Scalar-tensor perturbation mixing in Cosmology

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Going beyond linear theory

- The simplest approach to cosmological observables makes use of a background solution (the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker standard cosmological model) + small, linear perturbations superposed on it. While the need to go beyond linear theory is universally recognized when dealing with cosmic structure formation (Jeans instability), there are several other consequences of *going beyond the linear level* which have become a standard lore in cosmological analyses only during the last 20-30 years and others which are still poorly explored.

- Examples of deviations from linearity including tensor modes, such as:

    - production of gravitational waves (and vector modes) from density fluctuations (Matarrese et al. 1993, Matarrese et al. 1997; …; Perna et al. 2024)
    - second-order CMB B-mode production (Mollerach, Harari & Matarrese 2004)
    - production of magnetic fields from density fluctuations (Matarrese et al. 2004)
    - production of density perturbations (and vector modes) from gravitational waves (Bari et al. 2022, 2023).
    - modulation of GWs and GW backgrounds by scalar (and large-scale tensor) modes → anisotropies of the SGWB (Bartolo et al. 2019, ..). + GW wave-optics effects (Braga et al. 2024).
    - non-vanishing cross-correlations among seemingly independent observables (Ricciardone et al. 2021)

Perturbed FRW metric

- Perturbing the background metric to all orders (e.g. Matarrese, Mollerach & Bruni 1997)

scalars

$$g_{00} = -a^2(\tau)\left(1 + 2\sum_{r=1}^{+\infty}\frac{1}{r!}\,\psi^{(r)}\right) \qquad g_{0i} = a^2(\tau)\sum_{r=1}^{+\infty}\frac{1}{r!}\,\omega_i^{(r)}$$

vectors

$$g_{ij} = a^2(\tau)\left\{\left[1 - 2\left(\sum_{r=1}^{+\infty}\frac{1}{r!}\,\phi^{(r)}\right)\right]\delta_{ij} + \sum_{r=1}^{+\infty}\frac{1}{r!}\,\chi_{ij}^{(r)}\right\}$$

where $\chi_i^{(r)i} = 0$, and τ is the conformal time. The functions $\psi^{(r)}$, $\omega_i^{(r)}$, $\phi^{(r)}$, and $\chi_{ij}^{(r)}$ represent the r-th order perturbation of the metric.

$$\omega_i^{(r)} = \partial_i\omega^{(r)\|} + \omega_i^{(r)\perp}\,, \qquad \partial^i\omega_i^{(r)\perp} = 0 \qquad \chi_{ij}^{(r)} = \mathrm{D}_{ij}\chi^{(r)\|} + \partial_i\chi_j^{(r)\perp} + \partial_j\chi_i^{(r)\perp} + \chi_{ij}^{(r)\top}$$

$$\mathrm{D}_{ij} := \partial_i\partial_j - \frac{1}{3}\,\delta_{ij}\nabla^2\,. \qquad \partial^i\chi_{ij}^{(r)\top} = 0;$$

tensors

Towards a non-perturbative scheme on large scales

$$ds^2 = a^2 \left[-e^{2\Phi} d\eta^2 + e^{-2\Phi} \left(e^{\gamma}\right)_{ij} dx^i dx^j \right]$$

It is possible to split the tensor perturbations into the small-scale ripples h_{ij} and large-scale tensor modes H_{ij}

$$\gamma_{ij} \equiv h_{ij} + H_{ij}$$

Valbusa Dall'Armi et al. 2024;
Mierna et al., in preparation

Anisotropies of the Stochastic Gravitational-Wave Background (SGWB)

- A derivation of the angular power-spectrum of cosmological anisotropies, using a Boltzmann equation approach, has been obtained in [Alba & Maldacena 2016, Contaldi 2017, Bartolo et al. 2019a,b; 2020; Valbusa et al. 2020; Ricciardone et al. 2021; Schulze et al. 2023); see also Malhotra et al. 2022. Purely kinematic, non-perturbative approach (Mierna et al. in preparation).

- Anisotropies in the cosmological GW background are imprinted both at production (actually, when the relevant modes enter the Hubble radius) and by GW propagation through the large-scale scalar and tensor modes. Note that the first contribution is not present in the CMB radiation (as the universe is not transparent to photons before recombination), causing a potential dependence of the anisotropies on frequency.

- We showed that anisotropies in the inflation-produced SGWB are characterized by ubiquitous non-adiabatic scalar (and large-scale tensor) perturbations (Valbusa et al. 2023, 2024; Mierna et al. in preparation).

Inflationary initial conditions for anisotropies of SGWB - I

Valbusa Dall'Armi, Mierna, Matarrese & Ricciardone 2023, 2024

Perturbed metric including large-scale scalars, large and small-scale tensors

$$ds^2 = a^2 \left[-e^{2\Psi} d\eta^2 + e^{-2\Phi} \left(e^{\gamma}\right)_{ij} dx^i dx^j \right]$$

GW stress-energy tensor (Isaacson 1967). Covariant derivatives are w.r.t. slowly varying metric (Brill-Hartle average)

$$T^{\rm GW}_{\mu\nu} = \frac{1}{32\pi G} \left\langle \mathcal{D}_\mu \gamma^{\rm GW}_{\alpha\beta} \mathcal{D}_\nu \gamma^{{\rm GW}\,\alpha\beta} \right\rangle$$

This leads to the following multipoles

$$\delta_{\rm GW,0}(\eta_{\rm in}, \vec{k}, \hat{n}, q) = -2\Psi(\eta_{\rm in}, \vec{k}) + 4\Phi(\eta_{\rm in}, \vec{k}) + \frac{2}{3} H(\eta_{\rm in}, \vec{k}) \, ,$$

$$\delta_{\rm GW,1}(\eta_{\rm in}, \vec{k}, \hat{n}, q) = 0 \, ,$$

$$\delta_{\rm GW,2}(\eta_{\rm in}, \vec{k}, \hat{n}, q) = -\frac{2}{3} H(\eta_{\rm in}, \vec{k}) \, .$$

Neglecting the tensor perturbations, we can see that the contribution to the CGWB anisotropy from these inflationary initial conditions (IIC) enhances the angular power-spectrum compared to the adiabatic case,

$$\frac{C_\ell^{\rm IIC+SW}}{C_\ell^{\rm AD+SW}} \approx \left(\frac{4T_\Phi + [2 - n_{\rm gwb}(q)] T_\Psi}{[2 - n_{\rm gwb}(q)] T_\Psi} \right)^2 \, .$$

Inflationary initial conditions for anisotropies of SGWB - II

Valbusa Dall'Armi, Mierna, Matarrese & Ricciardone 2023, 2024

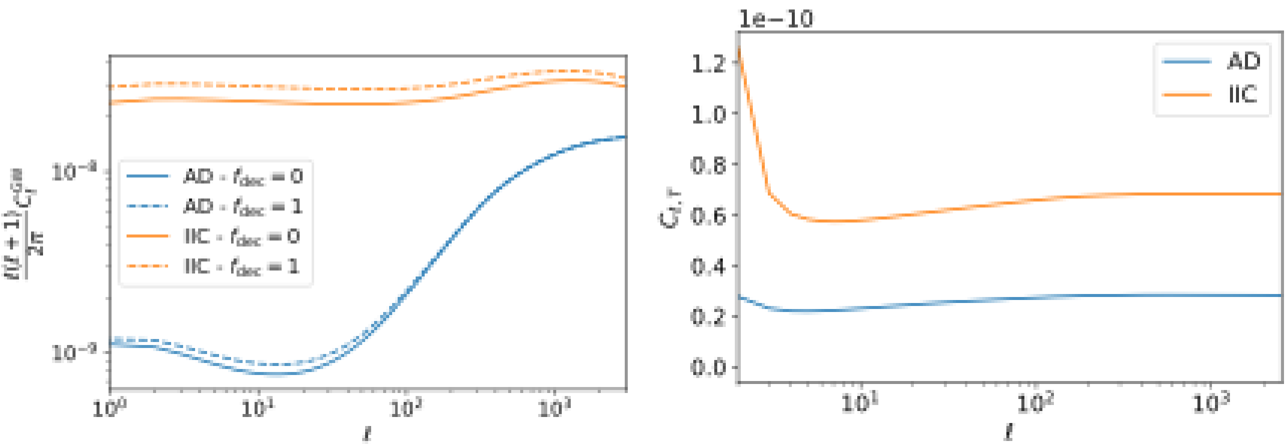
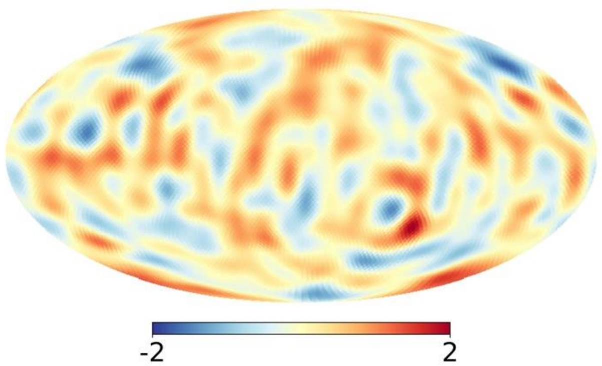


Figure 1. Left: plot of the angular power spectrum of the CGWB for adiabatic initial conditions (AD) and for inflationary initial conditions (IIC) with $n_{gwb} = 0.35$. Right: plot of the tensor contributions to the angular power spectrum for AD and IIC.

Mierna, Bartolo, Matarrese and Ricciardone 2024 derive SGWB anisotropies through a non-perturbative & entirely kinematic approach (as in Mollerach & Matarrese 1997 and Bartolo, Matarrese & Riotto 2005, for the CMB).

→ **See poster at this conference by Alina Mierna**

Cross-correlating SGWB and CMB



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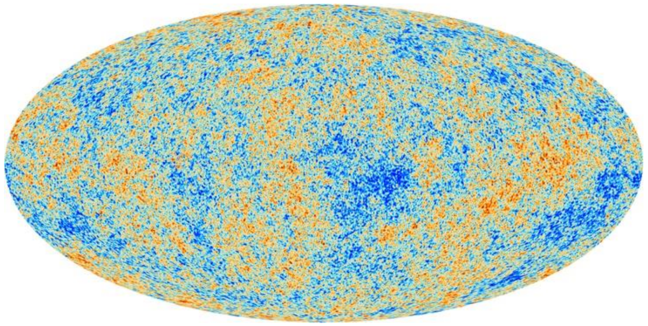
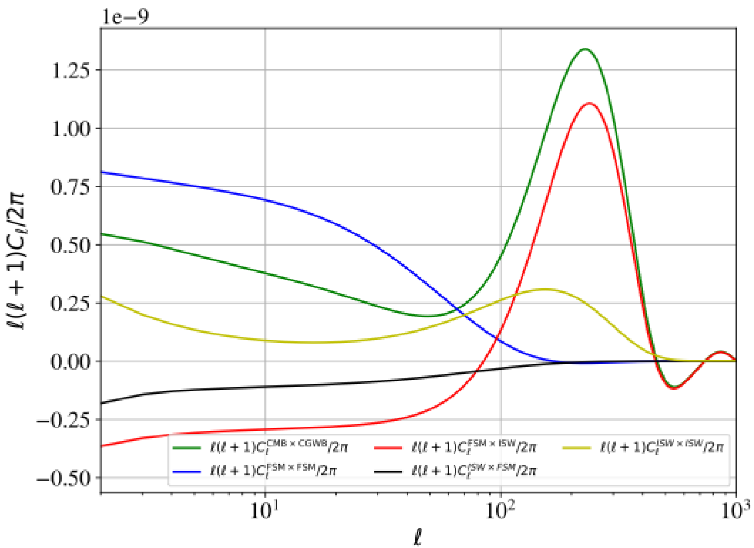
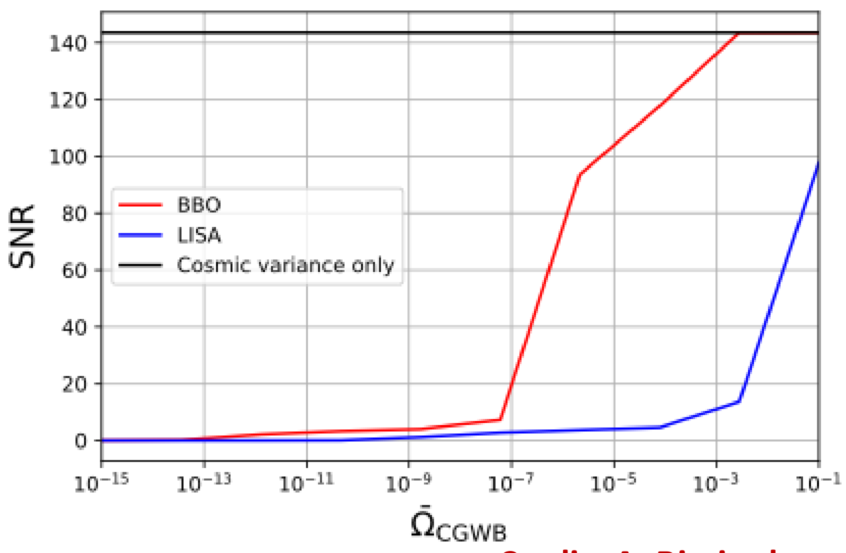


Ricciardone, Valbusa, Bartolo, Bertacca, Liguori, Matarrese 2021, PRL **127** 271301

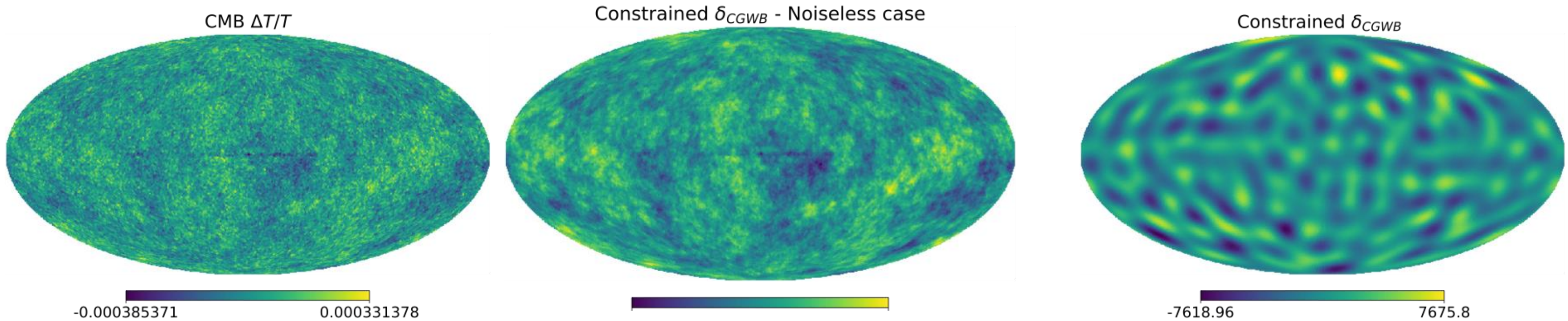




Credits A. Ricciardone

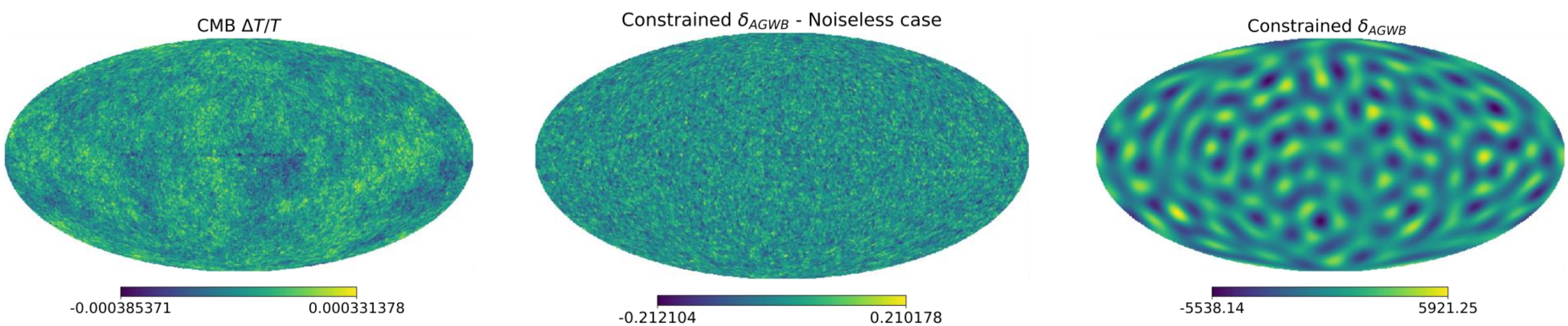
Constrained realizations

CGWB x CMB Planck SMICA map



$$h''_\lambda(\tau, \mathbf{k}) + 2\mathcal{H}h'_\lambda(\tau, \mathbf{k}) + k^2 h_\lambda(\tau, \mathbf{k}) = 4\mathcal{S}_\lambda(\tau, \mathbf{k}),$$

AGWB x CMB Planck SMICA map



Ricciardone, Valbusa, Bartolo, Bertacca, Liguori, Matarrese 2021, PRL **127** 271301

More power than previously assumed for primordial scalar and tensor modes

- For many years, based on extrapolation of CMB and galaxy scale observational data we assumed limited power both for scalar and tensor modes.

- In the last ~ 10 years our attitude changed, both for small-scale scalar modes (e.g. motivated by PBH formation) and large-scale tensor modes (e.g. motivated by axion-inflation models).

Non-Gaussianity in SIGW

Perna, Testini, Ricciardone & Matarrese, JCAP 05 (2024) 086. SIGW:

$$h''_\lambda(\mathbf{k},\eta) + 2\mathcal{H}h'_\lambda(\mathbf{k},\eta) + k^2 h_\lambda(\mathbf{k},\eta) = 4\mathcal{S}_\lambda(\mathbf{k},\eta)$$

with:

$$\begin{aligned}\mathcal{S}_\lambda(\mathbf{k},\eta) &\equiv -\varepsilon^\lambda_{lm}(\hat{\mathbf{k}})\mathcal{S}_{lm}(\mathbf{k},\eta) \\ &= \int \frac{d^3\mathbf{q}}{(2\pi)^{3/2}} Q_\lambda(\mathbf{k},\mathbf{q}) f(|\mathbf{k}-\mathbf{q}|, q, \eta)\mathcal{R}(\mathbf{q})\mathcal{R}(\mathbf{k}-\mathbf{q})\end{aligned}$$

We assume a NG model with 4 free parameters:

$$\begin{aligned}\mathcal{R}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) &+ \frac{3}{5} f_{\rm NL}(\mathcal{R}_g^2(\mathbf{x}) - \langle\mathcal{R}_g^2\rangle) + \frac{9}{25} g_{\rm NL}\mathcal{R}_g^3(\mathbf{x}) \\ &+ \frac{27}{125} h_{\rm NL}(\mathcal{R}_g^4(\mathbf{x}) - 3\langle\mathcal{R}_g^2\rangle^2) + \frac{81}{625} i_{\rm NL}\mathcal{R}_g^5(\mathbf{x}),\end{aligned}$$

We consider all the relevant non-Gaussian contributions up to fifth-order in the scalar seeds without any hierarchy; we derive the related GW energy density $\Omega_{GW}(f)$ and perform a Fisher matrix analysis to understand to which accuracy NG can be constrained with LISA

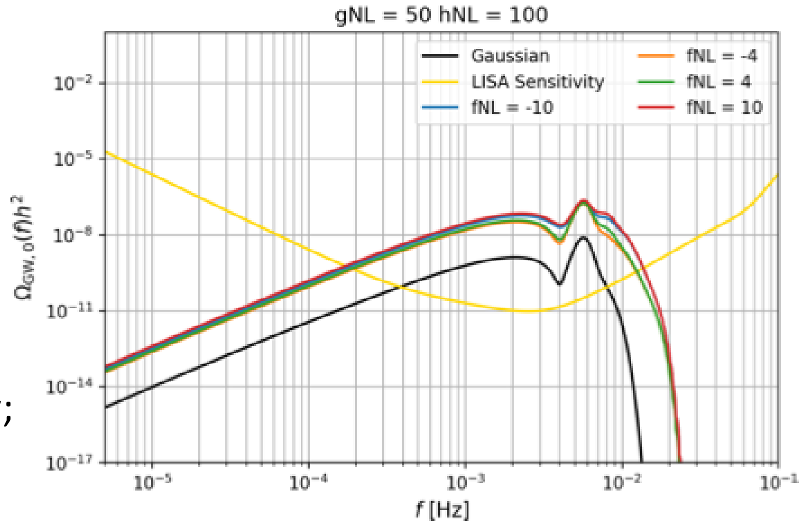


Figure 18. The figure shows the GW spectrum obtained for different values of $f_{\rm NL}$. In this case we consider non-vanishing values for $g_{\rm NL}$ and $h_{\rm NL}$ in order to account for their contributions to the GW spectrum.

SIGW in modified gravity

Kugarajh, Traforetti, Maselli, Matarrese & Ricciardone (to appear)

- We extend the calculation of SIGW to f(R) models of modified gravity (which lead to a "gravitational slip" term).
- As an example we take $f(R) = R + \alpha R^2$ expanding to order $\alpha \ll (H^2 \delta_m)^{-1}$

We obtain relevant new contributions to the SIGW

$$\left(1+12a^{-2}\alpha\left[\mathcal{H}'+\mathcal{H}^2\right]\right)\left(\chi_j^{i(2)''}+2\mathcal{H}\delta\chi_j^{i(2)'}+k^2\delta\chi_j^{i(2)}\right)+12a^{-2}\alpha\left[\mathcal{H}''+2\mathcal{H}\mathcal{H}'\right]\chi_j^{i(2)'}=S+\alpha\delta S$$

and to its power-spectrum.

- Non-negligible contribution at high frequences.
- Useful to constrain MG during radiation-dominated epoch.

Can gravitational waves seed cosmic structure formation? Yes (in principle)!

Bari, Ricciardone, Bartolo, Bertacca & Matarrese 2021 – Bari et al. 2022

The local GW energy density acts as a source for density perturbations (Tomita 1971; Matarrese, Mollerach & Bruni 1997), but see also more recent work by Zhang et al. 2014; Döring et al. 2021. The master (Raychaudhuri) equation reads

$$\delta^{(2)''} + \mathcal{H}\delta^{(2)'} - 4\pi G a^2 \overline{\rho}_m \delta^{(2)} = \frac{1}{2}\chi'^{ij}\chi'_{ij}$$

linear GWs

Whose homogeneous and sourced solutions are

$$\delta_h^{(2)} = c_1(x) D_+(\eta) + c_2(x) D_-(\eta)\,,$$

$$\delta_s^{(2)} = D_+(\eta)\int_0^\eta \frac{d\tilde{\eta} D_-(\tilde{\eta})}{W(\tilde{\eta})}\frac{1}{2}\chi'^{ij}\chi'_{ij} - D_-(\eta)\int_0^\eta \frac{d\tilde{\eta} D_+(\tilde{\eta})}{W(\tilde{\eta})}\frac{1}{2}\chi'^{ij}\chi'_{ij}\,.$$

These tensor induced scalar modes evolve like linear density perts. Inside the horizon, but unlike the latter, they vanish outside the horizon → they do not produce large-scale CMB anisotropies. But … they can contribute to seed LSS formation. And, they are highly non-Gaussian

Tensor induced density perturbations

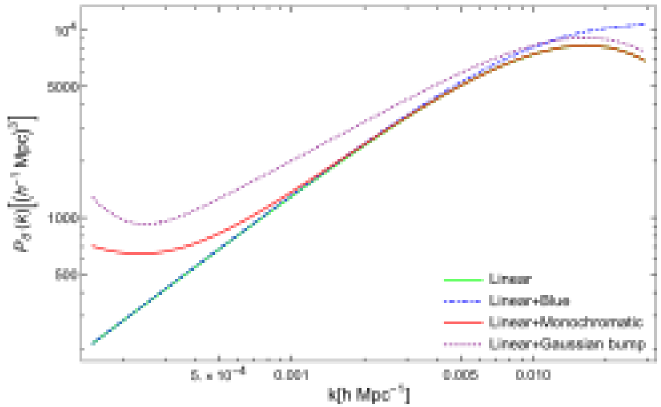


FIG. 1. Impact of different GW power spectra on the matter power spectrum: (i) blue-tilted ($A_T = 1.26 \times 10^{-10}, n_T = 0.32$, $k_* = k_{\mathrm{CMB}} = 0.01\ \mathrm{Mpc}^{-1}$), (ii) monochromatic ($A_T = 10^{-5}$, $k_* = 0.008\ \mathrm{Mpc}^{-1}$), and (iii) Gaussian bump ($A_T = 10^{-5}, \sigma = 2$, $k_p = 0.04\ \mathrm{Mpc}^{-1}$). The value of h is fixed at 0.68 [73].

In Bari et al. *PRL 129 (2022) 091301* we study the generation and evolution of second-order energy-density perturbations arising from primordial gravitational waves. Such ***tensor-induced scalar modes*** evolve as standard linear perturbations and may leave observable signatures in the LSS. We study the imprint on the matter power-spectrum of some primordial models predicting a large GW signal at high frequencies. This mechanism in principle allows to constrain/detect primordial GWs by looking at specific features in the matter/galaxy power-spectrum, thus allowing to probe them on a range of scales unexplored so far.

Prospects for LSS NG from tensor induced density perturbations

- Tensor-mode induced density perturbations δ arise from local fluctuations of the GW energy density, which is itself quadratic in the tensor field h_{ij} Because of that (If GW are Gaussian) our density field is chi-squared distributed, hence highly NG. This is an unusual case of NG preferentially in density perturbation rather than gravitational potential (with Jimenez, Verde & Kamionkowski in 1999 we named that case "model A" in contrast with "model B", which is the one usually assumes). Owing to the Central Limit Theorem, we should expect NG in CMB to be negligible for such NG (Scherrer 1995). We actually checked this fact for our model. Abdelaziz, Bari, Matarrese & Ricciardone 2024, in preparation.
- NEW: Scalar perturbations from GW in de Sitter inflation (Bertacca, Jimenez, Matarrese & Ricciardone, to appear): "Inflation without an inflaton".

GW modulation by scalar modes

- Bari, Bartolo, Domenech & Matarrese PRD 109 (2024) 023509; see also Picard & Malik 2023; Bari et al. in preparation
- Consider GWs propagating in a Universe perturbed by scalar modes in the radiation era. One can do the calculation at second order by keeping only mixed scalar-tensor modes (but, the linear evolution of tensor modes has nothing to do with their perturbative order at production!):

$$ds^2 = -e^{2\Phi}\,dt^2 + a^2 e^{-2\Psi}(e^\gamma)_{ij}dx^i dx^j,$$

- The GW equation of motion reads (leading order terms only):

$$\gamma''_{ij} + 2\mathcal{H}\gamma'_{ij} - \nabla^2\gamma_{ij} = 4\Phi\nabla^2\gamma_{ij} + 4\Phi'\gamma'_{ij},$$

- Interesting effects arise in case of primordial chiral GWs. Potential infrared divergence avoided by splitting the gravitational potential in long (perceived as a background by high-frequency GWs) and short modes.
- We also find that, for peaked primordial spectra, the low frequency tail of scalar-tensor induced GWs does not have a logarithmic running, in contrast to scalar- scalar induced GWs. There is also no resonant peak.

Gravitational waves in wave-optics - I

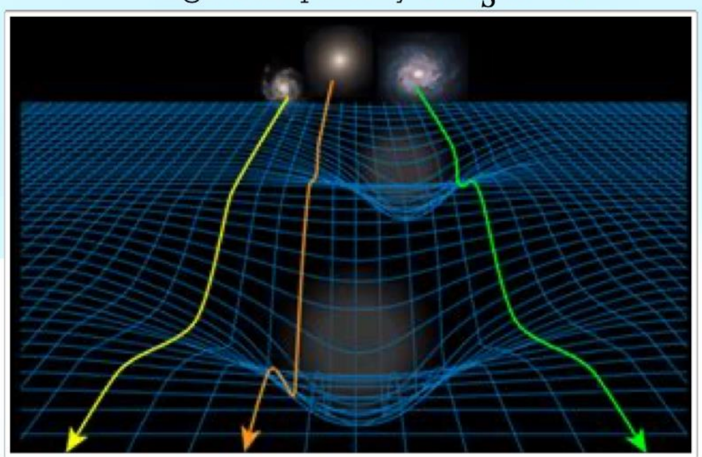
Braga, Garoffolo, Ricciardone, Bartolo & Matarrese arXiv:2405.20208 (JCAP, in press)

Incomplete list of papers on related issues: Nakamura & Deguchi 1999; Takahashi & Nakamura 2003; Dai et al. 2018; Cusin et al. 2019, 2020; Pizzuti et al. 2023; Pijnenburg et al. 2024; …

Optical regimes

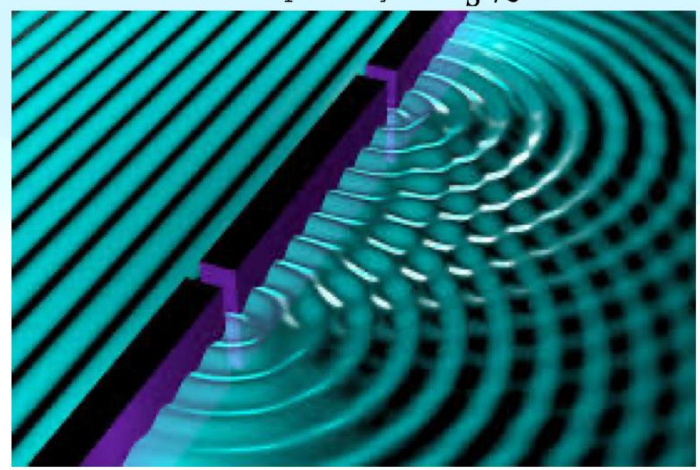
Geometric vs Wave optics

High Frequency: $\omega R_S \gg 1$



Ray description

Low Frequency: $\omega R_S \lesssim 1$



Wave effects

3.4 Wave effects

LISA CosGW: 2204.05434

GWs can be emitted at low frequencies ($w \lesssim 1$), allowing the observation of wave diffractive phenomena. For typical LISA sources, wave optics as in Eq. (15) needs to be considered for lenses with masses $M_L \sim 10^6 - 10^9 M_\odot$,

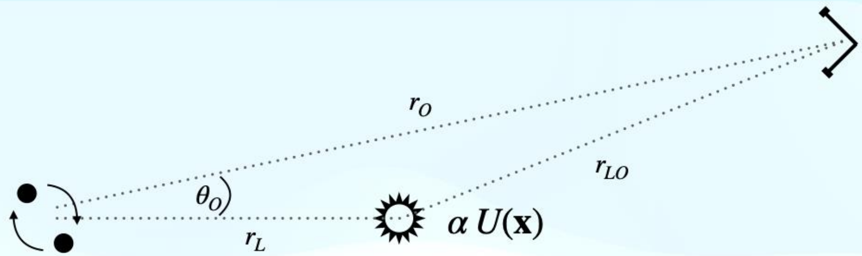
credits: Alice Garoffolo

Gravitational waves in wave-optics - II

Diffraction integral for a scalar wave

Nakamura&Deguchi 1999

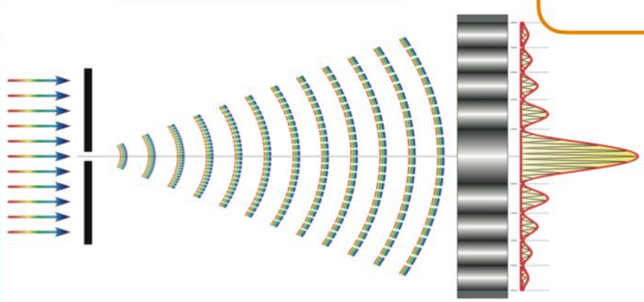
1. Klein-Gordon Eq.: $\left[\nabla^2 + \omega^2(1 - 4\alpha U)\right]\tilde{\Psi}_\omega(\mathbf{x}) = 0$
2. Amplification Factor: $F(\mathbf{x}) = \tilde{\Psi}_\omega / \tilde{\Psi}_\omega^{NL}$
3. Eikonal approximation: $|\partial_r^2 F| \ll |2i\omega\partial_r F|$
4. Schrödinger Eq.: $i\partial_r F = -\frac{1}{2\omega}\partial_{\boldsymbol{\theta}}^2 F + 2\alpha\omega U F$



$\omega = 1/\hbar$

Diffraction integral:

$$F(\vec{r}_O) = \int \mathcal{D}\boldsymbol{\theta}(r) \exp\left\{ i\omega \int_0^{r_O} dr \left[\frac{r^2}{2}|\dot{\boldsymbol{\theta}}|^2 - 2\alpha U(r, \boldsymbol{\theta}) \right] \right\}$$



Analogy between wave and quantum effects:
interference between all paths.
Geometric optics = classical limit

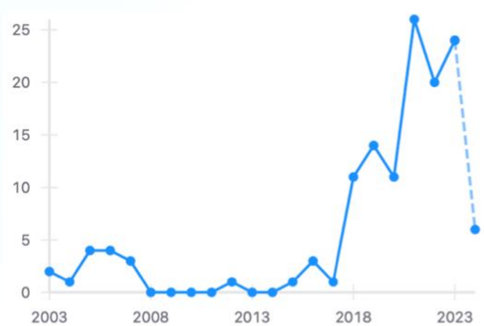
credits: Alice Garoffolo

Gravitational waves in wave-optics - III

Diffraction integral: Pros and Cons

PROs

1. Wave optics effects are frequency dependent
2. Easy high frequency limit
3. Non perturbative (strong lensing)
4. Already used for: lens parameter estimation, constraints PBH abundance, matter PS at small scales,...



Citation per year of Nakamura&Deguchi 1999

CONs

1. Eikonal: frequency lower bound $\omega \gg |\partial_r^2 F| / |\partial_r F|$
2. Scalar field: no polarization effects

Proper time path integrals

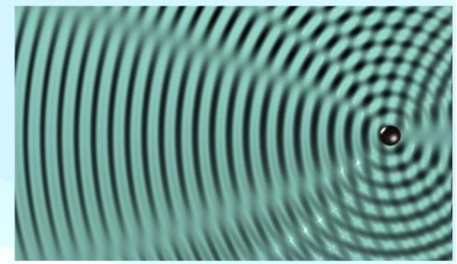
2405.20208

credits: Alice Garoffolo

Gravitational waves in wave-optics - IV

Proper time path integral

Finding a Schrödinger equation without Eikonal approximation



1. **Proper time:** $\tilde{\Psi}_\omega(\mathbf{x}) = -\frac{i}{\omega}\int_0^{+\infty} d\tau\, e^{i\omega\tau}\psi_\omega(\tau,\mathbf{x})$

2. **Schrödinger Eq.:** $\frac{i}{\omega}\frac{\partial\psi_\omega(\tau,\mathbf{x})}{\partial\tau} = \left[-\frac{\nabla^2}{\omega^2} + 4\alpha U\right]\psi_\omega(\tau,\mathbf{x})$

Proper time path integral:

$$\tilde{\Psi}_\omega(\mathbf{x}_f) = -\frac{i}{\omega}\int_0^{+\infty} d\tau\, e^{i\omega\tau}\int_{\mathbf{x}(\tau'=0)=\mathbf{x}_i}^{\mathbf{x}(\tau'=\tau)=\mathbf{x}_f}\mathcal{D}\mathbf{x}(\tau')\, e^{i\omega\int_0^\tau d\tau'\left[\frac{\dot{\mathbf{x}}^2}{4}-4\alpha U\right]}$$

$\omega = 1/\hbar$

Particle action

Integrate away τ

Sum over paths

Exact particle-like solution WITHOUT the need of Eikonal approximation

credits: Alice Garoffolo

Our findings + future prospects

1. $1/\omega \rightarrow 0$ limit to recover geometric optics
2. Eikonal assumption a posteriori to recover diffraction integral of Nakamura & Deguchi 1999
3. Perturbative expansion in a U
4. First-order solution for Coulomb-like potential
5. Extension to a massive scalar field
6. Polarization effects in a Kerr background
7. Extend to many sources → background. Extend to many lenses. Account for cosmic expansion (ongoing)

Concluding remarks

- The direct detection of the cosmological stochastic GW background (SGWB) both of primordial and astrophysical origin together with its anisotropies will provide a unique probe of the early Universe as well as of modified gravity theories.

- Cross-correlations of the SGWB with CMB and LSS will open a new window to search for new physical effects and constrain our cosmological and fundamental physics models.

- Going beyond linear evolution and allowing for scalar-tensor mixings opens several new unexpected phenomena which deserve being explored and fully exploited.