

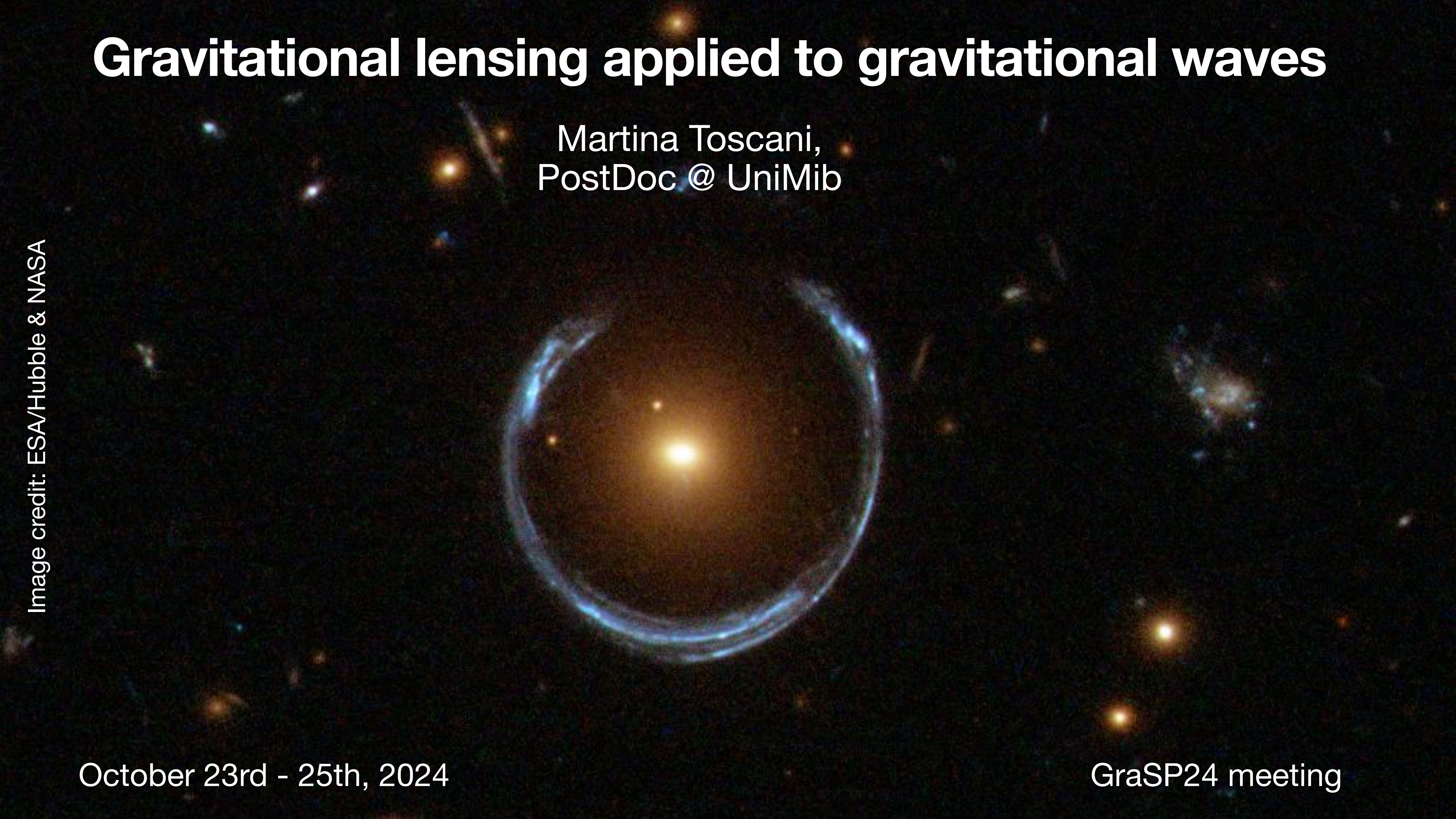
# Gravitational lensing applied to gravitational waves

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Image credit: ESA/Hubble & NASA

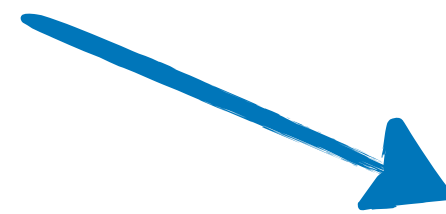
October 23rd - 25th, 2024

GraSP24 meeting



# Outline

- Lensing landscape and useful definitions
- Strong lensing applications within the LISA context
- Correct identification of lensed events



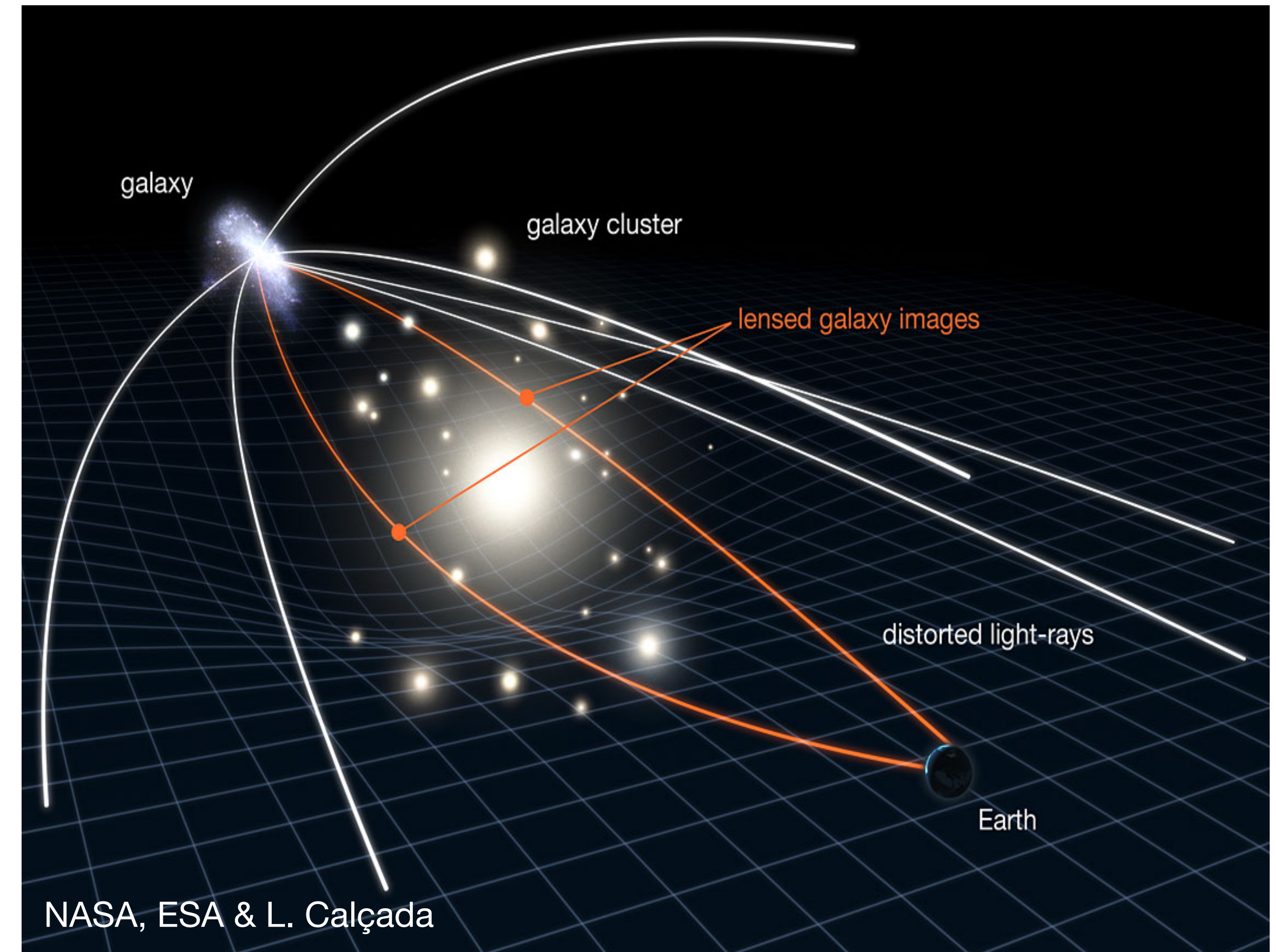
See the following talk!

# *What is gravitational lensing ?*

Consequence of deflecting light, waves or particles near concentrated mass distributions due to space-time curvature

Prediction of general relativity (also in Newton's theory)

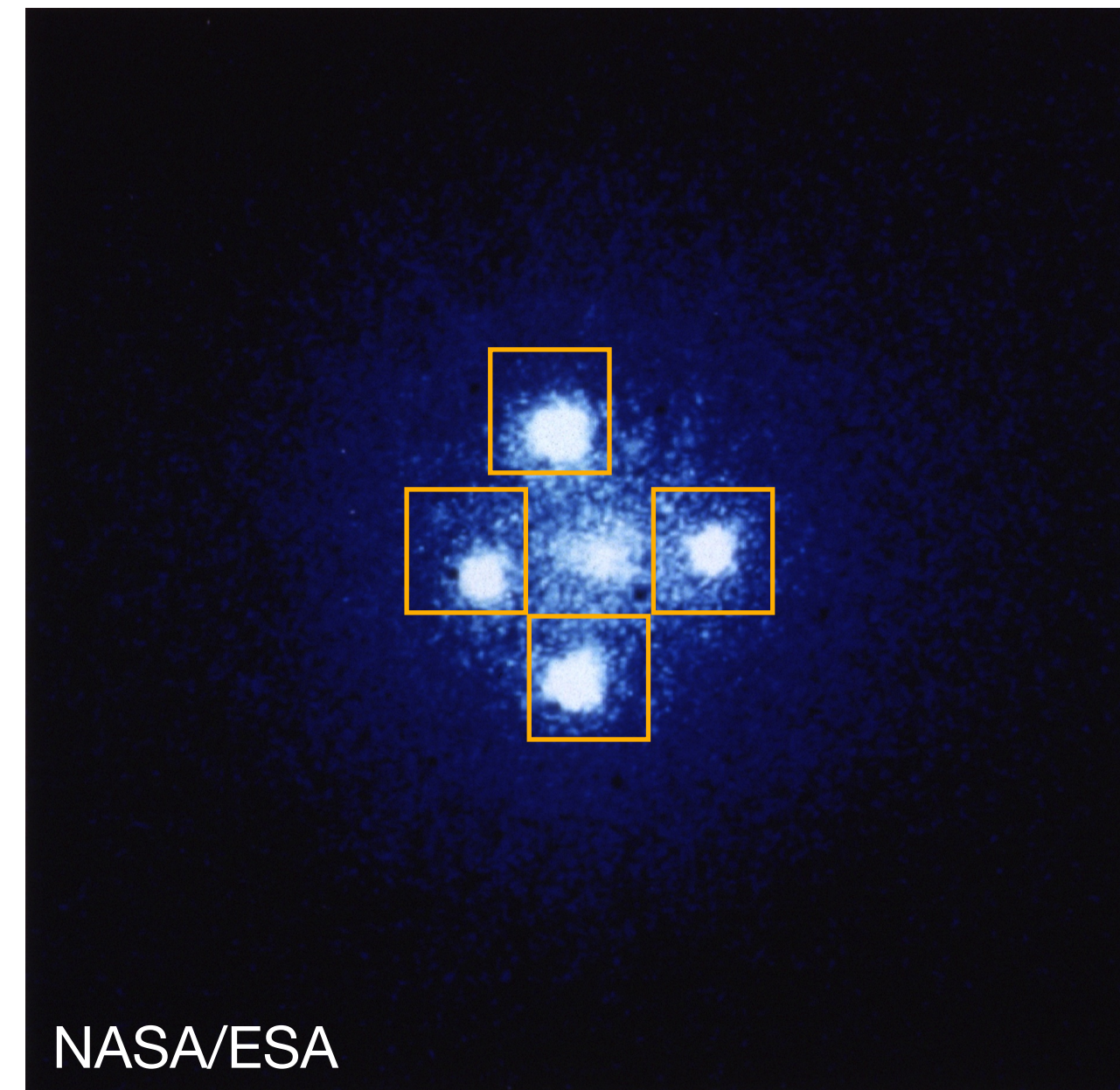
Observationally confirmed by Dyson, Eddington and Davidson during the Solar eclipse of 1919



# *What is gravitational lensing ?*

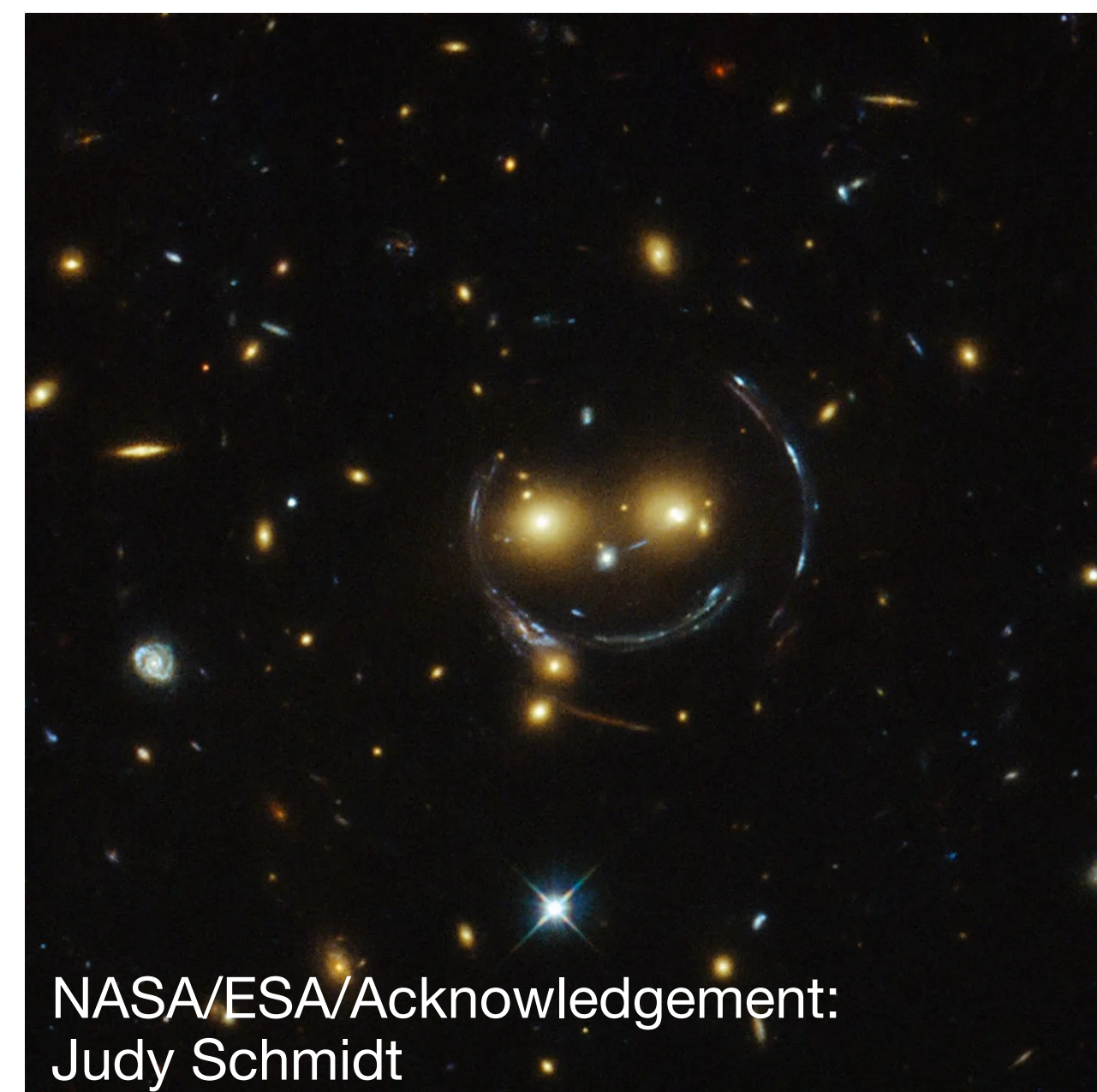
- Multiple images of quasars
- Magnified and distorted images of background galaxies
- Luminous arcs around galaxies

The same principle apply to electromagnetic (EM) waves and gravitational waves (GWs)



4 images same quasar

Einstein's cross



Hubble's happy face

Einstein's ring

# What is gravitational lensing ?

**Wave optics**

$$\lambda \gtrsim R_s \rightarrow M_L \lesssim 10^8 M_\odot \left( \frac{f}{\text{mHz}} \right)^{-1}$$

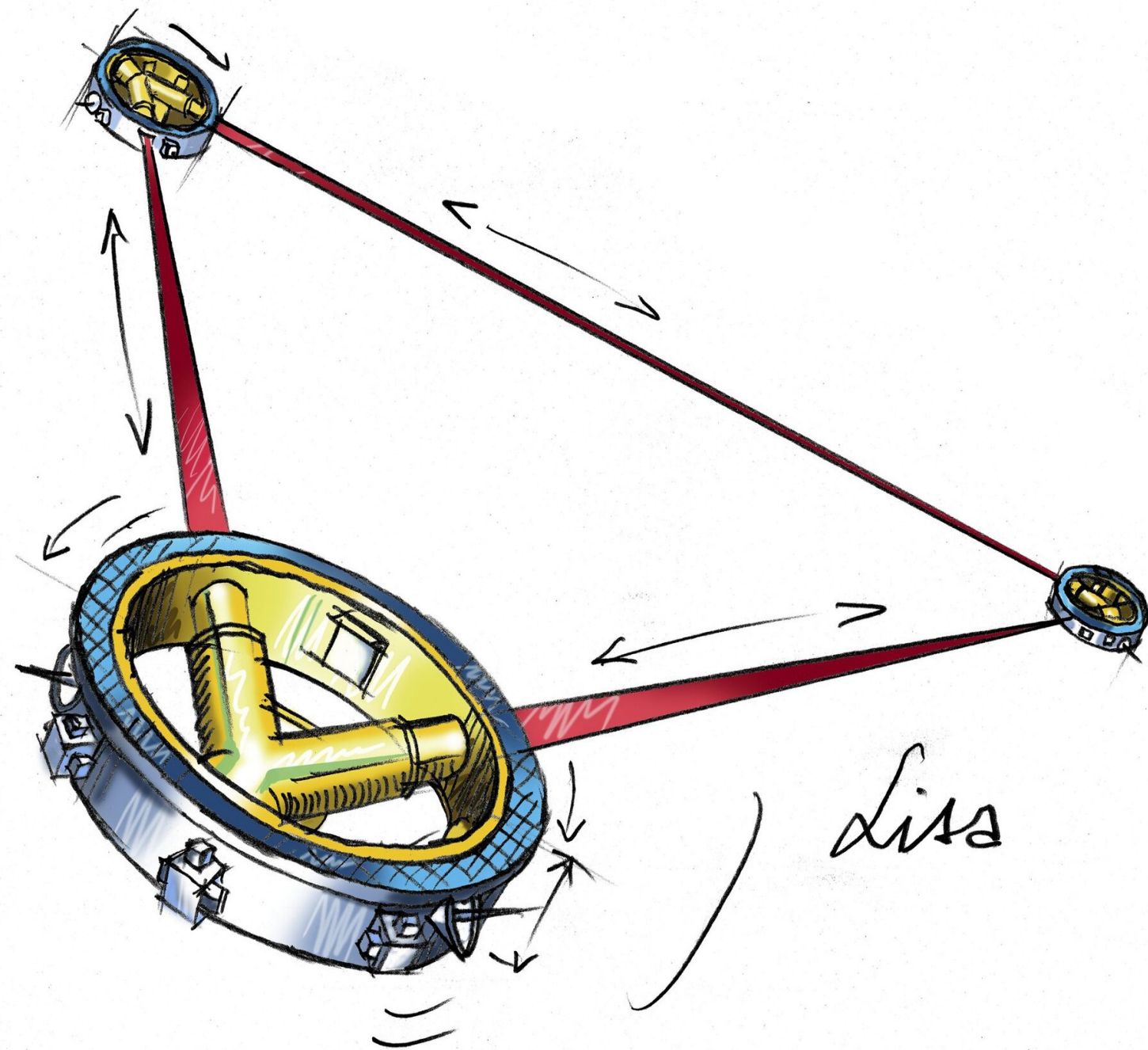
**Geometric optics**

$$\lambda \lesssim R_s \rightarrow M_L \gtrsim 10^8 M_\odot \left( \frac{f}{\text{mHz}} \right)^{-1}$$

$$10^{-4} \text{ Hz} \leq f \leq 10^{-1} \text{ Hz}$$

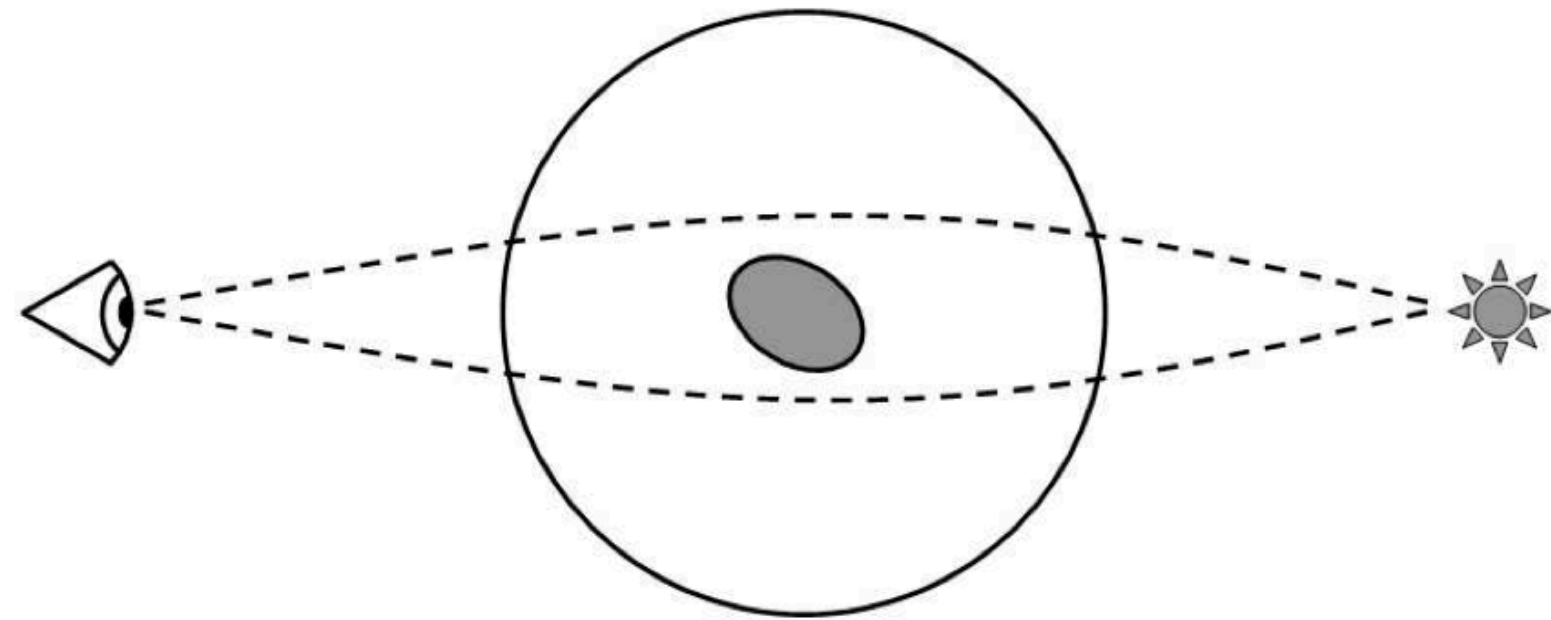
With lenses and galaxy clusters as lenses we can neglect diffraction

Esa-C. Vijoux



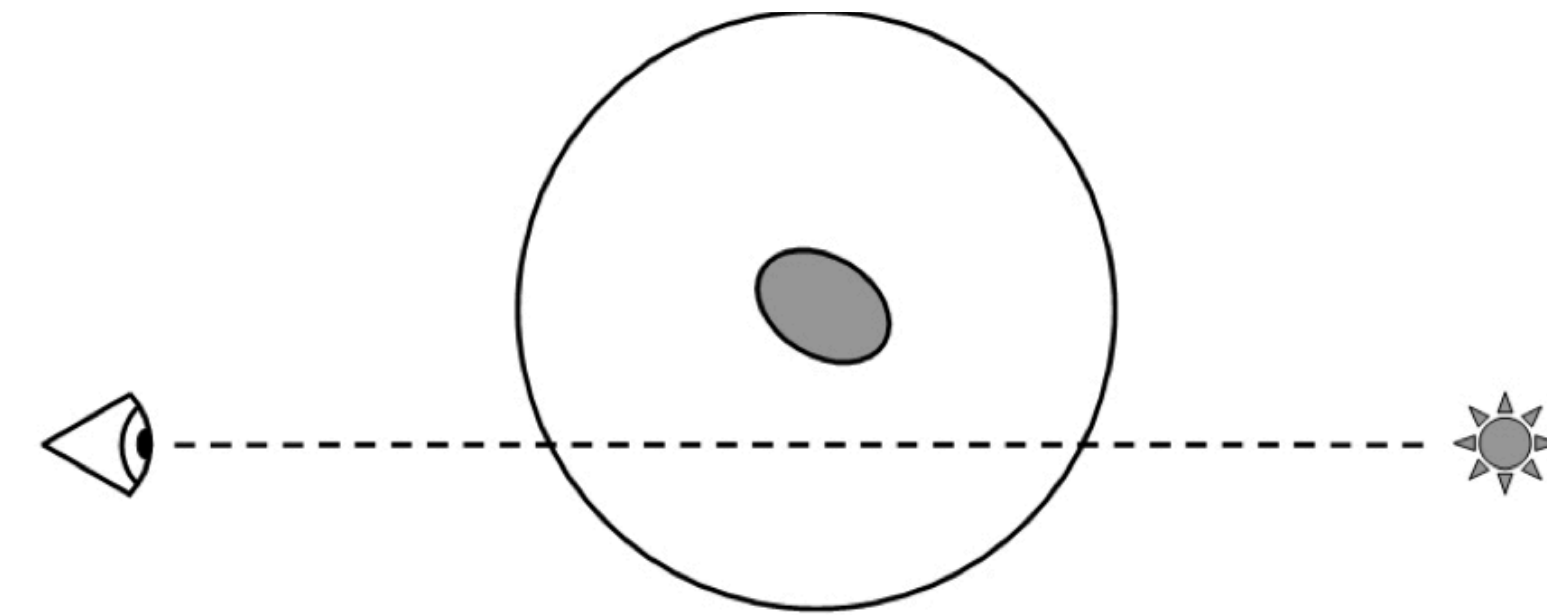
# *What is gravitational lensing ?*

Zackrisson and Riehm 2010



## **Strong lensing**

- Rare
- Individual detections
- High magnification and multiple images



## **Weak lensing**

- More common
- Statistical studies
- Slight distortions

# *Strong lensing: MBHBs*

Dai 2007, Sereno et al. 2010, Sereno et al. 2011, Ezquiaga et al. 2020, Goyal et al. 2020, Hannuksela et al. 2020, Cusin and Tamanini 2021, Wang 2021, Vijaykumar 2022, Wempe et al. 2022, Toscani et al. 2023, Toscani et al. 2024...

- Loudest sources
- Cosmological distances

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- Loudest sources
- Cosmological distances

Up to a few in LISA (Sereno et al 2010)

Multiple near-identical images

Difference only in

- amplitude
- overall phase
- arrival time

Same sky location

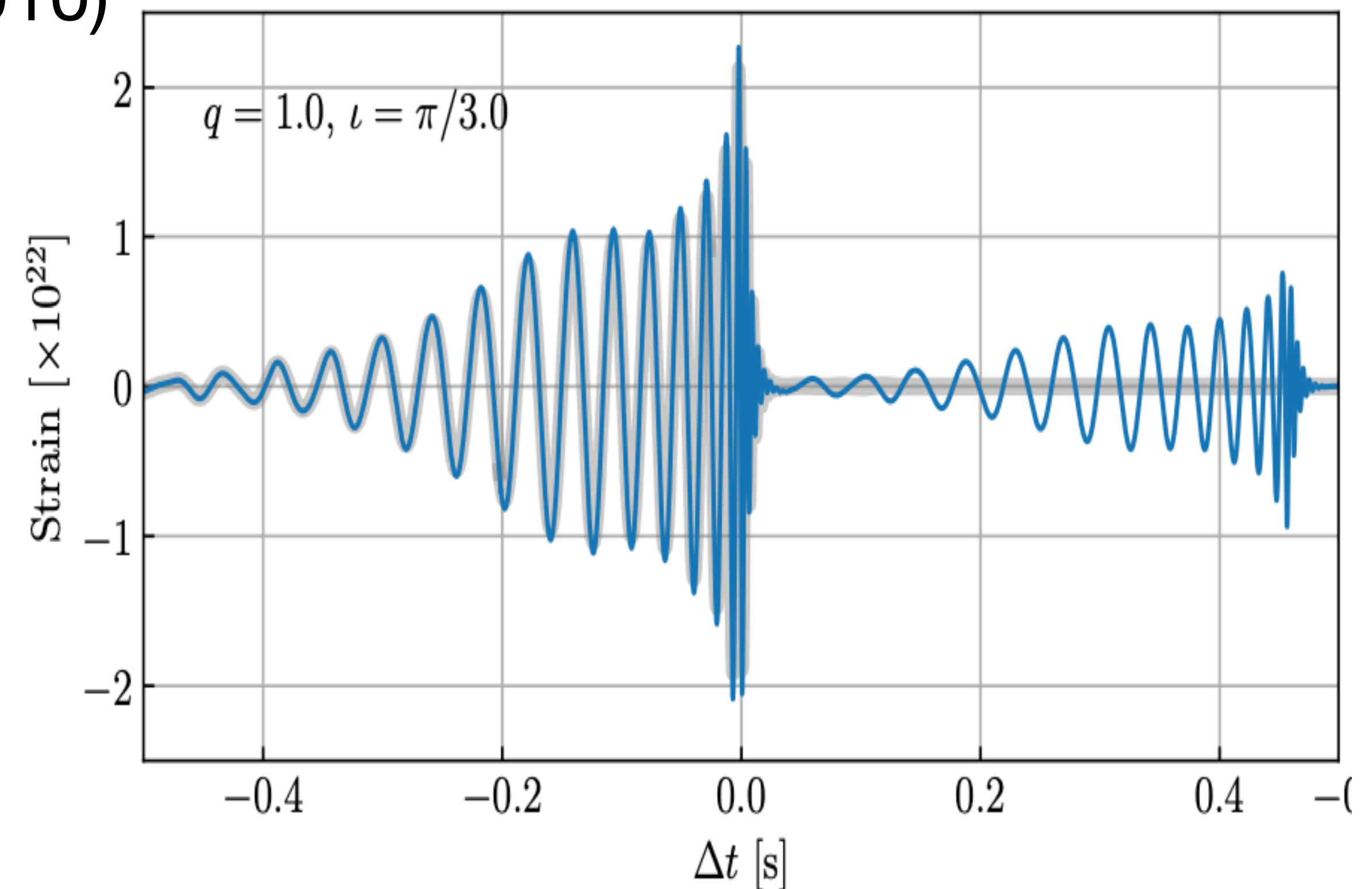


Image from Ezquiaga et al. 2020



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## Cosmological probes:

- strong lensing statistics:
  - Lens distribution
  - Luminosity distance from GWs
  - $\Omega_m, w$  which maximise likelihood
- time delay analysis:
  - Angular distances source, lens, observer
  - Luminosity distance from GWs
  - $H_0 (\Omega_m, w)$  which maximise likelihood

No need for EM counterparts

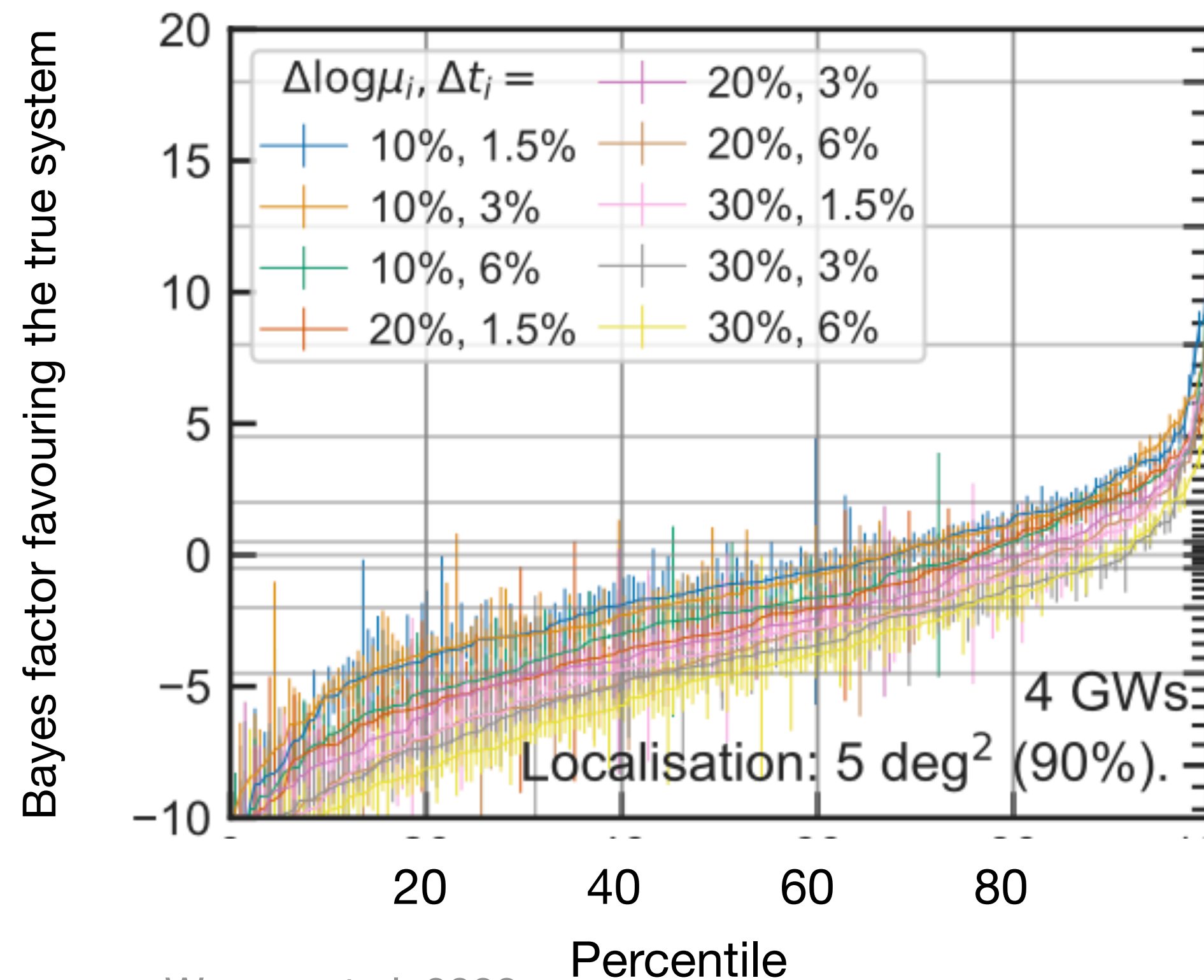
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Host galaxy will be lensed



Follow up EM observations



2 detected GWs: correct identification for 10ish% events

4 detected GWs: correct identification for 30ish% events

# *Strong lensing: EMRIs*

Dai 2007, Sereno et al. 2010, Sereno et al. 2011, Ezquiaga et al. 2020, Goyal et al. 2020, Hannuksela et al. 2020, Cusin and Tamanini 2021, Wang 2021, Vijaykumar 2022, Wempe et al. 2022, Toscani et al. 2023, Toscani et al. 2024...

- Excellent probes for strong field gravity
- Uncertain detection rates

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- Excellent probes for strong field gravity

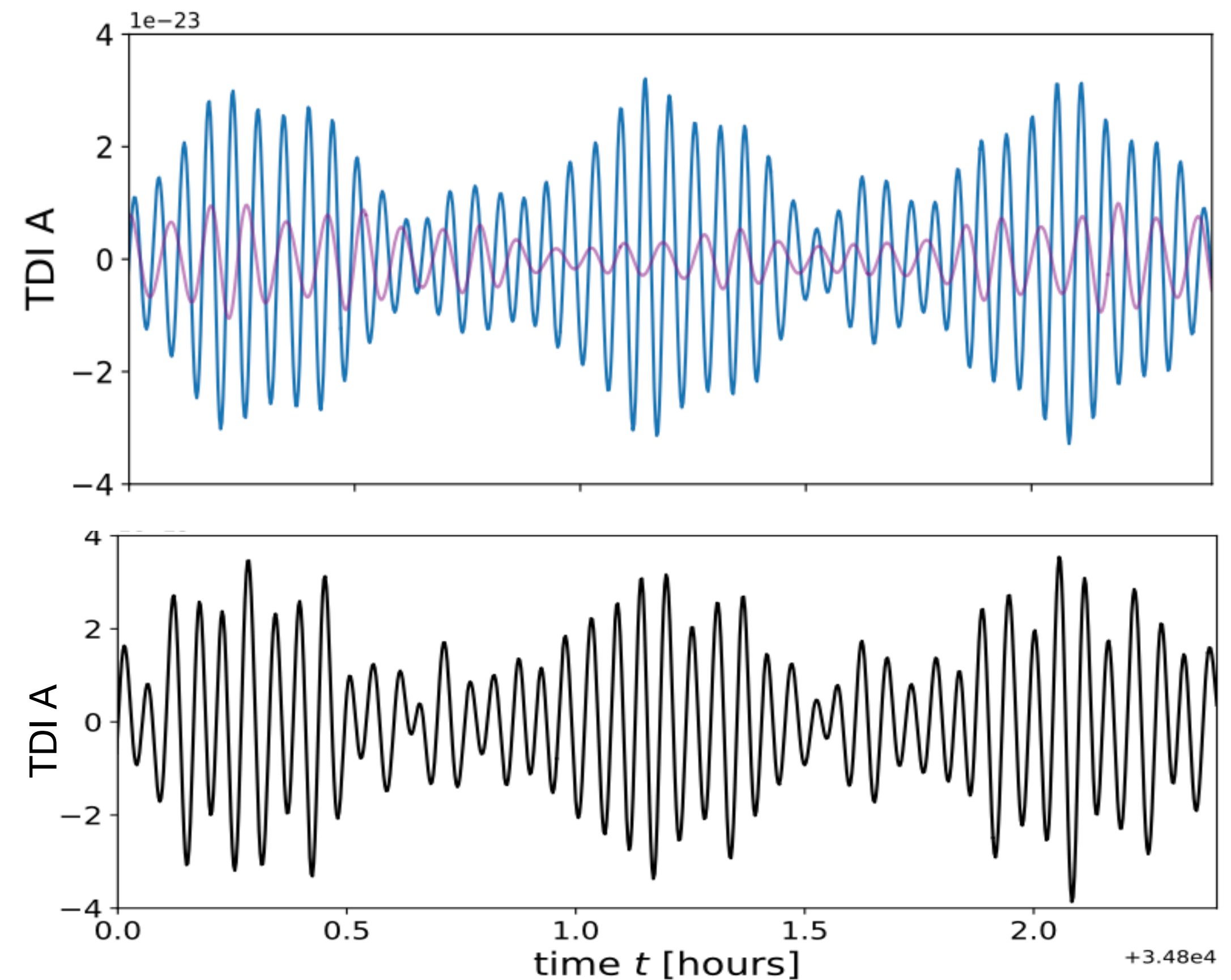
Up to 40 in LISA (Toscani et al. 2024)

Many observational cycles

Signal lasts longer than typical lensing time delay

Multiple images superimpose

- Uncertain detection rates



Toscani et al. 2024

# Strong lensing: EMRIs

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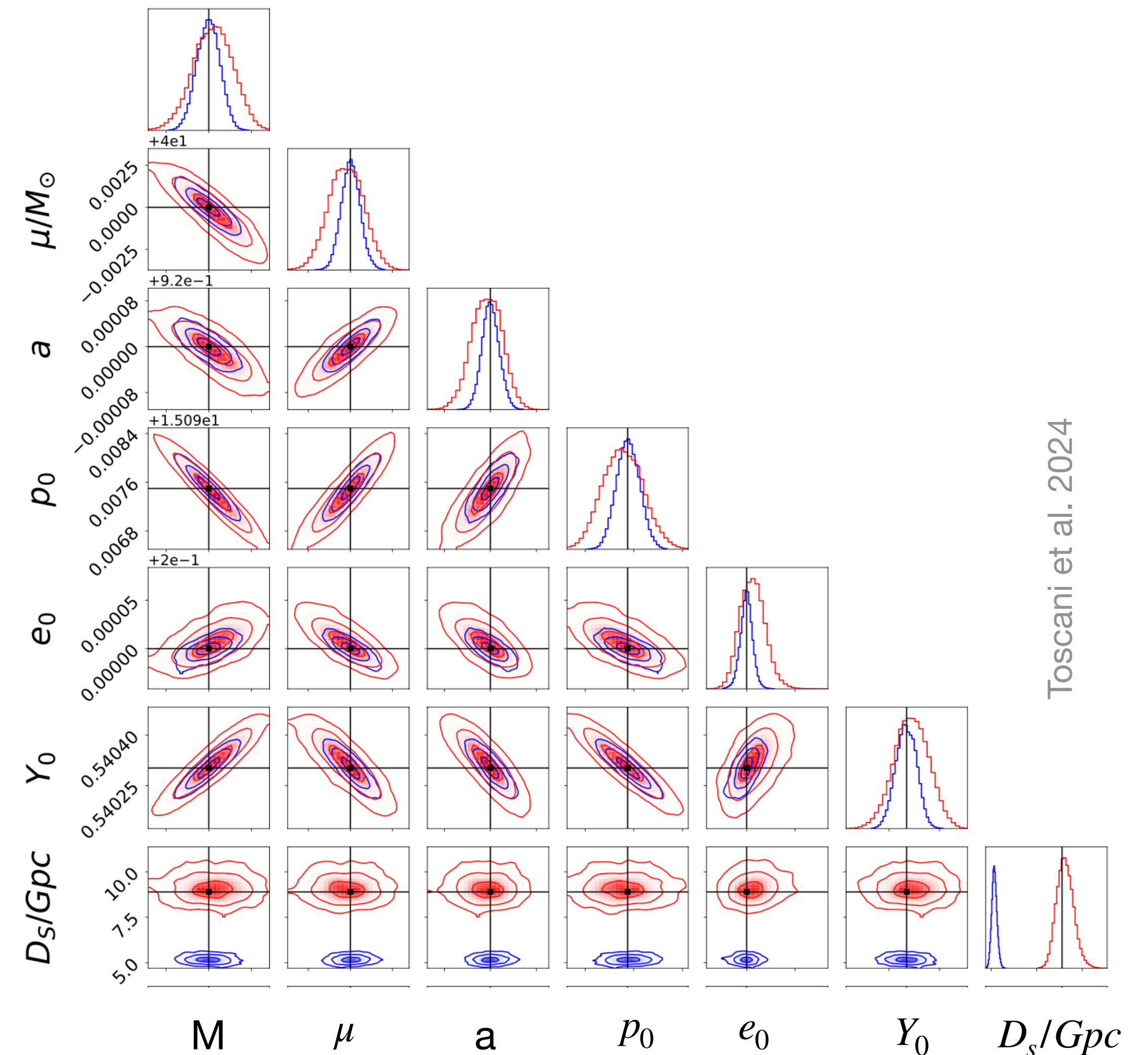
We have a LEMRI in the LISA data streams

Unlensed EMRI waveform to recover parameters

Time shift operation

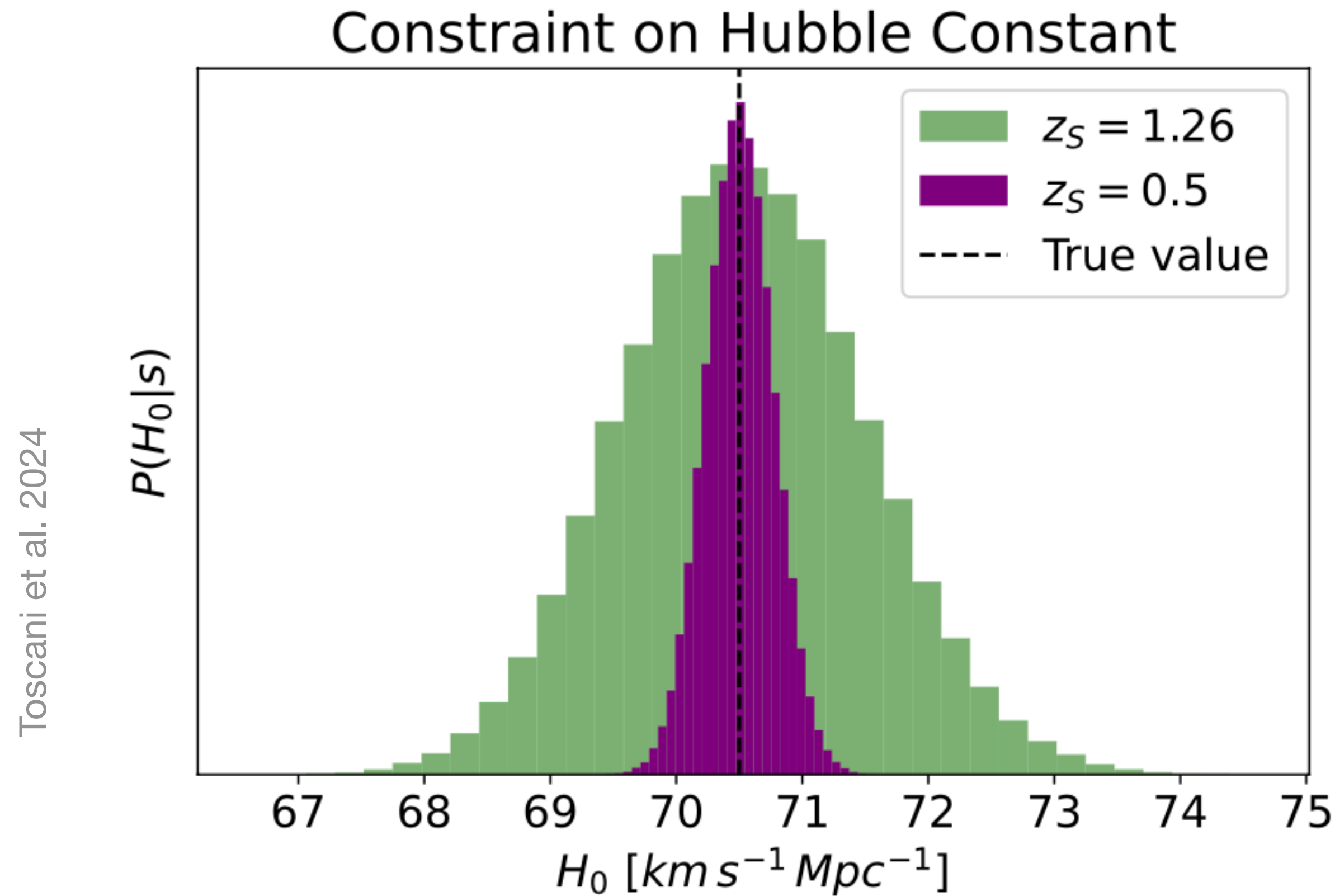
Maximise noise-weighted inner product

Robust technique



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Lens at 1      Relative error 1 %

Lens at 0.3      Relative error 0.4 %

$\Omega_m$  fixed

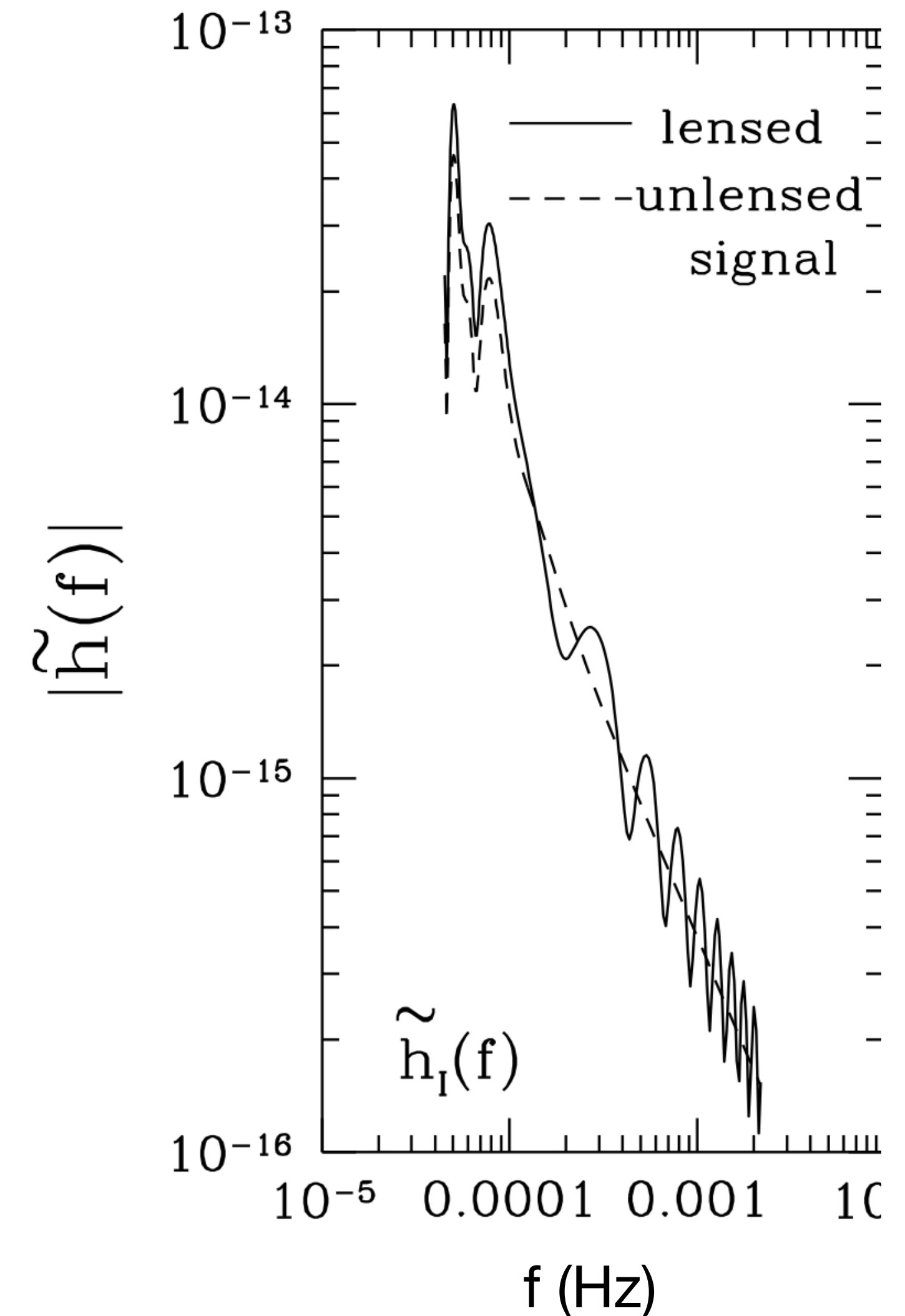
# Wave effects

Takahashi & Nakamura 2003, Gao et al. 2022, Caliskan et al. 2022, Garoffolo 2023

$$M_L \lesssim 10^8 M_\odot \left( \frac{f}{\text{mHz}} \right)^{-1}$$

Lenses like BHs, stars..

Oscillatory behaviour due to interference of multiple images



Takahashi & Nakamura 2003

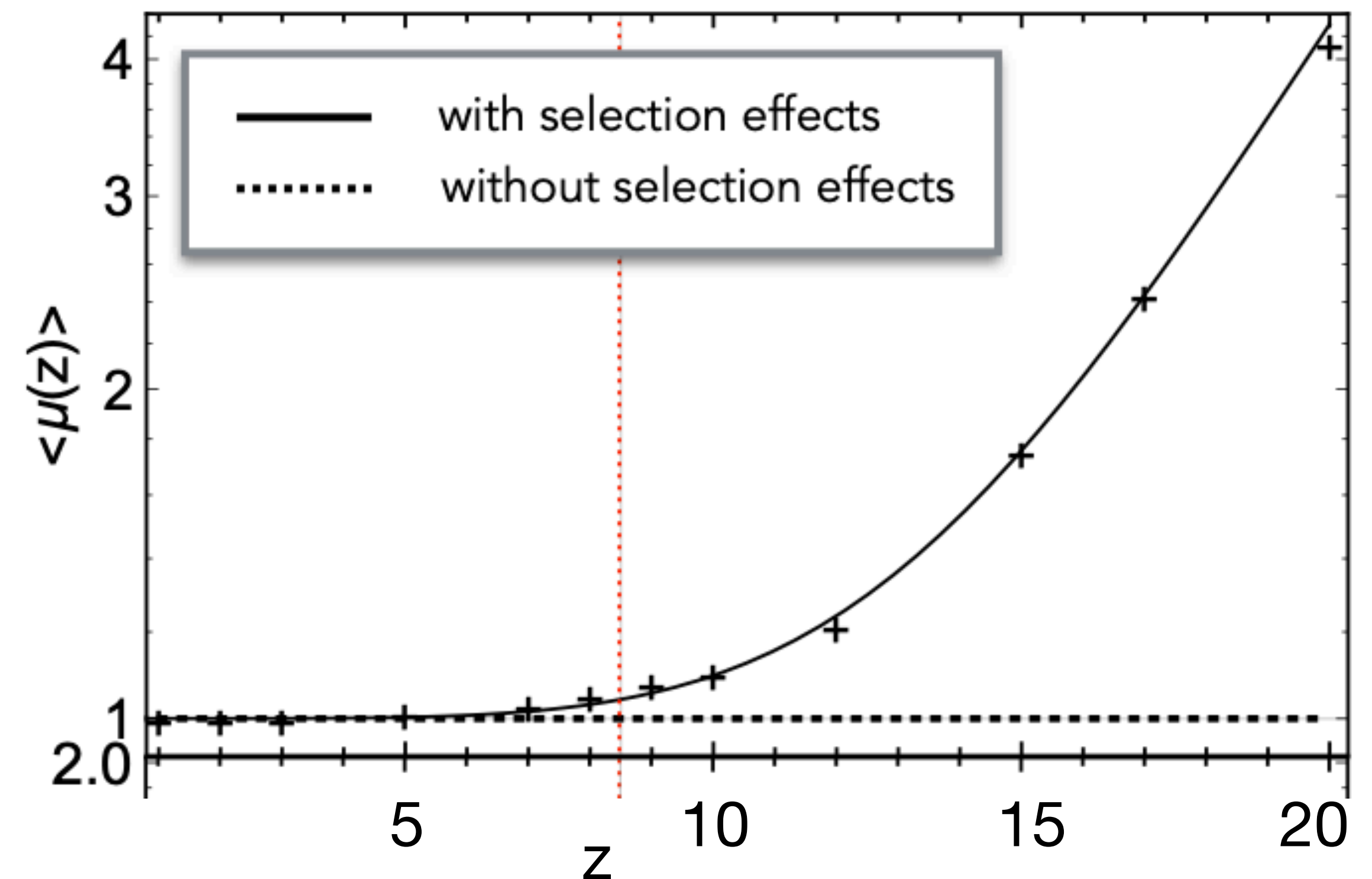
# Lensing selection effects

Dai 2007, Sereno et al. 2010, Sereno et al. 2011, Ezquiaga et al. 2020, Goyal et al. 2020, Hannuksela et al. 2020, Cusin and Tamanini 2021, Wang 2021, Vijaykumar 2022, Wempe et al. 2022, Toscani et al. 2023, Toscani et al. 2024...

$$h \propto \frac{\sqrt{\mu}}{d_L}$$

Selection effects shift mean magnification and introduce distance bias

- Distinguishing lensing events is crucial to retrieve correct information
- Different possible methodologies (parametric vs non-parametric, see Cheung et al. submitted)



Cusin & Tamanini 2021



# *Conclusions*

In the upcoming years we will see lensed GWs :-)!!

Useful to constrain cosmological parameters

Important to recognise lensing effect to properly reconstruct astrophysical properties of the source population

**THANKS FOR YOUR ATTENTION!!!**

# *Backup*

Cheung et al. 2024

- Benchmark model
- HDPGMM (Figaro by Rinaldi and Del Pozzo)
- Power load model
- Uniform model

Non parametric model performs the best!