Gravitational wave lensing beyond General Relativity: geometric optic expansion and dispersive phenomena

Nicola Menadeo

MAX-PLANCK-INSTITUT FÜR GRAVITATIONSPHYSIK (Albert-Einstein-Institut)

SSM Scuola Superiore Meridionale

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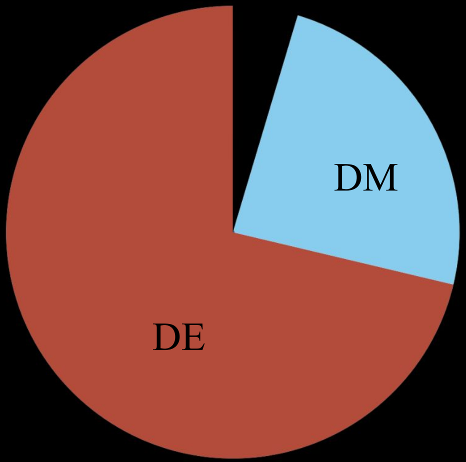
Outline

- Gravitational lensing in a nutshell
- General theory for gravitational radiation
- The point-like lens
- Outlooks

Gravitational lensing in a nutshell

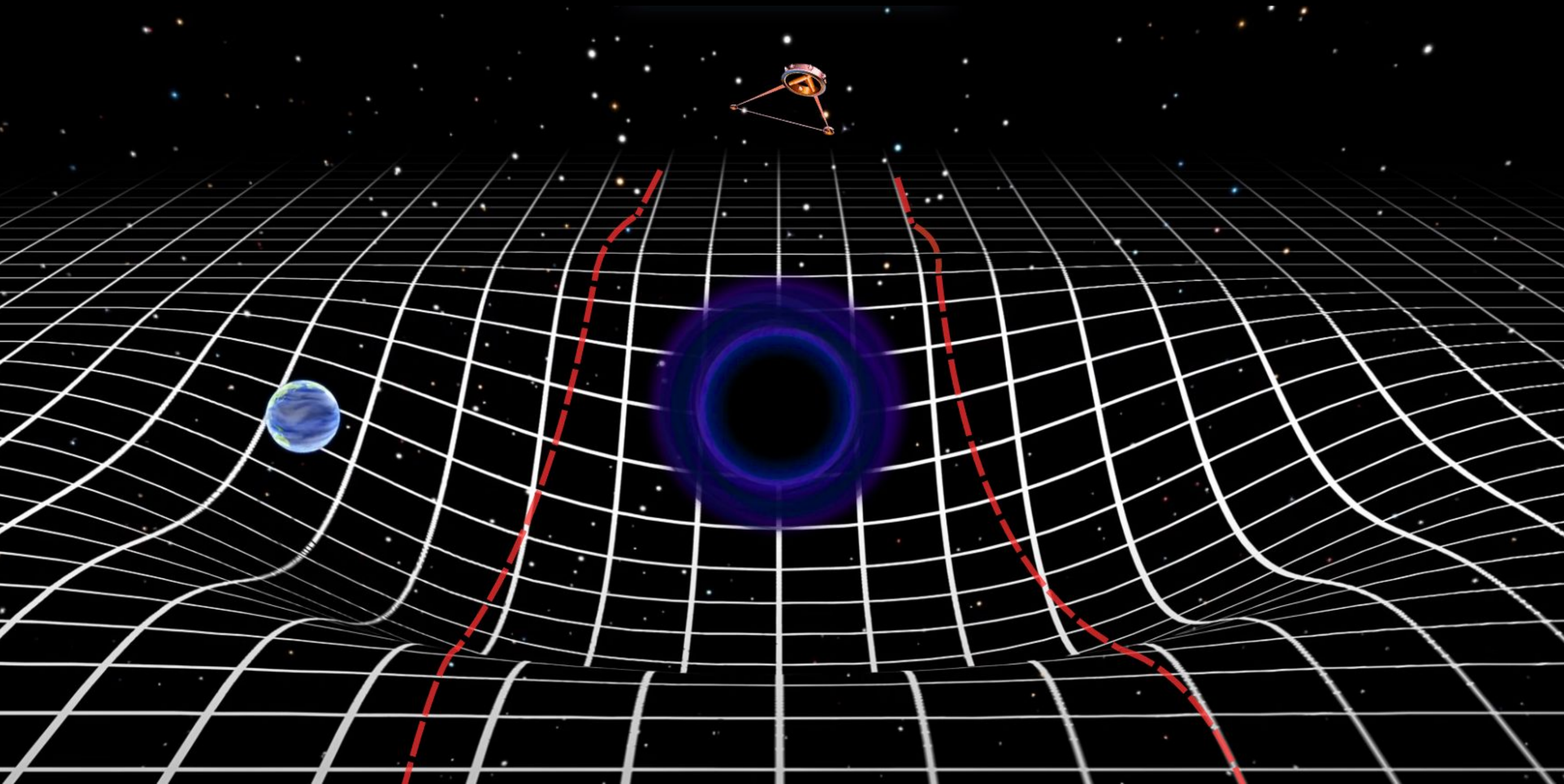
How much can we trust GR?

Why lensing of GWs?



Rich DM phenomenology
through EM Lensing

Gravitational lensing in a nutshell



Gravitational lensing in a nutshell

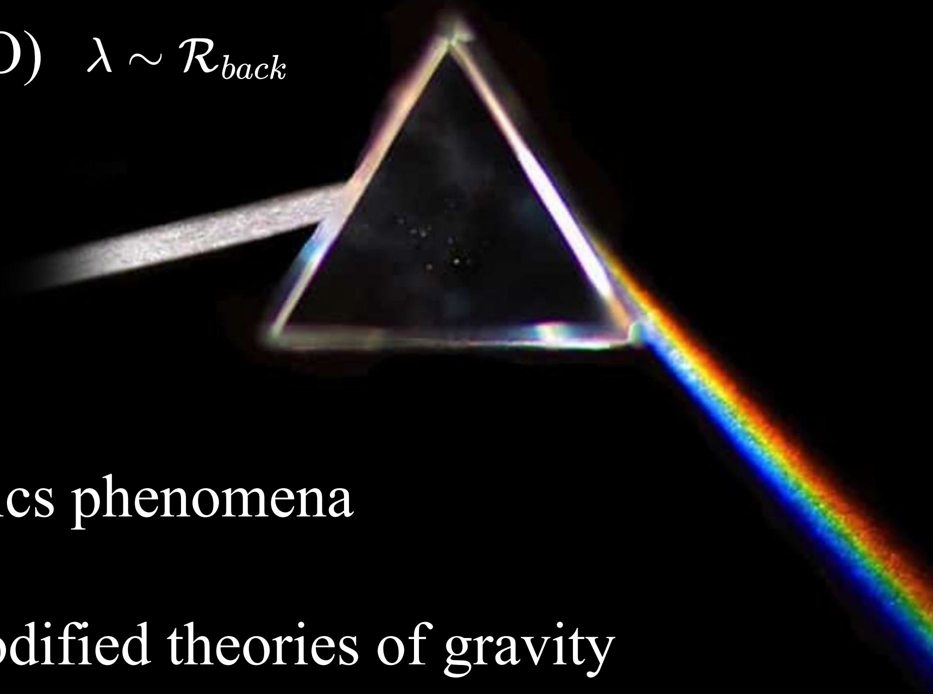
- Geometric optics (GO) $\lambda \ll \mathcal{R}_{back}$

Gravitational lensing in a nutshell

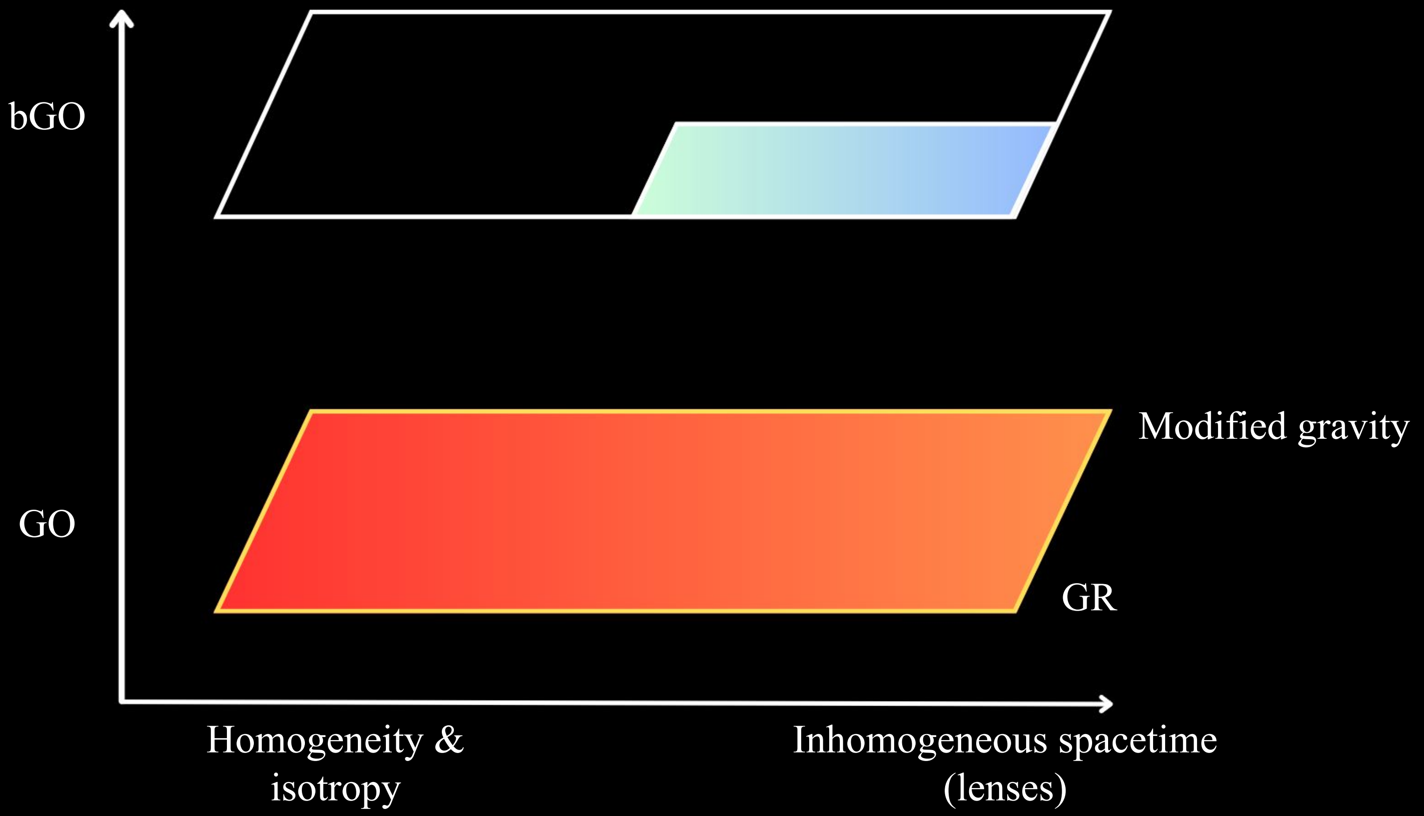
- Geometric optics (GO) $\lambda \ll \mathcal{R}_{back}$
- Beyond geometric optics (bGO) $\lambda \sim \mathcal{R}_{back}$

  (Cusin, Lagos, 2019)
  (Harte, 2019)
  (Dalang, Cusin, Lagos, 2021)

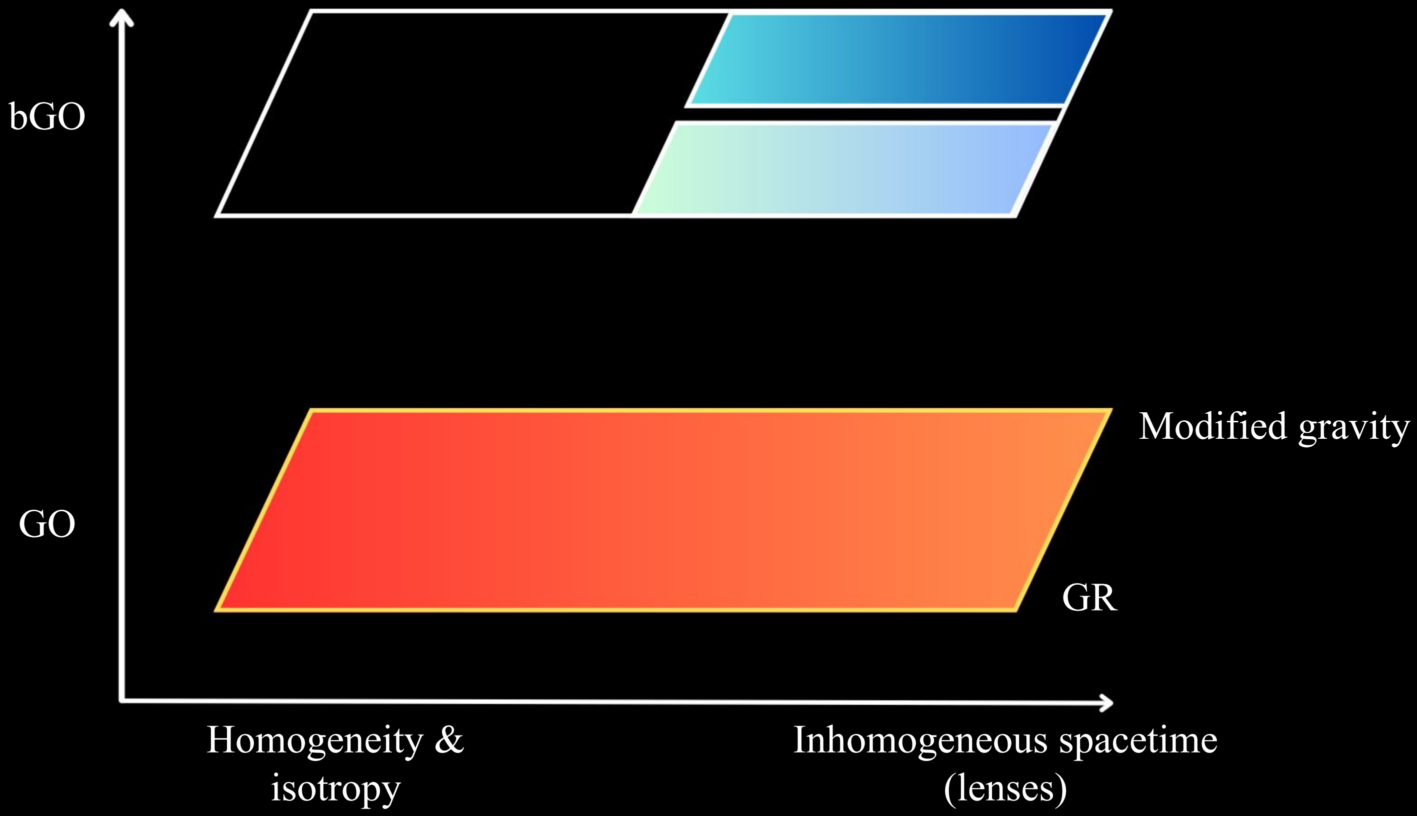
  - New interactions
  - New insights on wave optics phenomena
  - New framework to test modified theories of gravity



Gravitational lensing in a nutshell: state of the art



Gravitational lensing in a nutshell: state of the art



General theory for gravitational radiation

Perturbations in scalar-tensor theory

$$g^{\text{tot}}_{\alpha\beta} = g_{\alpha\beta} + h_{\alpha\beta},$$
$$\phi^{\text{tot}} = \bar{\phi} + \delta\phi.$$

Evolution equation of scalar and tensor waves

$$\mathbf{D}_{IJ} V^J = 0,$$
$$\mathbf{D}_{IJ} \equiv \mathcal{K}^{\gamma\rho}_{IJ} \nabla_\gamma \nabla_\rho + \mathcal{A}^{\gamma}_{IJ} \nabla_\gamma + \mathcal{M}_{IJ}$$

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Explicit system of coupled equations

$$\left[\begin{pmatrix} \mathsf{K}^{\alpha\beta\gamma\rho}_{\mu\nu} & \mathsf{K}^{\gamma\rho}_{\mu\nu} \\ \mathsf{K}^{\alpha\beta\gamma\rho} & \mathsf{K}^{\gamma\rho} \end{pmatrix} \nabla_\gamma \nabla_\rho + \begin{pmatrix} \mathsf{A}^{\alpha\beta\gamma}_{\mu\nu} & \mathsf{A}^{\gamma}_{\mu\nu} \\ \mathsf{A}^{\alpha\beta\gamma} & \mathsf{A}^{\gamma} \end{pmatrix} \nabla_\gamma + \begin{pmatrix} \mathsf{M}^{\alpha\beta}_{\mu\nu} & \mathsf{M}_{\mu\nu} \\ \mathsf{M}^{\alpha\beta} & \mathsf{M} \end{pmatrix} \right] \begin{pmatrix} h_{\alpha\beta} \\ \delta\phi \end{pmatrix} = 0$$

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Kinetic matrix

(Ezquiaga, Zumalacarregui, 2021)

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Kinetic matrix

Damping matrix

(Ezquiaga, Zumalacarregui, 2021)

(Dalang, Fleury +, 2021)

General theory for gravitational radiation

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Kinetic matrix — Damping matrix — Mass matrix

(Ezquiaga, Zumalacarregui, 2021) *(Dalang, Fleury +, 2021)* *(Menadeo, Zumalacarregui, 2024)*

General theory for gravitational radiation

Diagonalization process

$$\mathcal{K}^{\alpha\beta}_{IJ} e^I = \mathcal{G}^{\alpha\beta}_{I} e^I$$

- Not a trivial operation!
- Eigenvectors ⟶ propagating eigenstates
- Eigenvalues ⟶ propagation speed
- Jordan-Einstein frame mapping
- Definition of "*fully luminal*" theory

General theory for gravitational radiation

Short-wave expansion

$$V^J = e^{i\theta^J/\epsilon} \sum_n \epsilon^n A^{(n)J},$$

$$k_\mu^J \equiv \nabla_\mu \theta^J.$$

Amplitude decomposition

$$A^{(n)\,I} \equiv a^{(n)} e^I, \qquad e^I e^J = \delta^{IJ}$$

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$$A^{(n)\,I} \equiv a^{(n)} e^I, \qquad e^I e^J = \delta^{IJ}$$

$$\epsilon^{-2}: \qquad \mathcal{K}_{IJ}^{\alpha\beta} k_\alpha^J k_\beta^J = 0$$

$$\epsilon^{-1}: \qquad \left[\mathcal{K}_{IJ}^{\alpha\beta}\left(2k_\alpha^J \nabla_\beta + \nabla_\beta k_\alpha^J\right) + \mathcal{A}_{IJ}^{\alpha} k_\alpha^J\right] A^{(0)J} = 0.$$

$$\epsilon^{0}: \qquad \left[\mathcal{K}_{IJ}^{\alpha\beta}\left(2k_\alpha^J \nabla_\beta + \nabla_\beta k_\alpha^J\right) + \mathcal{A}_{IJ}^{\alpha} k_\alpha^J\right] A^{(1)J} = i\mathbf{D}_{IJ} A^{(0)J}.$$

General theory for gravitational radiation: Brans-Dicke

Action

$$S = \frac{1}{16\,\pi} \int d^4x \sqrt{-g} \left(\phi R - \frac{\omega}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right)$$

Diagonalization

$$\tilde{h}_{\alpha\beta} \equiv h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h - g_{\mu\nu} \frac{\delta\phi}{\bar{\phi}},$$

$$\nabla^\mu \tilde{h}_{\mu\nu} = 0.$$

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$$\left[\begin{pmatrix} \tilde{\mathsf{K}}^{\alpha\beta\gamma\rho}_{\mu\nu} & 0 \\ 0 & \tilde{\mathsf{K}}^{\gamma\rho} \end{pmatrix} \nabla_\gamma \nabla \rho + \begin{pmatrix} \tilde{\mathsf{A}}^{\alpha\beta\gamma}_{\mu\nu} & \tilde{\mathsf{A}}^{\gamma}_{\mu\nu} \\ 0 & \tilde{\mathsf{A}}^{\gamma} \end{pmatrix} \nabla \gamma + \begin{pmatrix} \tilde{\mathsf{M}}^{\alpha\beta}_{\mu\nu} & \tilde{\mathsf{M}}_{\mu\nu} \\ \tilde{\mathsf{M}}^{\alpha\beta} & \tilde{\mathsf{M}} \end{pmatrix} \right] \begin{pmatrix} \tilde{h}_{\alpha\beta} \\ \delta\phi \end{pmatrix} = 0$$

Fully luminal theory

General theory for gravitational radiation: Brans-Dicke

Short-wave expansion

$$\tilde{h}_{\mu\nu} = \left(\tilde{h}^{(0)}_{\mu\nu} + \epsilon\tilde{h}^{(1)}_{\mu\nu} + \ldots\right)e^{i\theta^T/\epsilon},$$

$$\delta\phi = \left(\delta\phi^{(0)} + \epsilon\delta\phi^{(1)} + \ldots\right)e^{i\theta^S/\epsilon},$$

$$k^{S,T}_{\mu} \equiv \nabla_{\mu}\theta^{S,T}.$$

Null tetrad basis

$$e^{\mu}_{A} \equiv \{k^{\mu}, m^{\mu}, l^{\mu}, n^{\mu}\},$$

Amplitude decomposition

$$\tilde{h}^{(n)}_{\mu\nu} \equiv \tilde{\alpha}^{(n)}_{AB}\Theta^{AB}_{\mu\nu},$$

General theory for gravitational radiation: Brans-Dicke

TT gauge

$$\epsilon^{-1}: \qquad k^{\mu}\tilde{h}^{(0)}_{\mu\nu} = 0,$$

$$\epsilon^{0}: \qquad k^{\mu}\tilde{h}^{(1)}_{\mu\nu} = i\nabla^{\mu}\tilde{h}^{(0)}_{\mu\nu}.$$

Geometric optics amplitude

$$\tilde{h}^{(0)}_{\mu\nu} = \tilde{\alpha}^{(0)}_{mm} m_{\mu} m_{\nu} + \tilde{\alpha}^{(0)}_{ll} l_{\mu} l_{\nu}$$

↓

L&R helicity polarizations

General theory for gravitational radiation: Brans-Dicke

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$$\epsilon^{-1}: \qquad k^\mu \tilde{h}^{(0)}_{\mu\nu} = 0,$$

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$$\tilde{h}^{(0)}_{\mu\nu} = \tilde{\alpha}^{(0)}_{mm} m_\mu m_\nu + \tilde{\alpha}^{(0)}_{ll} l_\mu l_\nu$$

↓

L&R helicity polarizations

Beyond geometric optics amplitude

$$\tilde{h}^{(1)}_{\mu\nu} = \boxed{\tilde{\alpha}^{(1)}_{mm} \bar{m}_\mu \bar{m}_\nu + \tilde{\alpha}^{(1)}_{ll} \bar{l}_\mu \bar{l}_\nu} + \boxed{\tilde{\alpha}^{(1)}_{nn} \bar{n}_\mu \bar{n}_\nu + \tilde{\alpha}^{(1)}_{nm} \bar{n}_{(\mu} \bar{m}_{\nu)} + \tilde{\alpha}^{(1)}_{nl} \bar{n}_{(\mu} \bar{l}_{\nu)}}$$

L&R helicity polarizations — novel components

General theory for gravitational radiation: Brans-Dicke

Geometric optics amplitude evolution

Scalar sector

$$\left[\tilde{\mathsf{K}}^{\alpha\beta}\left(2k_\alpha\nabla_\beta+\nabla_\alpha k_\beta\right)-k_\alpha\tilde{\mathsf{A}}^\alpha\right]\delta\bar{\phi}^{(0)}=0,$$

$$\delta\phi^{(0)}(\xi)=\delta\phi^{(0)}(\xi_s)\frac{D(\xi_s)}{\sqrt{\bar{\phi}(\xi_s)}}\left(\frac{\sqrt{\bar{\phi}(\xi)}}{D(\xi)}\right).$$

General theory for gravitational radiation: Brans-Dicke

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Gravitational sector

$$\left[\bar{\phi}\delta^\rho_\mu\delta^\sigma_\nu\mathcal{D}-2k_\alpha\tilde{\mathsf{A}}^{\alpha\beta\gamma}_{\mu\nu}\delta^\rho_\beta\delta^\sigma_\gamma\right]\tilde{h}^{(0)}_{\rho\sigma}=2\tilde{\mathsf{A}}^\alpha_{\mu\nu}k_\alpha\delta\phi^{(0)},$$

$$\tilde{\alpha}^{(0)}_{AB}(\xi)=\frac{\sqrt{\bar{\phi}(\xi_s)}D(\xi_s)}{\sqrt{\bar{\phi}(\xi)}D(\xi)}\tilde{\alpha}^{(0)}_{AB}(\xi_s).$$

(Dalang, Fleury +, 2021)

General theory for gravitational radiation: Brans-Dicke

Beyond geometric optics corrections

Scalar sector

$$\left[\tilde{\mathsf{K}}^{\alpha\beta}\left(2k_\alpha\nabla_\beta+\nabla_\beta k_\alpha\right)-k_\gamma\tilde{\mathsf{A}}^\gamma\right]\delta\bar{\phi}^{(1)}=i\left[\left(\tilde{\mathsf{K}}^{\alpha\beta}\nabla_\alpha\nabla_\beta+\tilde{\mathsf{A}}^\gamma\nabla_\gamma+\tilde{\mathsf{M}}\right)\delta\phi^{(0)}+\tilde{\mathsf{M}}^{\alpha\beta}\tilde{h}^{(0)}_{\alpha\beta}\right],$$

$$\delta\phi^{(1)}(\xi)=i\frac{\sqrt{\bar{\phi}(\xi)}}{D(\xi)}\int_{\xi_s}^{\xi}d\xi\frac{D(\xi)}{\bar{\phi}^{3/2}}\left[\left(\tilde{\mathsf{K}}^{\alpha\beta}\nabla_\alpha\nabla_\beta+\tilde{\mathsf{A}}^\gamma\nabla_\gamma+\tilde{\mathsf{M}}\right)\delta\phi^{(0)}+\tilde{\mathsf{M}}^{\alpha\beta}\tilde{h}^{(0)}_{\alpha\beta}\right].$$

General theory for gravitational radiation: Brans-Dicke

Beyond geometric optics corrections

Gravitational sector

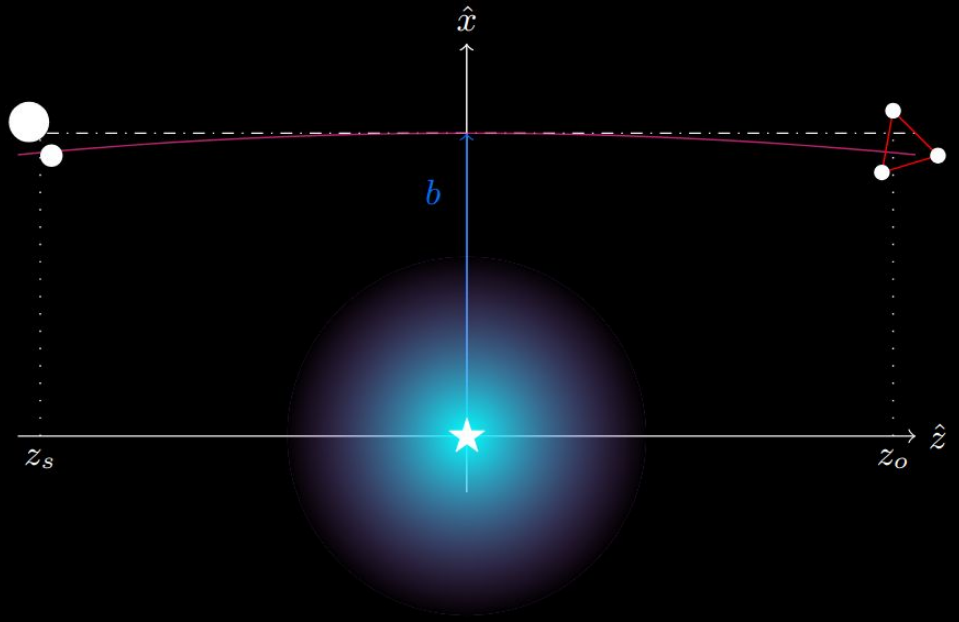
$$\left[\bar{\phi}\delta^{\rho}_{\mu}\delta^{\sigma}_{\nu}\mathcal{D} - 2k_{\alpha}\tilde{\mathsf{A}}^{\alpha\beta\gamma}_{\mu\nu}\delta^{\rho}_{\beta}\delta^{\sigma}_{\gamma}\right]\tilde{h}^{(1)}_{\rho\sigma} = i\tilde{\mathsf{F}}_{\mu\nu},$$

$$\tilde{\alpha}^{(1)}_{AB}(\xi) = \frac{i}{\sqrt{\bar{\phi}(\xi)D(\xi)}}\int_{\xi_s}^{\xi} d\xi \frac{D(\xi)}{\sqrt{\bar{\phi}}}\left(\hat{e}^{\mu}_{A}\hat{e}^{\nu}_{B}\tilde{\mathsf{F}}_{\mu\nu}\right)$$

The Point-like lens

Line element

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 - 2\Psi)d\vec{x}^2,$$



The Point-like lens

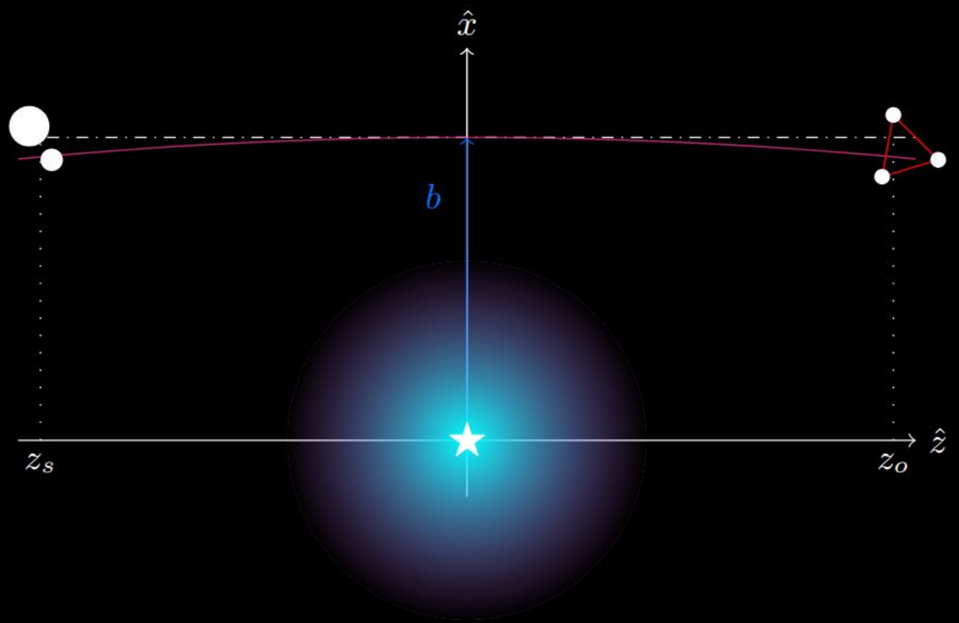
Line element

$$ds^2 = -(1+2\Psi)dt^2 + (1-2\Psi)d\vec{x}^2,$$

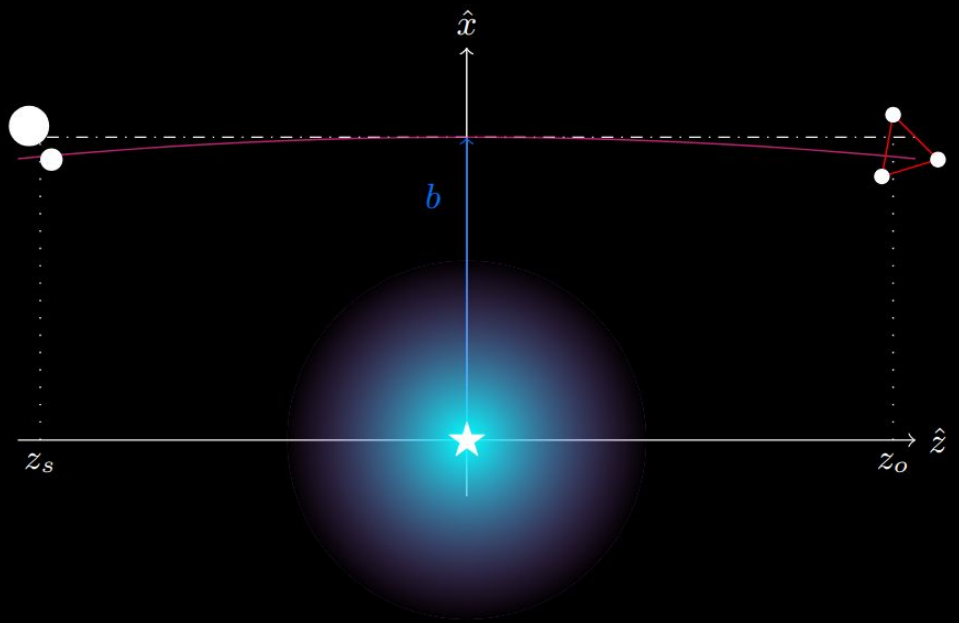
Perturbative approach

$$k^\mu = \bar{k}^\mu + \delta k^\mu,$$

$$h^{(n)}_{\mu\nu} = \bar{\alpha}^{(n)}_{AB}\Theta^{AB}_{\mu\nu} + \bar{\alpha}^{(n)}_{AB}\delta\Theta^{AB}_{\mu\nu} + \delta\alpha^{(n)}_{AB}\Theta^{AB}_{\mu\nu},$$



The Point-like lens



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$$h_{\mu\nu}^{(n)} = \bar{\alpha}_{AB}^{(n)}\Theta_{\mu\nu}^{AB} + \bar{\alpha}_{AB}^{(n)}\delta\Theta_{\mu\nu}^{AB} + \delta\alpha_{AB}^{(n)}\Theta_{\mu\nu}^{AB},$$

$$\int_{\xi_s}^{\xi_o} d\xi \longrightarrow \int_{-\infty}^{+\infty}\frac{dz}{\Omega}, \qquad D(z) = z - z_s,$$

$$\bar{\phi} \equiv \bar{\phi}_\infty + \frac{\beta M_L}{R}.$$

The Point-like lens

Beyond geometric optics corrections

Scalar sector

$$\delta\phi^{(1)}(z_o) = \frac{i}{\Omega}\frac{\sqrt{\bar{\phi}(z_o)}}{\bar{D}(z_o)}\left[F^{\phi}(b,\tau)\delta\phi^{(0)}(\xi_s) + F^{+}(b,\tau)\zeta_{+}^{(0)}(z_s) + F^{-}(b,\tau)\zeta_{-}^{(0)}(z_s)\right],$$

Gravitational sector

$$\begin{pmatrix} \delta\tilde{\alpha}_{mm}^{(1)}(z_o) \\ \delta\tilde{\alpha}_{ll}^{(1)}(z_o) \\ \delta\tilde{\alpha}_{nn}^{(1)}(z_o) \end{pmatrix} = \frac{i}{\sqrt{\bar{\phi}\,(z_o)}\bar{D}\,(z_o)}\begin{bmatrix} F_{mm}^{\phi}(b,\tau) & F_{mm}^{+}(b,\tau) & F_{mm}^{-}(b,\tau) \\ F_{ll}^{\phi}(b,\tau) & F_{ll}^{+}(b,\tau) & F_{ll}^{-}(b,\tau) \\ F_{nn}^{\phi}(b,\tau) & F_{nn}^{+}(b,\tau) & 0 \end{bmatrix}\begin{pmatrix} \delta\phi^{(0)}(z_s) \\ \zeta_{+}^{(0)}(z_s) \\ \zeta_{-}^{(0)}(z_s) \end{pmatrix}.$$

The Point-like lens

	$\delta\phi^{(1)}$	$\delta\alpha_{nn}^{(1)}$	$\delta\alpha_{mm}^{(1)}$	$\delta\alpha_{ll}^{(1)}$	$\delta\alpha_{nm}^{(1)}$	$\delta\alpha_{nl}^{(1)}$
GR	✗	✓	✗	✗	✗	✗
BD						

The Point-like lens

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GR	✗	✓	✗	✗	✗	✗
BD	✓	✓	✓	✓	✗	✗

The Point-like lens

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GR	✗	✓	✗	✗	✗	✗
BD	✓	✓	✓	✓	✗	✗

Scalar wave

The Point-like lens

	$\delta\phi^{(1)}$	$\delta\alpha_{nn}^{(1)}$	$\delta\alpha_{mm}^{(1)}$	$\delta\alpha_{ll}^{(1)}$	$\delta\alpha_{nm}^{(1)}$	$\delta\alpha_{nl}^{(1)}$
GR	✗	✓	✗	✗	✗	✗
BD	✓	✓	✓	✓	✗	✗

Different functional form w.t.r. to GR

The Point-like lens

	$\delta\phi^{(1)}$	$\delta\alpha^{(1)}_{nn}$	$\delta\alpha^{(1)}_{mm}$	$\delta\alpha^{(1)}_{ll}$	$\delta\alpha^{(1)}_{nm}$	$\delta\alpha^{(1)}_{nl}$
GR	✗	✓	✗	✗	✗	✗
BD	✓	✓	✓	✓	✗	✗

Direct impact on
L&R helicity polarizations

Outlooks

- First step towards the full GO framework
- Extension to more complex theories
    - Not “fully luminal theories”
- Different scalar field configurations
- More complex lensing scenarios

Outlooks

Thank you!

Backup slides

General theory for gravitational radiation: GR

Short-wave expansion

$$\tilde{h}_{\mu\nu} = \left(\tilde{h}^{(0)}_{\mu\nu} + \epsilon \tilde{h}^{(1)}_{\mu\nu} + \ldots\right) e^{i\theta^T/\epsilon},$$

$$k_\mu \equiv \nabla_\mu \theta^T.$$

ϵ^{-2}: $$k^\mu k_\mu = 0,$$

ϵ^{-1}: $$\left(2\, k^\alpha \nabla_\alpha + \nabla^\alpha k_\alpha\right) \tilde{h}^{(0)}_{\mu\nu} = \mathcal{D}\, \tilde{h}^{(0)}_{\mu\nu} = 0,$$ $$k^\mu \tilde{h}^{(0)}_{\mu\nu} = 0,$$

ϵ^0 : $$\mathcal{D}\, \tilde{h}^{(1)}_{\mu\nu} = i\left(\Box \tilde{h}^{(0)}_{\mu\nu} - 2R_{\alpha\mu\nu\beta} \tilde{h}^{(0)\alpha\beta}\right).$$ $$k^\mu \tilde{h}^{(1)}_{\mu\nu} = i\nabla^\mu \tilde{h}^{(0)}_{\mu\nu}.$$

General theory for gravitational radiation: GR

Geometric optics amplitude evolution

$$\epsilon^{-1}: \quad \tilde{\alpha}^{(0)}_{AB}(\xi) = \frac{D(\xi_s)}{D(\xi)} \tilde{\alpha}^{(0)}_{AB}(\xi_s)$$

$$\epsilon^{0}: \quad \tilde{\alpha}^{(1)}_{AB}(\xi) = \frac{i}{D(\xi)} \int_{\xi_s}^{\xi} d\xi \hat{e}^{\mu}_{A} \hat{e}^{\nu}_{B} D(\xi) \left[\Box \tilde{h}^{(0)}_{\mu\nu} - 2 R_{\alpha\mu\nu\beta} \tilde{h}^{(0)\alpha\beta} \right]$$