Constraining the quantum decoherence of inflationary gravitational waves

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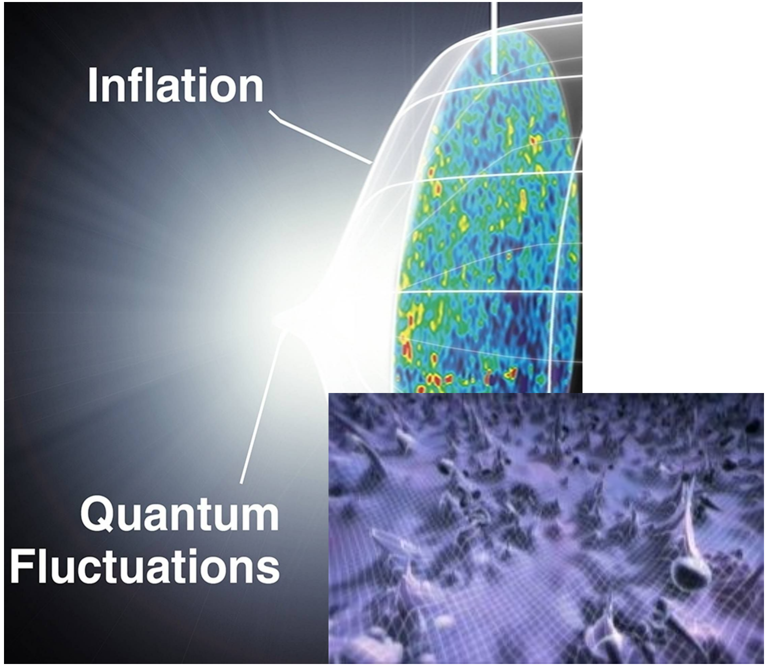
In collaboration with N. Bartolo

2408.02563

Hot big bang shortcomings

Hot big bang shortcomings

Inflationary solution



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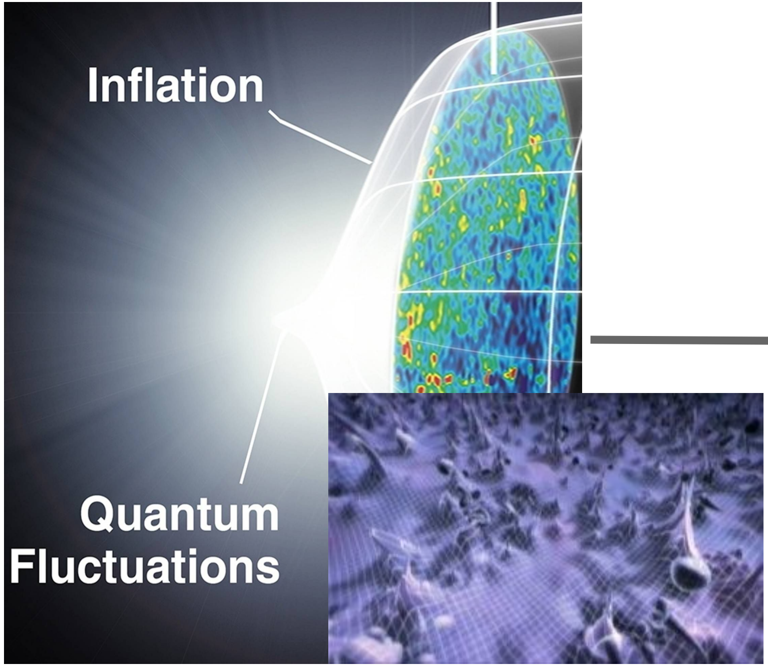
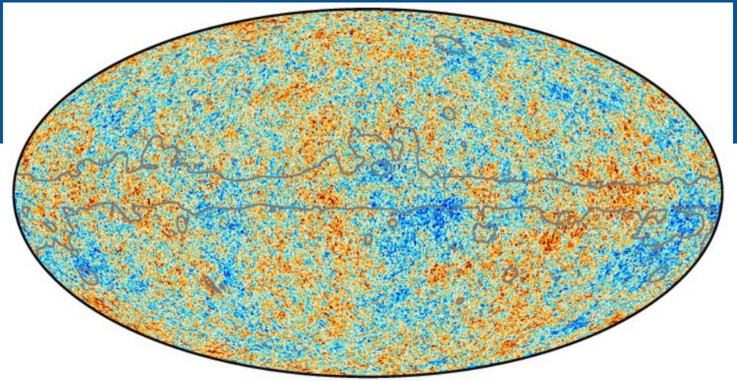
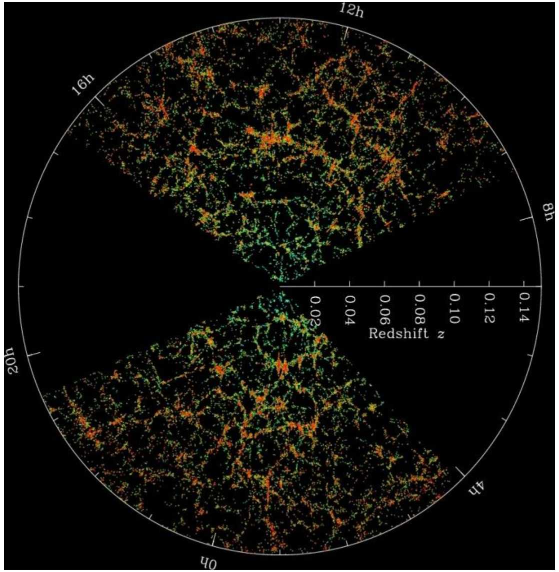


Planck 2018



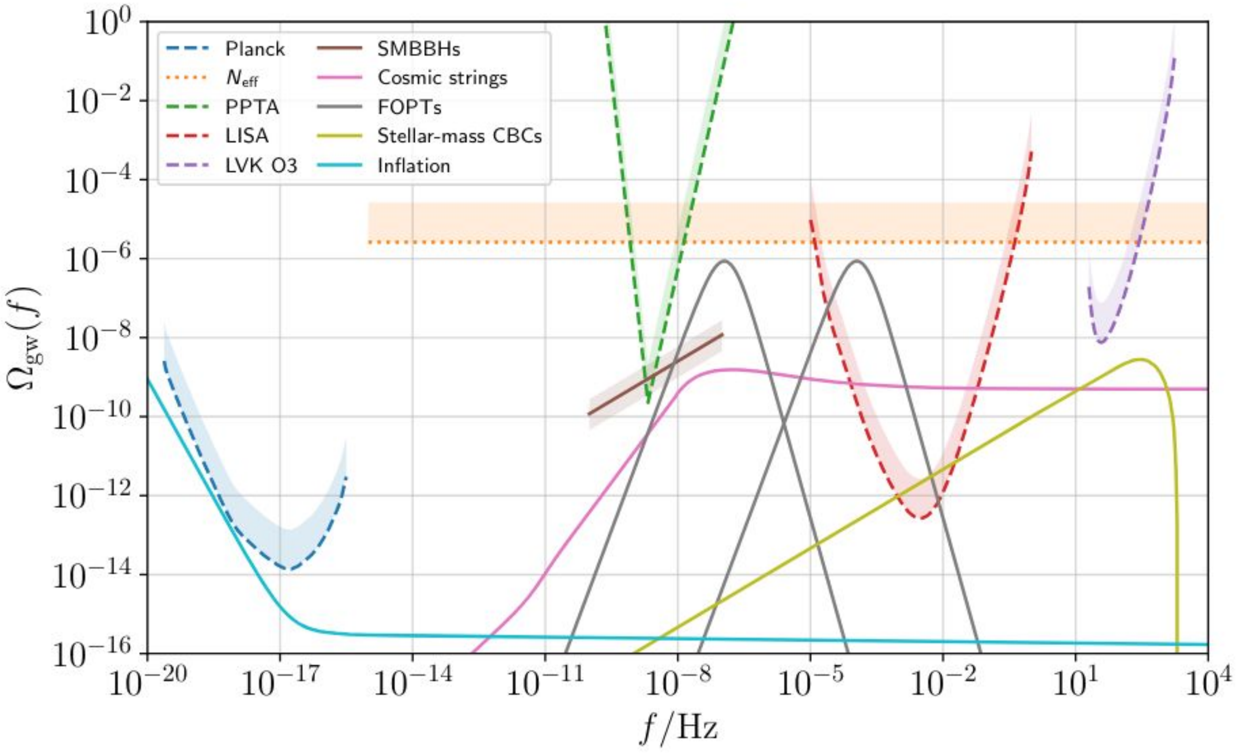
CMB

LSS



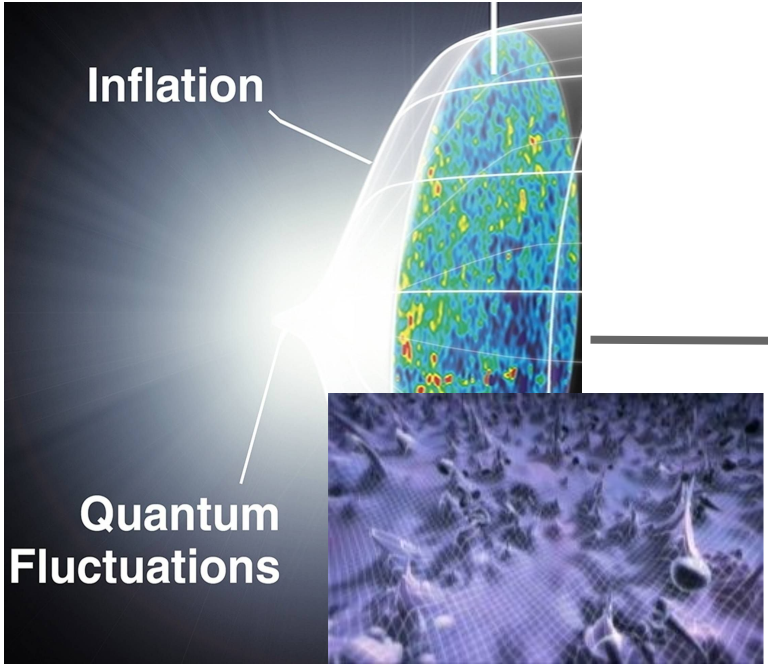
Sloan Digital Sky Survey

GWs from inflation



Renzini et al.
2202.00178 (Galaxies, 2020)

Quantum-to-classical



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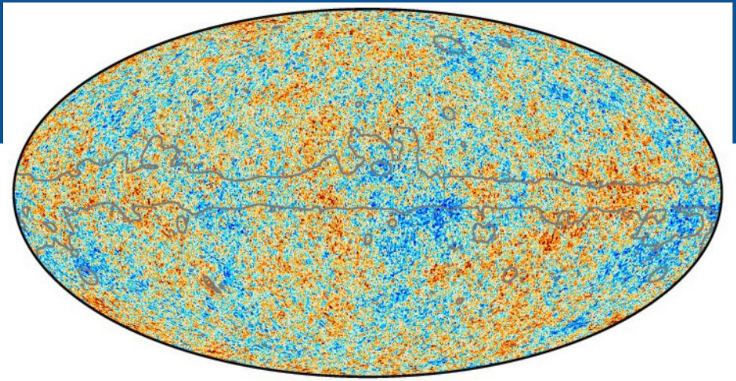

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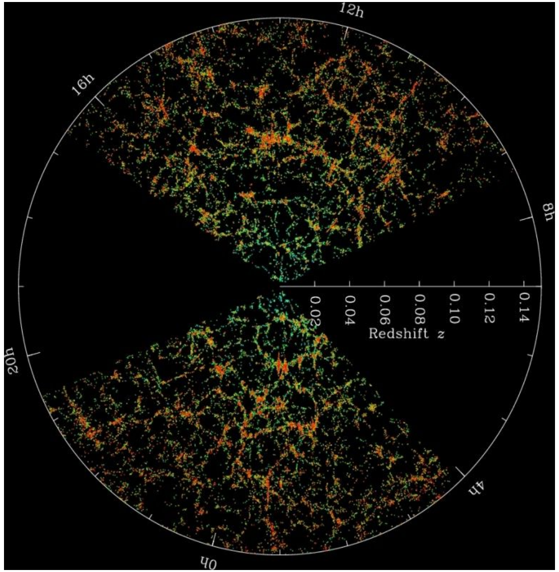

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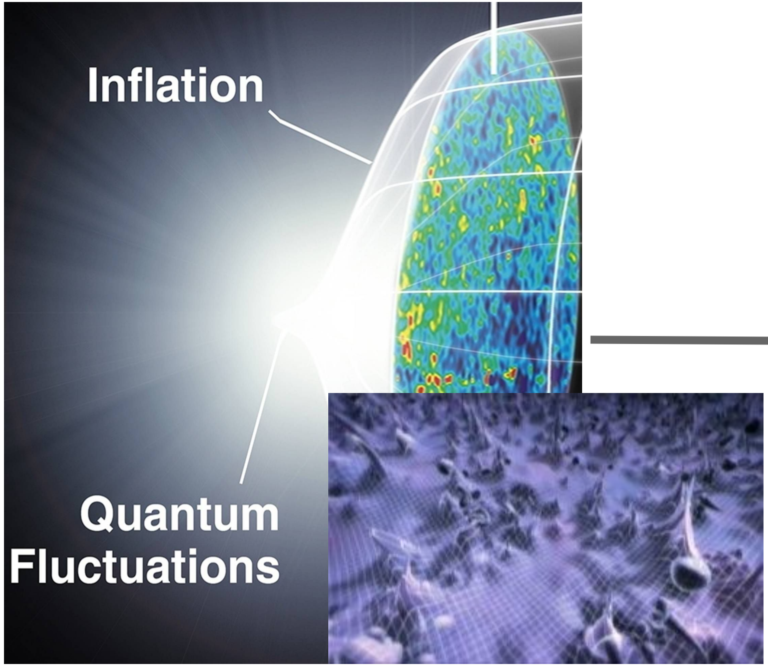

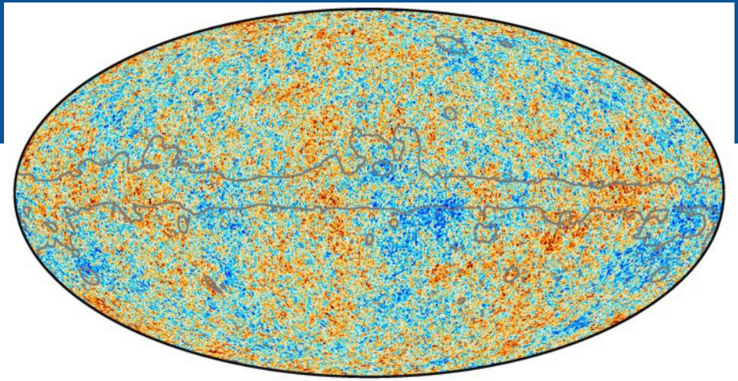

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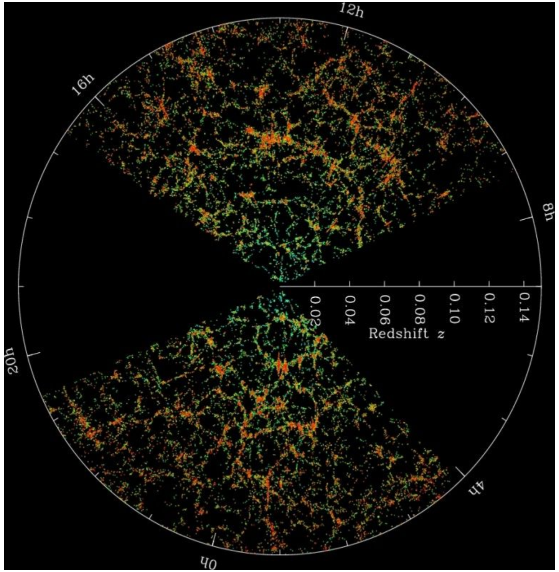
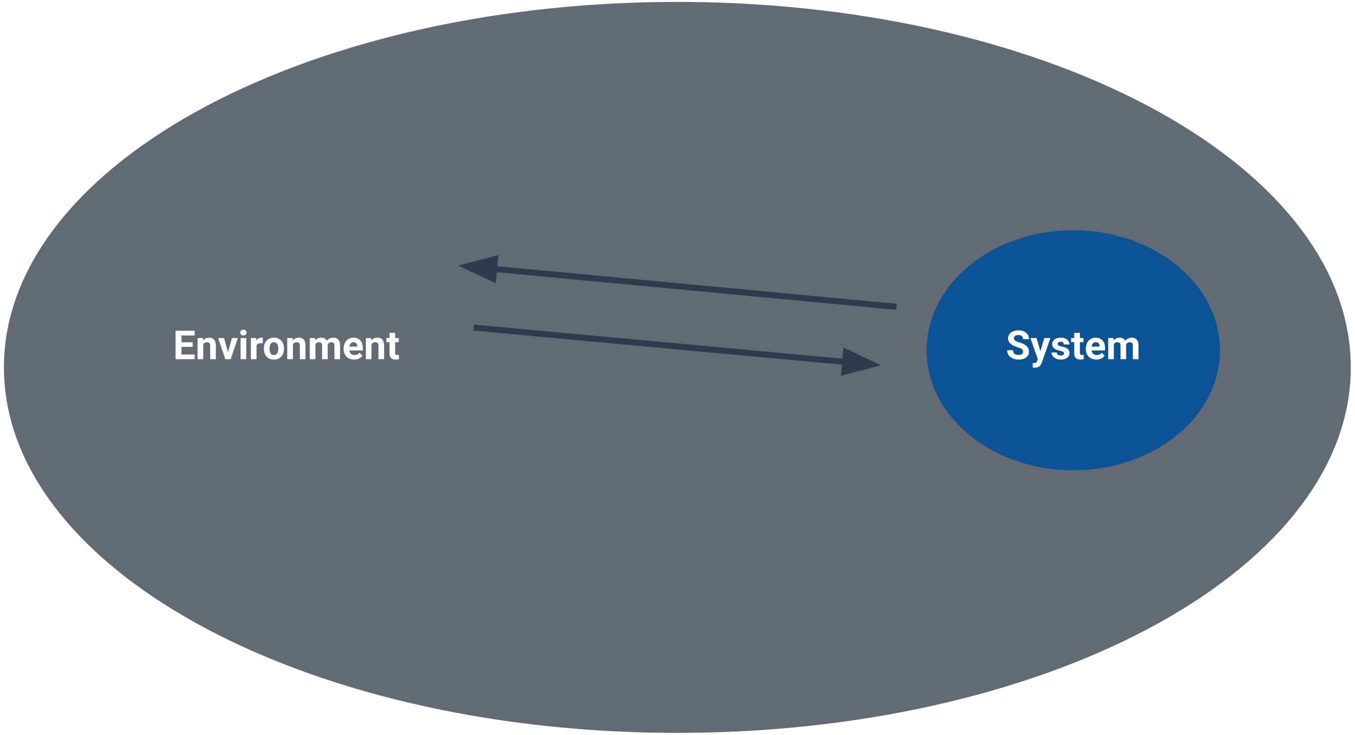


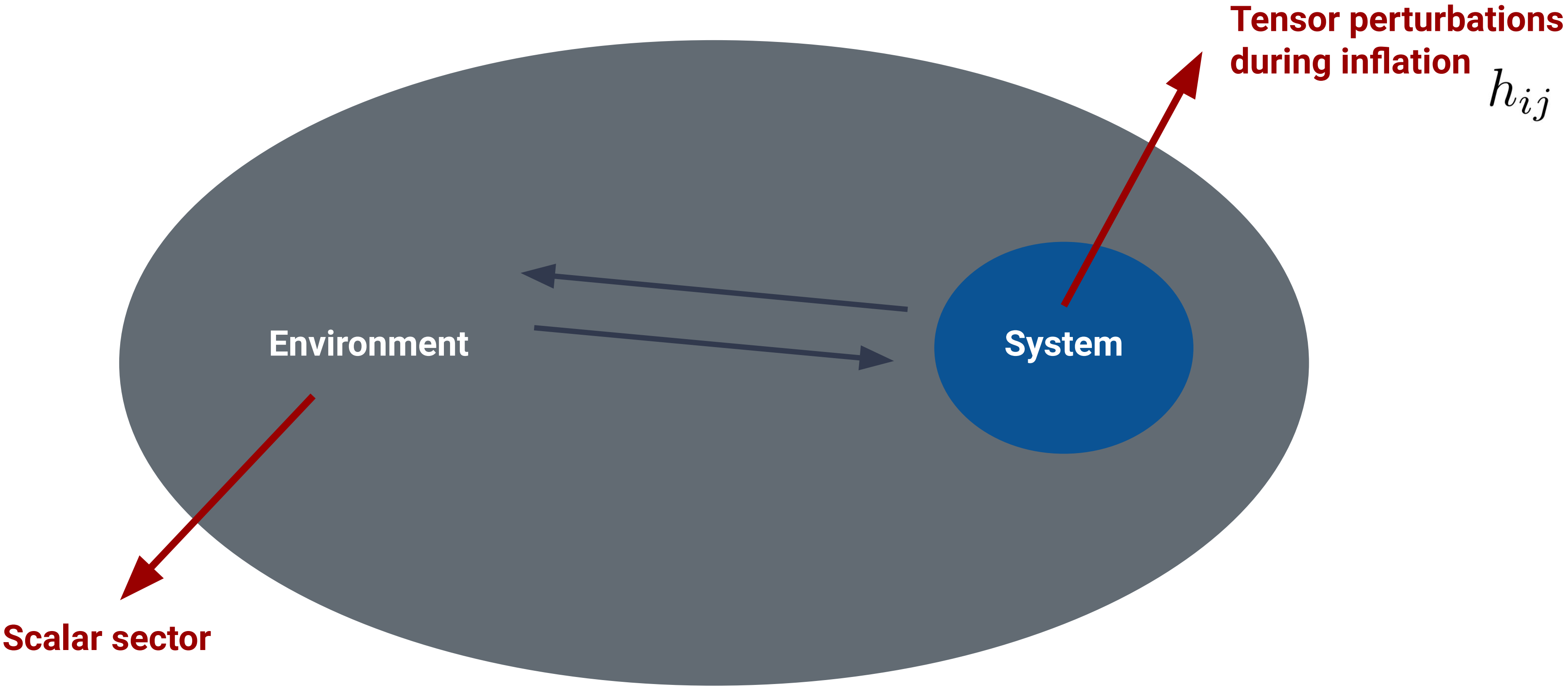
Sloan Digital Sky Survey

Quantum → ? → **Classical**

Quantum decoherence



Quantum decoherence



Two point correlation function

Two point correlation function:

$$\langle \hat{O}_{\boldsymbol{k}_1} \hat{O}'_{\boldsymbol{k}_2} \rangle = P_{OO'}(\boldsymbol{k_1})(2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2)$$

$\hat{h}_{\boldsymbol{k}_i}$ or $\hat{p}_{\boldsymbol{k}_i} = \hat{h}'_{\boldsymbol{k}_i}$

Power spectrum

Operators observable

Two point correlation function:

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$\hat{h}_{\boldsymbol{k}_i}$ or $\hat{p}_{\boldsymbol{k}_i} = \hat{h}'_{\boldsymbol{k}_i}$

Power spectrum

$$\langle \hat{O} \rangle = \begin{Bmatrix} \langle \hat{h}_{\boldsymbol{k}_1} \hat{h}_{\boldsymbol{k}_2} \rangle \\ \langle \hat{h}_{\boldsymbol{k}_1} \hat{p}_{\boldsymbol{k}_2} \rangle \\ \langle \hat{p}_{\boldsymbol{k}_1} \hat{h}_{\boldsymbol{k}_2} \rangle \\ \langle \hat{p}_{\boldsymbol{k}_1} \hat{p}_{\boldsymbol{k}_2} \rangle \end{Bmatrix}$$

Lindblad equation

Use the **Lindblad equation** which models the time evolution of the observable

Jerome Martin, Vincent Vennin
1801.09949 (JCAP, 2018)

$$\frac{\mathrm{d}\langle\hat{O}\rangle}{\mathrm{d}\eta} = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i[\hat{O}, \hat{H}_S] - \frac{\gamma}{2} \int \mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{y} C_R(\boldsymbol{x}, \boldsymbol{y}) \langle [\hat{O}, \hat{A}(\boldsymbol{x})], \hat{A}(\boldsymbol{y})] \rangle .$$

Information about the environment

System Hamiltonian

Information about the system

Time dependence

Use the **Lindblad equation** which models the time evolution of the observable

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Information about the environment

System Hamiltonian

Information about the system

- p time dependence of the interaction

$$\gamma = \gamma_* \left(\frac{a}{a_*}\right)^p$$

Cosmic scale factor, dependent on time

Interaction strength

Use the **Lindblad equation** which models the time evolution of the observable

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Information about the environment

System Hamiltonian

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Cosmic scale factor, dependent on time

- p time dependence of the interaction
- σ_γ interaction strength

Differential equation

$$\frac{\mathrm{d}\langle \hat{h}_{\boldsymbol{k}_1} \hat{h}_{\boldsymbol{k}_2} \rangle}{\mathrm{d}\eta}$$

$$\frac{\mathrm{d}\langle \hat{h}_{\boldsymbol{k}_1} \hat{p}_{\boldsymbol{k}_2} \rangle}{\mathrm{d}\eta}$$

$$\frac{\mathrm{d}\langle \hat{p}_{\boldsymbol{k}_1} \hat{h}_{\boldsymbol{k}_2} \rangle}{\mathrm{d}\eta}$$

$$\frac{\mathrm{d}\langle \hat{p}_{\boldsymbol{k}_1} \hat{p}_{\boldsymbol{k}_2} \rangle}{\mathrm{d}\eta}$$

→

$$P_{hh}''' + 4\omega^2 P_{hh}' + 4\omega\omega' P_{hh} = S(k,\eta)$$

Dispersion relation

Source function

- Compute source function $$S(k,\eta) \propto \frac{k^2}{\eta^4}$$

Decoherence power spectrum

- Compute source function $S(k, \eta) \propto \frac{k^2}{\eta^4}$
- Plug into known solution

$$P_{hh}(k) = 2\left[|h_{\boldsymbol{k}}(\eta)|^2 + 2\int_{-\infty}^{\eta} S(k, \eta')\mathfrak{Im}^2[h_{\boldsymbol{k}}(\eta')h^*_{\boldsymbol{k}}(\eta)]\mathrm{d}\eta'\right]$$

Sum over polarizations

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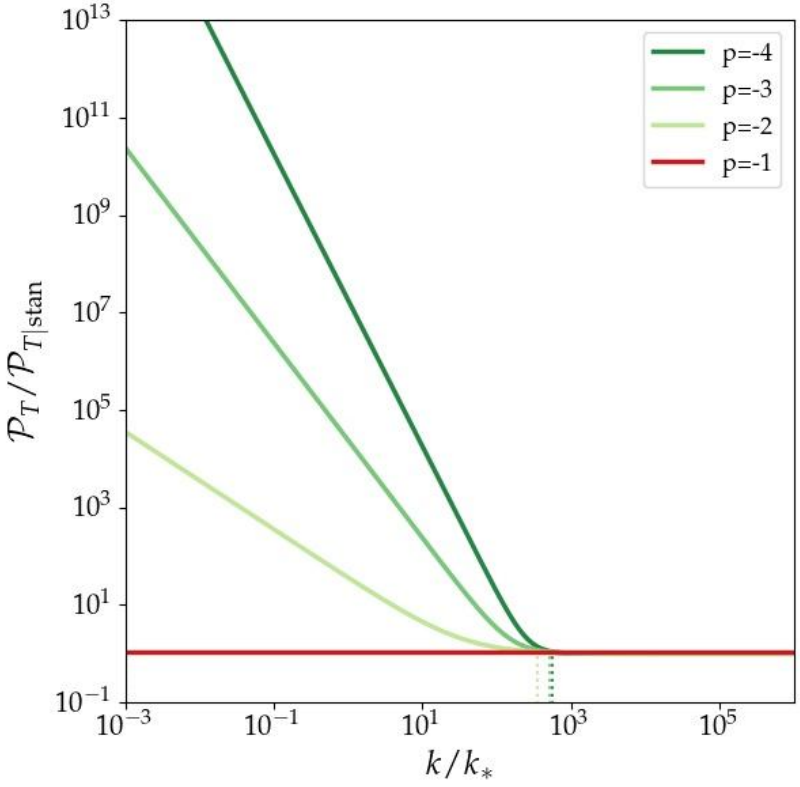
Sum over polarizations

$$\mathcal{P}_T(k) = \frac{k^3}{2\pi^2}\frac{32\pi\cdot 2P_{vv}}{M_{pl}^2 a^2} = \mathcal{P}_{T|\mathrm{stan}}[1 + \Delta\mathcal{P}_T(k)]$$

Power spectrum

Three cases:

- p<-1



Power spectrum

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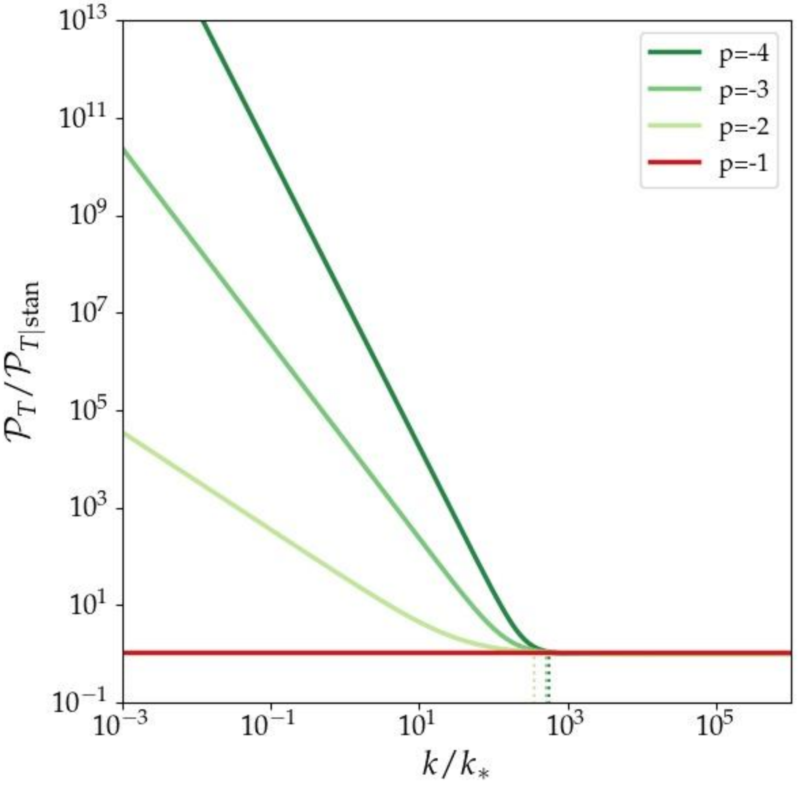

Power spectrum

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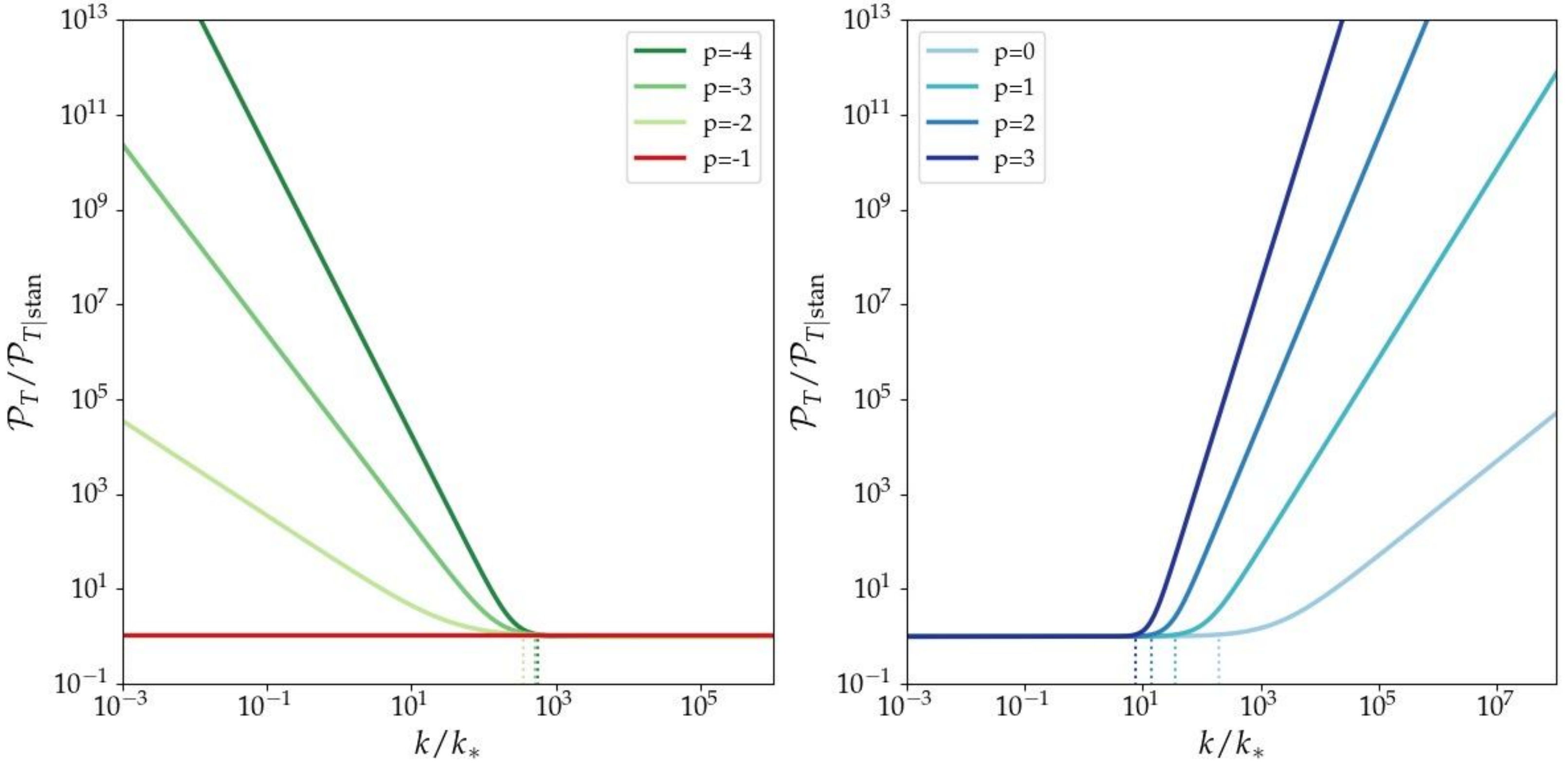
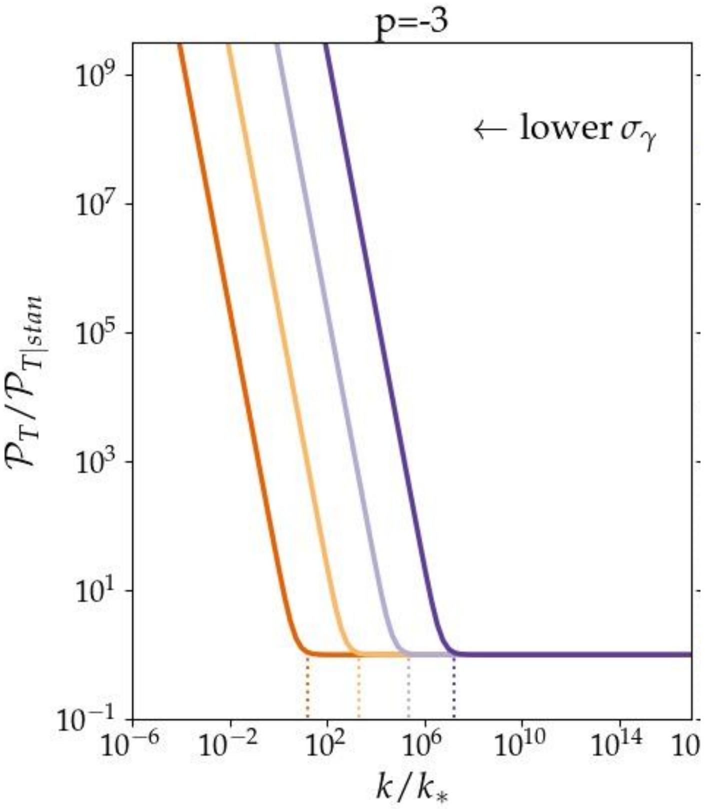


Interaction strength



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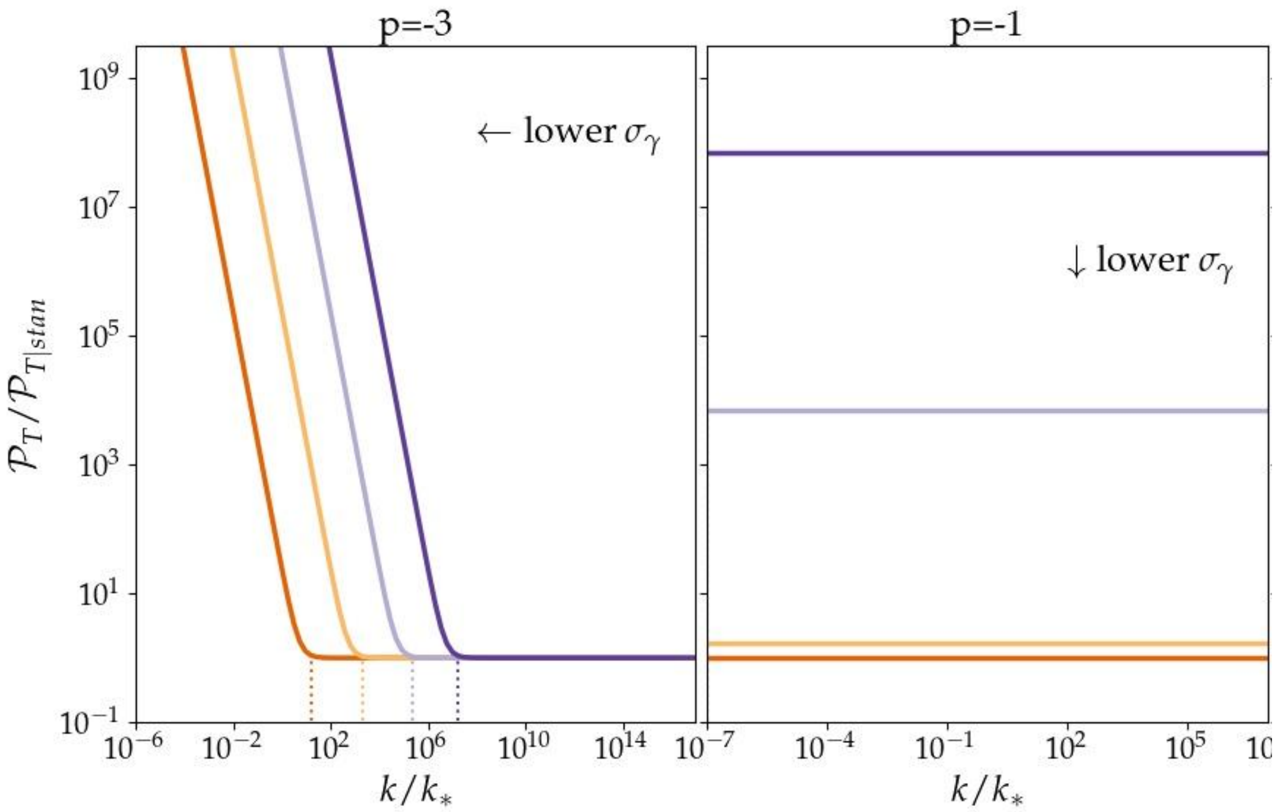

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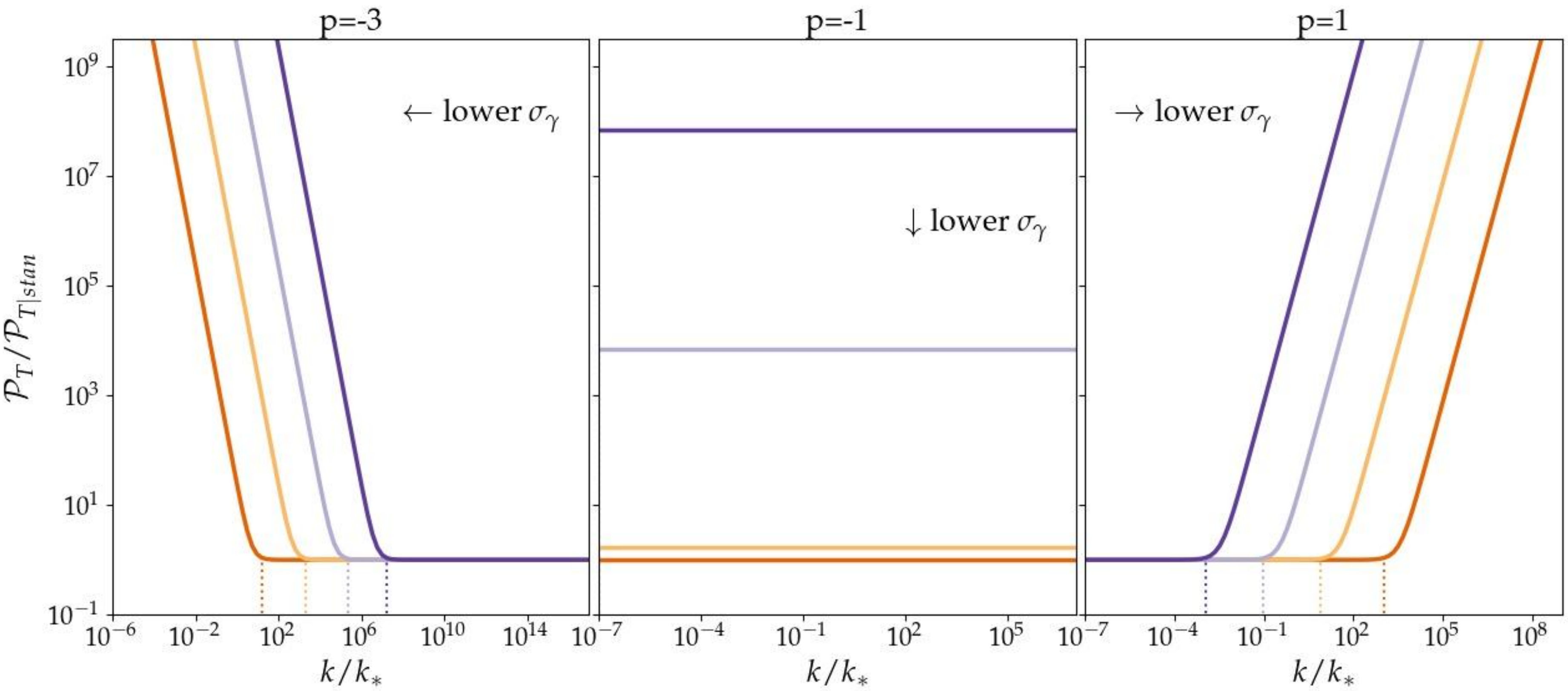

Cut-off mechanism

The mode needs to have crossed out of the environment correlation length in order to be affected by the environment

In addition, we require: $(H_* l_e) \ll 1$

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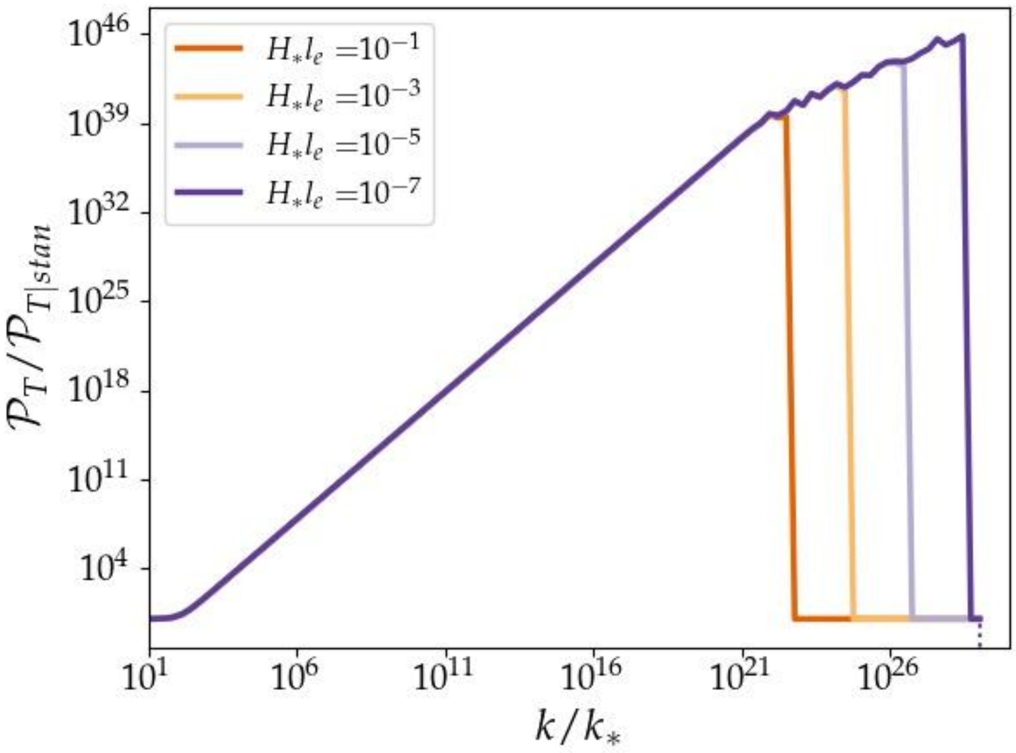

Observational constraints

Use observational data from PLANCK, BICEP/Keck array and LIGO-Virgo-KAGRA:

Giacomo Galloni et al.
2208.00188 (JCAP, 2023)

$$\mathcal{P}_T(k) = r_* A_s \left(\frac{k}{k_*} \right)^{n_T}$$

Tensorial spectral index:

$$-1.37 < n_T < 0.42 \quad \text{at } 95\% \text{ CL}$$

Tensor-to-scalar perturbation ratio:

$$r_* < 0.028$$

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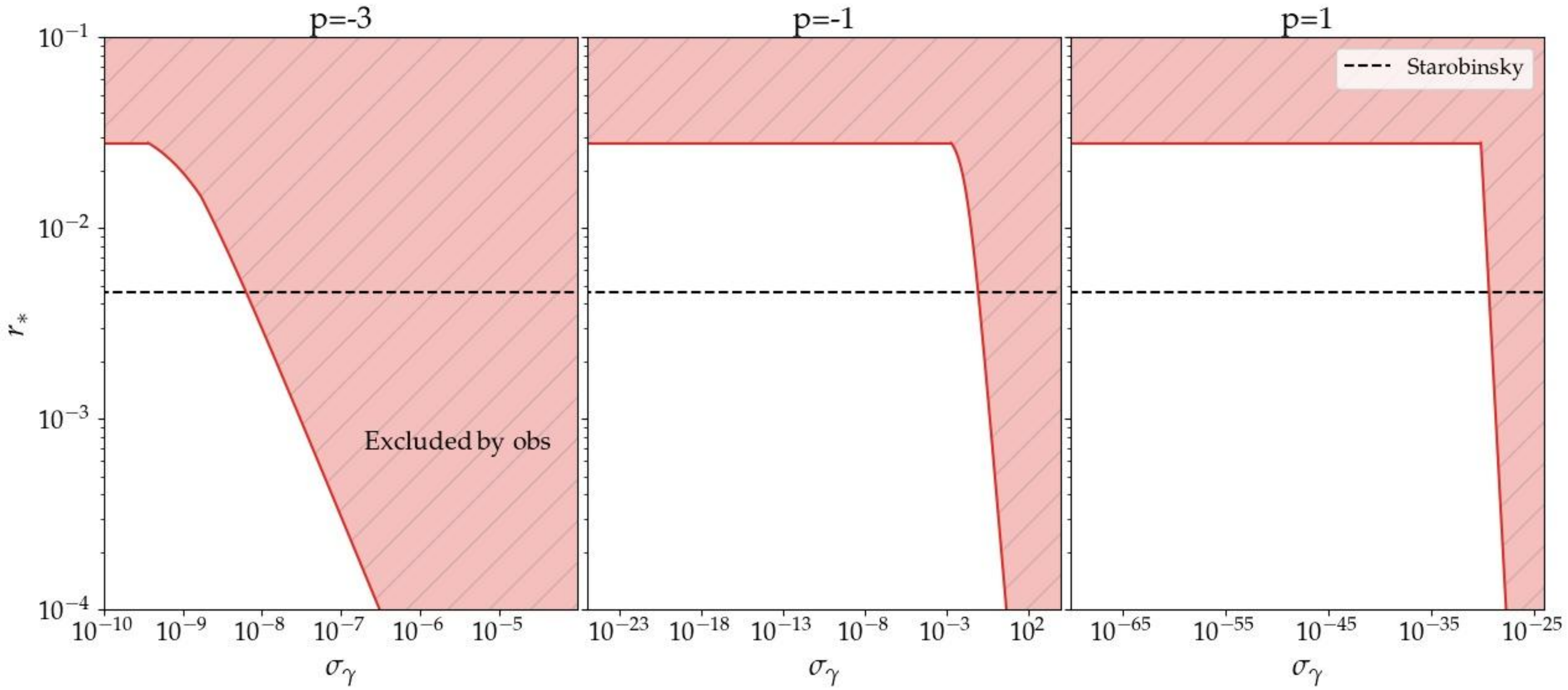

Decoherence criterion

Successful decoherence of scale k at the end of inflation requires: $\delta_{\boldsymbol{k}}(\beta^2\sigma_\gamma, p) \gg 1$ → Suppression of off-diagonal terms of the system density matrix

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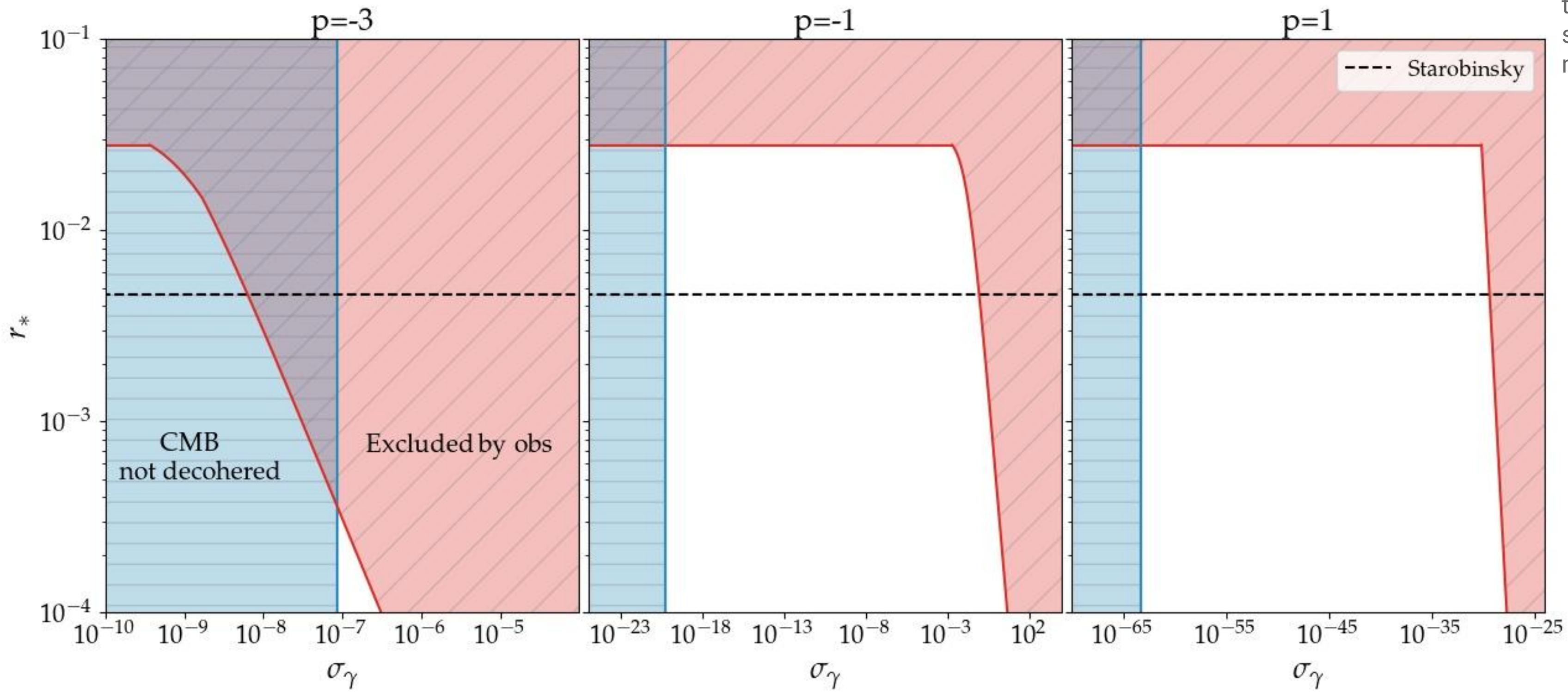
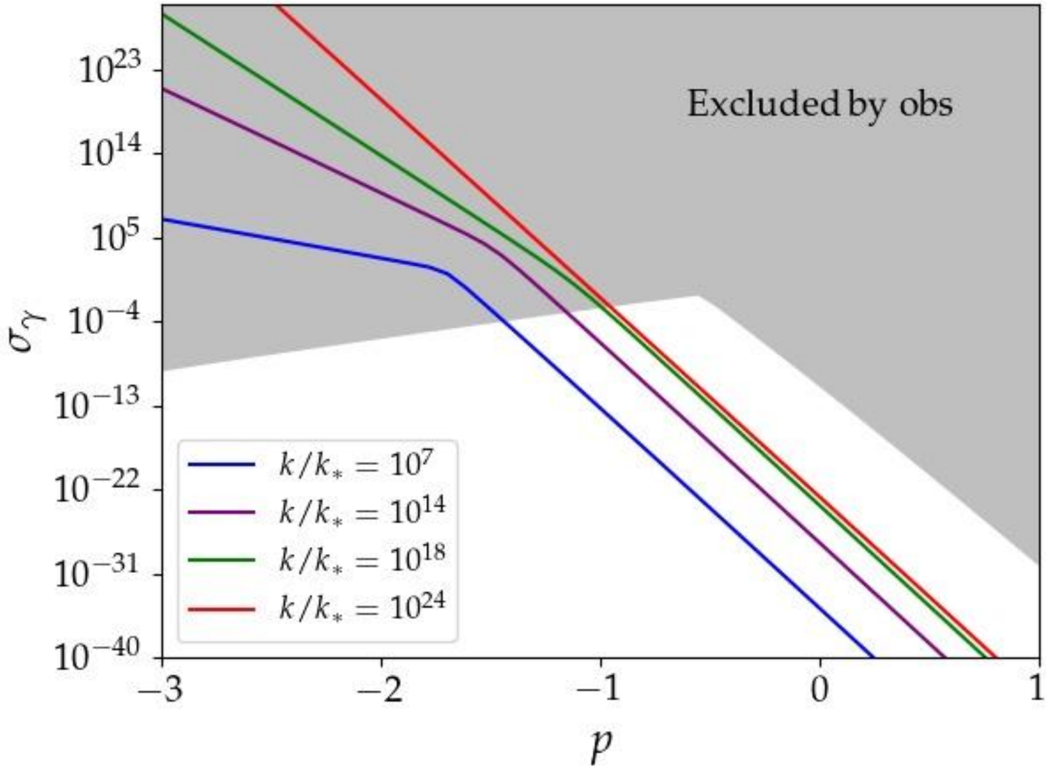


Quantum signatures

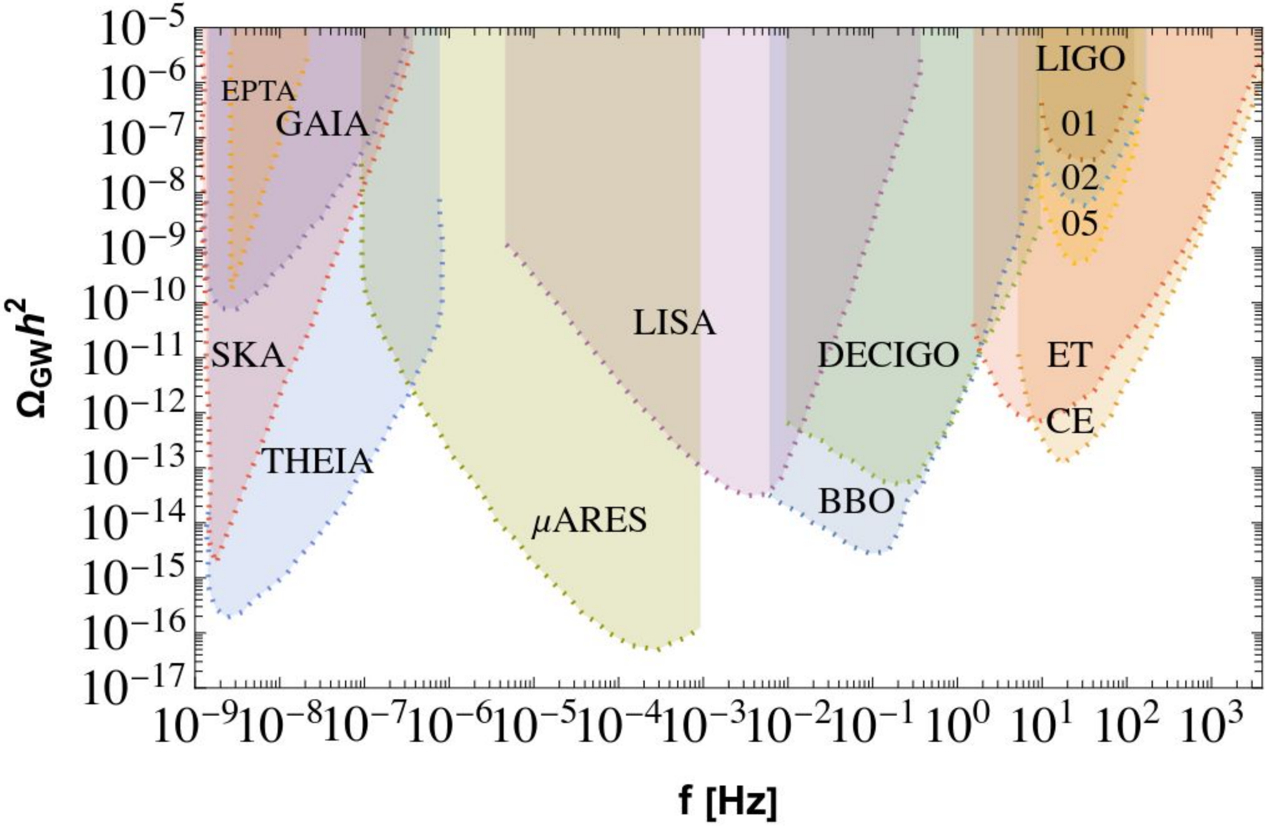


Future CMB detectors

Simons observatory

LiteBIRD

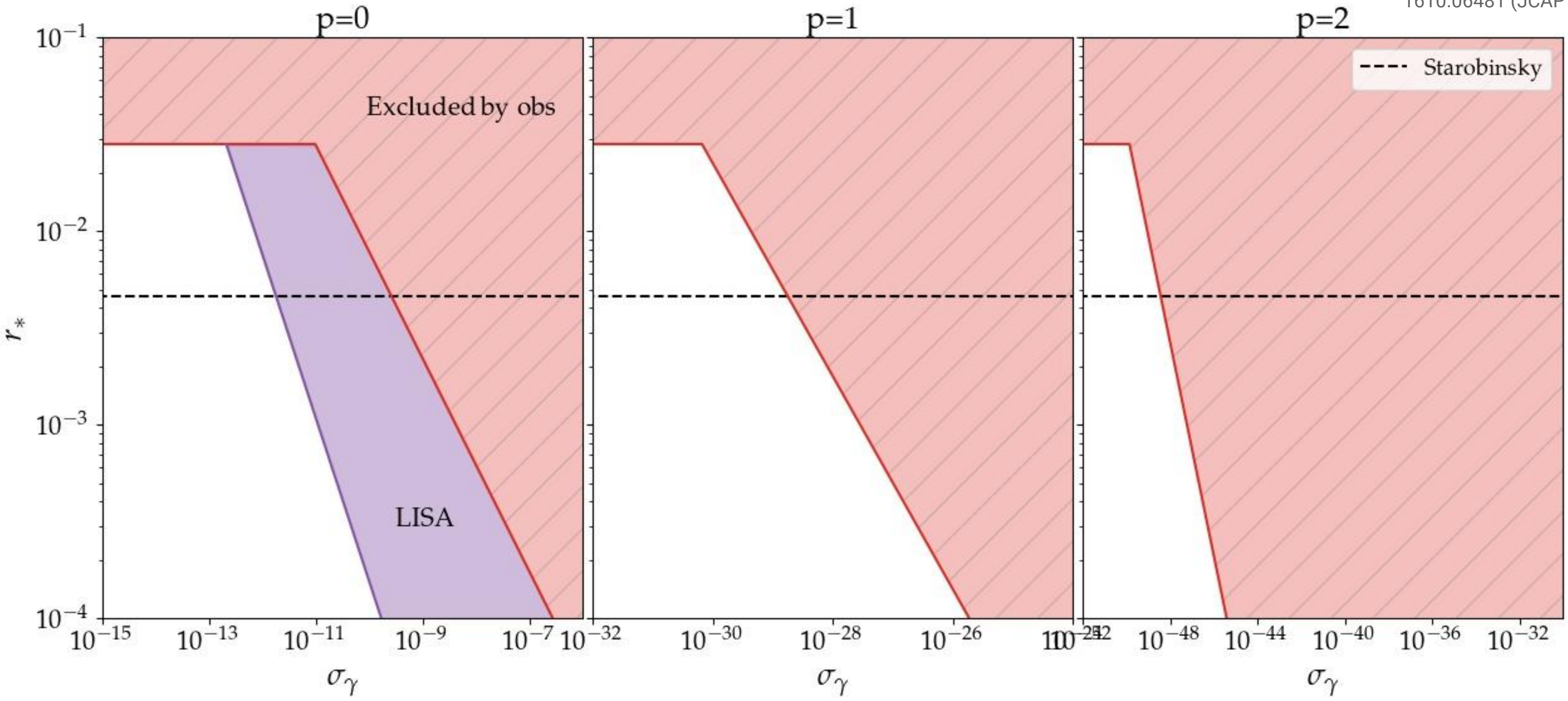
Future detectors



Rishav Roshan, Graham White (2401.04388)

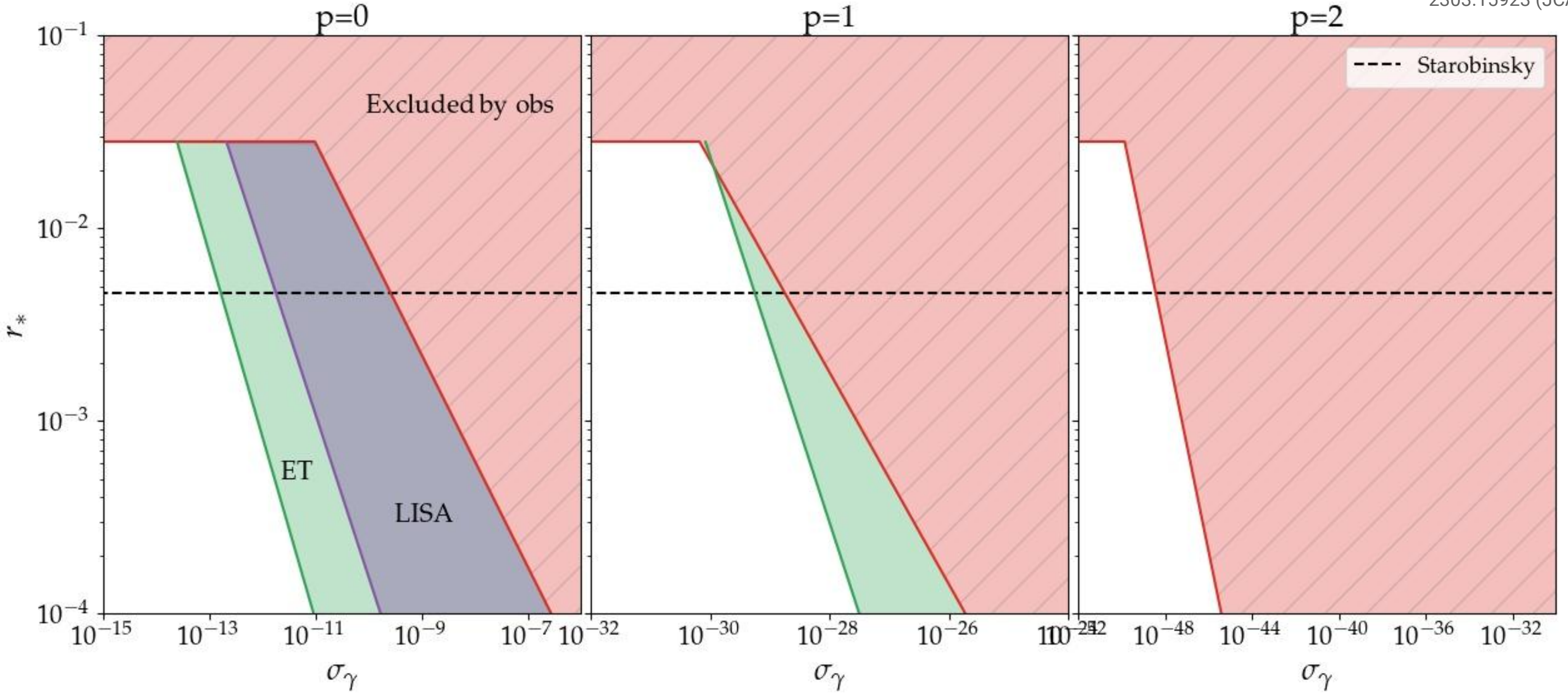
LISA

Bartolo et al.
1610.06481 (JCAP 12, 2016)



Einstein Telescope

Branchesi et al.
2303.15923 (JCAP, 2023)



Conclusions and future work

- The decoherence of gravitational waves can have a huge impact on their power spectrum, depending on it's parameters.
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- The decoherence of gravitational waves can have a huge impact on their power spectrum, depending on it's parameters.
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Thank you for your attention

Appendix

Quantum decoherence

- Topic is extensively researched, both from a particle physics and a cosmology point of view
- The main focus has been on scalar (i.e. density) perturbations

Observational constraints on quantum decoherence during inflation. Authors: Jerome Martin, Vincent Vennin (JCAP, 2018) → Compute the effect decoherence of scalar perturbations has on their power spectrum and how to constrain this

- Toy model for tensor perturbations: **Cosmic decoherence: primordial power spectra and non-Gaussianities. Authors: Aoumeur Daddi Hammou, Nicola Bartolo (JCAP, 2023)**

- With the advances in the detection of gravitational waves, it is time to extensively look into **the effect quantum decoherence has on the inflationary GW power spectrum**

Cut-off mechanism

$$P_{vv} = \sum_s v_{\boldsymbol{k}}^s(\eta) v_{\boldsymbol{k}}^{*s}(\eta) + \frac{\pi \gamma_* \beta^2 l_E^3 \bar{C}_R}{9 a_*^3 \sin(\nu\pi)^2} k^3 (-k\eta_*)^{p-3} (-k\eta) JI(-k\eta, \nu), \tag{3.13}$$

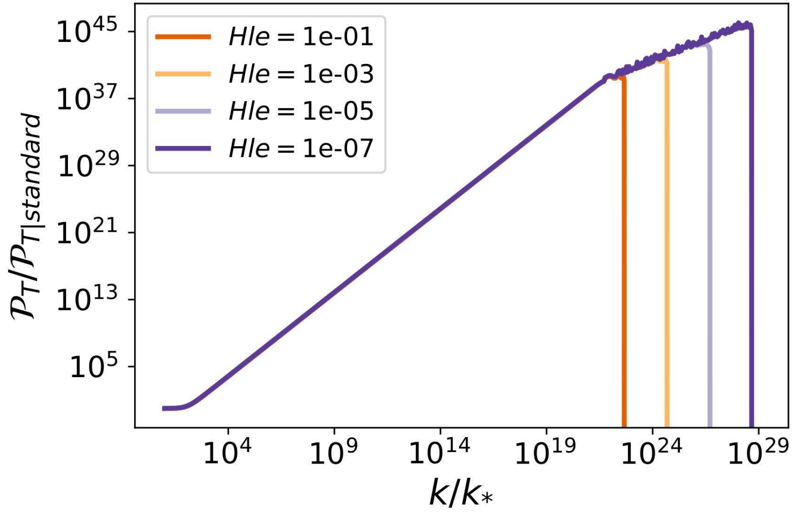
where we use

$$JI(-k\eta, \nu) = J_{-\nu}^2(-k\eta) I_1(\nu) - 2 J_{-\nu}(-k\eta) J_\nu(-k\eta) I_2(\nu) + J_\nu^2(-k\eta) I_1(\nu), \tag{3.14}$$

and the integrals I_1 and I_2 are defined by

$$I_1(\nu) \equiv \int_{-k\eta}^{(H_* l_E)^{-1}} \mathrm{d}z z^{2-p} \left(\frac{1}{z^2} - \frac{1}{(-k\eta_{IR})^2} \right) J_\nu^2(z),$$

$$I_2(\nu) \equiv \int_{-k\eta}^{(H_* l_E)^{-1}} \mathrm{d}z z^{2-p} \left(\frac{1}{z^2} - \frac{1}{(-k\eta_{IR})^2} \right) J_\nu(z) J_{-\nu}$$



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$$
\begin{aligned}
C_R(\tau) &= \sum_m \langle m| \left[\sum_n p_n |n\rangle\langle n|\tilde{R}(\tau)\tilde{R}(0) \right] |m\rangle \\
&= \sum_n p_n \langle n|\tilde{R}(\tau)\tilde{R}(0)|n\rangle \\
&= \sum_n p_n \langle n|e^{iH_{\mathrm{E}}\tau}\tilde{R}(0)e^{-iH_{\mathrm{E}}\tau}\tilde{R}(0)|n\rangle \\
&= \sum_{n,m,p,q} p_n \langle n|e^{iH_{\mathrm{E}}\tau}|m\rangle\langle m|\tilde{R}(0)|p\rangle\langle p|e^{-iH_{\mathrm{E}}\tau}|q\rangle\langle q|\tilde{R}(0)|n\rangle \\
&= \sum_{n,p} p_n e^{i(E_n-E_p)\tau}\langle n|\tilde{R}(0)|p\rangle\langle p|\tilde{R}(0)|n\rangle \\
&= \sum_{n,p} p_n e^{i(E_n-E_p)\tau}\left|\langle n|\tilde{R}(0)|p\rangle\right|^2 .
\end{aligned}
\qquad \text{(A.26)}
$$

In particular, one has $C_R(-\tau) = C_R^*(\tau)$. More specifically, one can see that $C_R(\tau)$ is a sum of exponentials oscillating at the Bohr frequencies of the environment. In the limit where the environment is large and contains an almost continuous set of energy levels, destructive interference occurs that quickly drives $C_R(\tau)$ to zero with a characteristic time t_c, $C_R(\tau) \simeq C_R(0)e^{-|\tau|/t_c}$.

Environmental correlation length

Furthermore, $C_R(\boldsymbol{x},\boldsymbol{y})$ is the same-time correlation function of the environment $\hat{R}$, defined by

$$C_R(\boldsymbol{x},\boldsymbol{y}) = \mathrm{Tr}_\mathrm{E}\big(\hat{\rho}_E\hat{R}(\boldsymbol{x})\hat{R}(\boldsymbol{y})\big). \tag{2.12}$$

Assuming that the environment is statistically homogeneous and isotropic, and that a single physical length scale l_E is involved, we take this to be a top-hat function, giving

$$C_R(\boldsymbol{x},\boldsymbol{y}) = \bar{C}_R\Theta\left(\frac{a|\boldsymbol{x}-\boldsymbol{y}|}{l_\mathrm{E}}\right), \tag{2.13}$$

where $\Theta(x)$ is 1 if $x<1$ and 0 otherwise, and $\bar{C}_R$ is a constant. In Fourier space this can be written as

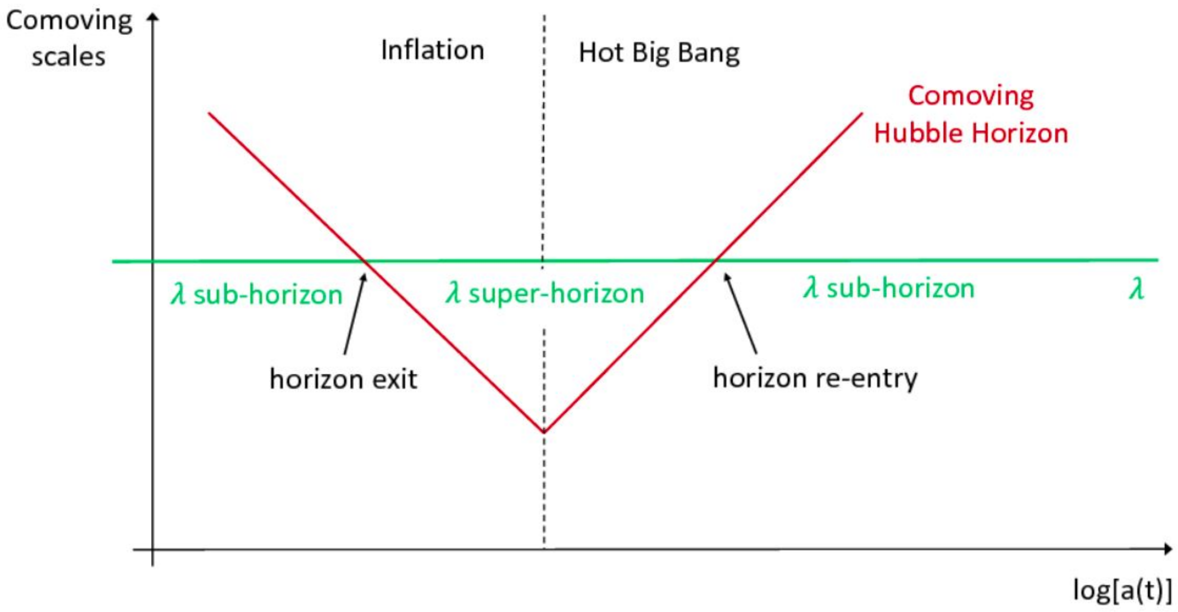
$$\tilde{C}_R(k) = \sqrt{\frac{2}{\pi}}\frac{\bar{C}_R}{k^3}\left[\sin\left(\frac{kl_\mathrm{E}}{a}\right) - \frac{kl_\mathrm{E}}{a}\cos\left(\frac{kl_\mathrm{E}}{a}\right)\right], \tag{2.14}$$

which can again be approximated by a top-hat function

$$\tilde{C}_R(k) \simeq \sqrt{\frac{2}{\pi}}\frac{\bar{C}_R l_\mathrm{E}^3}{3a^3}\Theta\left(\frac{kl_\mathrm{E}}{a}\right), \tag{2.15}$$

where the amplitude at the origin has been matched.

Comoving Hubble horizon



Computation power spectrum

To obtain the GW power spectrum from Eq. (2.11), we insert two-point correlators of the form $\langle\hat{O}\rangle = \langle\hat{O}_{\boldsymbol{k}_1}\hat{O}_{\boldsymbol{k}_2}\rangle$ with $\hat{O}_{\boldsymbol{k}_i} = \hat{v}^s_{\boldsymbol{k}_i}$ or $\hat{p}^s_{\boldsymbol{k}_i}$, where s refers to the polarization, finding:

$$\begin{aligned}
\frac{\mathrm{d}\langle\hat{v}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\rangle}{\mathrm{d}\eta} &= \langle\hat{v}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\rangle + \langle\hat{p}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\rangle \\
\frac{\mathrm{d}\langle\hat{v}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\rangle}{\mathrm{d}\eta} &= \langle\hat{p}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\rangle - \omega^2(k_2)\langle\hat{v}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\rangle \\
\frac{\mathrm{d}\langle\hat{p}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\rangle}{\mathrm{d}\eta} &= \langle\hat{p}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\rangle - \omega^2(k_1)\langle\hat{v}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\rangle \\
\frac{\mathrm{d}\langle\hat{p}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\rangle}{\mathrm{d}\eta} &= -\omega^2(k_2)\langle\hat{p}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\rangle - \omega^2(k_1)\langle\hat{v}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\rangle \\
&\quad + \beta^2\frac{4\gamma}{(2\pi)^{3/2}}\int \mathrm{d}^3\boldsymbol{k}\, k_1^l(k+k_1)_l k_2^a(k_2-k)_a \tilde{C}_R(|\boldsymbol{k}|)\langle\hat{v}^s_{\boldsymbol{k}+\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2-\boldsymbol{k}}\rangle\,.
\end{aligned} \tag{3.1}$$

The system is solved through a perturbative expansion in γ, and the environment correlator preserves statistical isotropy and homogeneity (see Eq. (2.7)). This means we have a statistically homogeneous and isotropic solution of the form

$$\langle\hat{O}_{\boldsymbol{k}_1}\hat{O}'_{\boldsymbol{k}_2}\rangle = (2\pi)^3 P_{OO'}(k_1)\delta(\boldsymbol{k}_1+\boldsymbol{k}_2). \tag{3.2}$$

In the last line of Eq. (3.1), the last term shows that we sum over the indices l and a. This allows us to take the dot product, giving

$$k_1^l(k+k_1)_l k_2^a(k_2-k)_a = (\boldsymbol{k}_1\cdot\boldsymbol{k})^2 + k_1^4 + 2k_1^2(\boldsymbol{k}_1\cdot\boldsymbol{k}). \tag{3.3}$$

Computation power spectrum

$$\begin{aligned}
\frac{\mathrm{d}P_{vv}(k)}{\mathrm{d}\eta} &= P_{vp}(k) + P_{pv}(k) \\
\frac{\mathrm{d}P_{vp}(k)}{\mathrm{d}\eta} = \frac{\mathrm{d}P_{pv}(k)}{\mathrm{d}\eta} &= P_{pp}(k) - \omega^2(k)P_{vv}(k) \\
\frac{\mathrm{d}P_{pp}(k)}{\mathrm{d}\eta} &= -\omega^2(k)\Big(P_{pv}(k) + P_{vp}(k)\Big) \\
&+ \beta^2 \frac{4\gamma}{(2\pi)^{3/2}} \int \mathrm{d}^3\boldsymbol{k}' \Big((\boldsymbol{k}\cdot\boldsymbol{k}')^2 + k^4 + 2k^2(\boldsymbol{k}\cdot\boldsymbol{k}')\Big)\tilde{C}_R(|\boldsymbol{k}'|)P_{vv}(|\boldsymbol{k}+\boldsymbol{k}'|).
\end{aligned} \tag{3.4}$$

These equations can be combined into a single third-order equation for P_{vv}

$$P_{vv}''' + 4\omega^2(k)P_{vv}' + 4\omega\omega' P_{vv} = S(\boldsymbol{k}, \eta), \tag{3.5}$$

where the source function $S(\boldsymbol{k}, \eta)$ is dependent on P_{vv} and defined as

$$S(\boldsymbol{k}, \eta) = \beta^2 \frac{8\gamma}{(2\pi)^{3/2}} \int \mathrm{d}^3\boldsymbol{k}' \Big((\boldsymbol{k}\cdot\boldsymbol{k}')^2 + k^4 + 2k^2(\boldsymbol{k}'\cdot\boldsymbol{k})\Big)\tilde{C}_R(|\boldsymbol{k}'|)P_{vv}(|\boldsymbol{k}+\boldsymbol{k}'|). \tag{3.6}$$

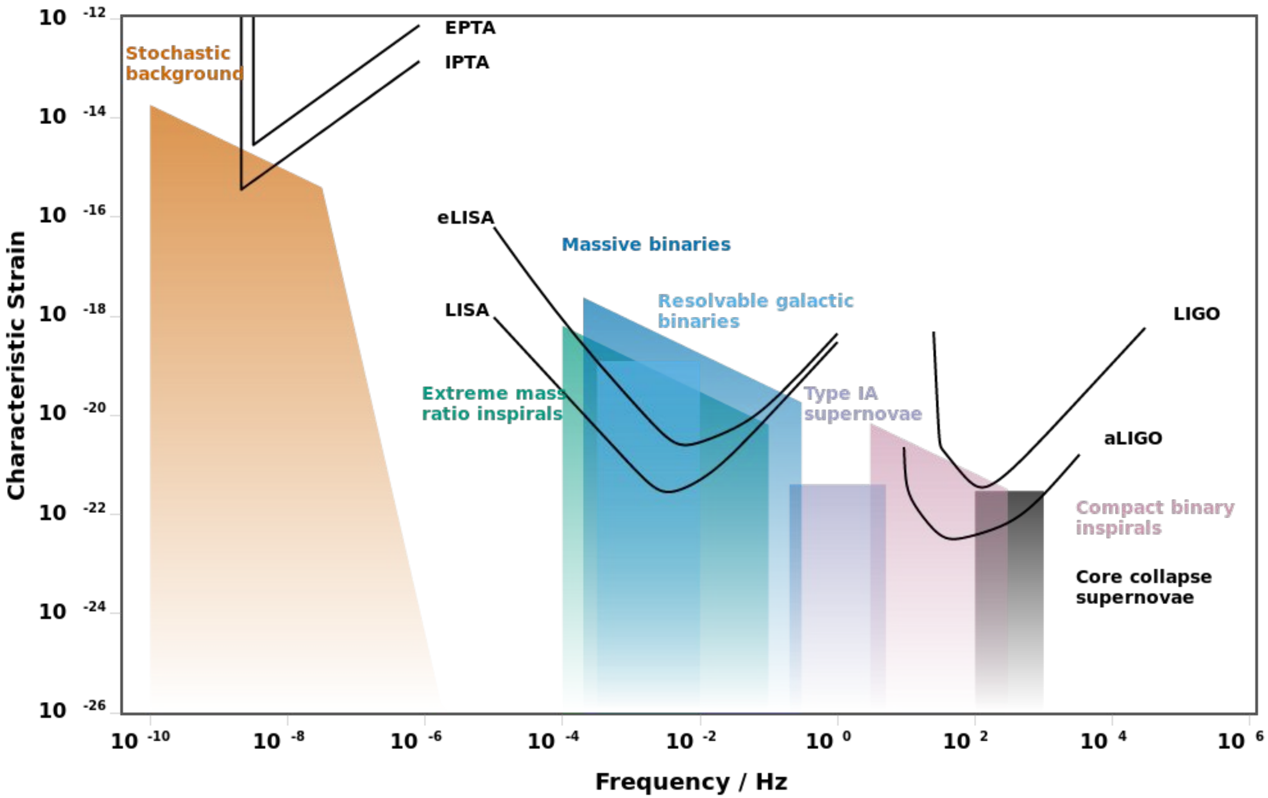
Power spectrum

$$\mathcal{P}_T(k) = \mathcal{P}_{T|\text{standard}}\left(1 + \frac{2\beta^2\sigma_\gamma}{9\sin^2(\nu\pi)}\left(\frac{k}{k_*}\right)^{p+1}\frac{J_{-\nu}^2(-k\eta)I_1(\nu) - 2J_{-\nu}(-k\eta)J_\nu(-k\eta)I_2(\nu) + J_\nu^2(-k\eta)I_1(\nu)}{[J_\nu^2(-k\eta) + Y_\nu^2(-k\eta)]}\right)$$

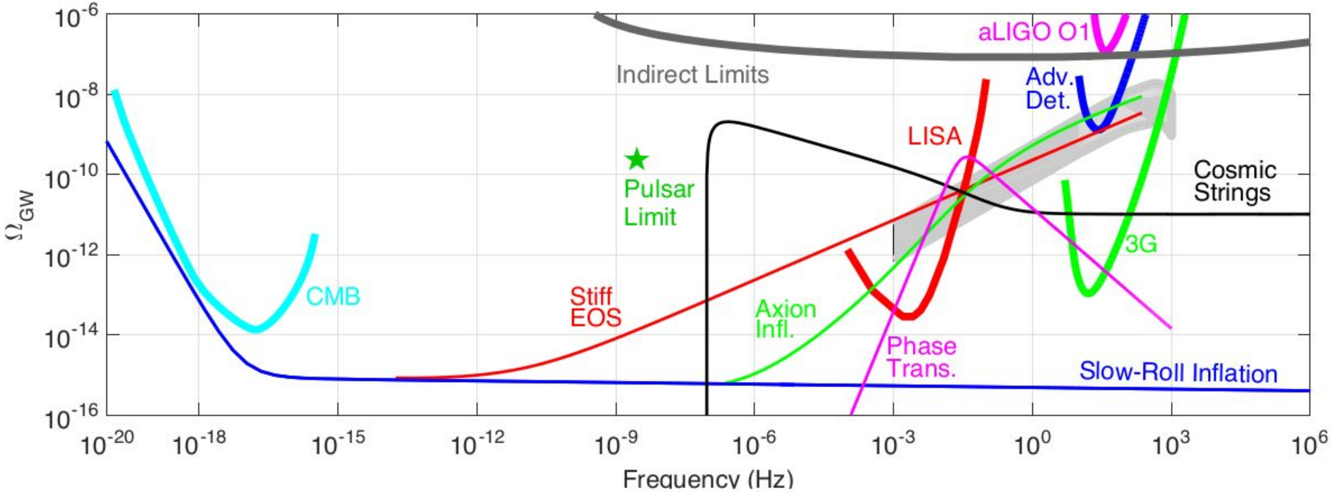
$$\beta^2\sigma_\gamma \equiv \frac{l_E^3\bar{C}_R\gamma_* k_*^4\beta^2}{a_*^3}$$

Defined such that our computation is analogue to the computation done by Jerome Martin and Vincent Vennin
https://arxiv.org/pdf/1801.09949.pdf

Future detectors



Future detectors



Assumptions

The Lindblad equation is based on the Born and Markov approximations which can be summarized as follows:

1. The environment evolves on a time scale that is much smaller than that of the system.
2. The back reaction of the system on the environment is negligible.
3. The influence of the environment on the system, which is clearly crucial here, can be treated perturbatively.

Derivation Lindblad equation

Let a system "S" be in interaction with some environment "E". The Hilbert space $\mathcal{E}$ of the full system can be written as the tensorial product of the Hilbert space of the system, $\mathcal{E}_S$, with the Hilbert space of the environment, $\mathcal{E}_E$, namely $\mathcal{E} = \mathcal{E}_S \otimes \mathcal{E}_E$. Then, the corresponding Hamiltonian reads[7]

$$H = H_0 + H_{\text{int}} = H_S \otimes \mathbb{I}_E + \mathbb{I}_S \otimes H_E + gH_{\text{int}} \,. \tag{A.1}$$

Here, H_S is the Hamiltonian of the system and acts in $\mathcal{E}_S$, while $\mathbb{I}_E$ is the identity operator acting in $\mathcal{E}_E$. In the same manner, H_E is the Hamiltonian of the environment and acts in $\mathcal{E}_E$, while $\mathbb{I}_S$ is the identity operator acting in $\mathcal{E}_S$. They represent the free Hamiltonian H_0 acting in the full space $\mathcal{E}$, while H_{int} is an interaction term. It carries a (supposedly small) dimensionless coupling parameter g characterising the strength of the interactions between the system and the environment.

The density matrix ρ of the full system (acting in the Hilbert space $\mathcal{E}$) obeys the unitary Liouville-von Neumann equation

$$i\frac{\mathrm{d}\rho}{\mathrm{d}t} = [H, \rho(t)] \,. \tag{A.2}$$

https://arxiv.org/pdf/1801.09949.pdf

Derivation Lindblad equation

Let us now restrict the analysis to the reduced density matrix of the system, $\tilde{\rho}_{\rm S}$. It is obtained from the full density matrix by tracing out the environment degrees of freedom, i.e.

$$\tilde{\rho}_{\rm S}(t) = {\rm Tr}_{\rm E}\left[\tilde{\rho}(t)\right] . \tag{A.9}$$

Let us recall that $\tilde{\rho}$ is an operator acting in $\mathcal{E}_{\rm S} \otimes \mathcal{E}_{\rm E}$ and, therefore, $\tilde{\rho}_{\rm S}$ is an operator acting in $\mathcal{E}_{\rm S}$ only. From Eq. (A.8), it obeys the equation

$$\begin{aligned}\tilde{\rho}_{\rm S}(t+\Delta t) - \tilde{\rho}_{\rm S}(t) = -ig\int_t^{t+\Delta t} {\rm d}t' {\rm Tr}_{\rm E}\left[\tilde{H}_{\rm int}(t'), \tilde{\rho}(t)\right] \\ - g^2\int_t^{t+\Delta t}{\rm d}t'\int_t^{t'}{\rm d}t''{\rm Tr}_{\rm E}\left[\tilde{H}_{\rm int}(t'), \left[\tilde{H}_{\rm int}(t''), \tilde{\rho}(t'')\right]\right] .\end{aligned} \tag{A.10}$$

Similarly to Eq. (A.9), we can define the reduced density operator of the environment, acting in $\mathcal{E}_{\rm E}$, by $\tilde{\rho}_{\rm E}(t) \equiv {\rm Tr}_{\rm S}\left[\tilde{\rho}(t)\right]$. It is important to stress that, in general, $\tilde{\rho}(t) \neq {\rm Tr}_{\rm E}\left[\tilde{\rho}(t)\right] \otimes {\rm Tr}_{\rm S}[\tilde{\rho}(t)]$, namely $\tilde{\rho}(t) \neq \tilde{\rho}_{\rm S}(t) \otimes \tilde{\rho}_{\rm E}(t)$, and one has instead

$$\tilde{\rho}(t) = \tilde{\rho}_{\rm S}(t) \otimes \tilde{\rho}_{\rm E}(t) + g^p \tilde{\rho}_{\rm correl}(t) . \tag{A.11}$$

This relation defines the quantity $\tilde{\rho}_{\rm correl}$, which describes the correlations between the system and the environment at time t. It satisfies[8] ${\rm Tr}_{\rm E}(\tilde{\rho}_{\rm correl}) = 0$ and ${\rm Tr}_{\rm S}(\tilde{\rho}_{\rm correl}) = 0$.

https://arxiv.org/pdf/1801.09949.pdf

Decoherence criterion

$$\delta_{\boldsymbol{k}} = \frac{1}{2}\int_{-\infty}^{\eta} S(\boldsymbol{k},\eta')P_{vv}(\boldsymbol{k},\eta')\mathrm{d}\eta'$$

↓

$$\beta^2\sigma_\gamma \gg \frac{9\sin^2(\nu\pi)}{2}\left(\frac{k}{k_*}\right)^{-p-1}\left[I_1(\nu,k,\eta)+I_1(-\nu,k,\eta)-2I_2(\nu,k,\eta)\cos(\nu\pi)\right]^{-1}.$$

The addition of a nonunitary term in the evolution equation of the density matrix of the system, that models the interaction with environmental degrees of freedom, leads to the dynamical suppression of its off-diagonal elements in the basis of the eigenstates of the interaction operator. This allows us to calculate the required interaction strength that leads to decoherence at the end of inflation.

$\delta_{\boldsymbol{k}}$ characterizes the additional decrease of the off-diagonal elements produced by the environment. Successful decoherence is characterized by the condition $\delta_{\boldsymbol{k}} \gg 1$

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$$f_{NL} = \frac{5}{18} \frac{B_{\zeta\zeta\zeta}(k,k,k)}{P_\zeta^2(k)}$$

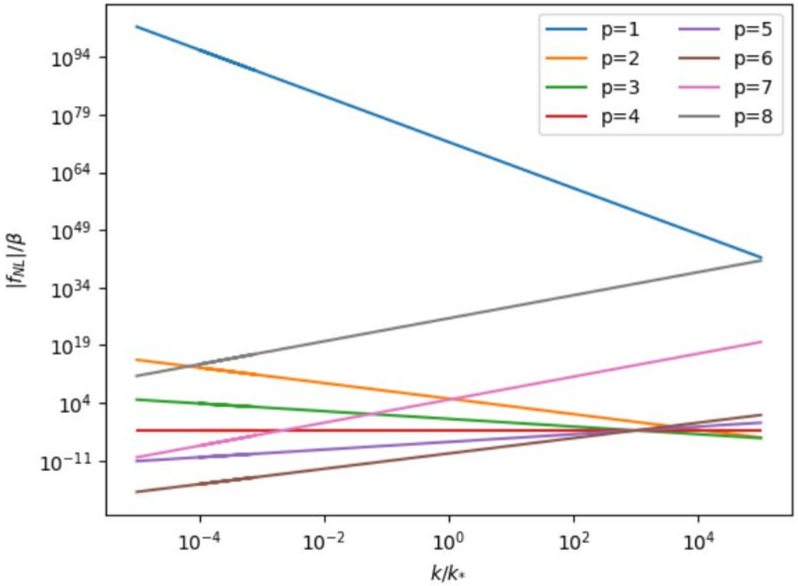

Figure 1. Bispectrum parameter f_{NL} absolute value rescaled by $\beta = \frac{\alpha k_\gamma^2}{k_*}$ as function of $\frac{k}{k_*}$ for the various values of parameter p. We choose $Hl_E = 10^{-3}$, and $N - N_* = 50$.

Cosmic decoherence: primordial power spectra and non-Gaussianities. Authors: Aoumeur Daddi Hammou, Nicola Bartolo (JCAP, 2023)