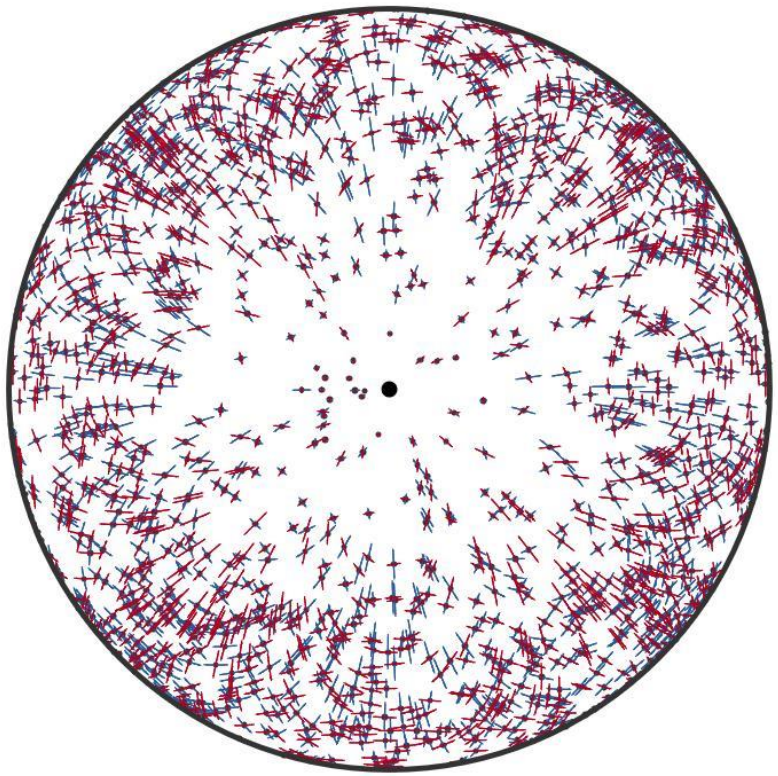
Detecting the Stochastic GWs Background with Astrometric angle correlations



Gravity Shape Pisa - 25/10/2024

Massimo Vaglio

SISSA

The Astrometric deflection

GW $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)$$

$$h_{00} = h_{0i} = 0\,, \qquad h_{ij} = A_{ij} e^{ik_\mu x^\mu} \qquad A^i_i = k^j A_{ij} = 0$$

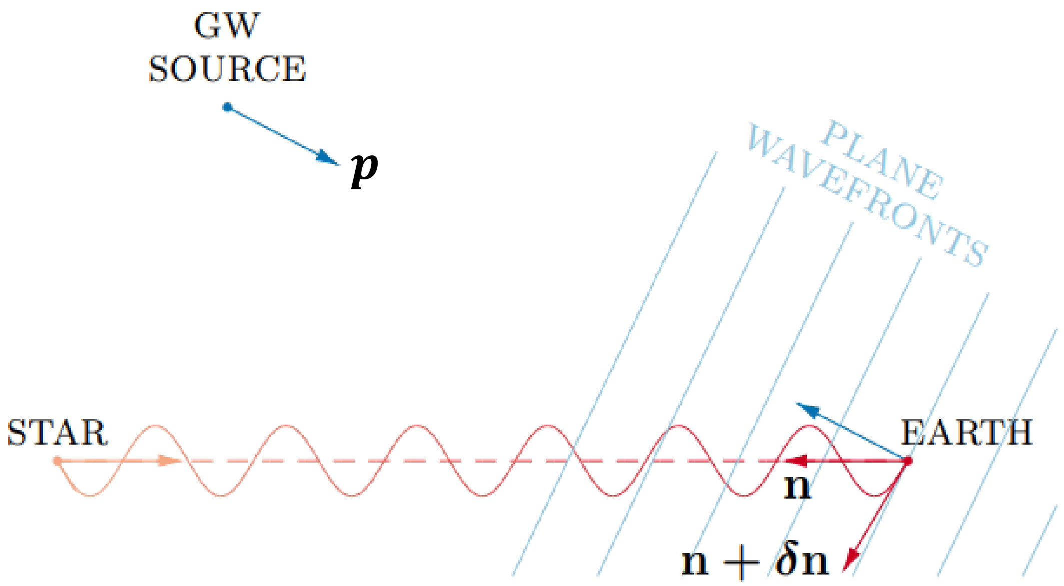
Plane wave TT-gauge

$$k^\mu = k^t(1, p^i) \qquad k^\mu k_\mu = 0 \implies \delta_{ij} p^i p^j = 1$$

Propagation direction

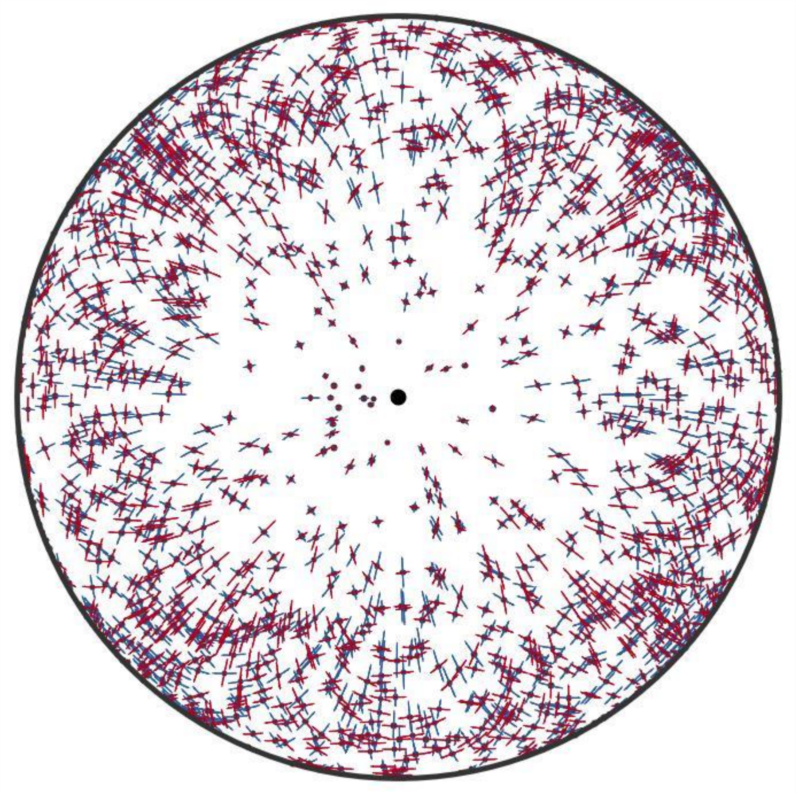
Tetrad basis $e_{(0)} = \partial_t + \mathcal{O}(h^2)\,, \quad e_{(i)} = \partial_i - \frac{1}{2} h^k_i \partial_k + \mathcal{O}(h^2)$

Photon wave vector $\sigma = \nu e_{(0)} + \nu n^i e_{(i)}$



Schematic view of the astrometric effect
Mihaylov et al. 2018

The Astrometric deflection



Astrometric deflection pattern for plus (red) and cross (blue) polarizations

$$\chi_1 = a^i \partial_i \,, \quad \chi_2 = b^i \partial_i \,, \quad \chi_3 = \frac{\partial}{\partial t} + p^i \partial_i . \qquad \vec{a}, \vec{b} \perp \vec{p}$$

Killing vector fields

$$-\sigma \cdot \chi_3 = \nu(e_{(0)} + n^i e_{(i)}) \cdot (e_{(0)} + p) = \nu - \nu n^i (p \cdot e_{(i)}) = \nu(1 - n \cdot p) = \text{const}$$

$$\sigma \cdot \chi_1 = \nu(e_{(0)} + n^i e_{(i)}) \cdot a = \nu n^i (a \cdot e_{(i)}) = \nu \left(n \cdot a + \frac{1}{2} h_{ij} n^i a^j \right) = \text{const}$$

$$\sigma \cdot \chi_2 = \nu(e_{(0)} + n^i e_{(i)}) \cdot b = \nu n^i (b \cdot e_{(i)}) = \nu \left(n \cdot b + \frac{1}{2} h_{ij} n^i b^j \right) = \text{const}$$

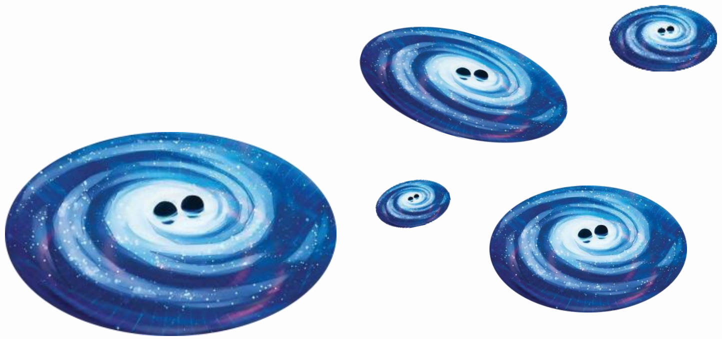
Astrometric shift

$$\delta n^i = \frac{n_i + p_i}{2(1 + n \cdot p)} h_{jk} n^j n^k - \frac{1}{2} h^i_j n^j + \mathcal{O}(h^2)$$

Book and Flanagan 2012

Stochastic GW Background

Astrophysical origin

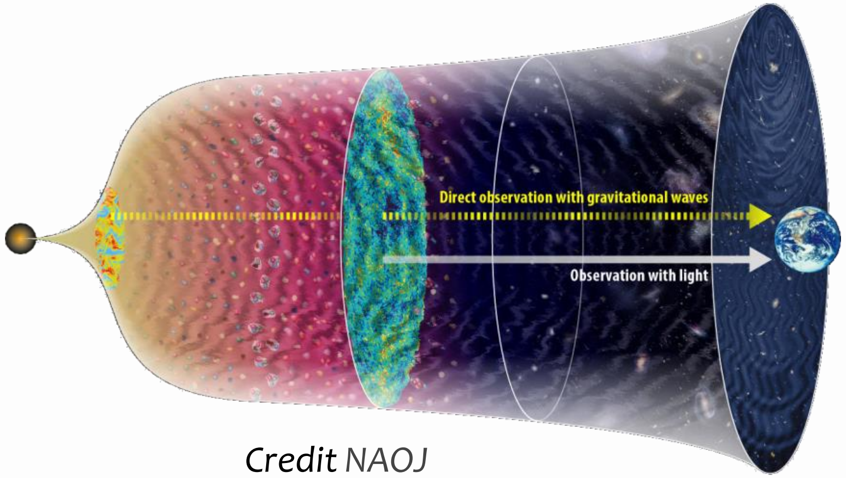


Credit NANOGrav

Incoherent superposition of many sources

- Coalescing binaries
- Supernovae
- Fast rotating/newly born NSs

Cosmological origin



Credit NAOJ

Predicted by many scenarios

- Inflation
- Phase transitions
- Cosmic strings

Faint evidence of a Stochastic Gravitational Wave Background from NANOgrav!

The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background – Agazie et al. 2023

Stochastic GW Background

Incoherent superposition of many waves
$$h(t,x) = \sum_{A=+,\times} \int df \int d\Omega_{\hat{p}} \tilde{h}_A(f,\hat{p}) e^{2i\pi f(t-\hat{p}x)}$$

$\tilde{h}_A(f,\hat{p})$ are stochastic variables

Spectral density of the SGWB

If the background is **stationary, isotropic, unpolarized**
$$\left\langle \tilde{h}_A(f,\hat{p})\tilde{h}_{A'}(f',\hat{p}') \right\rangle = \delta(f-f')\delta_{AA'}\frac{\delta^2(\hat{p},\hat{p}')}{4\pi}\frac{S_h(f)}{2}$$

Ensamble average

Noise pectral density

Because $s(t) = h(t) + n(t)$ and $\langle \tilde{n}(f) n(f') \rangle = \delta(f-f')\frac{S_n(f)}{2}$

The signal is seen as an additional noise!

Response to a SGWB

Similar problems were faced in the discovery of the CMB

«A measurement of excess antenna temperature at 4080 MHz», Penzias and Wilson 1965

Michele maggiore, Gravitational waves vol I

Cross-correlations $\langle s_1(t) s_2(t) \rangle$

Instead of matching the output to a given signal, one matches the otputs of different detectors

Matched Filtering for Deterministic Signals	Cross-Correlation for SGWB
Correlate noisy data with the template to extract the signal	Cross-correlate the data from two detectors to extract the common signal
Noise spectral density $S_n(f)$ of the detector suppresses noisy frequencies	Noise spectral densities $S_{n1}(f), S_{n2}(f)$ of the two detectors weight the correlation
Signal-to-noise ratio (SNR) quantifies detection	SNR from cross-correlation quantifies detection of SGWB

A quick look at Pulsar Timing Array

$$\sigma = \nu e_{(0)} + \nu n^i e_{(i)}$$

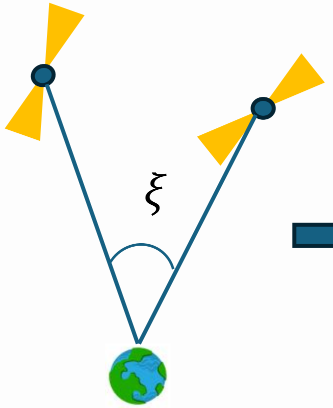
$$\frac{\delta\nu}{\nu} = \frac{1}{2(1+n\cdot p)} h_{ij} n^i n^j$$

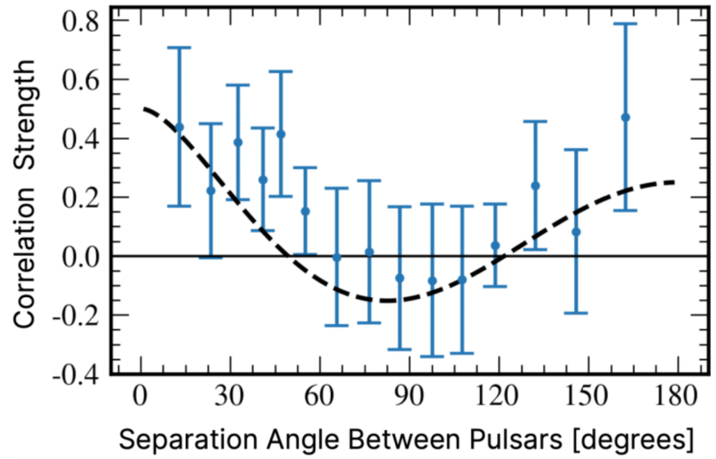
$$\delta n^i = \frac{p^i + n^i}{2(1+n\cdot p)} h_{ij} n^i n^j - \frac{1}{2} h^i_j n^j$$

$$\frac{\delta T}{T} = \sum_{A=+,\times} \int df \int d\Omega_{\hat{p}} \tilde{h}_A(f,\hat{p}) F_A(\hat{p},\hat{n}) e^{2i\pi f t}$$

$$F_A(\hat{p},\hat{n}) = \frac{e^A_{ij}(\hat{p}) n^i n^j}{2(1+n\cdot p)}$$

Analogous of antenna pattern functions





NANOGrav 2023

$$\left\langle \frac{\delta T_a}{T_a} \frac{\delta T_b}{T_b} \right\rangle = \sum_{A,A'=+,\times} \int df df' \int d^2\hat{p} d^2\hat{p}' \left\langle \tilde{h}_A(f,\hat{p}) \tilde{h}_{A'}(f',\hat{p}') \right\rangle F_A(\hat{p},n_a) F'_A(\hat{p}',n_b) e^{-2\pi i(f-f')t} = \int df \frac{S_h(f)}{2} \left[\int \frac{d\Omega_{\hat{p}}}{4\pi} \sum_{A=+\times} F_A(\hat{p},n_a) F_A(\hat{p},n_b) \right]$$

Hellings-Downs $\mathcal{H}(\xi)$

Astrometric correlations

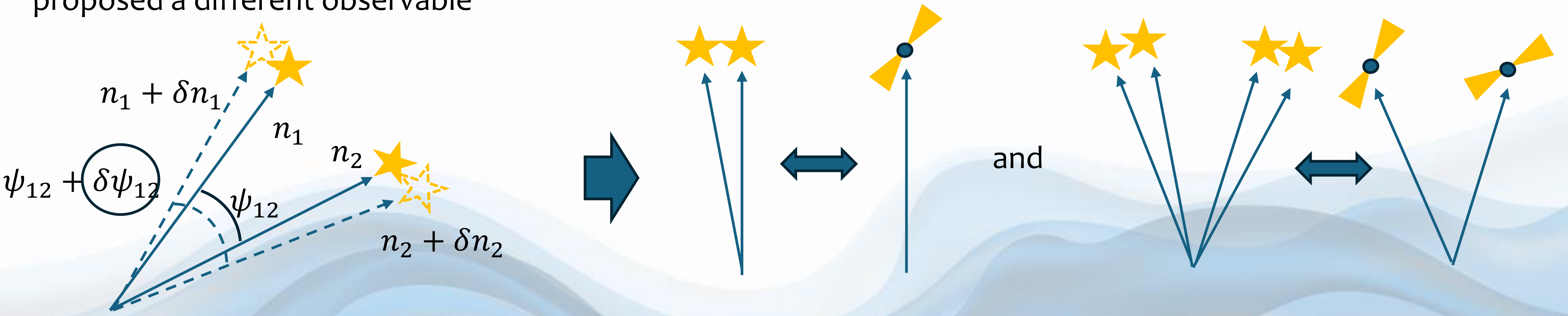
In the astrometric case the cross correlation is

Vectors

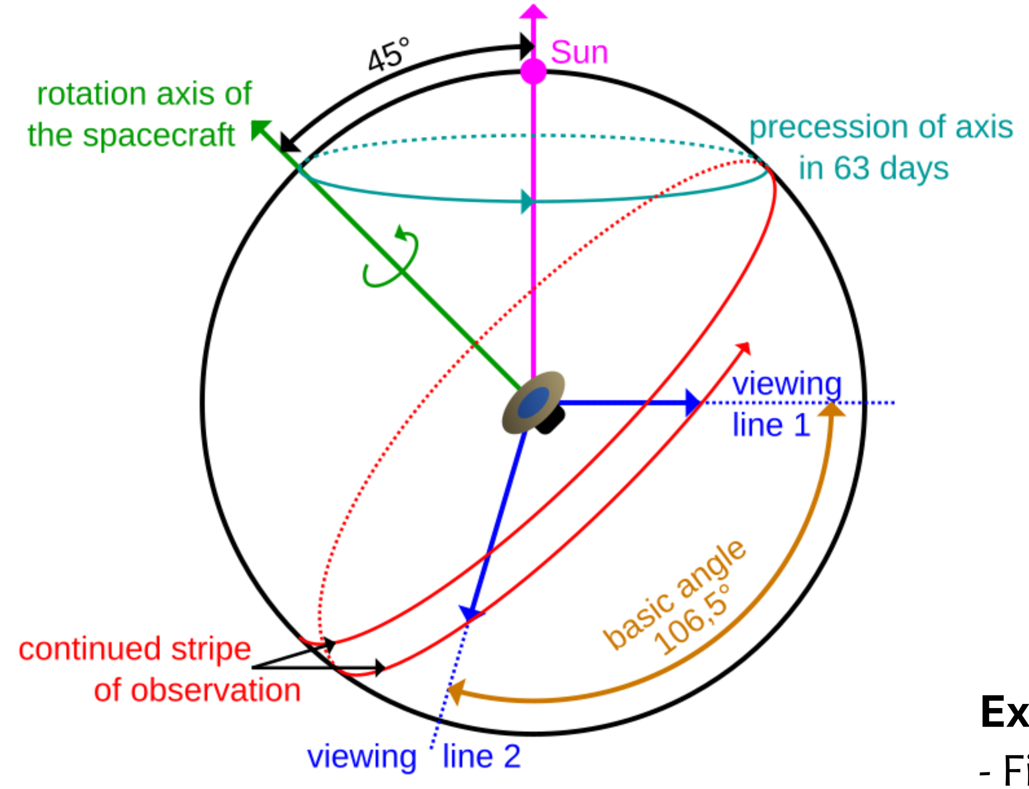
$$\langle \delta n_a^i \delta n_b^j \rangle \propto \Gamma^{ij}(n_a, n_b) \int df \frac{S_h(f)}{2}$$

Analogous of the Hellings-Downs

However, recently *Crosta et al. (Pinpointing gravitational waves via astrometric gravitational wave antennas, SciRep 2024)* proposed a different observable



Gaia mission

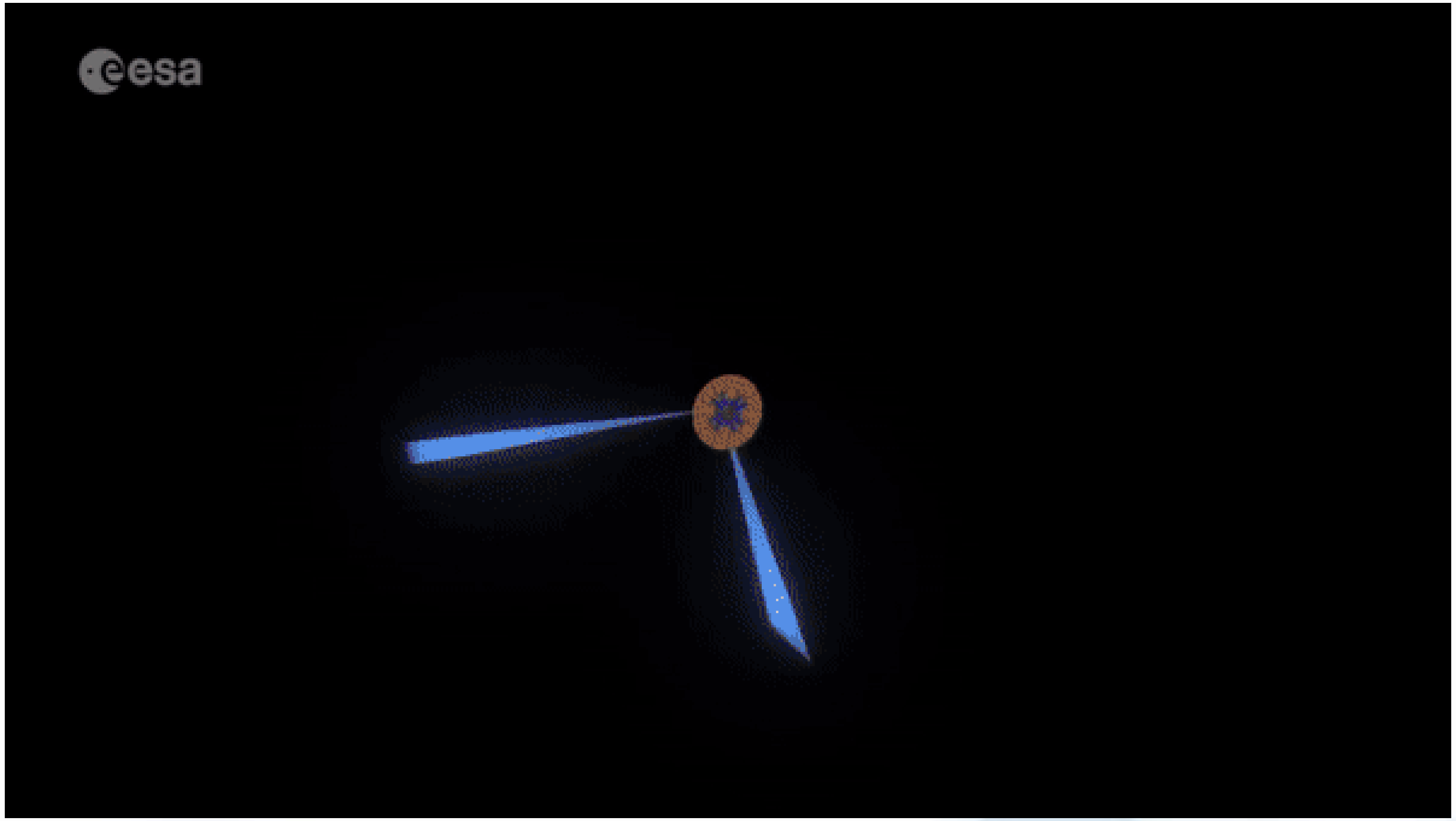


Launched by ESA in 2013, Gaia aims to create the most detailed **3D map of the Milky Way**.

Expected to operate until 2025:

- Field of view $\sim 1deg^2$
- Monitored each target around 70 times
- Measuring positions of about 1 billion stars

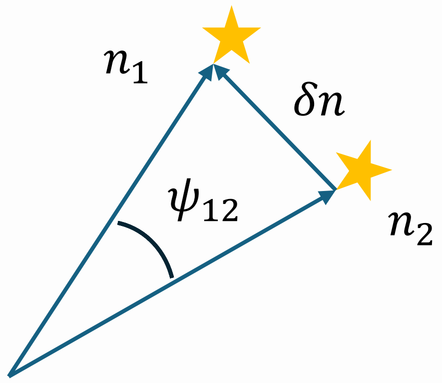
Gaia scanning method



Credit: B. Holl (University of Geneva, Switzerland), A. Moitinho & M. Barros (CENTRA – University of Lisbon), on behalf of DPAC, CC BY-SA IGO 3.0, CC BY-SA 3.0 igo,

From absolute to relative angles

$$\cos\psi_{12} = n_1 \cdot n_2$$



$n_1 \sim n_2 \sim n$

$\psi_{12} \ll 1$

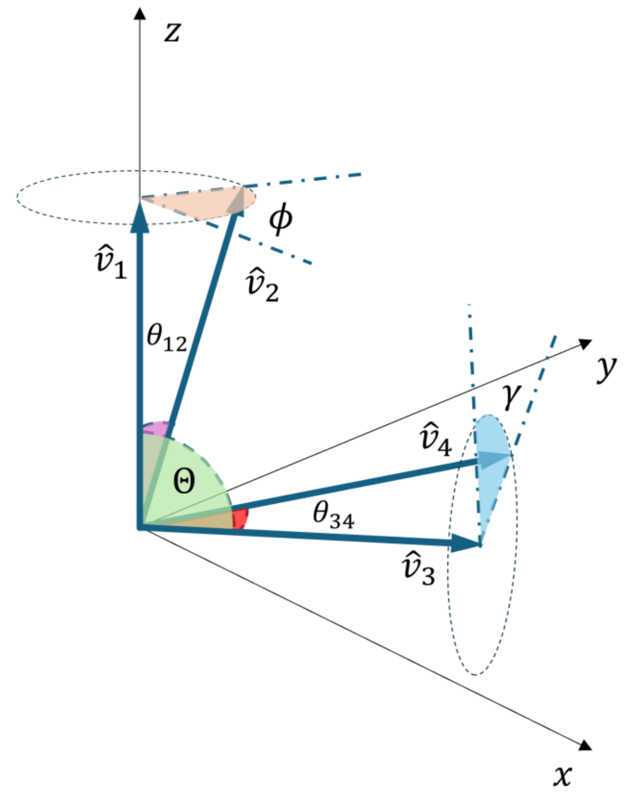
To order $\mathcal{O}(h)$: $\delta\cos\psi_{12} = n_0^1 \cdot \delta n^2 + \delta n^1 \cdot n_0^2$

$$= \frac{n_0^1 \cdot n_0^2 + p \cdot n_0^2}{2(1 + n_0^1 \cdot p)} h_{jk} n_0^{1j} n_0^{1k} + \frac{n_0^1 \cdot n_0^2 + p \cdot n_0^1}{2(1 + n_0^2 \cdot p)} h_{jk} n_0^{2j} n_0^{2k} - h_{ij} n_0^{1i} n_0^{2j}$$

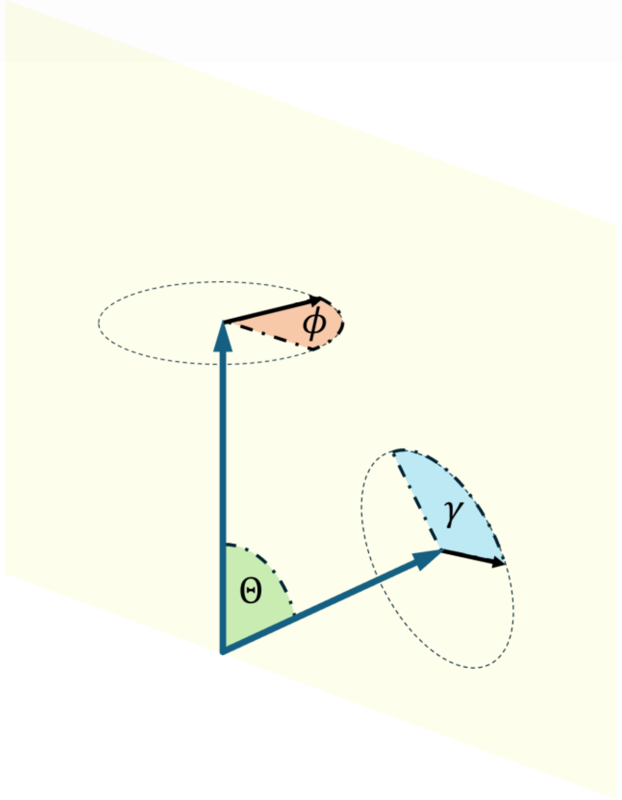
We can expand in small angles because Gaia's field of view $< 1\ \mathrm{deg}^2$

$$\delta\psi_{12}\psi_{12} \sim \psi_{12}^2 \frac{h_{ij}}{2}\left[\left(\delta\hat{n}^i - \frac{(\delta\hat{n}\cdot p)n^i}{(1+n\cdot p)}\right)\left(\delta\hat{n}^j - \frac{(\delta\hat{n}\cdot p)n^j}{(1+n\cdot p)}\right) - \frac{n^i n^j}{(1+n\cdot p)}\right] + \mathcal{O}(\psi_{12}^3)$$

From absolute to relative angles

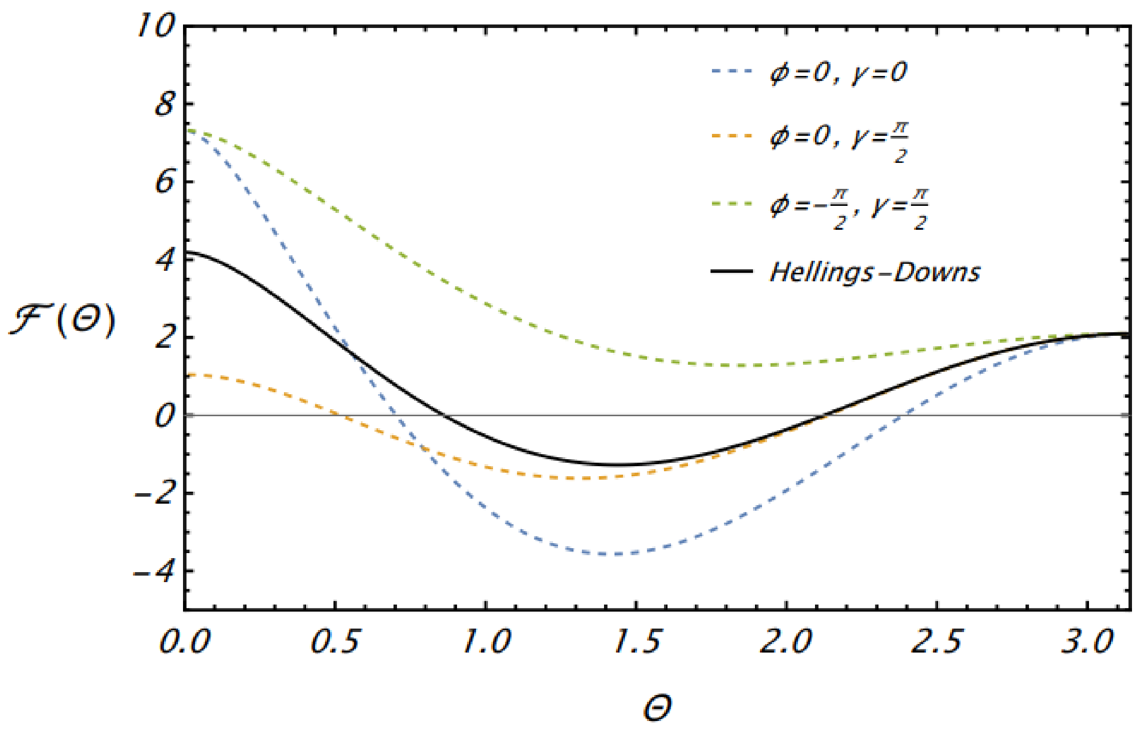


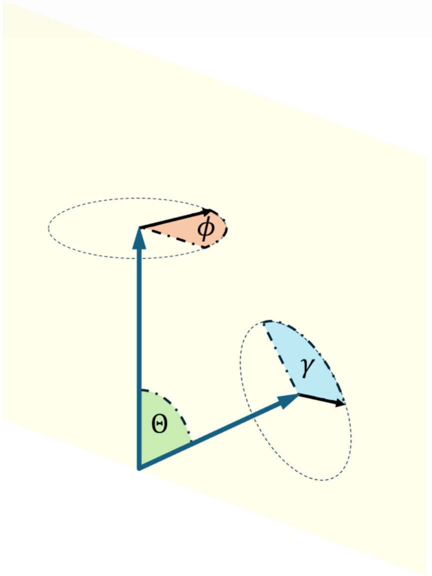
$\theta_{12}, \theta_{34} \to 0$



$$\left\langle \frac{\delta\psi_{12}}{\psi_{12}} \frac{\delta\psi_{34}}{\psi_{34}} \right\rangle \propto HDA(\Theta, \gamma, \phi)$$

Hellings-Donws analogue geometric factor





$$\mathcal{HDA}(\Theta,\Phi,\gamma) = \frac{\pi}{12(1+\cos\Theta)^2}\Bigg[22 + 31\cos\Theta + 10\cos(2\Theta) + \cos(3\Theta) - 15\cos(2(\gamma-\Theta+\Phi)) - 15\cos(2(\gamma+\Theta+\Phi)) + 12\cos(\Theta - 2(\gamma+\Phi))$$
$$+ 12\cos(\Theta+2(\gamma+\Phi)) + 24\log\sin\left(\frac{\Theta}{2}\right) + 24\cos(\gamma-\Phi)\cos(\gamma+\Phi)\sin^2\Theta + 6\Bigg(\cos(2(\gamma+\Phi))\left(9 + 4(11 - 12\cos\Theta)\log\sin\left(\frac{\Theta}{2}\right)\right)$$
$$+ 4\cos(\gamma-\Phi)\cos(\gamma+\Phi)\left(\cos\Theta + 4\log\sin\left(\frac{\Theta}{2}\right)\right)\sin^2\Theta + 2\log\sin\left(\frac{\Theta}{2}\right)\left(\cos\Theta - \cos(3\Theta) - 4\cos(2\Theta)\sin^2(\gamma+\Phi)\right)\Bigg)\Bigg]$$

Signal-to-noise ratio

The signal to noise ration can be estimated as

$$\mathrm{SNR}^2 \simeq 1.5\,\psi^2 \frac{N^2}{\sigma^4 \Delta t^2} \frac{T}{144\pi^4} \frac{h_{\mathrm{ref}}^4}{f_{\mathrm{ref}}^{4\gamma}} \frac{T^{1-4\gamma}}{1-4\gamma}$$

Proportional to ψ^2!

For a background with $$h_c = h_{\mathrm{ref}} \left(\frac{f}{f_{\mathrm{ref}}}\right)^{\gamma}$$

$h_{\mathrm{ref}} \to 3 \times 10^{-15}$

$\gamma \to -2/3$

$f_{\mathrm{ref}} \to 3 \times 10^{-8} Hz$

And assuming $N \sim 10^9, T = 15\ yr, \Delta t = 30\ days, \sigma = 20\mu as$

$$SNR \sim \psi \times 30$$

Conclusions

- Astrometric searches for a SGWB might be complementary to PTA observations in the nano-Hz band

- The two-point correlation functions in the «differential» setup depends on two additional angles that modulate the Hellings-Downs

- Taking relative angles as fundamental observables would reduce the uncertainty in the Gaia satellite orientation but at the price of having a smaller effect