Evaluating Summary Statistics with Mutual Information for Cosmological Inference

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In collaboration with

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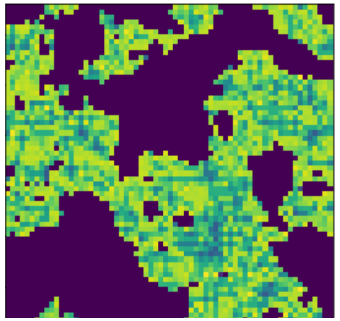
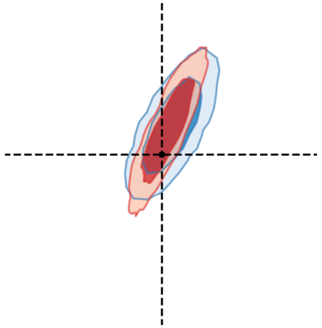
Statistical Inference in Cosmology

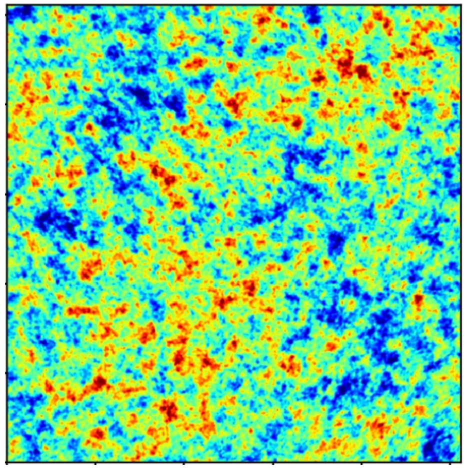
Observation → Summary Statistics → Inference → Constraint on models





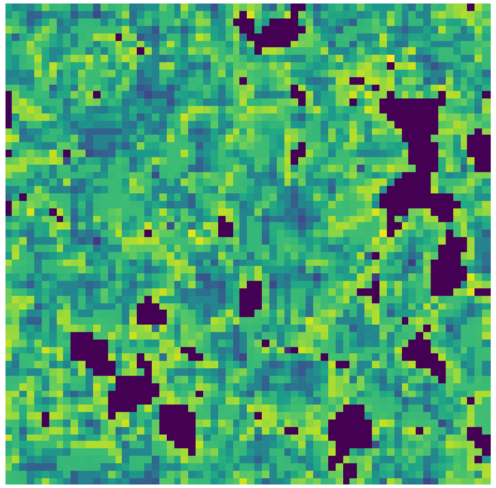
How to select optimal statistics ?

CMB



→ Power Spectrum

21cm

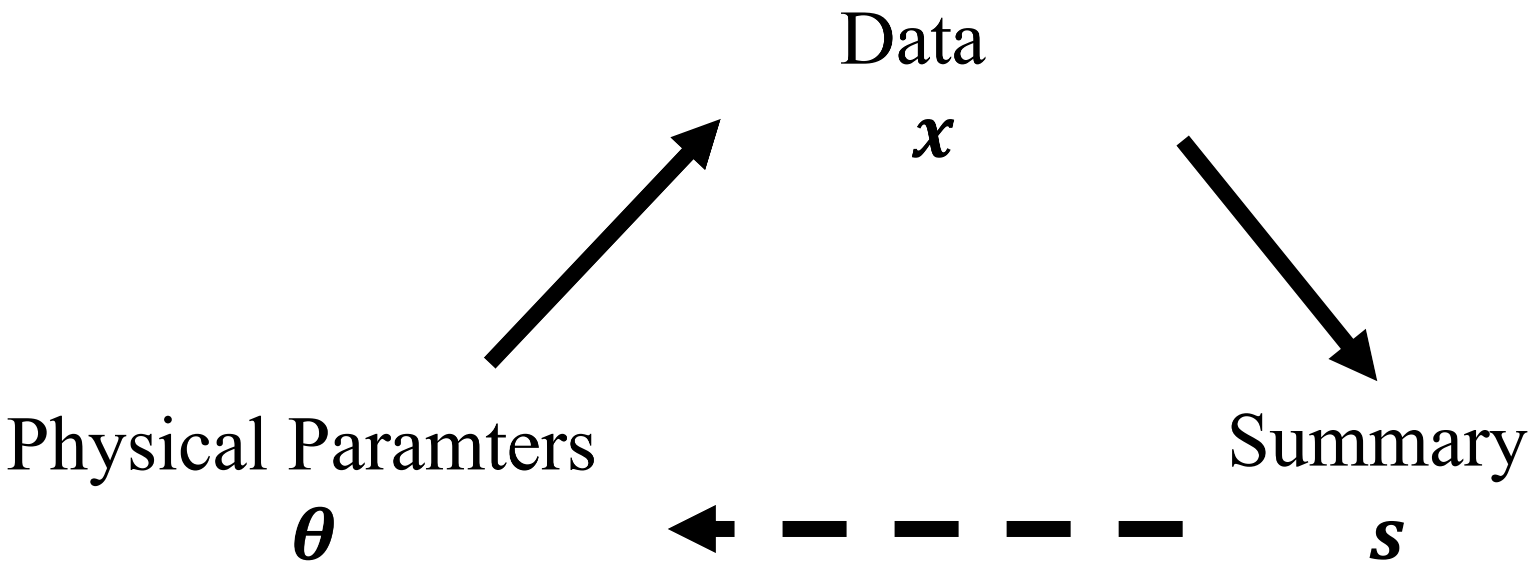


Spatial Correlation → Power Spectrum, Bispectrum

Morphological Properties → Minkowski Functionals / Bubble Size Distribution

Signal decomposition → Wavelet Transform

How to select optimal statistics ?



1. Through theoretic interpretation

2. Fisher Analysis

3. Running inferences with one mock observation

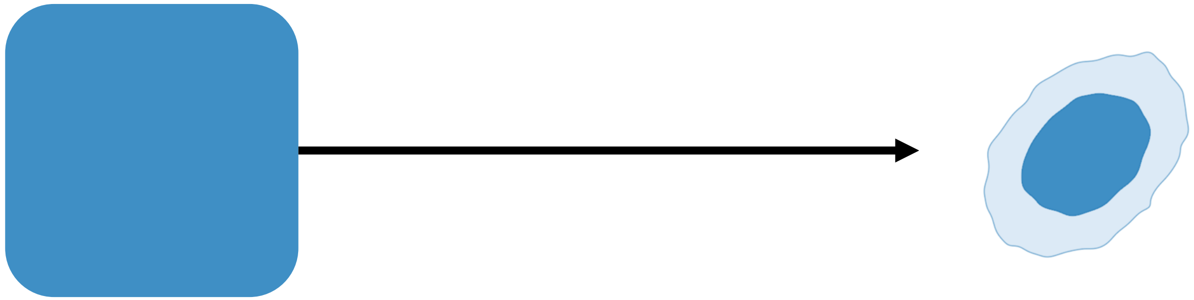
Method: Mutual information

Mutual information $$I(\theta; x) = \boldsymbol{D_{KL}}[p(\theta, x)||p(\theta)p(x)] = E_{p(x)}[\boldsymbol{D_{KL}}(p(\theta|x)||p(\theta))]$$

MI is a fundamental measure of Statistical Dependence.

It quatifys how much uncertainty is reduced in θ given x

Prior



Posterior

$\boldsymbol{p(\theta)}$ $\boldsymbol{p(\theta|x)}$

Method: Mutual information

Mutual information

$$I(\theta; x) = \boldsymbol{D_{KL}}[p(\theta, x)||p(\theta)p(x)] = E_{p(x)}[\boldsymbol{D_{KL}}(p(\theta|x)||p(\theta)]$$

MI is a fundamental measure of Statistical Dependence.

It quatifys how much uncertainty is reduced in θ given x

How to estimate mutual information?

$$I(\theta; x) \equiv E_{p(\theta,x)}\left[\log\frac{p(\theta|x)}{p(\theta)}\right] \approx E_{p(\theta,x)}\left[\log\frac{\boldsymbol{q(\theta|x)}}{p(\theta)}\right]$$

Variational Lower Bound

Summary Statistics Considered in this work

Power Spectrum

$$\langle \delta(k)\delta(k')\rangle = (2\pi)^3 \delta^D(k + k')P(k)$$

Bipectrum

$$\langle \delta(k_1)\delta(k_2)\delta(k_3)\rangle = (2\pi)^3 \delta^D(k_1 + k_2 + k_3)B(k_1, k_2, k_3)$$

Scattering Transform

$$S\delta[\lambda_1, \ldots, \lambda_k] = \left|\psi_{\lambda_k} \star \cdots |\psi_1 \star \delta|\right|$$

Use designed wavelets to convolve the field

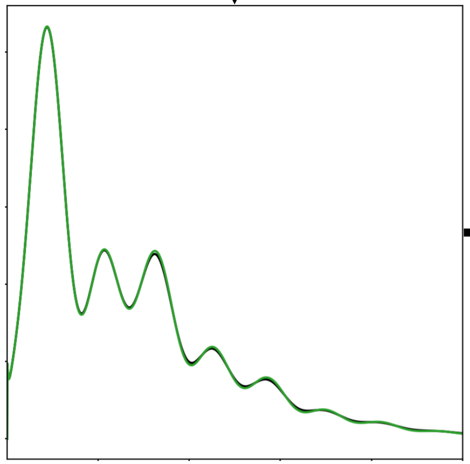
(Anden & Mallat 2014, Eickenberg et al. 2017,2018)

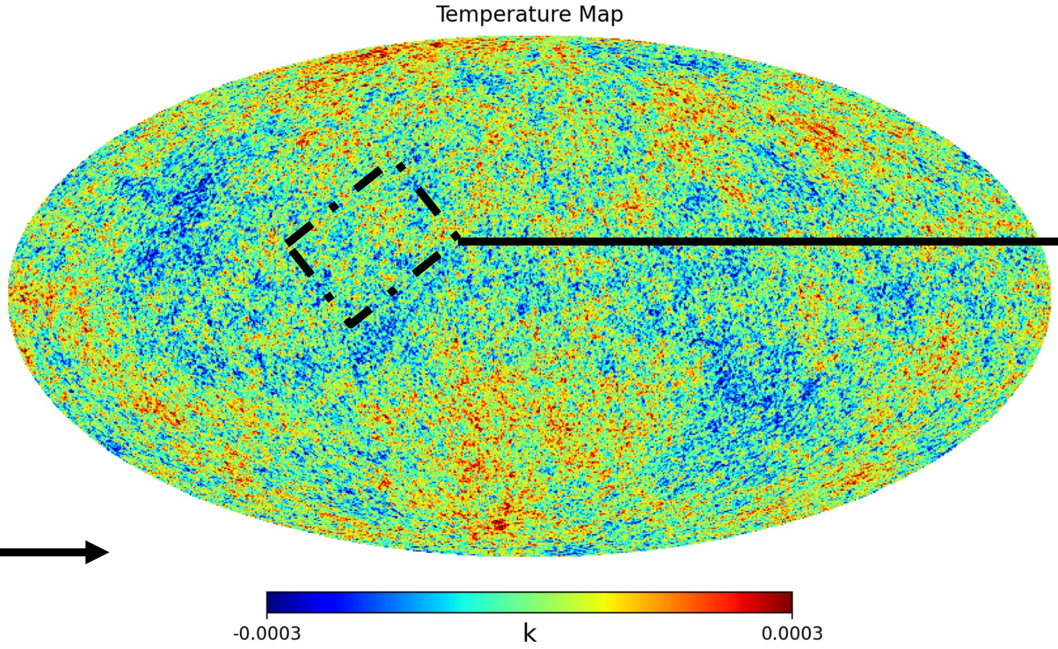
Experiments I:

Validate the method in CMB-like Gaussian random fields(GRF)

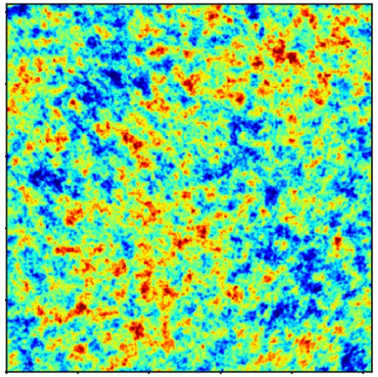
$\boldsymbol{\Omega_c} \in [\mathbf{0.21, 0.31}]$
$\boldsymbol{h} \in [\mathbf{0.64, 0.76}]$

Use ***CAMB*** to generate Power Spectral



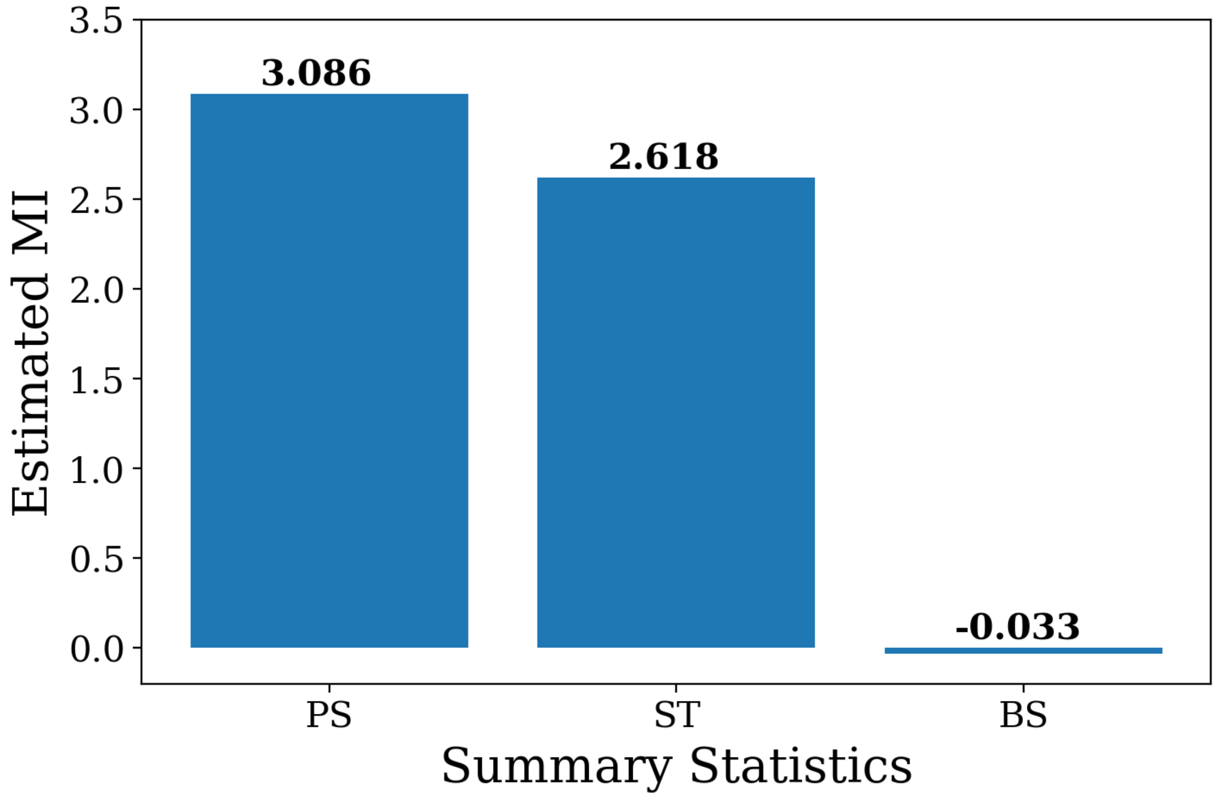


Use ***Healpy*** to generate CMB-like GRF



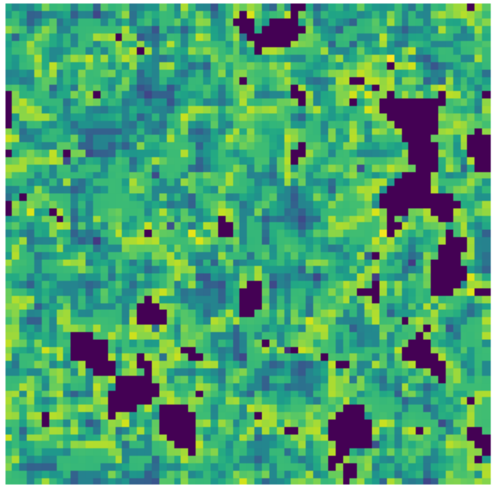
Patch for experiments

Experiments I:
Validate the method in CMB-like Gaussian random fields(GRF)



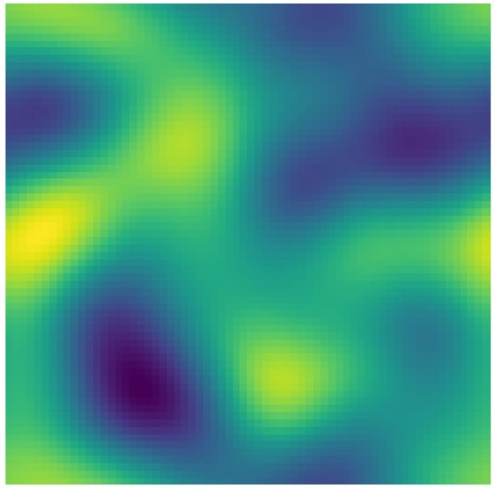
PS is optimal and BS contains no information

Experiments II:
Evaluate Statistics in an EoR inference Task

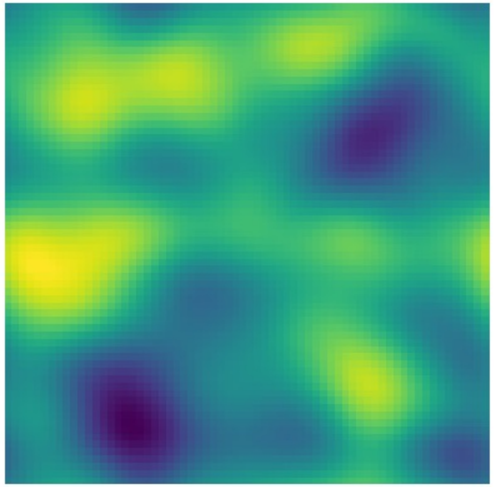


Pure 21cm signal (simulated by 21cmFAST)

Mesinger & Furlanetto (2007)



+Thermal Noise (SKA1-low configuration)



+Residual Foreground (GSM model +SVD)

Zheng et al. (2017)

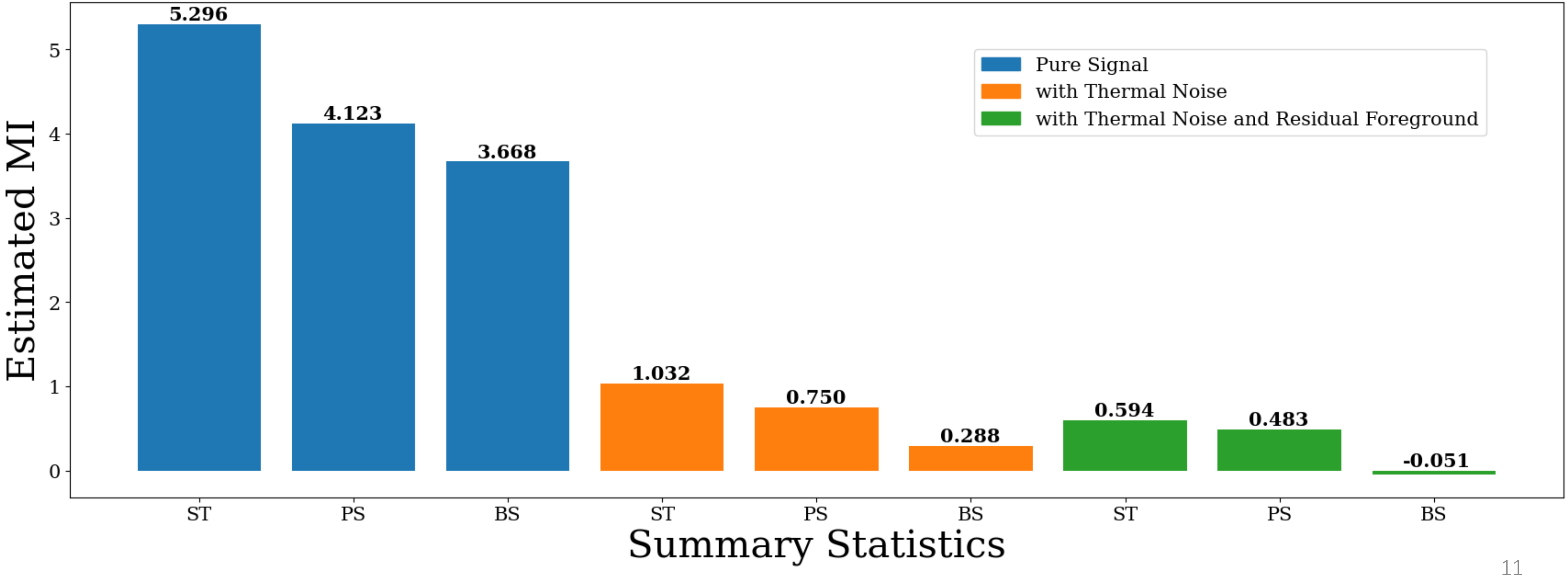
Reionization Parameters

ζ the ionizing efficiency

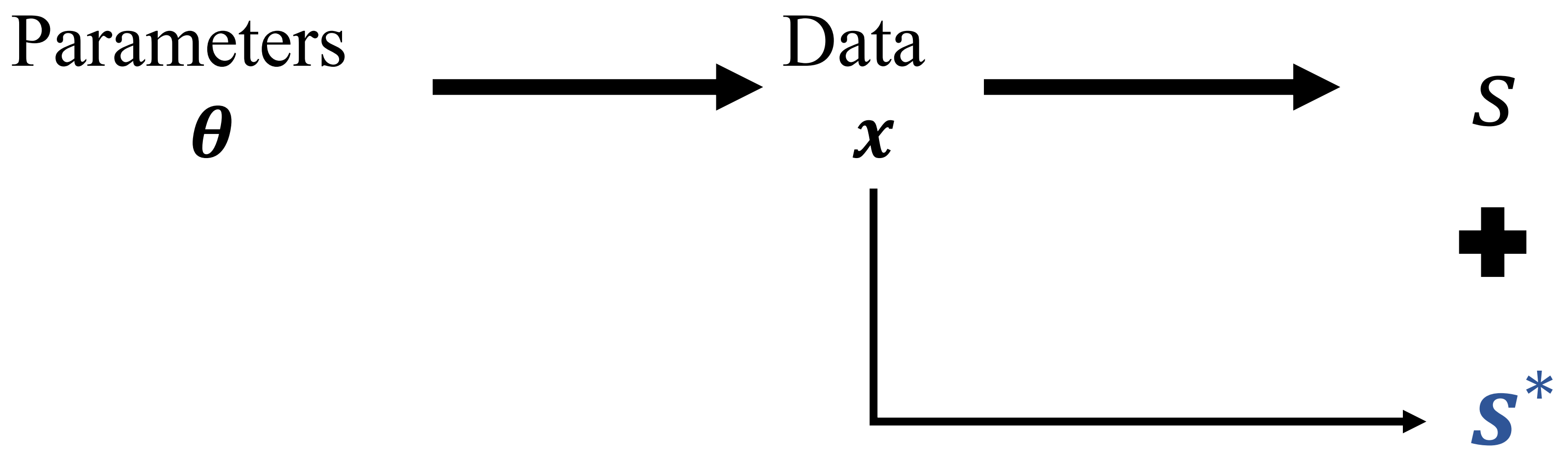
T_{vir} the minimum virial temperature of halos that host ionizing sources

Experiments: MI-based comparison of statistics in an EoR Inference task

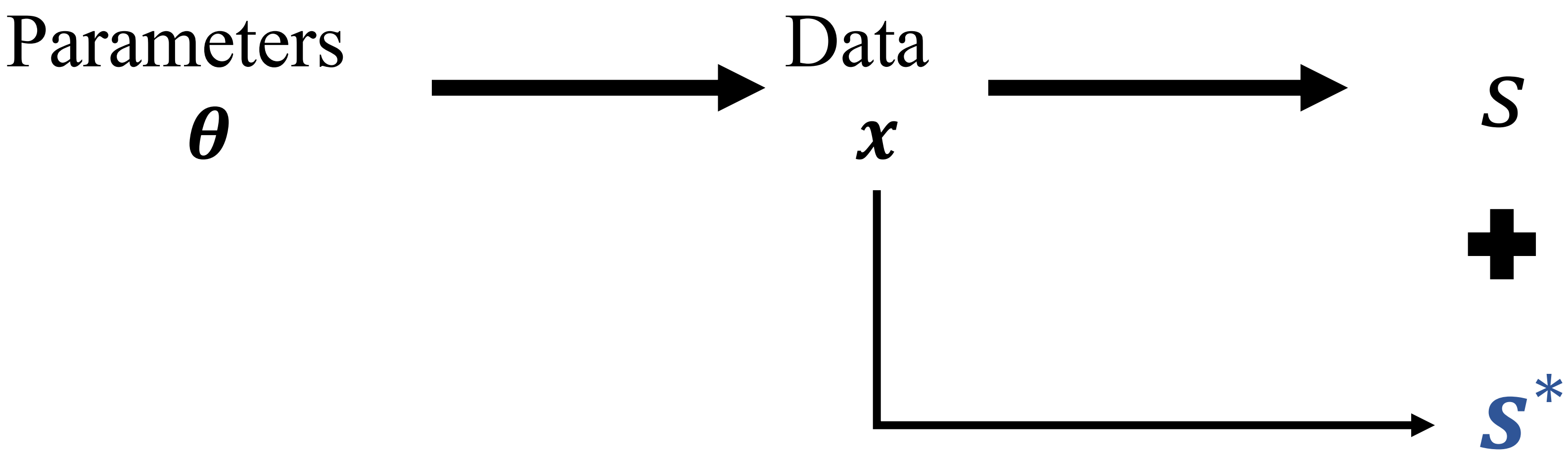
ST: Wavelet Transform **PS:** Power spectrum **BS:** Bispectrum



Mutual Information for evaluating Complementary Summary

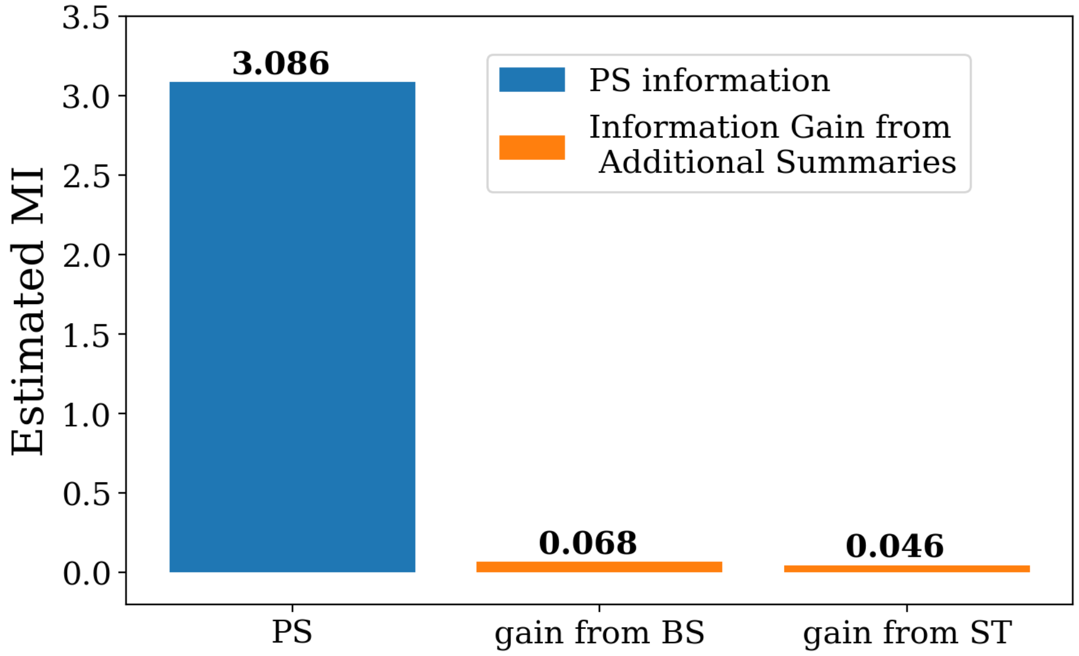


Mutual Information for evaluating Complementary Summary

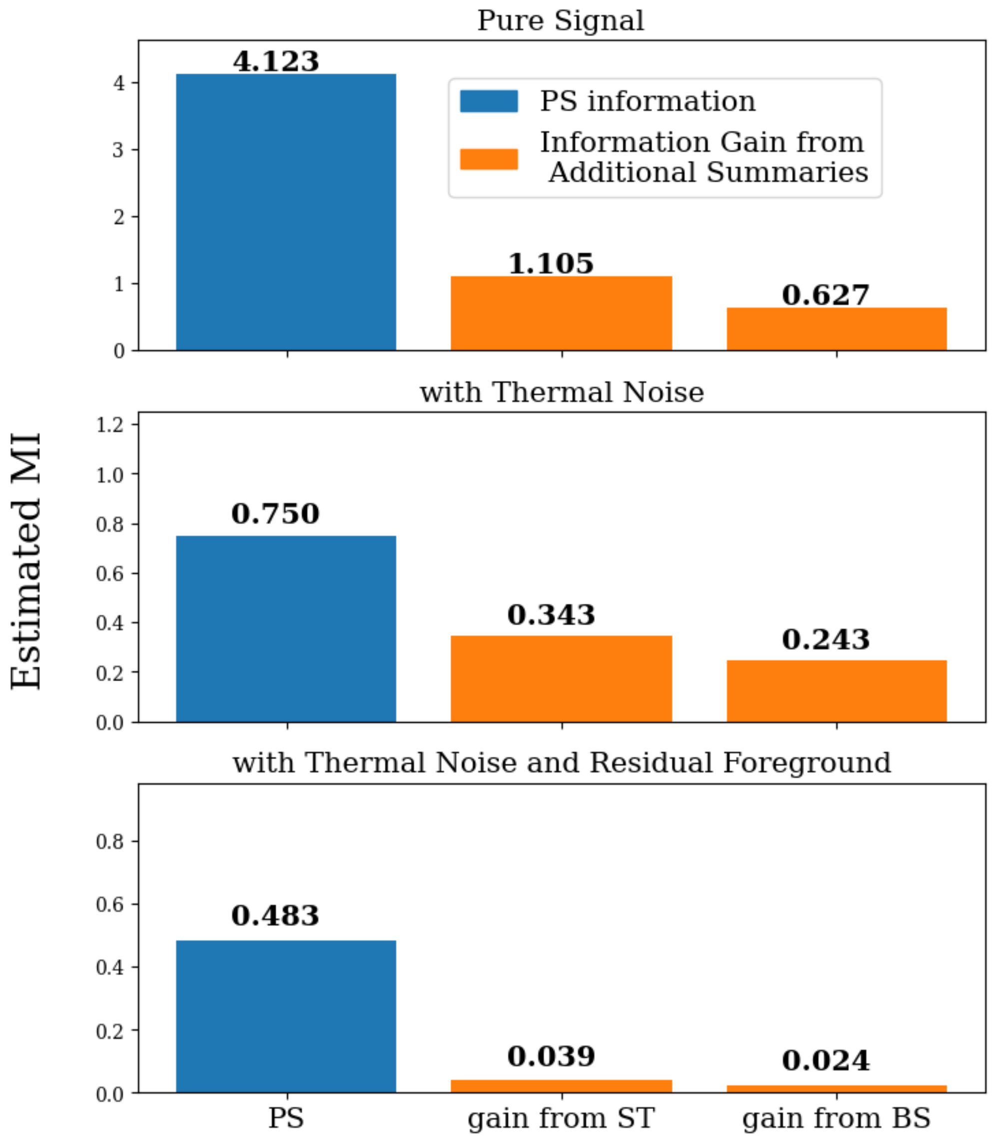


Conditional Mutual Information: $I(\theta; S^* | S)$

Mutual Information for evaluating Complementary Summary



CMB Experiment

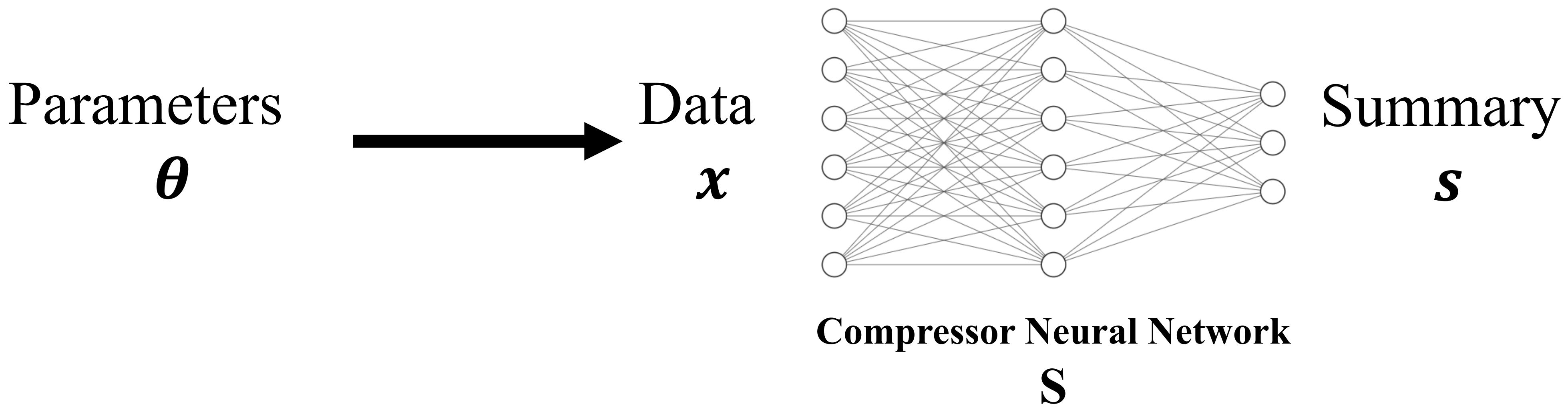


21cm Experiment

Mutual Information for Learning Summaries

1. Learn a new optimal summary by maximizing mutual information

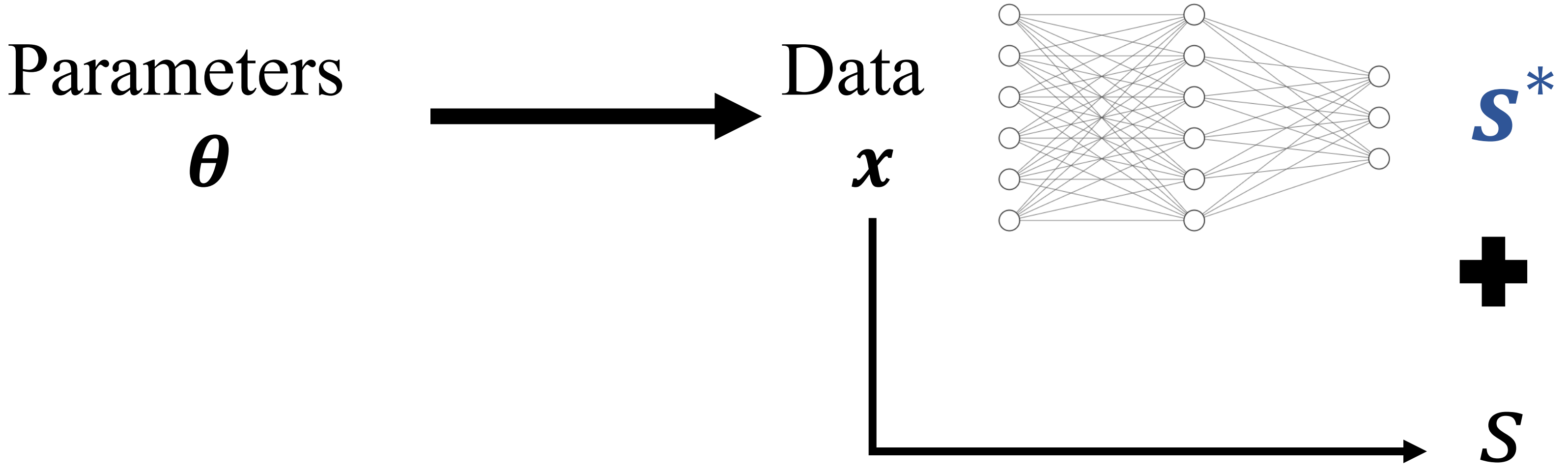
$$\max_{S} I(\theta; S(x))$$



Mutual Information for Learning Summaries

2. Learn a complementary summary by maximizing conditional mutual information

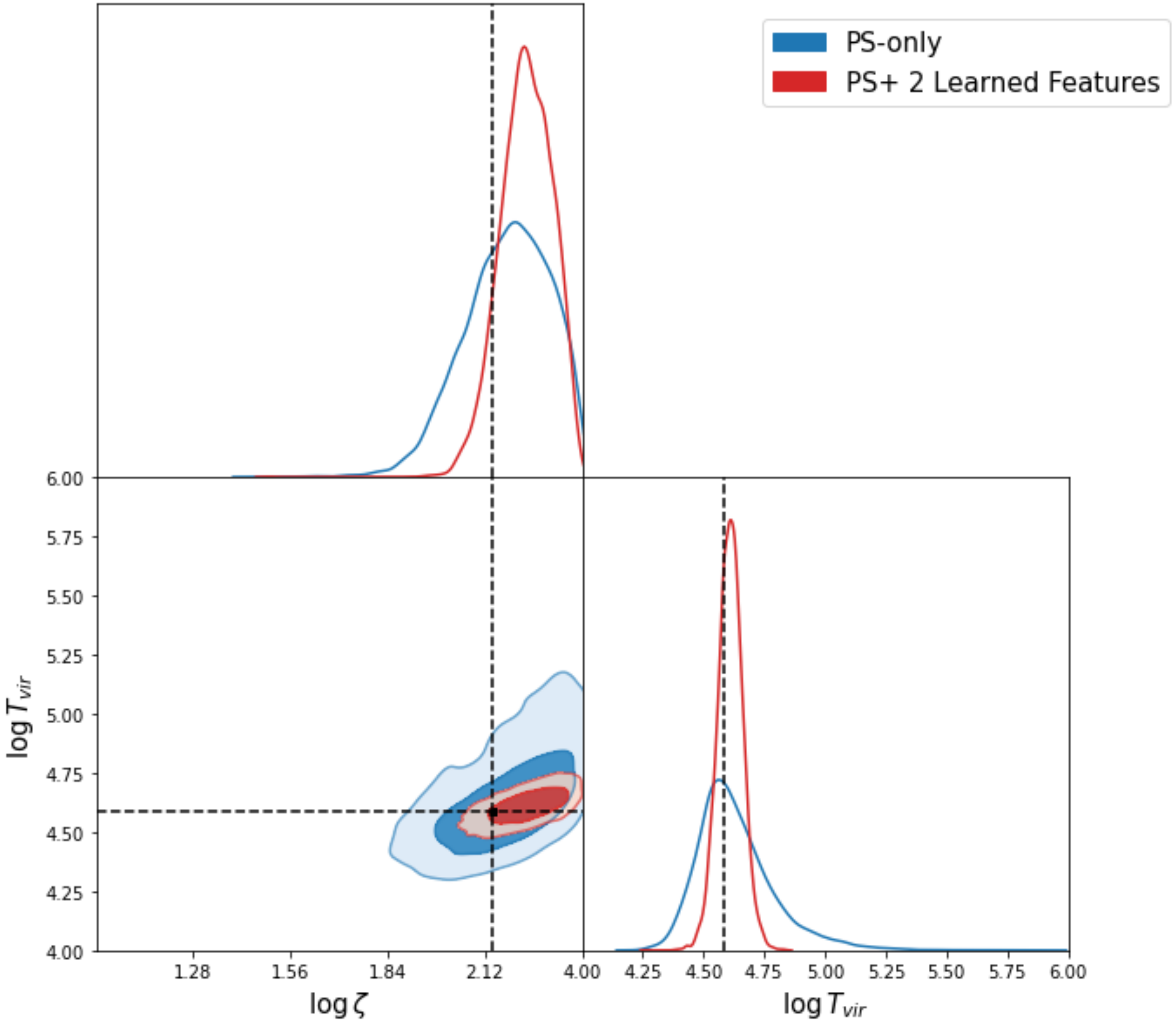
$$\max_{S} I(\theta; S^*(x) | S)$$



2. Learn a complementary summary by maximizing conditional mutual information

Learn two complementary features for Power Spectrum

Experiment:
Infer Reionization Parameter from 21cm images.



Summary

Mutual Information
- → Evaluate Information Content
- → Decompose Information
- → Learn new/complementary Summaries

More Details:

ICML 2023 AI4Astro Workshop: https://arxiv.org/abs/2307.04994

1–2 additional papers are expected to be released next month.

Part of the experimental results can be found on: https://github.com/suicee/MI4StatsEval