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Laboratorio di Acceleratori
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Transverse Diagnostics: beam size and emittance

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Tasks

- Investigate the performance of the CCD
  - Spatial calibration => dot grid target, USAF 1951 target
  - Resolution => Siemens star, USAF 1951 target
- Measure the emittance of the laser beam
  - Measure spot sizes for different distances of the lens
  - Analyse the horizontal profiles as function of the lens position
  - Calculate the laser beam emittance
    - Compare the two methods (i.e. two-points and «quadrupole» scan)

Introduction

- Diagnostics is the *organ of sense* of an accelerator
  - Instrumentation for daily check
    - profile measurements, charge, beam position, …
  - Instrumentation for commissioning and accelerator development
    - Emittance, bunch length, energy measurements (and more and more)

Motivation

- Particle beam properties in the transverse phase space are characterized by the **transverse beam emittance**
  - **Key parameter** both for light sources (**spectral brilliance**) and colliders (**luminosity**)

Measure for phase space density of photon flux

$$B = \frac{\#photons}{[sec][mm^2][mrad^2][0.1\%BW]}$$

connection to machine parameters

$$B \propto \frac{I}{\varepsilon_x \varepsilon_y}$$ [A/(m*rad)²]

Measure for the collider performance

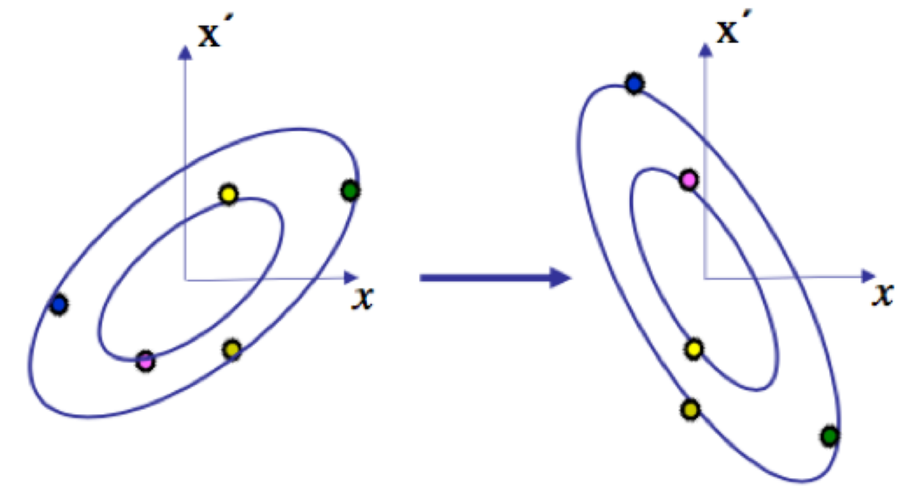
$$\dot{N} = L\sigma$$

connection to machine parameters

$$L \propto \frac{I_1 I_2}{\varepsilon}$$ [cm⁻²s⁻¹]

Transverse Emittance

- **Projection of phase space volume**
  - Separate horizontal, vertical and longitudinal plane
- **Linear forces** (*Liouville's theorem*)
  - Any particle moves on an ellipse in phase space $(x, p_x) \rightarrow (x, x')$
  - ellipse rotates in magnets and shears along drifts
    - but area is preserved: emittance



$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

(α, β, γ, ε: Courant-Snyder or Twiss parameters)

- **Transformation along accelerator**
  - Knowledge of the magnetic structure (**beam optics**) ➔ transformation from initial (i) to final (f) location

Single particle transformation

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

Transformation of optical functions

$$\begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}_f = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1 + R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}_i$$

Transverse Emittance Ellipse

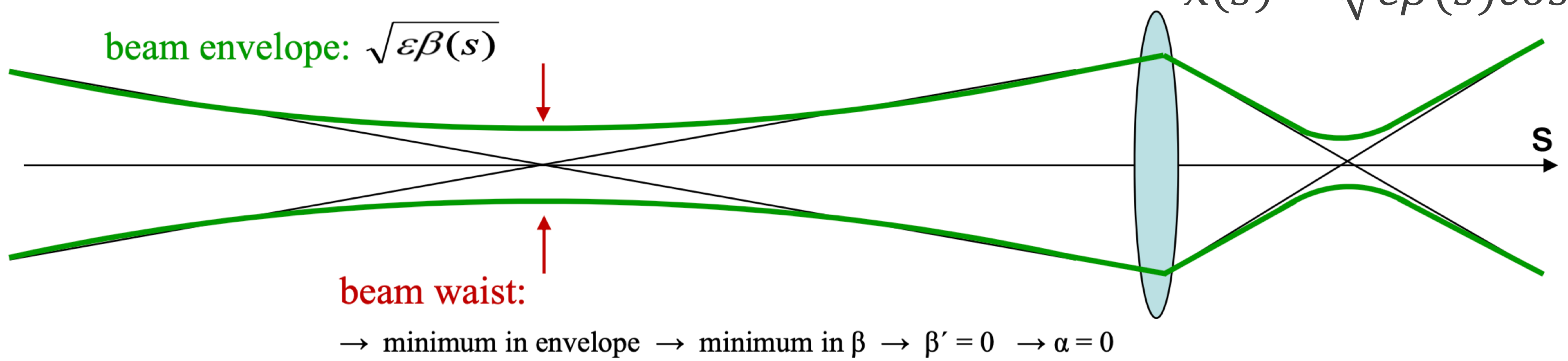
$$\alpha(s) = -\frac{\beta'(s)}{2}$$

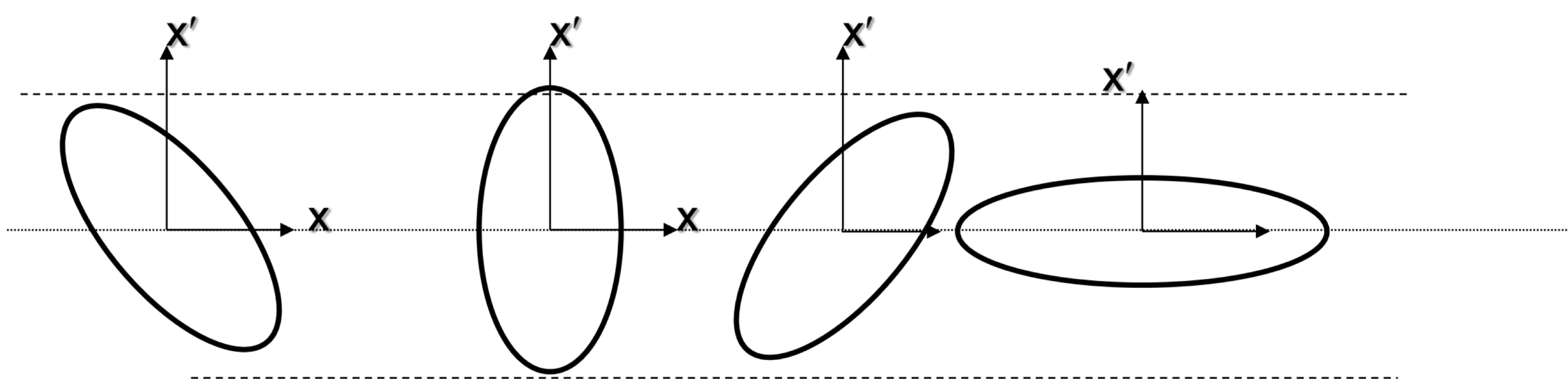
$$\gamma(s) = \frac{1+\alpha^2(s)}{\beta(s)}$$

- Propagation along the accelerator
  - Change of ellipse shape and orientation → area is preserved

$$\varepsilon = \gamma(s)x(s)^2 + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$

$$x(s) = \sqrt{\varepsilon\beta(s)}cos(\Psi(s)+\Phi))$$





Transverse Emittance Ellipse

- The transverse emittance is described either in the form of an **ellipse equation** via the Courant-Snyder or **Twiss parameters** as

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

- or as **statistical definition (**P.M. Lapostolle, IEEE Trans. Nucl. Sci. NS-18, No.3 (1971) 1101**)**

$$\varepsilon_{rms} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle x x'\rangle^2}$$

  - **characterization of beam charge distribution by its 2nd statistical moments**

rms size

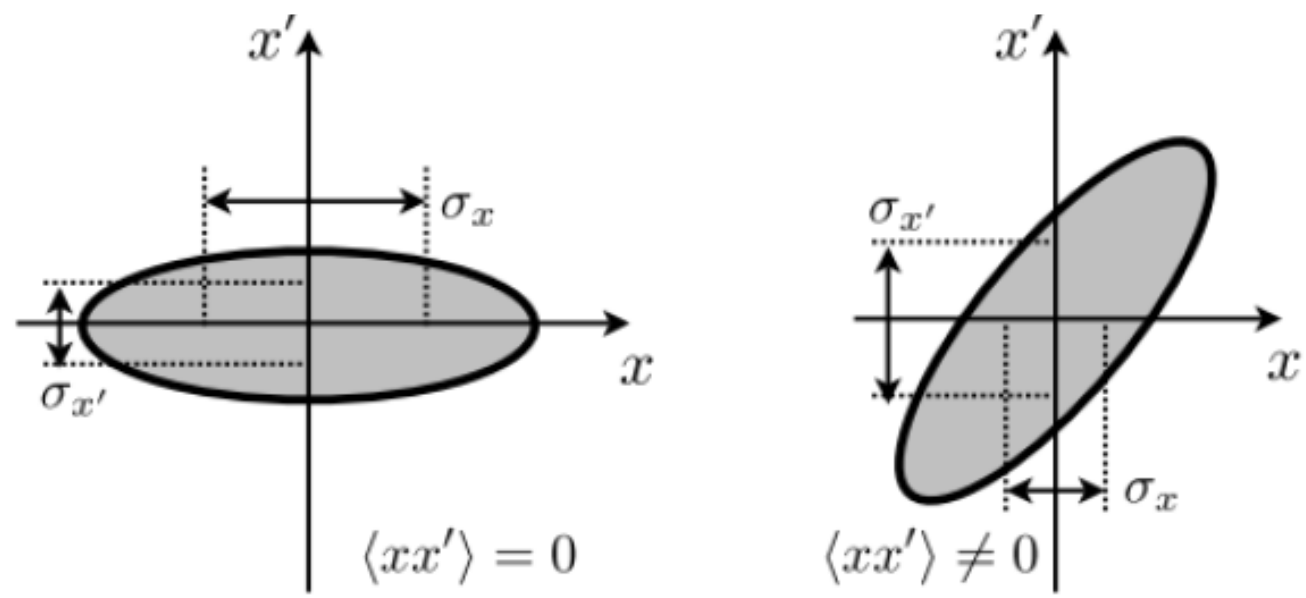
$$\sigma_x^2(z) = \langle x^2\rangle = \frac{1}{N_e}\sum_j x_j^2$$

rms divergence

$$\sigma_{x'}^2(z) = \langle x'^2\rangle = \frac{1}{N_e}\sum_j x_j'^2$$

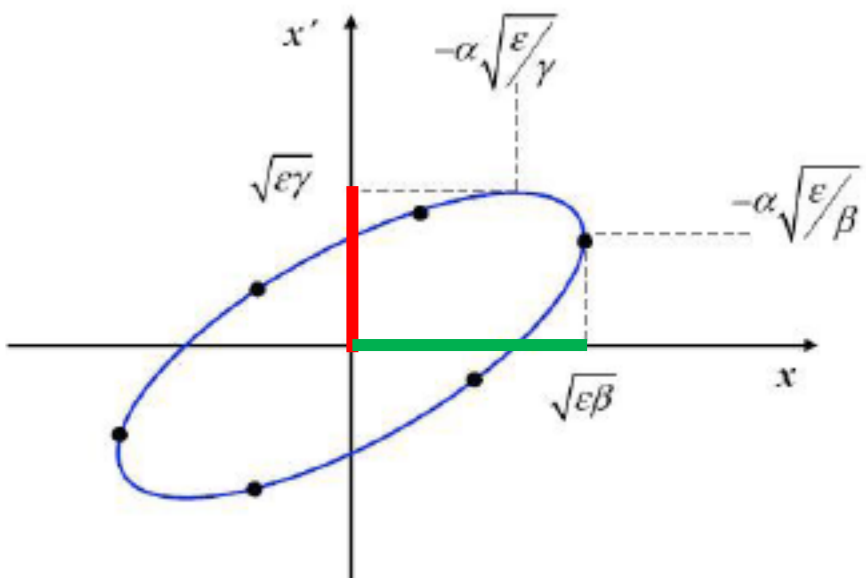
correlation

$$\langle x x'\rangle = \frac{1}{N_e}\sum_j x_j x_j'$$



Emittance and Beam Matrix

- The emittance itself is not directly measured
- The measurable quantities are the projections onto both axes, i.e. beam size or beam divergence
- Beam matrix based schemes, e.g. **Twiss parameters** or mapping of the phase space
  - exploit the transfer properties of the beam matrix



Let assume uncoupled motion: 2D sub-space

Beam matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2}$$

Beam Matrix based Measurements

The emittance is determined by measuring at least 3 matrix elements.

The **observable** is the **rms beam size** $\sigma = \sqrt{\Sigma_{11}} = \sqrt{\varepsilon\beta} = \sqrt{\langle x^2 \rangle}$

Σ_{12} and Σ_{22} must be inferred from beam profiles taken under various transport conditions,

Transformation of beam matrix $\Sigma^b = R\Sigma^a R^T$

therefore the knowledge of transport matrix R is required $R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$

Since $\Sigma(s)$ → determination of Σ required at same location.

Beam size measurements for **at least 3 different matrix elements** are required in order to solve for the 3 independent unknown parameters: ε, β and α.

$$\Sigma_{11}^f = R_{11}^2 \Sigma_{11}^i + 2R_{11}R_{12}\Sigma_{12}^i + R_{12}^2\Sigma_{22}^i$$

measurement: profiles

known: transport optics

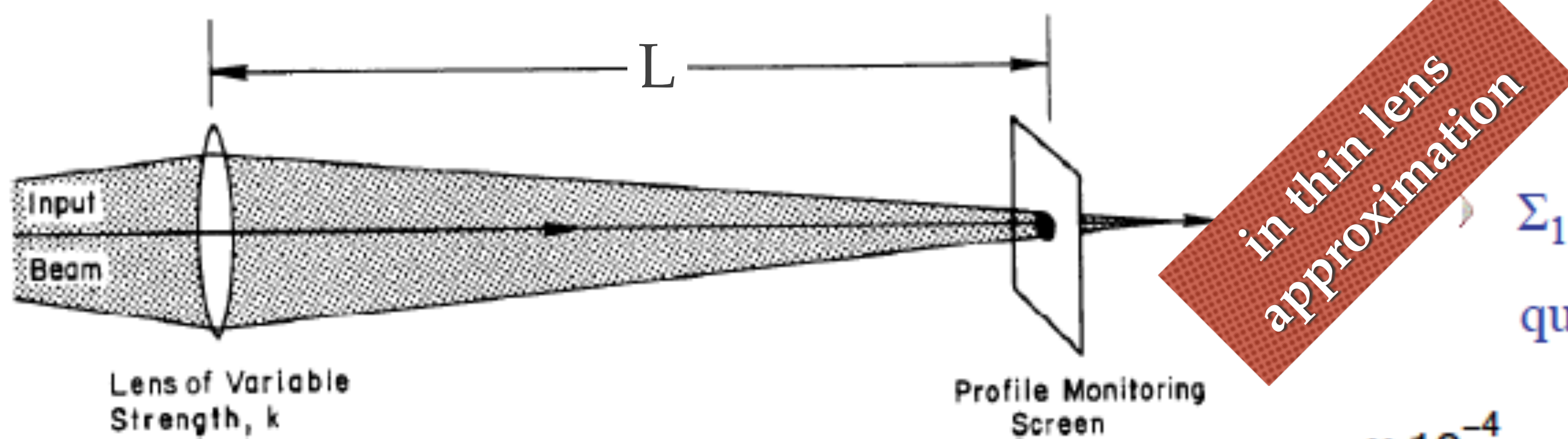
deduced: beam matrix elements at initial location

Beam Matrix based Measurements

„quadrupole scan" method

- use of variable quadrupole strengths

→ change quadrupole settings and measure beam size in profile monitor located downstream



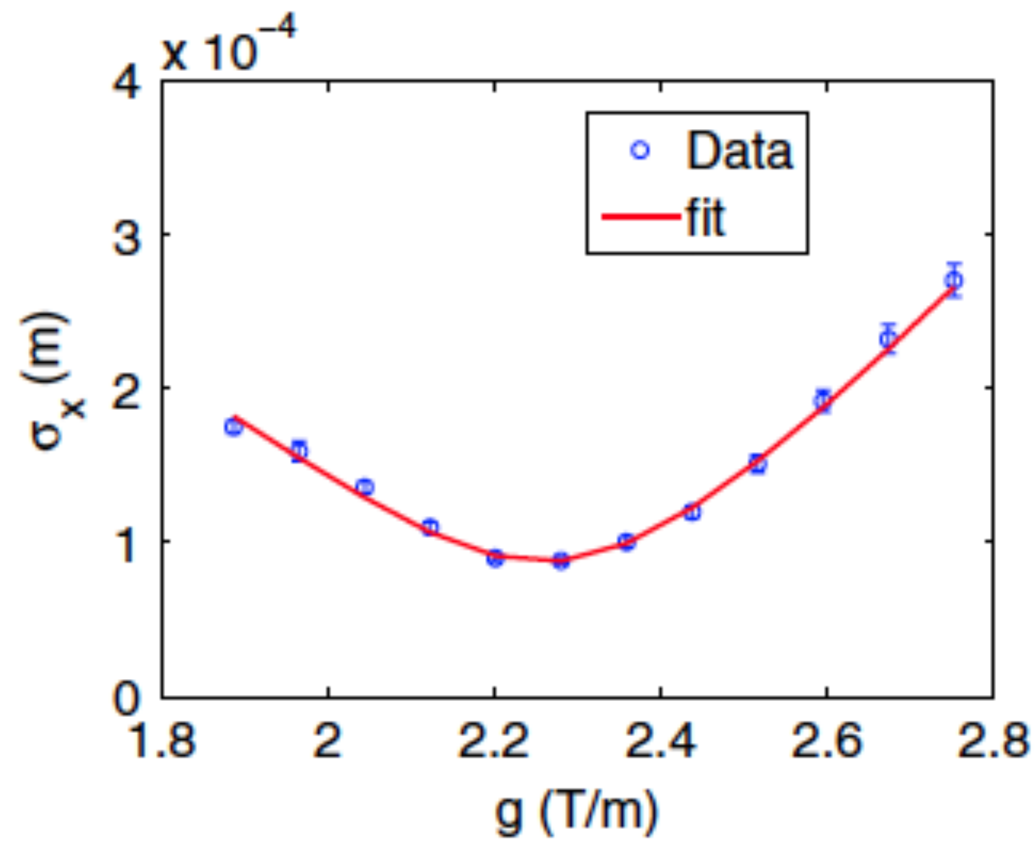
Q (f = 1/K) **S** (drift space)

- Σ_{11} depends quadratically on quadrupole field strength

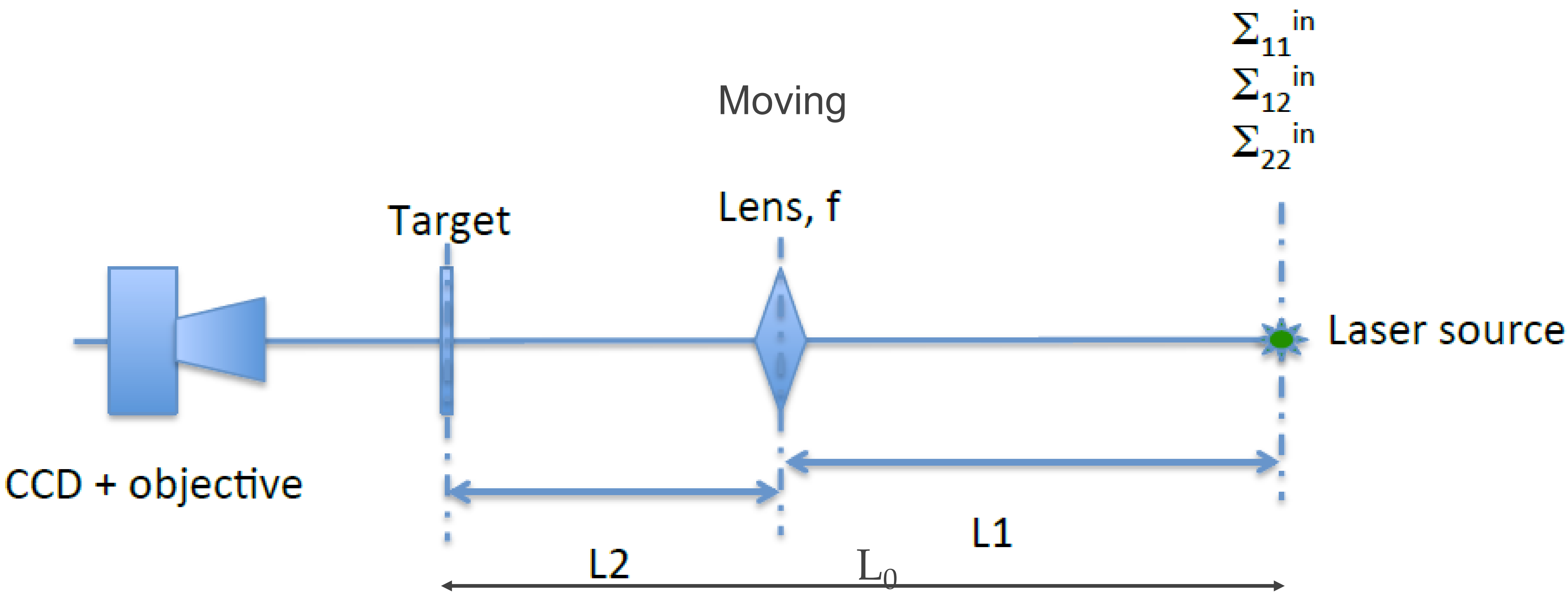
$$R_{drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$R_{quad} = \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$$

$$R_{Total} = R_{drift} R_{quad}$$



Transport Matrix



$$R_{tr} = R(L_2) \cdot R(f) \cdot R(L_1)$$

Emittance of the perfect Gaussian laser beam

$$\epsilon_{gauss} = \frac{\lambda}{4\pi}$$

Two-points Method

For the thin lens approximation we can evaluate the emittance by only two points

In **focal plane** $L = f$

$$\Sigma_{11}^{f} = \cancelto{0}{R_{11}^{2}}\Sigma_{11}^{i} + 2\cancelto{0}{R_{11}}R_{12}\Sigma_{12}^{i} + R_{12}^{2}\Sigma_{22}^{i} \quad\Rightarrow\quad \Sigma_{22}^{in} = \frac{\sigma_{\perp,waist}^{2}}{f^{2}}$$

At the same time **this point is the waist** of the beam:

$$\Sigma_{12}^{f} = 0 \quad\Rightarrow\quad \Sigma_{12}^{in} = \frac{\sigma_{\perp}^{2}}{f^{2}}(2f - L_0)$$

After that You already know two out of three coefficients, thus to find the third one, we can simply use one more point, i.e. basically **any point**:

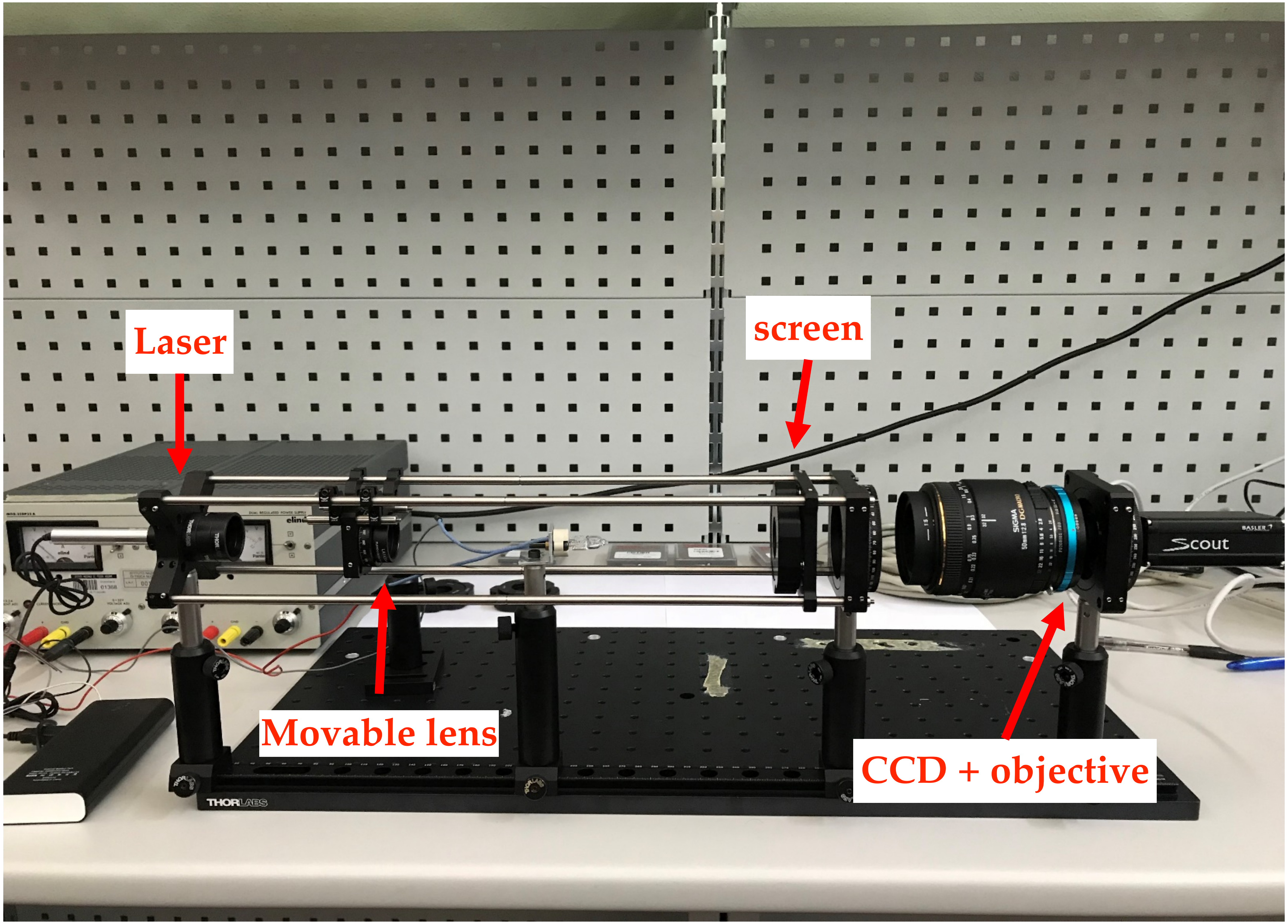
$$\Sigma_{11}^{f} = R_{11}^{2}\Sigma_{11}^{i} + 2R_{11}R_{12}\Sigma_{12}^{i} + R_{12}^{2}\Sigma_{22}^{i}$$

“Ciò che dobbiamo imparare a fare, lo impariamo facendolo.”

– *Aristotele*

Experimental Setup

Test setup

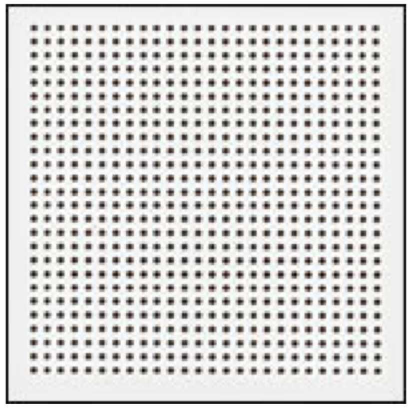


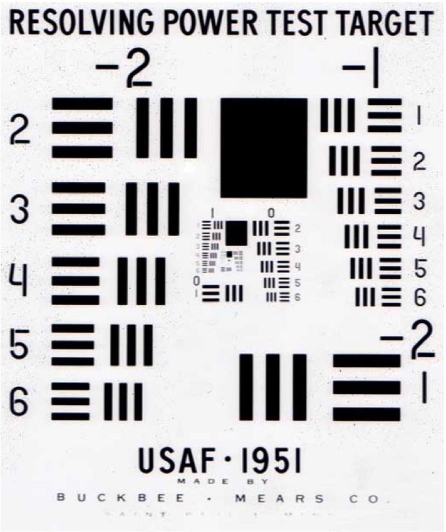
Experimental Components

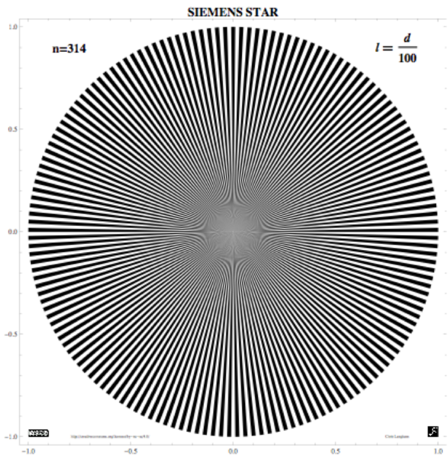
Item	LDM635
Wavelength, Typical	**532 nm**
Wavelength, Min/Max	625 - 645 nm
Beam Diameter	3.5 mm
Power	4.7 mW

Item	LDM635
CCD camera	Basler ACE 2 Basic
resolution	2448 x 2048 px
pixel size	2.74 x 2.74 µm
Size of the matrix	6.7 x 5.6 mm mm

- In the same position of the screen characterize:
  - **calibration**
    - dot grid target (**spacing: 0.5 mm**)
  - **resolution**
    - USAF 1951-target
  - **focusing**
    - Siemens star (n = 314, l=d/100)







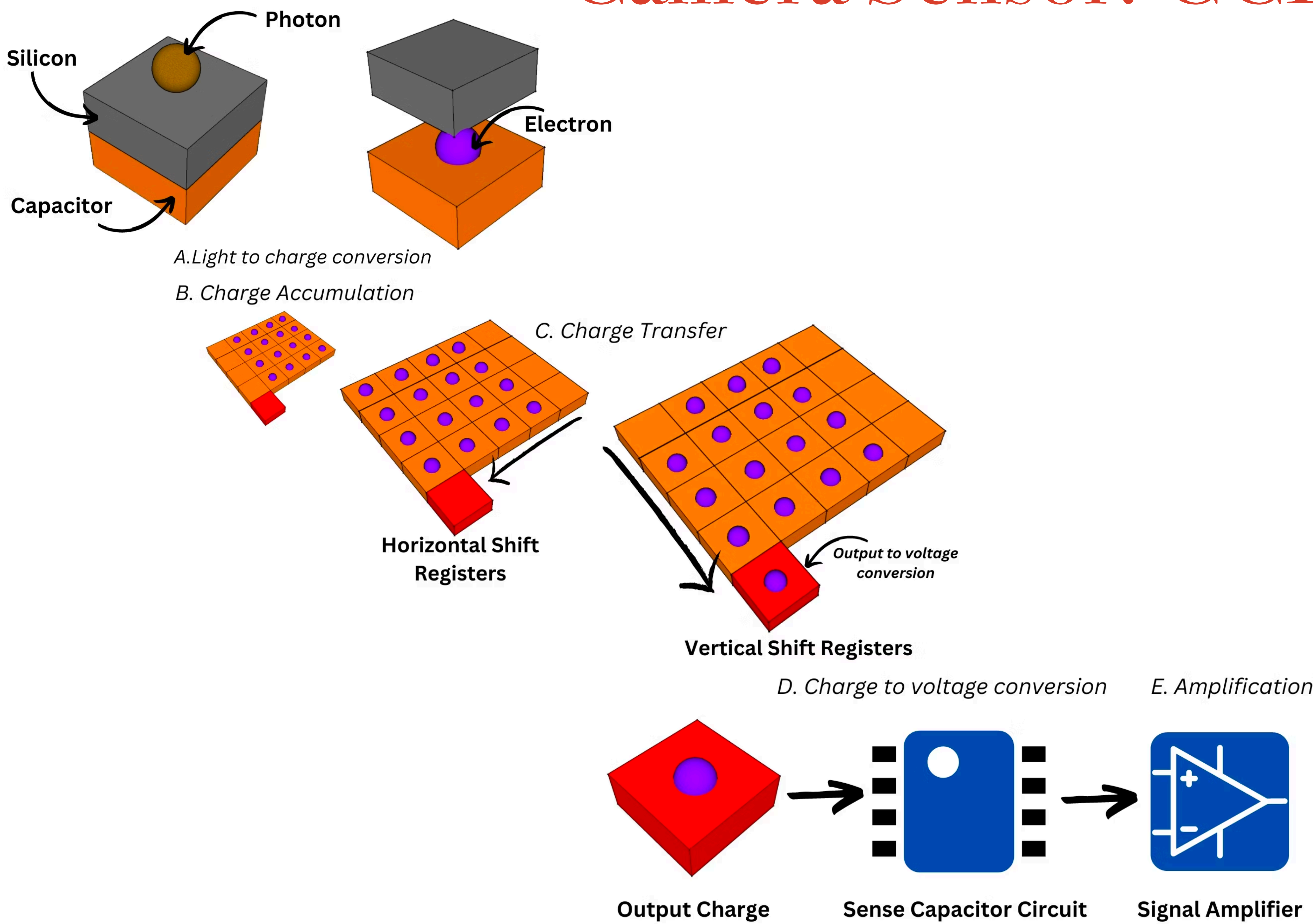
Camera Sensor Working Principle

CCD and CMOS differ in terms of manufacturing process and signal readout method

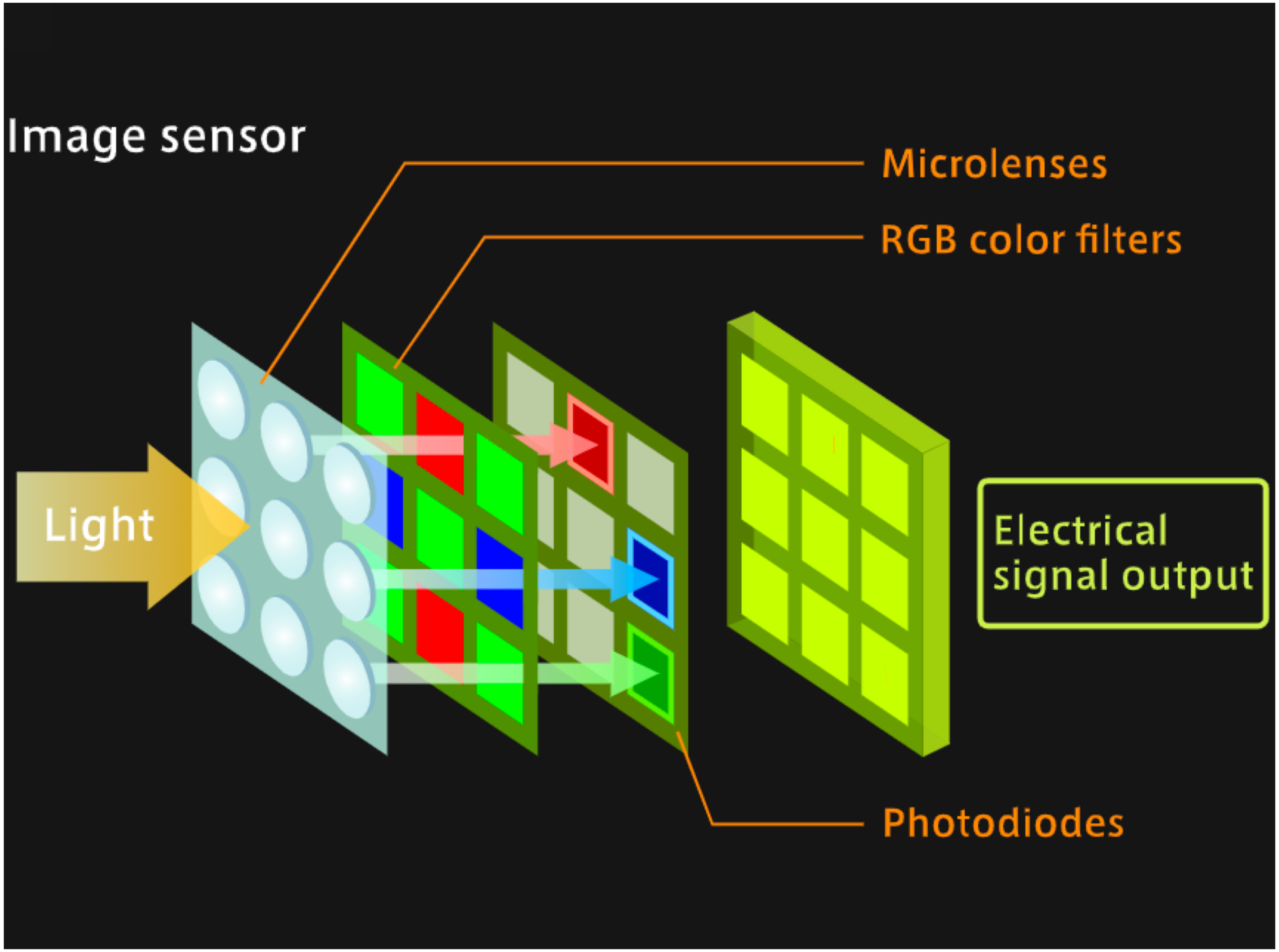
- Both **CCD (charge-coupled device) and CMOS (complementary metal-oxide semiconductor)** image sensors have to
  - **convert light into electrons**
    - a 2-D array of thousands or millions of tiny solar cells
  - **read the** value (**accumulated charge**) of each cell in the image.
    - CCD device: charge transported across the chip and read at one corner of the array ➔ An analog-to-digital converter turns each pixel's value into a digital value.
    - CMOS device: several transistors at each pixel amplify and move the charge using traditional wires ➔ each pixel can be read individually.

- Because of the manufacturing differences, there have been some noticeable differences between CCD and CMOS sensors
  - CCD sensors create high-quality, low-noise images.
  - CMOS sensors, traditionally, are more susceptible to noise

Camera Sensor: CCD

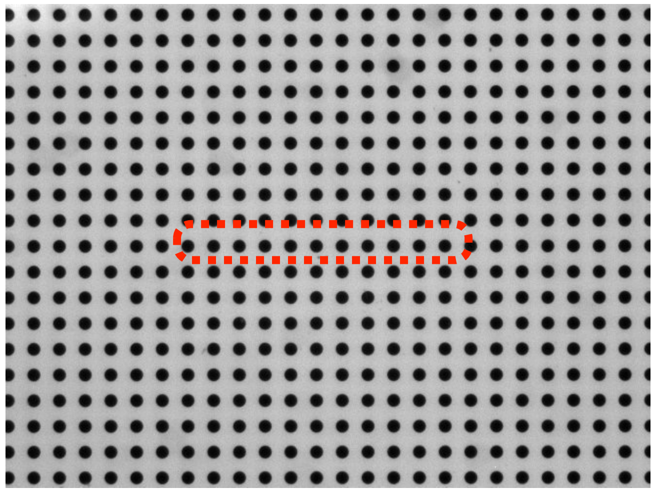


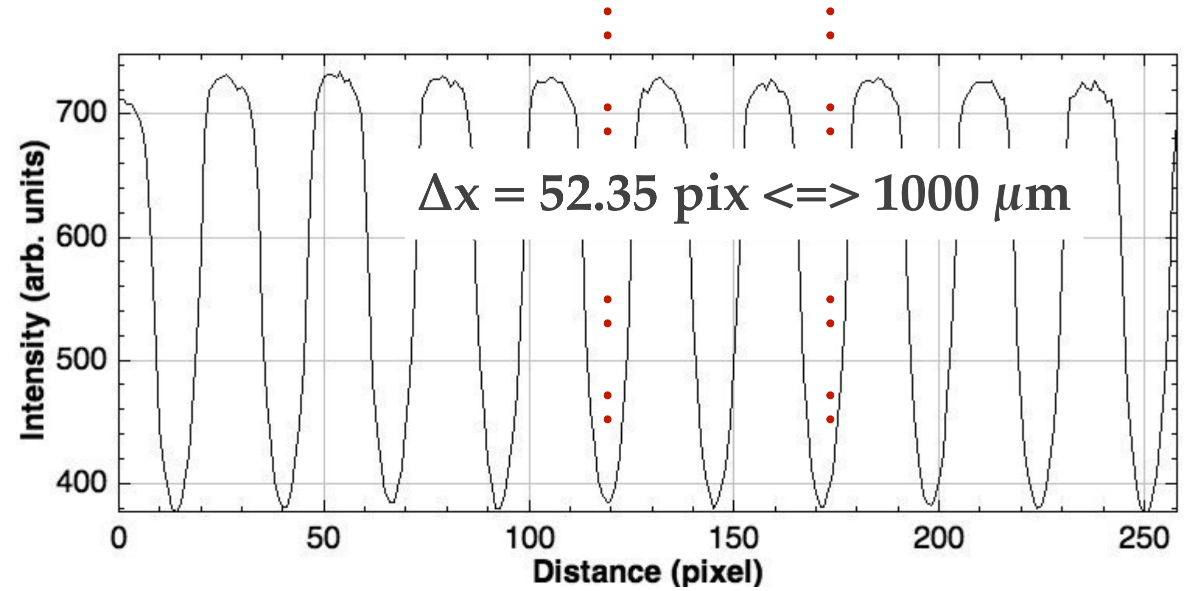
Camera Sensor: CMOS



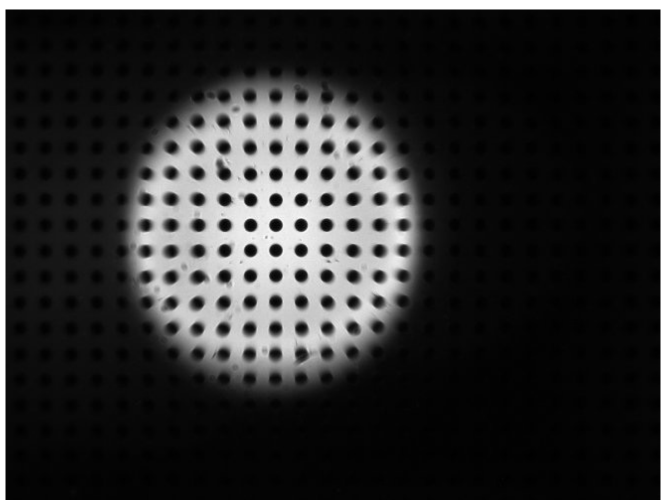
Dot grid target

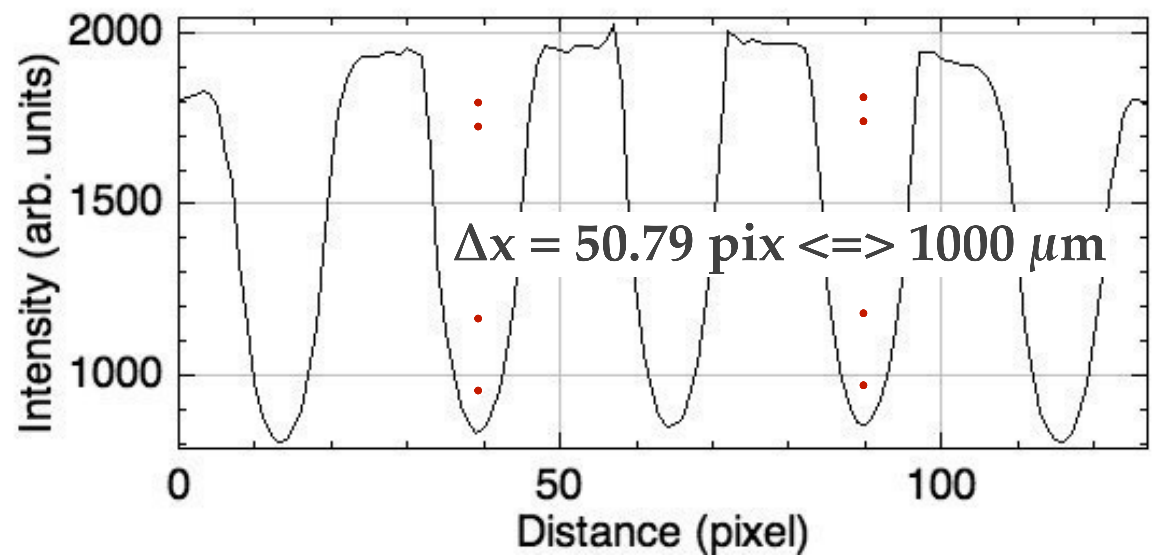
The importance of uniform illumination



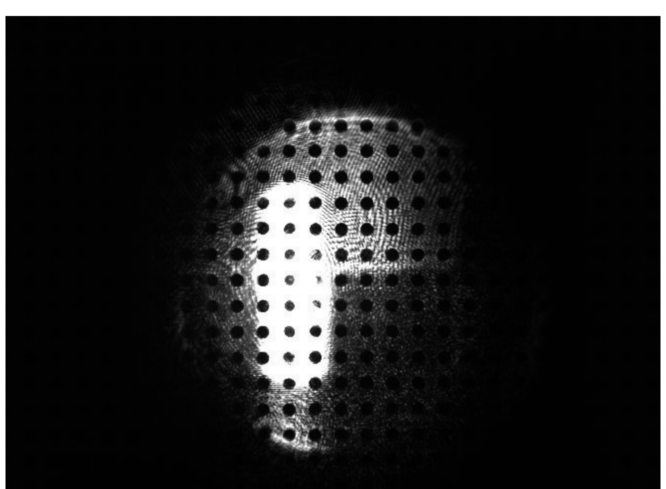


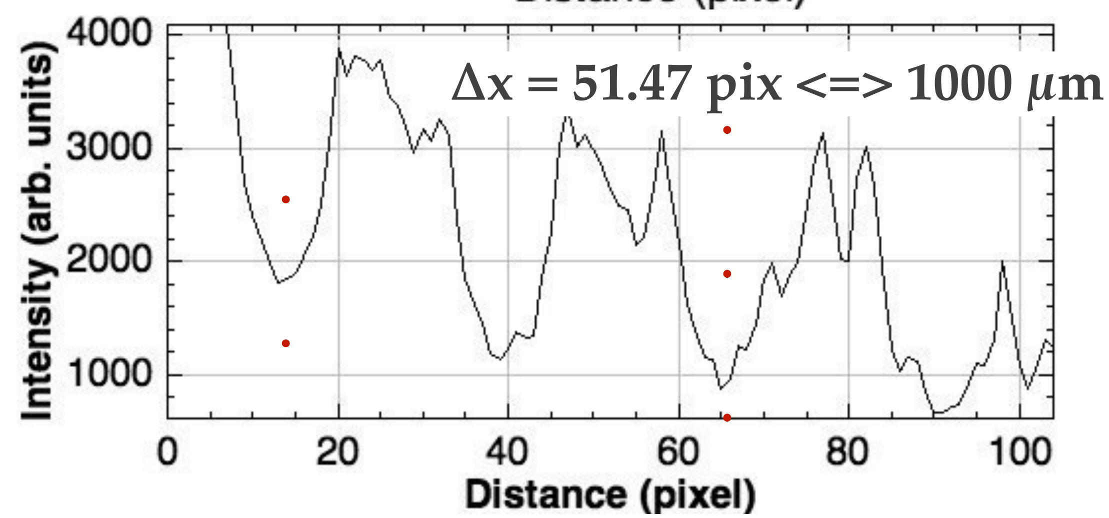
19.1 μm/pix





19.69 μm/pix

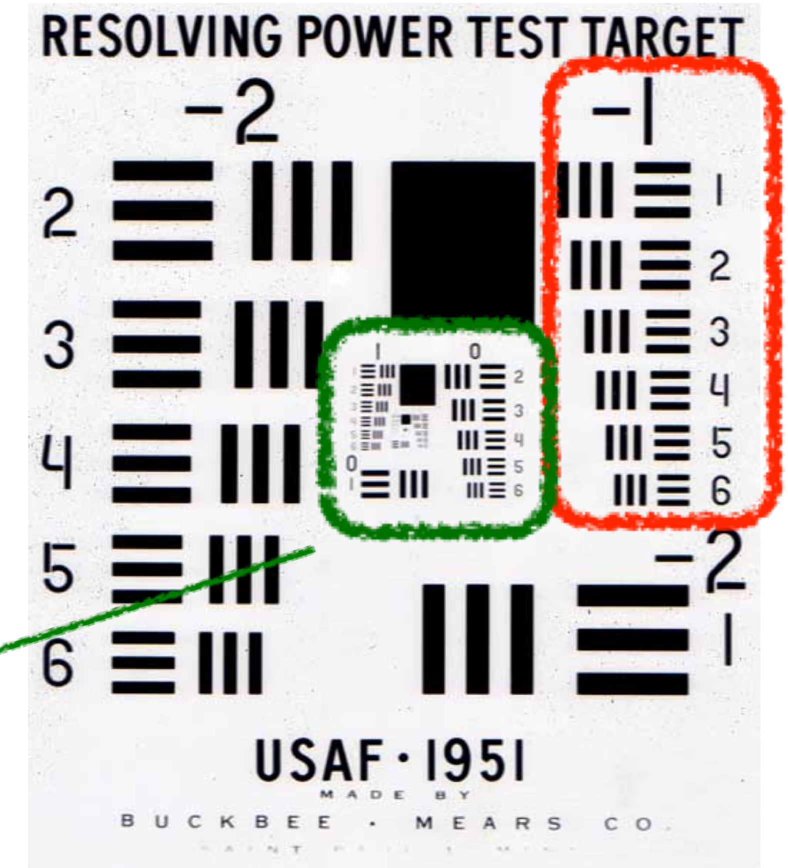




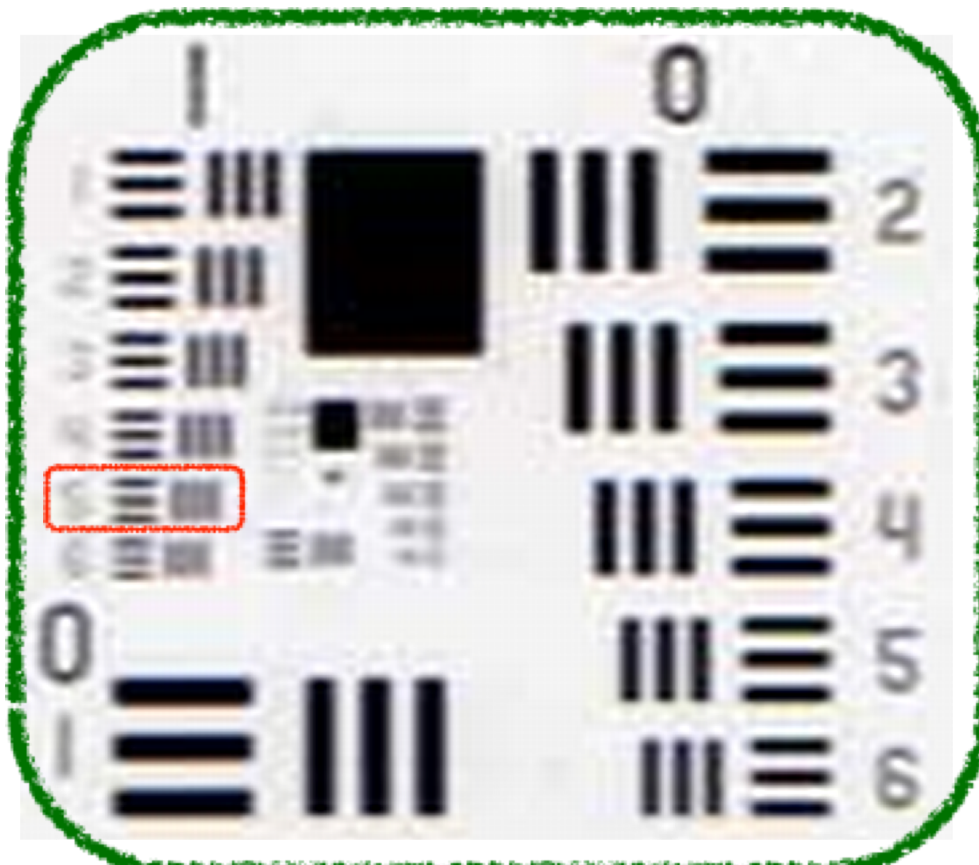
19.43 μm/pix

USAF 1951-target

- To compare system performances with theoretical ones
  - convert spatial frequency of the target to spatial frequency in the image plane



Group "-1" with the elements "1-6"



Group 1: element 5 is still resolved; element 6 can't be resolved (the bars blur)

USAF 1951-target

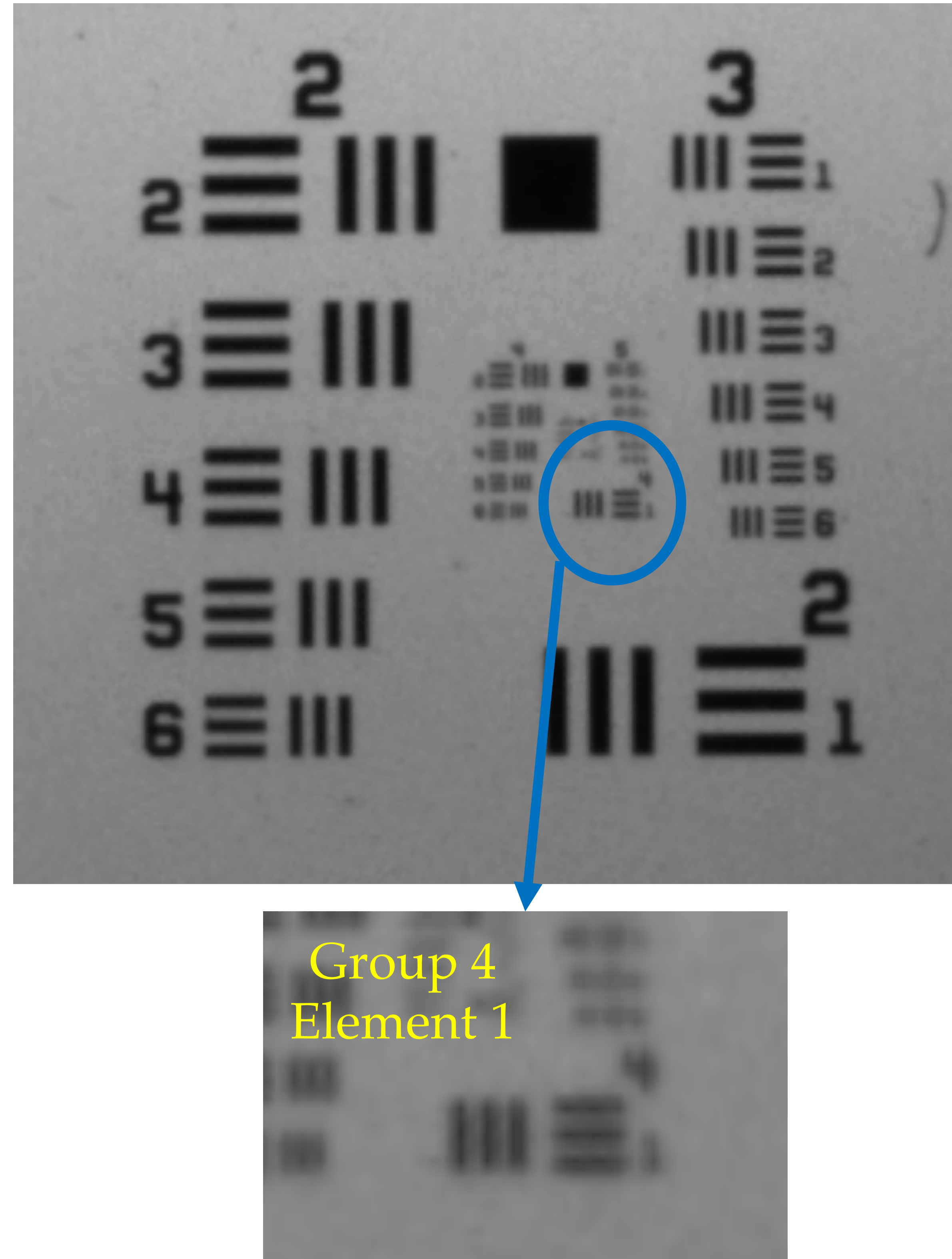
$$Resolution\left(\frac{line\ pair}{mm}\right) = 2^{Group+\left(\frac{Element-1}{6}\right)}$$

Target resolution, R

Element	Group Number									
	-2	-1	0	1	2	3	4	5	6	7
1	0.250	0.500	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00
2	0.280	0.561	1.12	2.24	4.49	8.98	17.95	36.0	71.8	144.0
3	0.315	0.630	1.26	2.52	5.04	10.10	20.16	40.3	80.6	161.0
4	0.353	0.707	1.41	2.83	5.66	11.30	22.62	45.3	90.5	181.0
5	0.397	0.793	1.59	3.17	6.35	12.70	25.39	50.8	102.0	203.0
6	0.445	0.891	1.78	3.56	7.13	14.30	28.50	57.0	114.0	228.0

Values are in *line pairs/mm*

USAF 1951-target: Example



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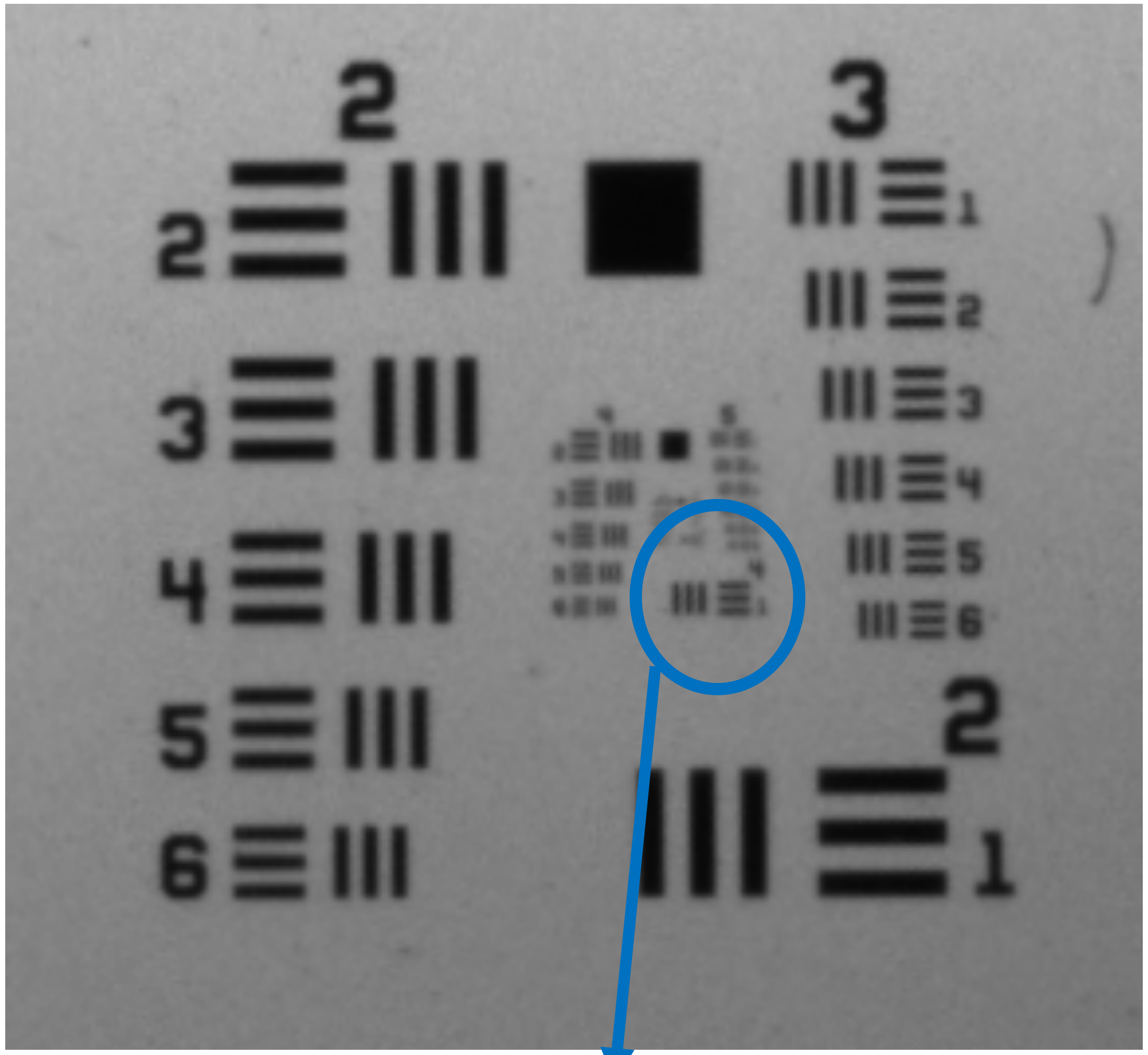
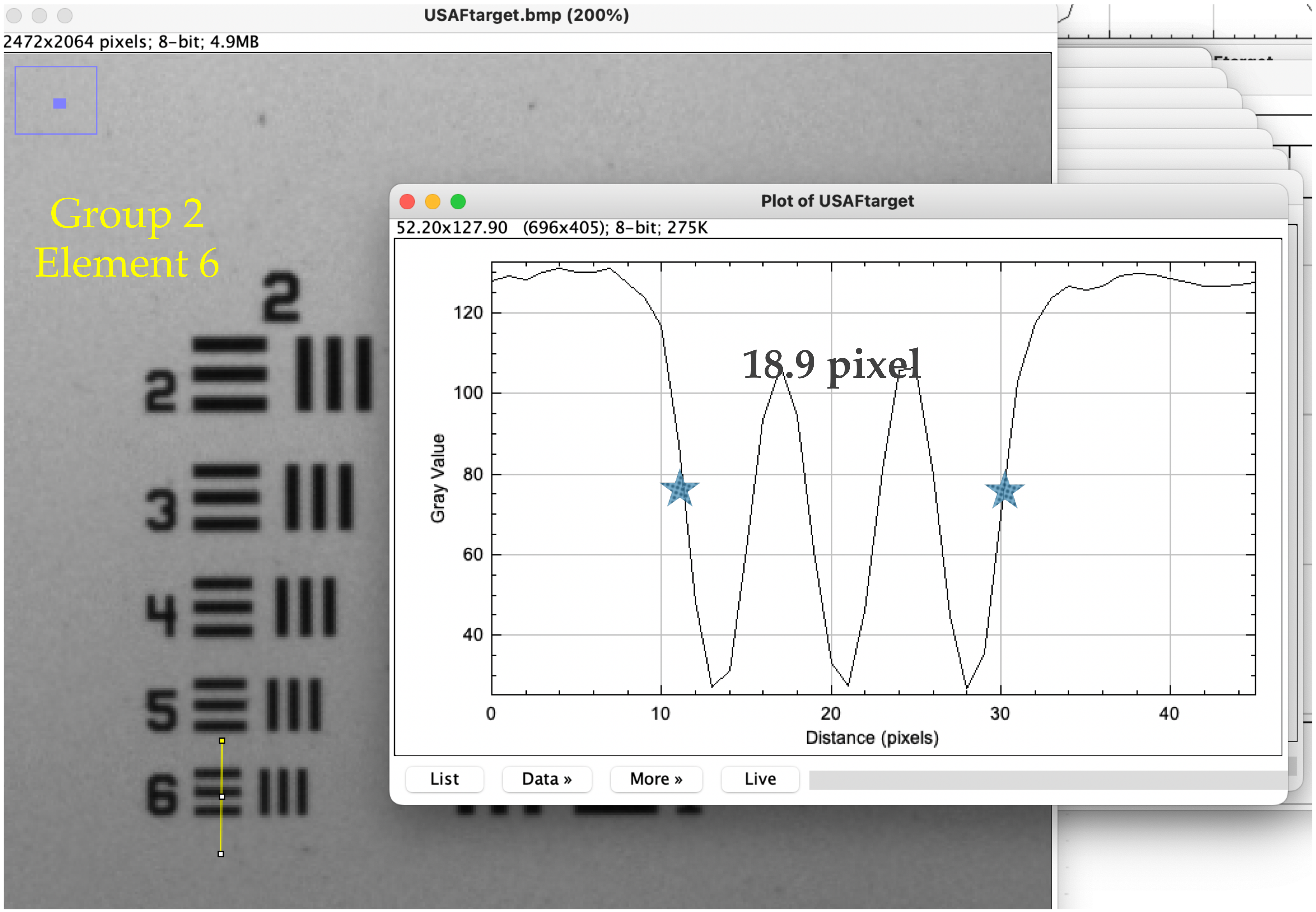
USAF 1951-target: Example



16 line pair/mm ➔ 16*9.16 pix/2500 um ➔ 17 um/pix

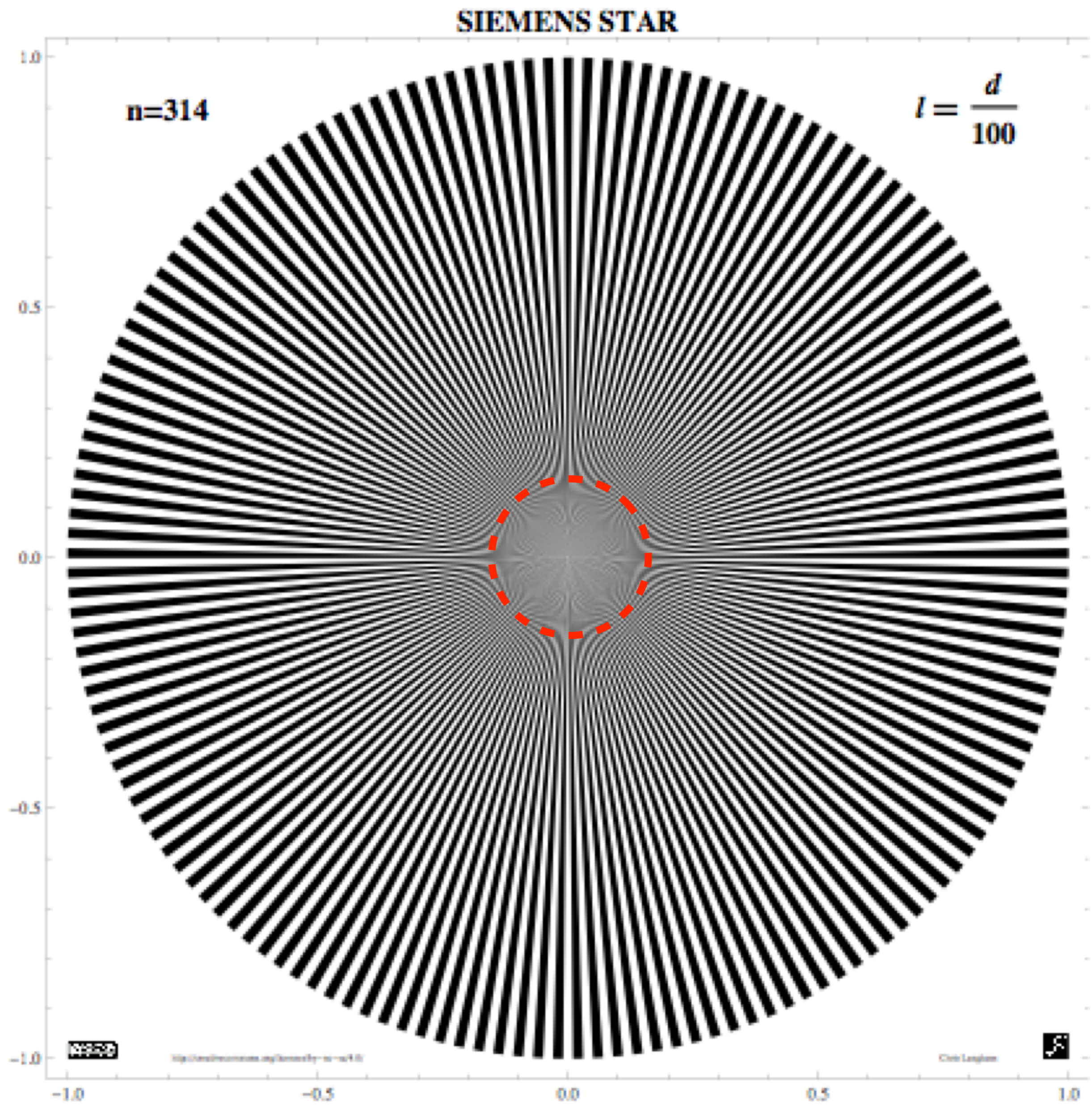
USAF 1951-target: Example



7.13 line pair/mm ➔ 7.13*18.9 pix/2500 um ➔ 18.55 um/pix

Siemens star

- It consists of alternating black and white thin "pie shaped" segments: moving towards the center of the star, the lines get closer and closer together.
  - The higher the resolution of the system generating the star pattern, the closer to the center of the star they will appear to merge.



circle of confusion

Siemens star

Sector Star Targets

Click to Enlarge
Close Up of the R1L1S3P Sector Star Pattern

Sector star targets, also known as Siemens star targets, consist of a number of dark bars that increase in thickness as they radiate out from a shared center. The blank spaces between the bars can themselves be thought of as light bars, and they are designed to be the same thickness as the dark bars at any given radial distance. Theoretically, the bars meet only at the exact middle point of the target. Some sector star targets, including all those sold on this page, have a blank center circle that cuts the bars off before they touch. However, depending on the resolution of the optical system through which the targets are viewed, the bars will appear to touch at some distance from the center. By measuring this distance, the user is able to define the resolution of the optical system.

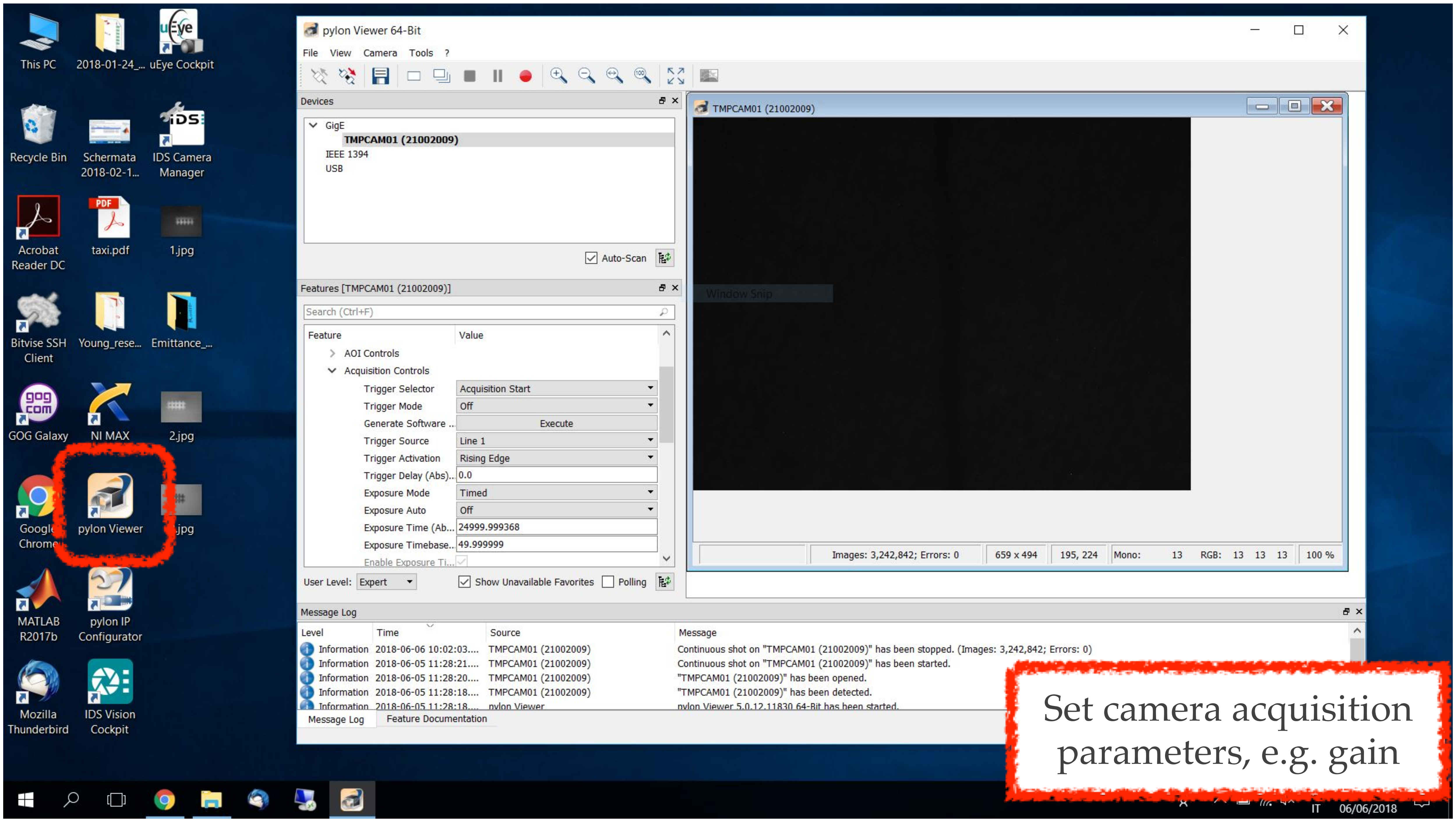
To calculate the resolution at any given radial distance from the center of the sector star, start by calculating the thickness of a line pair, or one dark bar and one light bar, at that radius. This can be done using the formula for the chord length, given below, where *r* is the radial distance from the center. The angle Θ is the number of degrees covered by one pair of light and dark bars and is equal to 360° divided by the total number of bars. Once the thickness of the line pair is calculated, the resolution is the reciprocal of the thickness.

$$c = 2r * sin(\frac{\theta}{2}) \quad Resolution = \frac{1}{c}$$

Thorlabs offers two dedicated sector star targets (R1L1S2P and R1L1S3P) and three targets that include sector stars along with other patterns (R1L3S5P, R1L1S1P, and R1L1S1N). The table below summarizes the sector star pattern on each target.

Item #	Pattern Type	Sector Star Pattern Outer Diameter	Center Circle Diameter	Number of Bars	Resolution at Outer Diameter	Resolution at Center Circle
R1L1S2P	Positive	10 mm	200 µm	36 Over 360°	1.15 lp/mm	57.5 lp/mm
R1L1S3P				72 Over 360°	2.29 lp/mm	115 lp/mm
R1L3S5P	Positive	2 mm	100 µm	36 Over 360°	5.75 lp/mm	115 lp/mm
R1L1S1P	Positive	2 mm	20 µm	36 Over 360°	5.75 lp/mm	575 lp/mm
R1L1S1N	Negative					

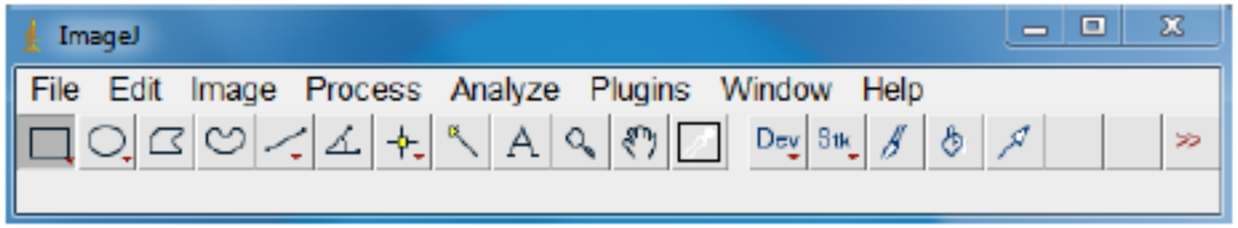
CCD readout



Set camera acquisition parameters, e.g. gain

ImageJ Introduction

press icon access to start panel

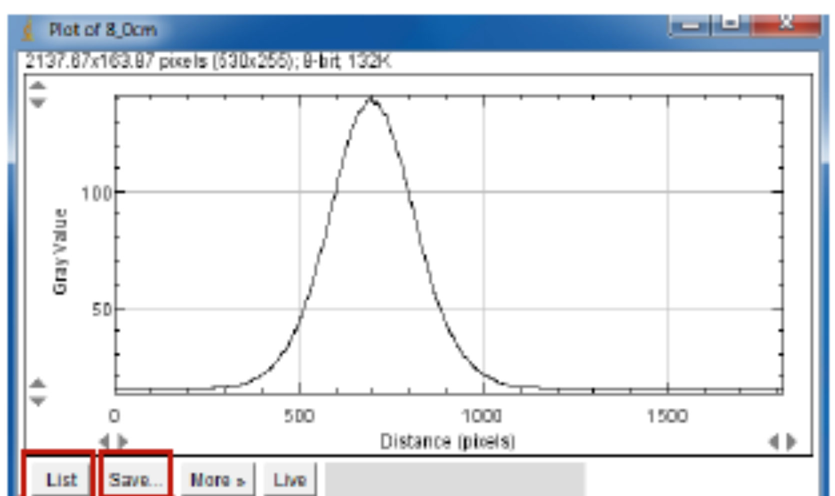


load image file → File → Open (Shortcut: Ctrl + O)

select ROI: in start panel: select left button (below "File"), usually already pre-selected

then with left mouse button: draw rectangular ROI

plot horizontal projection → Analyze → Plot Profile (Shortcut: Ctrl + k)



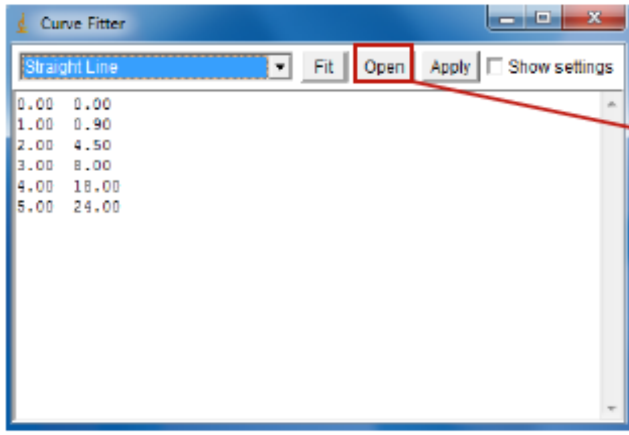
list data points

save data as Excel sheet (required for profile fitting)

ImageJ Introduction

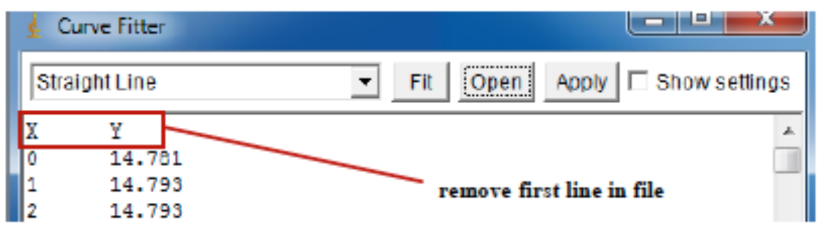
profile fitting → Analyze → Tools → Curve Fitting…

- load profile data:



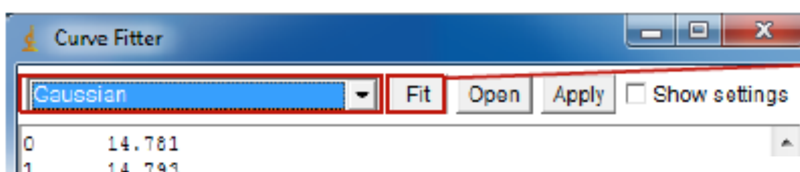
load data file

- delete bad data:



remove first line in file

- select fit function:

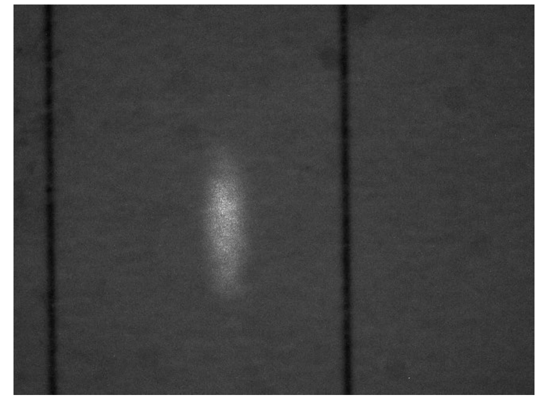


fit profile data

$$y = a + (b-a)\cdot e^{-\frac{(x-c)^2}{2d^2}}$$

fit results:

Signal - Background



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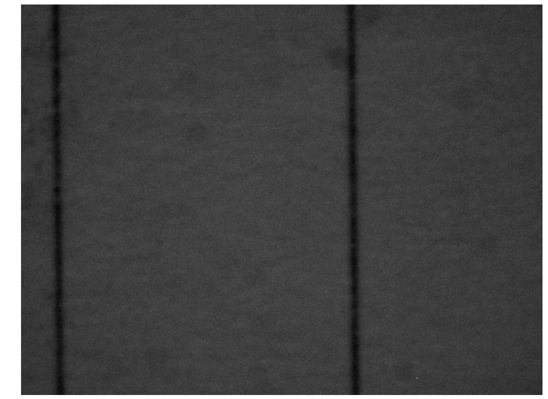

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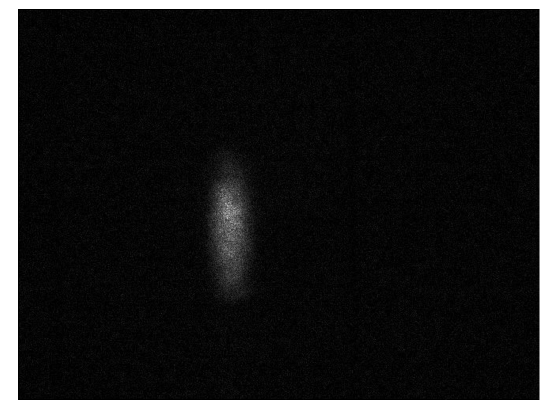

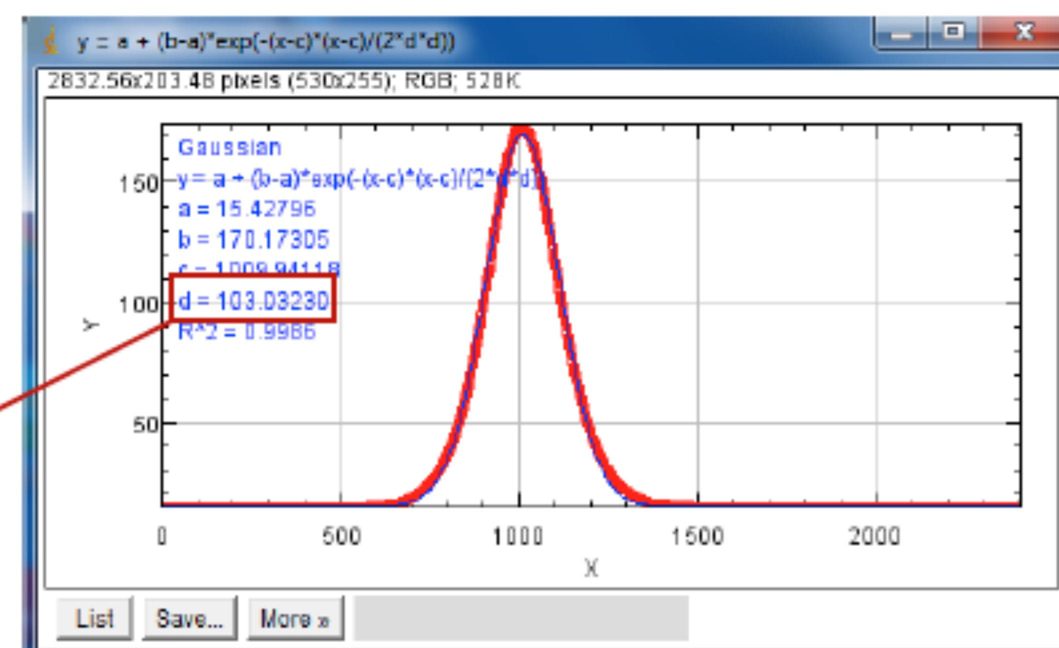

1 σ-width (in pixel)

Acknowledgements

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