Probing the regularization of spacetime singularities

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Gravity shape Pisa, 25 Oct. 2024

SDU | ħQTC
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General Relativity

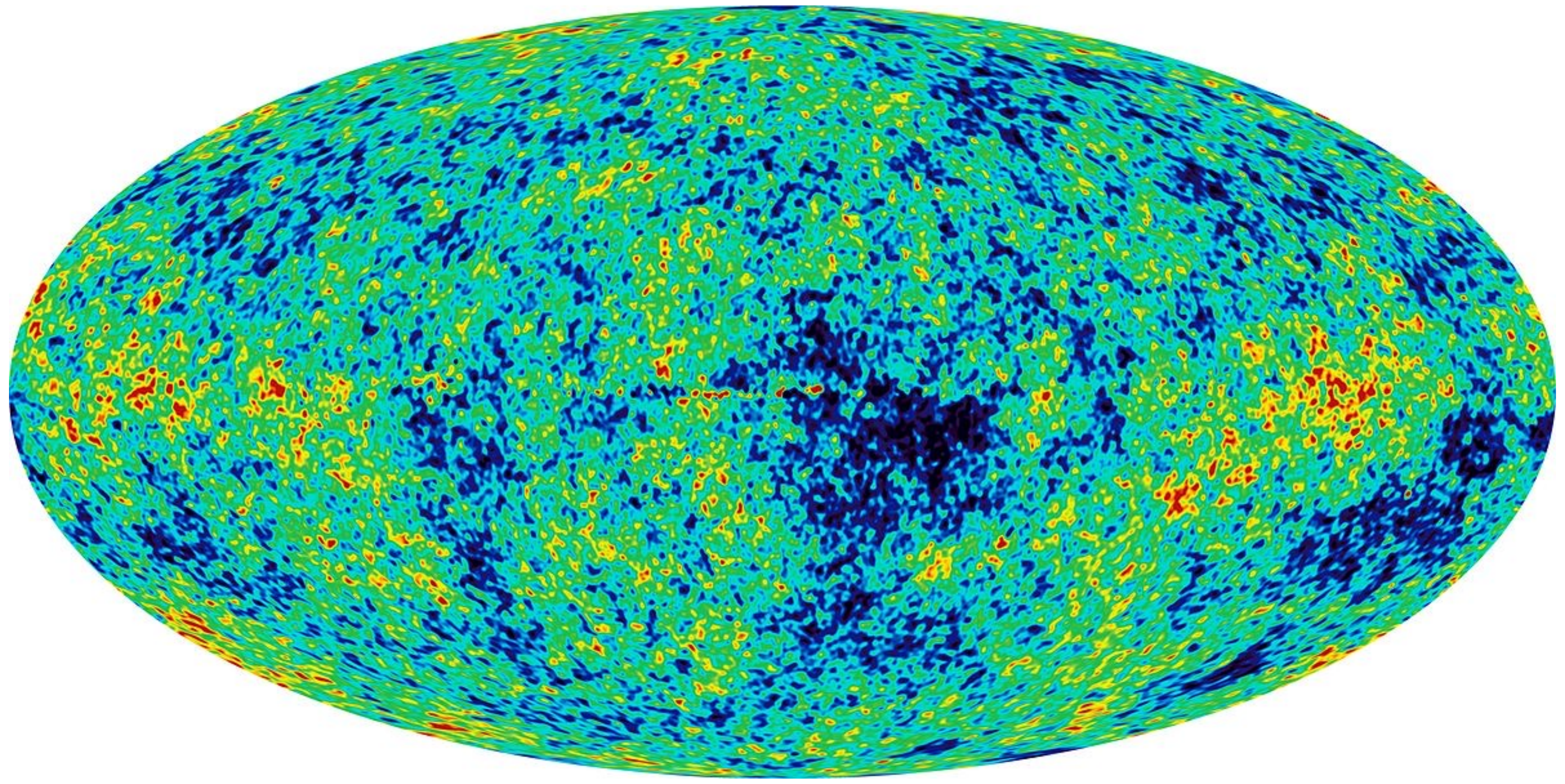
Gravitational collapse

Cosmological evolution

Singularity theorems

Black Hole singularities

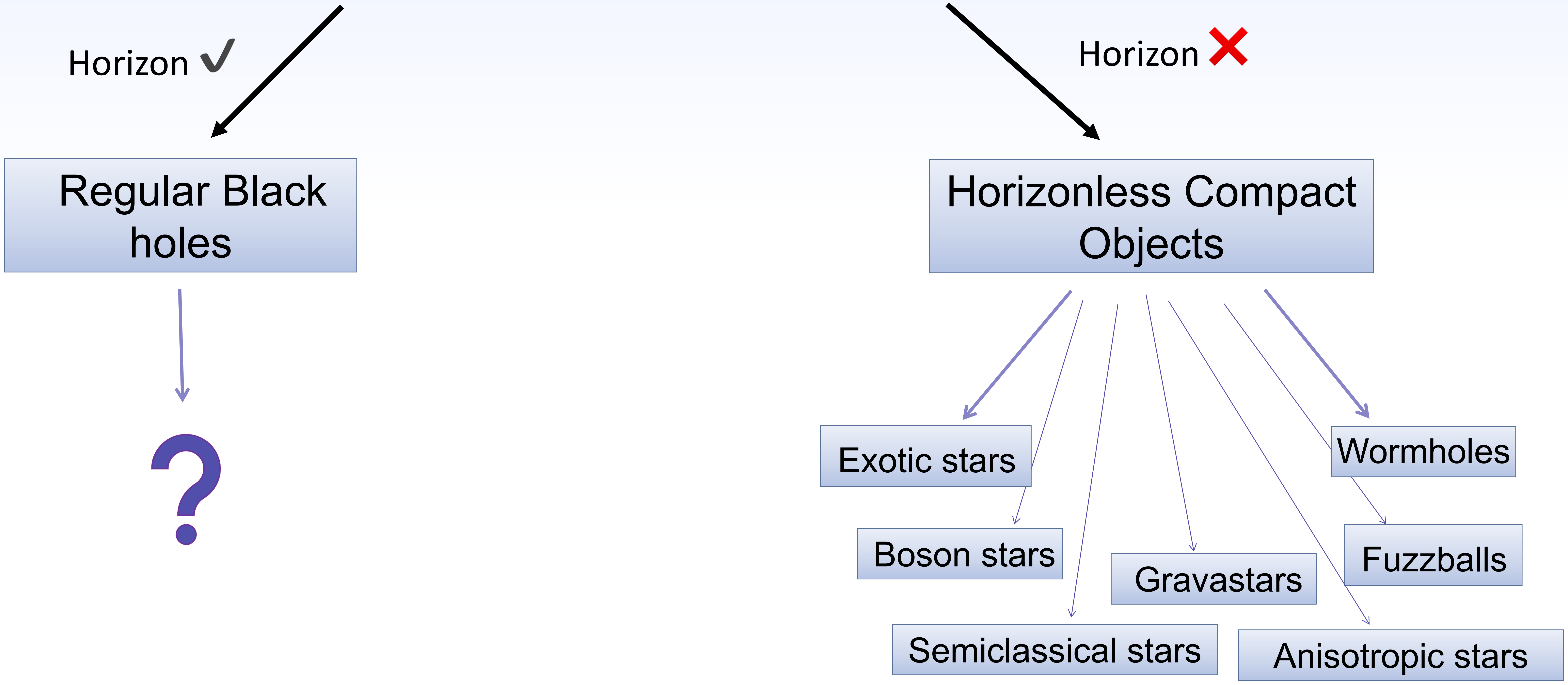
Big Bang singularity



But singularities are considered the demise of General Relativity!
A singular space-time is **geodesic incomplete**:
there exist at least one geodesic that cannot be extended beyond a finite proper time or affine parameter (particles seem to disappear from existence!)

Black holes Mimickers

In a complete theory of (**quantum) gravity** we expect the formation of **spacetime singularities to be prevented.**
There are basically two possible **alternatives to singular black holes** to describe the ultra-compact objects that we see in the sky

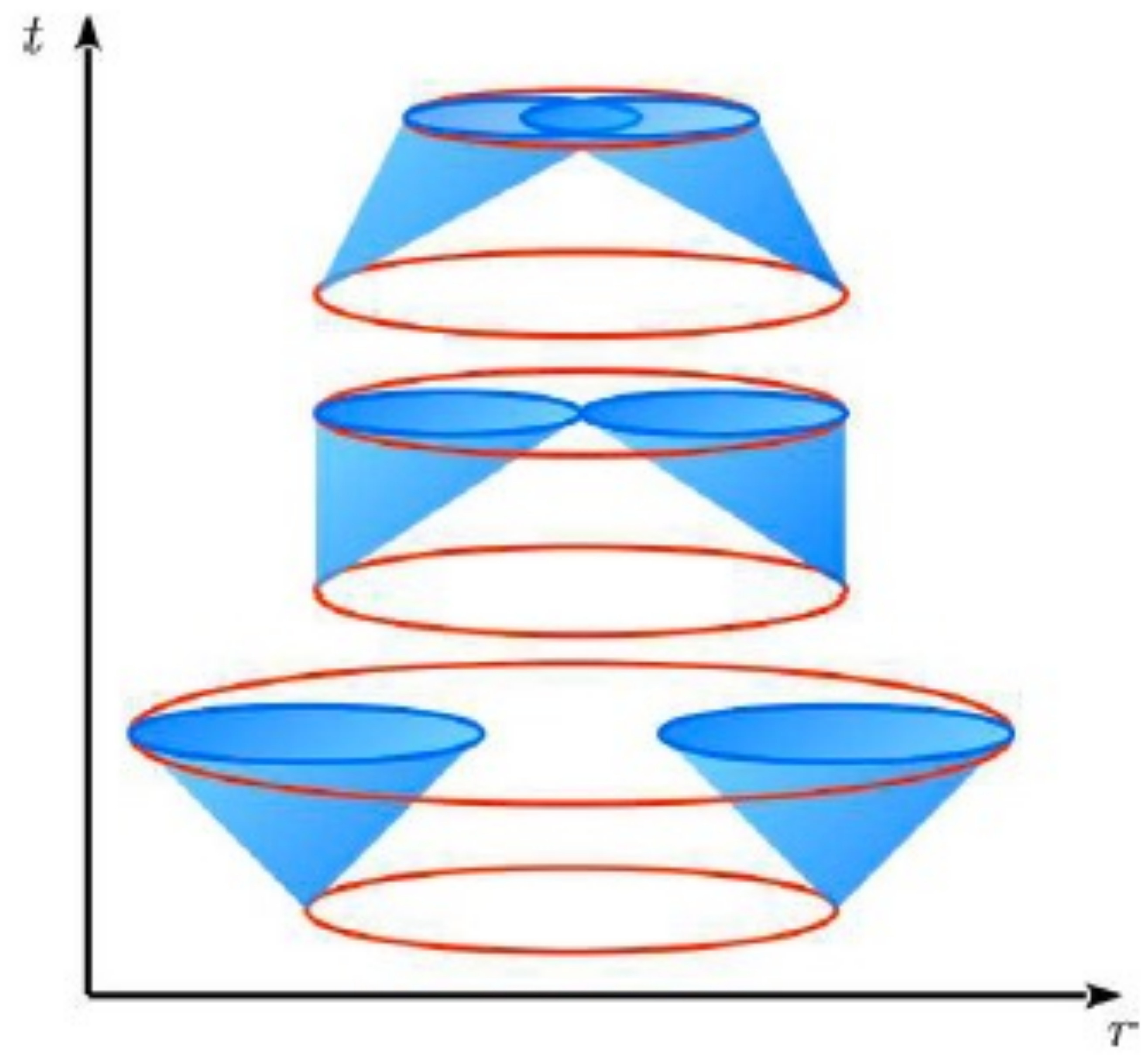


Beyond the Penrose Theorem

From the **Penrose Theorem**
if **Einstein equations** holds, a **non-compact Cauchy surface** is present and the **Null Energy condition** is respected, when a **trapped surface (a horizon)** is formed there is no way to escape the formation of a **singular focusing point**

The **expansion θ** of a congruence of geodesics tells you how much **a cloud of particles expands or contracts isotropically** as it moves along the congruence.

When both the expansion of null **ingoing θ_-** and **outgoing θ_+** geodesics become negative,
a **trapped surface (a horizon) is formed**!



Beyond the Penrose Theorem

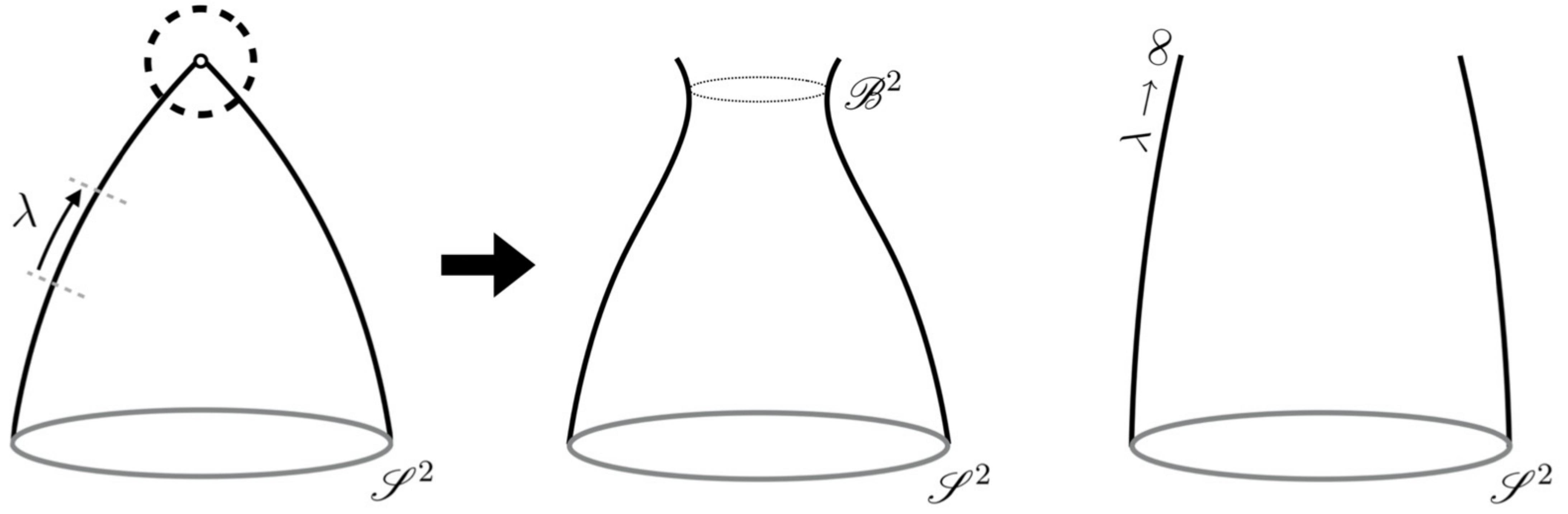
Penrose Theorem

$\theta_+ < 0$ in some points

GR + NEC + non-compact Cauchy surface

Singular focusing point!

$\theta_+ \to -\infty$



Violation of the Penrose Theorem

$\theta_+ < 0$ in some points

~~GR + NEC + non-compact Cauchy surface~~

Defocusing point at which the outgoing expansion changes sign again

$\boldsymbol{\theta_+ = 0}$

Then we can classify the possible effective **regular geometry** on the basis of what is happening to the ingoing expansion $\boldsymbol{\theta_-}$ at the defocusing point!

Black holes Mimickers

There are basically two possible **alternatives to singular black holes** to describe the ultra-compact objects that we see in the sky

Horizon ✓

Regular Black holes

Horizon ✗

Horizonless Compact Objects

Multiple horizons

(At the defocusing point θ_{-} remains negative)

The singularity is replaced by an inner horizon shelding a non-singular core.

Hidden wormholes

(At the defocusing point $\theta_{-} \geq 0$)

The singularity is replaced by a wormhole throat hidden inside a trapping horizon

Exotic stars

Wormholes

Boson stars

Gravastars

Fuzzballs

Semiclassical stars

Anisotropic stars

*we are considering spherical symmetric objects

Two families of (spherically symmetric) Black holes Mimickers

both can **interpolate between regular black holes and horizonless compact objects***

$$ds^2 = -e^{-2\phi(r)} f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad f(r) = 1 - \frac{2m(r)}{r}.$$

From Multiple horizons to Exotic stars (varying ℓ)

$$\phi(r) = 0$$

and

$$m(r) = M\frac{r^3}{r^3 + 2M\ell^2} \quad \text{(Hayward metric)}$$

or

$$m(r) = M\frac{r^3}{(r^2 + \ell^2)^{3/2}} \quad \text{(Bardeen metric)}$$

or…

From Hidden wormholes to traversable Wormholes (varying ℓ)

$$\phi(r) = \frac{1}{2}\log\left(1 - \frac{\ell^2}{r^2}\right)$$

and

$$m(r) = M\left(1 - \frac{\ell^2}{r^2}\right) + \frac{\ell^2}{2r} \quad \text{(Simpson-Visser)}$$

*Carballo-Rubio et al. 2023

The effect of the regularization on the ringdown signal

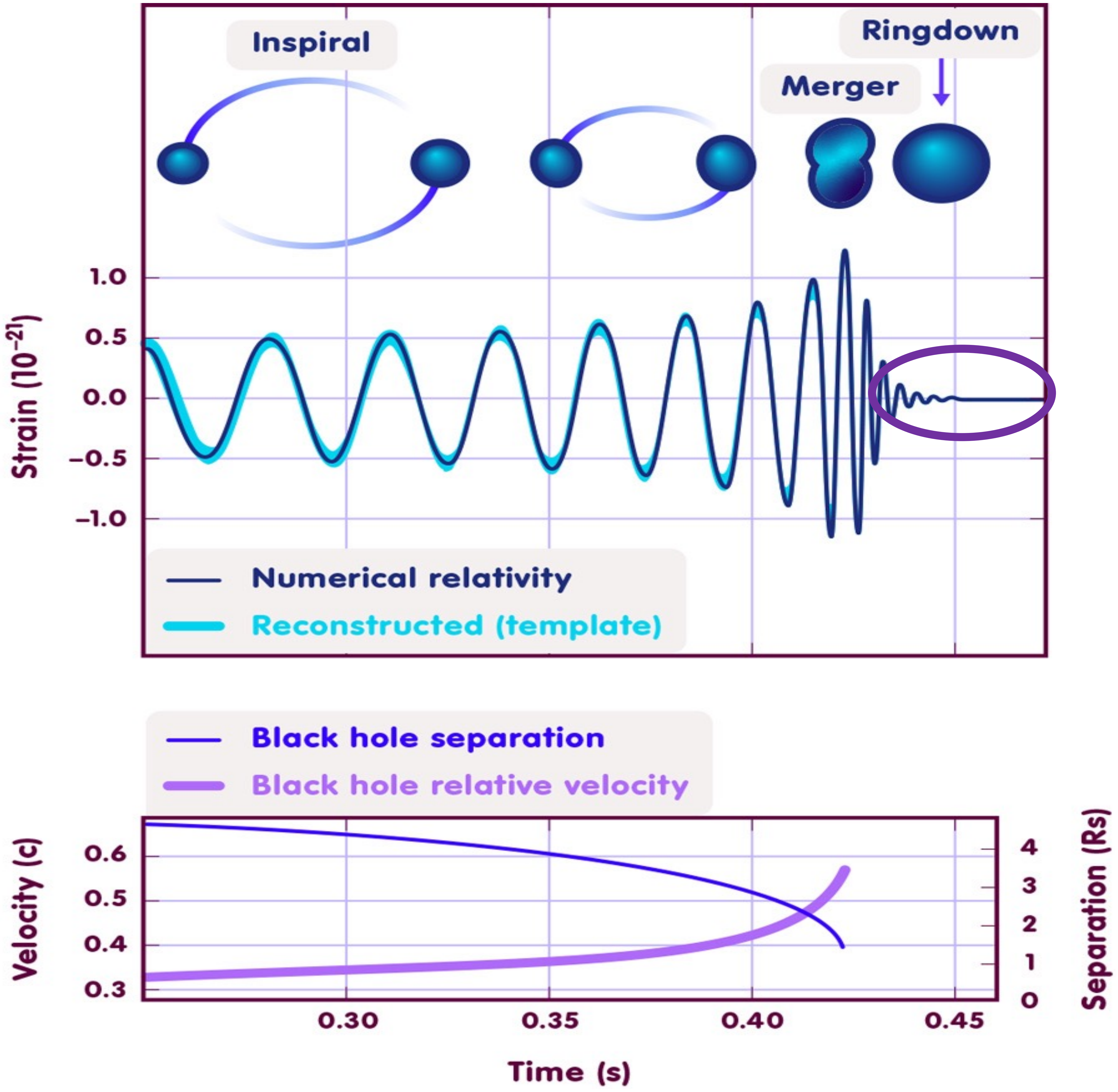
The ringdown signal

The ringdown is caused by the characteristic oscillations of the final **peturbed** object

It can be modelled as a series of damped sinusoid at certain characteristic frequencies: the **quasi normal modes**

$$\frac{d^2\psi(r)}{dr_*^2} + \left(\omega^2 - V(r)\right)\psi(r) = 0$$

Final stable metric + a linear perturbation ψ



LIGO / Redesign: Daniela Leitner

$$ds^2 = -e^{-2\phi(r)} f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad f(r) = 1 - \frac{2m(r)}{r}$$

Study of gravitational perturbations

We interpret the model as a solution of GR + nonlinear electrodynamics:

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{16\pi}R - \frac{1}{4\pi}\mathcal{L}(F)\right),$$

With

$$F = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \frac{\ell^2}{2\,r^4}$$

and

$$\mathcal{L}(F) = \frac{m'(r)}{r^2} \neq F$$

Important issue:

axial gravitational perturbations are actually coupled with polar perturbations of the magnetic field!

Study of test field perturbations

We obtain the equations of motion for spin s fields assuming that the they do not change the stress-energy tensor (at least) to first order

↓

No need to interpret the effective stress-energy tensor as some form of matter

Detectability

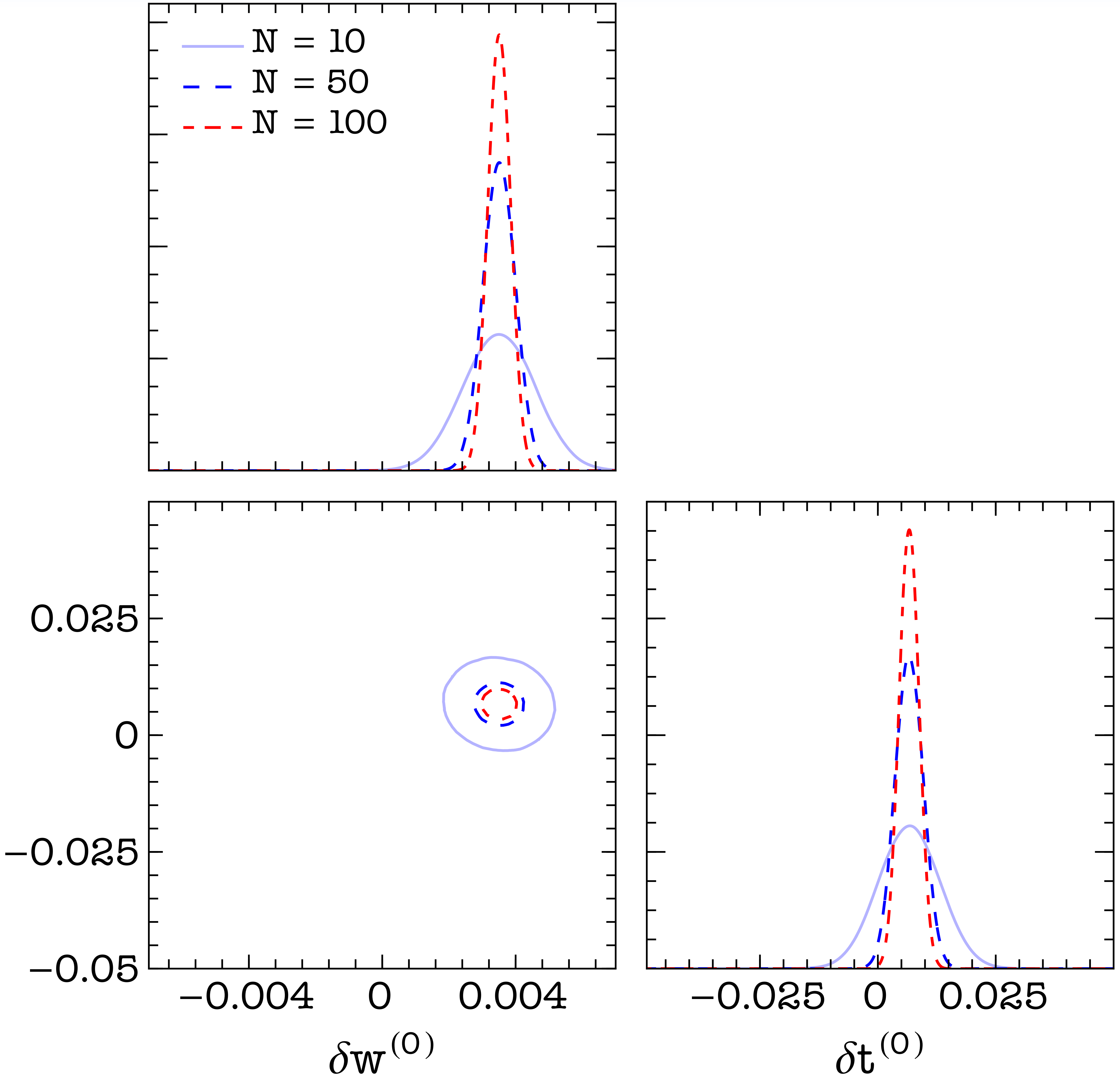
Parspec framework* at order 0 in the spin

A data analysis framework for the GW ringdown of BHs in modified theories of gravity

$$\omega_i := Re\,[\omega_i] = \frac{1}{M_i}\omega_{Kerr}^{(0)}\,(1+\gamma_i\delta\omega^{(0)})$$
$$\tau_i := \frac{1}{Im[\omega_i]} = M_i\,\tau_{Kerr}^{(0)}\,(1+\gamma_i\delta\tau^{(0)})$$

- We simulate N observations of ringdown signals from regular BHs binary merger
- Isolating the dependence of the corrections on the masses of the sources we can combine different observations to obtain more precise results on $\delta\omega$ and $\delta\tau$
- Through a Monte Carlo Markov chain we obtain the posterior probability distribution for $\delta\omega$ and $\delta\tau$

*Maselli et al. 2019



From the observations of the ringdown **of 0(100) RBHs** with **SNR∼100**
we can exclude the GR hypothesis at **90% confidence** level
for macroscopic values of ℓ/M $(0(10^{-1})$

but remember this is at **order 0 in the spin…**

Echoes and the effect of breakreaction

Echoes

Consider a perturbation around a spherically symmetric BH or an **horizonless** compact object

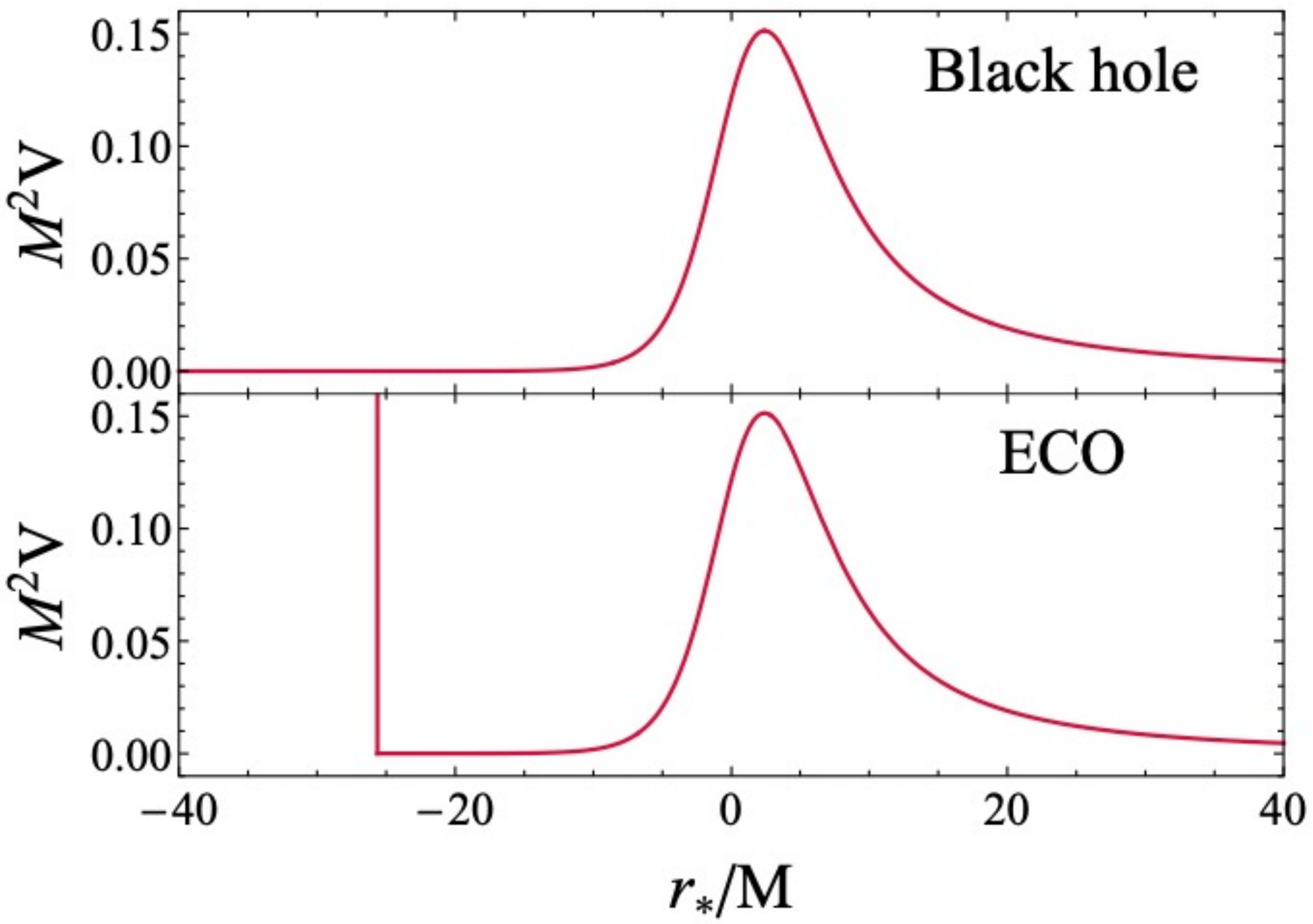
At linear level the field equation is:

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_l(r)\right]\Psi_{lm}(t,r)=0$$

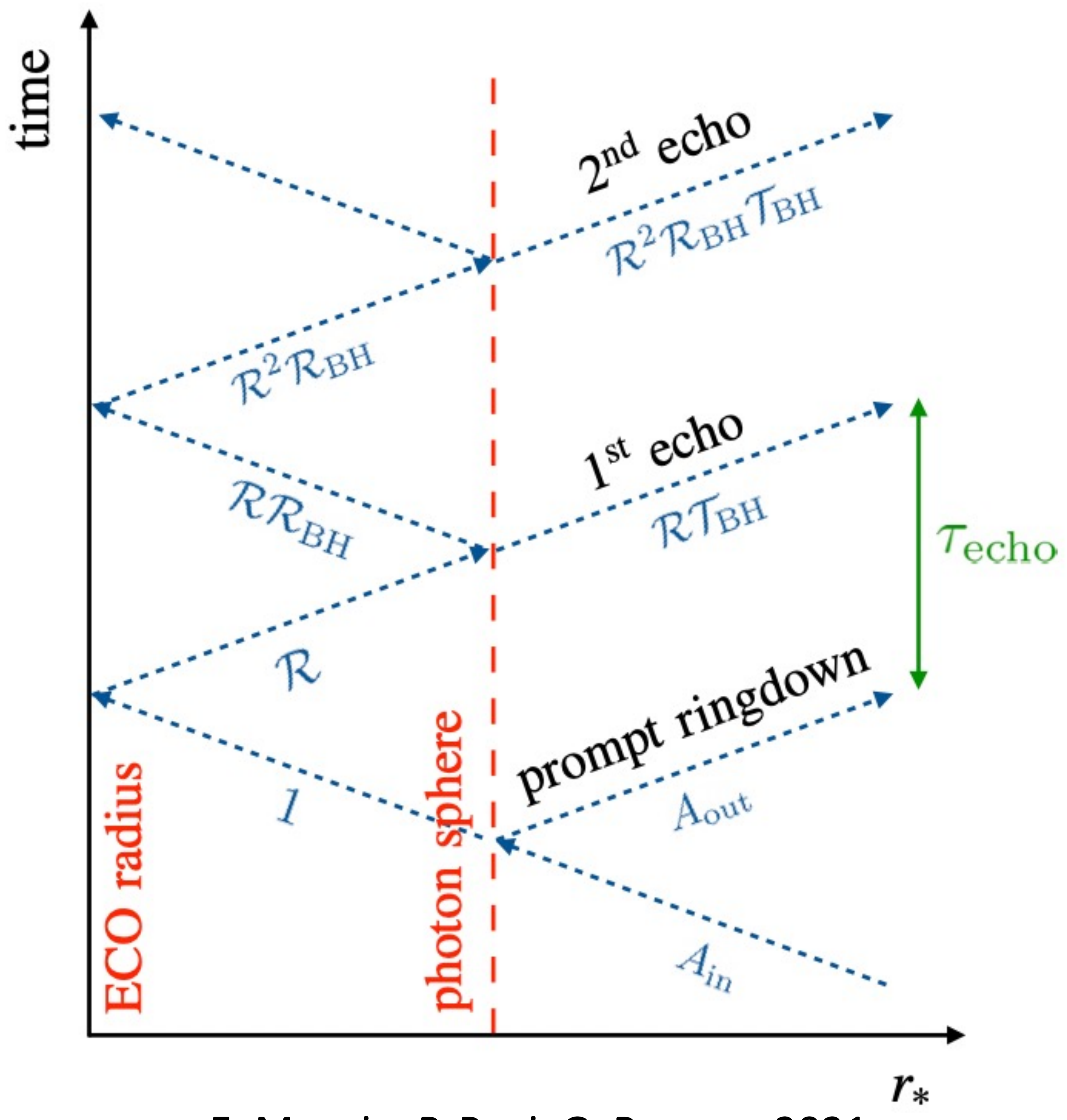
But the potential is very different in the two cases

↓

We see echoes (smaller copies) of the original signal



E. Maggio, P. Pani, G. Raposo, 2021



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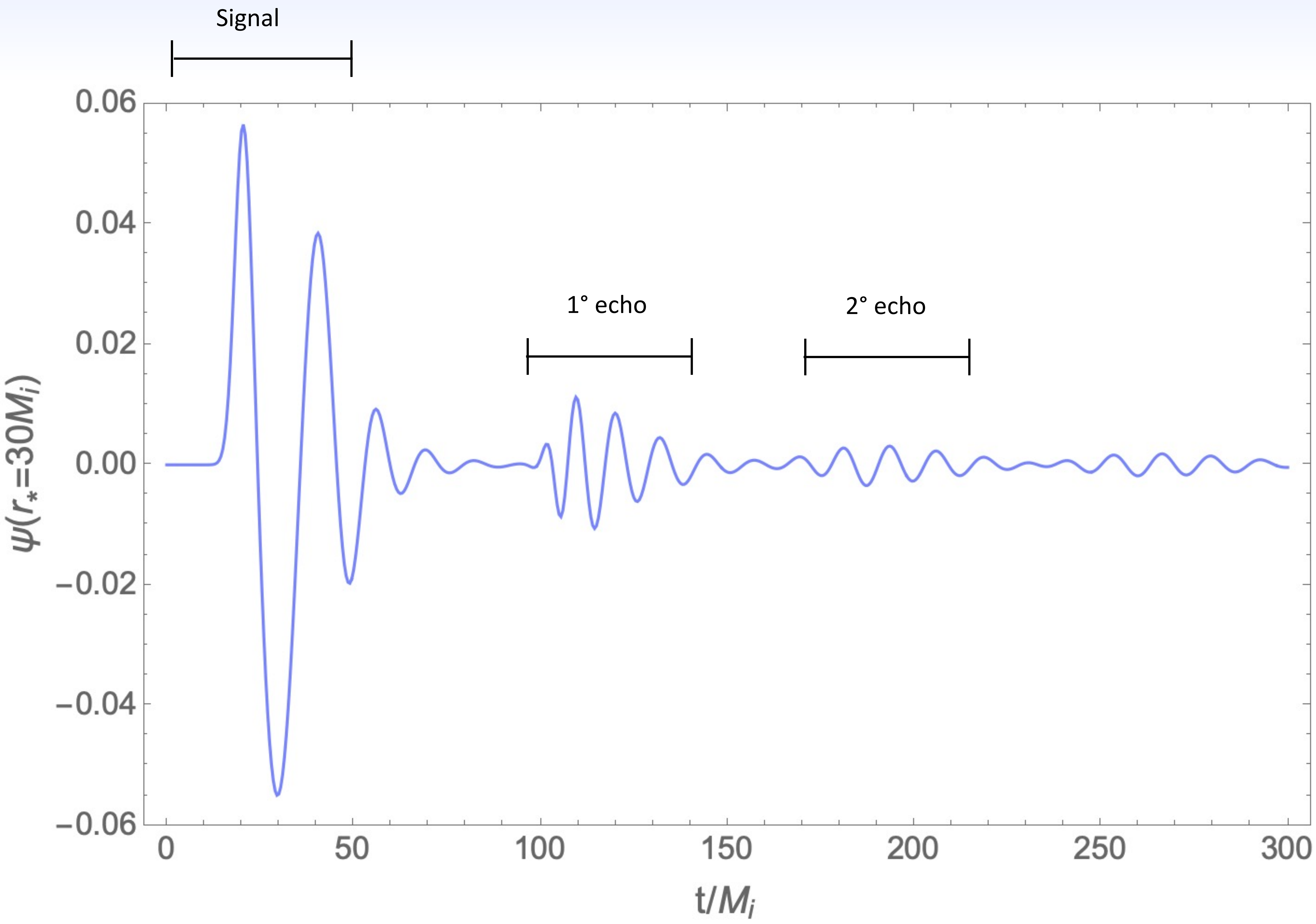


Time delay

Defining the compactness parameter as:

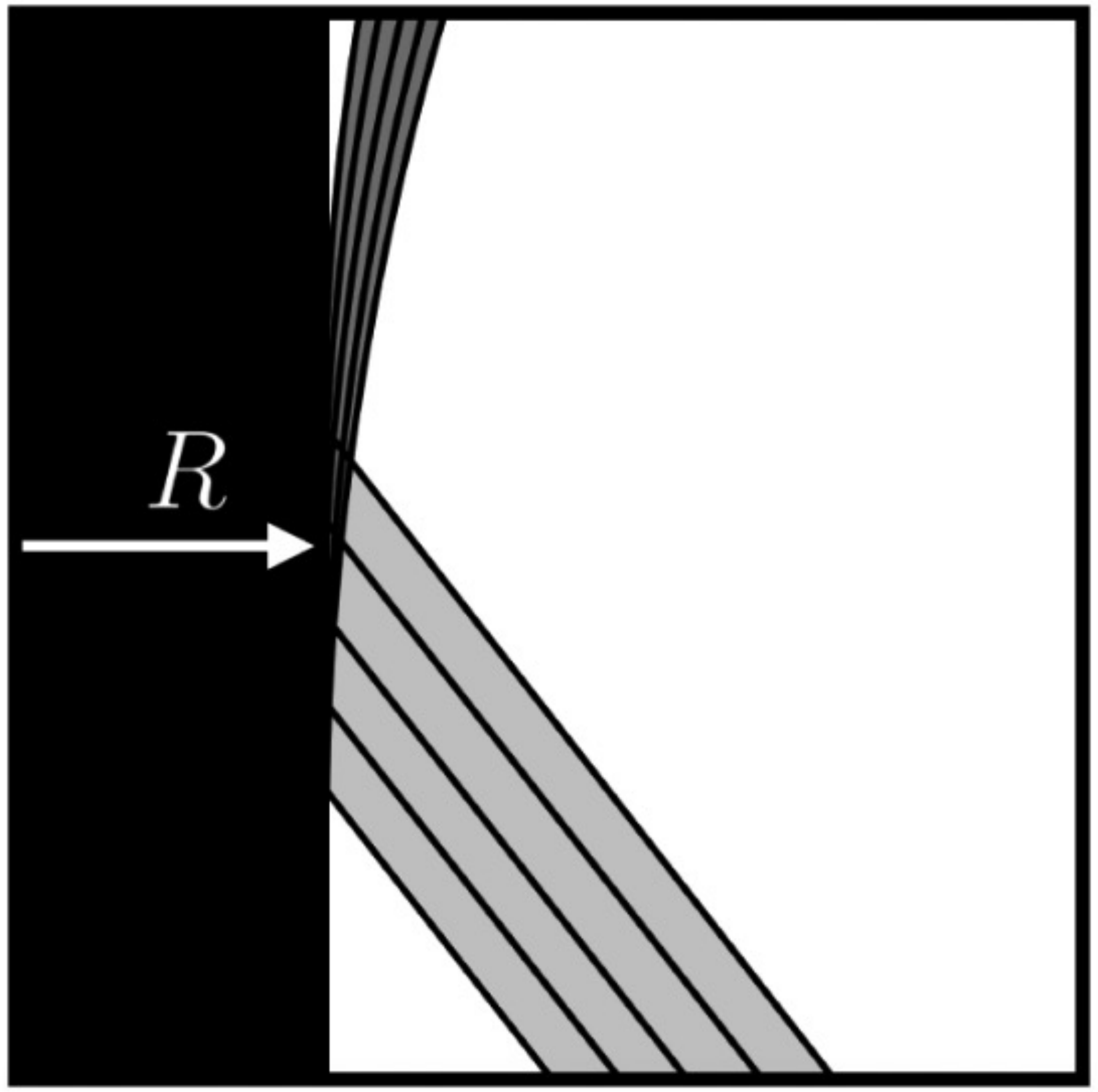
$$\sigma = \frac{r_0}{2M} - 1$$

↓

$$\Delta t_{echo} = 2 \int_{r_0=2M(\sigma+1)}^{r_{peak}\sim 3M} g_{rr} dr = 2 \int_{r_0=2M(\sigma+1)}^{r_{peak}\sim 3M} \frac{dr}{1 - \frac{2M}{r}} \simeq 2M(1 - 2\sigma - 2\ln(2\sigma))$$

The logarithmic dependence on σ would allow to detect even Planckian corrections ($\sigma \sim l_{Planck}/\mathrm{M}$) at the horizon scale

Limits of linear approximation



Peeling of outgoing geodesic
The accumulation of geodesics around the gravitational radius produces high densities

Instability
Lightring and Ergoregion instability!
They can be quenced if **absorption** is taken into account

↓

Non linear interactions should be taken into account

Absorption beyond the test field limit

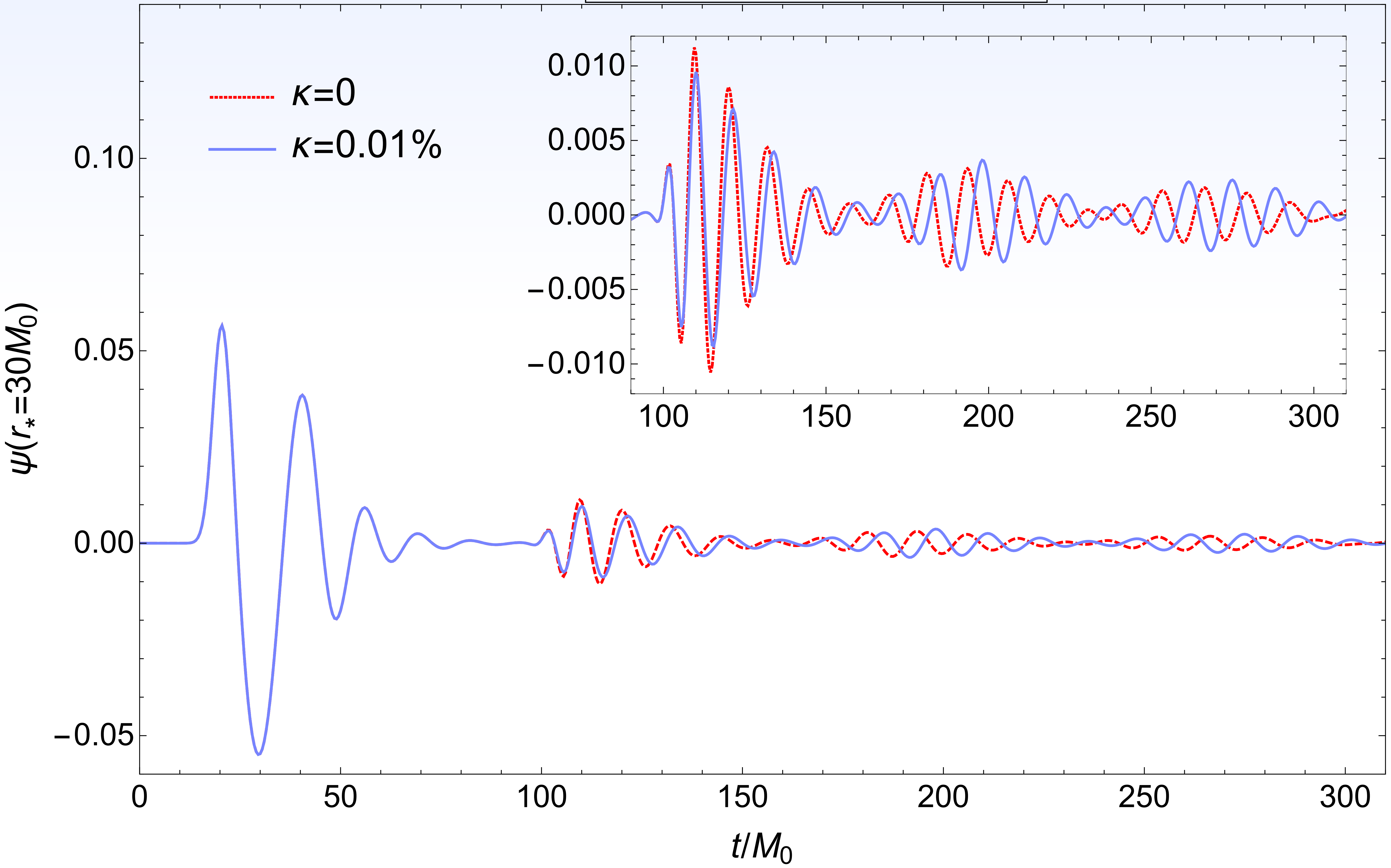
Partial absorption of the first echo → $M_0 \rightarrow M = M_0 + \Delta E_{1st\ ehco}$

→ $\sigma_{2nd\ echo} \ll \sigma_{1st\ echo}$ → $\Delta t_{2nd\ echo} > \Delta t_{1st\ echo}$

Absorption coefficient κ

For high compact object very small ΔM causes big changes in the compactness!

We lose the main feature of echoes signal: the periodicity!



Probing the core?

Until now we have assumed that perturbations are reflected at the surface or completly lost inside the object, it is instead physically reasonable to **allow for GWs to travel through the object**, and consequently to carry out information about its internal structure

Semiclassical stars

$$G^{\mu}{}_{\nu} = 8\pi \left(T^{\mu}{}_{\nu} + M_{\rm P}^2 \langle \hat{T}^{\mu}{}_{\nu} \rangle \right) ,$$

constant-density perfect fluid

Boulware vacuum

$$ds^2 = -f(r)dt^2 + q(r)dr^2 + r^2 d\Omega^2,$$

≈Schwarzschild exterior

$$f = 1 - \frac{2M}{r} + \frac{M_{\rm P}^2 M^2}{90\pi r^4} + O\left(\frac{M_{\rm P}^2 M^3}{r^5}\right),$$

$$q = \left[1 - \frac{2M}{r} - \frac{M_{\rm P}^2 M^2}{6\pi r^4} + O\left(\frac{M_{\rm P}^2 M^3}{r^5}\right)\right]^{-1}$$

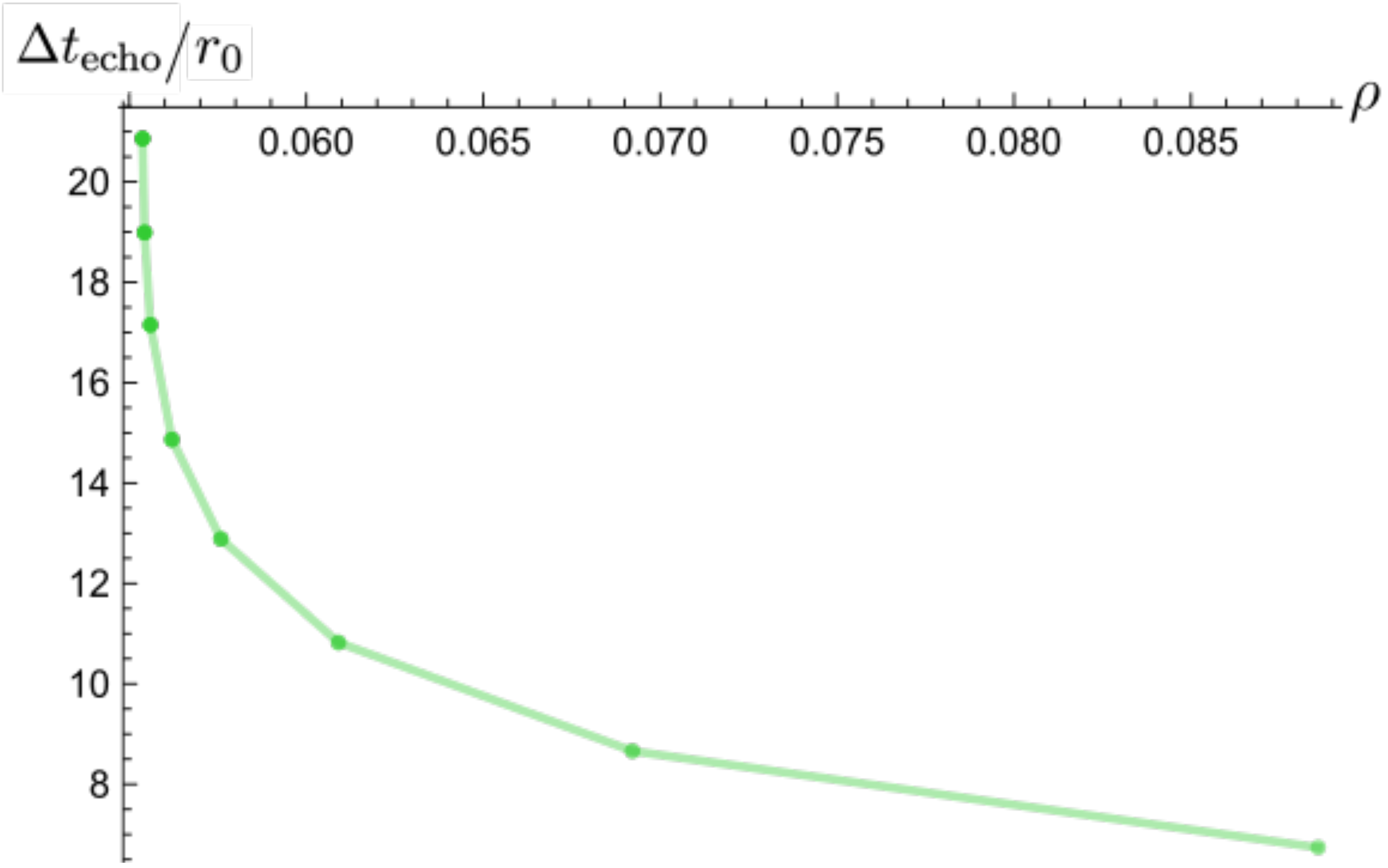
Regular interior that depends on the choice of the density and the compactness

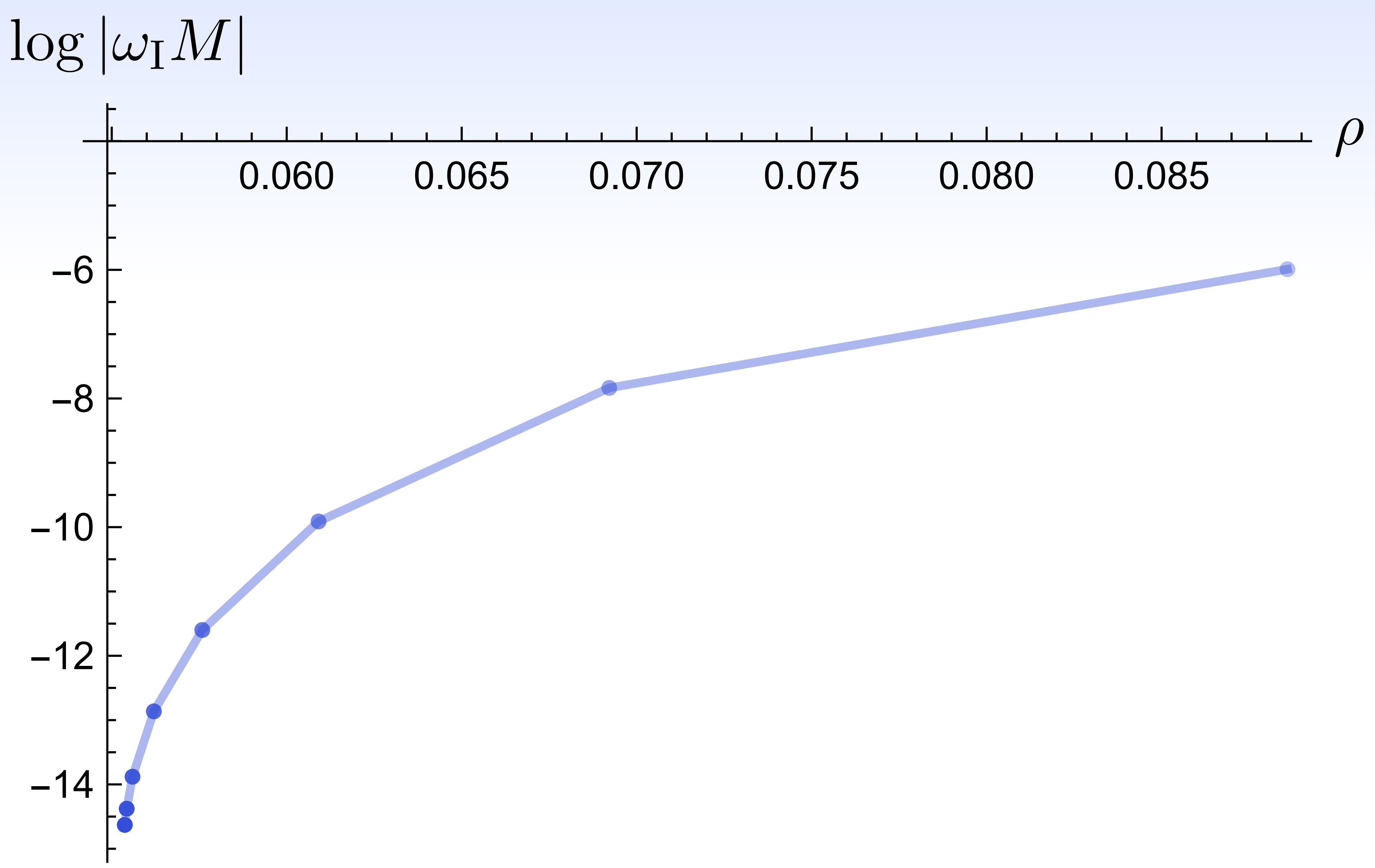
$$q = 1 + q_2 r^2 + O(r^3), \quad f = f_0 + f_2 r^2 + O(r^3)$$

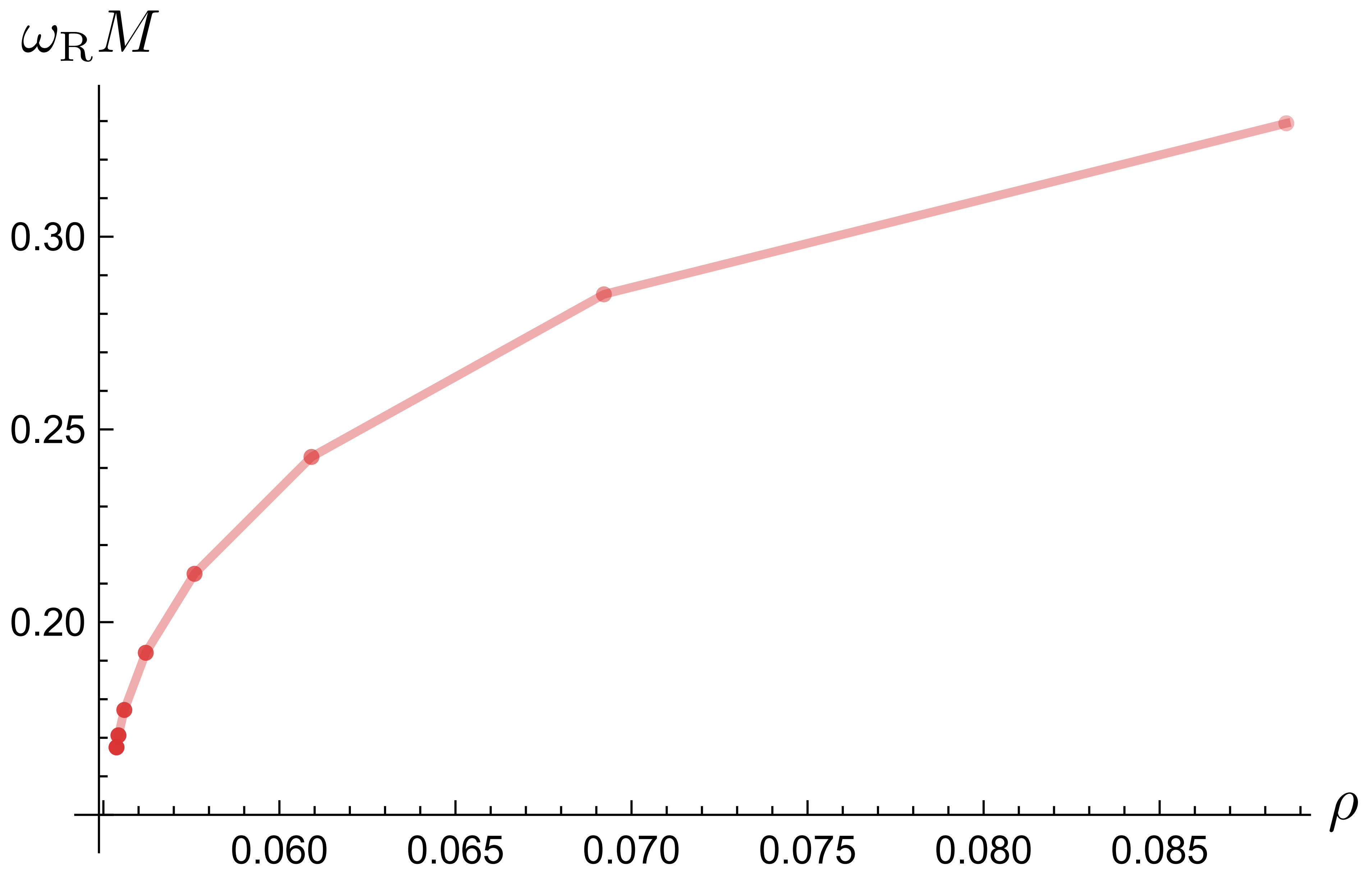
According to the sign of q_2 and f_2 we have De Sitter, Anti De Sitter or mixed center!

We can study the dependence of the ringdown on the internal structure of the star!

$$\Delta t_{\rm echo} \approx 2M - 4M\sigma - 4M\ln(2\sigma) + t_{\rm int}$$



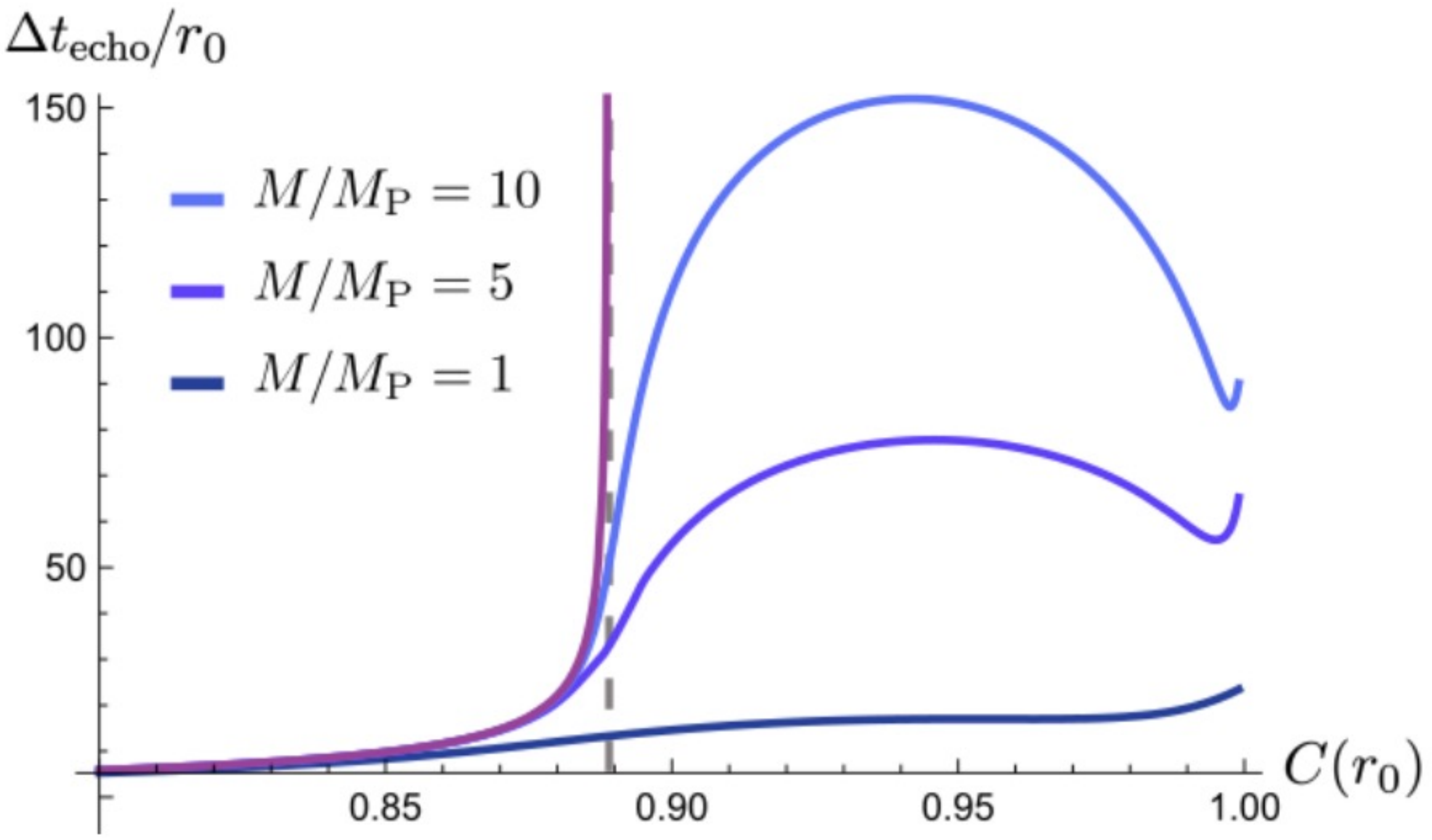




Extremely delayed echoes

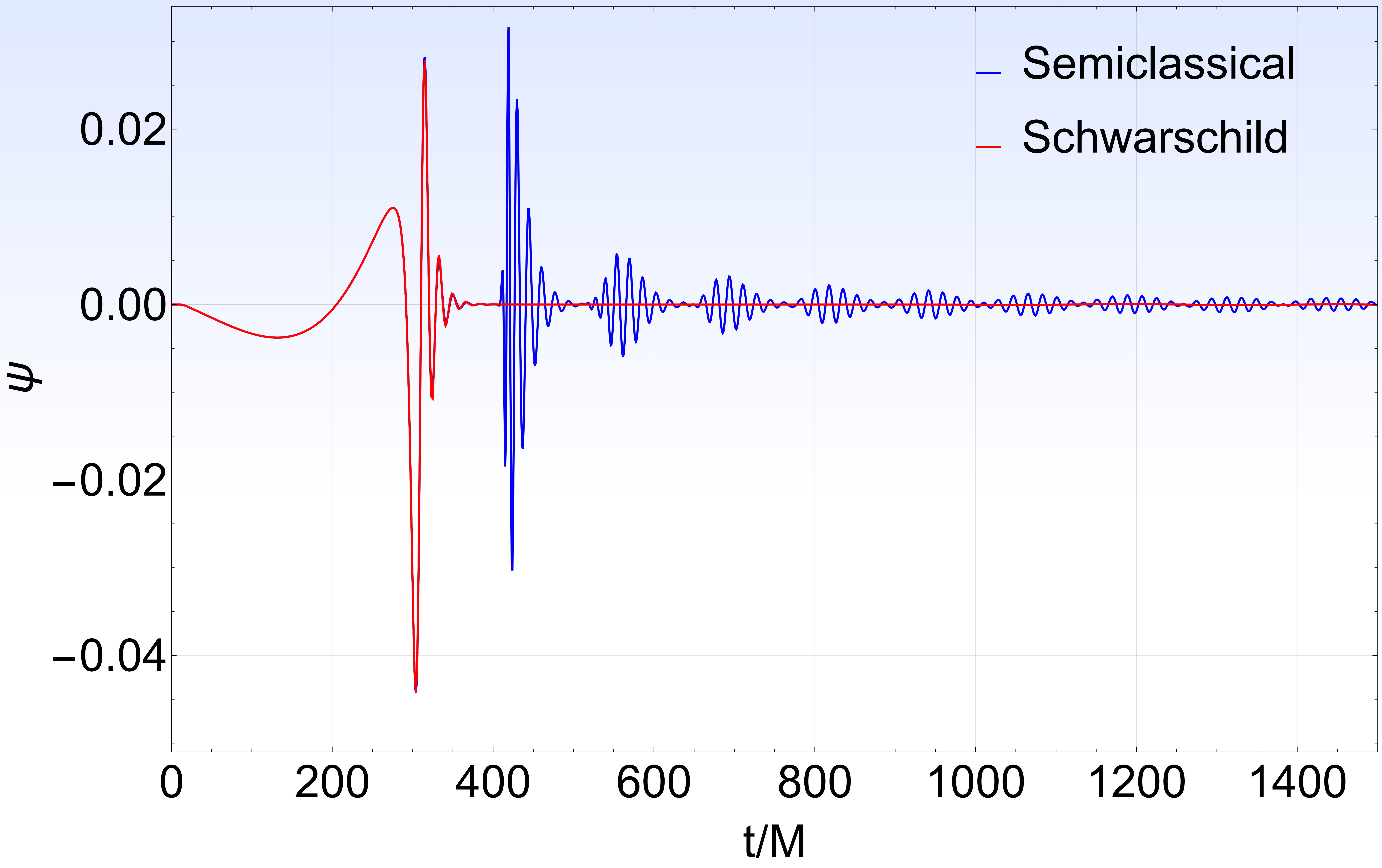
$$\Delta t_{\rm echo} \approx 2M - 4M\sigma - 4M\ln(2\sigma) + t_{\rm int}$$

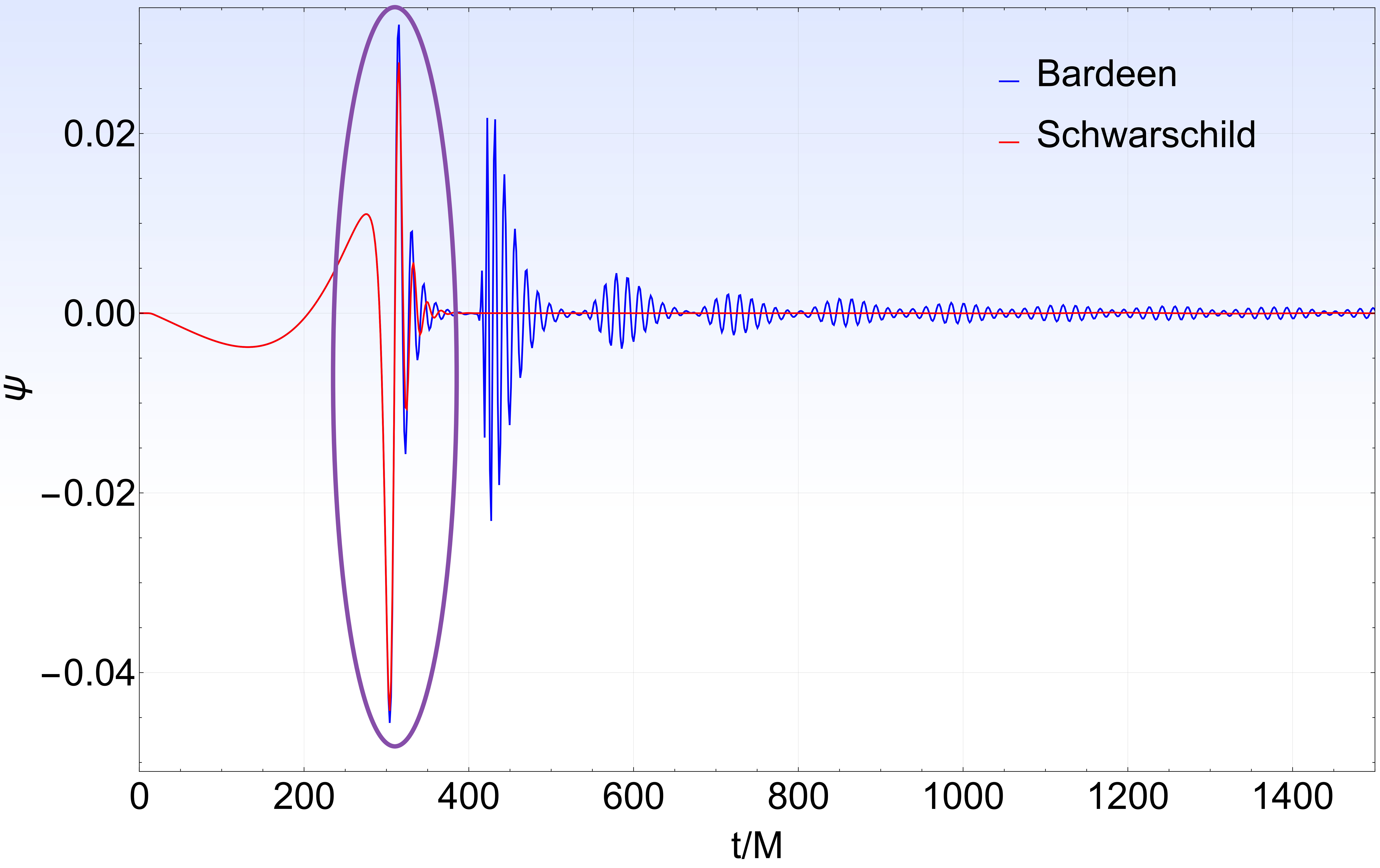
It depends also on M/M_{planck} (no scale invariance!)



We have observable time delays (order of seconds) only for objects with masses of order $\sim 10^{-17} - 10^{-15}\ M_\odot$!

See also Zimmerman et al. 2023


Semiclassical
Schwarschild
0.02
0.00
−0.02
−0.04
ψ
0
200
400
600
800
1000
1200
1400
t/M



Bardeen
Schwarschild
0.02
0.00
−0.02
−0.04
ψ
0
200
400
600
800
1000
1200
1400
t/M

Conclusions

We are able to connect possible unknown (quantum) physics to phenomenology and to potentially observable effects in the GWs signal!

So, can we probe the new physics responsible for singularity regularizations?

If this new physics is "quantum" in the sense of $\ell \sim \ell_{plack}$

we have **too small corrections** in the ringdown signal and **probably too delayed echoes**

Luckily physics is complicated!

- $\ell \neq \ell_{planck}$?
- Small (primordial) regular objects?
- Models with a shorter interior light crossing time?
- Partial reflection of GWs at the surface of ECOs?
- Searches for individual echoes?

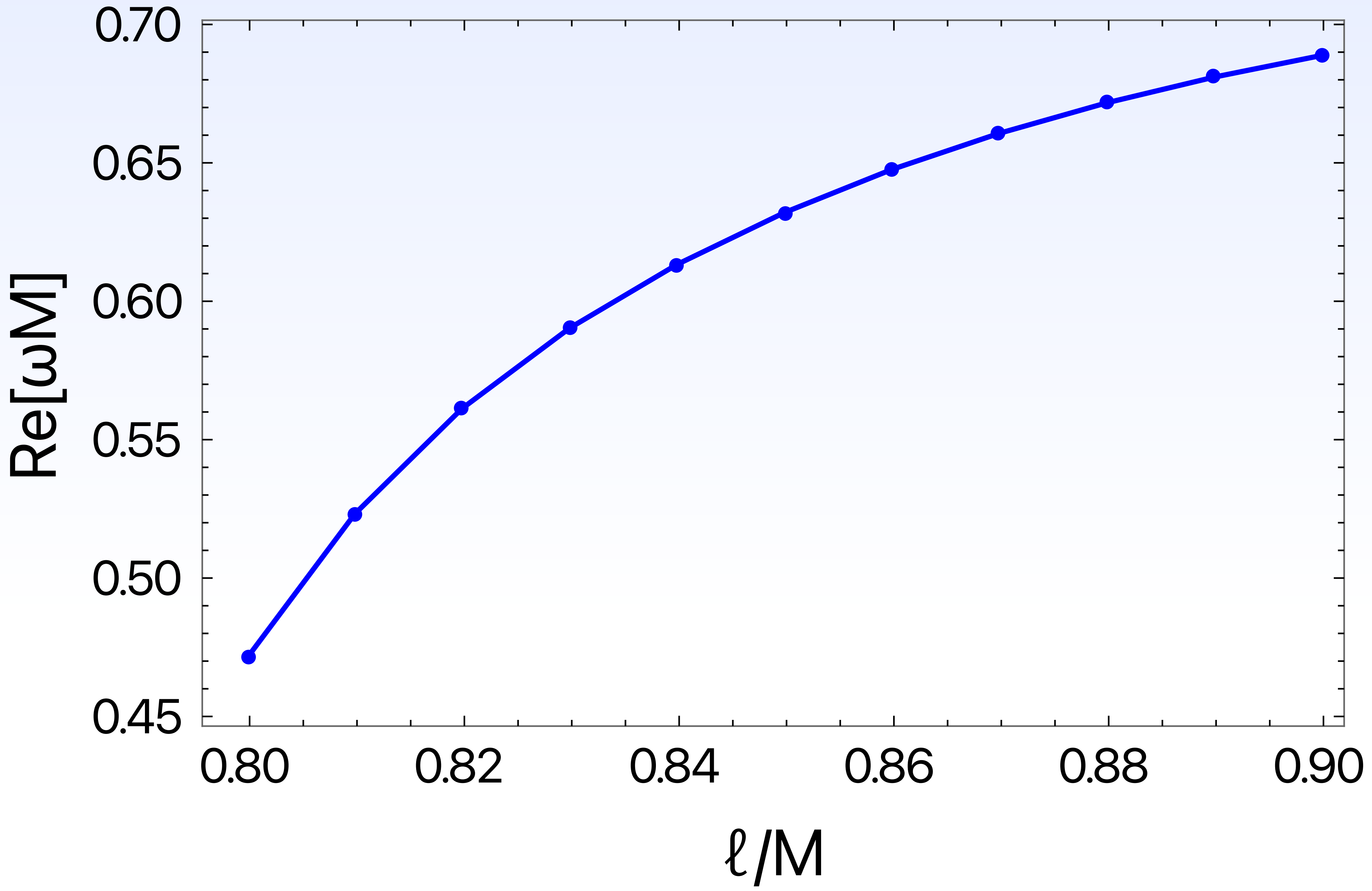
- But also noise, systematics, enviroment, backreaction effects...

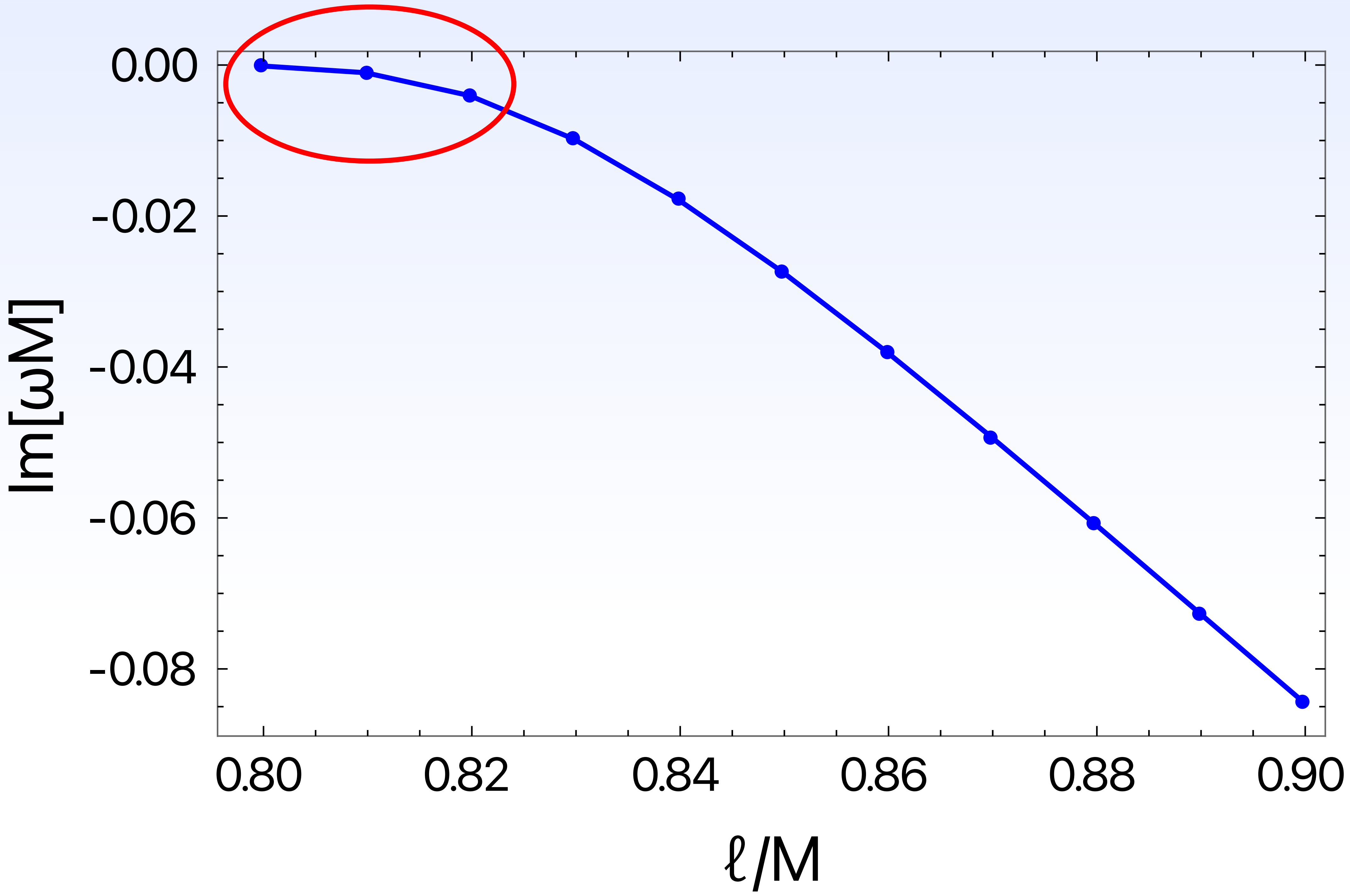
Thank You!

Back up slides

Horizonless compact object branch

Boundary conditions: regular at the center, purely outgoing at infinity





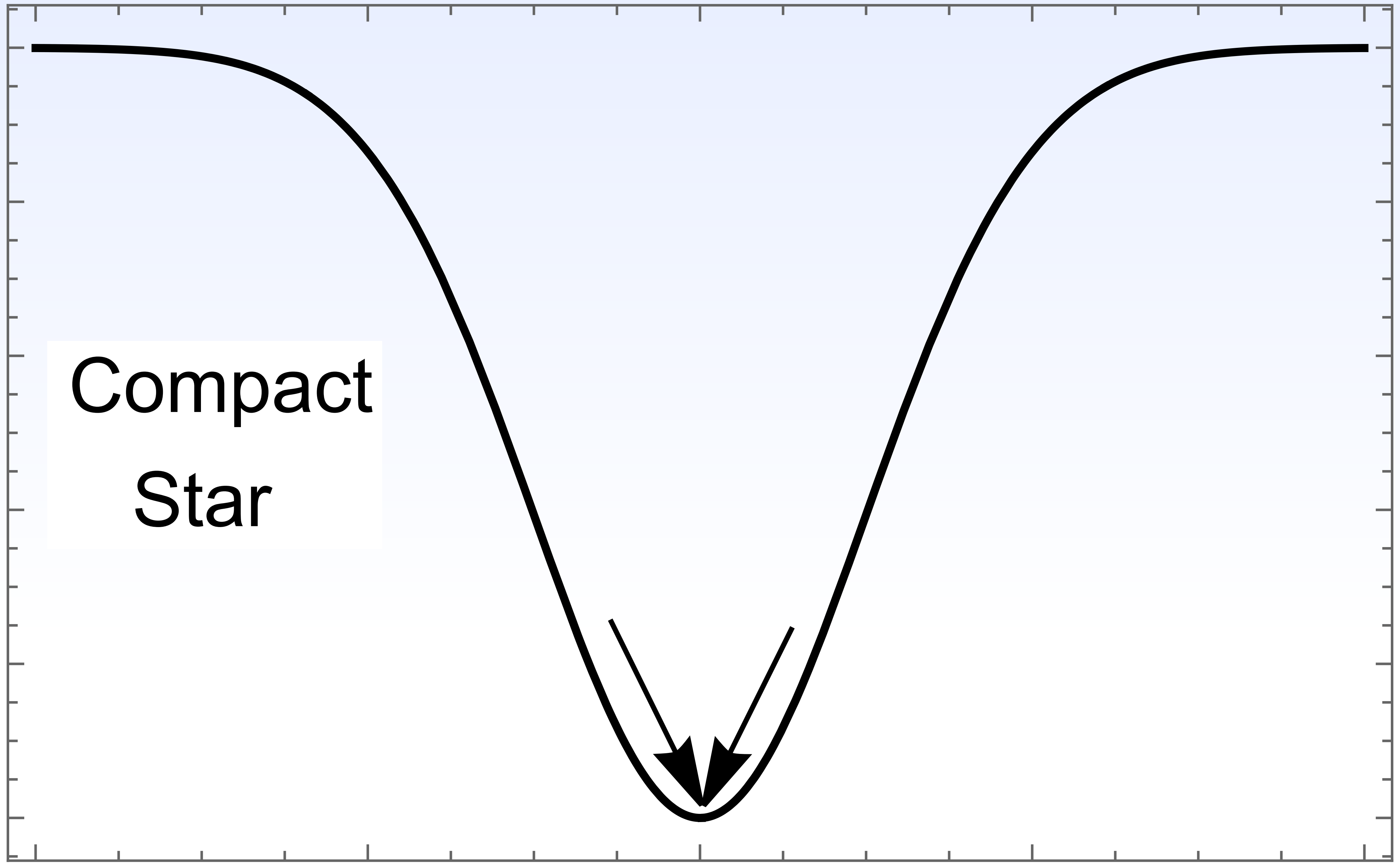
$e^{i\omega t} = e^{iRe[\omega]t}\, e^{-Im[\omega]t}$

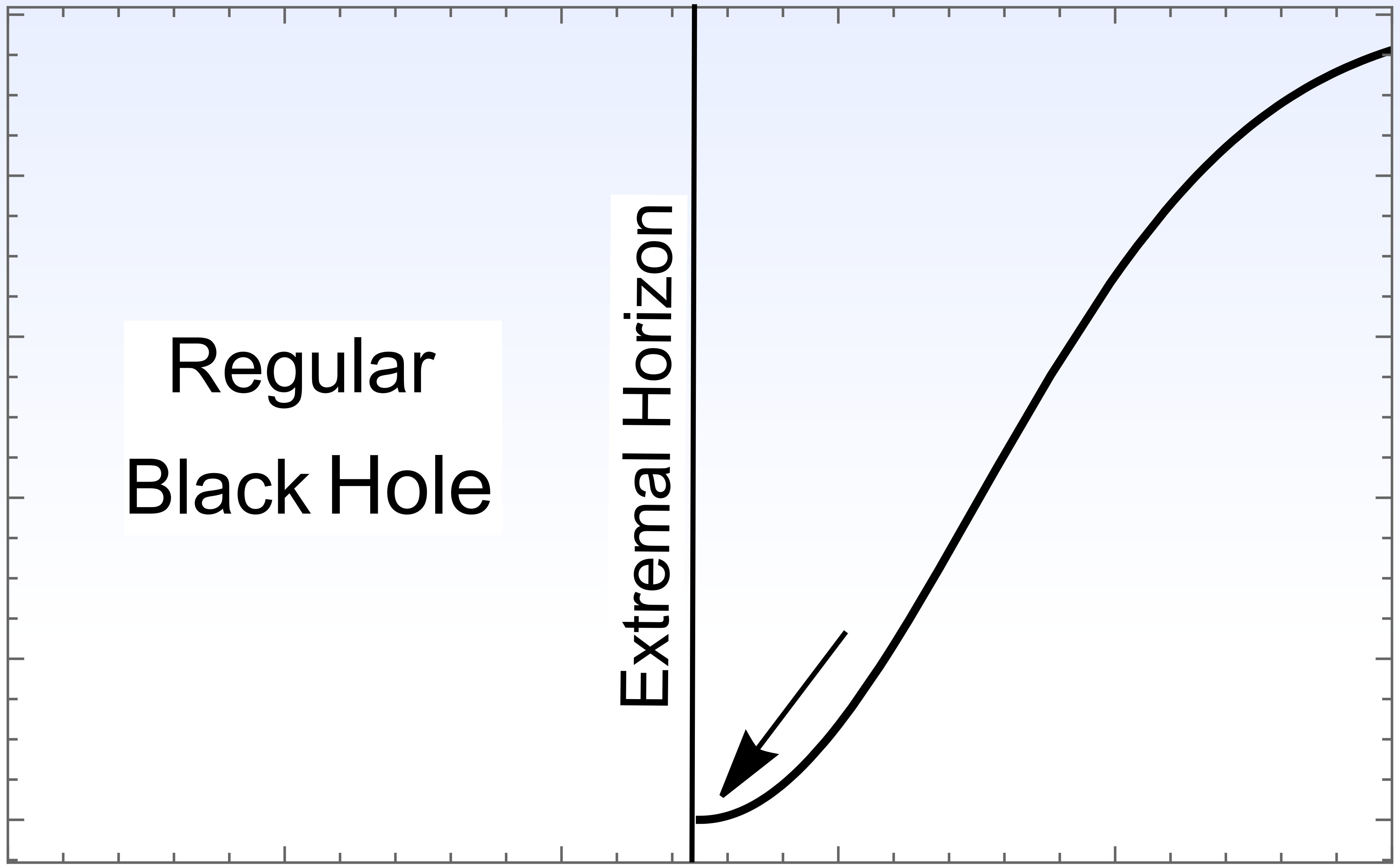
$\tau = \frac{1}{Im[\omega]}$ is the damping time

Small $Im[\omega]$ → **long living modes** connected with the presence of a **stable lightring!**

Lightring (and Aretakis) instabilities

$$\frac{d^2\psi(r)}{dr_*^2} + \left(\omega^2 - V(r)\right)\psi(r) = 0$$





The perturbation accumulates near the minimum of the potential causing possible non-linear instabilities

The “stable lightring” is already present in the extremal RBH case!
Connection to the Aretakis Instability?

very good question…

Expansion at constant compactness

$$\frac{r_0(t)}{2\,M(t)} = \sigma_0 + 1$$

$$\Delta t_{echo} \sim constant$$

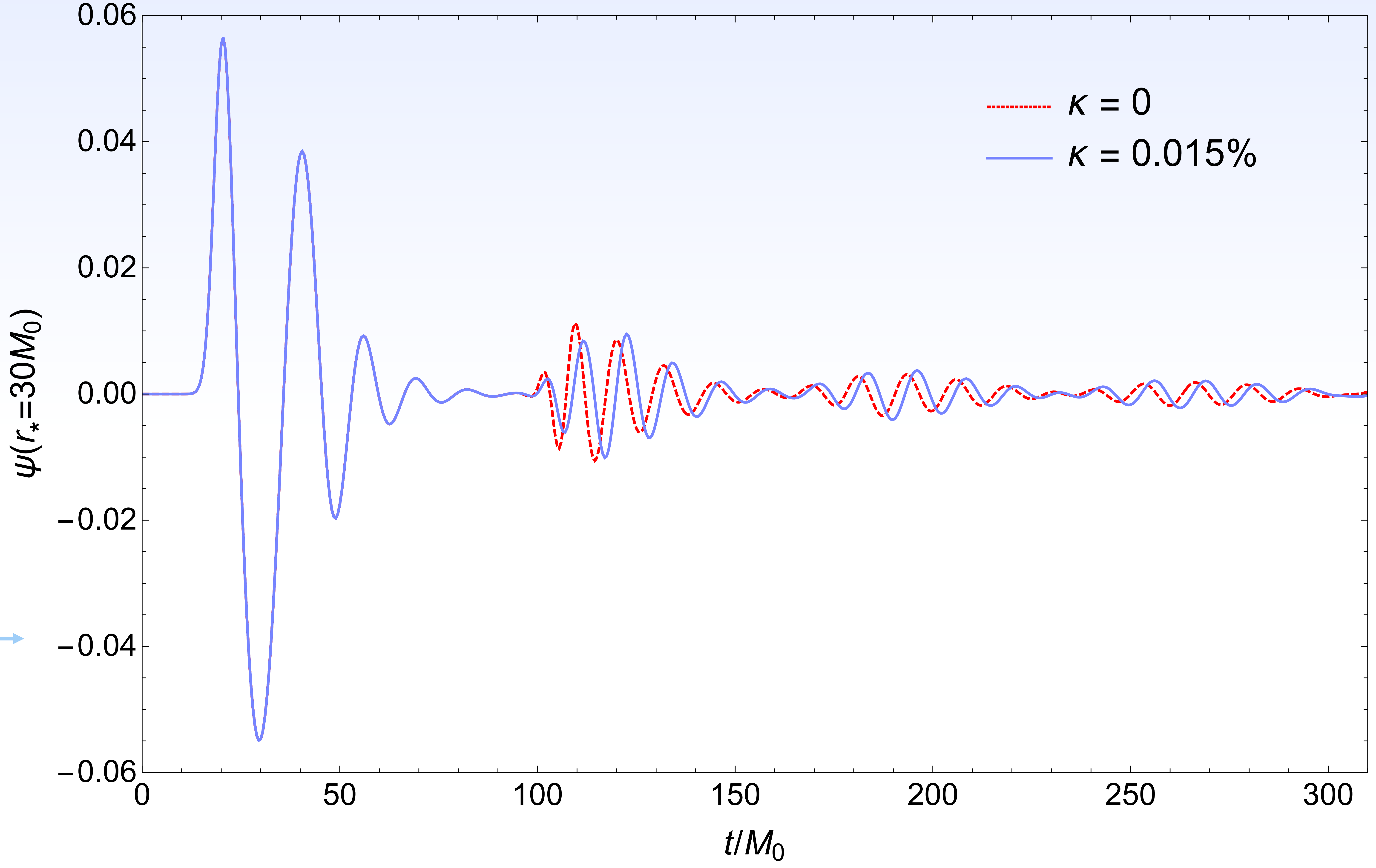
BUT

- **It can requires superluminal motion of the surface**

- There might be a transient phase

- Other, more general model of expansion can be possible

Transient phase $\tau \sim 65 M_0 > \Delta t_{echo}$



Partial absorption of the first echo → $M_0 \rightarrow M = M_0 + \Delta E_{1st\ ehco}$

→ $\sigma_{2nd\ echo} \ll \sigma_{1st\ echo}$ → $\Delta t_{2nd\ echo} > \Delta t_{1st\ echo}$

For example, if $\Delta M = (5 \cdot 10^{-8}) M_0$:

$$\sigma_0 = \frac{r_0}{2M_0} - 1 = \frac{2\,M_0(1+10^{-7})}{2\,M_0} - 1 = 10^{-7}$$

$$\sigma_f = \frac{r_0}{2M_f} - 1 = \frac{2\,M_0(1+10^{-7})}{2\,M_0(1+5\cdot 10^{-8})} - 1 = 5\cdot 10^{-8}$$

$$\Delta t_0 \sim -4\ln(2\,\sigma_0) = 61.7\,M_0$$

$$\Delta t_f \sim -4\ln\left(2\,\sigma_f\right) = 64.5\,M_0$$

For high compact object
very small ΔM causes big changes
in the compactness!

- One simple effective metric can interpolate between spherically symmetric **regular BHs** and **horizonless compact objects**

- We found that the **quasi-normal modes** of these objects **deviates** from the Schwarschild ones showing that we can potentially probe the inner structure of compact objects

- These deviations from the spectrum of singular BHs seems to be **detectable** with the next Generation of GW detectors stacking multiple events

- **Echoes** could be a powerful probe of new physics but **non-linear effects** and **propagation within the object's interior** should be taken into account, as they can drastically affect the signal and our cability to detect it!