal{B}(B_s \to \mu^+\mu^-)$ or ϵ_K

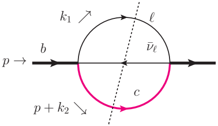
- A resolution of the puzzle is central given the perspective sensitivities at Belle II and LHCb

Main challenges for the future

- Data are one of the most important ingredient
  - $\Rightarrow$ Belle and Belle II exhausted their available datasets, but Belle II is taking data according to the foreseen expectations
  - $\Rightarrow$ LHCb started producing results, but we don't have data yet that we can use for phenomenological analysis
- From the theory point of view, progress has been done but more is needed
  - $\Rightarrow$ We need to reconcile different approaches
  - $\Rightarrow$ How do we estimate uncertainties and how can we go beyond the current status?

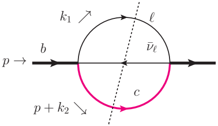
Inclusive decays

Theory framework



$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T \left\{ \mathcal{H}_{\mathrm{eff}}^\dagger(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} |B(p)\rangle$$

Theory framework



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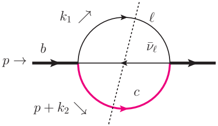
$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

Theory framework



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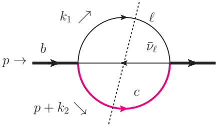
- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p) | \mathcal{O}_{n+3,i} | B(p) \rangle$ are non perturbative
  - $\Rightarrow$ They need to be determined with non-perturbative methods, e.g. Lattice QCD
  - $\Rightarrow$ They can be extracted from data
  - $\Rightarrow$ With large n, large number of operators

Theory framework



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loss of predictivity

Theory framework for $B \to X_c \ell \bar{\nu}$

Double expansion in $1/m$ and α_s

$$\Gamma_{sl} = \Gamma_0 f(\rho) \Big[1 + a_1 \left(\frac{\alpha_s}{\pi}\right) + a_2 \left(\frac{\alpha_s}{\pi}\right)^2 + a_3 \left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_\pi^2}{m_b^2} + \left(g_0 + g_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_G^2(m_b)}{m_b^2} + d_0 \frac{\rho_D^3}{m_b^3} - g_0 \frac{\rho_{LS}^3}{m_b^3} + \ldots \Big]$$

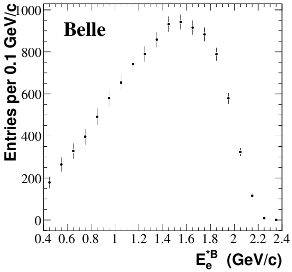
- The coefficients are known
- $\mu_\pi^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v (i\vec{D})^2 b_v | B \rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | B \rangle_\mu$

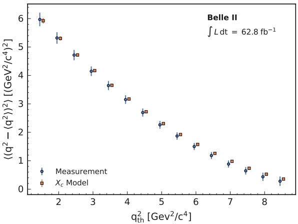
  $\Rightarrow$ No Lattice QCD determinations are available yet
- Use for the first time of α_s^3 corrections **[Fael, Schönwald, Steinhauser, '20]**
- Ellipses stands for higher orders

  $\Rightarrow$ proliferation of terms and loss of predictivity

How do we constrain the hadronic parameters?

We need information from kinematic distributions



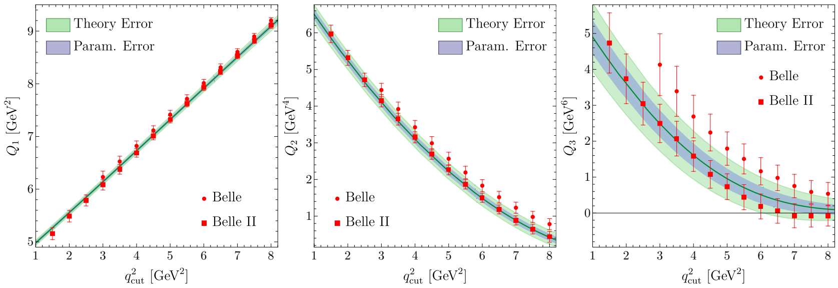


- Traditional method: Extract the hadronic parameters from moments of kinematic distributions in E_l and M_X
- New idea: Use q^2 moments to exploit the reduction of free parameters due to RPI [Fael, Mannel, Vos, '18, Bernlochner et al, '22]
- Measurements of branching fractions are needed and are at the moment quite old
- Can we do it on the lattice? [Gambino, Hashimoto, '20, '23, Hashimoto, Jüttner, et al, '23]

Global fit

[MB, Capdevila, Gambino, '21, Finauri, Gambino, '23]

	$m_b^{\rm kin}$	$\overline{m}_c$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	$10^2{\rm BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$	$\chi^2_{\rm min}(/{\rm dof})$
without	4.573	1.092	0.477	0.306	0.185	-0.130	10.66	42.16	22.3
q^2-moments	0.012	0.008	0.056	0.050	0.031	0.092	0.15	0.51	0.474
Belle II	4.573	1.092	0.460	0.303	0.175	-0.118	10.65	42.08	26.4
	0.012	0.008	0.044	0.049	0.020	0.090	0.15	0.48	0.425
Belle	4.572	1.092	0.434	0.302	0.157	-0.100	10.64	41.96	28.1
	0.012	0.008	0.043	0.048	0.020	0.089	0.15	0.48	0.476
Belle &	4.572	1.092	0.449	0.301	0.167	-0.109	10.65	42.02	41.3
Belle II	0.012	0.008	0.042	0.048	0.018	0.089	0.15	0.48	0.559



About QED effects in inclusive decays

Why do we care about QED Effects?

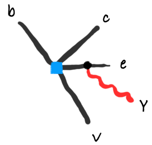
- We want to match the theory description with the experimental measurements that are always affected by photon emissions
- The MC PHOTOS accounts for QED effects, reporting results which can be compared with the non-radiative theory predictions
- PHOTOS knows only about real emission and obtains the virtual part by normalisation

$$\frac{d\Gamma}{dz dx} = \mathcal{F}^{(0)}(\omega_{\text{virtual}} + \omega_{\text{real}}) \Rightarrow \int dx (\omega_{\text{virtual}} + \omega_{\text{real}}) = 1$$

$\boxed{\textbf{Are virtual corrections under control?}}$

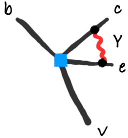
Leading contributions

1. Collinear logs: captured by splitting functions



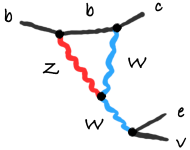
$$\sim \frac{\alpha_e}{\pi} \log^2 \left(\frac{m_b^2}{m_e^2} \right)$$

2. Threshold effects or Coulomb terms



$$\sim \frac{4\pi\alpha_e}{9}$$

3. Wilson Coefficient



$$\sim \frac{\alpha_e}{\pi} \left[\log \left(\frac{M_Z^2}{\mu^2} \right) - \frac{11}{6} \right]$$

Branching ratio

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- The total branching ratio is not affected by large logs due to KLN theorem
- The large corrections are from the Wilson Coefficient and the threshold effects

$$\frac{\Gamma}{\Gamma^{(0)} g(\rho)} = 1 + \frac{\alpha}{\pi}\left[\ln\left(\frac{M_Z^2}{m_b^2}\right) - \frac{11}{6} + 5.516(14)\right]$$

$$= 1 + \underbrace{1.43\% - 0.44\%}_{\text{Wilson Coefficient}} + \underbrace{1.32\%}_{\text{Threshold effects}} = 1 + 2.31\%$$

- Large shift of the branching ratio of the same order of the current error on V_{cb}
- How do we incorporate in the current datasets?
- Moments are less sensitive because they are normalised

Global fit + QED

[Finauri, Gambino, '23]

- Implementation of QED corrections are analysis dependent
- BaBar provides branching fractions with and without radiation

$$R_{\mathrm{QCD}}^{\mathrm{new}} = \zeta_{\mathrm{QED}} R_{\mathrm{QCD}}^{\mathrm{Babar}}$$

$\Rightarrow$ ζ_{QED} accounts for the misalignment between the corrected BaBar results and the results from the full $\mathcal{O}(\alpha_e)$ computation

m_b^{kin}	$\overline{m}_c(2\,\mathrm{GeV})$	μ_π^2	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.090	0.453	0.288	0.176	−0.113	10.62	41.95
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48

- The central value shifts slightly
- Belle II data are needed to understand how to apply the correction
- Can we go beyond scalar QED?

Exclusive decays

Exclusive matrix elements

$$\langle H_c|J_\mu|H_b\rangle = \sum_i S_\mu^i \mathcal{F}_i$$

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S_\mu^i \mathcal{F}_i$$

form factor

scale Λ_{QCD}

independent

Lorentz structures

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

form factor

scale Λ_{QCD}

independent

Lorentz structures

Form factors determinations

- Lattice QCD
- QCD SR, LCSR

only points at specific kinematic points

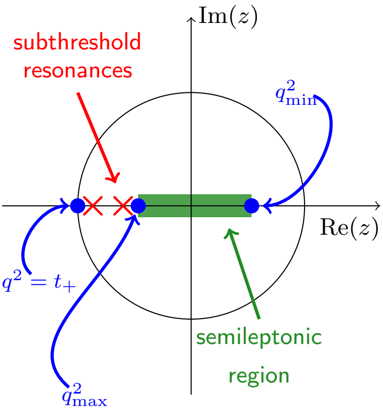
Form factors parametrisations

- HQET (CLN + improvements) $\Rightarrow$ reduce independent degrees of freedom
- Analytic properties $\rightarrow$ BGL

data points needed to fix the coefficients of the expansion

The z-expansion and unitarity

[Boyd, Grinstein, Lebed, '95, Caprini, Lellouch, Neubert, '98]



- in the complex plane form factors are real analytic functions
- q^2 is mapped onto the conformal complex variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- q^2 is mapped onto a disk in the complex z plane, where $|z(q^2, t_0)| < 1$

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

$$\sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

[Alternative method: 2105.02497 and following]

The Heavy Quark Expansion in a nutshell

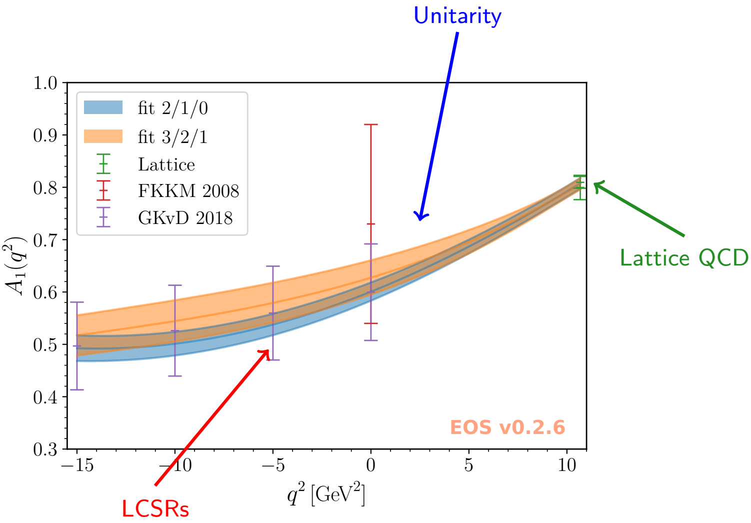
The HQE exploits the fact that the b and c quarks are heavy

- Double expansion in $1/m_{b,c}$ and α_s
- The HQE symmetries relate $B^{(*)} \to D^{(*)}$ form factors
- At $1/m_{b,c}$ drastic reduction of independent degrees of freedom

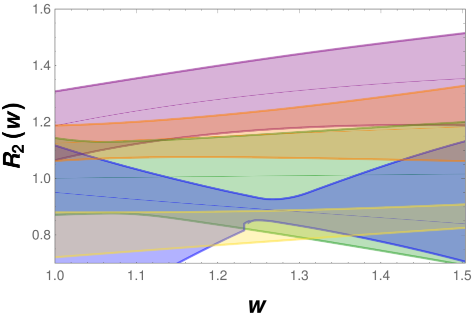
With current precision we know we have to go beyond the $1/m_{b,c}$ order and we use the following form

$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi}\right) \xi + \frac{\Lambda_{\text{QCD}}}{2m_b} \sum_j c_{ij} \xi^j_{\text{SL}} + \frac{\Lambda_{\text{QCD}}}{2m_c} \sum_j d_{ij} \xi^j_{\text{SL}} + \left(\frac{\Lambda_{\text{QCD}}}{2m_c}\right)^2 \sum_j g_{ij} \xi^j_{\text{SSL}}$$

- Total of 10 independent structures to be extracted from data
- We use the conformal mapping $q^2 \mapsto z(q^2)$ to include bounds and have a well-behaved series


Unitarity
Lattice QCD
LCSRs
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
$A_1(q^2)$
fit 2/1/0
fit 3/2/1
Lattice
FKKM 2008
GKvD 2018
EOS v0.2.6
−15
−10
−5
0
5
10
q^2 [GeV^2]

$B \to D^*$ after 2021



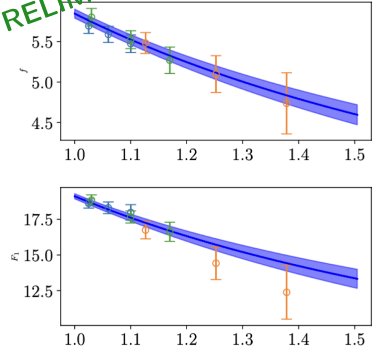
- FNAL/MILC '21
- HQE@$1/m_c^2$
- Exp data (BGL)
- JLQCD '23
- HPQCD '23

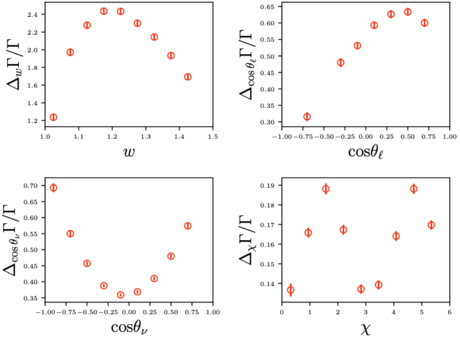
- Are the Lattice QCD datasets compatible?
- What's the source of the discrepancy with HQET? [MB, Harrison, Jung, ongoing]
- Why are experimental data so different from LQCD data?

Can we combine the LQCD results?

[MB, A.Jüttner, in preparation]

PRELIMINARY!



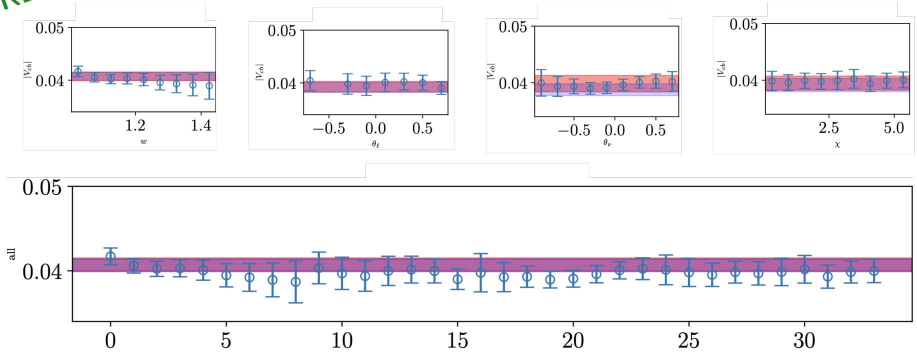


- A combined fit to all possible LQCD data is possible in the BGL approach
- How does it compare with data?
  - New Belle and Belle II datasets available!
- How can we extract V_{cb}?

V_{cb} from JLQCD and Belle II data

[MB, A.Jüttner, in preparation]

PRELIMINARY!



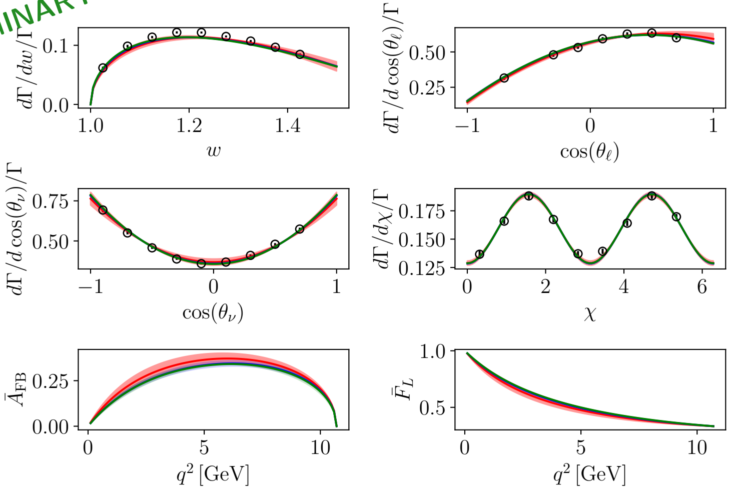
- With JLQCD results and Belle II datasets V_{cb} is flat throughout the bins
- The combination needs to account for correlations
- The statistical procedure to do it has to be carefully defined

[See also:2310.03680]

Comparison with kinematic distributions

[MB, A.Jüttner, in preparation]

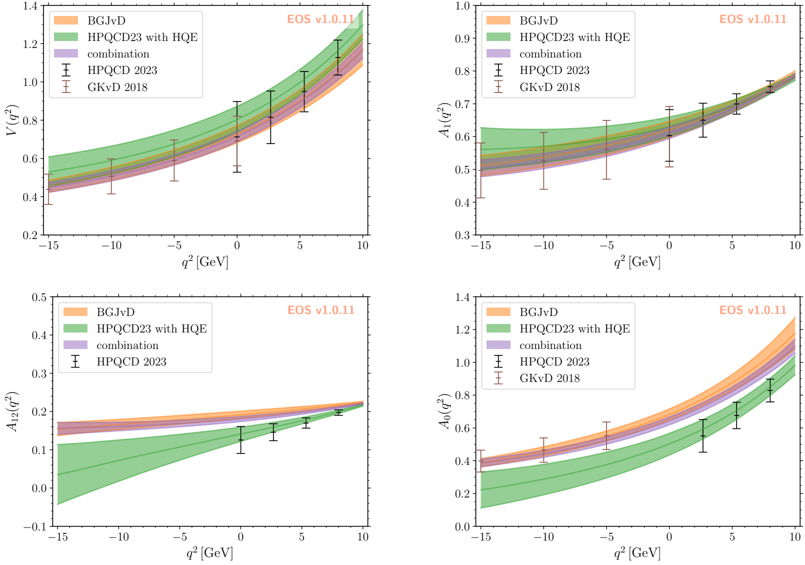
PRELIMINARY!



- Fits are all acceptable
- Theory and Experiment agree on the shapes

What can we learn from the HQE?

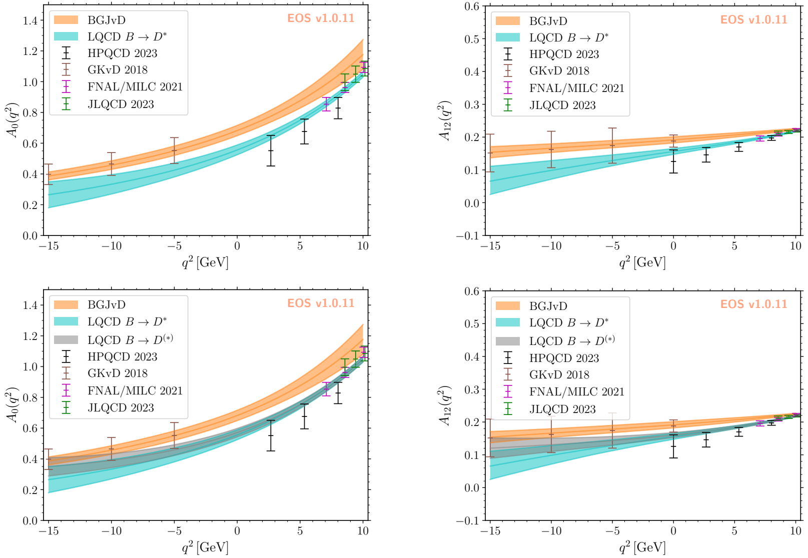
[MB, J. Harrison, M. Jung, in preparation]



$\Rightarrow$ V and A_1 drive the V_{cb} determination and they are quite well compatible

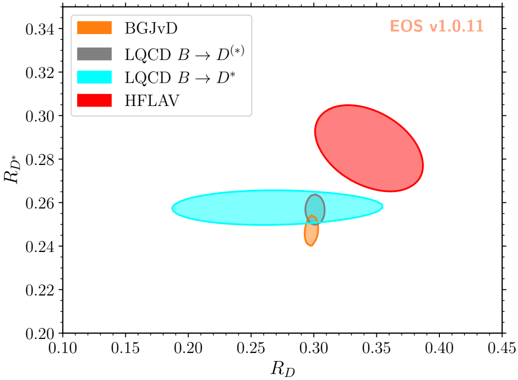
HQE with lattice only

[MB, J. Harrison, M. Jung, in preparation]



$\Rightarrow$ Motivates a joint $B \to D^{(*)}$ LQCD analysis

Predictions



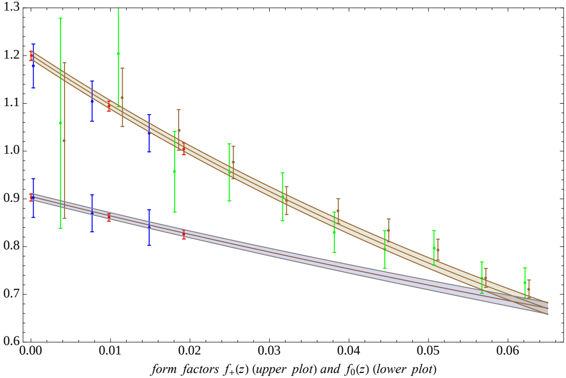
$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})}$$

- The predictions for $R_{D^{(*)}}$ change quite drastically combining different datasets
- The combined $B \to D^{(*)}$ fit yield values in between the HQE and the $B \to D^*$ only fit

$$B \to D$$

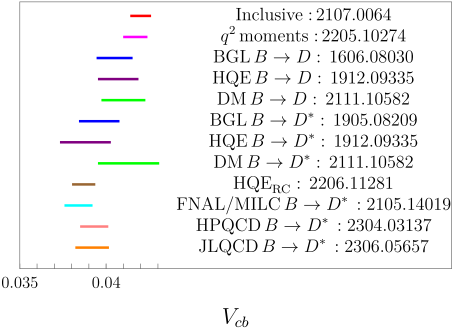
[Bigi, Gambino, '16]

- Belle+Babar data and HPQCD+FNAL/MILC Lattice points



$$|V_{cb}| = (40.49 \pm 0.97) \times 10^{-3}$$

Pheno Status 1

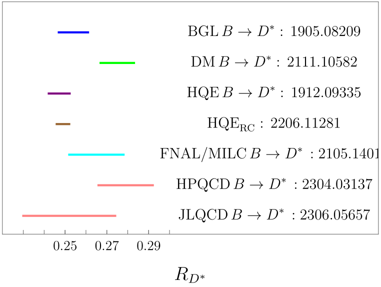


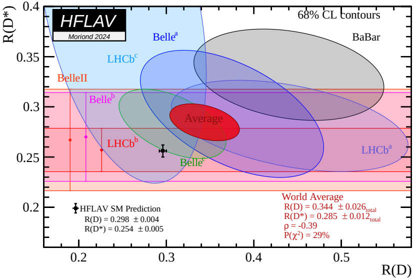
- The inclusive determination is solid
- No evident issues for $B \to D$
- Spread between inclusive and exclusive up to $3 - 4\sigma$
- Work in progress for the theory predictions of $B \to D^*$ to understand the various tensions

  $\Rightarrow$ Do we have to correct for QED?

- New experimental data are available are under scrutiny

Pheno status 2





- New Lattice QCD results point to larger values for R_{D^*}

  $\Rightarrow$ Difference in the slopes is crucial and has to be understood

- No change in R_D, where Lattice QCD results, LCSRs, HQET and experimental data agree very well with each other

Other open problems

- The QED issue is present also for exclusive modes

  $\Rightarrow$ One calculation available for $B \to D$ only

  $\Rightarrow$ The $B \to D^*$ case is much more involved

  $\Rightarrow$ How do we reconcile the threshold effects between the exclusive and the inclusive?

$$\mathcal{B}(B \to X_c \ell\nu) = \mathcal{B}(B \to D\ell\nu) + \mathcal{B}(B \to D^*\ell\nu) + \mathcal{B}(B \to D^{**}\ell\nu) + \dots$$

- The exclusive V_{cb} from $B \to D^*$ is roughly determined by one form factor which agrees quite well in different determinations

  $\Rightarrow$ Even with more precise LQCD data this won't be resolved if not made worse by smaller uncertainties

- Concerning the inclusive determination, new branching fractions measurements are welcome

  $\Rightarrow$ Can LHCb have a say concerning $B_s \to X_c \ell\bar{\nu}$ and $\Lambda_b \to X_c \ell\bar{\nu}$?

Conclusions

- V_{cb} is a fundamental parameter that drives predictions for many processes
- At the current status, there is a significant difference between inclusive and exclusive determinations
- The inclusive determination is solid, different datasets yield very compatible results, the only caveat is the branching fraction measurement
- The exclusive determination is more messy

  $\Rightarrow$ New Lattice QCD determination disagree among themselves and with experimental data

  $\Rightarrow$ The solution is not clear yet, work in progress in many directions

  $\Rightarrow$ This is a combined theory+experimental problem, only synergy between communities can shed light on this puzzle

Appendix

$B \to D^{(*)}$ form factors

- 7 (SM) + 3 (NP) form factors
- Lattice computation for $q^2 \neq q^2_{\text{max}}$ only for $B \to D$
- Calculation usually give only a few points
- q^2 dependence must be inferred
- Conformal variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- $t_+ = (m_B + m_{D^{(*)}})^2$ pair production threshold
- $t_0 < t_+$ free parameter that can be used to minimise $|z_{\text{max}}|$
- $|z| \ll 1$, in the $B \to D$ case $|z| < 0.06$

The HQE parametrisation 1

- Expansion of QCD Lagrangian in $1/m_{b,c} + \alpha_s$ corrections

[Caprini, Lellouch, Neubert, '97]

- In the limit $m_{b,c} \to \infty$: all $B \to D^{(*)}$ form factors are given by a **single** Isgur-Wise function

$$F_i \sim \xi$$

- at higher orders the form factors are still related $\Rightarrow$ **reduction** of free parameters

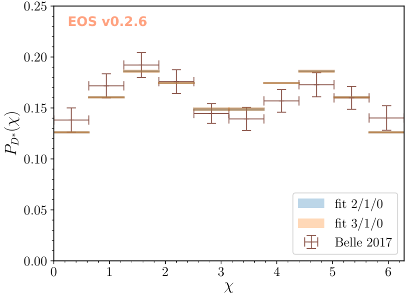
$$F_i \sim \left(1 + \frac{\alpha_s}{\pi}\right)\xi + \frac{\Lambda_{\mathsf{QCD}}}{2m_b}\xi^i_{\mathsf{SL}} + \frac{\Lambda_{\mathsf{QCD}}}{2m_c}\xi^i_{\mathsf{SL}}$$

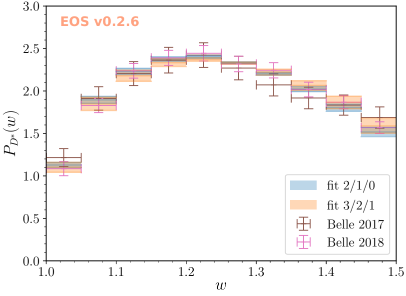
  - at this order 1 leading and 3 subleading functions enter
  - ξ^i are not predicted by HQE, they have to be determined using some other information

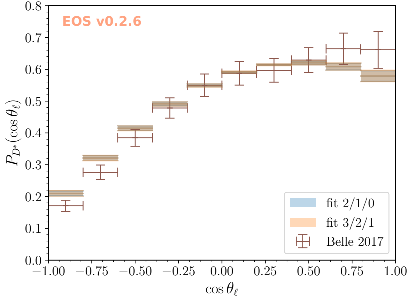
The HQE parametrisation 2

- Important point in the HQE expansion: $q^2 = q^2_{\text{max}}$
- At this point Luke's Theorem applies: the subleading corrections vanish for some form factors
- The leading Isgur-Wise function is normalised: $\xi(q^2 = q^2_{\text{max}}) = 1$
- **Problem**: contradiction with lattice data!
- $1/m_c^2$ corrections **have to be systematically included** **[Jung, Straub, '18, MB, M.Jung, D.van Dyk, '19]**
  - well motivated also since $\alpha_s/\pi \sim 1/m_b \sim 1/m_c^2$

Comparison with kinematical distributions



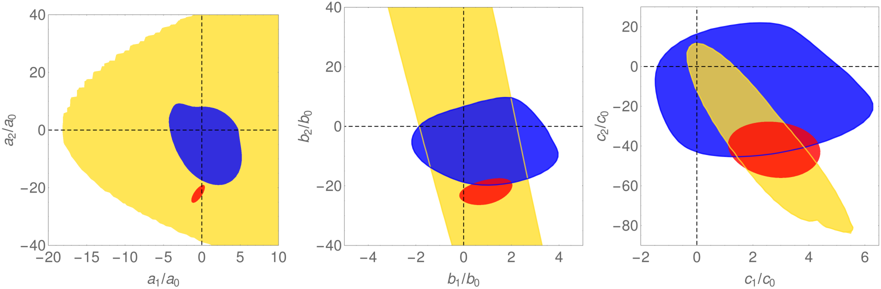




good agreement with kinematical distributions

Fit stability

- BGL fit to Belle 2017 and 2018 data (yellow)
- HQE fit 2/1/0 (red)
- HQE fit 3/2/1 (blue)



- compatibily of HQE fit with data driven one
- 2/1/0 underestimates massively uncertainties

$3/2/1$ is our nominal fit

HQET in a nutshell

- In HQET it is convenient to work with **velocities** instead of momenta
- Instead of q^2 we use the dimensionless variable $w = v_B \cdot v_{D^*}$
- When the $B(b)$ decays such that the $D^*(c)$ is at rest in the $B(b)$ frame

$$v_B = v_{D^*} \qquad \Rightarrow \qquad w = 1$$

- The brown muck doesn't realise that anything changed
- At zero recoil, the leading IW function is **normalized**

$$\xi(w = 1) = 1$$

BGL vs CLN parametrisations

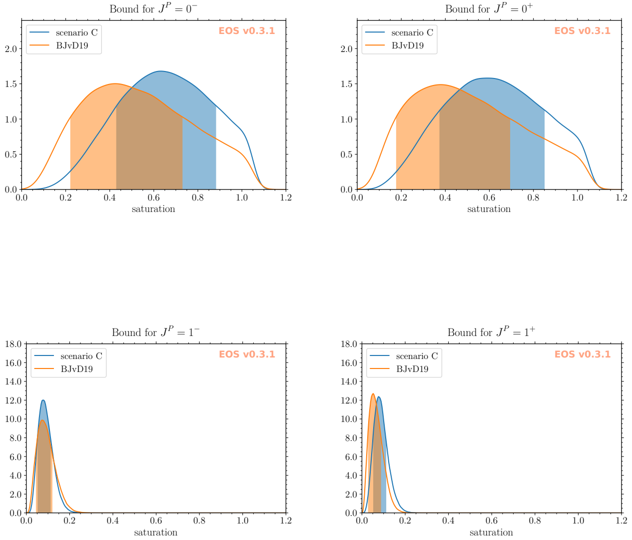
CLN [Caprini, Lellouch, Neubert, '97]

- Expansion of FFs using HQET
- $1/m_{b,c}$ corrections included
- Expansion of leading IW function up to 2nd order in $(w-1)$

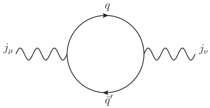
BGL [Boyd, Grinstein, Lebed, '95]

- Based on analyticity of the form factors
- Expansion of FFs using the conformal variable z
- Large number of free parameters

Results: unitary bounds



Unitarity Bounds



$$= i \int d^4x \, e^{iqx} \langle 0 | T \left\{ j_\mu(x), j_\nu^\dagger(0) \right\} | 0 \rangle = (g_{\mu\nu} - q_\mu q_\nu) \Pi(q^2)$$

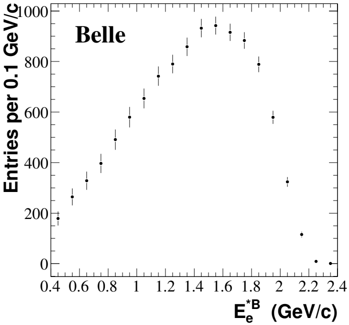
- If $q^2 \ll m_b^2$ we can calculate $\Pi(q^2)$ via perturbative techniques $\Rightarrow \chi(0)$
- Dispersion relations link Im $\left(\Pi(q^2)\right)$ to sum over matrix elements

$$\sum_i |F_i(0)|^2 < \chi(0)$$

[Boyd, Grinstein,Lebed, '95
Caprini, Lellouch, Neubert, '97]

- The sum runs over <u>all</u> possible states hadronic decays mediated by a current $\bar{c}\Gamma_\mu b$
  - The unitarity bounds are more effective the most states are included in the sum
  - The unitarity bounds introduce correlations between FFs of different decays
  - $B_s \to D_s^{(*)}$ decays are expected to be of the same order of $B_{u,d} \to D_{u,d}^{(*)}$ decays due to $SU(3)_F$ simmetry

How do we constrain the OPE parameters?



- Lepton energy and hadronic invariant mass distributions can be used to extract non perturbative information
- Moments of the kinematic distributions

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell,\mathrm{cut}}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\mathrm{cut}}}}$$

$$R^* = \frac{\int_{E_\ell > E_{\ell,\mathrm{cut}}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int dE_\ell \frac{d\Gamma}{dE_\ell}}$$

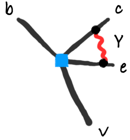
- Similar definition for hadronic mass moments

- The moments give access to the distribution, but not to the normalisation
- They admit an HQE as the rate

$\Rightarrow$ No $\mathcal{O}(\alpha_s^3)$ terms are known yet

The inclusive case

- If wrt QCD the hadronic and leptonic system are separated, QED corrections mix them
  - ⇒ Defining fully inclusive observables is harder
  - ⇒ Analogy with experiments is essential



- The OPE is still valid for the total decay width
- At the differential level, this is generally not true
  - ⇒ Large contributions factorise wrt to tree-level
  - ⇒ Useful to go beyond NLO

Two calculation approaches

1. Splitting Functions

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi}\,\bar{L}_{b/e}\int_y^{1-\rho}\frac{dx}{x}\,P_{ee}^{(0)}\left(\frac{y}{x}\right)\left(\frac{d\Gamma}{dx}\right)^{(0)}$$

$\log(m_b^2/m_e^2)$ plus distribution

- Correction vanishes for the inclusive branching fraction
- Suitable for evaluating $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha/m_b^n)$ corrections

2. Full $\mathcal{O}(\alpha)$ corrections

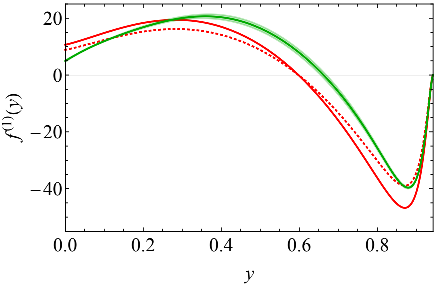
- Access all corrections, not only the one that factorise
- Real corrections are computationally expensive
  - $\Rightarrow$ Cuba library employed to carry out the 4-body integration
  - $\Rightarrow$ Phase space splitting used to reduce the size of the integrands

Lepton Energy spectrum

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- We compute bins in the lepton energy using the full $\mathcal{O}(\alpha)$ calculation
- We compare them to the results given by the splitting functions
- The difference the two calculations for the lepton energy spectrum and obtain a full analytic formula for the radiative corrections

  $\Rightarrow$ Relatively small, easy-to-use formula to obtain branching fractions, lepton energy moments w/o cuts

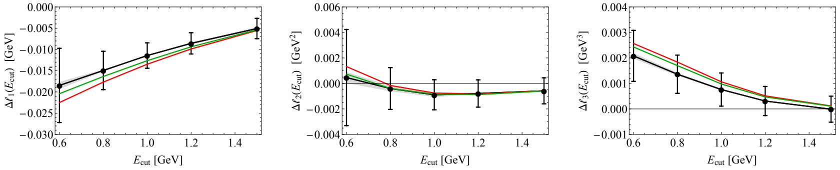


$$f^{(1)}(y) = \frac{\bar{L}_{b/e}}{2}\, f^{(1)}_{\rm LL}(y) + \Delta f^{(1)}(y)$$

Comparison with data

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- Babar provides data with and without applying PHOTOS to subtract QED effects
  - ⇒ Perfect ground to test our calculations
  - ⇒ Not the same for Belle at the moment, could be possible for future analysis?



- The moments, since they are normalised, are not affected by the large threshold corrections
- The agreement with BaBar is very good

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell,\mathrm{cut}}} dE_\ell\, E_\ell^n \,\frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\mathrm{cut}}}}$$

QED for exclusive decays

- For $B^0 \to D^+ \ell \bar{\nu}$, the threshold effects were calculated and are $1 + \alpha\pi$

  [Ginsberg, '66, De Boer, Kitahara, Nisandzic, '18]

- For $B^0 \to D^{*+} \ell \bar{\nu}$, the threshold effects might have a different structure because the hadronic matrix element is different

  $\Rightarrow$ To verify explicitly

- Structure-dependent terms are unknown, but maybe something is doable in the HQE?

- How do we reconcile the threshold effects between the exclusive and the inclusive?

$$\mathcal{B}(B \to X_c \ell \nu) = \mathcal{B}(B \to D \ell \nu) + \mathcal{B}(B \to D^* \ell \nu) + \mathcal{B}(B \to D^{**} \ell \nu) + \ldots$$