COSMOLOGY OF THE QCD AXION

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The (Minimal) QCD Axion

$$\mathscr{L}_{\text{SM}} \supset \theta_{\text{strong}} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Why CP-violation in QCD is tiny $(\bar{\theta}_{\text{strong}} \ll 1)$?

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- Why CP-violation in QCD is tiny ($\bar{\theta}_{\rm strong} \ll 1$)?
- QCD Axion solution: promote $\theta_{\rm strong}$ to a dynamical field $\rightarrow \dfrac{a}{f_a}$
- *Axion* potential minimized at $a = \bar{\theta}_{\rm strong} = 0$ (CP conserving)

Chiral rotations

$$\mathscr{L}_{\rm SM} \supset \theta_{\rm strong} \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} - (\bar{q}_L M_q e^{i\theta_q} q_R + h.c.)$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

- We could rotate away the phase: $q_L' = q_L e^{-i\theta_q/2}$, $q_R' = q_R e^{i\theta_q/2}$

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- Redefinition $\Longrightarrow$ $\bar{\theta}_{strong} = \theta_{strong} + 2\theta_q$
- $\bar{\theta}$ is the physically invariant quantity
- Neutron electric dipole (NEDM) constraint: $\bar{\theta}_{strong} \leq 7 \times 10^{-12}$

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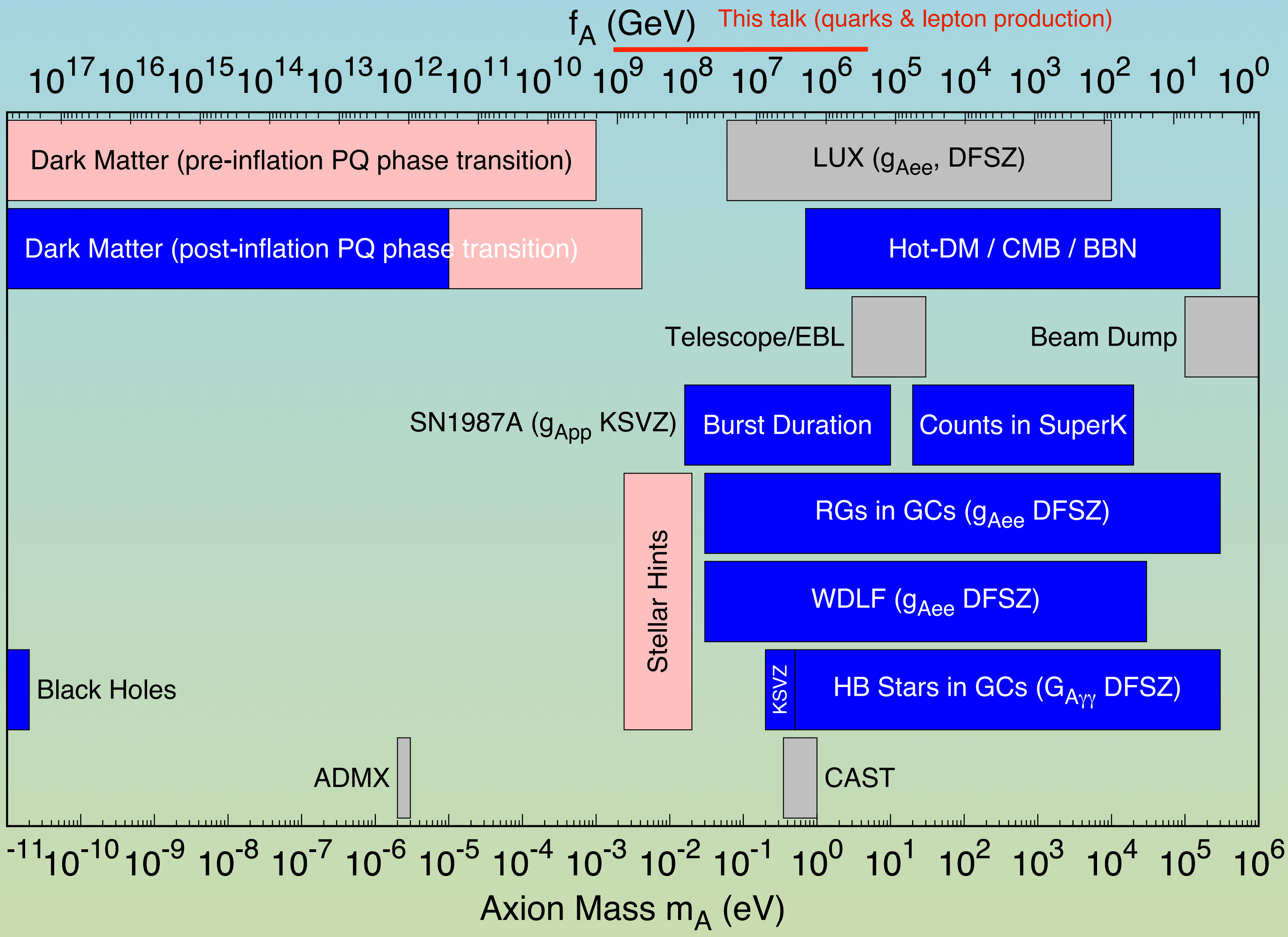
- Light scalar particle, $m_a \approx \Lambda_{QCD}^2 / f_a \approx 0.57 eV \left(\frac{10^7 GeV}{f_a}\right)$
- Large f_a required *(Cosmology, Supernovae, star cooling…)*

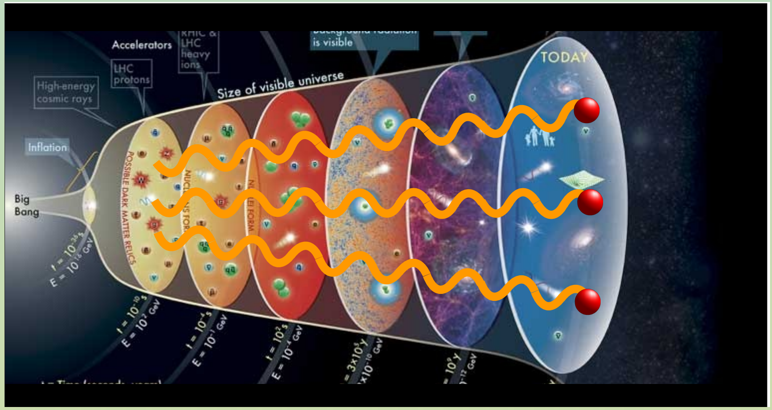


f_A (GeV)
This talk (quarks & lepton production)
10^17 10^16 10^15 10^14 10^13 10^12 10^11 10^10 10^9 10^8 10^7 10^6 10^5 10^4 10^3 10^2 10^1 10^0
Dark Matter (pre-inflation PQ phase transition)
LUX (g_Aee, DFSZ)
Dark Matter (post-inflation PQ phase transition)
Hot-DM / CMB / BBN
Telescope/EBL
Beam Dump
SN1987A (g_App KSVZ)
Burst Duration
Counts in SuperK
Stellar Hints
RGs in GCs (g_Aee DFSZ)
WDLF (g_Aee DFSZ)
Black Holes
KSVZ
HB Stars in GCs (G_Aγγ DFSZ)
ADMX
CAST
10^-11 10^-10 10^-9 10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 10^0 10^1 10^2 10^3 10^4 10^5 10^6
Axion Mass m_A (eV)

Axion Cosmology

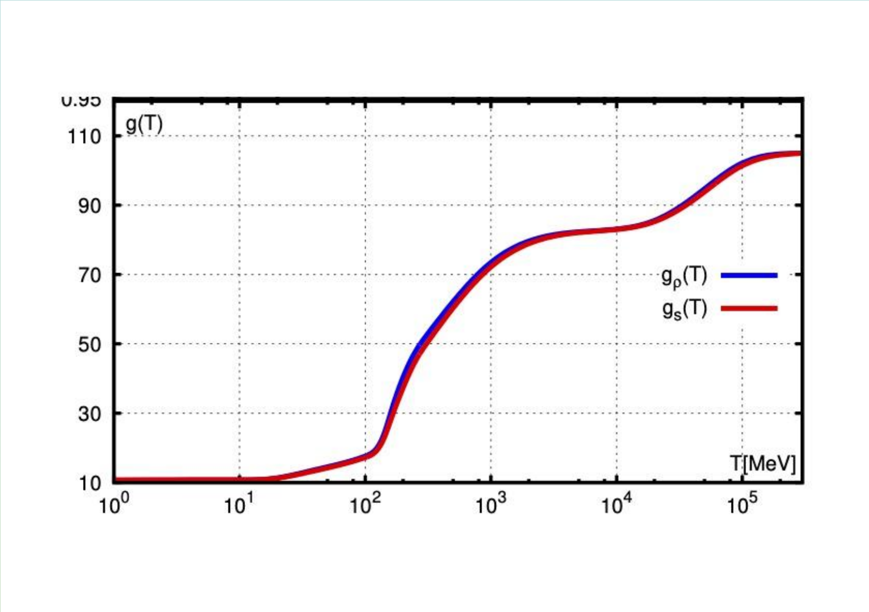
$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \dots$$

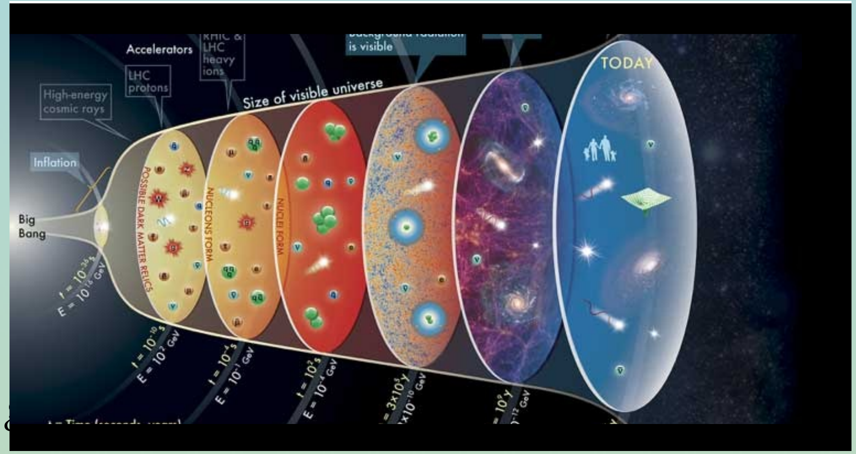
- Dynamical explanation of $\theta_{\mathrm{strong}} \ll 1$
- Light scalar particle, $m_a \approx \Lambda_{QCD}^2/f_a \approx 0.57eV\left(\frac{10^7 GeV}{f_a}\right)$
- Two populations of cosmological relic axions:
  - "Cold axions" candidate for Cold Dark matter
  - "Thermal axions" (``Hot-DM"): relativistic at production, Become non-relativistic later small part of dark matter (like relic neutrinos)



Relic light particles in Cosmology

- Primordial plasma, g_* degrees of freedom and temperature T





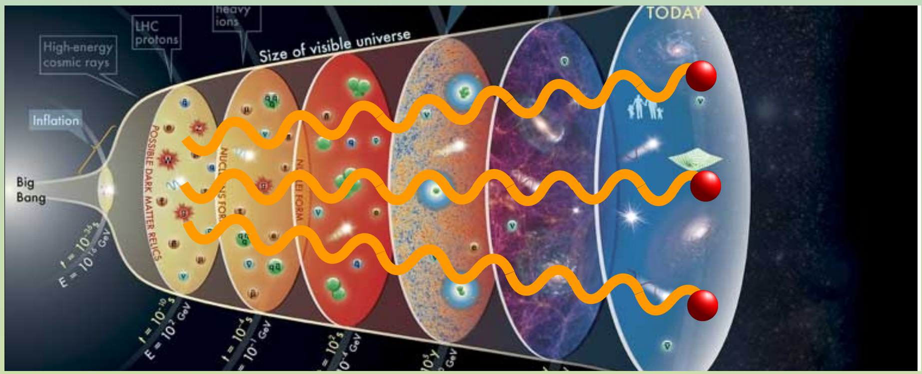
Total plasma energy density: $\rho_{\rm TOT} \propto g_* T^4$

$$g_* \equiv \sum_{i=\text{RELATIVISTIC BOSONS}} g_i + \frac{7}{8} \sum_{i=\text{RELATIVISTIC FERMIONS}} g_i$$

- Conservation of entropy: $g_*^{1/3} T \propto 1/a$
- When a species becomes non-relativistic (e.g. $e^+ - e^-$ at $T \ll m_e$) g_* decreases
  T slightly “increases” (plasma gets slightly “heated”)

Relic light particles in Cosmology

- Light particles with small interaction ("thermalization rate" $\Gamma = n \cdot \langle \sigma v \rangle$), (e.g. neutrinos, axions)



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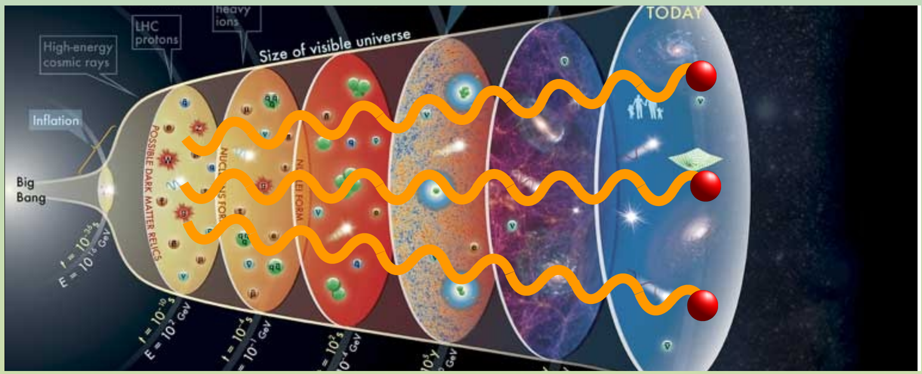
- Compare with Hubble rate ($H \equiv \dot{a}/a$): $\Gamma \gg H \implies$ equilibrium

- If Particle Decouples ($\Gamma \ll H$) below some Temperature T_{DEC}, its distribution freezes at its "own temperature" and freely evolves, $\rho_P \propto T_P^4$, with $T_P = T_{\mathrm{DEC}}/a$



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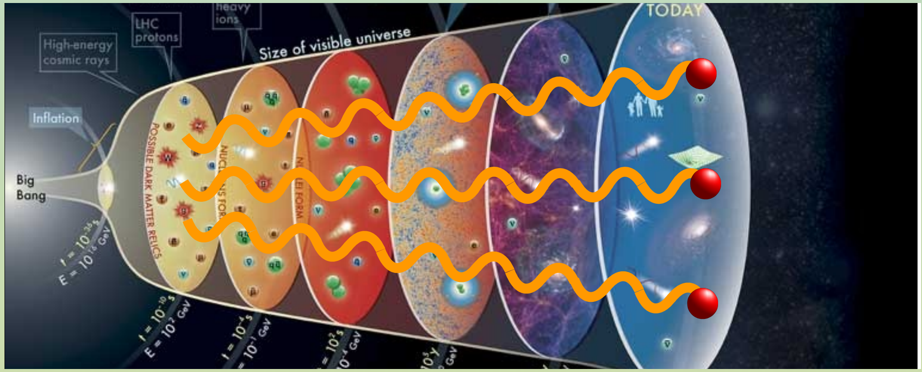
- Compared to plasma (photons) it does NOT get heated after decoupling of other particles

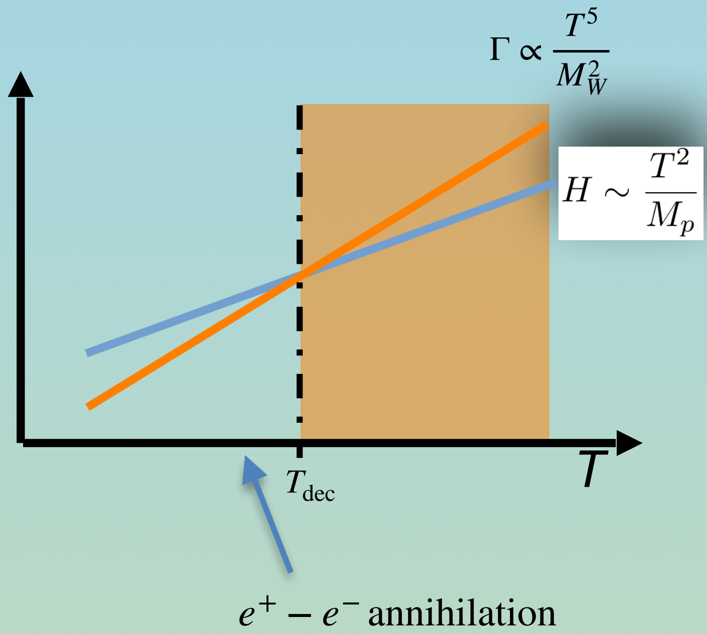
$$\rho_P/\rho_\gamma \propto T_P^4/T^4 \propto 1/g_{*DEC}^{4/3}$$

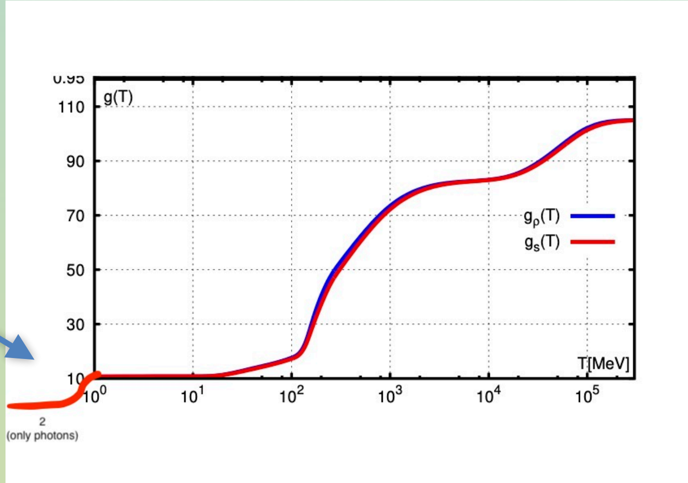


Example: Relic Neutrinos

- Neutrinos decouple at $T \approx MeV$, not heated by $e^+ - e^-$ annihilation

$$\frac{\rho_\nu}{\rho_\gamma} \propto \frac{1}{g_{*,DEC}^{4/3}} = \left(\frac{4}{11}\right)^{4/3}, \qquad T_\nu \approx 0.7\, T_\gamma \approx 1.96\,\mathrm{K}$$





Example: Cosmic Neutrino Background

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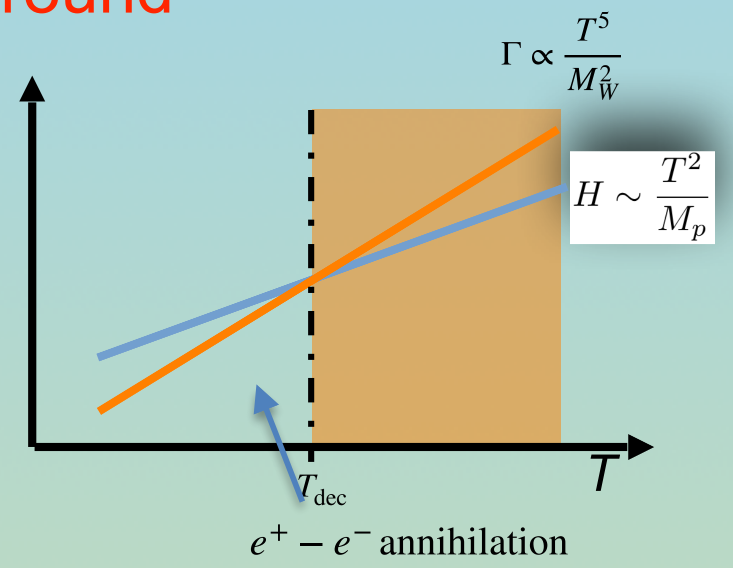


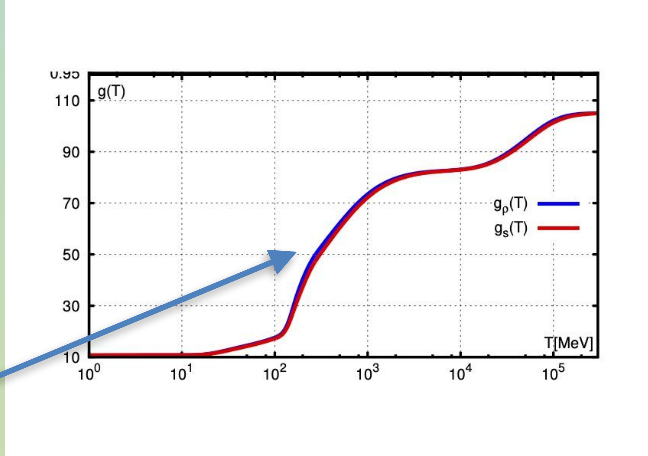
- Any light particle (axions,…) can do the same.

- Traditional parameterization as "extra neutrinos species":

$$\Delta N_{\rm eff} \equiv \left(\frac{8}{7}\right)\left(\frac{11}{4}\right)^{4/3} \frac{\rho_P}{\rho_\gamma}\Big|_{\rm CMB}$$

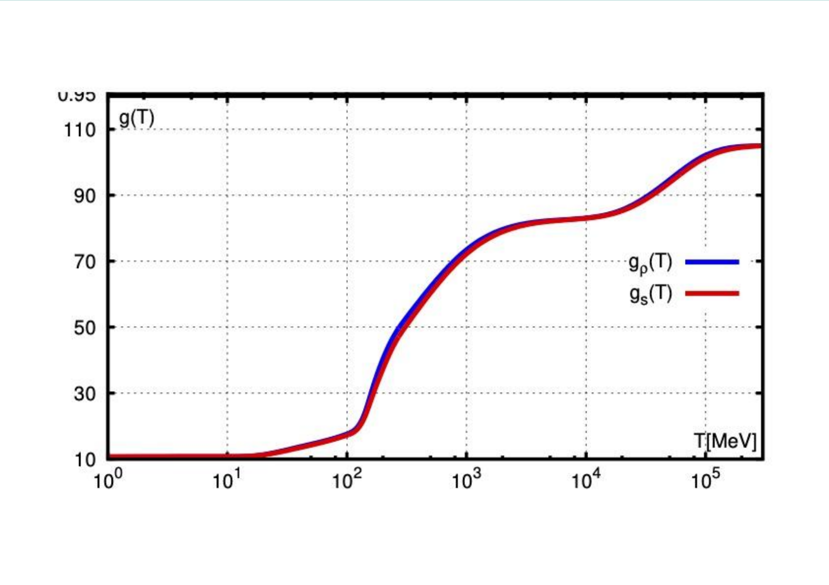
- Relic abundance suppressed as:

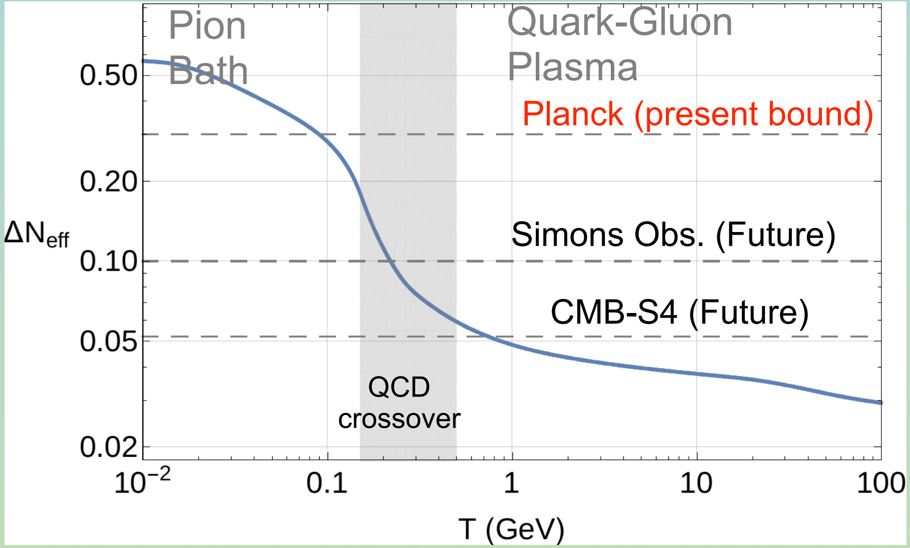
$$\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}\Big|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}$$



Example: Relic Scalars

- Relic abundance $\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}$





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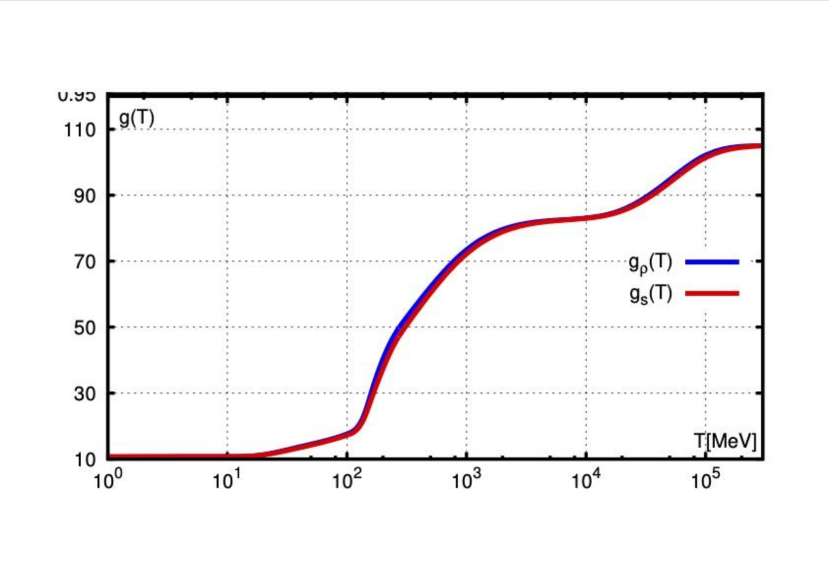

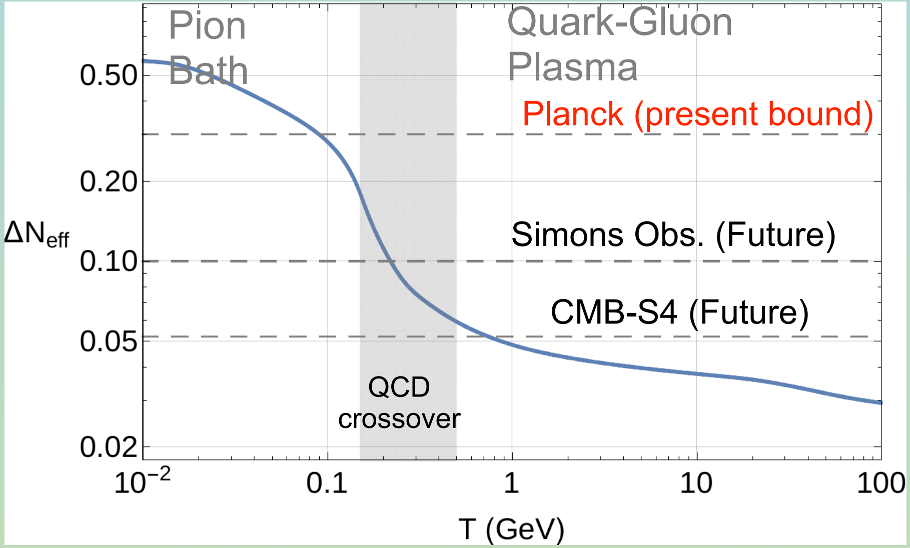

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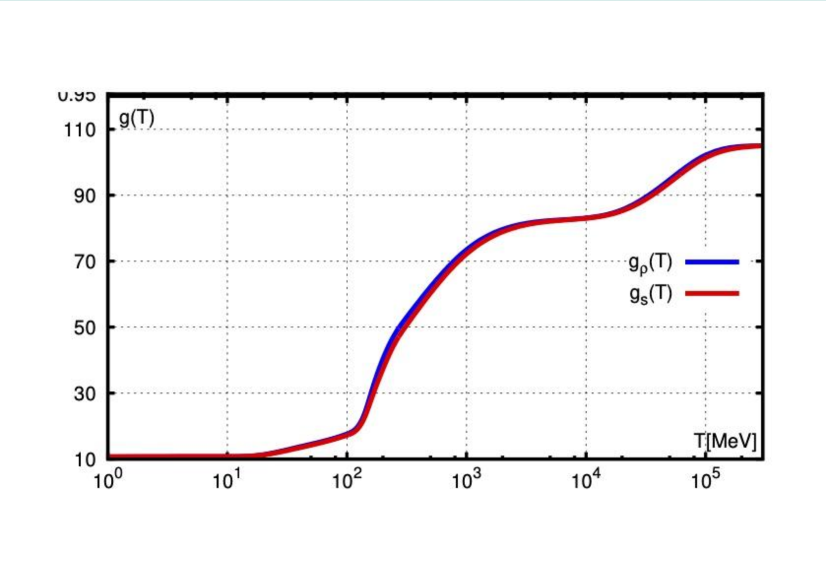

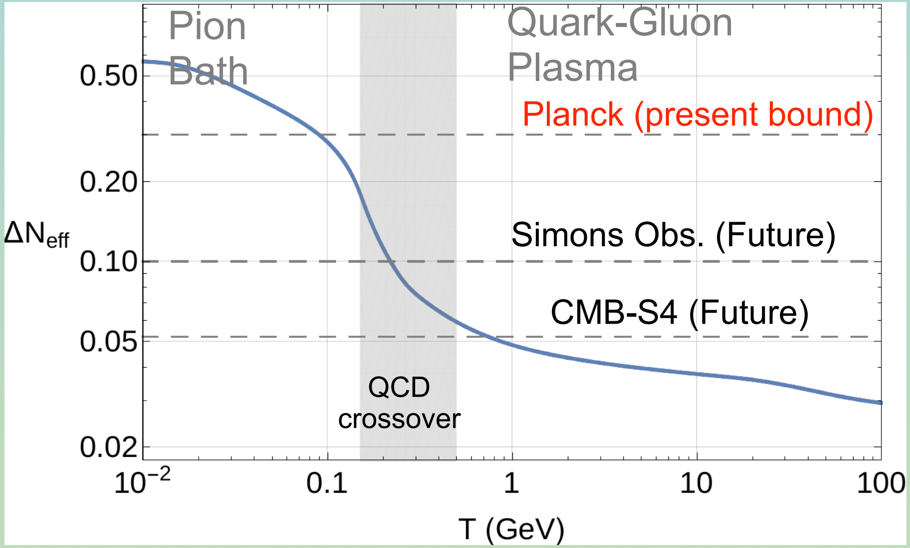

- Main effect: Extra "radiation" at CMB time ($T \approx 0.1eV$) ⇨ $\Delta N_{\rm eff}$ affects CMB spectra
- If massive ($m \lesssim 0.1\text{eV}$) becomes non-relativistic after CMB time ⇨ adds to Dark Matter and affects its fluctuations (same as neutrino mass!)

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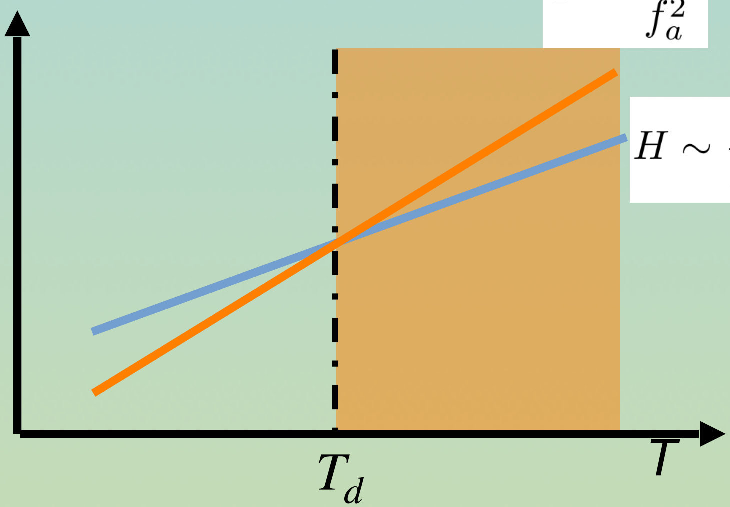
Axion-Gluon scatterings ($T \gtrsim T_{QCD}$)

$$\Gamma \sim \frac{T^3}{f_a^2}$$



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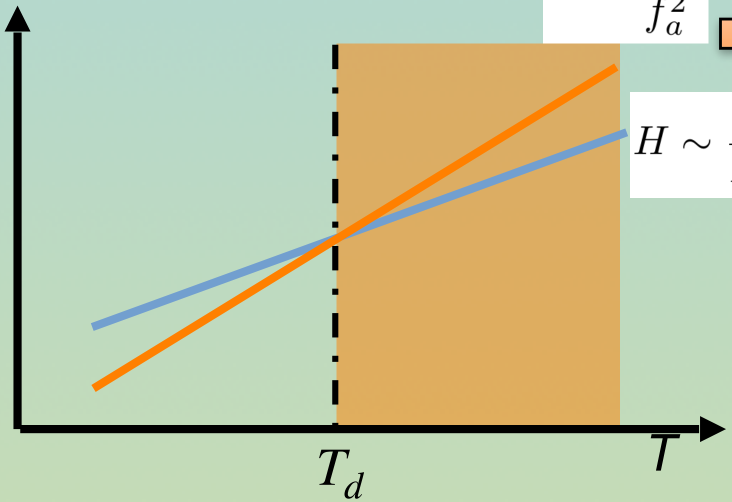

$$H \sim \frac{T^2}{M_p}$$

$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8 \text{ GeV}}\right)^2$$

Cosmic Axion Background

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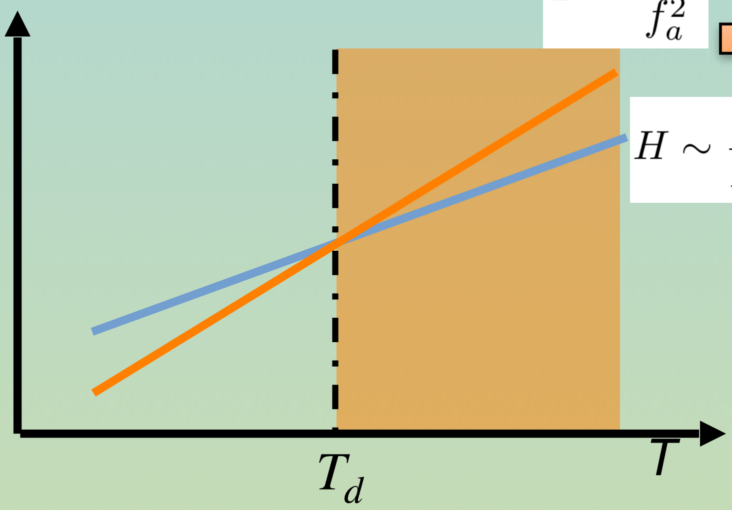

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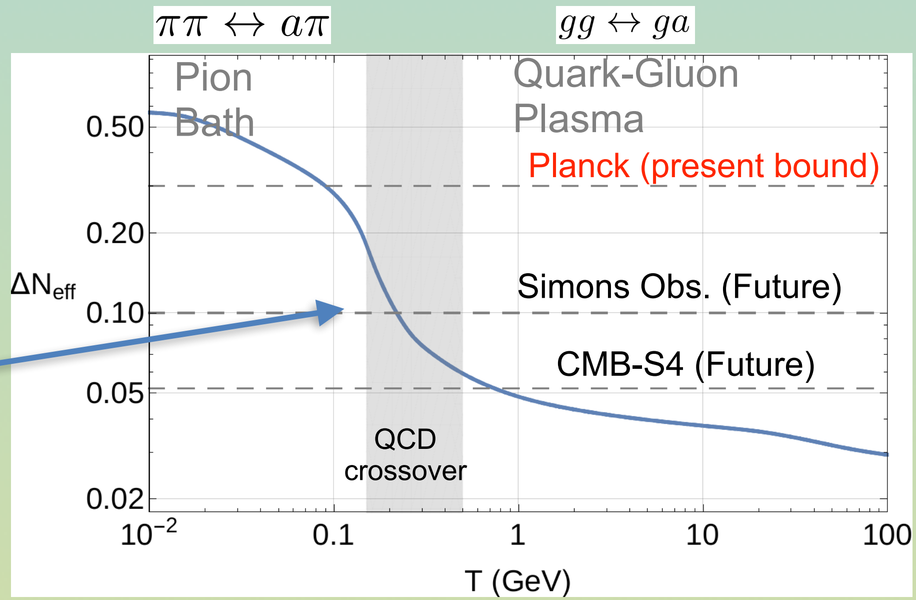
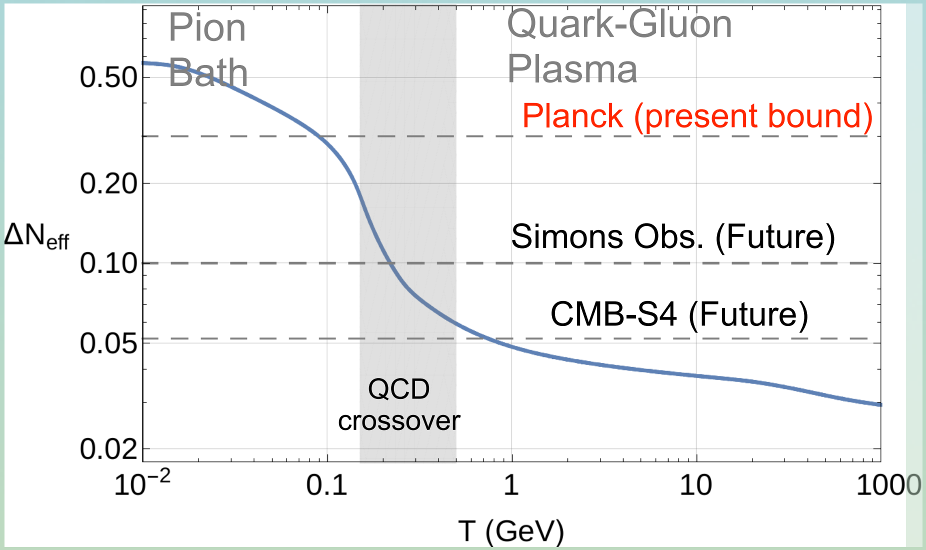


$$\Delta N_{\text{eff}} \propto \frac{\rho_P}{\rho_\gamma}|_{\text{CMB}} \propto \frac{1}{g_{*,DEC}^{4/3}}$$

Axion ΔN_{eff} has a long history:

$\pi\pi \leftrightarrow a\pi$ $\quad$ $gg \leftrightarrow ga$



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"Standard" treatments:

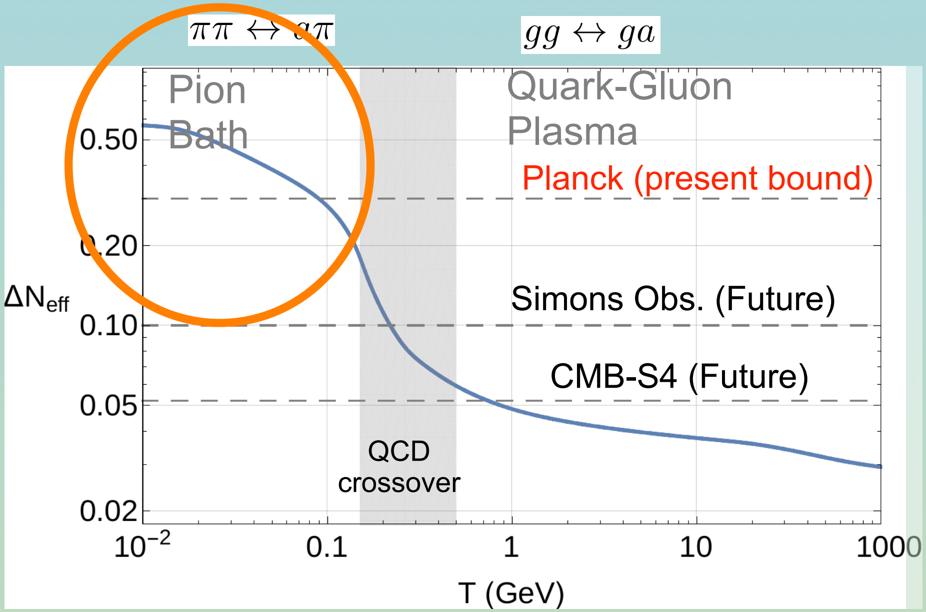
1. Instantaneous decoupling ($\Gamma = H$)
2. Single Boltzmann Eq.for abundance Y.

$$\frac{dY}{d\log x} = (Y^{\text{eq}} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right) \qquad (x \equiv m/T)$$

Axion $\Delta N_{\rm eff}$ has a long history:

Our work: Improving present bounds from pion scatterings

(A.N., Rompineve, Villadoro, PRL '23)



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Momentum-dependent Boltzmann Equation and Thermalization Rate Γ

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

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(Detailed balance, plasma particles in equilibrium)

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Perturbatively, due to scatterings with pions: $\pi\pi \leftrightarrow a\pi$

$$\Gamma^{<} = \frac{1}{2E}\int \left(\prod_{i=1}^{3} \frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i}\right) f_1^{\rm eq} f_2^{\rm eq}(1 + f_3^{\rm eq})(2\pi)^4 \delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu)|\mathcal{M}|^2_{2\leftrightarrow 2}$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO chiral perturbation theory rate
(Chang Choi '93)

(Used in all previous cosmological bounds)

$$|\mathcal{M}^{\mathrm{LO}}|^2 = \theta_{a\pi}^2 \frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4}$$

$$\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d}\frac{f_\pi}{2f_a}$$

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→ breaks down at $T \gtrsim 60$ MeV !

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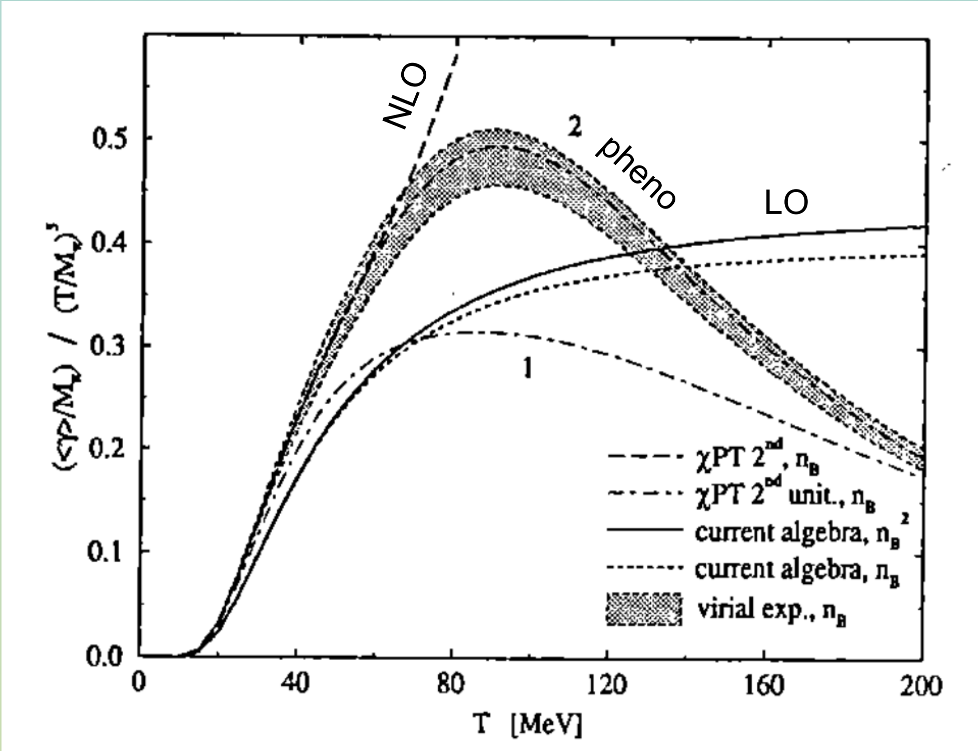

Schenk '94

1. Axion-pion mixing

General form of low energy axion QCD Lagrangian
(non-derivative axion coupling rotated in the mass matrix)

$$\mathcal{L} = \bar{q} \left(i\partial\!\!\!/ + \frac{c_0}{2f_a} \partial\!\!\!/ a \, \gamma_5 \right) q - \bar{q}_L M_a q_R + h.c. \, , \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i \frac{a}{2f_a}}$$

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Below QCD scale (chiral perturbation theory)

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + 2B_0(M_a U^\dagger + U M_a^\dagger)\right] + \ldots \qquad U \equiv \exp(i\vec{\pi}\cdot\vec{\sigma}/f_\pi)\,,$$

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$$V(\pi_0, a) = -B_0 f_\pi \left[m_u \cos\left(\frac{a}{2f_a} - \frac{\pi_0}{f_\pi}\right) + m_d \cos\left(\frac{a}{2f_a} + \frac{\pi_0}{f_\pi}\right)\right]$$

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$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right)q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}}$$

Below QCD scale (chiral perturbation theory)

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + 2B_0(M_a U^\dagger + U M_a^\dagger)\right] + \ldots \qquad U \equiv \exp(i\vec{\pi}\cdot\vec{\sigma}/f_\pi)\,,$$

$$V(\pi_0, a) = -B_0 f_\pi\left(m_u \cos\left(\frac{a}{2f_a} - \frac{\pi_0}{f_\pi}\right) + m_d \cos\left(\frac{a}{2f_a} + \frac{\pi_0}{f_\pi}\right)\right)$$

Diagonalization of the quadratic V

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\rm phys} + \sin(\theta_{a\pi})\,a_{\rm phys} \simeq \pi^0_{\rm phys} + \theta_{a\pi}a_{\rm phys}$$

(Axion-pion mixing)

$$\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d}\frac{f_\pi}{2f_a}$$

1. The Thermalization Rate Γ

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$$\frac{\partial_\mu a}{2f_a} j^\mu_A \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

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@ all orders in χPT

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e.g. @ LO

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2\frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4}$$

$$|\mathcal{M}_{\pi\pi}^{\rm LO}|^2 = \frac{s^2+t^2+u^2-4m_\pi^4}{f_\pi^4}$$

$\lesssim 10\%$

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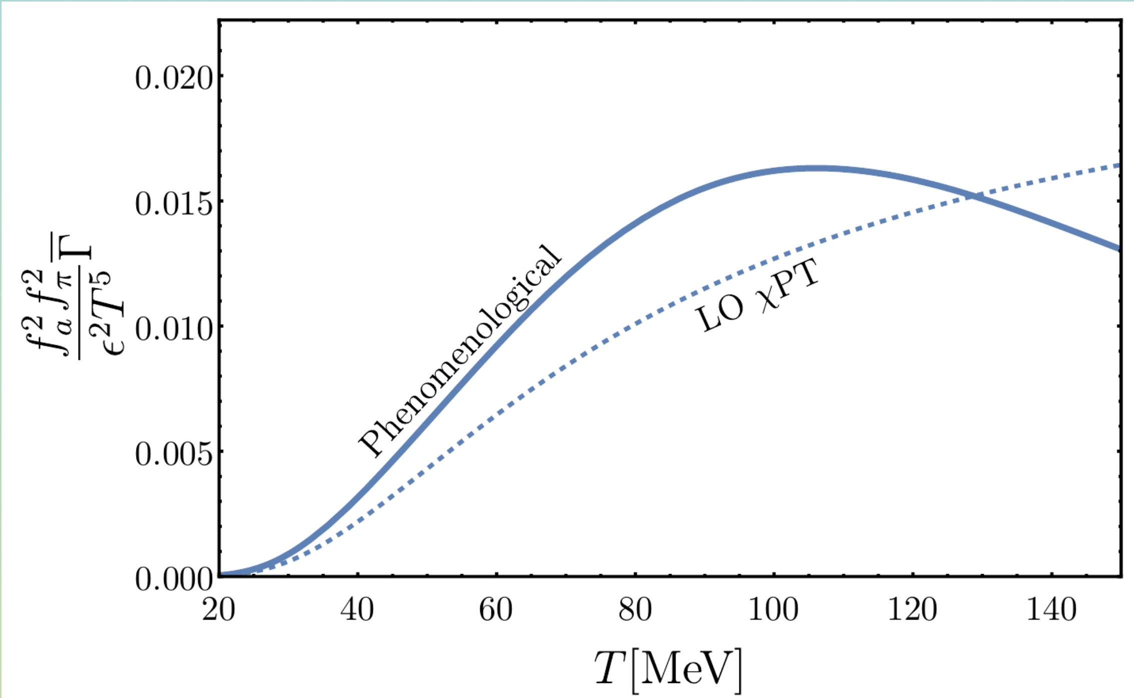
$\lesssim 10\%$

- At 1-loop we explicitly checked that leading order in $\mathcal{O}(m_\pi^2/s)$ $\pi\pi \leftrightarrow a\pi$ (Di Luzio, Martinelli, Piazza '21) reproduced from $\pi\pi \leftrightarrow \pi\pi$

1. The Axion Thermalization Rate Γ (from pions): our result



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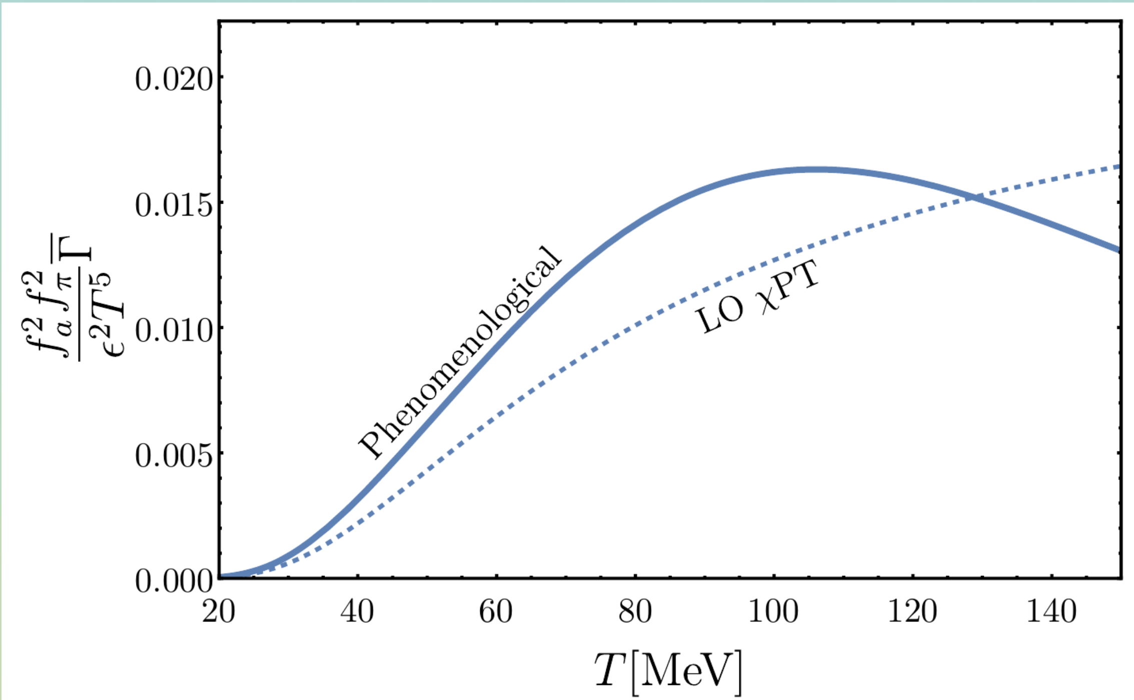

In reasonable agreement with:
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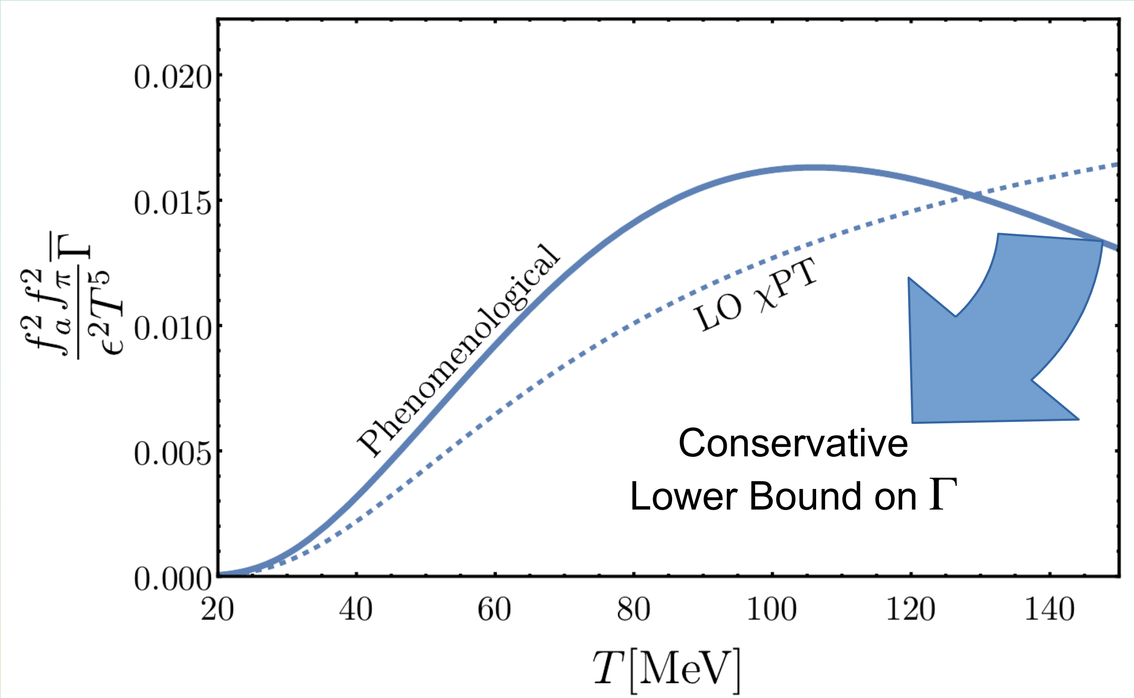

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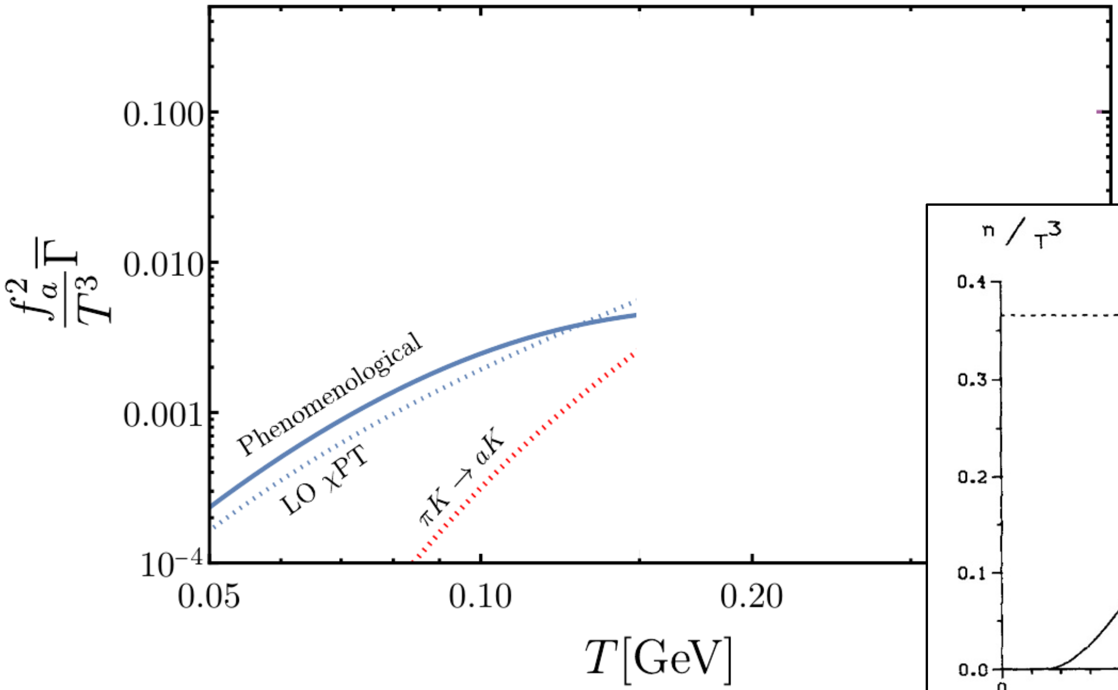
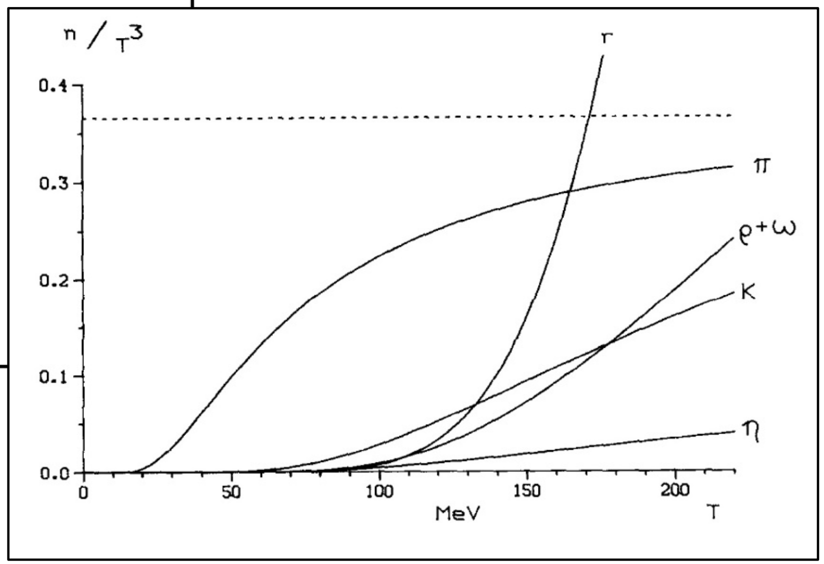
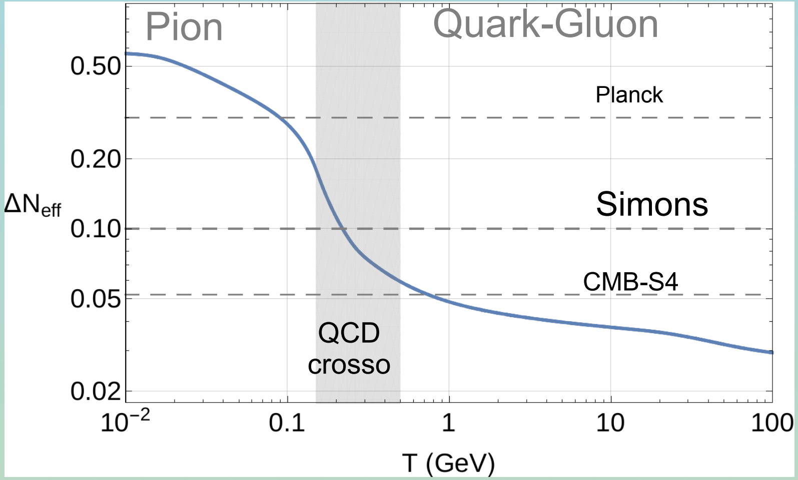
1. The Thermalization Rate Γ
(Possible other channels: Kaons,…)





Gerber Leutwyler '89

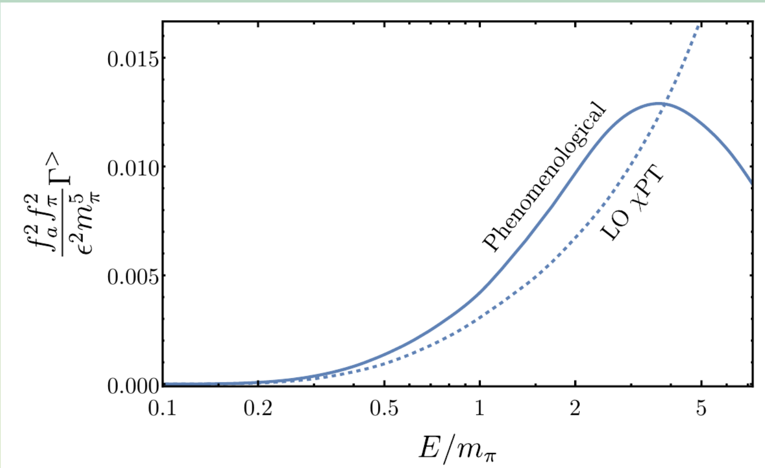
2. Momentum Dependence



Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

High momenta k decouple later than low k



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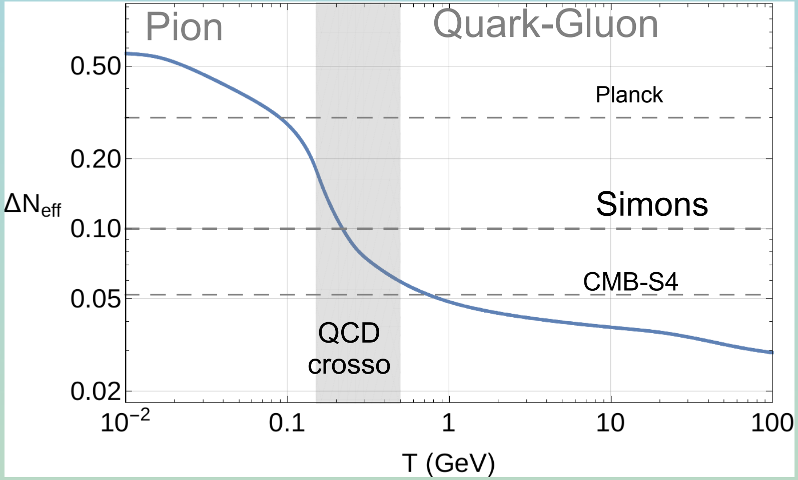

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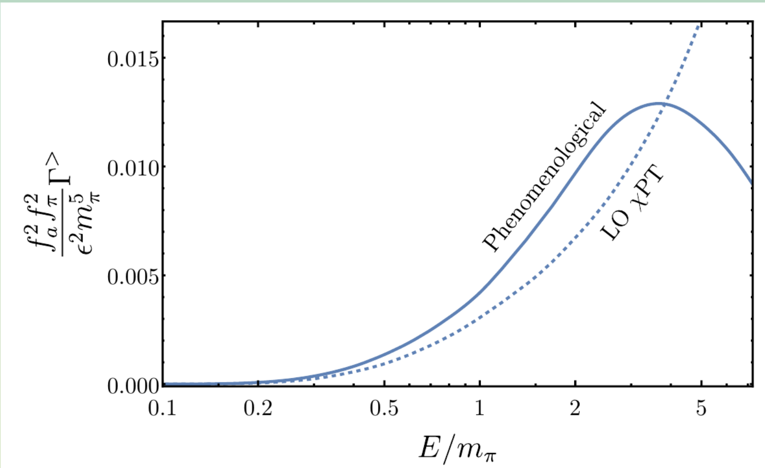
They see a lower g_* → More abundant



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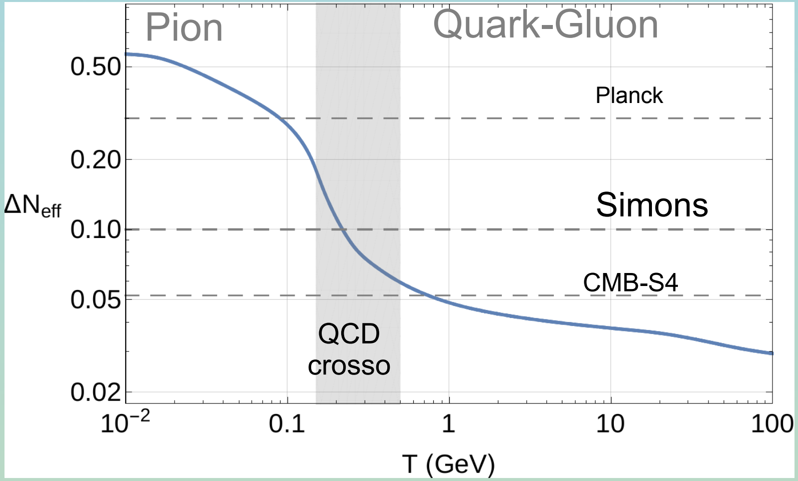

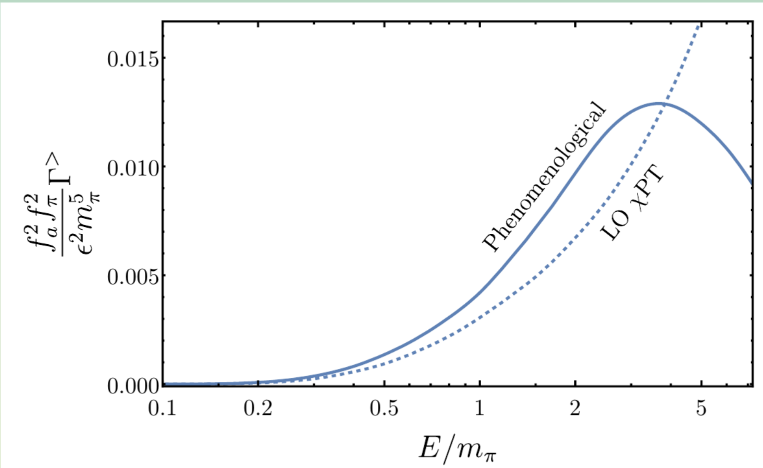

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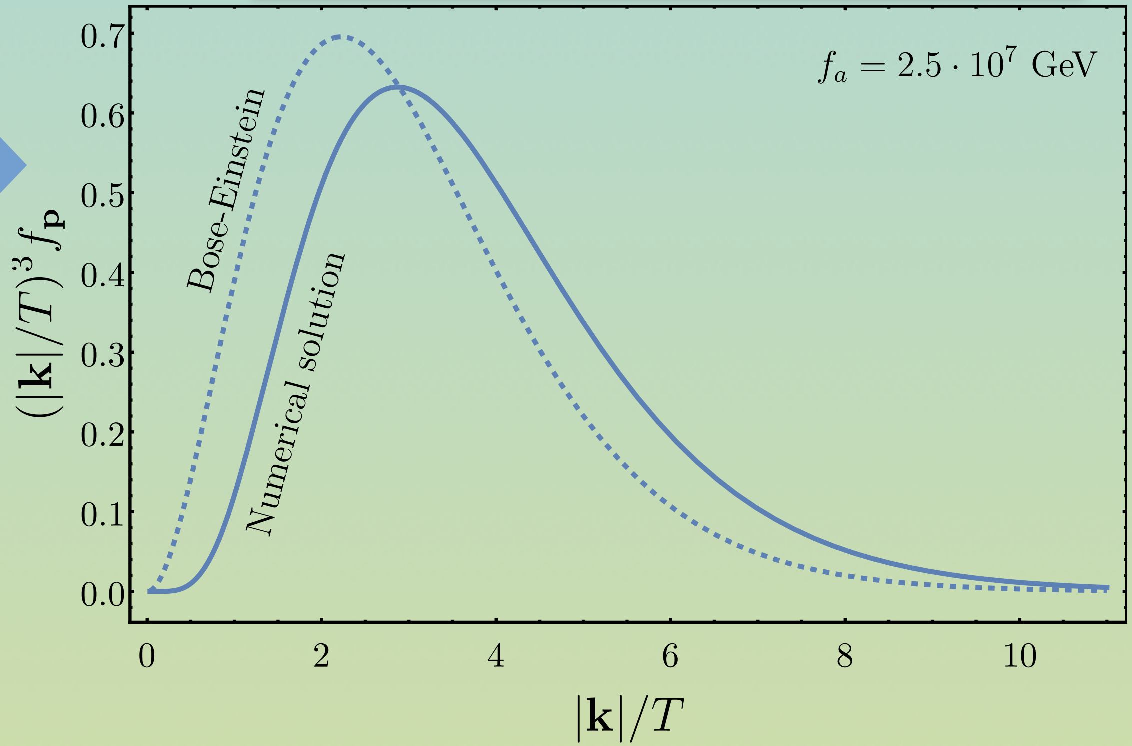
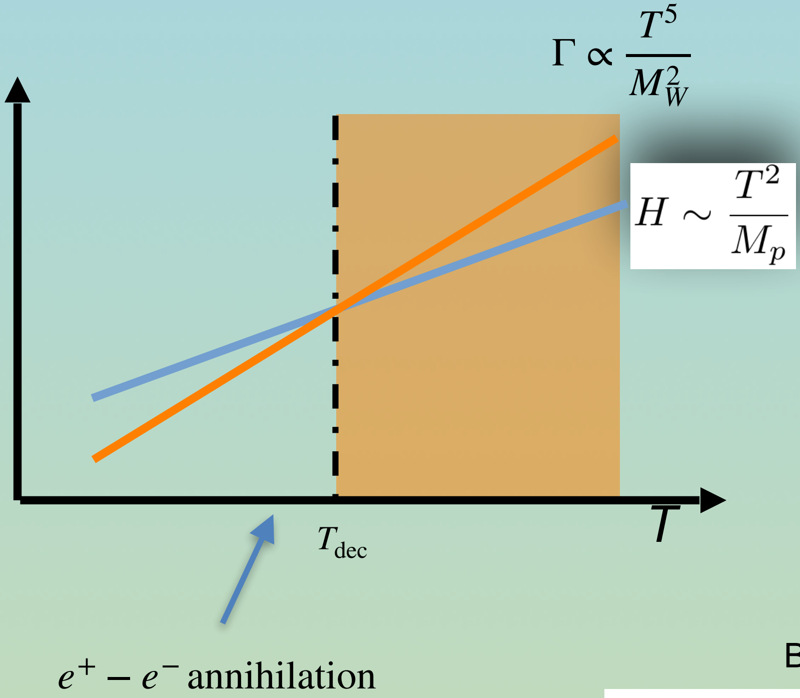
~ 40% enhanced total abundance



2. Momentum Dependence: Neutrinos



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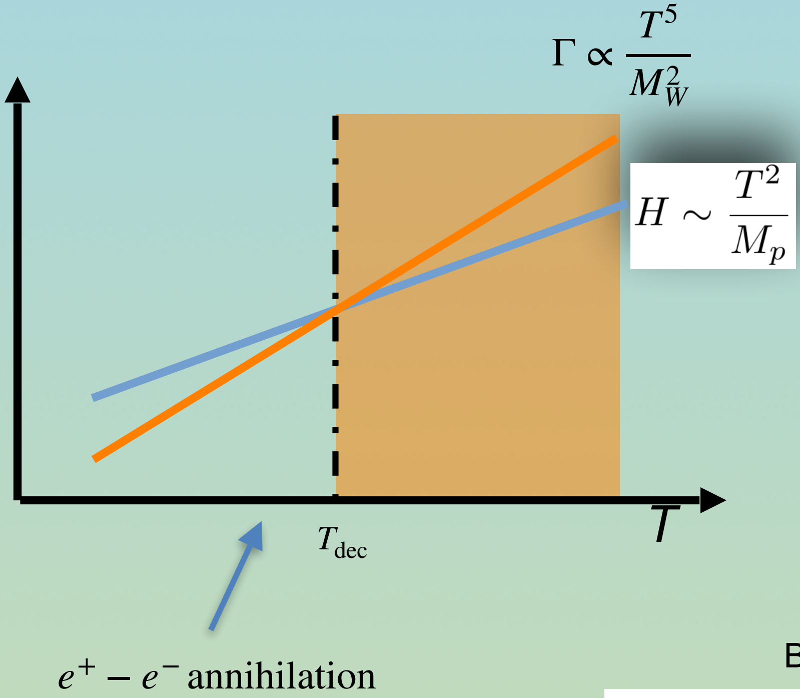

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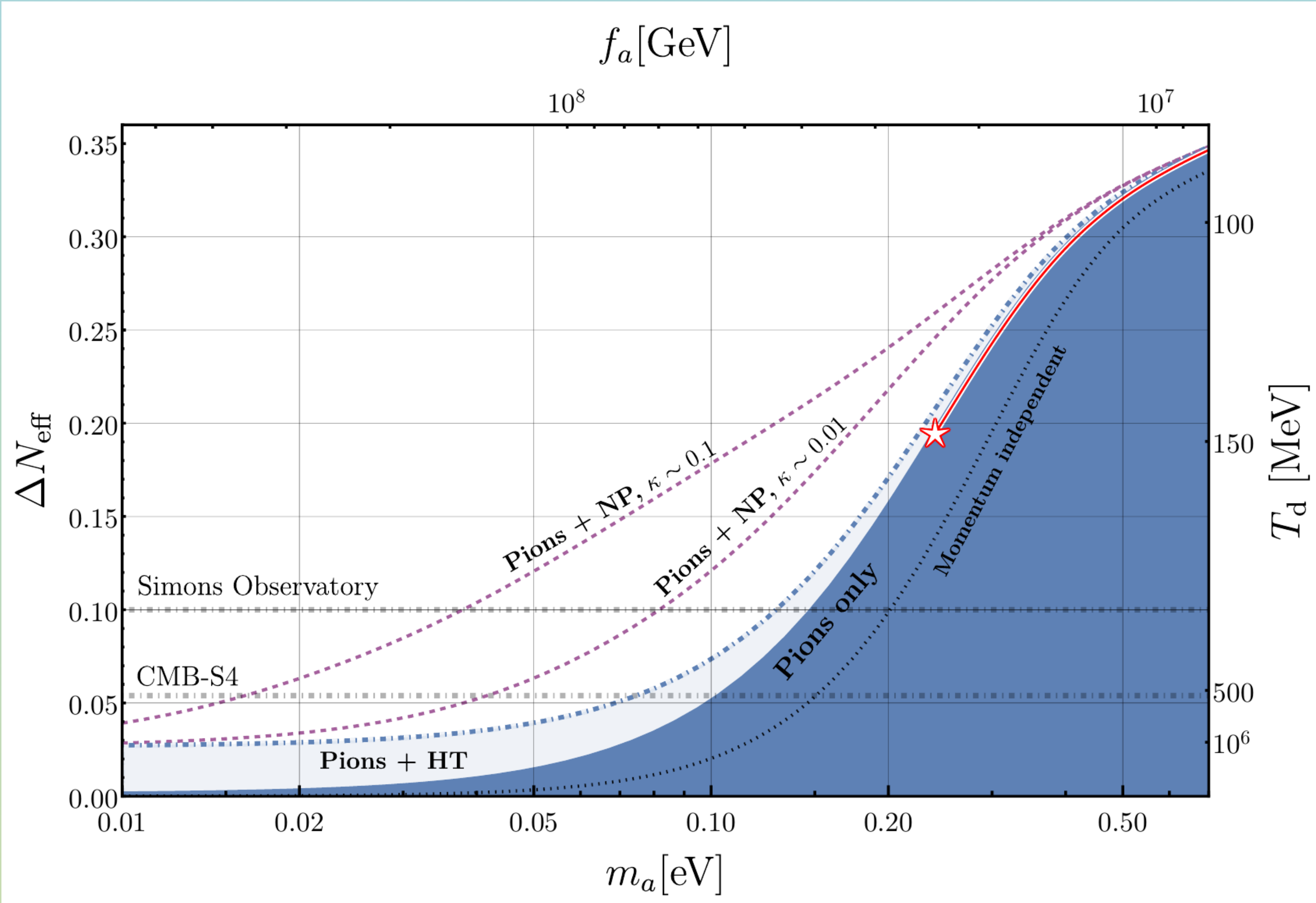
But m_e and $T_{\rm dec}$ are more separated → Only ~ 1% enhancement

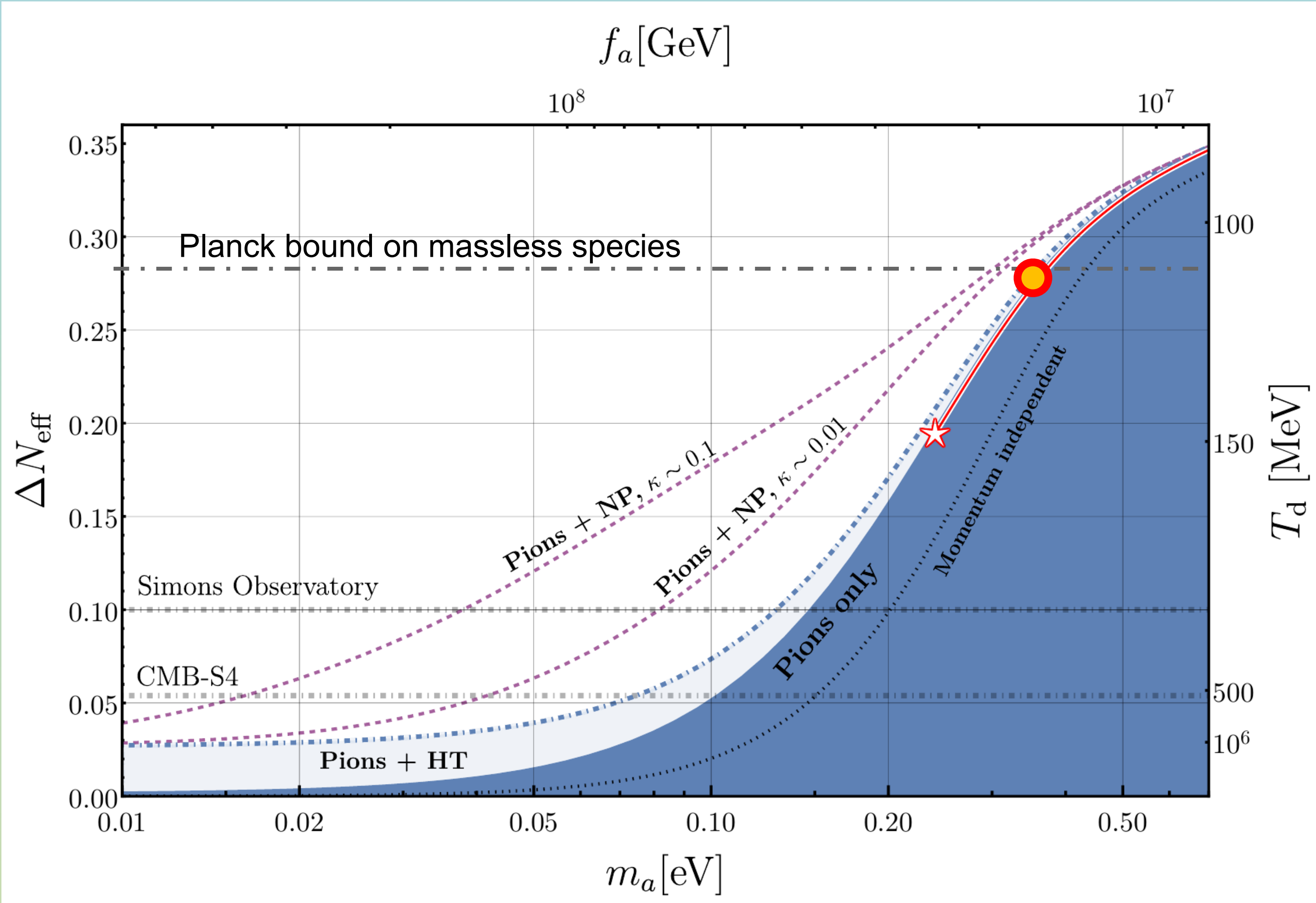
$N_{\rm eff} \approx 3.044$

Present bound+Future Reach



Planck18+BAO+Pantheon

Present bound+Future Reach



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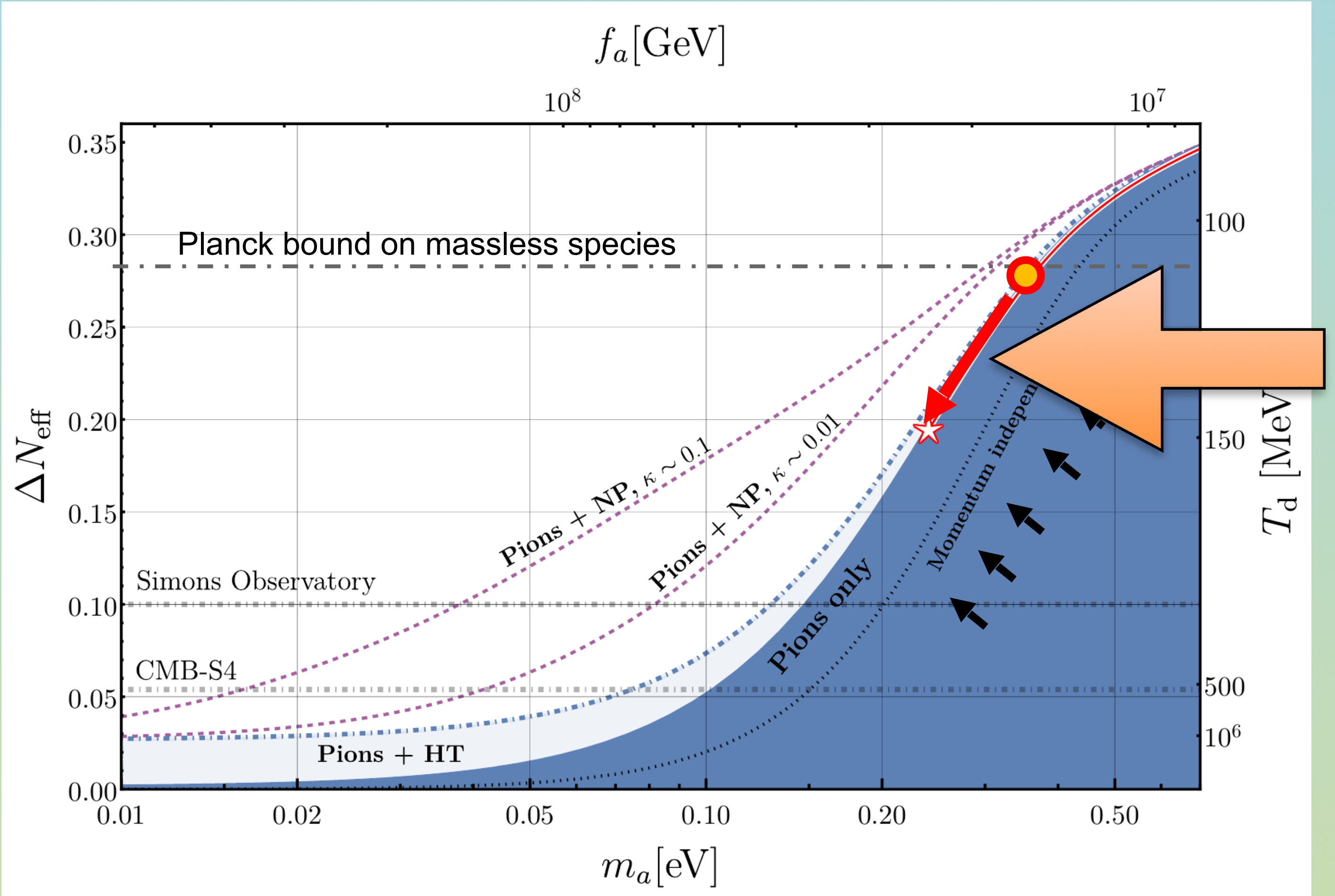
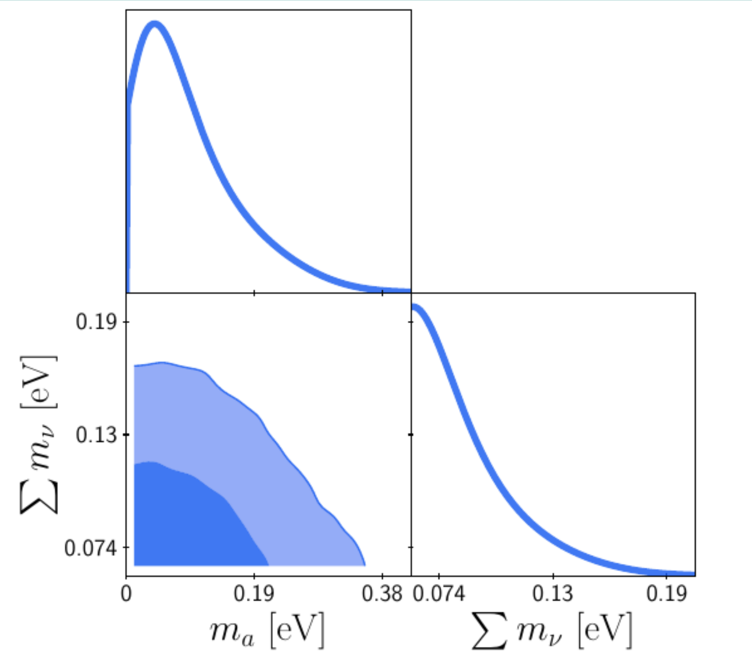


Effect of massive relic

(Free streaming, suppresses Matter Power Spectrum on small scales, like neutrinos)

Planck18+BAO+Pantheon

3. Combined cosmological Fit
(Λ_{CDM} + massive neutrinos + *axions*)



$$m_a \leq 0.24 \text{ eV}$$

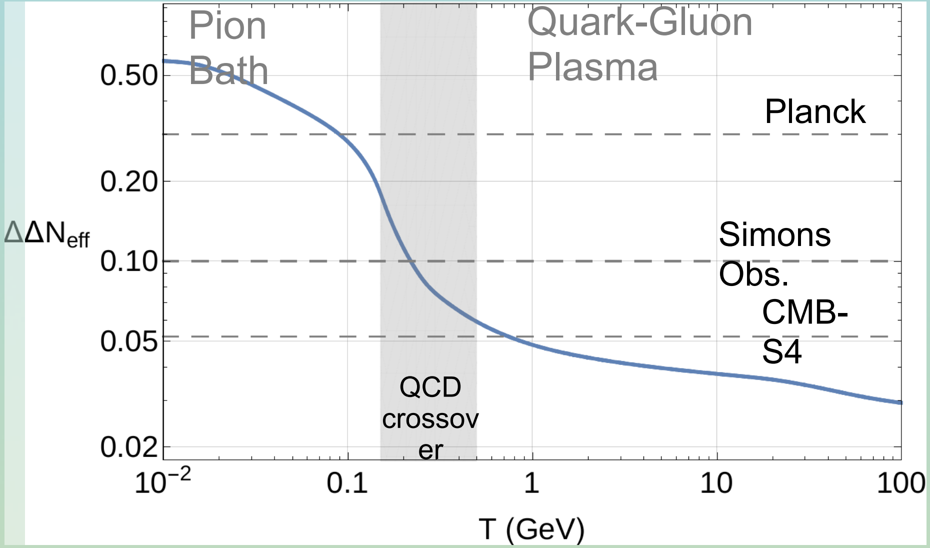
$$f_a \geqslant 2.4 \cdot 10^7 \text{ GeV}$$

$\Leftrightarrow$

$$\Delta N_{\text{eff}} \lesssim 0.19$$

- Assuming 3 neutrinos with unknown total mass $\sum m_\nu$

Future Reach



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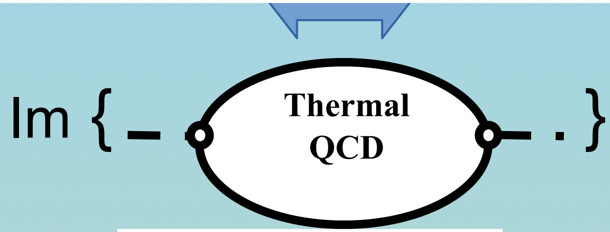

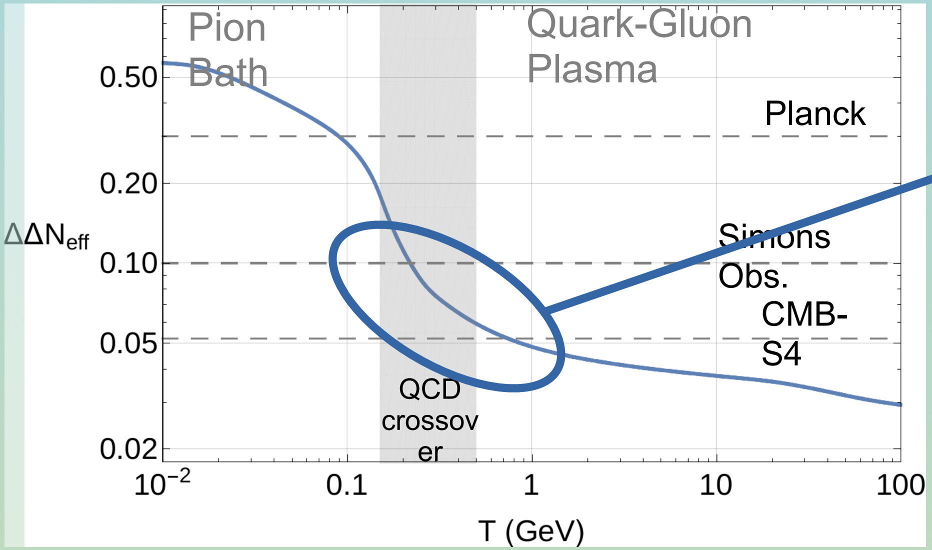

Non-Perturbative

$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\rm top}}{2Ef_a^2}$$

$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^{\mu}x_{\mu}} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^{\mu})\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$

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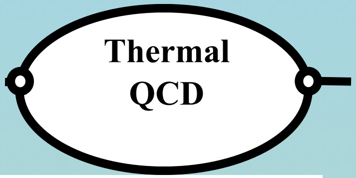

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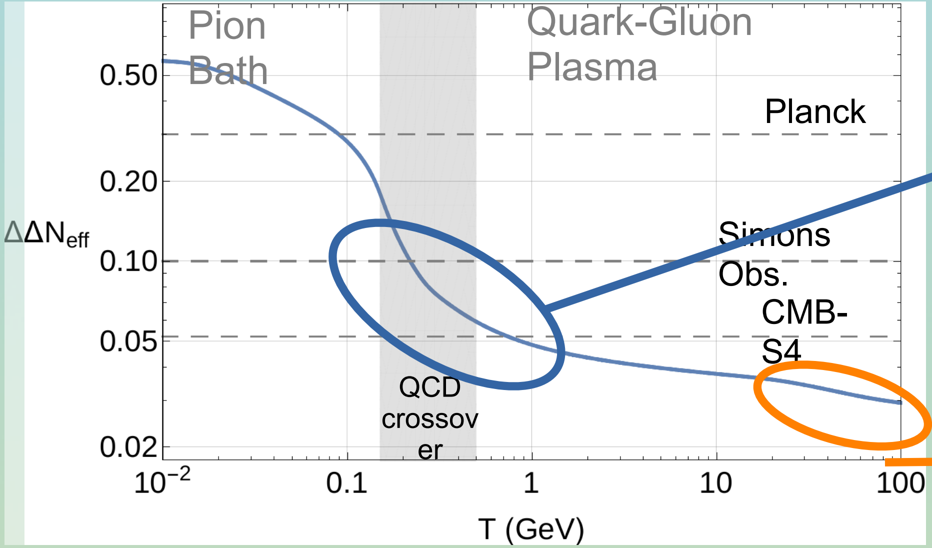


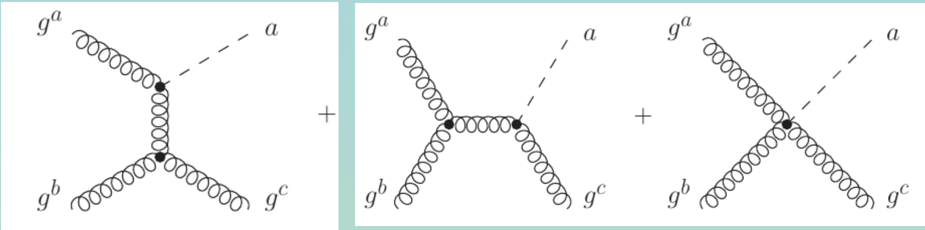
Non-Perturbative

$gg \leftrightarrow ga$

Perturbative ?

$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

High Temperatures Regime



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Masso, Rota, Zsembinszki ‘02
Graf, Steffen ‘10

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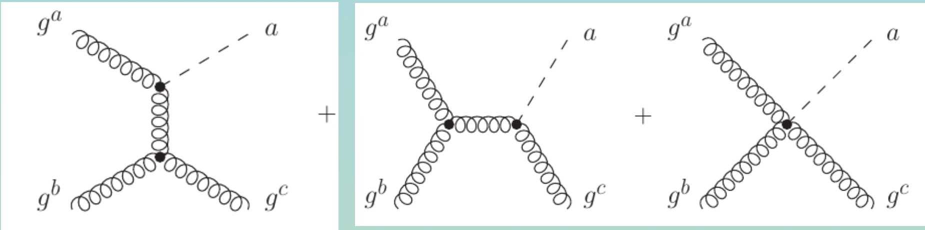

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$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

for $g_s \ll 1$

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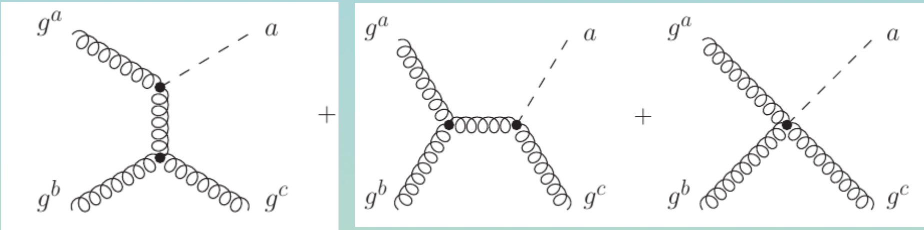

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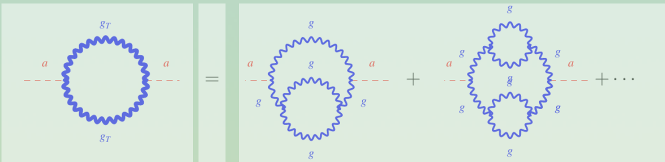
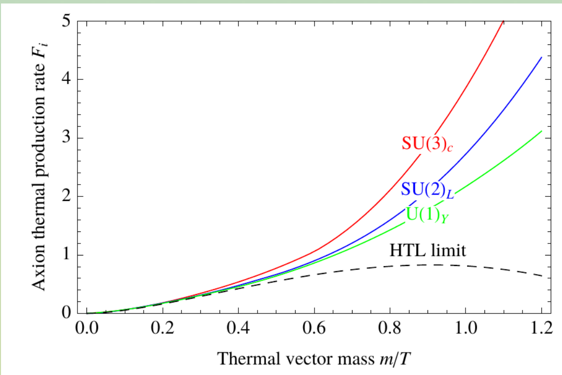
Unphysical negative F_3 cured by
Salvio, Strumia, Xue '13





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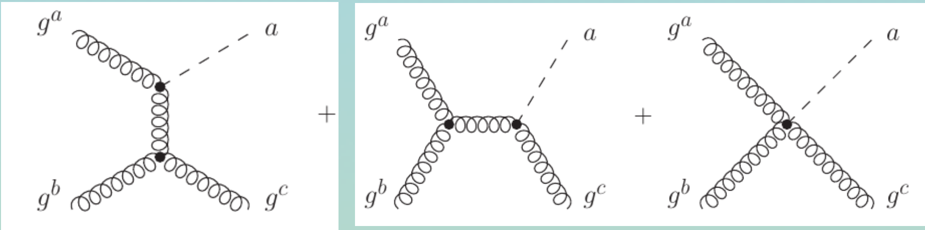

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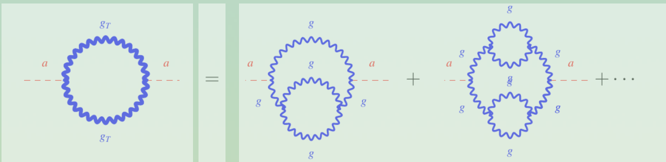
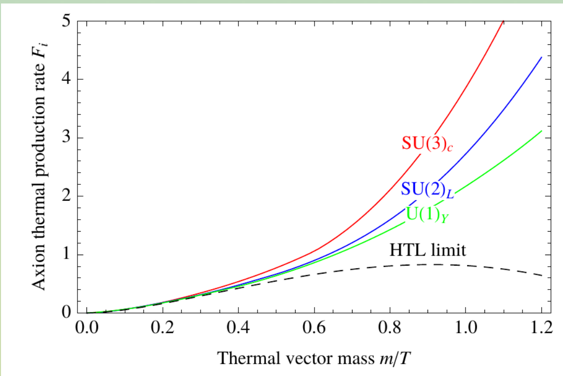
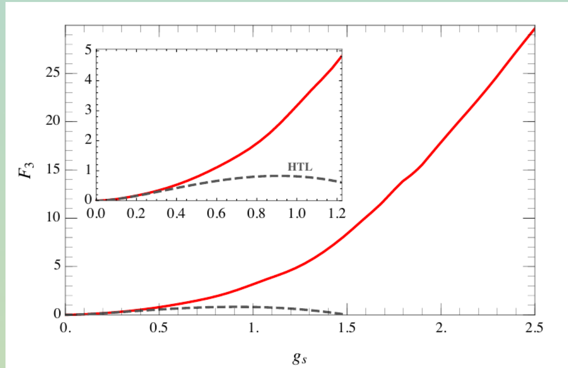




Recently D’Eramo, Hajkarim, Yun (‘21):
extrapolated F_3 from Salvio et al. to $g_s > 1$
(Beyond regime of validity?)



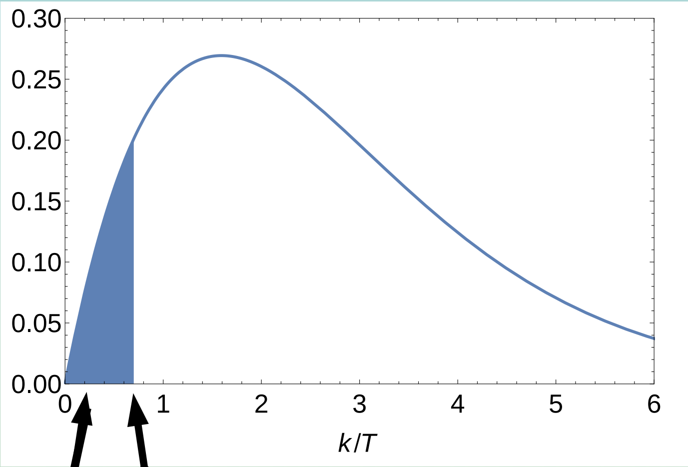
*Matching gluon to pions through QCD crossover?

Pion-axion: suppressed by $\theta_{a\pi} \propto \dfrac{m_u - m_d}{m_u + m_d}$, gluon is not

Pion rates not monotonic with T

Rates could have sudden jumps, as g_* does

High Temperatures Regime



$k \sim m_e \sim g_s T$

$k \sim m_m \sim g_s^2\, T$

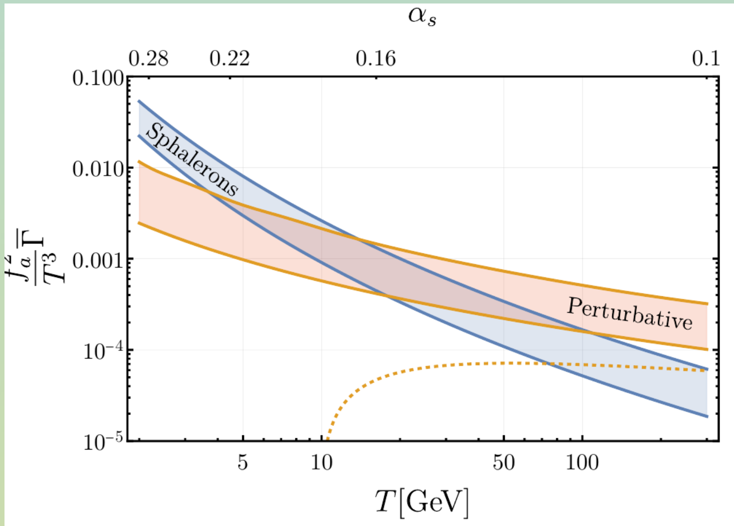
$\# \sim 1\,/\,g_s^2$

$@\, g_s \ll 1$:

large occupation numbers: T/m_g at small k

→ dominated by semi-classical

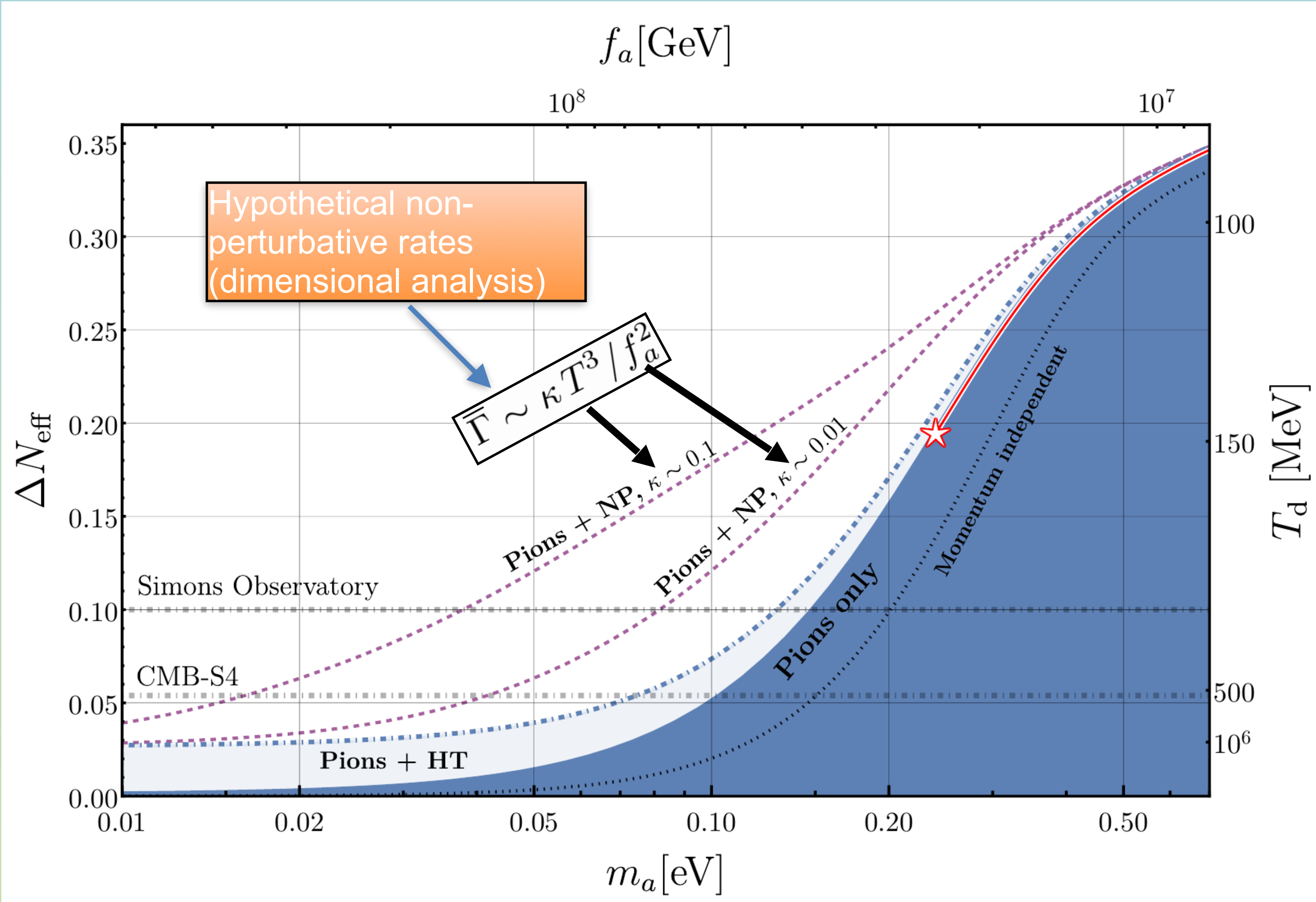
[non-linear YM equations - dissipation from strong sphalerons]



$$\Gamma_{\rm sphal} \simeq \frac{(N_c\alpha_s)^5 T^3}{f_a^2}$$

(Adapted from: Moore, Tassler '10, but derived only at $k_{\rm axion} = 0$!)

Future Reach ($T_{\rm DEC} \gtrsim T_c$ region)



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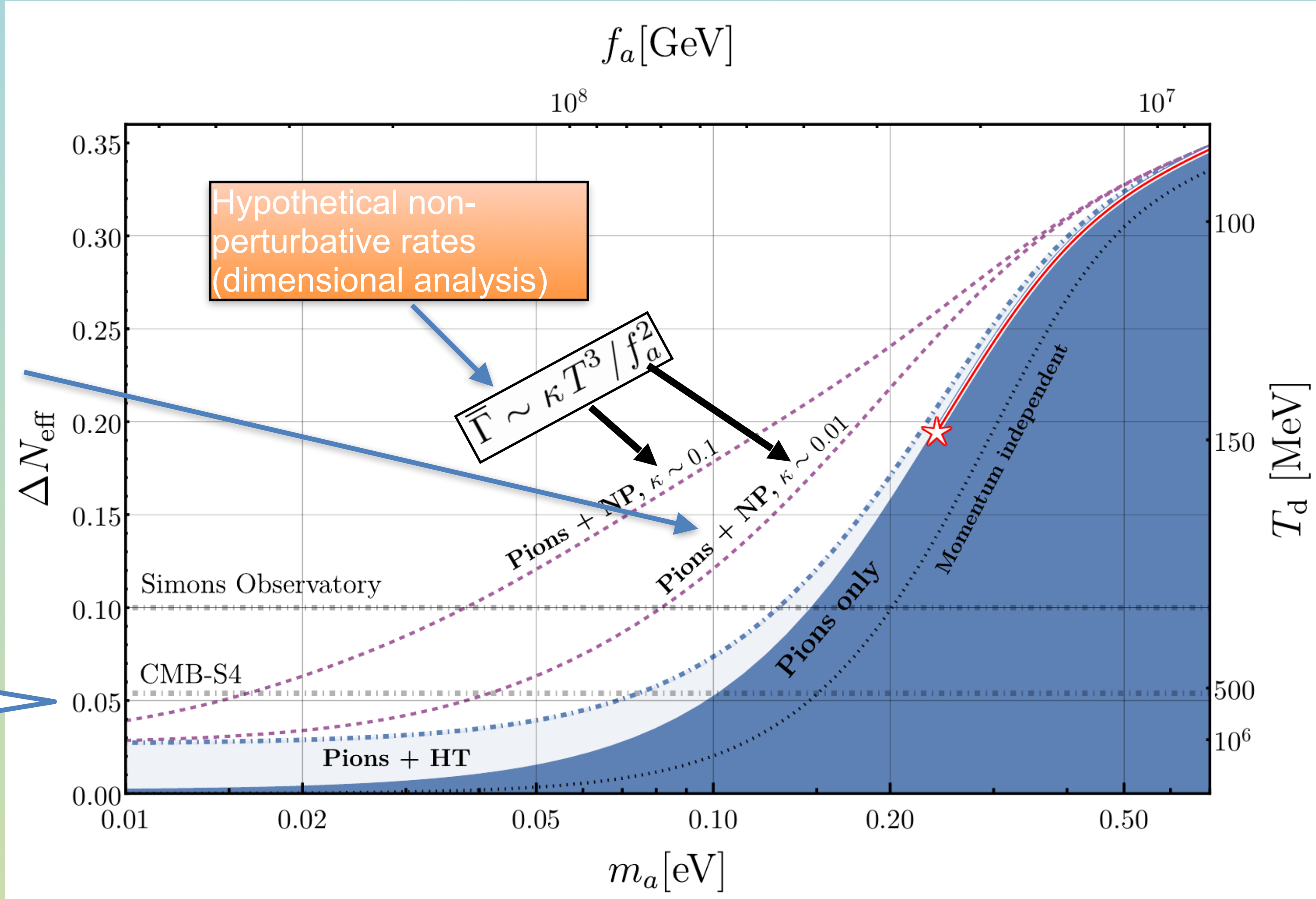
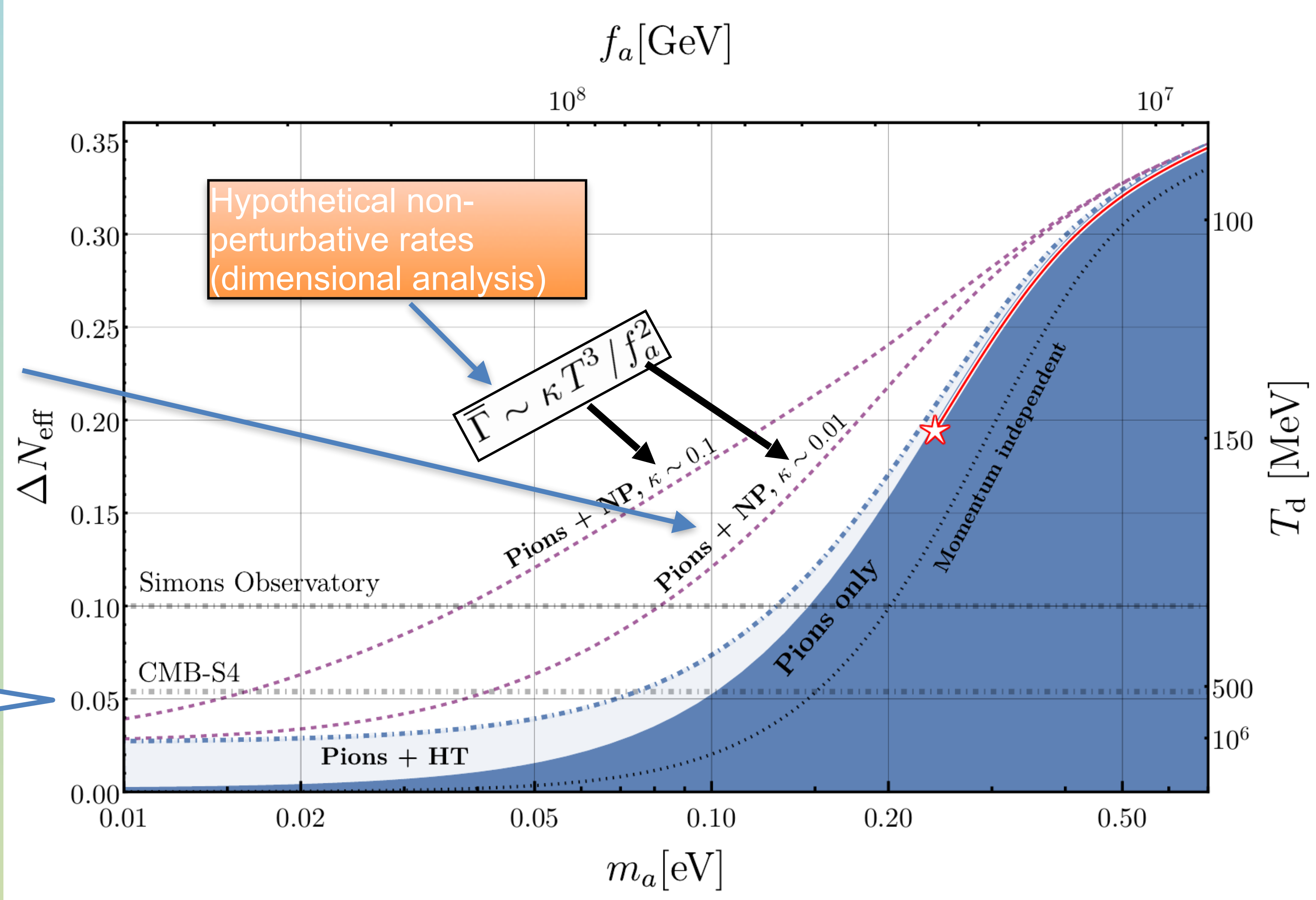


Planck+Euclid!

(Brinckmann et al. '19)

Future Reach ($T_{\rm DEC} \gtrsim T_c$ region)

Consistent with very recent Lattice QCD simulation (Bonanno et al. e-Print: 2308.01287) on $\Gamma_{\rm sphal}$



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- We derived pion-axion rates reliable also at T>60 MeV and upper bound on m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like QCD axions)
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- Importance of momentum dependence on Boltzmann equation @ around QCD scale
- Non-perturbative rates at $T \sim T_c$ crucial for upcoming CMB experiments $\Delta N_{\rm eff} \sim 0.1$

Conclusions:

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Thank you!

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- If a is directly coupled to SM heavy quarks (c, b, t):

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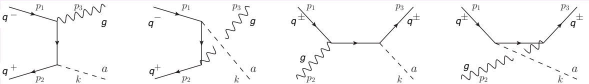

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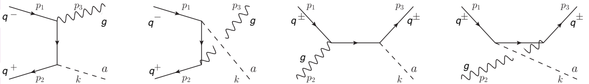

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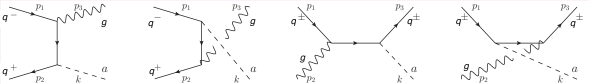

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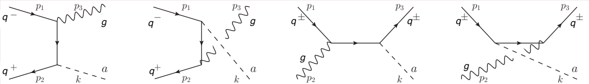

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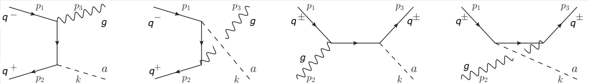

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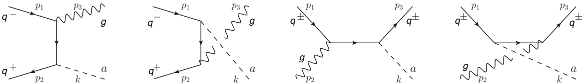

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- Range $10^9\mathrm{GeV} \gtrsim f/c_i \gtrsim 10^7\mathrm{GeV}$ [5]
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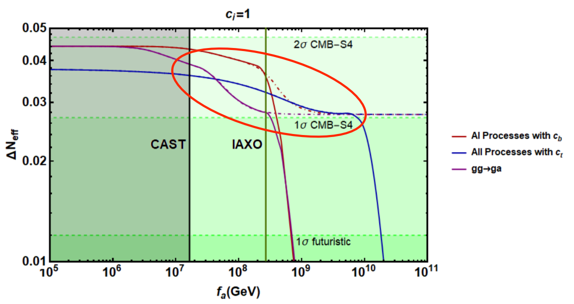
- Solving Boltzmann equations for n_a:

(R.Ferreira & A.N., PRL 2018; F.Arias-Aragon et al. JCAP, 2021)



$$10^9\mathrm{GeV} \gtrsim f/c_i \gtrsim 10^7\mathrm{GeV} \quad , 5\times 10^{-3}\mathrm{eV} \lesssim m_a \lesssim 0.5\mathrm{eV}$$

QCD Axion through $N_{\rm eff}$

- Potentially larger for c-quark: $N_{\rm eff} \lesssim 0.05 - 0.06$ (but uncertain)

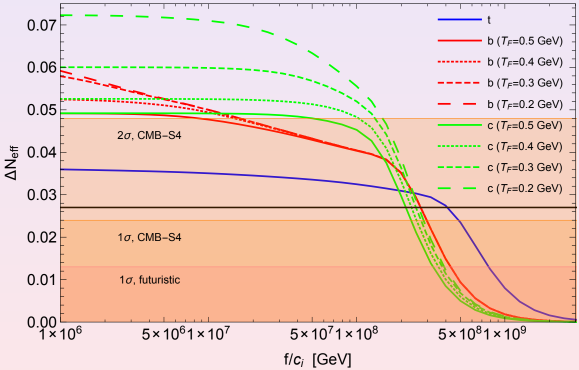


Figure: R.Ferreira & A.N., PRL 2018.

Hot Axions via Quark Decays

- $a - q$ interaction can be flavor non-diagonal

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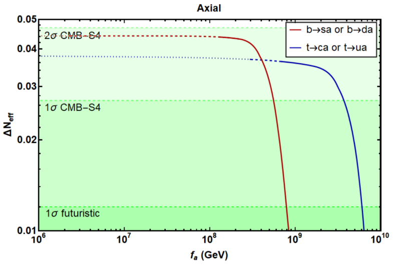

Figure: F.Arias-Aragon, F.D'Eramo, R.Z.Ferreira, A. N , L.Merlo, JCAP 2021.

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- The same can be done with leptons (μ and τ) [6]
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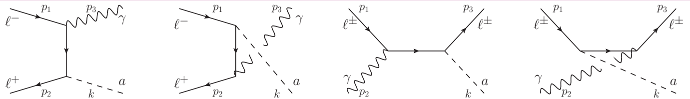

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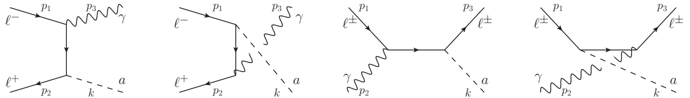

- Slightly smaller f/c_ℓ
- Ratio peaks at $\boxed{T \approx m_\ell}$ $\Longrightarrow$ Larger N_{eff}

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- Smaller $f/c_i \lesssim \text{few} \cdot 10^7$ GeV
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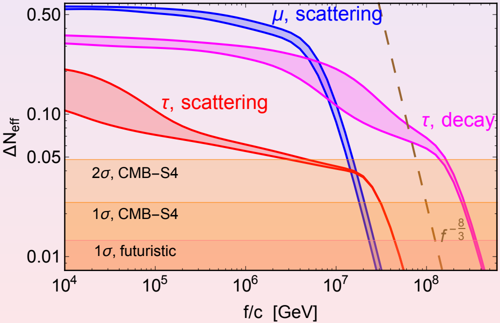

- Caveat: μ scattering constrained by SN cooling at $f/c_\mu \gtrsim 10^8 GeV$ (Bolling et al. PRL 2020, Croon et al. JHEP 2021)

Axion-Pion coupling

- DFSZ model: *a* couples to u-type and d-type quarks,
- KSVZ model: no coupling to SM fermions

$$\textbf{DFSZ}: \quad c_u^0 = \frac{1}{3}\cos^2(\beta)\,, \quad c_d^0 = \frac{1}{3}\sin^2(\beta)\,,$$
$$\textbf{KSVZ}: \quad c_u^0 = c_d^0 = 0\,,$$

Conclusions

1. If $f \lesssim \mathcal{O}(10^9)$ GeV, coupling with quarks and leptons (with $c_i = \mathcal{O}(1)$) dominates over $\frac{\alpha_s}{8\pi} \frac{a}{f} G\tilde{G}$
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5. Non-diagonal couplings $\implies$ production via Decays more efficient $(f \lesssim \mathcal{O}(10^{10})$ GeV$)$

EXTRA SLIDES

- AXION as COLD DARK MATTER

AXION AS COLD DARK MATTER

- The axion can form a coherent condensate with high occupation numbers
- Can be described by classical fields
- Example: start at 'initial time' with homogenous $a(t_i) = a_0 \neq 0$ over our horizon
- $V(a) = \Lambda_{\rm QCD}^4 \left[1 - \cos\left(\frac{a}{f_a}\right)\right]$ at Temperatures below $\Lambda_{\rm QCD}$

Coherent oscillations $\ddot{a} + 3H\dot{a} + V'(a) = 0$

Approximately $\ddot{a} + 3H\dot{a} + m_a^2 a = 0$

Frozen when $m_a \ll H$ (early times)

Oscillates in time like matter when $m_a \gtrsim H$ (late times)

AXION AS COLD DARK MATTER

- How to start at ‘initial time’ with homogenous $a(t_i) = a_0 \neq 0$ over our horizon?
- Inflation can make the field ~ almost homogeneous
- Inflation stretches the field gradients decay classically during inflation

``PRE-INFLATIONARY” SCENARIO

AXION AS COLD DARK MATTER

- The axion arises from a Complex Scalar (``KSVZ” models):
- $V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2 \qquad v = f_a \ (N_{\mathrm{DW}} = 1)$

- The symmetry is broken during inflation $\Phi = ve^{i\theta} = f_a e^{i\frac{a}{v}}$,
- A scalar field in inflation has quantum fluctuations of order H_i
- If very small ($H_i \ll f_a$)

- $\theta(t_i) = a(t_i)/v$ is a random value in $(-\pi, \pi)$, almost homogenous in our horizon

``PRE-INFLATIONARY” SCENARIO

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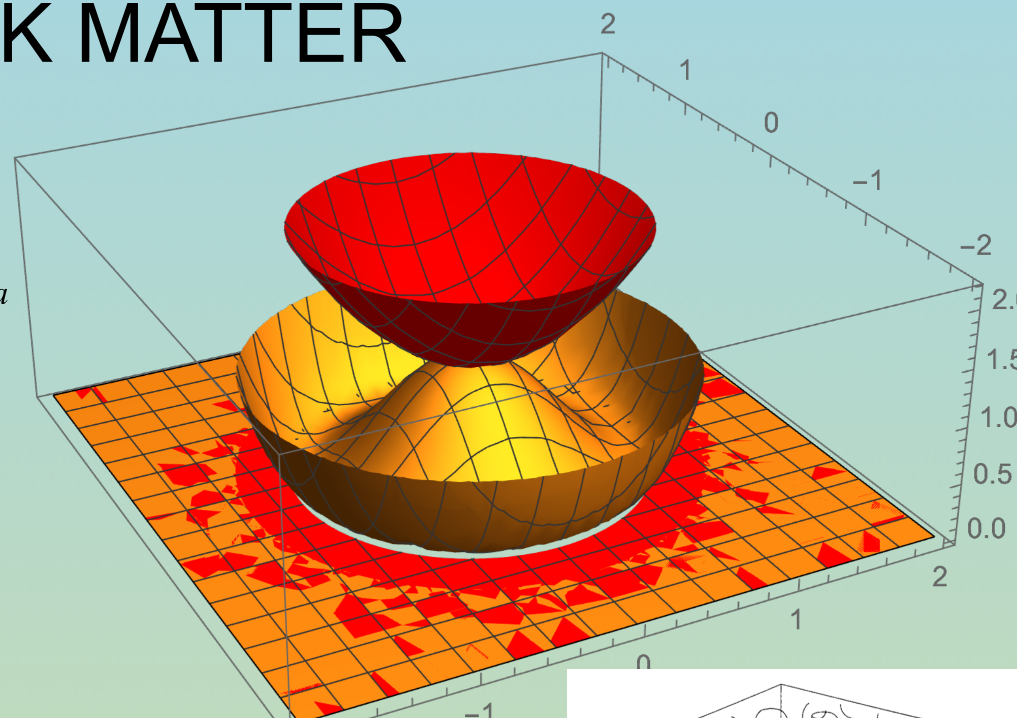


- Another possible scenario: If the Universe reaches $T = f_a$
- At $T > f_a$ the symmetry is restored $v(\Phi, T) \approx T^2 |\Phi|^2$
- At $T \approx f_a$ the symmetry gets broken

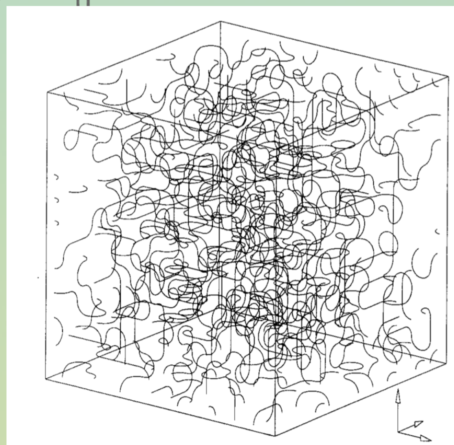
``POST-INFLATIONARY" SCENARIO

The field falls randomly → Strings form when the phase wraps from 0 to 2π

Network of strings forms

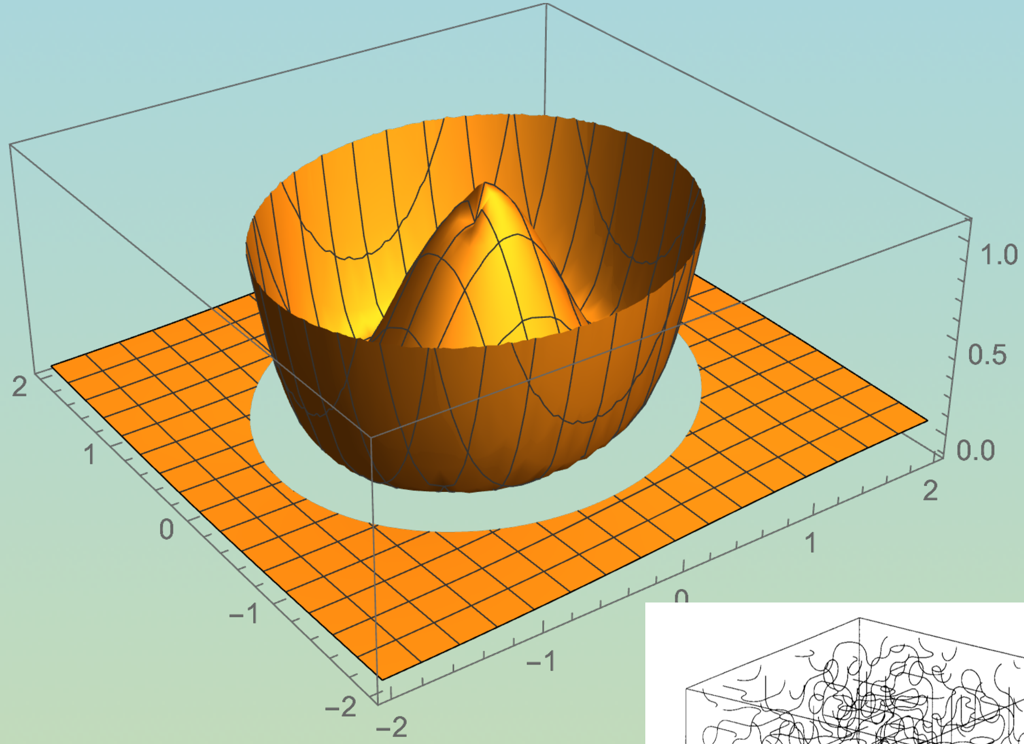
After initial transient → "scaling" behavior

O(1) string per Hubble volume



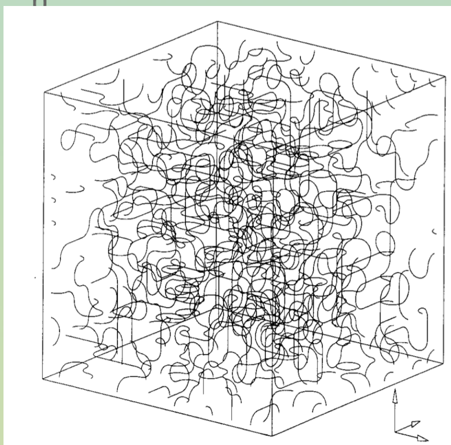
AXION AS COLD DARK MATTER

- Much later the potential gets tilted
- $V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2 + \Lambda_{QCD}\left(1 - \cos(\frac{a}{f_a})\right)$



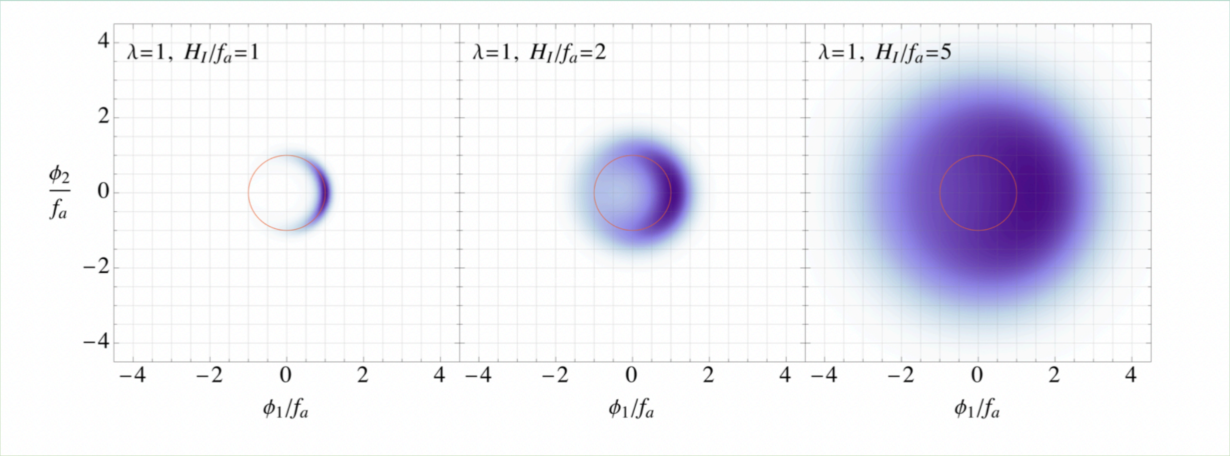
The field goes in the only minimum (after forming domain walls at $a/f_a \approx \pi$)

Strings and walls decay into (cold?) axions,
which add to Cold Dark Matter



AXION AS COLD DARK MATTER

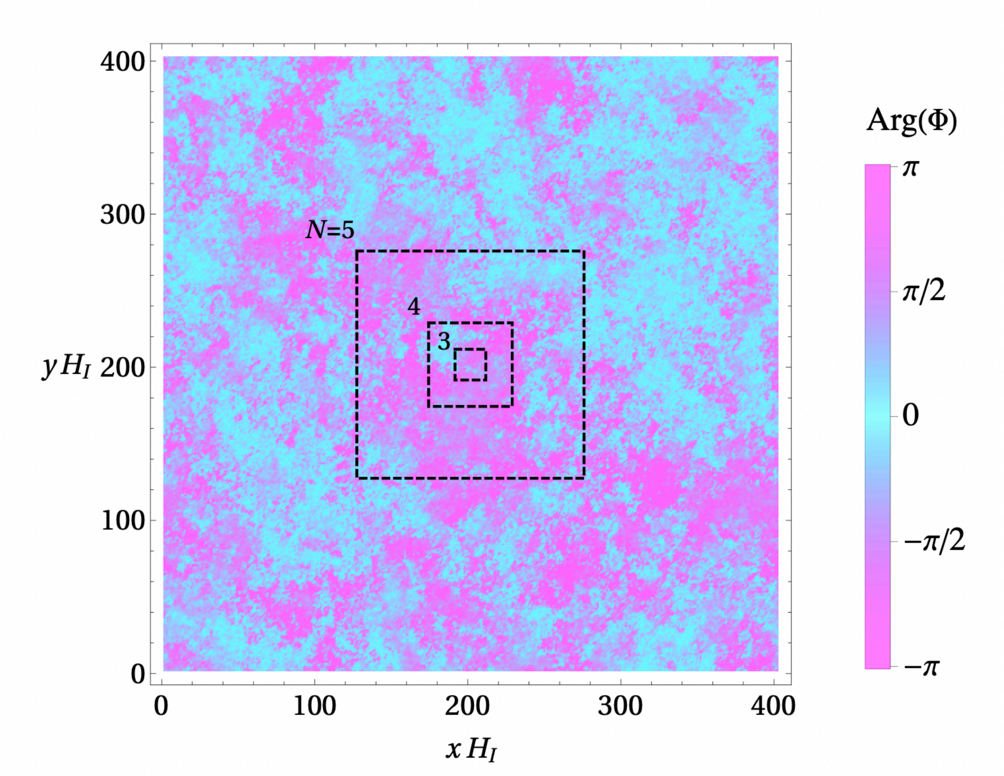
- THIRD POSSIBILITY: "STOCHASTIC INFLATIONARY SCENARIO"
- $H_i \gtrsim f_a$ large fluctuations during inflation (see Lyth 1992, Lyth & Stewart 1992)
- Both the angular and the radial field have large fluctuations



- Strings form due to large inflationary fluctuations
- If Temperature is never large enough after inflation ($T < f_a$) Symmetry is NOT restored after inflation

AXION AS COLD DARK MATTER

- On small patches: angle θ almost constant
- On large patches: can wrap from 0 to 2π
- Strings form, separated by a length $d = e^{N_s}/H_I$
- $N_s \approx 10/\sqrt{\lambda}$
- If $N_s \gtrsim 60$ field coherent in our entire horizon
- If $N_s < 60$ Strings separated by macroscopic length d



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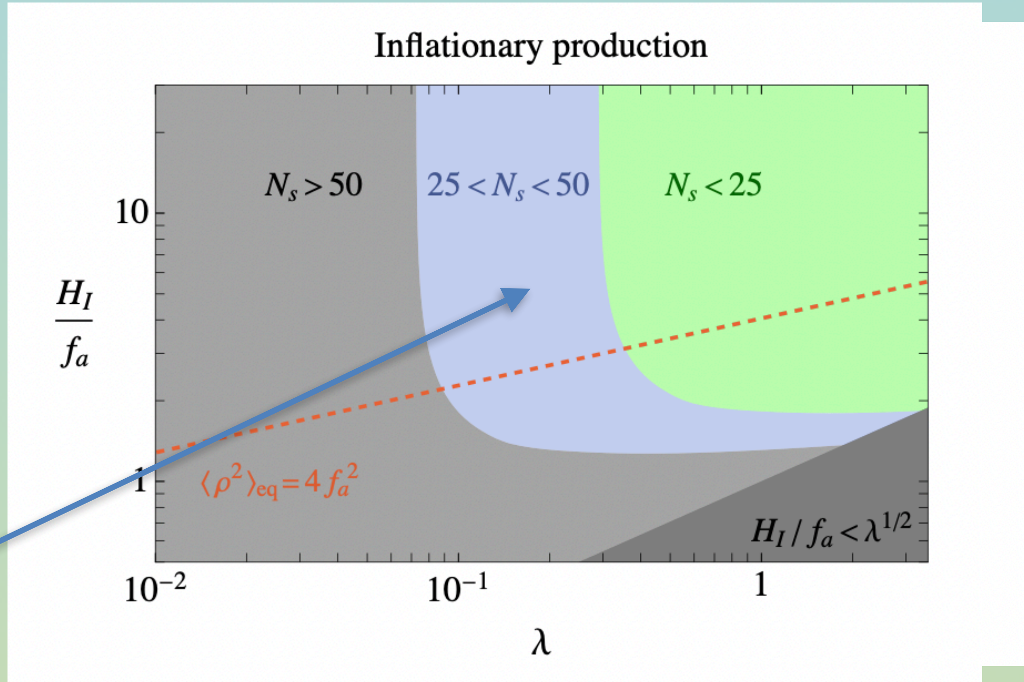
-
- If 25< $N_s < 60$ strings reenter the horizon after QCD phase transition: "LATE STRINGS" (NEW phenomenology)



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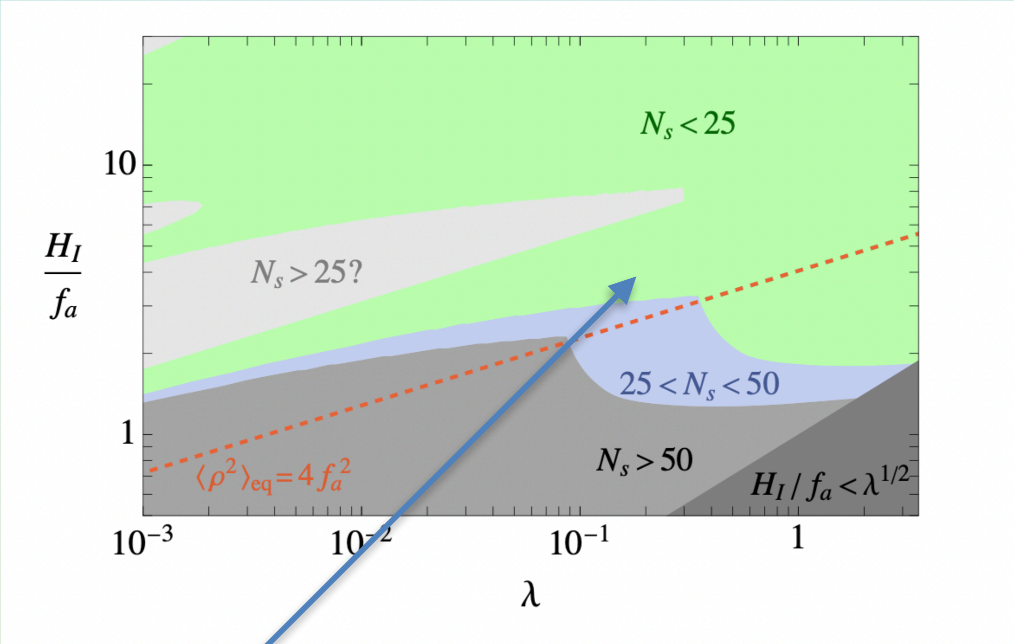
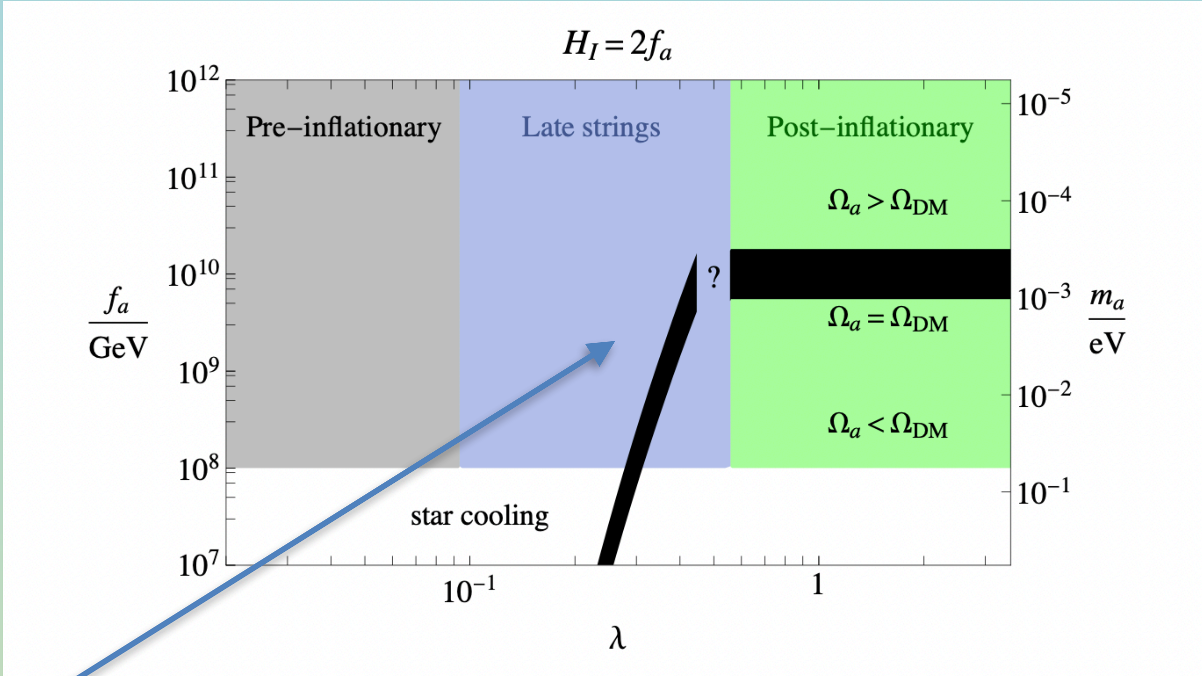


Overshoot mechanism after inflation:
<u>if the field starts high in the potential,</u>
<u>it can roll on the opposite side</u>

EARLY STRING FORMATION

AXION AS COLD DARK MATTER



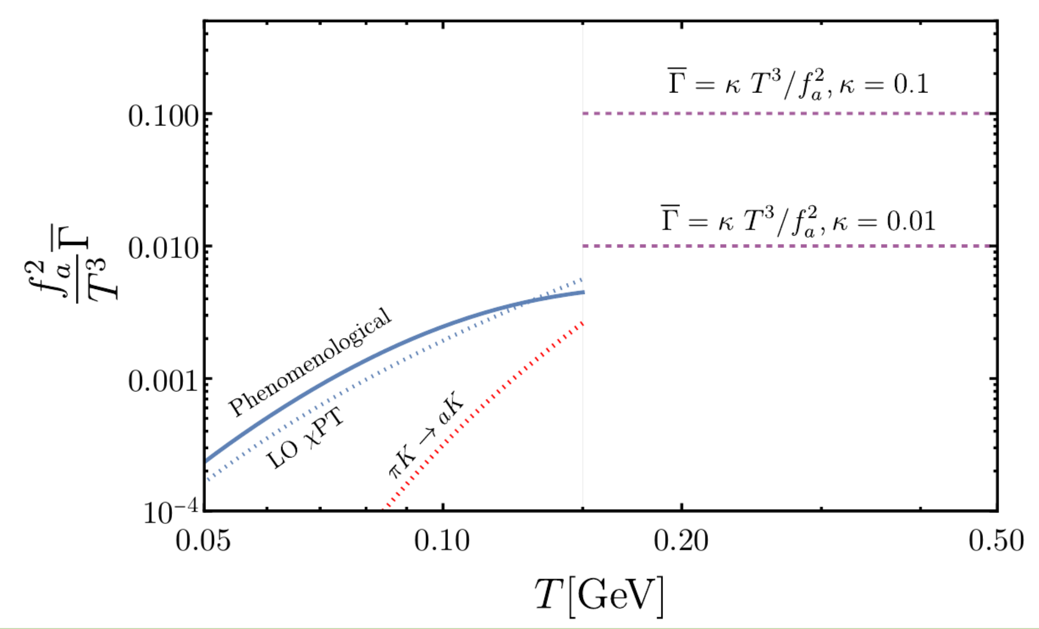
Standard post-inflationary:
Uncertainty from string simulations,
but close to $f_a \sim 10^{10} - 10^{11} GeV$
$(m_a \sim 10^{-3} - 10^{-4} eV)$

Late strings scenario: enhanced abundance.

Smaller f_a possible (down to astrophysical bound)

Strong Sphaleron-like contribution to Axion rate

$$\overline{\Gamma}_{\rm sphal} = \frac{1}{n^{\rm eq}} \int^{|\mathbf{k}|<|\mathbf{k}_s|} \frac{d^3\mathbf{k}}{(2\pi)^3 2E} \frac{\Gamma_{\rm sphal}}{f_a^2} e^{-E/T} = \frac{(N_c\alpha_s)^5 T^3}{4\zeta_3 f_a^2} \left(1 - \left(1 + \frac{|\mathbf{k}_s|}{T}\right) e^{-|\mathbf{k}_s|/T}\right)$$



$$\Gamma^{>}_{\rm top}(E = |\mathbf{k}| < |\mathbf{k}_s|) \simeq \Gamma_{\rm sphal} \simeq (N_c\alpha_s)^5 T^4$$

$$|\mathbf{k}_s| \sim N_c\alpha_s T$$

The Thermal Width:

Challenge for Lattice QCD: <u>Compute Γ_k for $T > T_c$</u>

Existing Attempts (at k=0) e.g.

Moore, Tassler '10 : Classical SU(N) simulations

Kotov '18 ,

Altenkort et al. '20,

Mancha, Moore '22 : Quantum Euclidean (plus modeling)

$$\Gamma_{\mathrm{sphal}} = 2T \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

$$G(\tau) = \int \mathrm{d}^3x \, \langle q(\vec{0},0) q(\vec{x},\tau)\rangle = -\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, \rho(\omega) \frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)}$$

Important to exploit upcoming experiments!