Holographic match of $1/N$ corrections to the BPS entropy of AdS_5 black holes

Alejandro Ruipérez

Università di Roma Tor Vergata

Department of Physics, Genova U. - 13 March 2024

Joint work with:

D. Cassani and E. Turetta

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Black holes and quantum gravity

Most fundamental puzzles in **quantum gravity** are related to **black holes**:

- Breakdown of GR near curvature singularities

Hawking, Penrose

- Information loss inherent to black hole evaporation

Hawking

- Microscopic interpretation of black hole **entropy**:

$$\boxed{\mathcal{S} = \frac{A_{\mathcal{H}}}{4G} + \ldots \stackrel{?}{=} \log d_{\text{micro}}}$$

String theory has allowed us to reproduce the Bekenstein-Hawking entropy (and beyond) from microstate counting

Strominger, Vafa + ...

Microstate counting of supersymmetric AdS black holes

We will discuss the microstate counting of **AdS black holes** in the controlled setup of **supersymmetric holography**

Holography allows us to study **quantum gravity in AdS** space via a dual **CFT**

Use **AdS/CFT** to address the microstate counting problem

$$\boxed{\text{Black hole} \quad = \quad \begin{array}{c}\text{ensemble of states}\\ \text{in quantum gravity}\end{array} \overset{\text{AdS/CFT}}{=} \begin{array}{c}\text{ensemble of states}\\ \text{in the dual CFT}\end{array}}$$

Remarkable progress in last years reproducing the BH entropies of AdS black holes

Benini, Hristov, Zaffaroni 15

Cabo-Bizet, Cassani, Martelli, Murthy 18; Choi, Kim, Kim, Nahmgoong 18; Benini, Milan 18

Precision holography

Goal of this talk: discuss how this match can be improved to account for subleading contributions in the large-N expansion (**precision holography**)

Focus on supersymmetric $\mathbf{AdS}_5$ black holes

Gutowski, Reall; Kunduri, Lucietti, Reall
Chong, Cvetic, Lu, Pope

- **CFT:** Cardy-like limit works at **finite** $N \to$ corrected entropy and charges
- **Bulk:** match CFT predictions by adding higher-derivative terms in $\mathcal{L}_{\text{bulk}}$

Anomalies will guide us in this endeavour; crucial role in this story

$\to$ First steps towards exact **quantum black hole entropy**

Further motivations to study higher-derivative gravity

- **Resolution of the horizon of small black holes via α' corrections**

Major historical role in string theory: finite microscopic entropy but singular horizon in supergravity $\Rightarrow \mathcal{S}_{\mathrm{BH}} = 0$?

$$S_{\mathrm{BH}} \neq \log d_{\mathrm{micro}}$$

Puzzle expected to be solved by α' corrections [Sen 95] , still under debate...

Dabholkar 05 + ... ; Cano, AR, Ramírez 18 + ... + Massai, AR, Zatti 23

Intriguing connections to the string/black hole correspondence, swampland arguments (species scale), etc. Chen, Maldacena, Witten; Vafa+ ; Lust, Dvali, Gómez

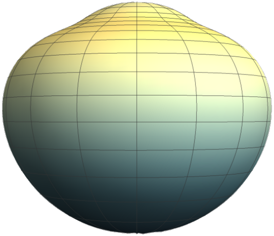
Further motivations to study higher-derivative gravity

- **EFT corrections to Kerr** Endlich, Gorbenko, Huang, Senatore 17; Cano, AR 19

  Higher-derivative terms capture generic modifications of the gravitational interaction at low energies

  How is the Kerr solution modified? Deviations wrt to GR from GWs data?

  Theoretical prejudices against this possibility recently argued not to be correct for near-extremal black holes... Horowitz, Kolanowski, Remmen, Santos



Plan of the talk

Summary of relevant AdS_5 black holes

- AdS_5 black holes of minimal supergravity:
  - General (thermal) solution: depends on 4 parameters $\leftrightarrow E, Q_R, J_1, J_2$

    Chong, Cvetic, Lu, Pope
  - BPS limit with $J_1 \neq J_2$: depends on 2 parameters $\leftrightarrow Q_R, J_1, J_2$ + constraint

    Chong, Cvetic, Lu, Pope
  - BPS limit with $J_1 = J_2 \equiv J$: depends on 1 parameter $\leftrightarrow Q_R, J$ + constraint

    Gutowski, Reall

- Multi-charge AdS_5 black holes of matter-coupled supergravity:
  - More general (thermal) solution: depends on $3 + n$ parameters $\leftrightarrow E, Q_I, J$

    $\to$ only known for the $\mathrm{U}(1)^3$ model: $C_{IJK} \sim |\epsilon_{IJK}|$, $n = 2$

    Chong, Cvetic, Lu, Pope
  - BPS limit with $J_1 = J_2 \equiv J$: depends on $1+ n$ parameter $\leftrightarrow Q_I, J$ + constraint

    $\to$ only known when the scalar manifold is a symmetric space

$$C^{IJK} C_{J(LM} C_{PQ)K} = \frac{1}{27} \delta^I_{(L} C_{MNP)}$$

Kunduri, Lucietti, Reall

Counting BPS states: the superconformal index

Consider a $\mathcal{N}=1$ SCFT on the spatial manifold S^3 and a supercharge $\mathcal{Q}$ satisfying the commutation relations

$$[J_1, \mathcal{Q}] = [J_2, \mathcal{Q}] = \frac{1}{2}\mathcal{Q}\,, \qquad [Q_I, \mathcal{Q}] = -r_I \mathcal{Q}$$

where Q_I, with $I = 1, \ldots, n+1$, are linear combinations of the superconformal **R-charge** and n Abelian **flavour charges**

The field theory quantity of interest for us is the **superconformal index** $\mathcal{I}$ (supersymmetric part. funct. on $S^1_\beta \times S^3$)

$$\mathcal{I} = \mathrm{Tr}\, e^{\pi i(1+n_0)F} e^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\} + \boldsymbol{\omega_1} J_1 + \boldsymbol{\omega_2} J_2 + \boldsymbol{\varphi^I} Q_I}$$

Kinney, Maldacena, Minwalla, Raju; Romelsberger

Supersymmetry implemented via ($n_0 = \pm 1$ in this talk)

$$\boldsymbol{\omega_1 + \omega_2 - 2r_I\varphi^I = 2\pi i n_0}$$

The multi-charge Cardy-like limit of the index

The **black hole saddle** can be isolated by taking a **Cardy-like limit**

Cassani, Komargodski; Choi, J. Kim, S. Kim, Nahmgoong + ...

In the flavoured case the expression for the Cardy-like limit of the index is

$$-\log \mathcal{I} = \frac{k_{IJK}\varphi^I\varphi^J\varphi^K}{6\,\omega_1\omega_2} - k_I\varphi^I\frac{\omega_1^2+\omega_2^2-4\pi^2}{24\,\omega_1\omega_2} + \dots$$

$$\omega_1+\omega_2-2r_I\varphi^I = \pm 2\pi i$$

- Fully controlled by **anomalies**:

$$k_{IJK} = \mathrm{Tr}\, Q_I Q_J Q_K \qquad\qquad k_I = \mathrm{Tr}\, Q_I$$

- For holographic SCFTs:

$$k_{IJK} = k^{(0)}_{IJK} + k^{(1)}_{IJK} + \dots \qquad\qquad k_I = 0 + k^{(1)}_I + \dots$$

Black hole entropy via Legendre transform

- Extremization principle: $(I \equiv -\log \mathcal{I} - \dots)$

$$\boxed{\begin{array}{c} \mathcal{S} = \operatorname{ext}_{\{\omega_1,\omega_2,\varphi,\Lambda\}} \left[-I - \omega_1 J_1 - \omega_2 J_2 - \varphi^I Q_I - \Lambda \left(\omega_1 + \omega_2 - 2 r_I \varphi^I - 2\pi i \right) \right] \\ -\dfrac{\partial I}{\partial \omega_i} = J_i + \Lambda \,, \qquad -\dfrac{\partial I}{\partial \varphi^I} = Q_I - 2 r_I \Lambda \,, \qquad \omega_1 + \omega_2 - 2 r_I \varphi^I = 2\pi i \end{array}}$$

Hosseini, Hristov, Zaffaroni; Cabo-Bizet, Cassani, Martelli, Murthy

- $I = I\left(\omega_i, \varphi^I\right)$ can be written as an homogeneous function of degree 1 $\Rightarrow$

$$\mathcal{S} = 2\pi i \Lambda|_{\text{ext}}$$

- Λ satisfies a polynomial equation with real coefficients depending on the charges Q_I and angular momenta J_i

- $\operatorname{Im} \mathcal{S} = 0 \quad \Leftrightarrow \quad (\Lambda^2 + X)(\text{rest}) = 0 \quad \Rightarrow \quad$ condition on Q_I and J_i

Field theory predictions: universal case

The **universal case** is recovered by setting $Q_I = r_I Q_R$ for all I's:

- Condition on Q_I and J_i:

$$\left[3Q_R + 4\,(2\,\mathtt{a} - \mathtt{c})\right]\left[3Q_R^2 - 8\mathtt{c}\,(J_1 + J_2)\right]$$
$$= Q_R^3 + 16\,(3\mathtt{c} - 2\mathtt{a})\,J_1 J_2 + 64\mathtt{a}\,(\mathtt{a} - \mathtt{c})\,\frac{(Q_R + \mathtt{a})(J_1 - J_2)^2}{Q_R^2 - 2\mathtt{a}(J_1 + J_2)}$$

- Corrected **BPS entropy**

$$\mathcal{S} = 2\pi\sqrt{X} = \pi\sqrt{3Q_R^2 - 8\mathtt{a}(J_1 + J_2) - 16\,\mathtt{a}(\mathtt{a} - \mathtt{c})\,\frac{(J_1 - J_2)^2}{Q_R^2 - 2\mathtt{a}(J_1 + J_2)}}$$

Cassani, AR, Turetta 22

$\mathtt{a}, \mathtt{c}$ are the Weyl anomaly coefficients:

$$\mathtt{a} = \frac{3}{32}\left(3k_{RRR} - k_R\right)\,, \qquad \mathtt{c} = \frac{1}{32}\left(9k_{RRR} - 5k_R\right)$$

Flavoured case: assumptions

The Legendre transform can be implemented under suitable assumptions on the anomaly coefficients: Cassani, Papini 19; Cassani, AR, Turetta 24

- $k^{(0)}_{IJK}$ satisfies the **'magic property'**

$$k^{(0)\,IJK}k^{(0)}_{J(LM}k^{(0)}_{PQ)K} = \frac{4}{9}\mathtt{a}^{(0)}\delta^{I}_{(L}k^{(0)}_{MPQ)}$$

- Relation between cubic and linear coefficients

$$k_{IJK} = k^{(0)}_{IJK} + k_{(I}r_J r_{K)}$$

The latter condition is satisfied quite generally by $\mathcal{N}=1$ **quiver theories** with gauge group $\mathrm{SU}(N)^{\nu}$ describing D3-branes probing the tip of a Calabi-Yau cone:

$$k_I = -\nu\, r_I$$

Field theory predictions: flavoured case

$J_1 = J_2 \equiv J$ **case** Cassani, AR, Turetta 24

- Black hole entropy:

$$\mathcal{S} = 2\pi\sqrt{p_1 + \frac{\nu}{12}\left[2\mathsf{a}^{(0)} + 5p_2 - \frac{2p_1\left(p_1 - p_2^2 + 2Jp_2\right)}{\mathsf{a}^{(0)}\left(p_1 + J^2\right)}\right]}$$

- Non-linear constraint among the charges:

$$p_0 - p_1 p_2 = \frac{\nu}{6}\left[\frac{5}{2}\left(p_1 + p_2^2\right) + \mathsf{a}^{(0)}\left(p_2 - J\right) + \frac{p_1\left(p_2 - J\right)\left(p_1 + p_2^2\right)}{\mathsf{a}^{(0)}\left(p_1 + J^2\right)}\right]$$

where

$$p_2 = -12k^{(0)IJK} r_I r_J Q_K - 2\mathsf{a}^{(0)}$$

$$p_1 = 6k^{(0)IJK} r_I Q_J Q_K - 4\mathsf{a}^{(0)} J$$

$$p_0 = -k^{(0)IJK} Q_I Q_J Q_K - 2\mathsf{a}^{(0)} J^2$$

Application to $\mathbb{C}^3/\mathbb{Z}_\nu$ orbifold theories

- Motivation: embeddings in string/M-theory of dual AdS_5 BHs only known for type IIB on S^5 or S^5/Γ orbifolds, dual to $\mathrm{SU}(N)$ $\mathcal{N}=4$ SYM or the $\mathbb{C}^3/\Gamma$ orbifold theories

- Choosing a 'democratic' basis such that $r_I = \frac{1}{2}$ for $I = 1, 2, 3$, the **'t Hooft anomalies** are given by

$$k_{IJK} = \frac{\nu N^2}{2} |\epsilon_{IJK}| - \frac{\nu}{8}\,, \qquad k_I = -\frac{\nu}{2}\,, \qquad I = 1, 2, 3$$

- The exact superconformal R-symmetry is $\mathcal{R} = \frac{2}{3}\,(Q_1 + Q_2 + Q_3)$ and the Weyl anomaly coefficients are

$$\mathtt{a} = \frac{\nu N^2}{4} - \frac{3\nu}{16}\,, \qquad \mathtt{c} = \frac{\nu N^2}{4} - \frac{\nu}{16}$$

Application to $\mathbb{C}^3/\mathbb{Z}_\nu$ orbifold theories

Black hole entropy:

$$\mathcal{S} = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_1Q_3 - 4\,\mathtt{a}J + \frac{2(\mathtt{c}-\mathtt{a})}{3\mathtt{a}}\,\frac{\mathcal{U}(1,2,3)+\mathcal{U}(2,3,1)+\mathcal{U}(3,1,2)}{Q_1Q_2+Q_2Q_3+Q_1Q_3-4\,\mathtt{a}J+J^2}}$$

$$\mathcal{U}(1,2,3) \,\equiv\, \left[Q_1Q_2 - J(Q_3+2\mathtt{a})\right](Q_1-Q_2)^2$$

Non-linear constraint:

$$\left[Q_1+Q_2+Q_3+2(2\mathtt{a}-\mathtt{c})\right](Q_1Q_2+Q_2Q_3+Q_3Q_1-4\mathtt{c}J) - Q_1Q_2Q_3 - 2(3\mathtt{c}-2\mathtt{a})J^2$$
$$+\frac{2(\mathtt{c}-\mathtt{a})}{3\mathtt{a}}\,\frac{\mathcal{T}(1,2,3)+\mathcal{T}(2,3,1)+\mathcal{T}(3,1,2)}{Q_1Q_2+Q_2Q_3+Q_1Q_3-4\mathtt{a}J+J^2} \,=\, 0\,,$$

$$\mathcal{T}(1,2,3) \,\equiv\, \left[(3Q_1+3Q_2-2Q_3-2J)Q_3 - 6\mathtt{a}J\right](Q_1+Q_2+2\mathtt{a})(Q_1-Q_2)^2$$

Higher-derivative supergravity

Given the field theory predictions, we aim at matching them holographically

The first step is to identify the gravity theory accounting for the corrections $\rightarrow$ include **higher-derivative terms**

Possible strategies:

- Consider higher-derivative terms directly in ten or eleven dimensions
- **Effective approach**: construct a higher-derivative 5D effective action that matches the (corrections to the) **'t Hooft anomalies** holographically

2^{nd} approach boils down to **supersymmetrization of the Chern-Simons terms**

$$\boldsymbol{k_{IJK}}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda \quad \text{and} \quad \boldsymbol{k_I}\epsilon^{\mu\nu\rho\sigma\lambda}R_{\mu\nu\alpha\beta}R_{\rho\sigma}{}^{\alpha\beta}A^I_\lambda$$

Higher-derivative supergravity

Procedure followed to obtain the four-derivative theory:

- Start from off-shell supergravity and add the relevant four-derivative invariants
- Integrate out the auxiliary dofs $\rightarrow$ effective action for the propagating dofs

Main features of the theory:

- $\mathrm{U}(1)_R$ FI gauging $\boldsymbol{g_I}$ $\rightarrow$ non-trivial scalar potential; no hypers nor tensor mult.

- Two couplings $\boldsymbol{\alpha\lambda_I}$ and $\boldsymbol{\alpha\tilde{\lambda}_{IJK}}$ controlling four- and two-derivative corrections:

$$\mathcal{L}_{\text{eff}} \subset R + \frac{1}{4} C^{(\alpha)}_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} F^I_{\mu\nu} F^J_{\rho\sigma} A^K_\lambda + \alpha\lambda_I X^I \mathcal{X}_{\text{GB}} + \frac{\alpha\lambda_I}{2} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A^I_\lambda$$

$$C^{(\alpha)}_{IJK} \equiv C_{IJK} - 6\alpha\lambda_{(I} g_J g_{K)} + \alpha\tilde{\lambda}_{IJK}$$

The four-derivative minimal supergravity action

$$\mathcal{L} = \boldsymbol{c_0} R + 12\boldsymbol{c_1} g^2 - \frac{\boldsymbol{c_2}}{4} F^2 - \frac{\boldsymbol{c_3}}{12\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda$$
$$+ \boldsymbol{\lambda_1 \alpha} \left(\mathcal{X}_{\rm GB} - \frac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \frac{1}{8} F^4 - \frac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \right)$$

$\mathcal{X}_{\rm GB} \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$ is the Gauss-Bonnet invariant

Two-derivative corrections parametrized by $c_i = 1 + \alpha g^2 \delta c_i$:

$$\delta c_0 = 4\boldsymbol{\lambda_2}\,, \qquad \delta c_1 = -10\boldsymbol{\lambda_1} + 4\boldsymbol{\lambda_2}\,, \qquad \delta c_2 = 4\boldsymbol{\lambda_1} + 4\boldsymbol{\lambda_2}\,, \qquad \delta c_3 = -12\boldsymbol{\lambda_1} + 4\boldsymbol{\lambda_2}$$

Cassani, AR, Turetta 22

Higher-derivative corrections to black hole thermodynamics

Do corrections to black hole thermodynamics agree with the field theory predictions?

How do we compute them? Several strategies can be followed:

- **Solve the corrected EOMs and then compute the thermodynamics**; doable for asymptotically-flat black holes, seems hopeless for AdS ones...

  Cano, Chimento, Meessen, Ortín, Ramírez, AR, Zatti, 2018-2022

- **Find corrected near-horizon geometry and compute Page charges**; useful both for asymptotically flat and AdS black holes (closely related to Sen formalism)

- **Extract the thermodynamics via evaluation of the Euclidean on-shell action**; this is the shortest and most effective strategy. No need to know the corrected solution

  Reall, Santos 19

Euclidean quantum gravity

- The **grand-canonical partition function** $\mathcal{Z}\left(\beta, \Omega_i, \Phi^I\right)$ is computed by the Euclidean path integral with (anti-)periodic boundary conditions

$$\mathcal{Z}\left(\beta, \Omega_i, \Phi^I\right) \simeq e^{-I\left(\beta, \Omega_i, \Phi^I\right)}$$

- $I\left(\beta, \Omega_i, \Phi^I\right)$ should then be identified with $\beta\times$(grand-canonical potential), leading to the **quantum statistical relation**

$$I = \beta E - \mathcal{S} - \beta \Omega_i J_i - \beta \Phi^I Q_I$$

Gibbons, Hawking

- Because of the master formula of the **AdS/CFT correspondence**:

$$\boxed{I\left(\beta, \Omega_i, \Phi^I\right) = -\log \mathcal{Z}_{\text{CFT}}\left(\beta, \Omega_i, \Phi^I\right)}$$

Matching the Cardy regime of the index: universal case

- Take general solution with $\boldsymbol{J_1 \neq J_2}$, **impose supersymmetry** and evaluate I

  $\to$ complicated expression in terms of the parameters of the solution

- In terms of the right variables, $\boldsymbol{\omega_i \equiv \beta(\Omega_i - \Omega_i^*), \varphi \equiv \beta\,(\Phi - \Phi^*)}$:

$$\boldsymbol{I = \frac{2\pi}{27G}\left[1 - 4(3\lambda_1 - \lambda_2)\alpha\right]\frac{\varphi^3}{\omega_1\omega_2} + \frac{2\pi\alpha}{3G}\lambda_1\varphi\frac{\omega_1^2 + \omega_2^2 - 4\pi^2}{\omega_1\omega_2}}$$

  $\to$ **matches CFT prediction** after using the holographic dictionary:

$$\boldsymbol{k_{RRR}} = \frac{4\pi}{9G}\left[1 - 4\left(3\boldsymbol{\lambda_1} - \boldsymbol{\lambda_2}\right)\alpha\right] \qquad \boldsymbol{k}_R = -\frac{16\pi\alpha\boldsymbol{\lambda}_1}{G}$$

- It improves the match of Cabo-Bizet, Cassani, Martelli, Murthy 18 to subleading order in the large-N expansion

Results in the universal case: entropy and charges

Entropy and charges from $I(\beta, \Omega, \Phi)$: Cassani, AR, Turetta 22

- BPS entropy

$$\mathcal{S} = \pi \sqrt{3Q_R^2 - 8\mathtt{a}(J_1 + J_2) - 16\,\mathtt{a}(\mathtt{a} - \mathtt{c}) \frac{(J_1 - J_2)^2}{Q_R^2 - 2\mathtt{a}(J_1 + J_2)}}$$

(also: Bobev, Dimitrov, Reys, Vekemans 22)

- Non-linear constraint among the charges

$$\begin{aligned} &[3Q_R + 4\,(2\,\mathtt{a} - \mathtt{c})]\left[3Q_R^2 - 8\mathtt{c}\,(J_1 + J_2)\right] \\ &= Q_R^3 + 16\,(3\mathtt{c} - 2\mathtt{a})\,J_1 J_2 + 64\mathtt{a}\,(\mathtt{a} - \mathtt{c}) \frac{(Q_R + \mathtt{a})(J_1 - J_2)^2}{Q_R^2 - 2\mathtt{a}(J_1 + J_2)} \end{aligned}$$

In perfect agreement with field theory predictions!

Direct match of the entropy

- Wald's entropy:

$$\mathcal{S} = -2\pi \int_{\mathcal{H}} \mathrm{d}^3x \sqrt{\gamma}\, \mathcal{P}^{\mu\nu\rho\sigma} n_{\mu\nu} n_{\rho\sigma}$$

- Page electric charge:

$$Q_R = -\int_{\mathcal{H}} \left(\star\mathcal{F} - \frac{c_3}{\sqrt{3}} F \wedge A - \frac{2\lambda_1 \alpha}{\sqrt{3}} \Omega_{\mathrm{CS}} \right)$$

- Angular momentum:

$$J = \int_{\mathcal{H}} \boldsymbol{\epsilon}_{\mu\nu} \left[-2\nabla_\sigma \mathcal{P}^{\mu\nu\sigma\rho} \eta_\rho + \mathcal{P}^{\mu\nu\sigma\rho} \nabla_\sigma \eta_\rho + \frac{1}{2} \iota_\eta A \left(\mathcal{F}^{\mu\nu} + \frac{c_3}{3\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} A_\rho F_{\sigma\lambda} \right) \right]$$

Cassani, AR, Turetta 23

Direct match of the entropy

Cassani, AR, Turetta 23; Cano, David 24

- Corrected near-horizon solution & previous formulae → corrected entropy and charges

- Perfect agreement with on-shell action method (modulo frame ambiguities in the electric charge)

- Microcanonical form of the entropy relying entirely on the near-horizon geometry

$$\mathcal{S} = \pi\sqrt{3Q_R^2 - 8\mathtt{a}(J_1 + J_2) - 16\,\mathtt{a}(\mathtt{a} - \mathtt{c})\frac{(J_1 - J_2)^2}{Q_R^2 - 2\mathtt{a}(J_1 + J_2)}}$$

Asymptotically-flat black holes

Take $g_I \to 0$ limit of the supersymmetric on-shell action + linear constraint:

$$\boldsymbol{I = \frac{\pi}{G}\frac{C_{IJK}\varphi^I\varphi^J\varphi^K}{\omega_+^2-\omega_-^2} + \frac{2\pi}{G}\alpha\,\lambda_I\varphi^I\,\frac{3\omega_+^2+\,\omega_-^2}{\omega_+^2-\omega_-^2}}\,, \qquad \boldsymbol{\omega_+ = \pm 2\pi i}$$

Verify this for 3-charge black holes with $\boldsymbol{\omega_- = 0}$ in the $\mathrm{U}(1)^3$ model

Cvetic, Youm

Identify **supersymmetric non-extremal saddle**:

$$\boldsymbol{E = Q_1 + Q_2 + Q_3} \qquad \Leftrightarrow \qquad \boldsymbol{\omega_+ = \pm 2\pi i}$$

→ **on-shell action** of this configuration **agrees** with the above prediction

Corrected entropy of the BMPV black hole

$\mathcal{S}$ can be obtained again via Legendre transform:

$$\boldsymbol{\mathcal{S}} = \sqrt{C^{IJK}Q_I Q_J \left(Q_K + \frac{18\pi\alpha}{G}\lambda_K\right) - \frac{\pi}{4G}J_-^2\left(1 + \frac{24\pi\alpha}{G}\frac{C^{IJK}Q_I Q_J \lambda_K}{C^{IJK}Q_I Q_J Q_K}\right)}$$

The constraint on $\boldsymbol{Q_I}$ and $\boldsymbol{J_\pm}$ boils down to $\boldsymbol{J_+ = 0}$ (well known)

- The static case reproduces the well-known shift on the charges
- Prediction for the corrected entropy of the **BMPV black hole**

Summary of main results

- Finite-N corrections to the entropy and charges of AdS_5 black holes: CFT predictions + holographic match in higher-derivative supergravity
- Cardy regime of the index = supersymmetric on-shell action
  - Universal case: checked in full generality
  - Flavoured (multi-charge) case: checked for the $\mathrm{U}(1)^3$ model and for random choices of the higher-derivative couplings $\boldsymbol{\lambda_I}$, $\boldsymbol{\widetilde{\lambda}_{IJK}}$
- Evidence that complex saddles of asymptotically-flat BHs also compute an index
  - Prediction for the corrected entropy of the BMPV black hole
- General lessons
  - There are a number of strategies that render the study of higher-derivative corrections much more tractable
  - In the holographic context: they allow us to extract valuable information beyond the strict large-N limit and to perform precision tests of holography

Future directions

- Final form the supersymmetric on-shell action strongly suggests one may be able to derive it using topological arguments
  - non-renormalization theorem (only CS terms contribute)?
  - Equivariant localization seems the right tool for these purposes...

    Benetti Genolini, Gauntlett, Sparks; Martelli, Zaffaroni

- Several new lines of research in the asymptotically-flat case:
  - Attractor mechanism of complex saddles

    Boruch, Iliesiu, Murthy, Turiaci
  - Gravitational index of the heterotic string and small black holes?

    Chowdhury, Sen, Shanmugapriya, Virmani; Chen, Murthy, Turiaci
  - Match with α' corrections in string theory?
  - Match with microstate counting of the BMPV black hole?

    Maldacena, Moore, Strominger; Dabholkar, Gomes, Murthy, Sen; Murthy, Castro
  - ...

THANKS

Supersymmetric on-shell action

Impose **supersymmetry**, **keeping $\boldsymbol{\beta}$ finite**, and then evaluate the on-shell action:

$$E = \Omega_1^* J_1 + \Omega_2^* J_2 + \Phi^{*I} Q_I \qquad \Leftrightarrow \qquad \omega_1 + \omega_2 - \frac{3}{\sqrt{2}} g_I \varphi^I = \pm 2\pi i$$

$$\omega_i \equiv \beta \left(\Omega_i - \Omega_i^*\right) \qquad\qquad \varphi^I \equiv \beta(\Phi^I - \Phi^{*I})$$

I becomes a function of $\boldsymbol{\omega_i}$ and $\boldsymbol{\varphi^I}$, but not of β!

$$\begin{aligned} I &= \beta \left(E - \Omega_1^* J_1 - \Omega_2^* J_2 - \Phi^{*I} Q_I\right) - \mathcal{S} - \boldsymbol{\beta \left(\Omega_i - \Omega_i^*\right)} J_i - \boldsymbol{\beta(\Phi^I - \Phi^{*I})} Q_I \\ &= -\mathcal{S} - \boldsymbol{\omega_i} J_i - \boldsymbol{\varphi^I} Q_I \end{aligned}$$

$\rightarrow$ **expected to match the index in the Cardy-like limit**