

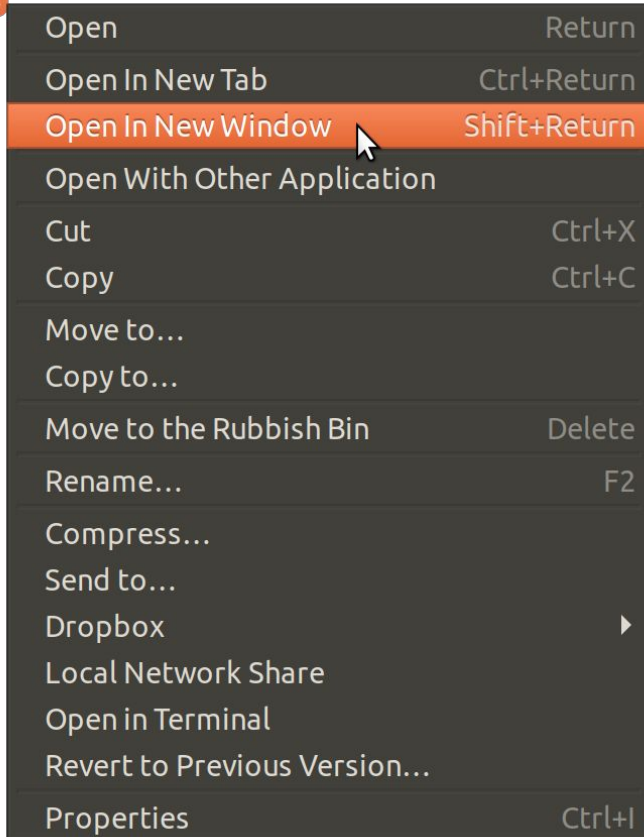
The precision science of **extreme mass-ratio inspirals**

Christopher Berry • cplberry.com • [@cplberry](https://twitter.com/cplberry)

GraSP24 • 24 October 2024 • [arXiv:1903.03686](https://arxiv.org/abs/1903.03686) [arXiv:2212.06166](https://arxiv.org/abs/2212.06166)



Image: LIGO/Caltech/MIT/Sonoma State/Aurore Simonnet



An undiscovered source

Massive black holes and stellar companions

Precision measurement

Intricate orbits in the strong field

Black hole astrophysics

Using a population to build understand

An undiscovered source

Spoilers

Signals encode
information about
their sources

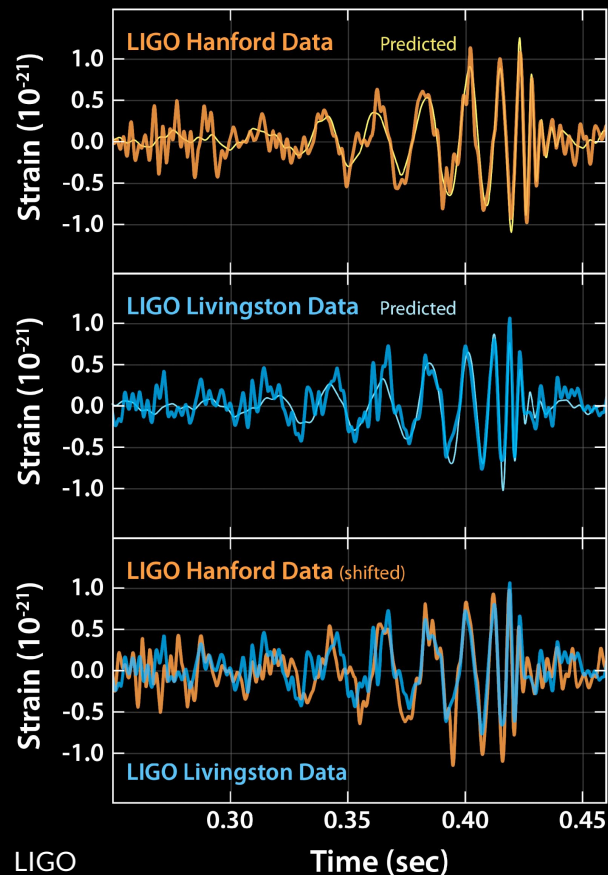
GW150914
parameter estimation
arXiv:1602.03840

GW150914
astrophysical
implications
arXiv:1602.03846

14 September 2015 we
observed gravitational waves

The signal came from the
coalescence of a **near-equal**
mass binary black hole

This material is based upon work supported by
NSF's LIGO Laboratory which is a major facility fully
funded by the National Science Foundation



LIGO

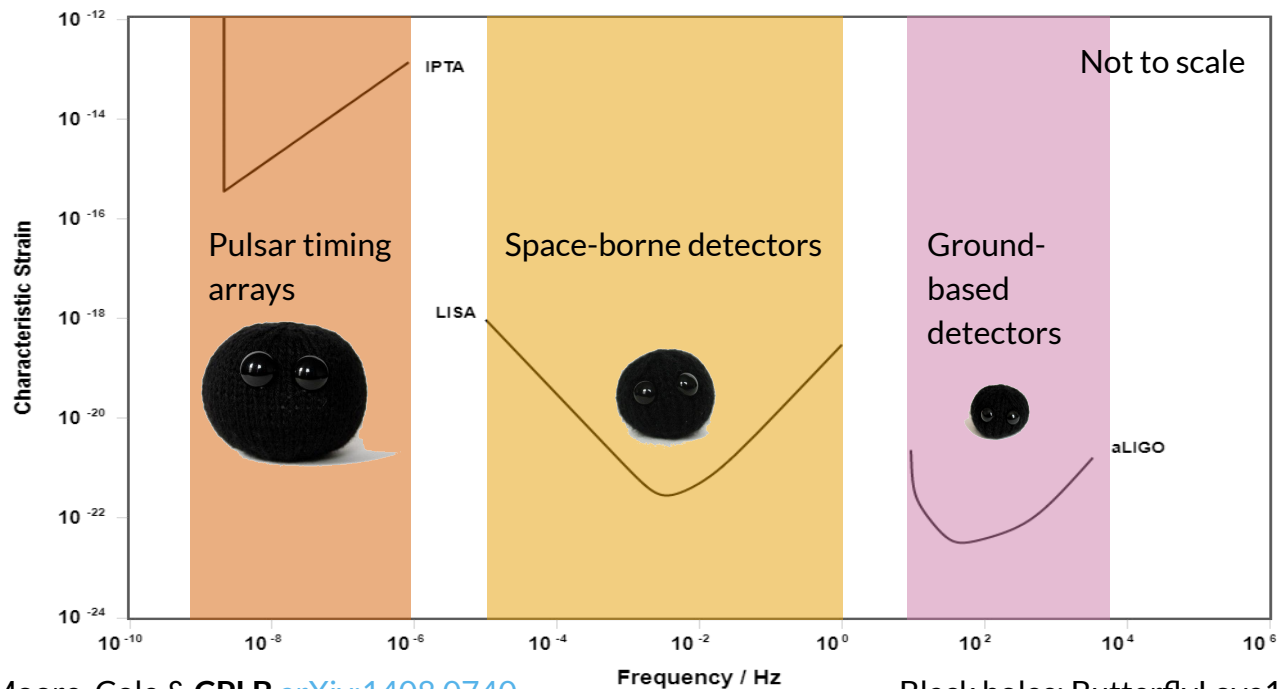
Gravitational-wave spectrum

Different technologies used for different frequency ranges

Currently, **LIGO**, **Virgo** and **KAGRA** observe at highest frequencies

LISA is due for launch in 2030s

Pulsar timing arrays observe at lowest frequencies



Moore, Cole & CPLB [arXiv:1408.0740](https://arxiv.org/abs/1408.0740)

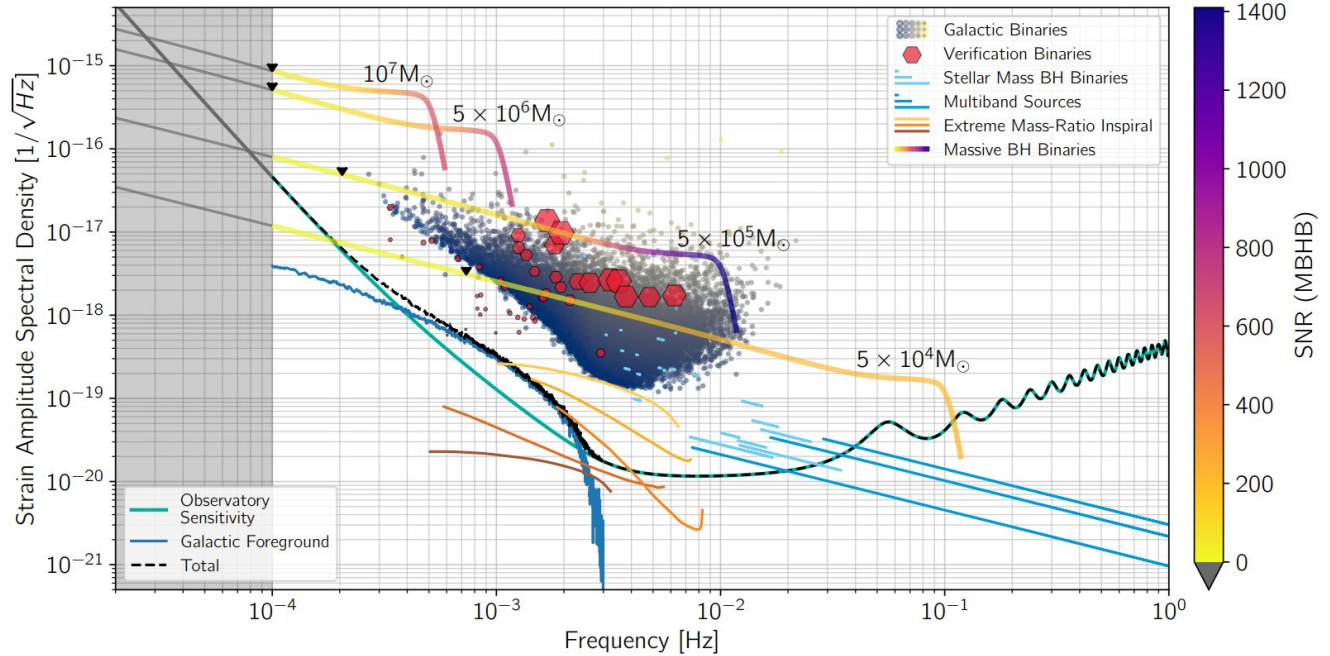
Black holes: ButterflyLove1

LISA

LISA mission accepted
by ESA in January,
launch planned for 2035

LISA can contribute to a
wide range of
astrophysics
[arXiv:2203.06016](https://arxiv.org/abs/2203.06016)

Data analysis will be
extremely complicated
lisa-ldc.lal.in2p3.fr



LISA [arXiv:2402.07571](https://arxiv.org/abs/2402.07571)

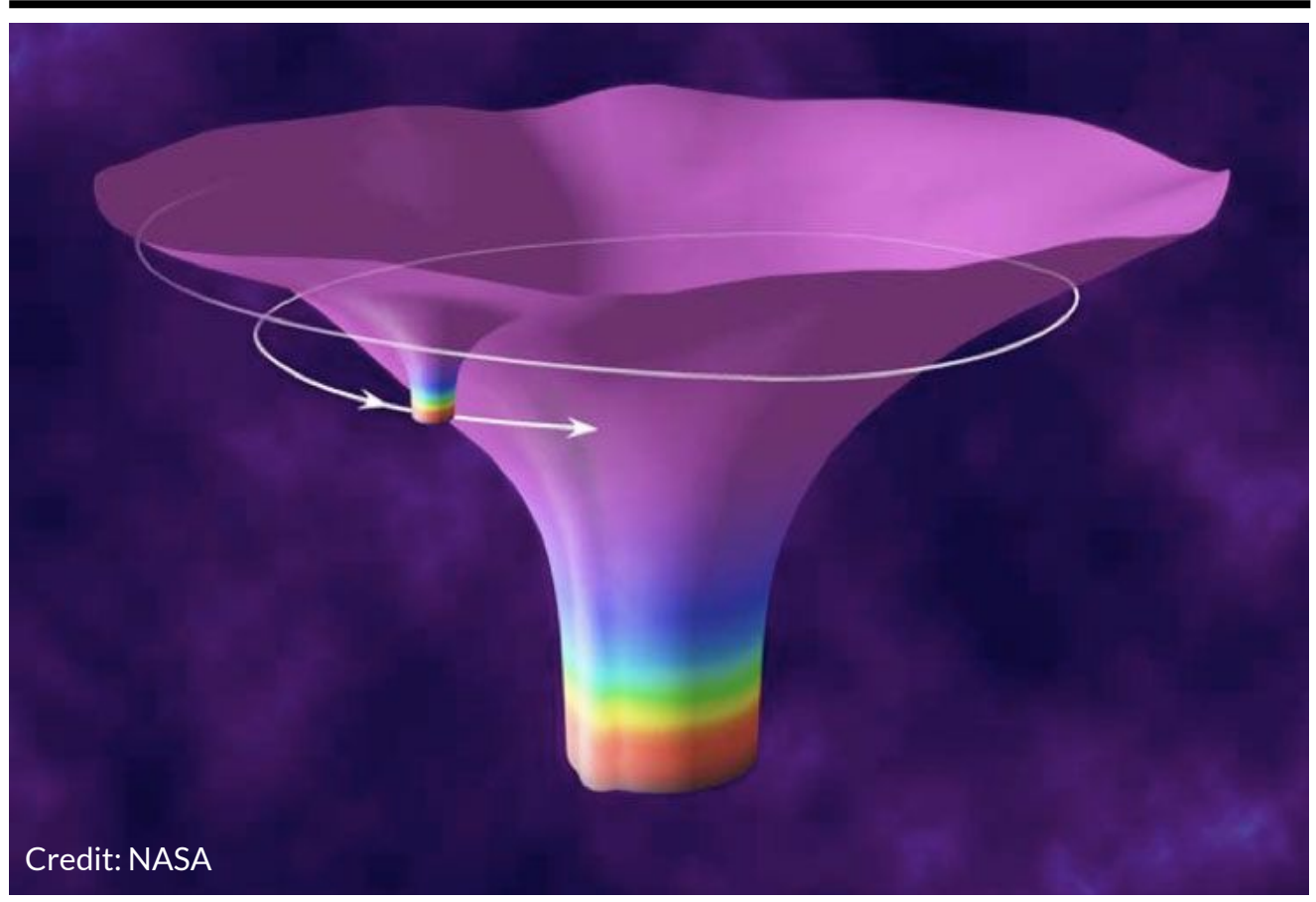
Extreme mass-ratio

inspirals (EMRIs) source
massive black hole +
stellar-mass companion.
Mass ratio $q \sim 10^{-5}$
[arXiv:astro-ph/0703495](https://arxiv.org/abs/astro-ph/0703495)

Intermediate mass-ratio
inspirals (IMRIs) $q \sim 10^{-3}$

Binary extreme
mass-ratio inspirals
(b-EMRIs) source
massive black hole +
stellar-mass binary
[arXiv:1801.05780](https://arxiv.org/abs/1801.05780)

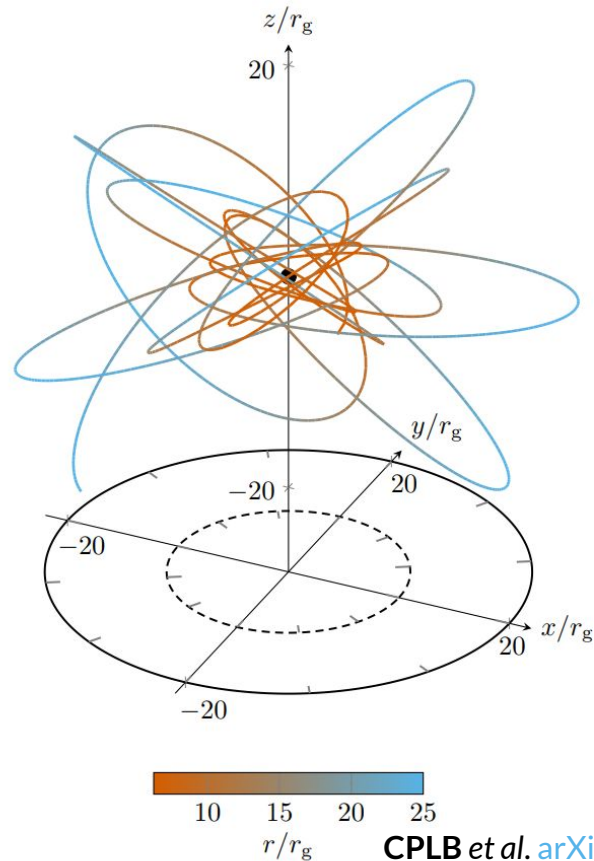
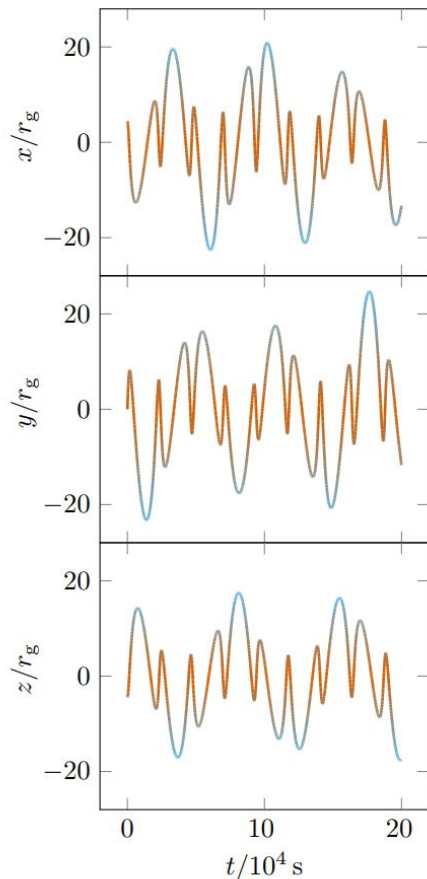
Extremely large
mass-ratio inspirals
(XMRI) source massive
black hole +
subsolar-mass
companion $q < 10^{-8}$
[arXiv:1903.10871](https://arxiv.org/abs/1903.10871)



EMRIs complete many orbits in the strong-field regime. Number of orbits $\sim 1/q$

Waveforms have a complicated frequency structure, e.g., [Speri et al. arXiv:2307.12585](#)

Complexity encodes information about the background spacetime, e.g., [Barack & Cutler arXiv:gr-qc/0310125](#)



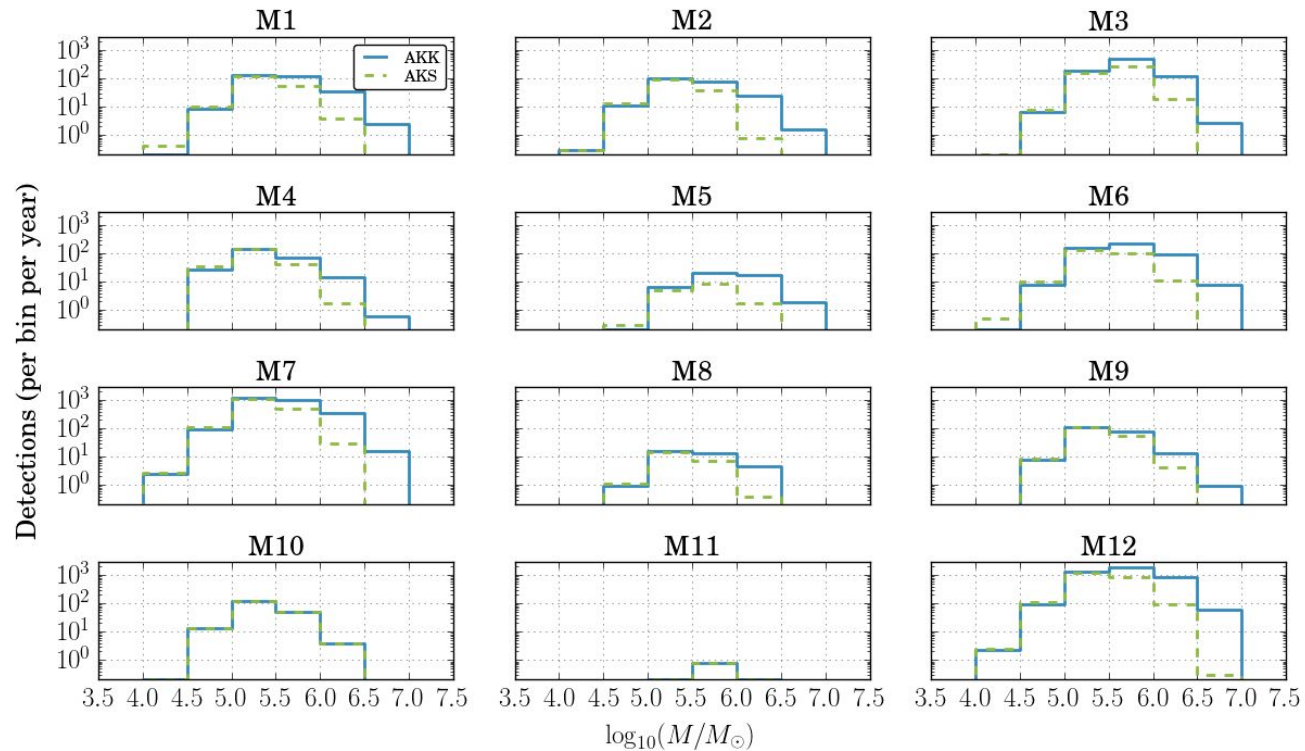
CPLB et al. [arXiv:1903.03686](#)

Precision measurement

EMRI populations are uncertain. Rates range from ~ 1 to 10^3 per year

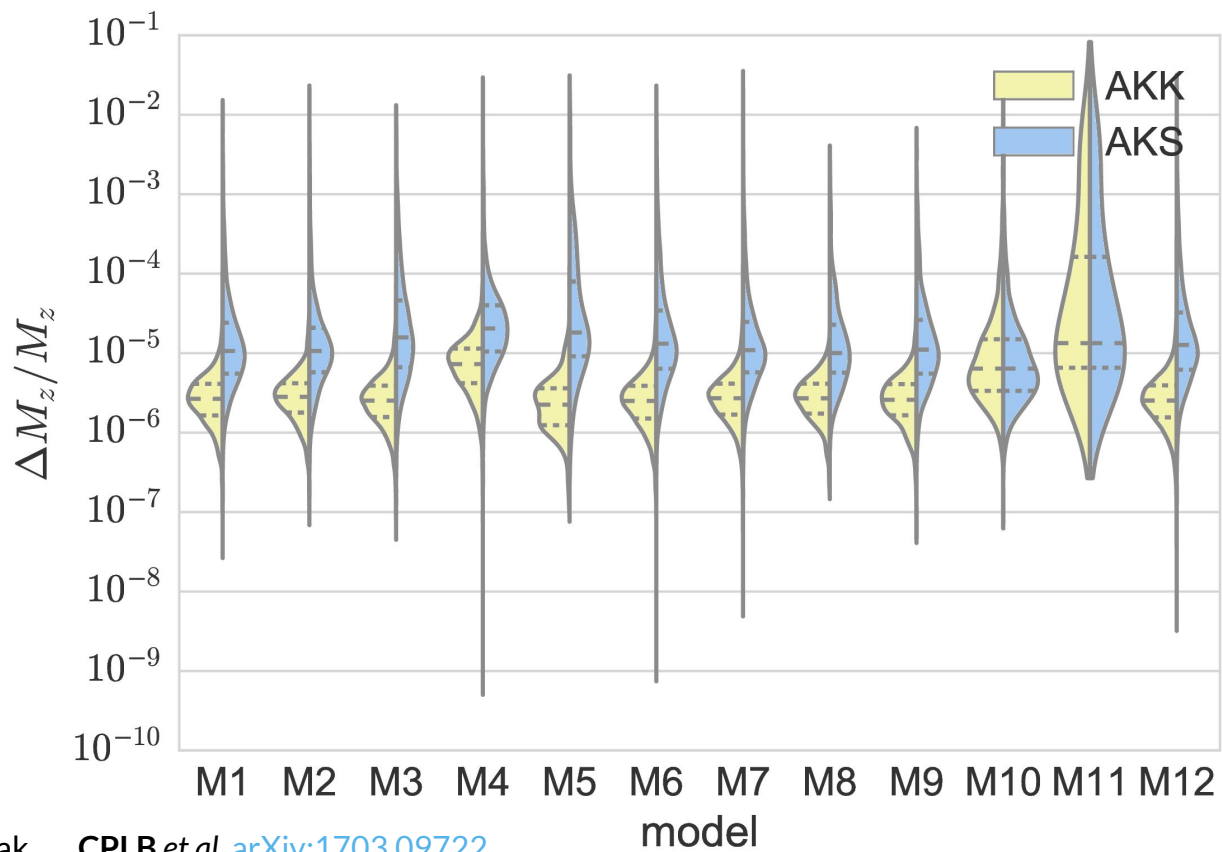
Results are sensitive to choice of waveform

AKK analytic kludge Kerr
AKS analytic kludge Schwarzschild (pessimistic)



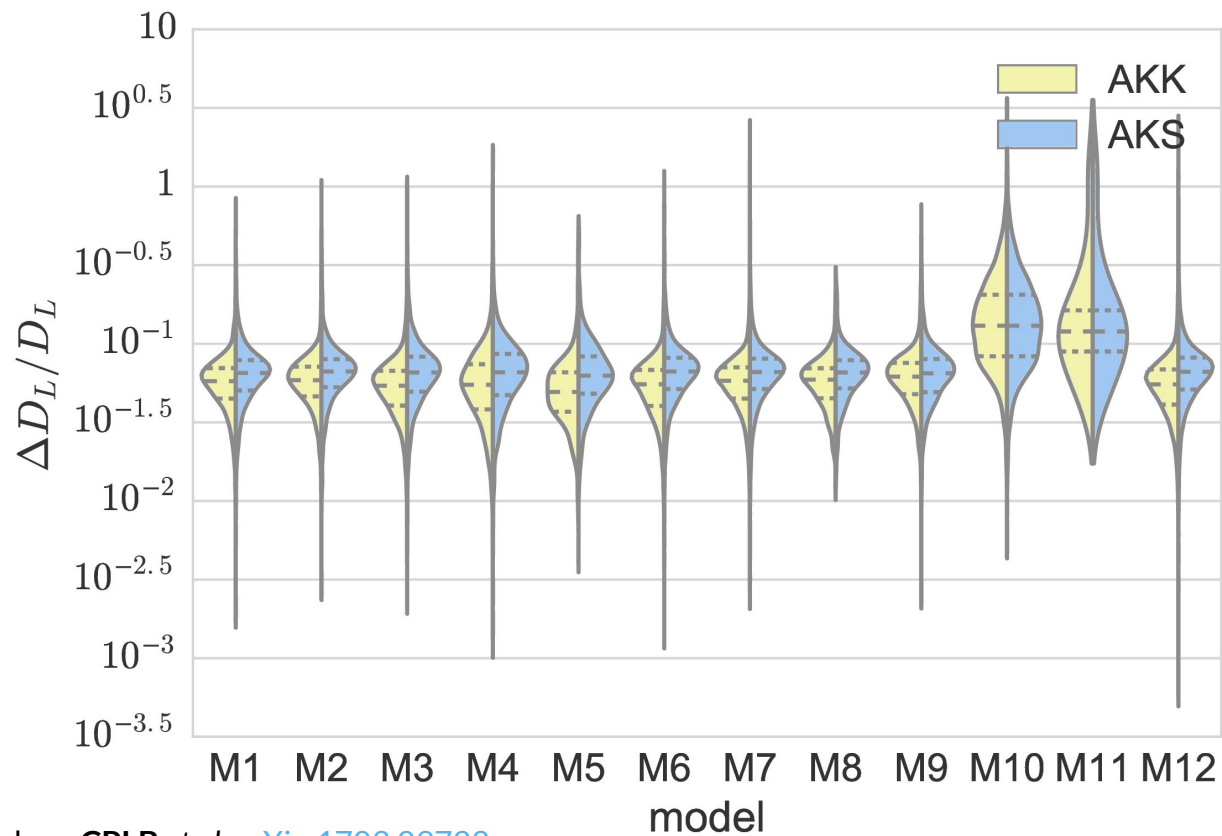
Babak, ..., CPLB *et al.* [arXiv:1703.09722](https://arxiv.org/abs/1703.09722)

Massive black hole
mass sets orbital
frequencies



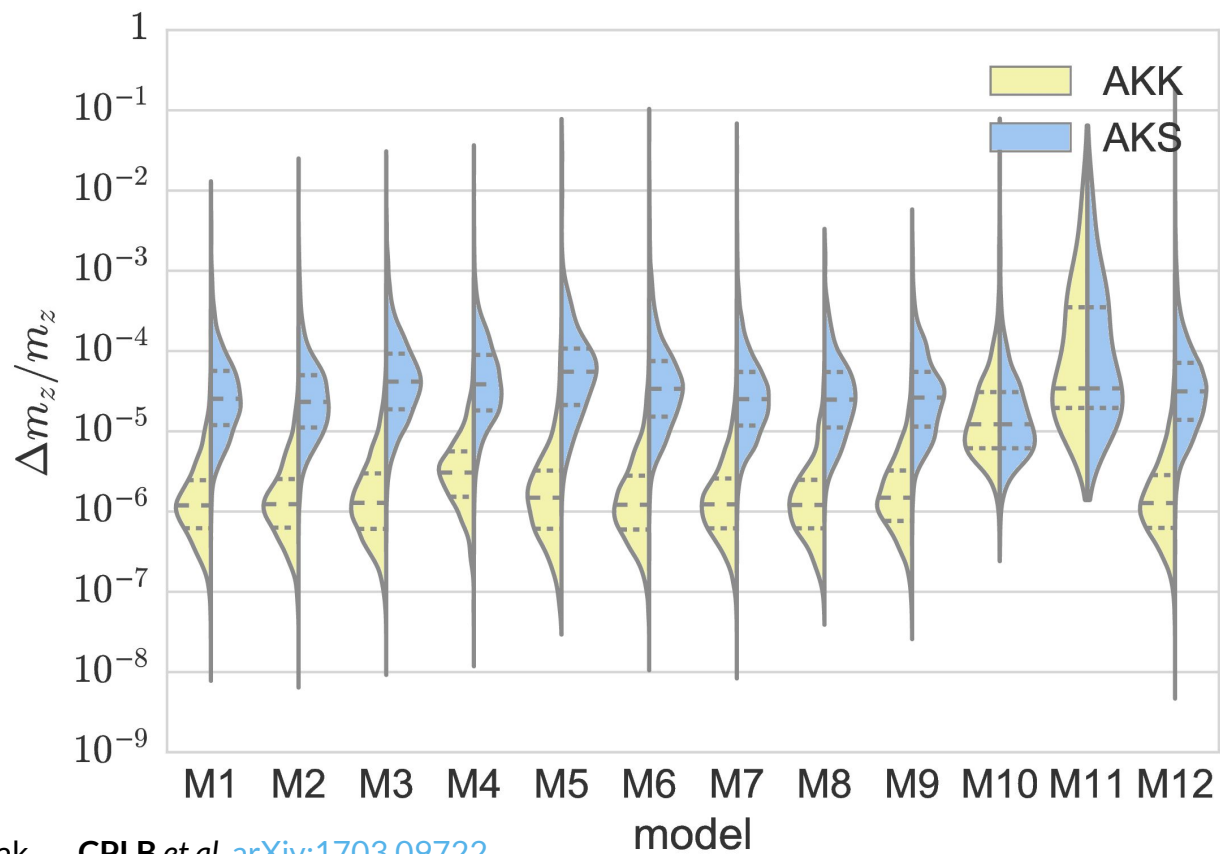
Babak, ..., CPLB et al. [arXiv:1703.09722](https://arxiv.org/abs/1703.09722)

Distance sets
amplitude



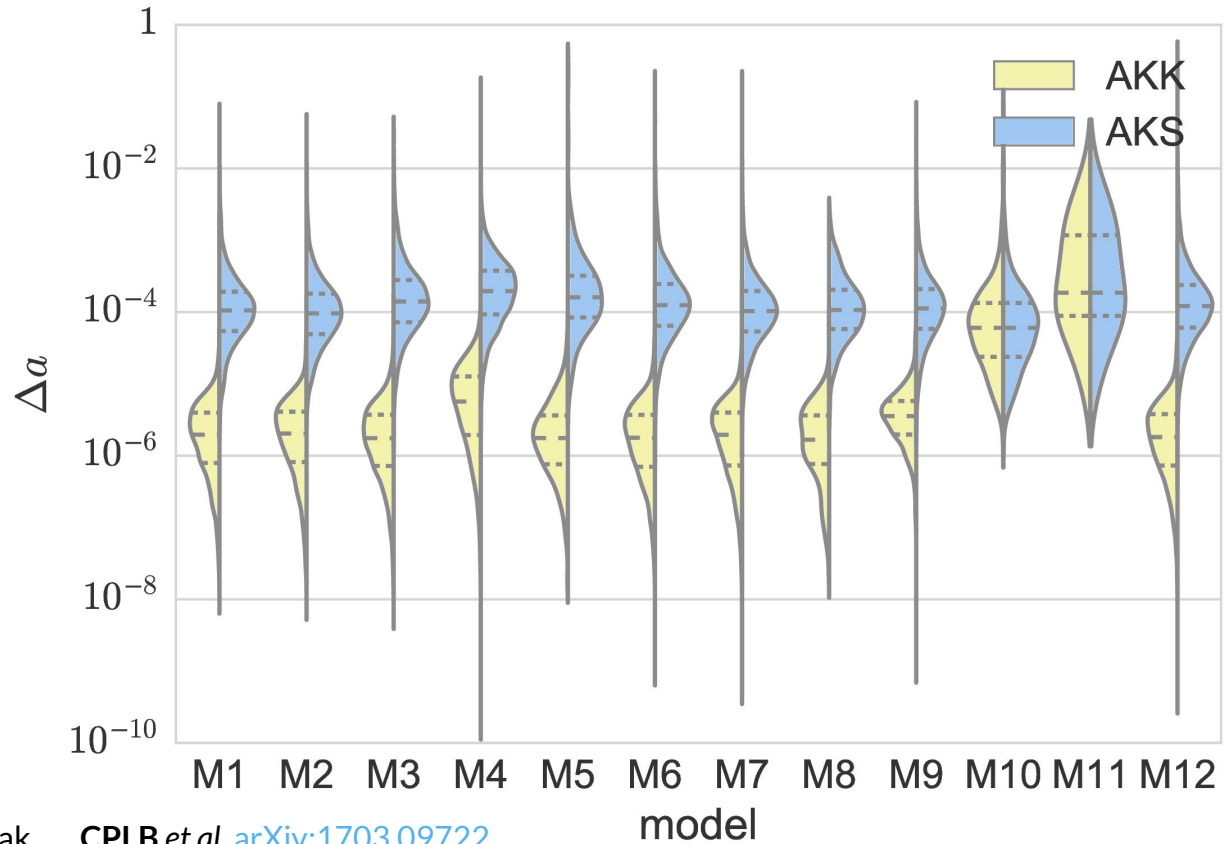
Babak, ..., CPLB et al. [arXiv:1703.09722](https://arxiv.org/abs/1703.09722)

Companion mass sets
inspiral rate and
amplitude



Babak, ..., CPLB et al. [arXiv:1703.09722](https://arxiv.org/abs/1703.09722)

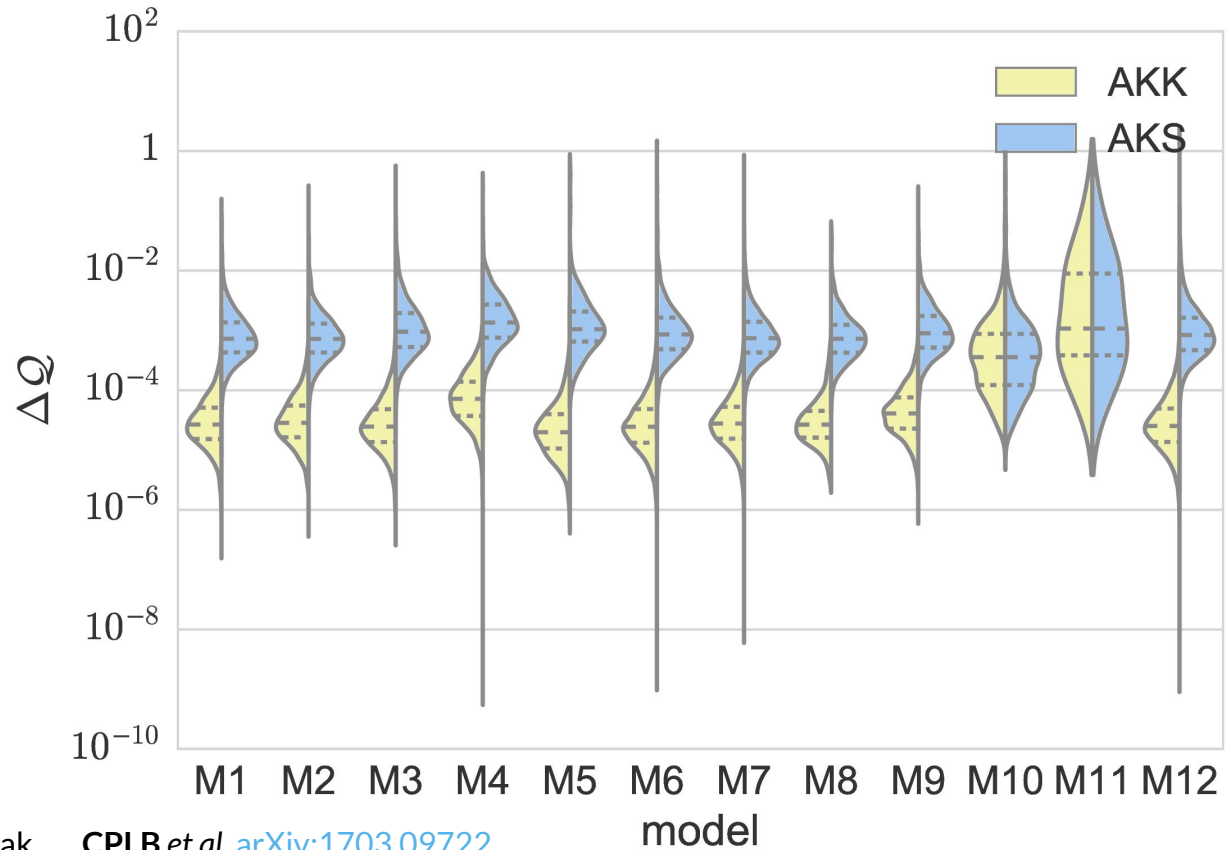
Massive black hole
spin influences
spacetime close to the
black hole



Babak, ..., CPLB et al. [arXiv:1703.09722](https://arxiv.org/abs/1703.09722)

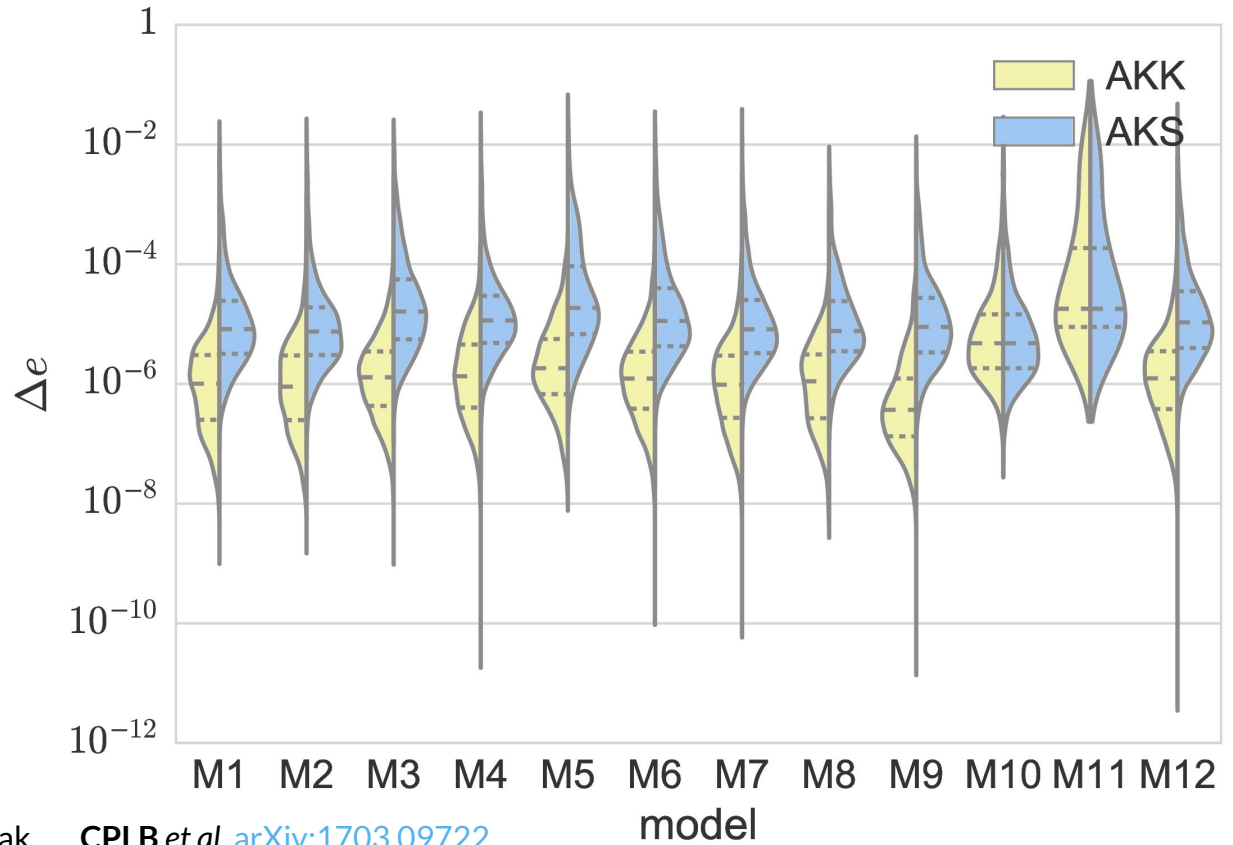
Deviations in the quadrupole moment would identify a non-Kerr spacetime

EMRIs can provide precision tests of general relativity, e.g., [Cárdenas-Avendaño & Sopuerta arXiv:2401.08085](#)



Babak, ..., CPLB *et al.* [arXiv:1703.09722](#)

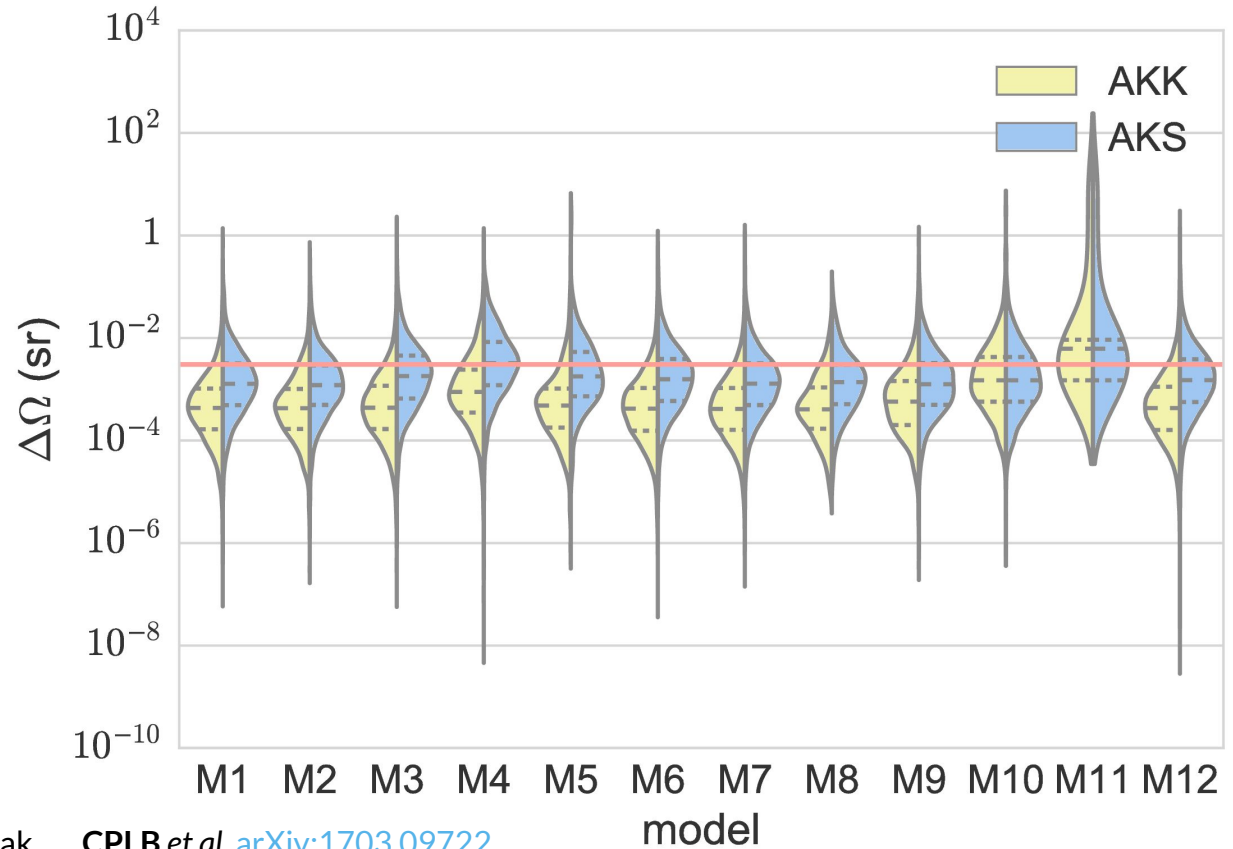
Eccentricity
influences set of
frequencies



Babak, ..., CPLB et al. [arXiv:1703.09722](https://arxiv.org/abs/1703.09722)

Sky location
important for
searching for
electromagnetic
counterparts and
association with host
galaxy and cosmology

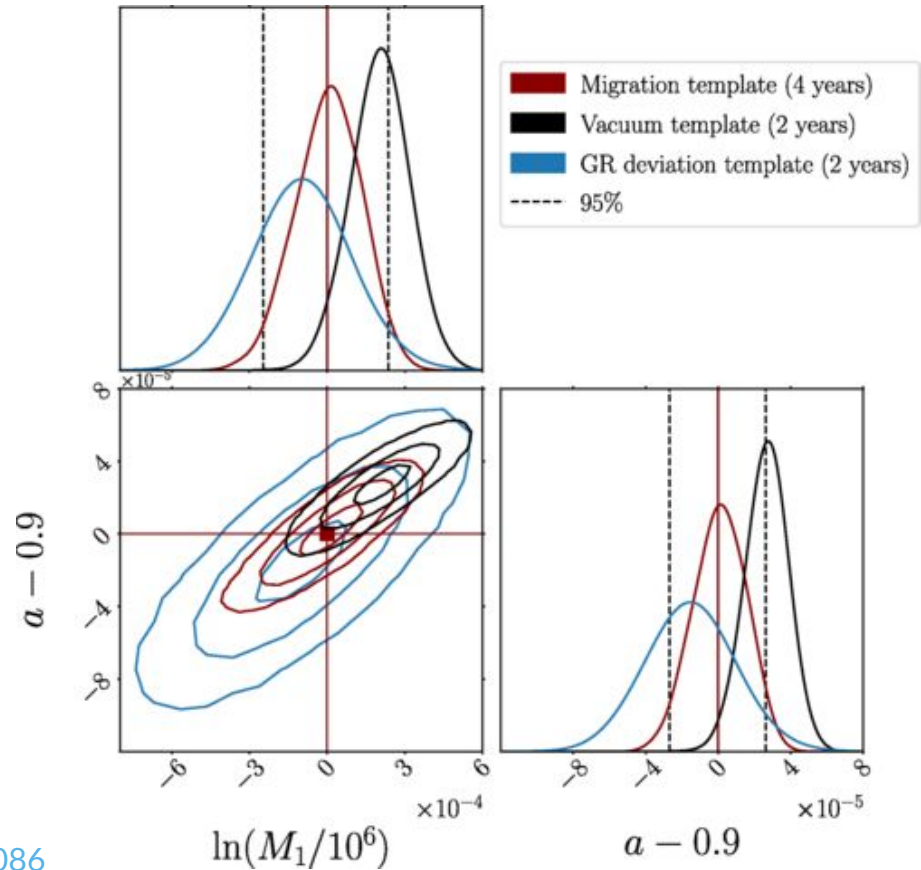
Measurement of the
cosmological
parameters
[Laghi et al.](#)
[arXiv:2102.01708](#)



Babak, ..., CPLB et al. [arXiv:1703.09722](#)

Using up-to-date
equatorial waveforms

Can measure
environmental or
non-Kerr effects



Speri et al. [arXiv:2207.10086](https://arxiv.org/abs/2207.10086)

Black hole astrophysics

EMRI formation

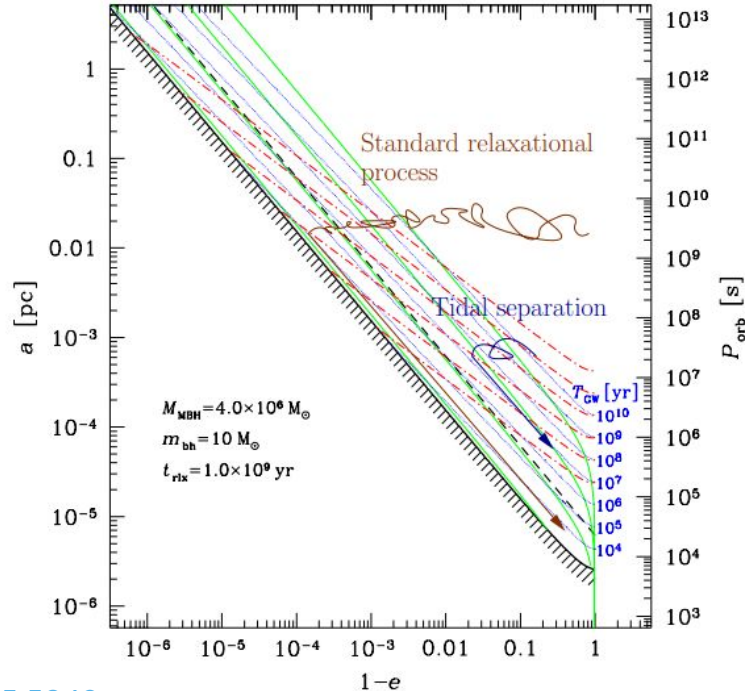
How to get companions close enough?

Dynamical friction and two-body scattering
arXiv:1205.5240

Scattering from cliff-diving orbits
arXiv:2304.13062

Disc-assisted wet EMRIs
arXiv:2104.01208

Supernova kicks
arXiv:1902.04581



Amaro-Seoane [arXiv:1205.5240](https://arxiv.org/abs/1205.5240)

Population inference

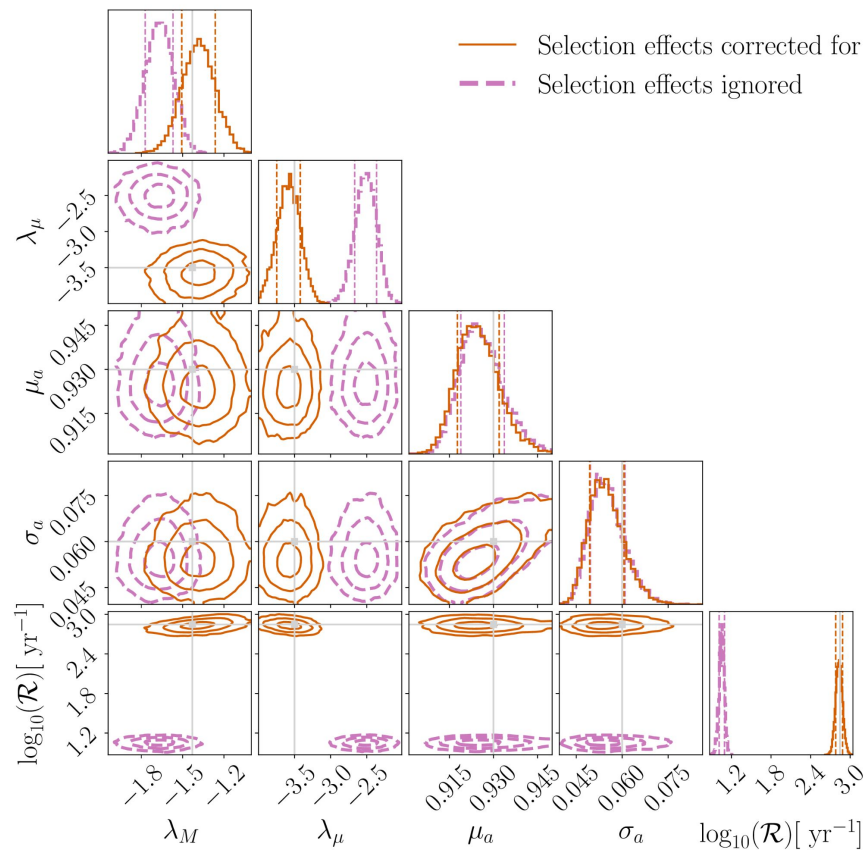
requires:

1. Parameter estimates for each signal
2. A population model
3. Accounting for selection effects

How to account for selection effects

[Mandel *et al.*](#)

[arXiv:1809.02063](#)



Chapman-Bird, CPLB & Woan

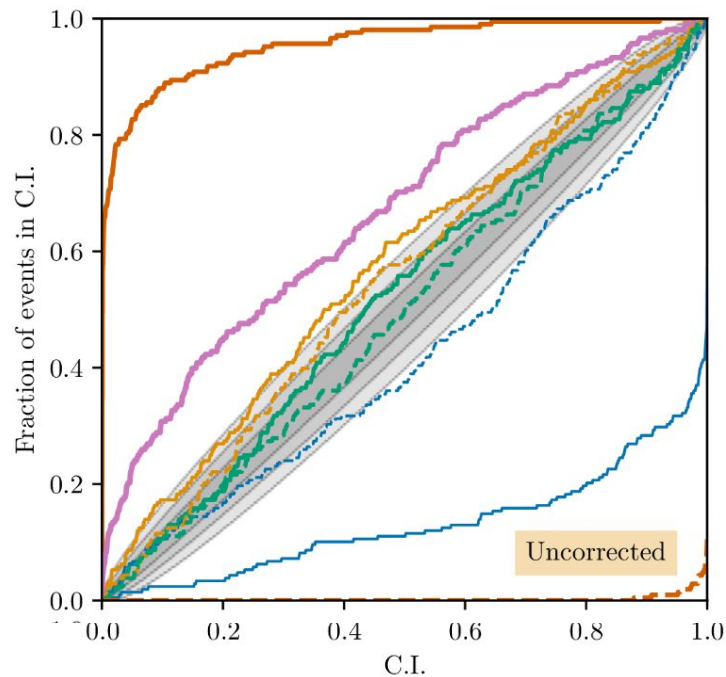
[arXiv:2212.06166](#)

Validation

Not accounting for selection effects leads to **biases**

Linear interpolation is insufficient

Our neural network approach works



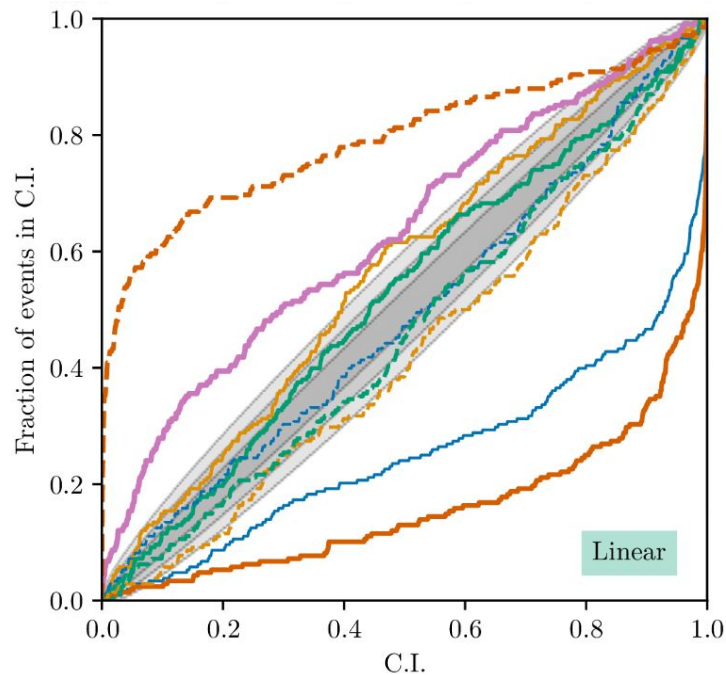
Chapman-Bird, **CPLB** & Woan [arXiv:2212.06166](https://arxiv.org/abs/2212.06166)

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Chapman-Bird, CPLB & Woan [arXiv:2212.06166](https://arxiv.org/abs/2212.06166)

Validation

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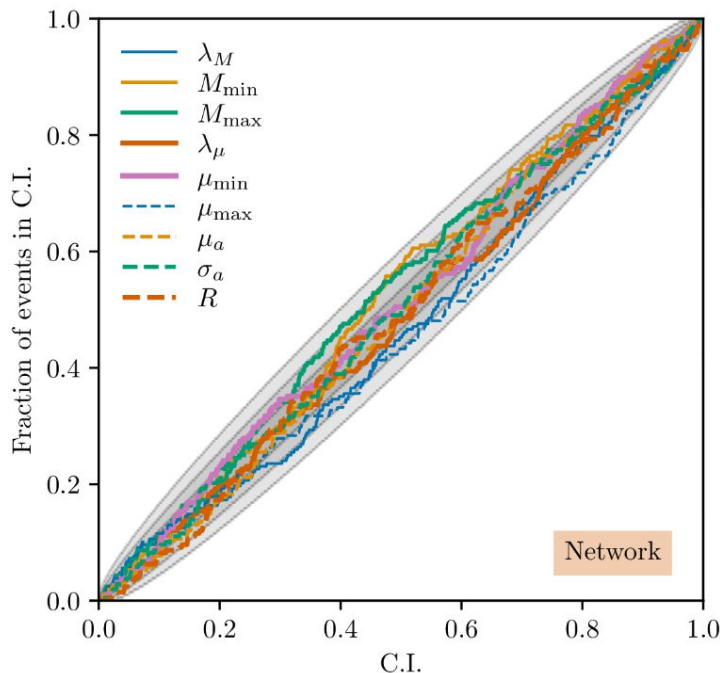
Linear interpolation is insufficient

Our **neural network** approach works

And is quick enough to run **hundreds** of population inferences!

poplar code

[DOI:10.5281/zenodo.7573034](https://doi.org/10.5281/zenodo.7573034)



Chapman-Bird, **CPLB** & Woan [arXiv:2212.06166](https://arxiv.org/abs/2212.06166)

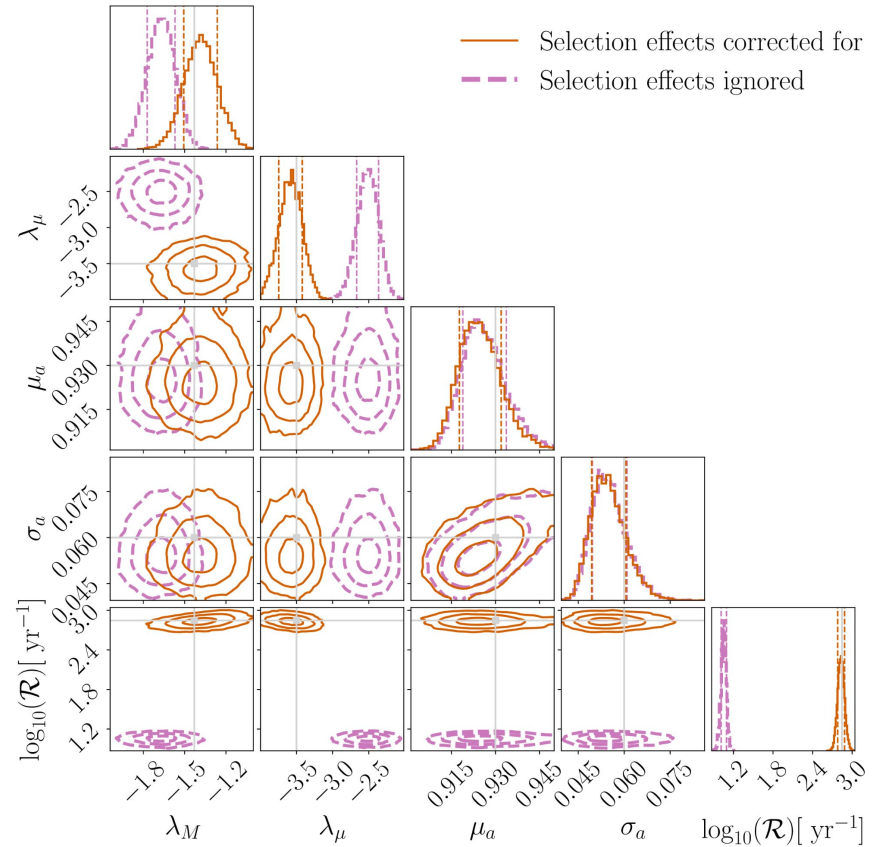
For sources with
redshift < 6 and an
optimistic event rate

MBH mass function
slope: 9%

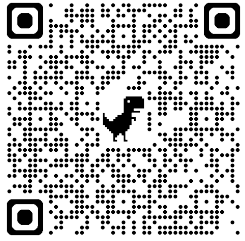
CO mass function
slope 5%

Width of the MBH
spin magnitude
distribution 10%

Event rate 12%

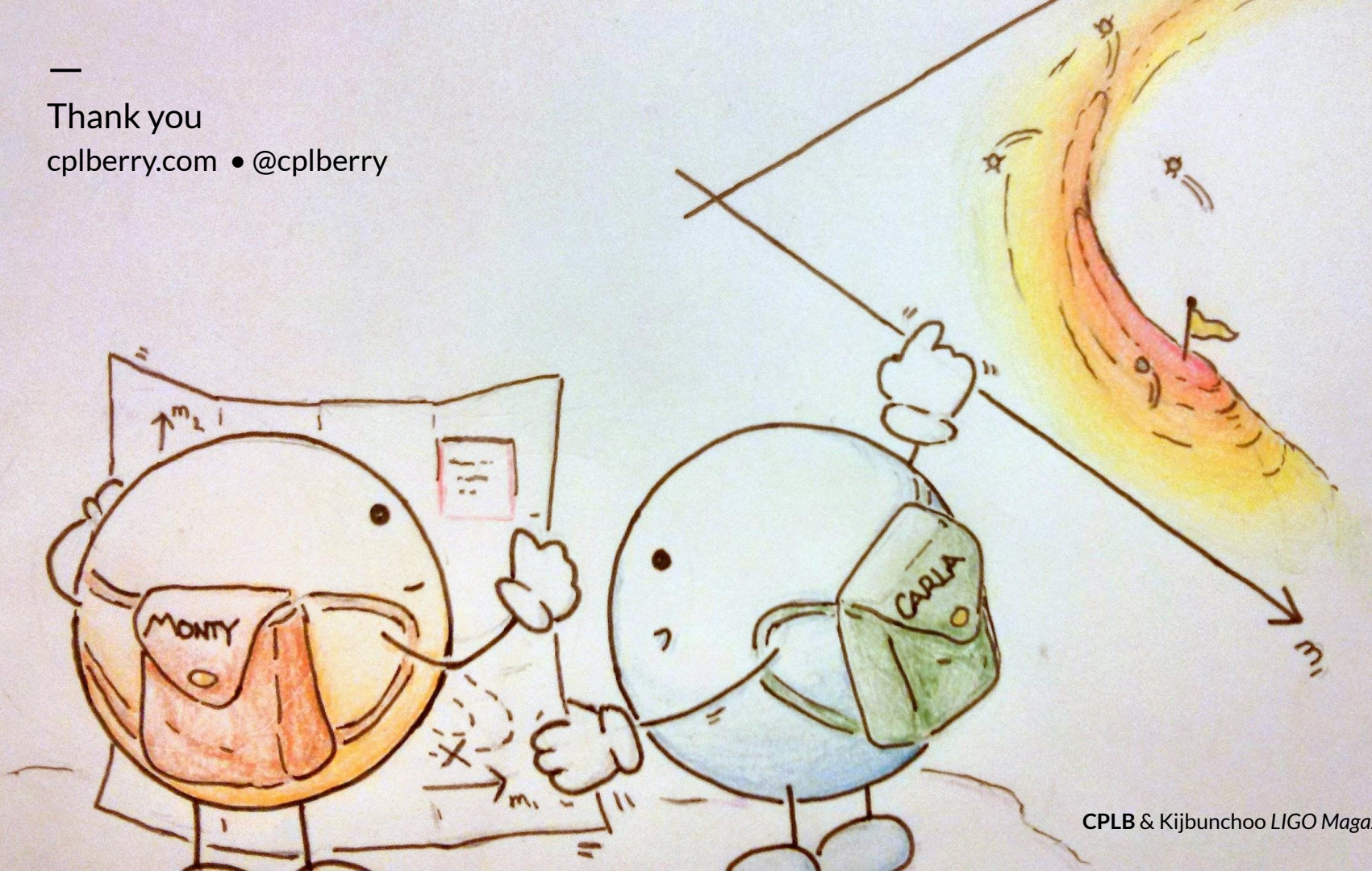


Chapman-Bird, **CPLB** & Woan
[arXiv:2212.06166](https://arxiv.org/abs/2212.06166)



1. **Extreme mass-ratio inspirals** have intricate waveforms
2. They enable **precision measurements** of individual sources, and a unique insight into the population
3. Significant work is needed to develop LISA data analysis

—
Thank you
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ALIA Bender, Begelman & Gair
2013

TianQin Luo *et al.*
arXiv:1512.02076

Taiji Ruan *et al.*
arXiv:1807.09495

DECIGO Kawamura *et al.*
arXiv:2006.13545

TianGO Kuns *et al.*
arXiv:1908.06004

GADFLI McWilliams
arXiv:1111.3708

gLISA Tinto *et al.*
arXiv:1410.1813

SAGE Lacour *et al.*
arXiv:1811.04743

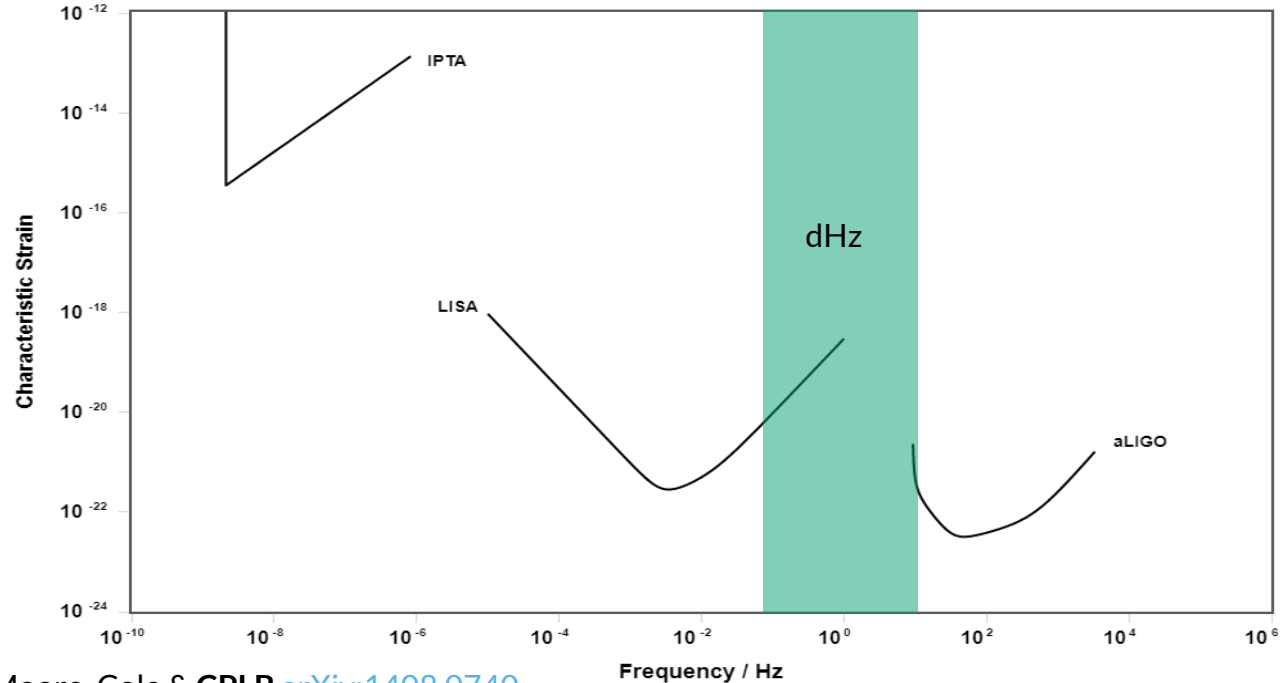
MAGIS Graham *et al.*
arXiv:1711.02225

AEDGE Abou El-Neaj *et al.*
arXiv:1908.00802

Big Bang Observer Crowder &
Cornish arXiv:gr-qc/0506015

DO-Conservative & DO-Optimal
Arca Sedda, **CPLB** *et al.*
arXiv:1908.11375

Gravitational-wave spectrum



Moore, Cole & **CPLB** [arXiv:1408.0740](https://arxiv.org/abs/1408.0740)

Results depend upon waveforms. Rapid development in recent years

LISA

[arXiv:2311.01300](https://arxiv.org/abs/2311.01300)

EMRI waveforms calculated using **gravitational self-force**, e.g., [Barack & Pound](#)

[arXiv:1805.10385](https://arxiv.org/abs/1805.10385)

Fast EMRI Waveform (FEW) package available

[Chua et al.](#)

[arXiv:2008.06071](https://arxiv.org/abs/2008.06071)

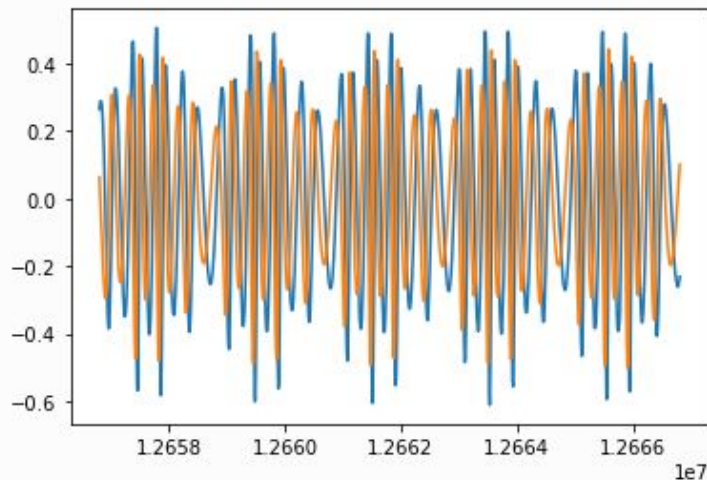
[Katz et al.](#)

[arXiv:2104.04582](https://arxiv.org/abs/2104.04582)

FEW package

```
[5]: plt.plot(t[-1000:], wave.real[-1000:])  
plt.plot(t[-1000:], wave.imag[-1000:])
```

```
[5]: [<matplotlib.lines.Line2D at 0x7fd9853f850>]
```



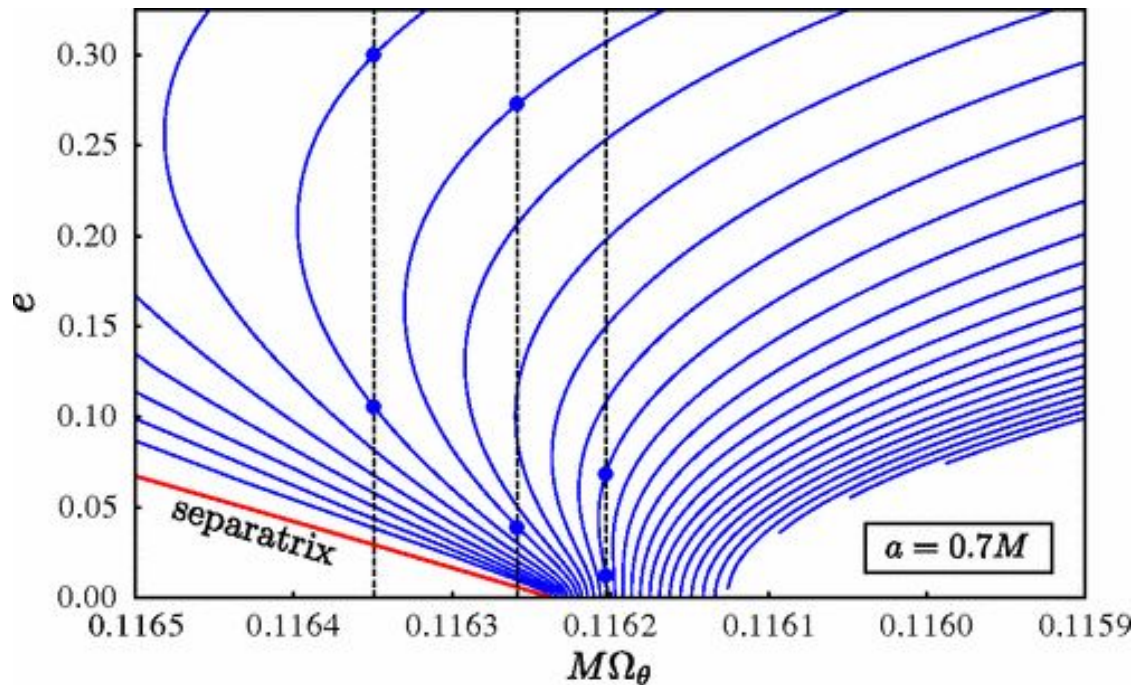
[Katz et al.](#)

bhptoolkit.org/FastEMRIWaveforms/

EMRI frequencies

EMRIs are typically described by 3 frequencies: radial, polar and azimuthal

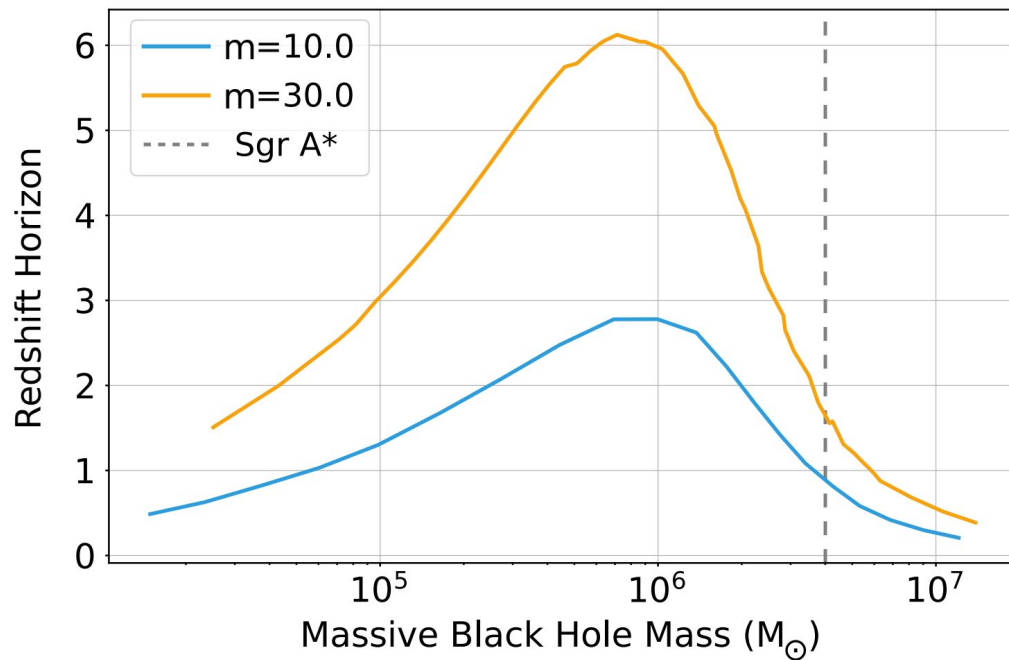
Orbital frequencies in Kerr
Schmidt
[arXiv:gr-qc/0202090](https://arxiv.org/abs/gr-qc/0202090)



Warburton, Barack & Sago [arXiv:1301.3918](https://arxiv.org/abs/1301.3918)

EMRI horizon

EMRIs can be seen to
cosmological redshifts

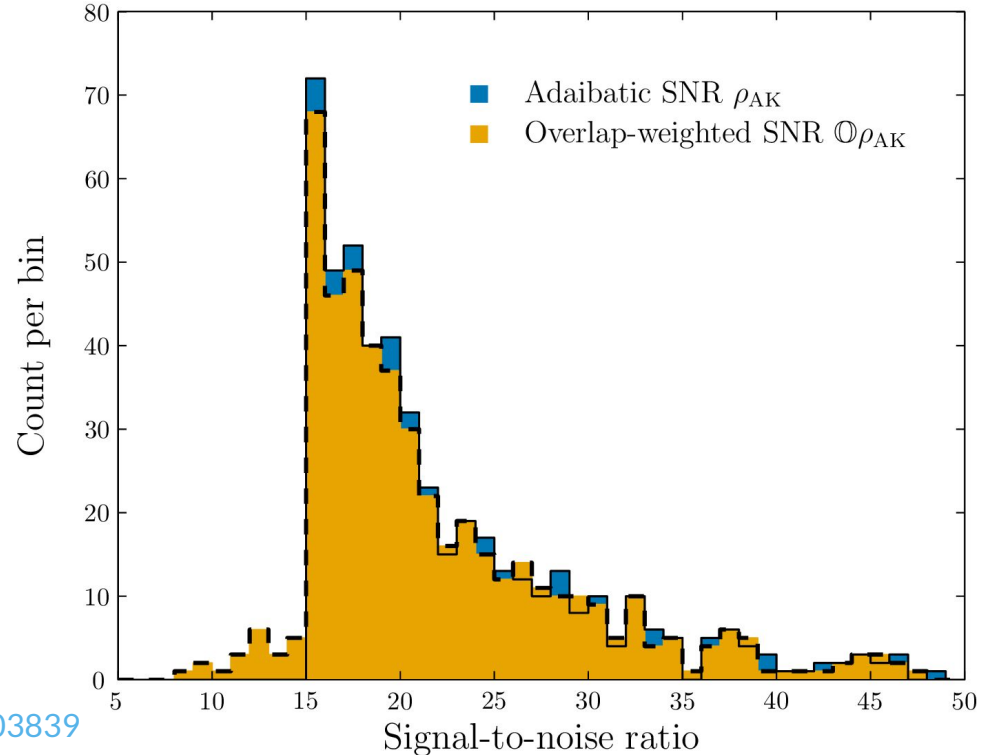


LISA [arXiv:2402.07571](https://arxiv.org/abs/2402.07571)

Resonances

Transient resonances
need to be accounted
for in evolution
[Flanagan & Hinderer](#)
[arXiv:1009.4923](#)

Most resonances are
late in inspiral, and
less important for low
eccentricity orbits



CPLB *et al.* [arXiv:1602.03839](#)

Population models

EMRI populations are uncertain. Rates range from ~ 1 to 10^3 per year

Results are sensitive to choice of waveform

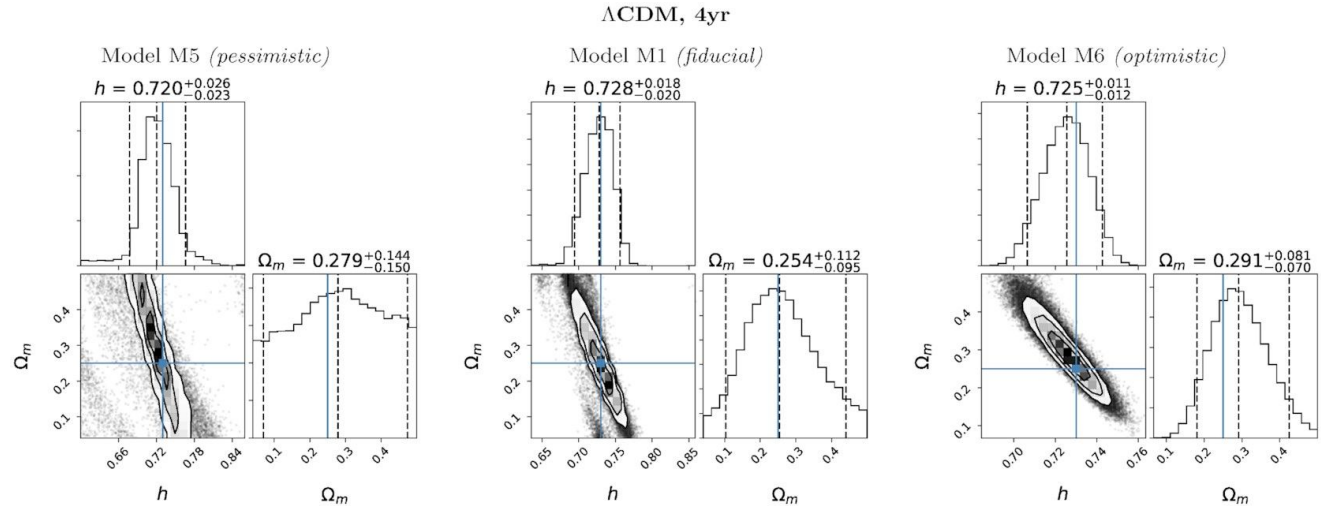
AKK analytic kludge
Kerr
AKS analytic kludge
Schwarzschild
(pessimistic)

Model	Mass	MBH	Cusp	$M-\sigma$	N_p	CO	EMRI rate [yr^{-1}]		
	function	spin	erosion	relation		mass [M_\odot]	Total	Detected (AKK)	Detected (AKS)
M1	Barausse12	a98	yes	Gultekin09	10	10	1600	294	189
M2	Barausse12	a98	yes	KormendyHo13	10	10	1400	220	146
M3	Barausse12	a98	yes	GrahamScott13	10	10	2770	809	440
M4	Barausse12	a98	yes	Gultekin09	10	30	520 (620)	260	221
M5	Gair10	a98	no	Gultekin09	10	10	140	47	15
M6	Barausse12	a98	no	Gultekin09	10	10	2080	479	261
M7	Barausse12	a98	yes	Gultekin09	0	10	15800	2712	1765
M8	Barausse12	a98	yes	Gultekin09	100	10	180	35	24
M9	Barausse12	aflat	yes	Gultekin09	10	10	1530	217	177
M10	Barausse12	a0	yes	Gultekin09	10	10	1520	188	188
M11	Gair10	a0	no	Gultekin09	100	10	13	1	1
M12	Barausse12	a98	no	Gultekin09	0	10	20000	4219	2279

Babak, ..., CPLB *et al.* [arXiv:1703.09722](https://arxiv.org/abs/1703.09722)

Cosmological
parameter
measurements from
dark siren analysis

Initially suggested in
[MacLeod & Hogan](#)
[arXiv:0712.0618](#)



Laghi *et al.* [arXiv:2102.01708](#)

Machine learning

Two levels of emulation:

1. **Signal SNRs** to learn detection threshold as a function of source parameters
2. **Population detection probability** as a function of population hyperparameters

poplar code

[DOI:10.5281/zenodo.7573034](https://doi.org/10.5281/zenodo.7573034)

Setting	SNR MLP	$\alpha(\lambda)$ MLP
Number of (hidden) layers	10	8
Neurons per layer	128	128
Activation function	SiLU	SiLU
Rescaling	Unit normal	Unit normal
Optimiser	Adam	Adam
Learning rate	5×10^{-4}	5×10^{-4}
Batch size	10^4	10^5
Max epochs	10^5	10^3
Loss function	L1	L1

Chapman-Bird, **CPLB** & Woan [arXiv:2212.06166](https://arxiv.org/abs/2212.06166)

Astrophysical distribution

BGP = binned

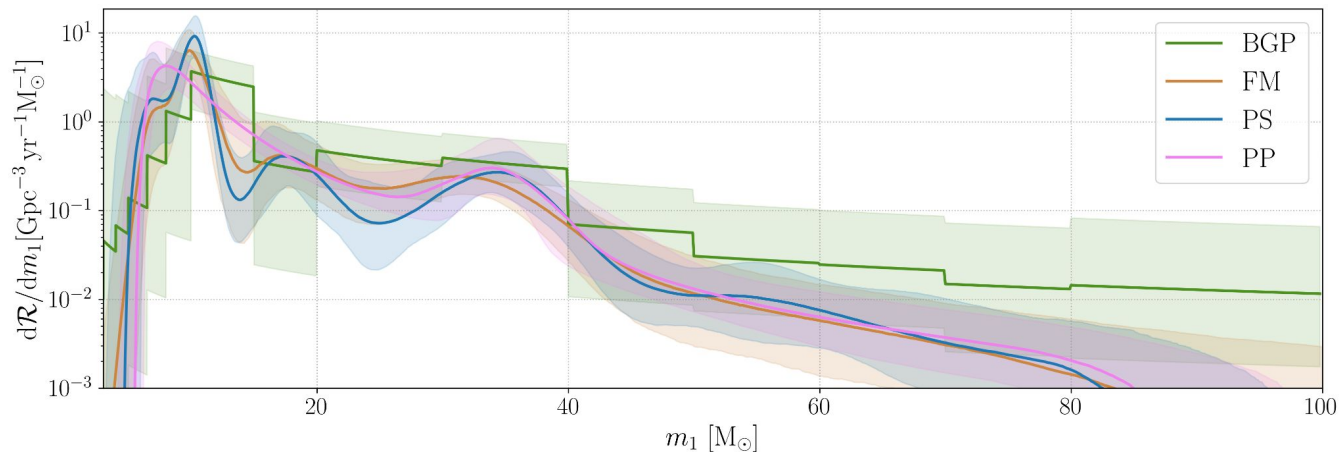
Gaussian process

FM = flexible mixtures

Gaussian kernels

PS = power-law plus
spline

PP = power-law plus
(Gaussian) peak



LVK [arXiv:2111.03634](https://arxiv.org/abs/2111.03634)