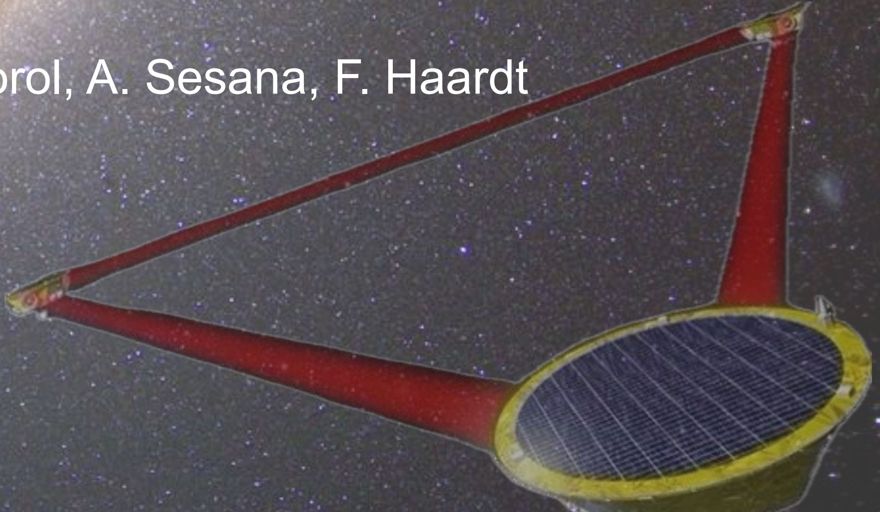
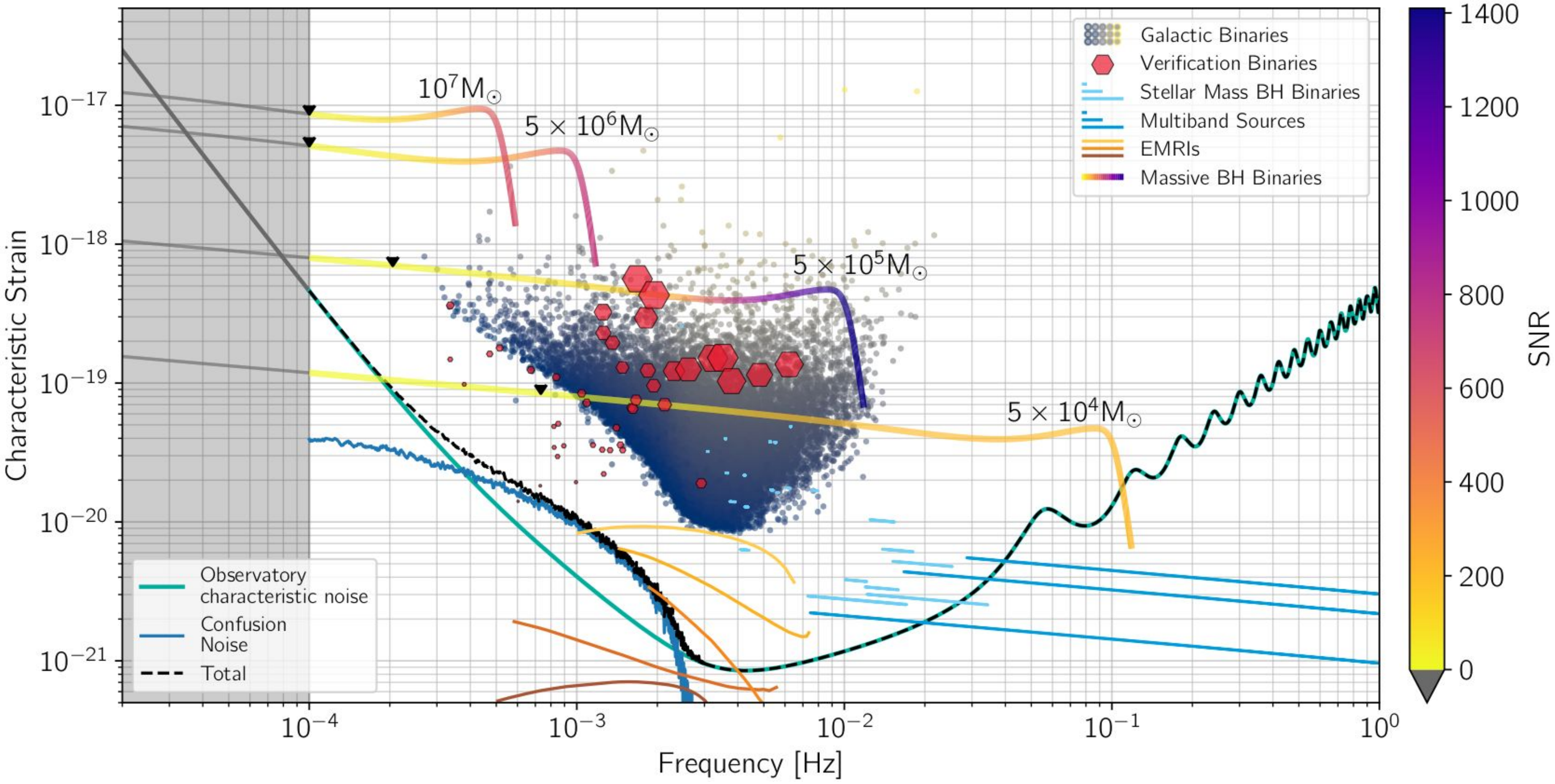
CYCLOSTATIONARY PROCESSES IN LISA

Pozzoli, Buscicchio, et al (2410.08274)
Pozzoli, Buscicchio, Klein (in prep)

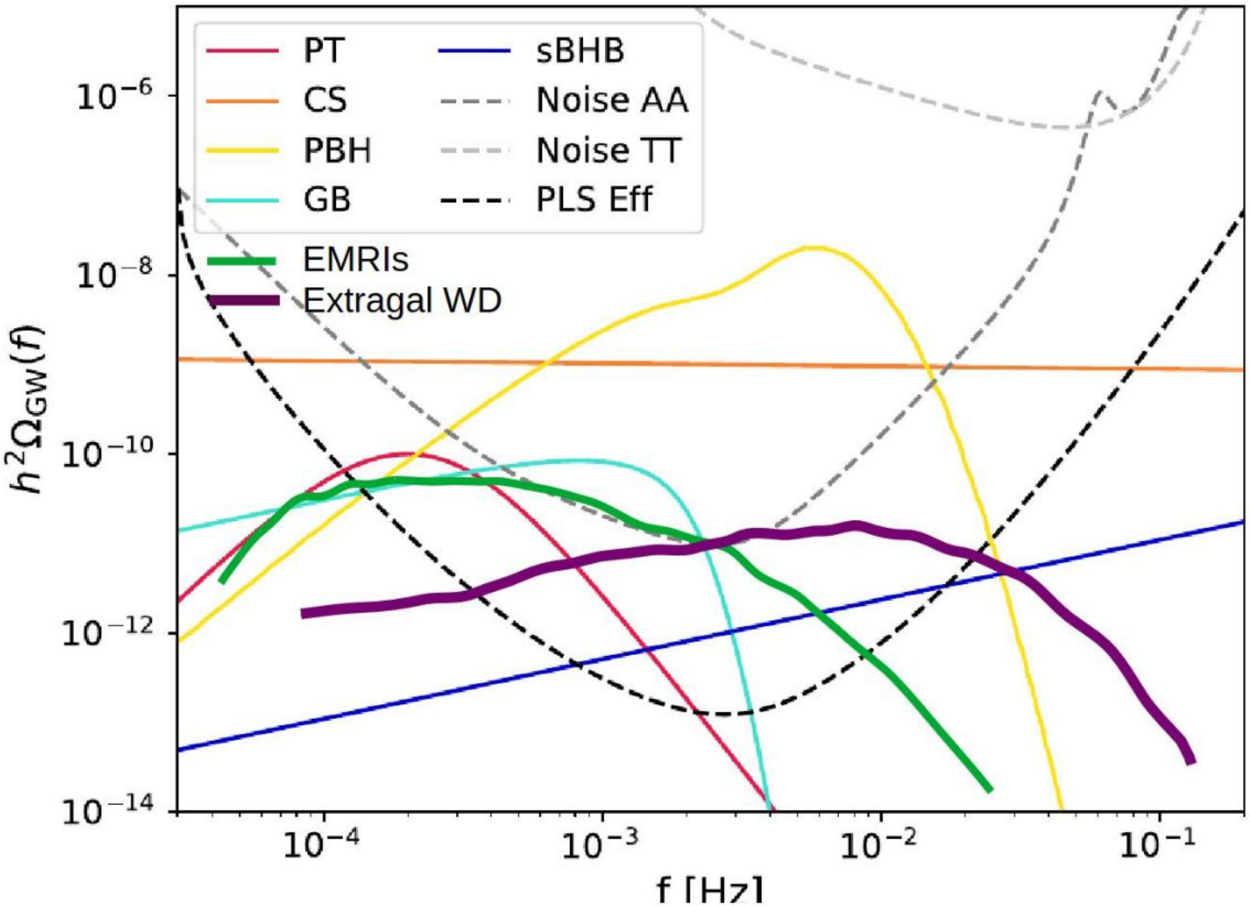
Speaker: Federico Pozzoli
Co-Authors: R. Buscicchio, A. Klein, V. Korol, A. Sesana, F. Haardt

GraSP24, 24/10/24

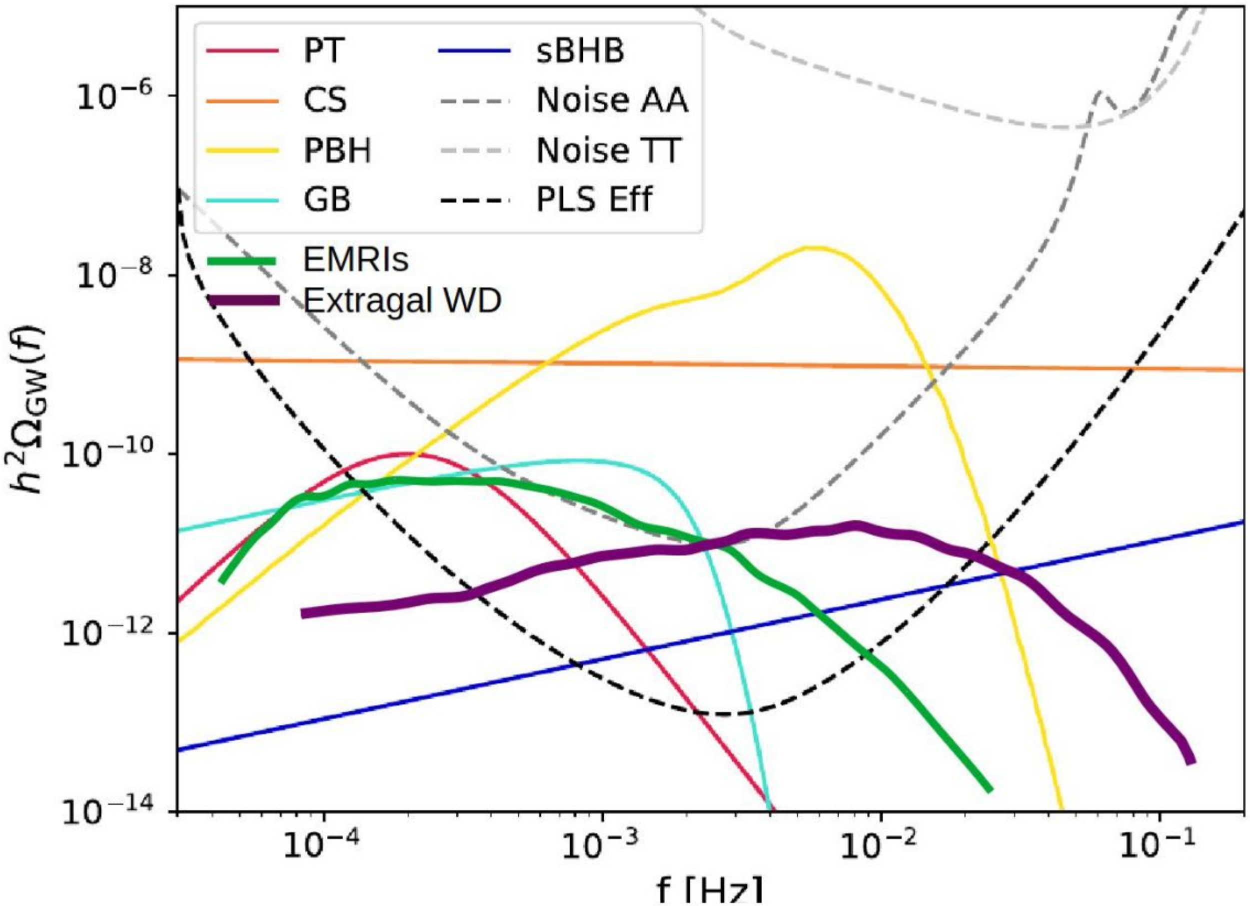




Galactic Binaries
Verification Binaries
Stellar Mass BH Binaries
Multiband Sources
EMRIs
Massive BH Binaries
$10^7 M_\odot$
$5 \times 10^6 M_\odot$
$5 \times 10^5 M_\odot$
$5 \times 10^4 M_\odot$
Observatory characteristic noise
Confusion Noise
Total
Characteristic Strain
Frequency [Hz]
SNR



PT
CS
PBH
GB
sBHB
Noise AA
Noise TT
PLS Eff
EMRIs
Extragal WD
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
$h^2\Omega_{GW}(f)$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
f [Hz]



Cosmo:
Caprini+24
Auclair+19
Bartolo+19

Astro:
Nelemans 09
Babak+23
Pozzoli+23
Hofman+24

SEARCHING BACKGROUND IN LISA -CHALLENGES

NOISE SIGNAL

$$\Sigma(f, f') = \Sigma_n(f, f') + \Sigma_{\mathrm{GW}}(f, f')$$

SEARCHING BACKGROUND IN LISA -CHALLENGES

$$\Sigma(f, f') = \boxed{\Sigma_n(f, f')} + \Sigma_{\rm GW}(f, f')$$

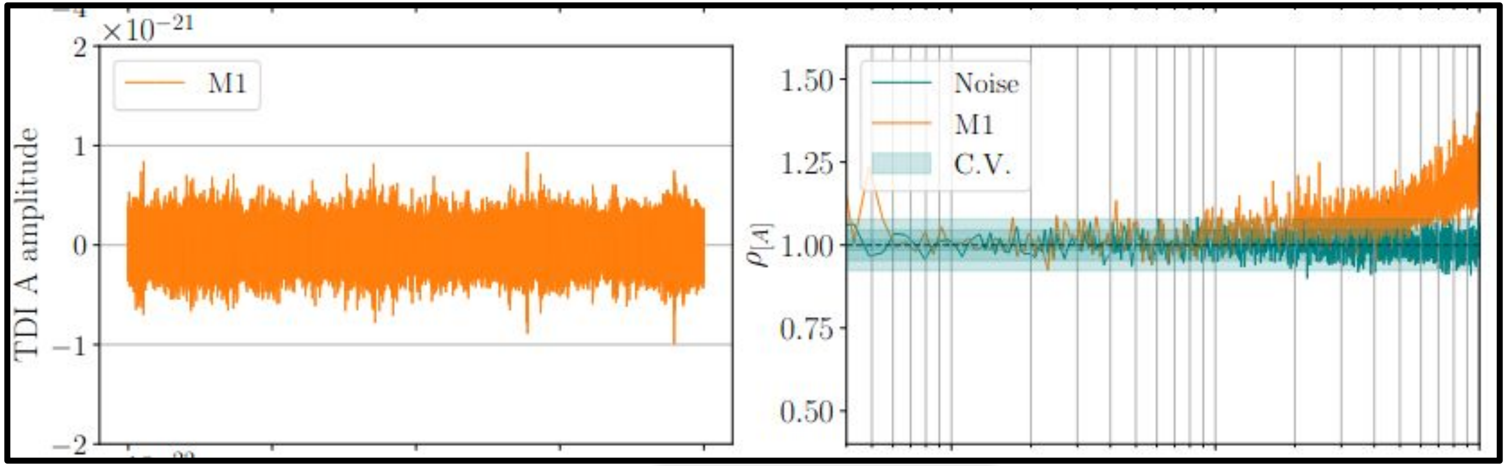
- Non-Stationarity (glitches, …) (Alvey+24)
- Noise Uncertainties (Muratore+23)
- Correlation between datastreams (Hartwig+23)
- …

SEARCHING BACKGROUND IN LISA -CHALLENGES

$$\Sigma(f, f') = \Sigma_n(f, f') + \Sigma_{\rm GW}(f, f')$$

- Non-stationarity, Anisotropy, Non-Gaussianity
- Overlapping signals
- Uncertainties in the Models (both Astro&Cosmo)
  —> Model Flexibility

SEARCHING BACKGROUND IN LISA -CHALLENGES



- Non-stationarity, Anisotropy, Non-Gaussianity

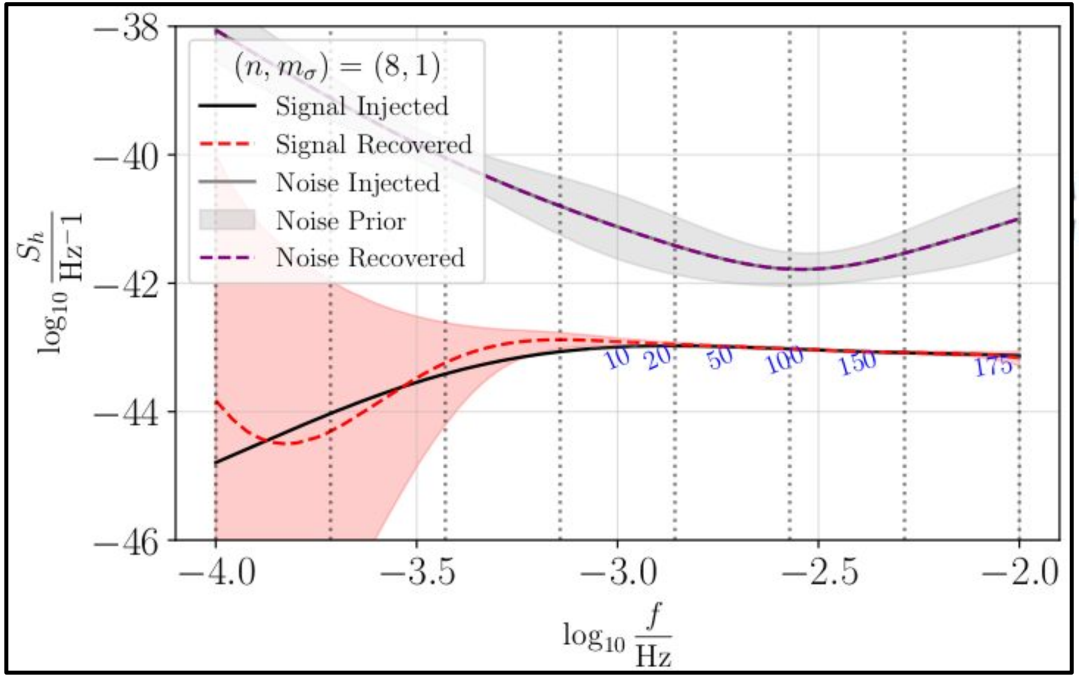
*Piarulli, Buscicchio, **Pozzoli**+24
Non-Gaussianity for EMRI SGWB
*Buscicchio+24
Non-Gaussianity for Galactic foreground

SEARCHING BACKGROUND IN LISA -CHALLENGES

$$\Sigma(f,f') = \Sigma_n(f,f') + \Sigma_{\rm GW}(f,f')$$

- Non-stationarity, Anisotropy, Non-Gaussianity
- Overlapping signals
- Uncertainties in the Models (both Astro&Cosmo)
  —> Model Flexibility

SEARCHING BACKGROUND IN LISA -CHALLENGES



Pozzoli+24: a flexible parametrization based on Gaussian Process Theory

- Uncertainties in the Models (both Astro&Cosmo) —> Model Flexibility

SEARCHING BACKGROUND IN LISA -CHALLENGES

$$\Sigma(f,f') = \Sigma_n(f,f') + \Sigma_{\rm GW}(f,f')$$

TODAY

- Non-stationarity, Anisotropy, Non-Gaussianity
- Overlapping signals
- Uncertainties in the Models (both Astro&Cosmo)
  —> Model Flexibility

CYCLOSTATIONARY PROCESSES

Cyclostationary processes are stochastic processes whose statistical properties are periodic in time

$$E\left[X(t)\right] = m(t) = m(t+T)$$
$$E\left[X(t')X(t)\right] = \Sigma(t',t) = \Sigma(t'+T, t+T)$$

CYCLOSTATIONARY PROCESSES

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$$B(t,\tau) = \Sigma(t',t)$$
$$B(t,\tau) = \sum_{n=-\infty}^{+\infty} B_n(\tau) e^{2\pi i \frac{nt}{T}}$$

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$\longrightarrow$

$$C(f,f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f'+f}{2}\right)\delta\left(f-f'+\frac{n}{T}\right)$$

CYCLOSTATIONARY PROCESSES

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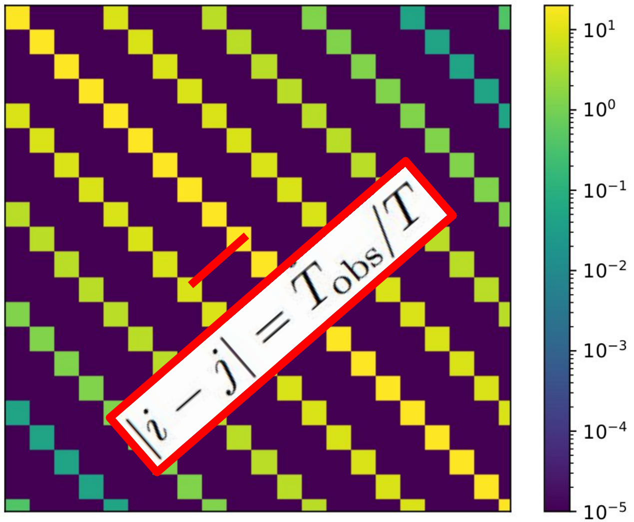

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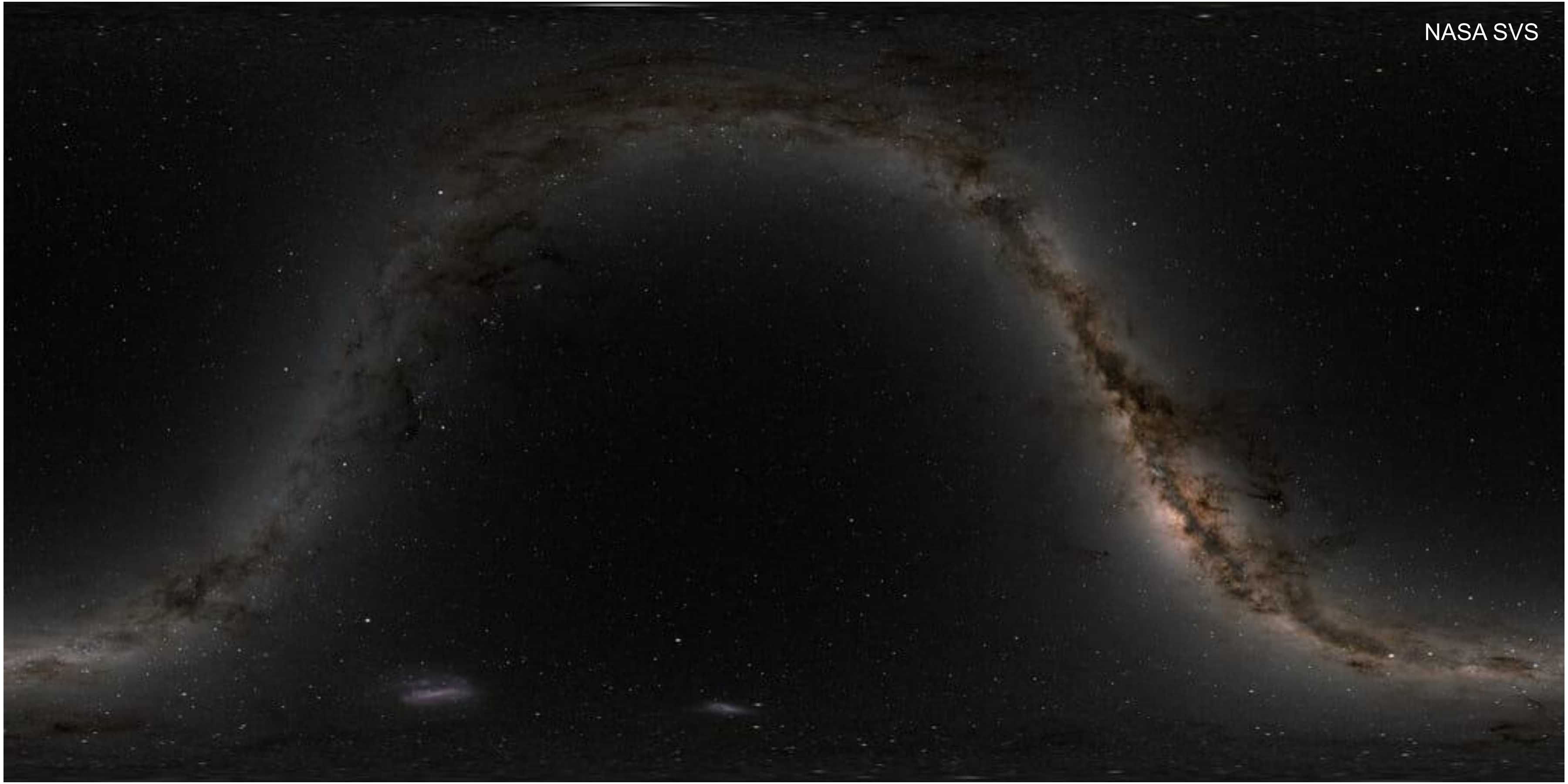
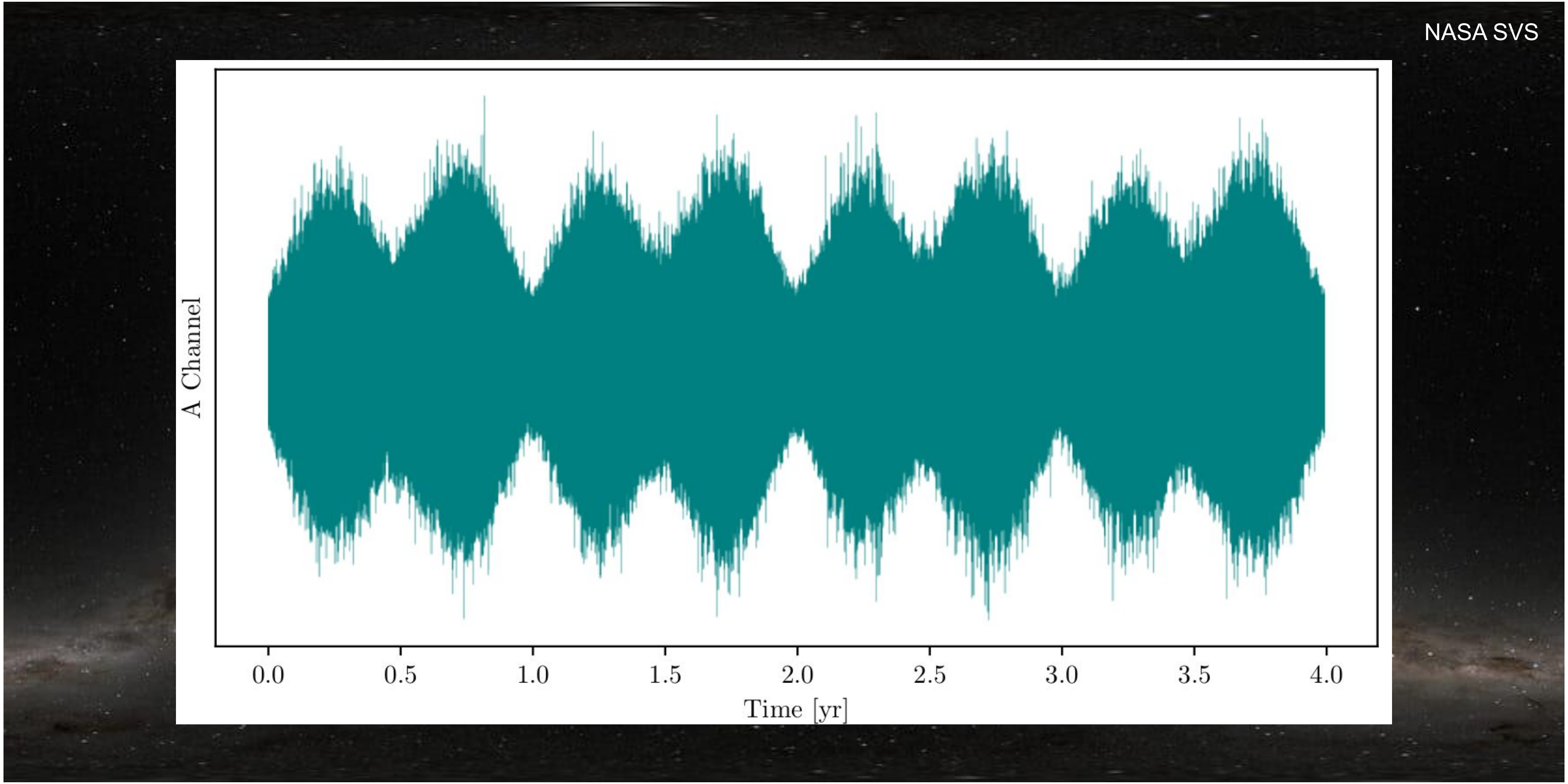
CYCLOSTATIONARITY IN LISA



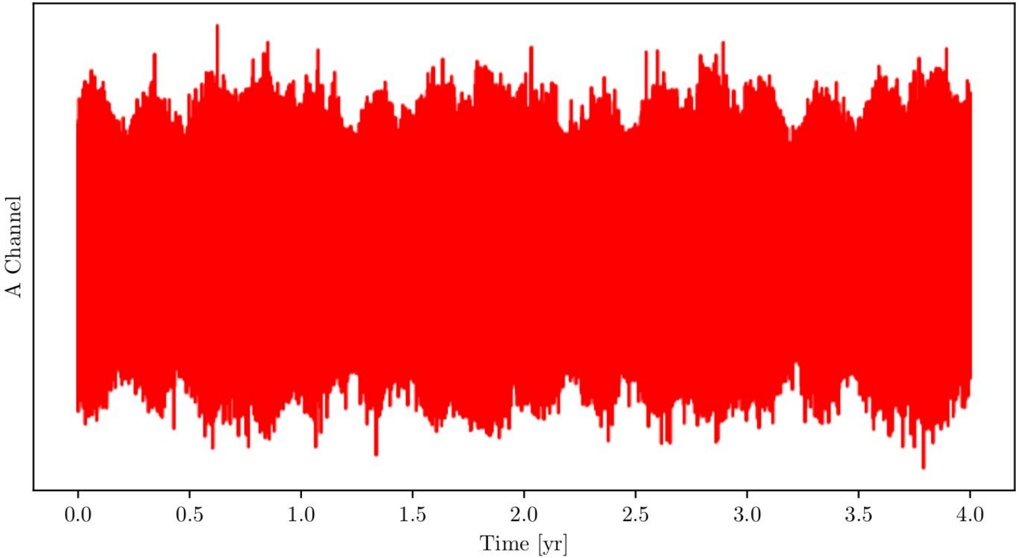
CYCLOSTATIONARITY IN LISA



CYCLOSTATIONARITY IN LISA

Unresolved DWDs in Milky Way Satellite (e.g., LMC, SMC, Sagittarius,...) and in nearby Galaxies (e.g., Andromeda) contribute to a SGWB

LMC



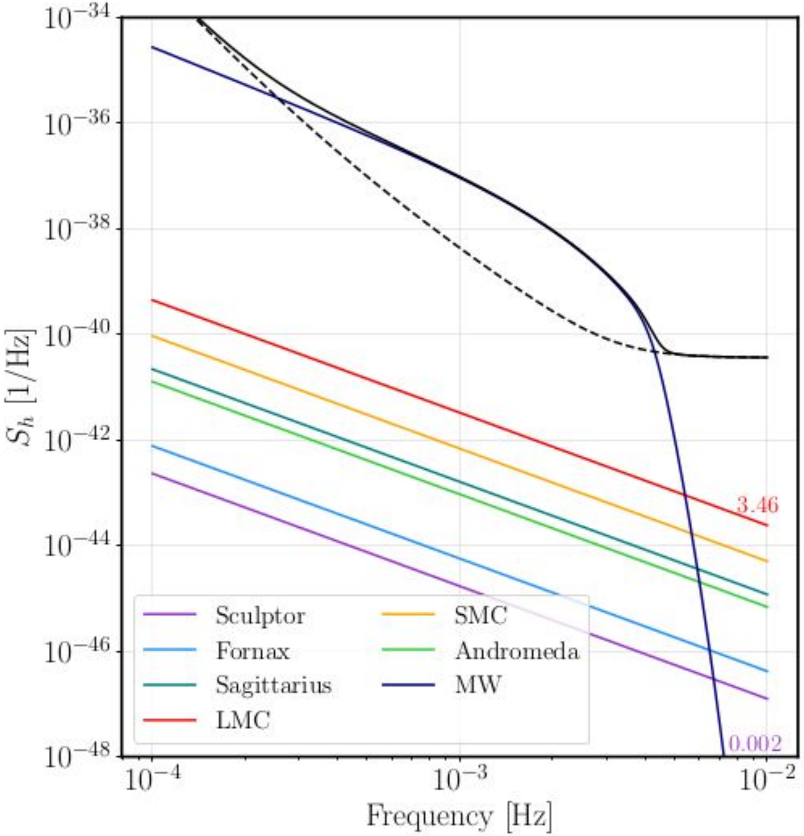
$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h \left(\frac{f' + f}{2} \right) \delta \left(f - f' + \frac{n}{T} \right)$$

ASTROPHYSICAL SPECTRUM

$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f' + f}{2}\right) \delta\left(f - f' + \frac{n}{T}\right)$$

Fourier coefficient of
MODULATION

ASTROPHYSICAL SPECTRUM



$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f'+f}{2}\right)\delta\left(f - f' + \frac{n}{T}\right)$$

Milky Way Foreground

(Karnesis+21)

$$S_h(f) = \frac{A}{2} f^{-7/3} e^{-(f/f_1)^{\alpha_{\mathrm{MW}}}}\left(1+\tanh\left(\frac{f_{\mathrm{knee}}-f}{f_2}\right)\right)$$

Satellite Background

$$S_h(f) = A_{\mathrm{sat}}\left(\frac{f}{10^{-3.5}\mathrm{Hz}}\right)^{\gamma}$$

$$\gamma = -(9+3\alpha)/3$$

ASTROPHYSICAL SPECTRUM

Korol+22

Amplitude of GW Inspiral

$$S_h(f) = \int d\mathcal{M}_c p(\mathcal{M}_c) \int df_s p(f_s) \delta(f - f_s) \frac{(G\mathcal{M}_c)^{10/3}}{(c^4 D)^2} (\pi f_s)^{4/3}$$

Satellite Background

$$S_h(f) = A_{\text{sat}} \left(\frac{f}{10^{-3.5}\text{Hz}} \right)^{\gamma}$$

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ASTROPHYSICAL SPECTRUM

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DWDs in a satellite have all the same distance

Satellite Background

$$S_h(f) = A_{\rm sat} \left(\frac{f}{10^{-3.5} {\rm Hz}} \right)^{\gamma}$$

$$\gamma = -(9 + 3\alpha)/3$$

ASTROPHYSICAL SPECTRUM

Korol+22

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Due to Fourier Transform of cos
In Inspiral waveform

Satellite Background

$$S_h(f) = A_{\rm sat} \left(\frac{f}{10^{-3.5}\rm Hz} \right)^{\gamma}$$

$$\gamma = -(9 + 3\alpha)/3$$

ASTROPHYSICAL SPECTRUM

Korol+22

Amplitude of GW Inspiral

$$S_h(f) = \int d\mathcal{M}_c p(\mathcal{M}_c) \int df_s p(f_s) \delta(f - f_s) \frac{(G\mathcal{M}_c)^{10/3}}{(c^4 D)^2} (\pi f_s)^{4/3}$$

Maoz+18 Binary Separation distribution is a power law with slope α + 4

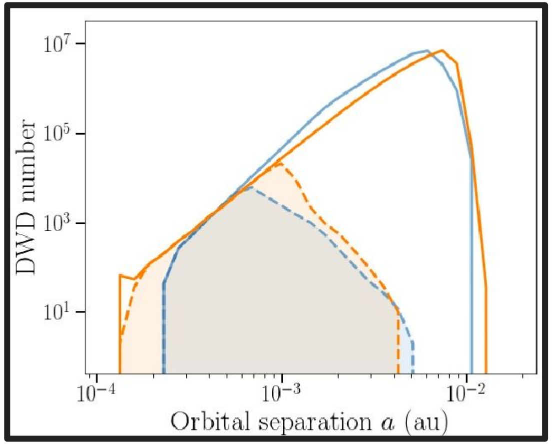
Satellite Background

$$S_h(f) = A_{\rm sat} \left(\frac{f}{10^{-3.5}\rm Hz} \right)^{\gamma}$$

$$\gamma = -(9 + 3\alpha)/3$$

$$\alpha \approx -1.3$$

Based on spectroscopic observation

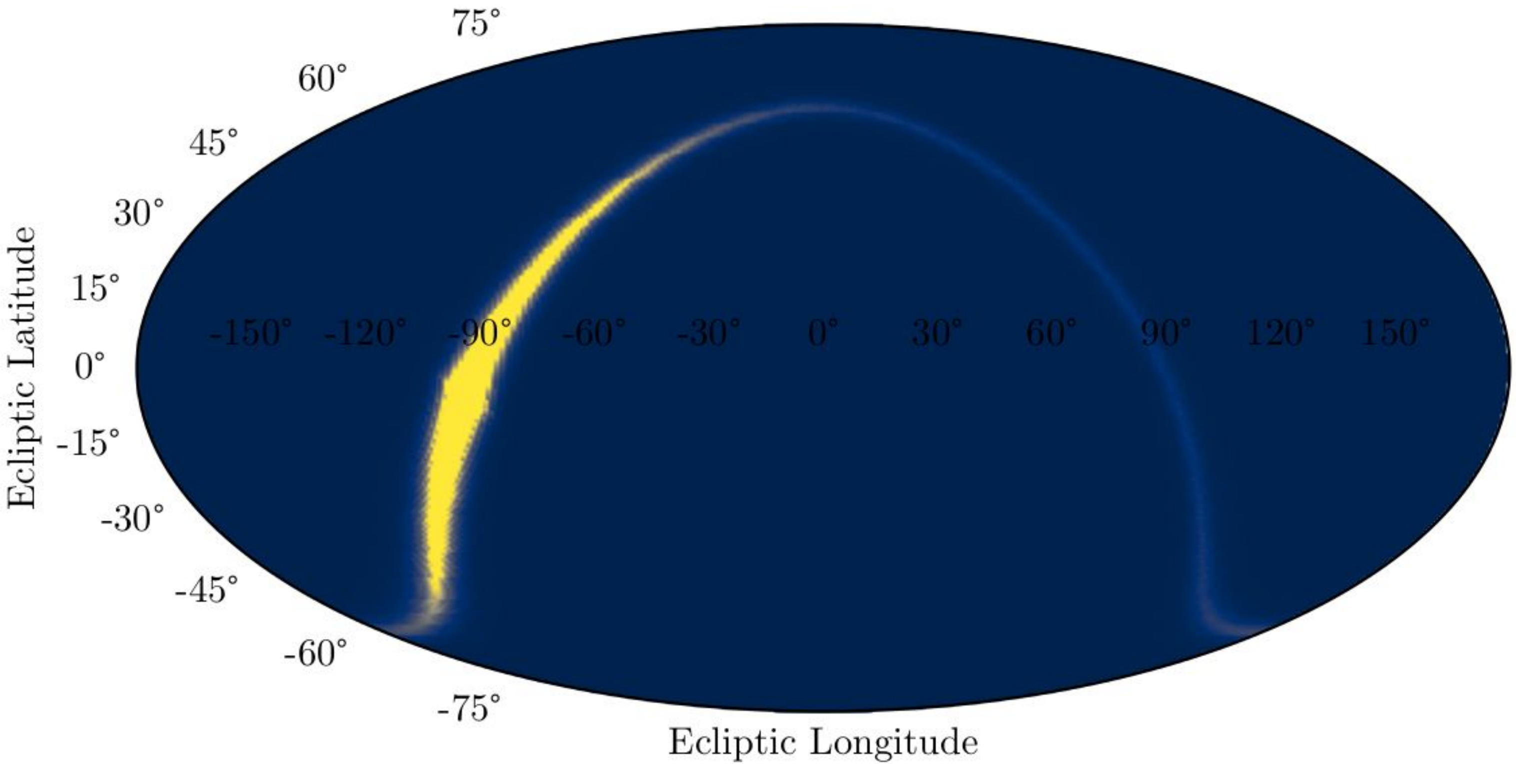


MODULATION

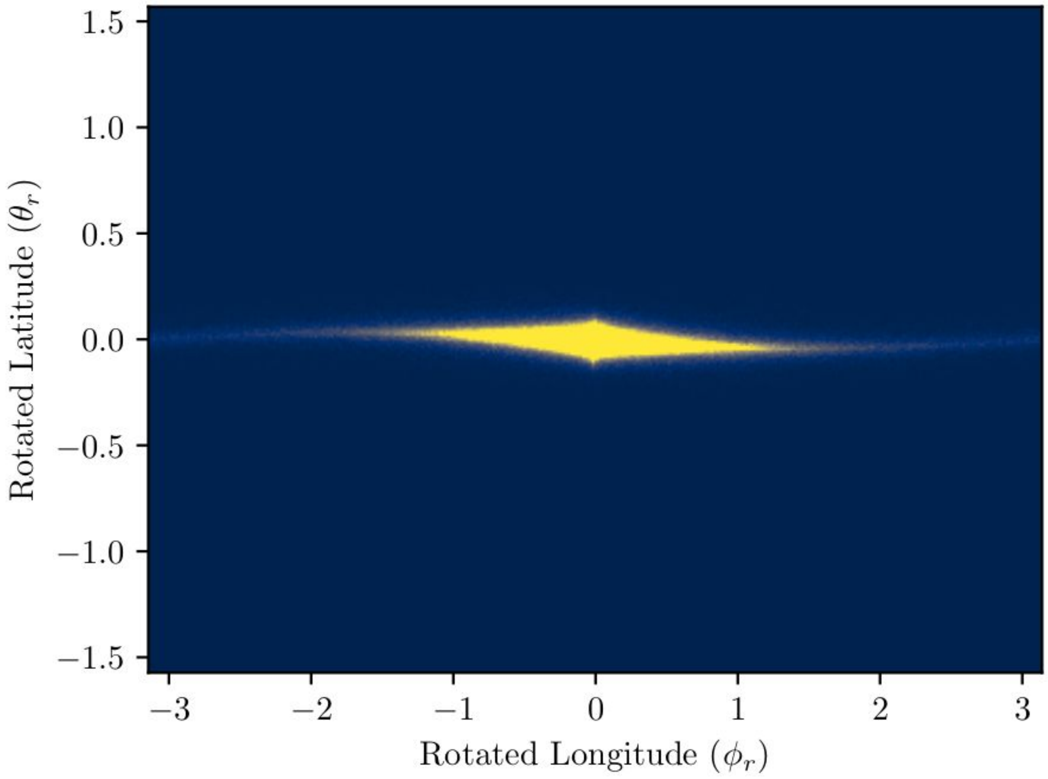
We have to average the time domain signal in LISA over the probability distribution of the sources in the sky

$$\int d\lambda \int d\beta \cos\beta p(\lambda, \beta) h^2(t, \lambda, \beta)$$

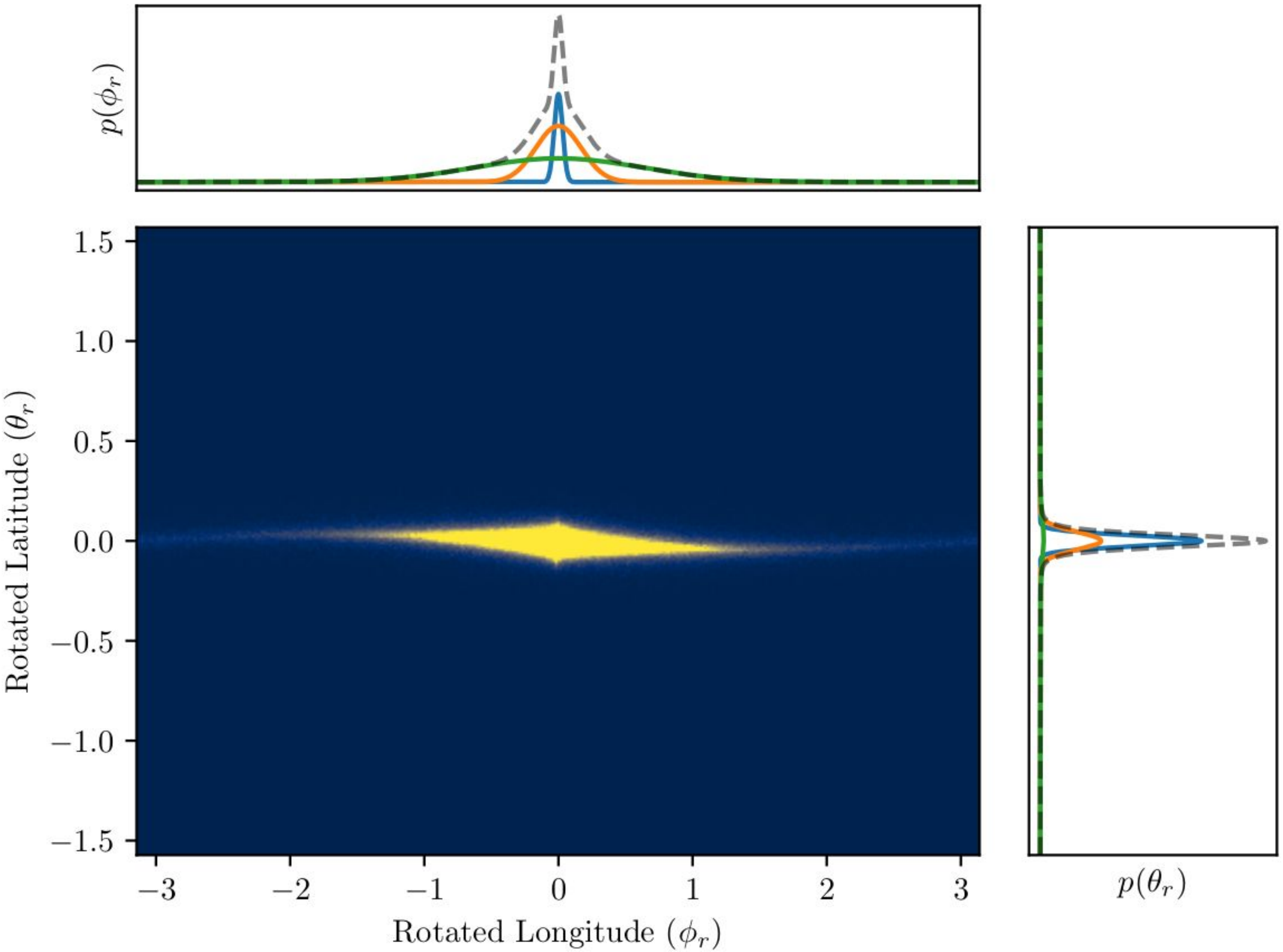
MODULATION


75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-150°
-120°
-90°
-60°
-30°
0°
30°
60°
90°
120°
150°
Ecliptic Latitude
Ecliptic Longitude

MODULATION



Rotated Latitude (θ_r)
Rotated Longitude (ϕ_r)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−3
−2
−1
0
1
2
3


$p(\phi_r)$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Rotated Latitude (θ_r)
−3
−2
−1
0
1
2
3
Rotated Longitude (ϕ_r)
$p(\theta_r)$

MODULATION

We have to average the time domain signal in LISA over the probability distribution of the sources in the sky

$$\int d\lambda \int d\beta \cos\beta p(\lambda,\beta) h^2(t,\lambda,\beta)$$

MODULATION

We have to average the time domain signal in LISA over the probability distribution of the sources in the sky

The problem reduces to resolve integral like

$$\int_{\mathcal{R}} d\theta_r \int_{\mathcal{R}} d\phi_r p\left(\theta_r\right) p\left(\phi_r\right) e^{im\theta_r} e^{in\phi_r}$$

MODULATION

We have to average the time domain signal over the probability distribution of the sources in the sky

$$\int_{\mathcal{R}} d\theta_r \int_{\mathcal{R}} d\phi_r p\left(\theta_r\right) p\left(\phi_r\right) e^{im\theta_r} e^{in\phi_r} = \varphi_{\theta_r}(m)\varphi_{\phi_r}(n)$$

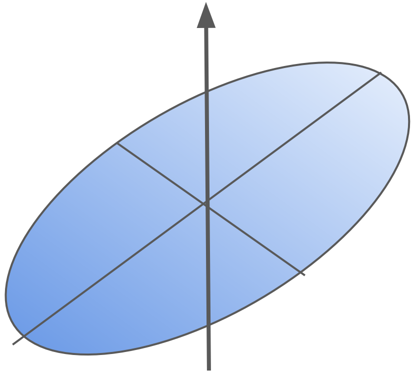
The solution is well-know for a large set of probability distribution, and it is called
CHARACTERISTIC FUNCTION

MODULATION

We relate the signal modulation to the properties of the distribution

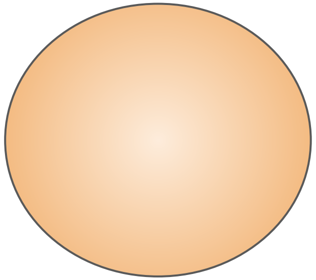
Milky Way Modulation Parameters:

- Center Coordinates of distribution
- Rotation Angle
- Gaussian Variances (Sizes of distribution)



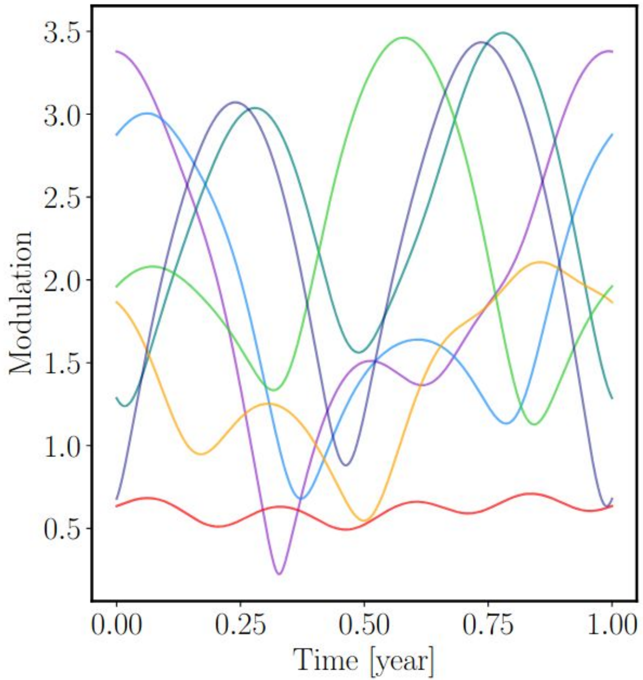
Satellite Modulation Parameters:

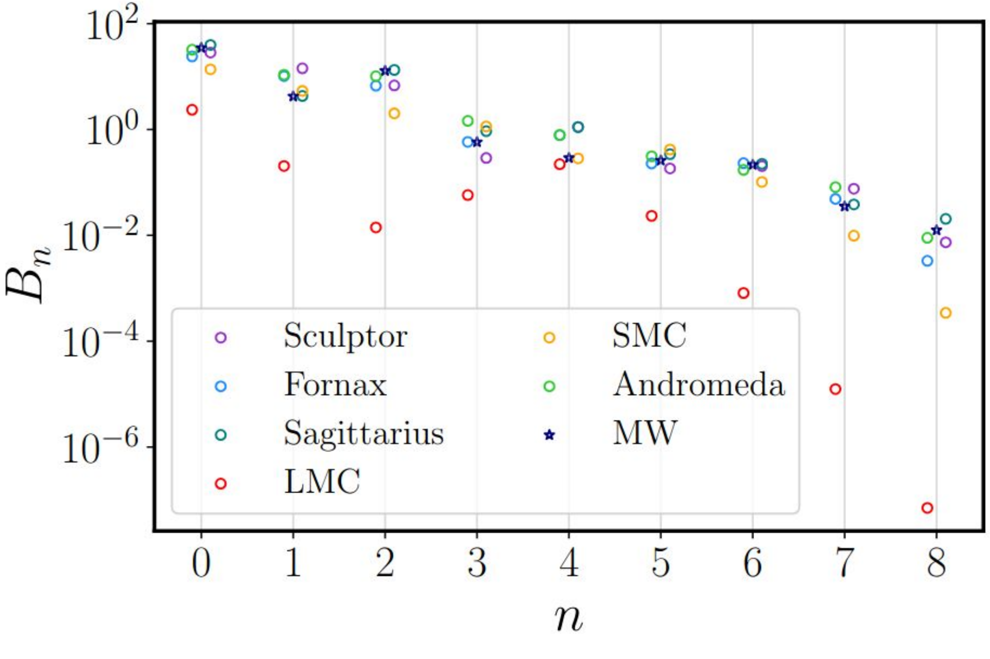
- Center Coordinates of distribution
- Gaussian Variance (Size of distribution)



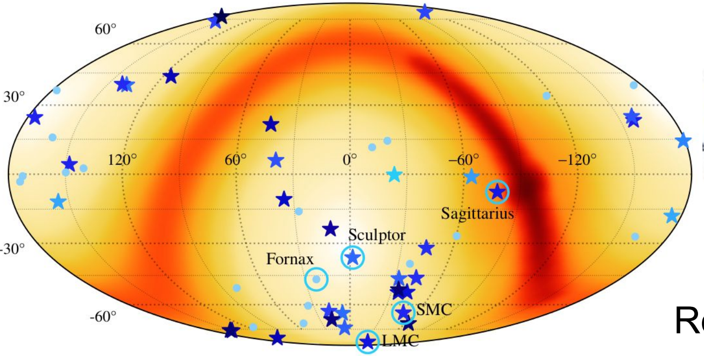
$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h \left(\frac{f' + f}{2} \right) \delta \left(f - f' + \frac{n}{T} \right)$$

Fourier Coefficient of Modulation

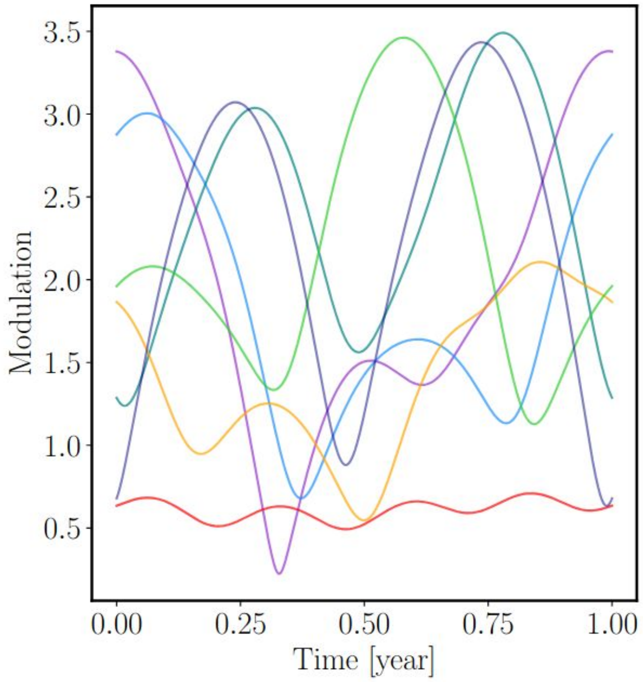


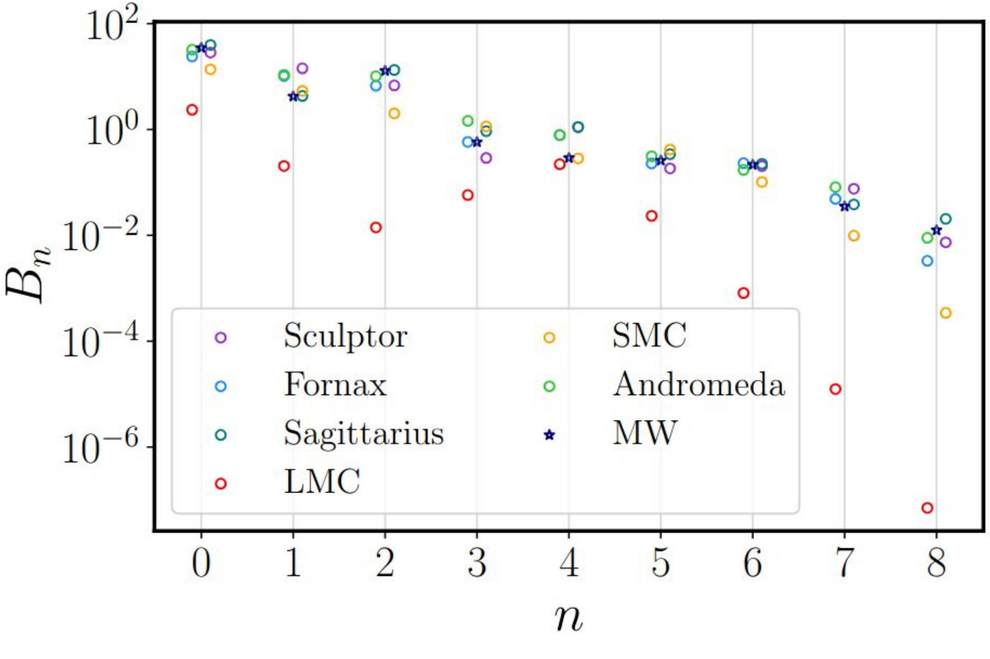


$C(f, f')$ … $f - f' + \frac{n}{T})$



Roebber+20





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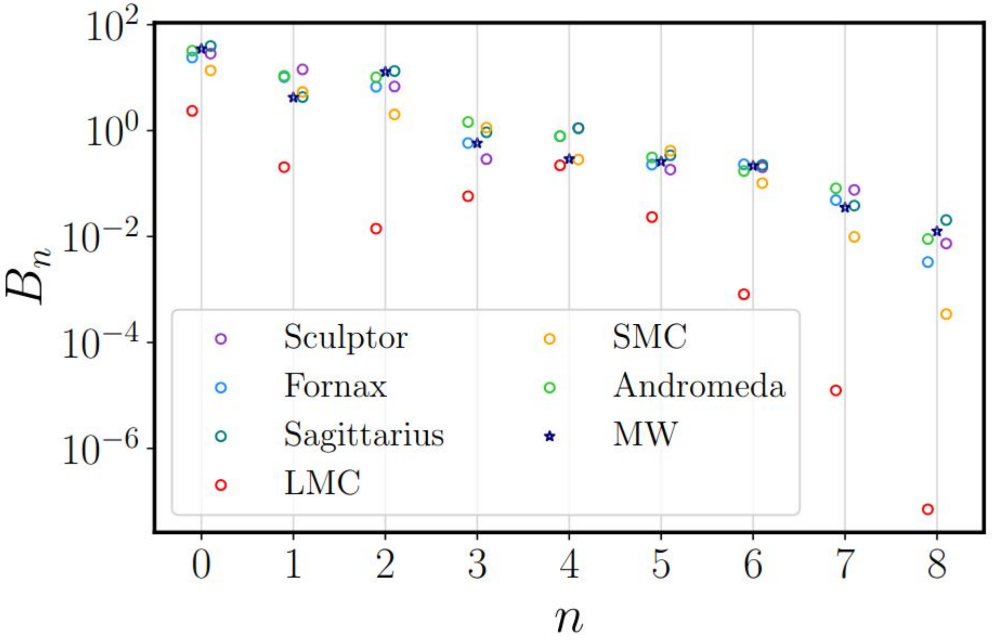
Fourier Coefficient of Modulation

The modulation is **primarily** influenced by **latitude**, while the impact of **size** is a **secondary effect.**



CYCLOSTATIONARY MODEL

Likelihood

$$\log \mathcal{L}(\tilde{d}|\boldsymbol{\theta} = \{\boldsymbol{\theta}_{\mathrm{MW}}, \boldsymbol{\theta}_{\mathrm{sat}}, \boldsymbol{\theta}_{\mathrm{n}}\}) \propto - \sum_{i=A,E} \frac{1}{2} \log(\det [\Sigma_{\mathrm{d}}]_{\mathrm{i}}) + \frac{1}{2} \tilde{\mathrm{d}}_{\mathrm{i}}^{\mathrm{T}} [\Sigma_{\mathrm{d}}]_{\mathrm{i}}^{-1} \tilde{\mathrm{d}}_{\mathrm{i}}$$

$$[\Sigma_{\mathrm{d}}]_i = (\Sigma_{\mathrm{MW}}(\boldsymbol{\theta}_{\mathrm{MW}}) + \Sigma_{\mathrm{sat}}(\boldsymbol{\theta}_{\mathrm{sat}}) + \Sigma_{\mathrm{n}}(\boldsymbol{\theta}_{\mathrm{n}}))_i$$

Parameter

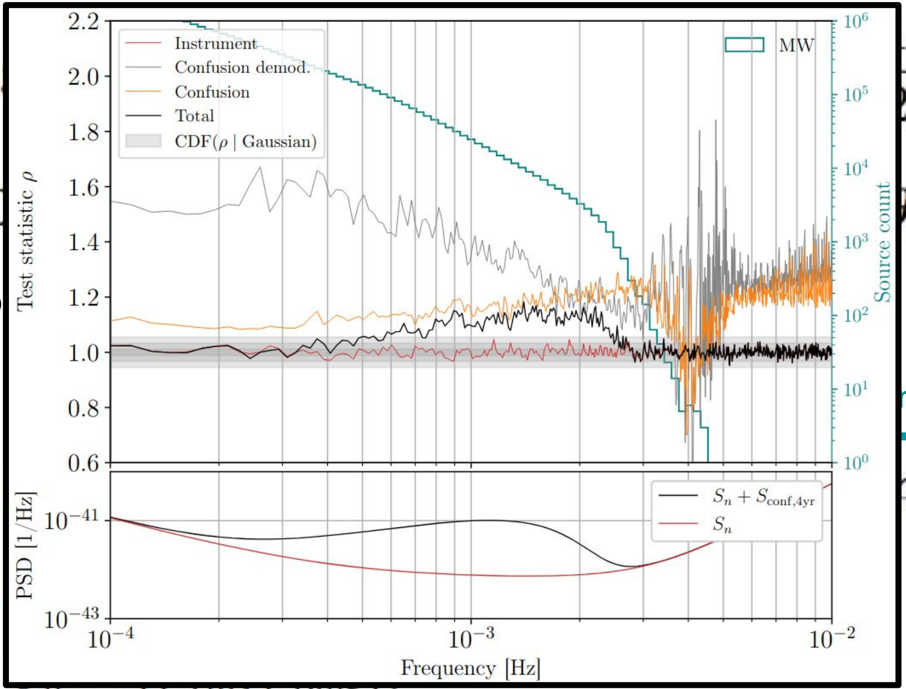
Spectrum — Modulation

- $\boldsymbol{\theta}_{\mathrm{MW}} = \{\mathcal{A}_{\mathrm{MW}}, \alpha, f_{\mathrm{knee}}, f_2, f_1, \lambda, \sin\beta, \sigma_1, \sigma_2, \psi\}$
- $\boldsymbol{\theta}_{\mathrm{sat}} = \{\mathcal{A}_{\mathrm{sat}}, \gamma, \lambda, \sin\beta, \sigma\}$;
- $\boldsymbol{\theta}_n = \{\mathcal{P}_{\mathrm{tm}}, \mathcal{P}_{\mathrm{oms}}\}$.

CYCLOSTATIONARY MODEL

Likelihood

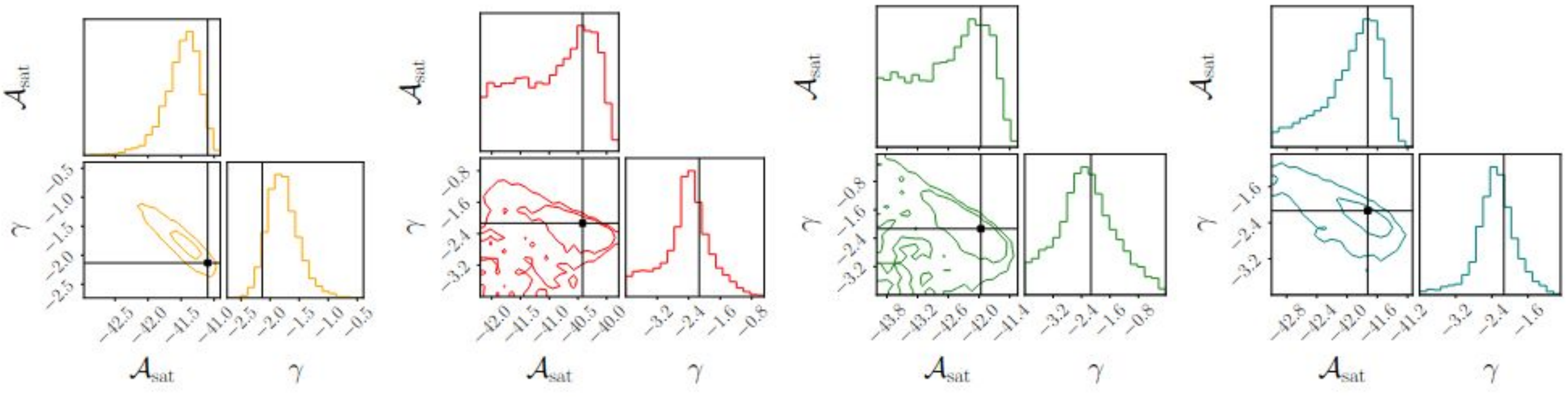
$$\ldots \log(\det[\Sigma_\mathrm{d}]_\mathrm{i}) + \frac{1}{2}\tilde{\mathrm{d}}_\mathrm{i}^\mathrm{T}\,[\Sigma_\mathrm{d}]_\mathrm{i}^{-1}\,\tilde{\mathrm{d}}_\mathrm{i}$$



Buscicchio+24

RESULTS - Satellite (Mock) + Noise

With our modulation parametrization, **we can place physically informed prior**.
We assume **perfect knowledge of the satellite's sky position**, as they are already well-determined through EM observations.



SMC LMC Andromeda Sagittarius

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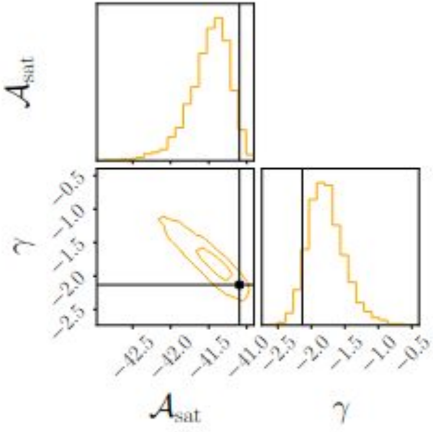
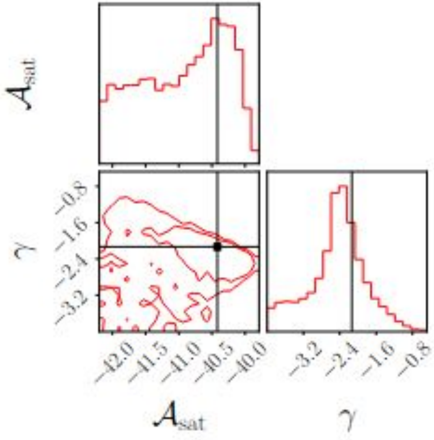
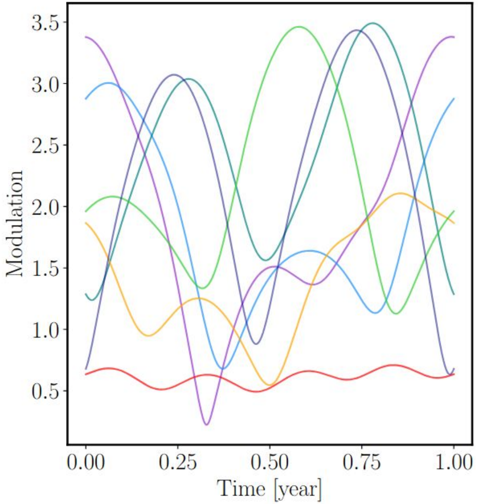
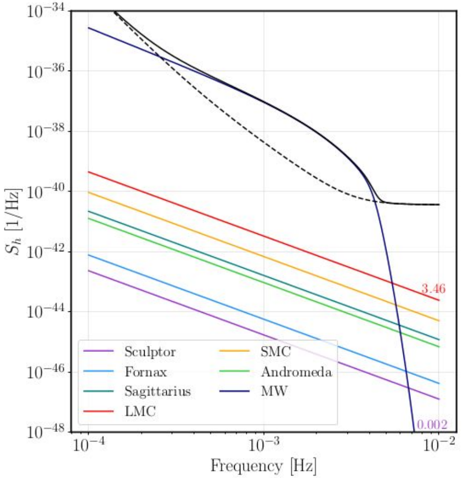


SMC

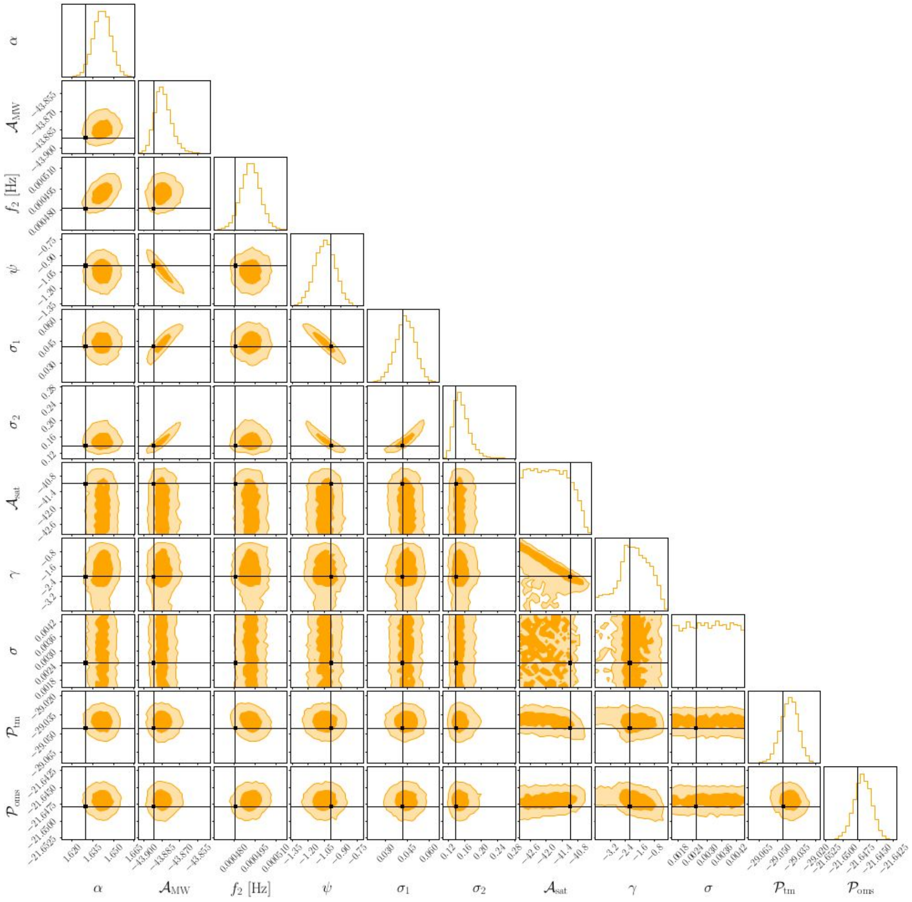


LMC





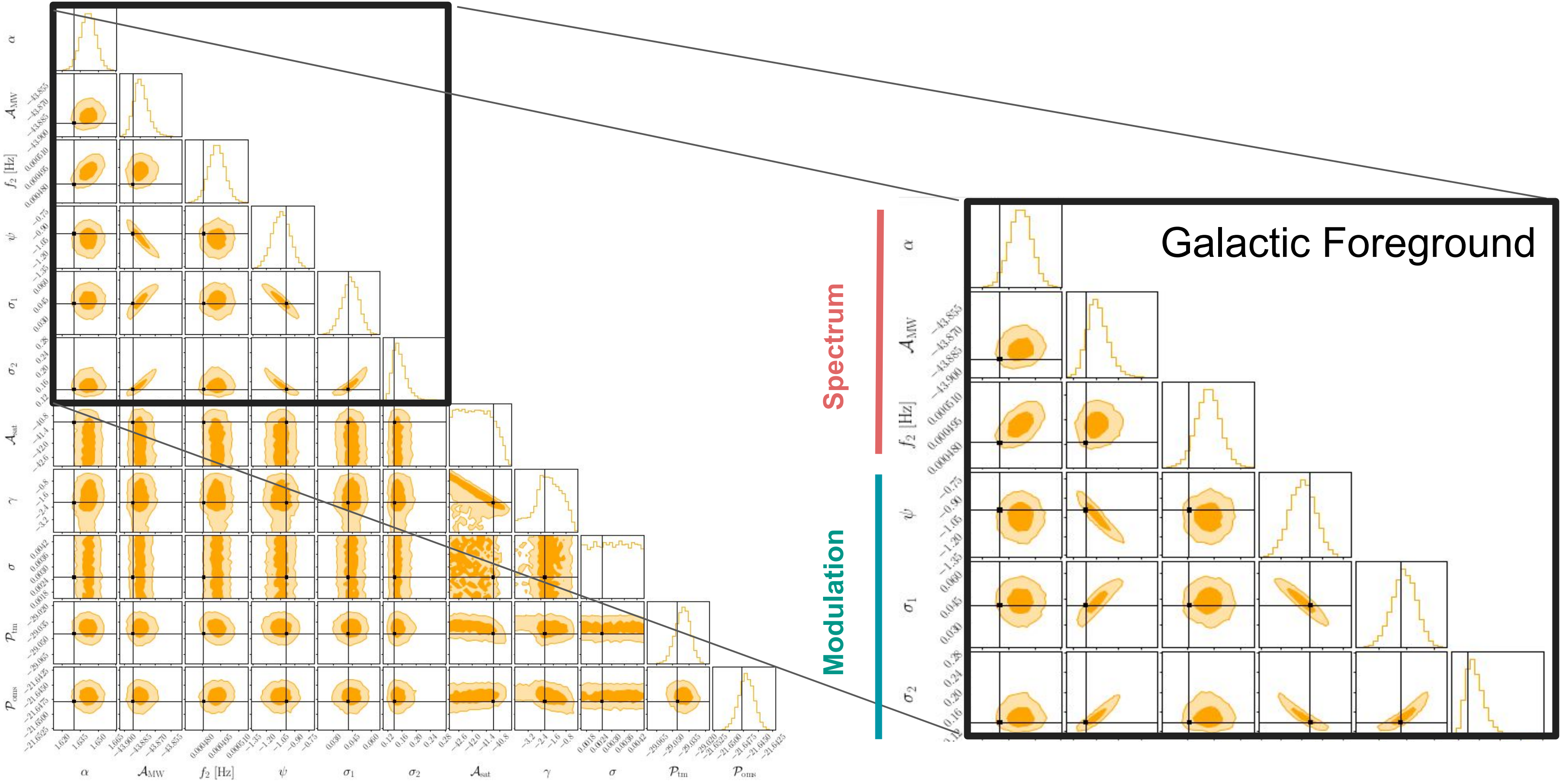
RESULTS - Satellite (Mock) + Noise + MW (Mock)



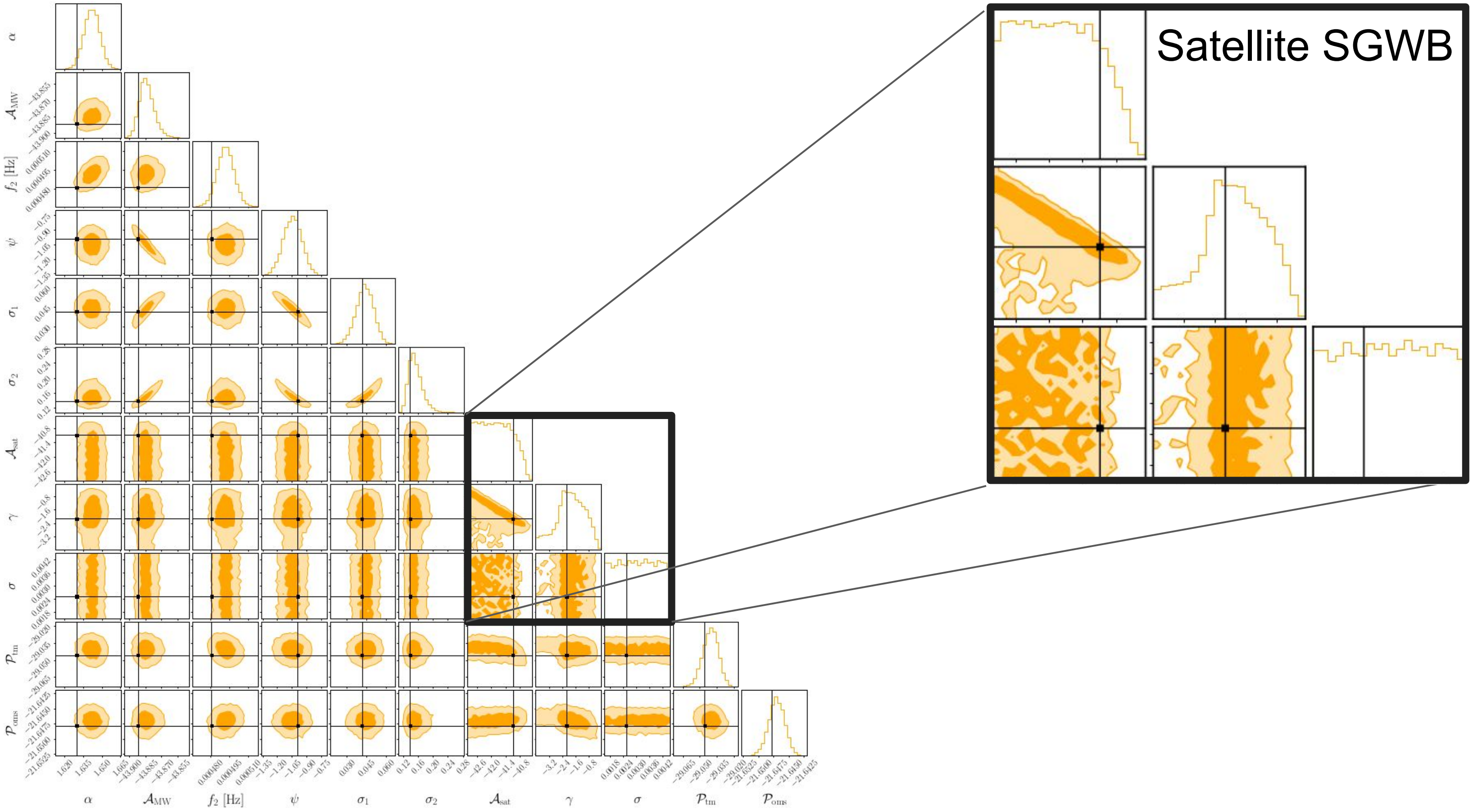
We are able to recover the MW (both the modulation and spectrum)

MW compromises the satellite detection (as expected)

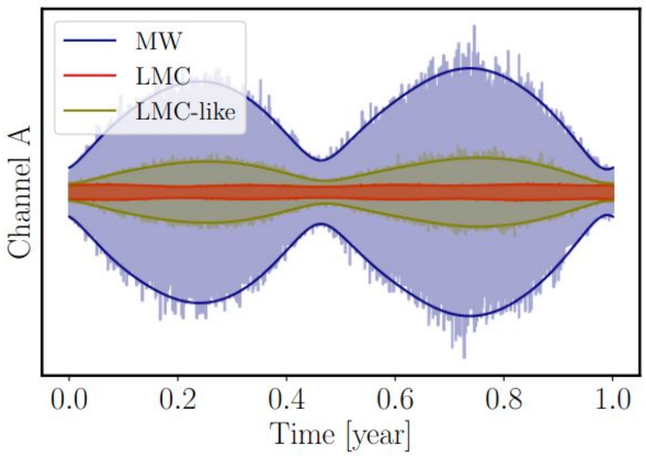
RESULTS - Satellite (Mock) + Noise + MW (Mock)

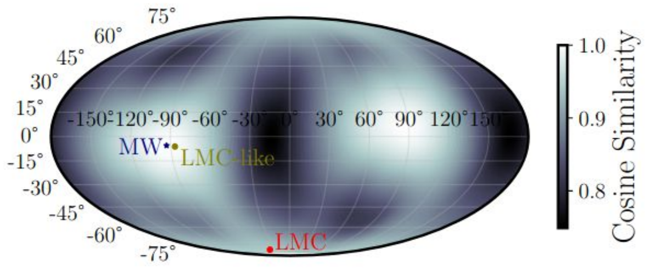


RESULTS - Satellite (Mock) + Noise + MW (Mock)



RESULTS - Hidden Satellite





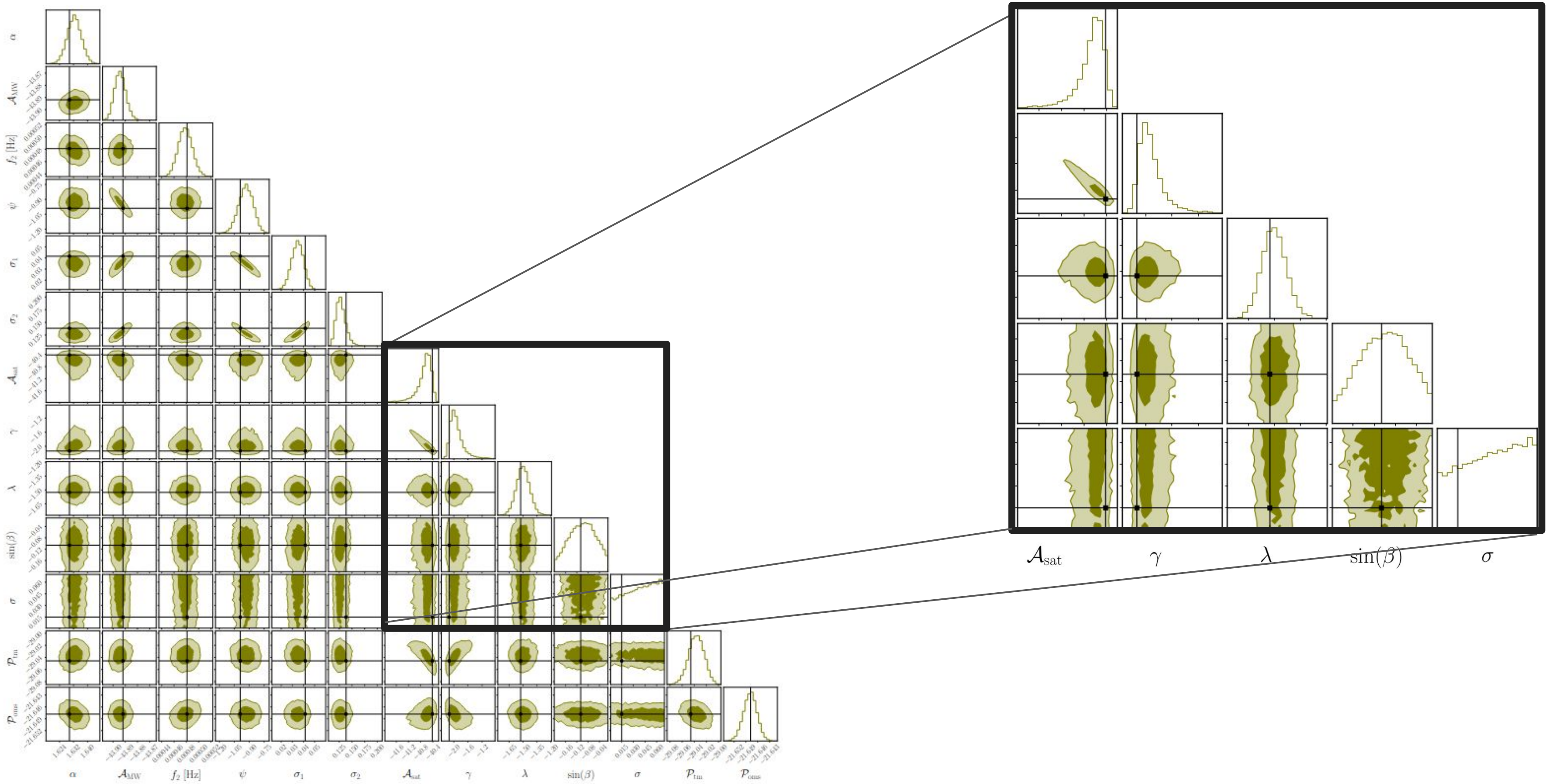
Unlike EM radiation, GW are not obscure by gas and dust

Thus, LISA has the potential to observe beyond the galactic plane (Zone of Avoidance)

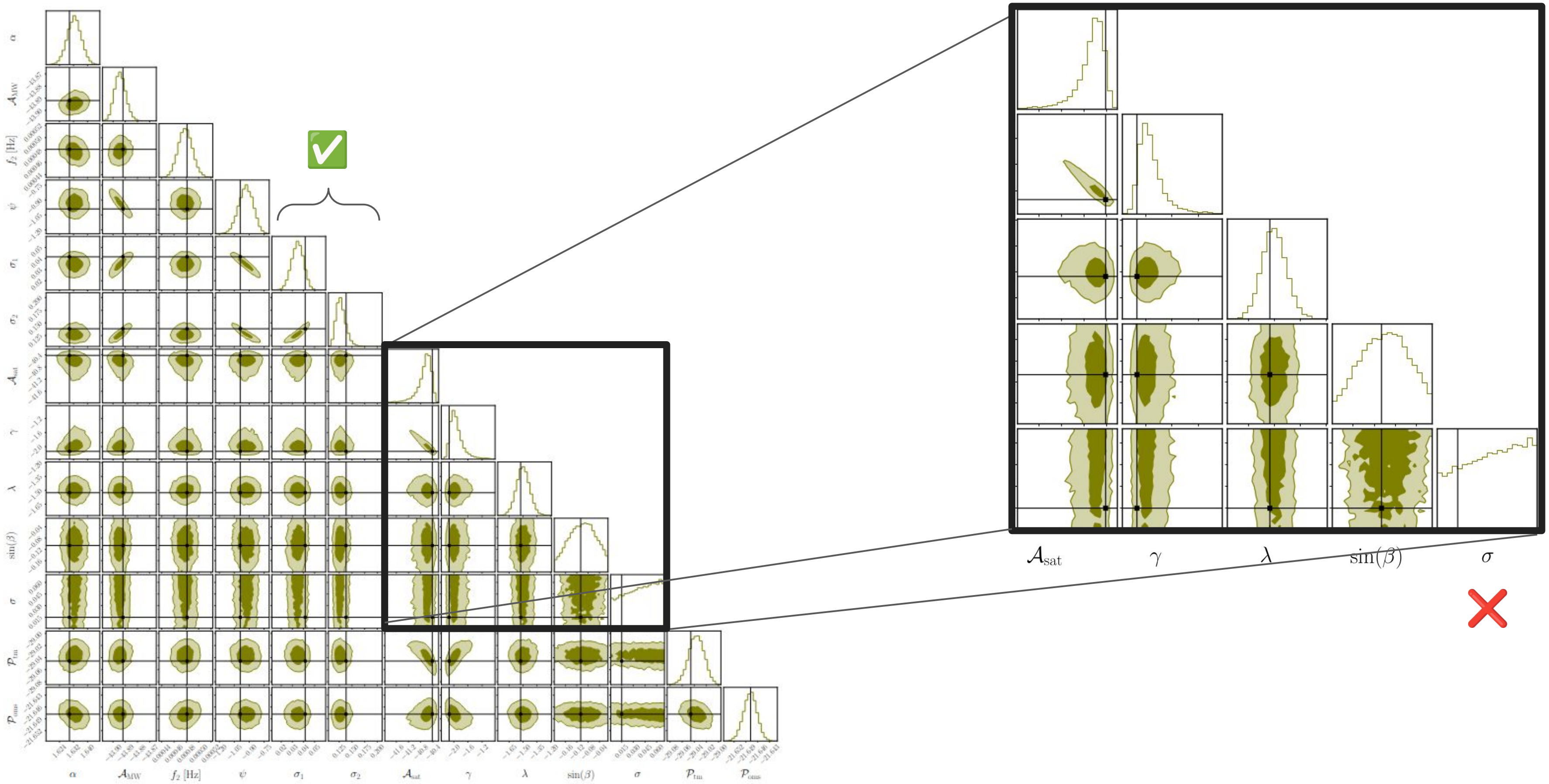
We consider an **LMC-like satellite behind the Milky Way** (i.e. same Astrophysical spectrum -> same total mass and distance)

Are we able to observe it?

RESULTS - Hidden Satellite



RESULTS - Hidden Satellite



CONCLUSION

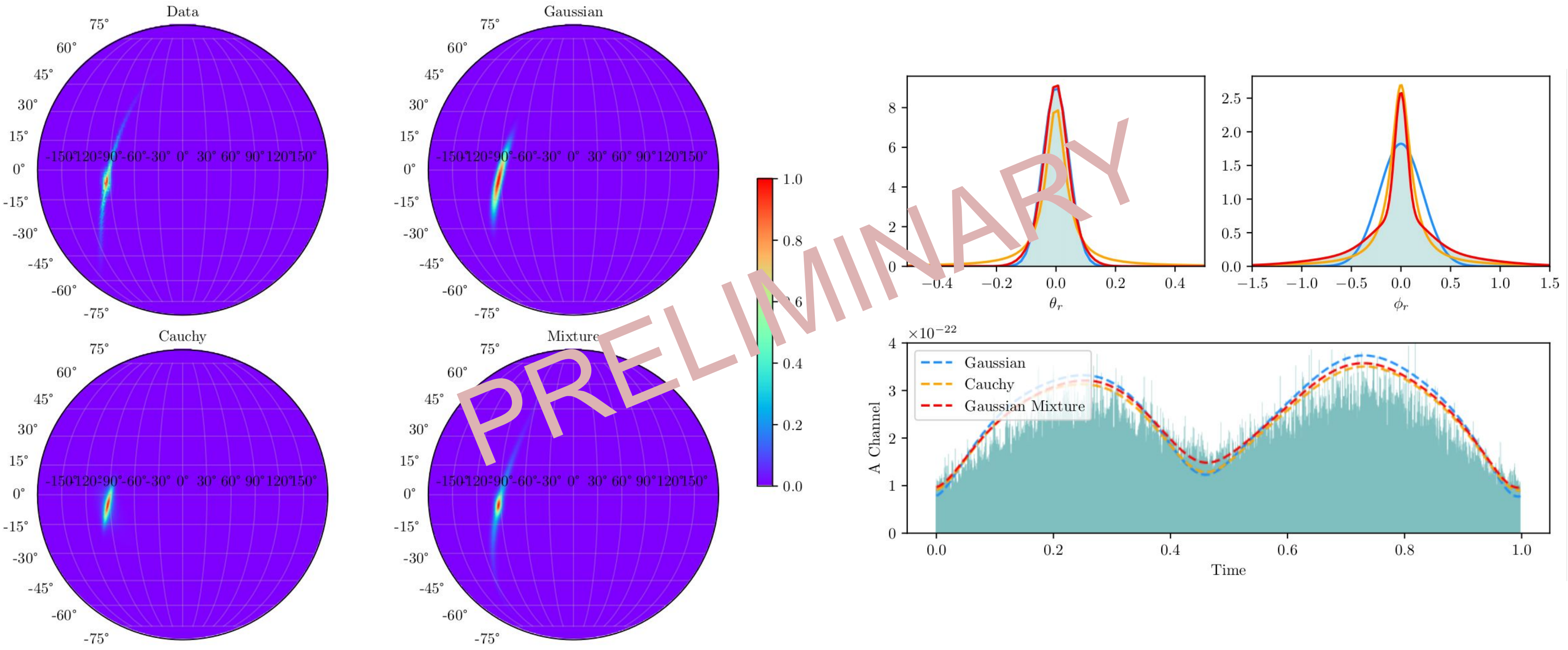
- We introduce a novel method to address anisotropy from astrophysical SGWB.

- Detection of MW satellite strongly depends on the interplay between the spectrum and modulation.

- We could have access to Zone of Avoidance with LISA behind Milky Way

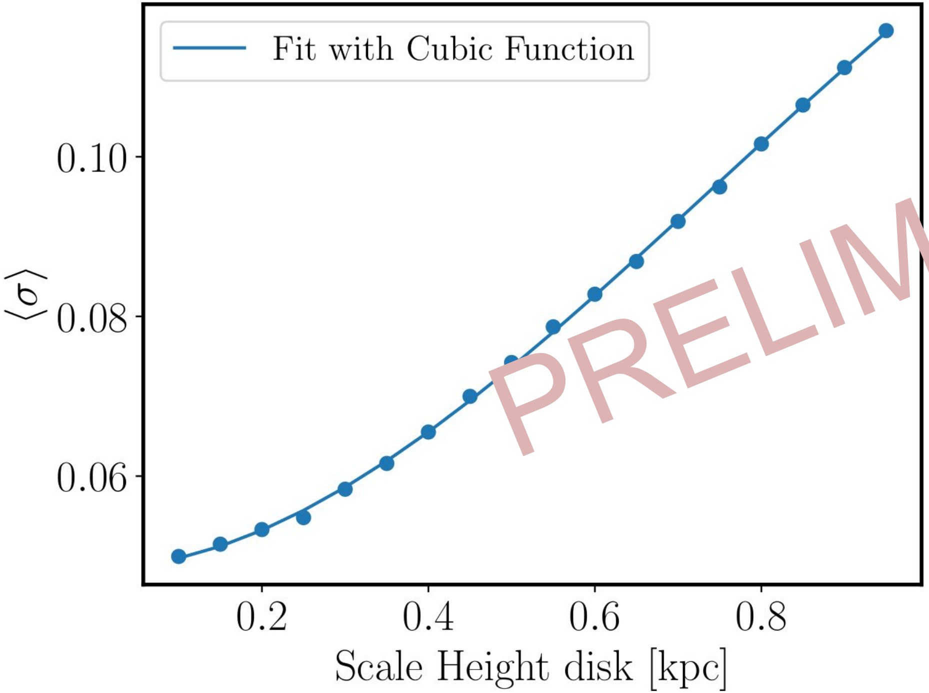
- Study Milky Way Morphology and structure with DWD foreground

WHAT's NEXT



Pozzoli, Buscicchio, Klein in prep

WHAT's NEXT

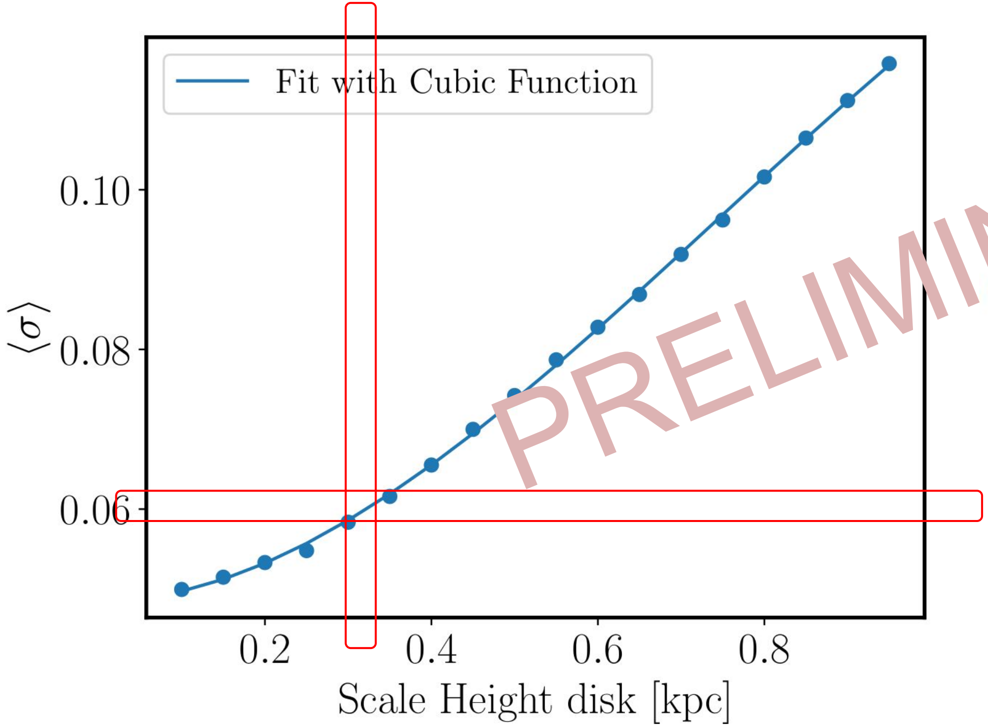


PRELIMINARY

$$\rho(r)\rho(z) \propto \exp\left(-r/r_h\right) \exp\left(-z/z_h\right)$$

Pozzoli, Buscicchio, Klein in prep

WHAT's NEXT



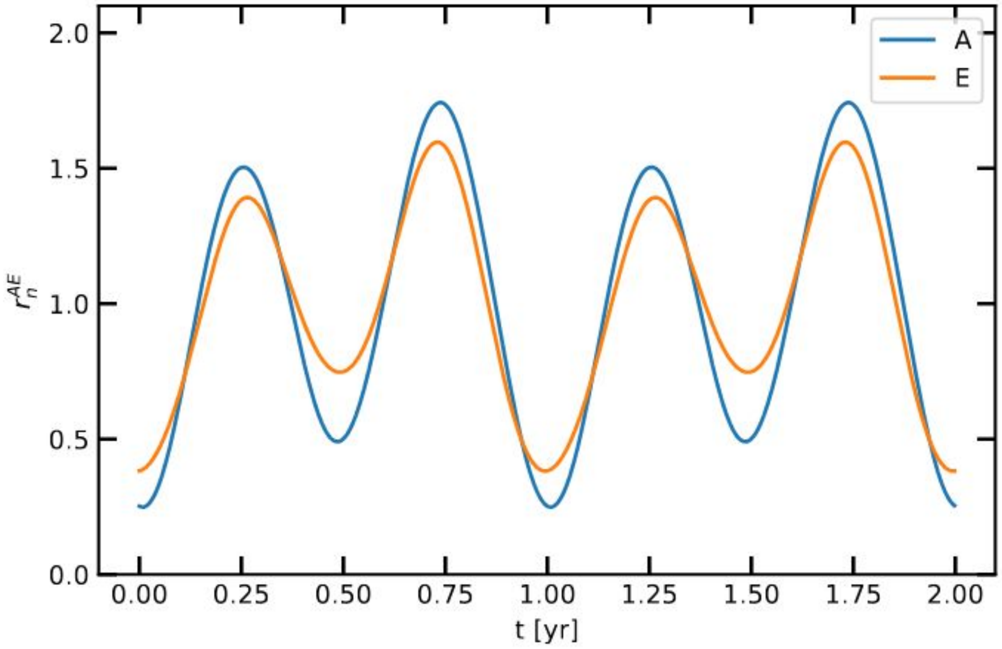
$$\rho(r)\rho(z) \propto \exp\left(-r/r_h\right)\exp\left(-z/z_h\right)$$

Pozzoli, Buscicchio, Klein in prep

BACKUP SLIDES

MODULATION

Digman&Cornish (2022) provide a phenomenological fit based on a realization of MW foreground



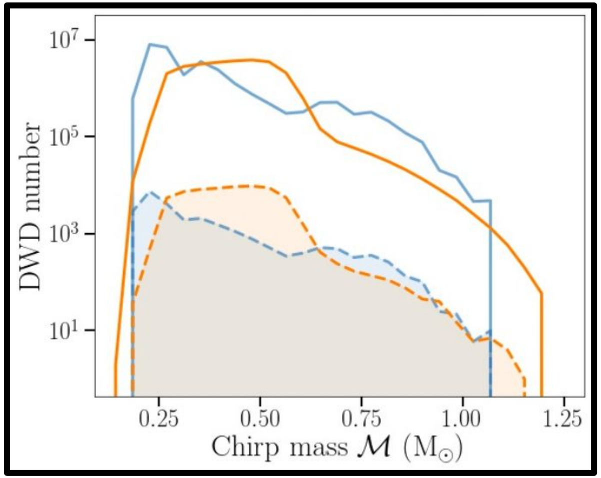
ASTROPHYSICAL SPECTRUM

Korol+22

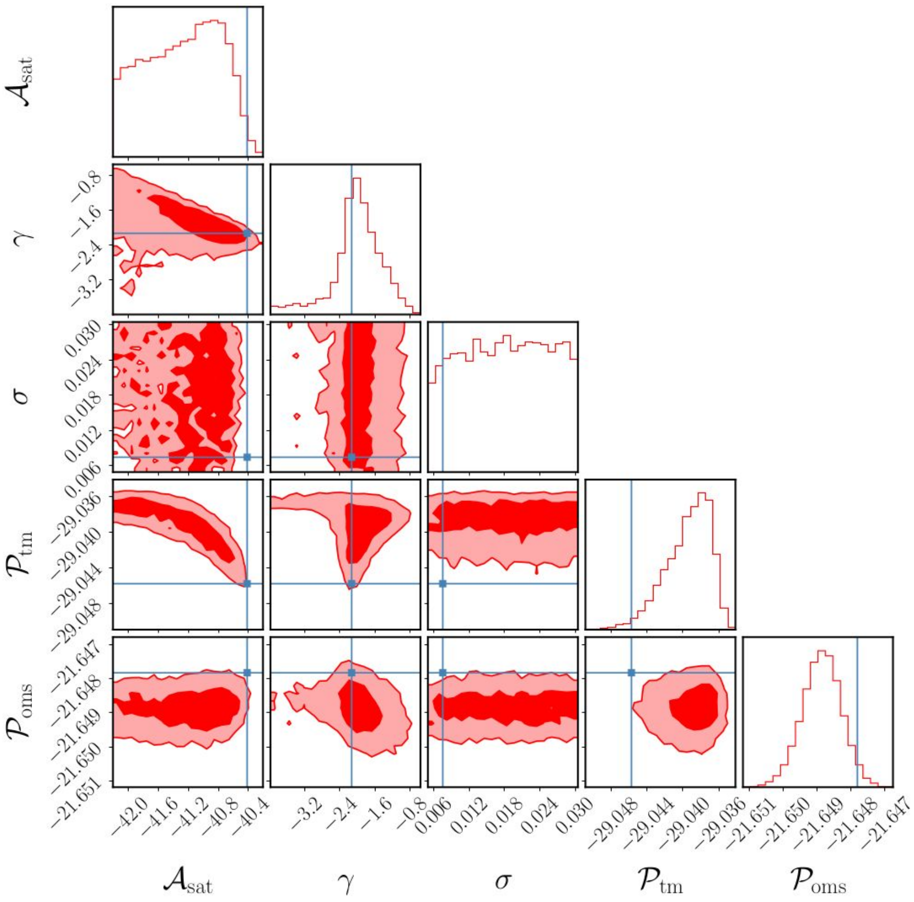
$$S_h(f) = \int d\mathcal{M}_c p(\mathcal{M}_c) \int df_s p(f_s)\delta(f - f_s)\frac{(G\mathcal{M}_c)^{10/3}}{(c^4 D)^2}(\pi f_s)^{4/3}$$

Primiray Mass m1: Gaussian Mixture based on SDSS spectroscopic observation (Kepler+15)

Secondary Mass m2: Flat distribution [0.15 M⊚, m1]



RESULTS - Satellite (Realistic) + Noise



LMC from catalog generated with Stellar Population Synthesis code (Korol+24)

We fix the sky position of LMC in the modulation model