New Theoretical Perspective on Axions

Surjeet Rajendran, The Johns Hopkins University

(With David E. Kaplan and Tom Melia)

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$$\mathcal{L} \supset \theta_0 G \tilde{G} + m_Q \bar{Q} Q + \frac{a}{f_a} G \tilde{G} \implies \frac{a}{f_a} + \bar{\theta} = 0$$

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Consequence: Strong CP can only be solved by dynamical methods i.e. axion!

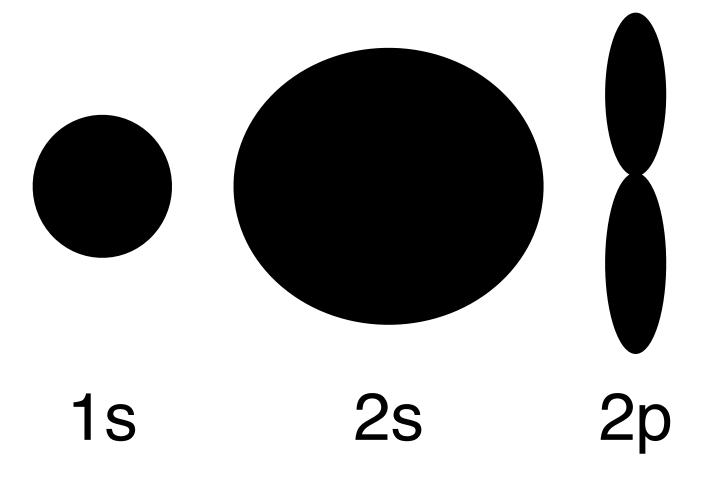
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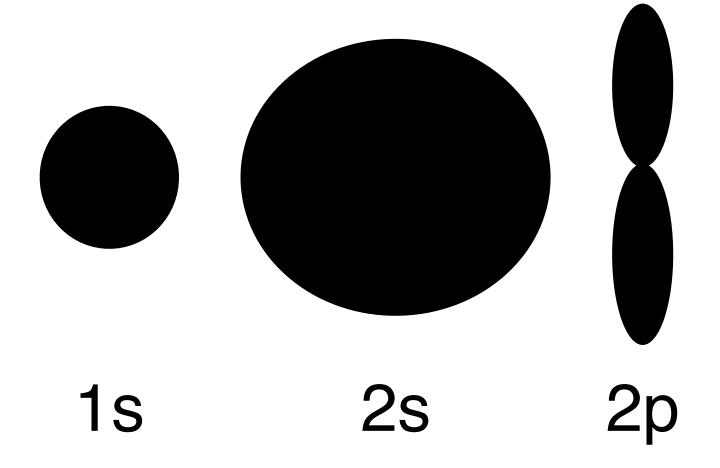
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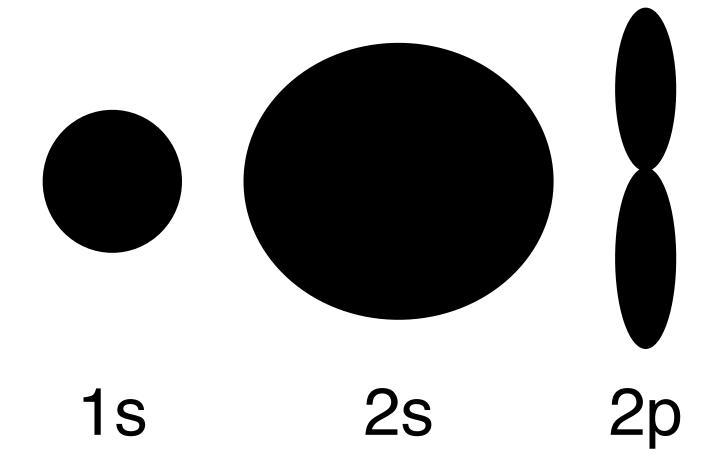


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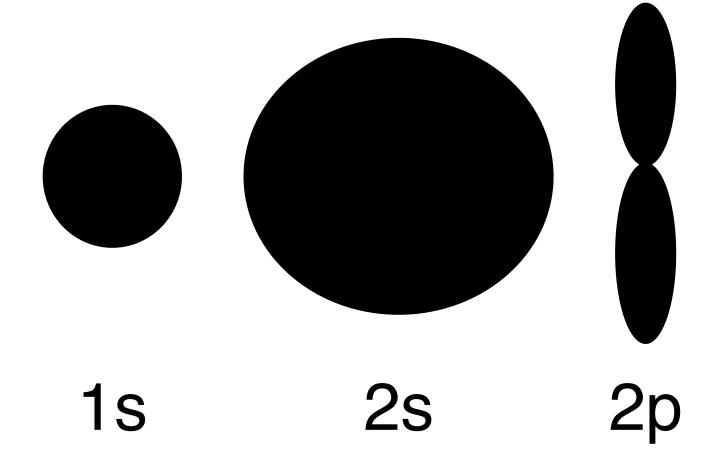
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Hydrogen Produced by a number of high energy processes in early universe

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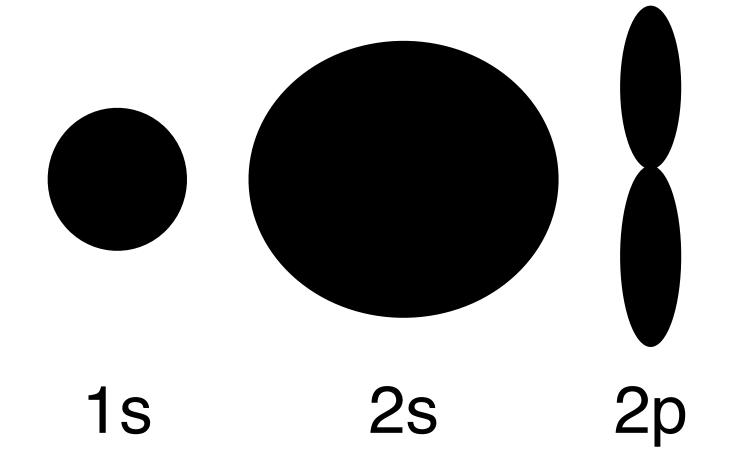
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Not a problem - states decay to Is

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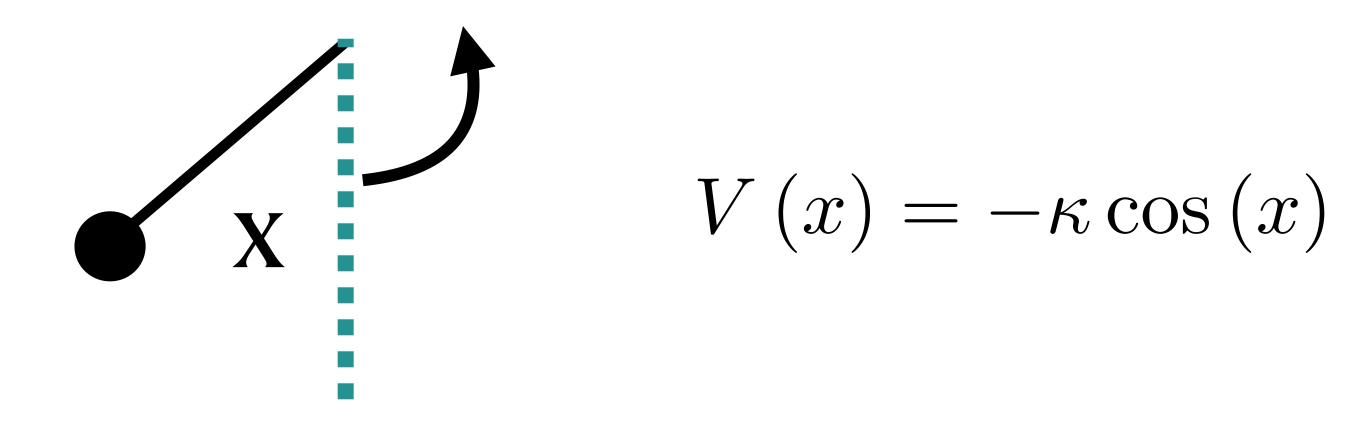
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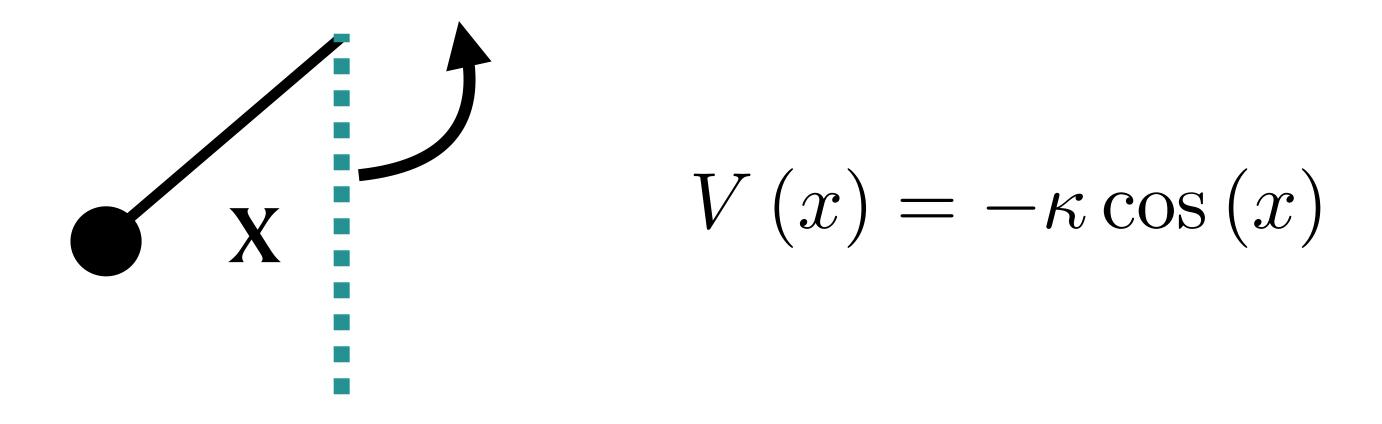
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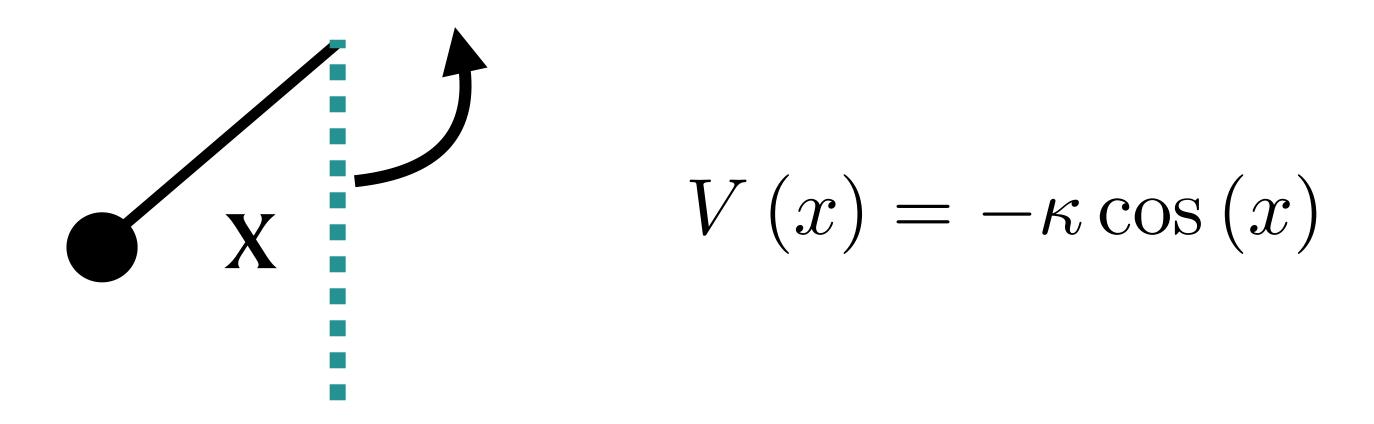
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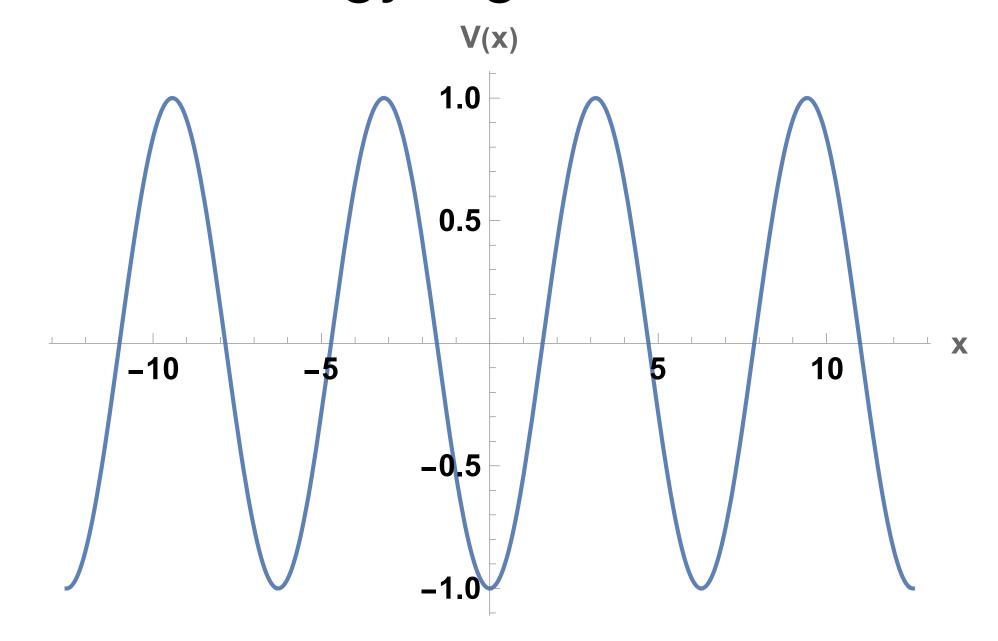
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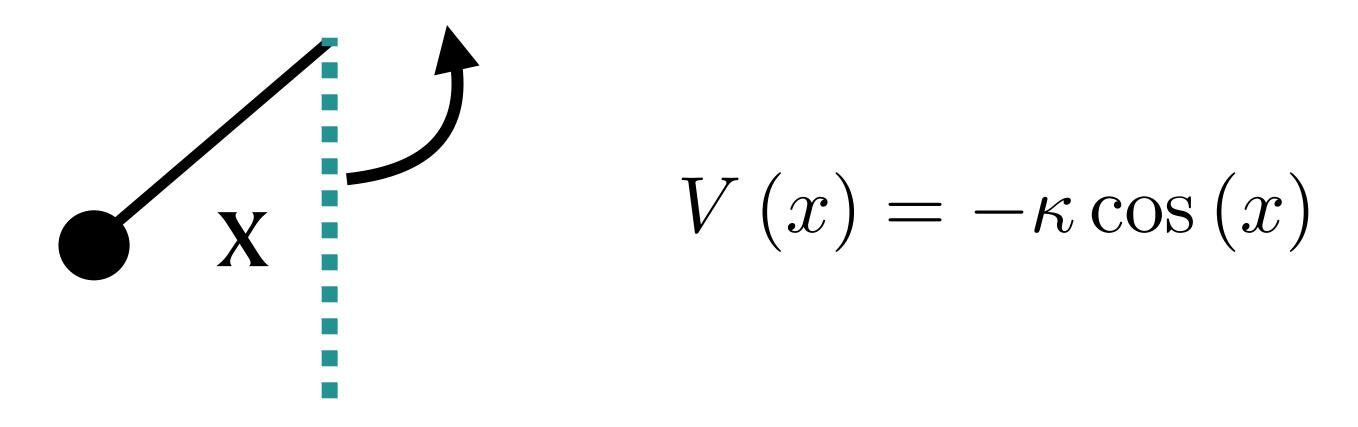
Only solutions are dynamical - e.g. massless hidden photon allowing states to decay, collisions between atoms

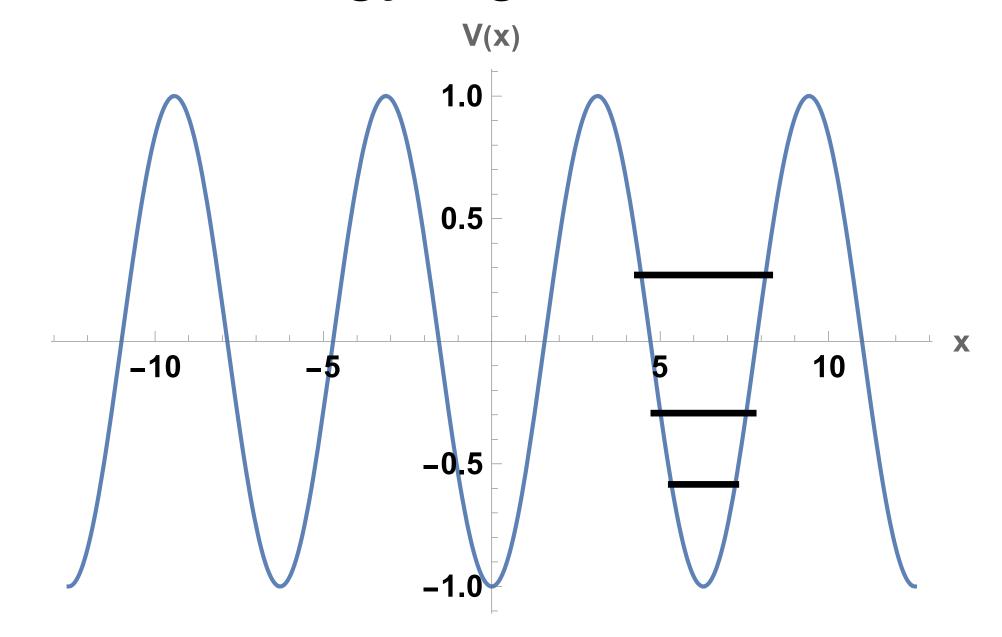


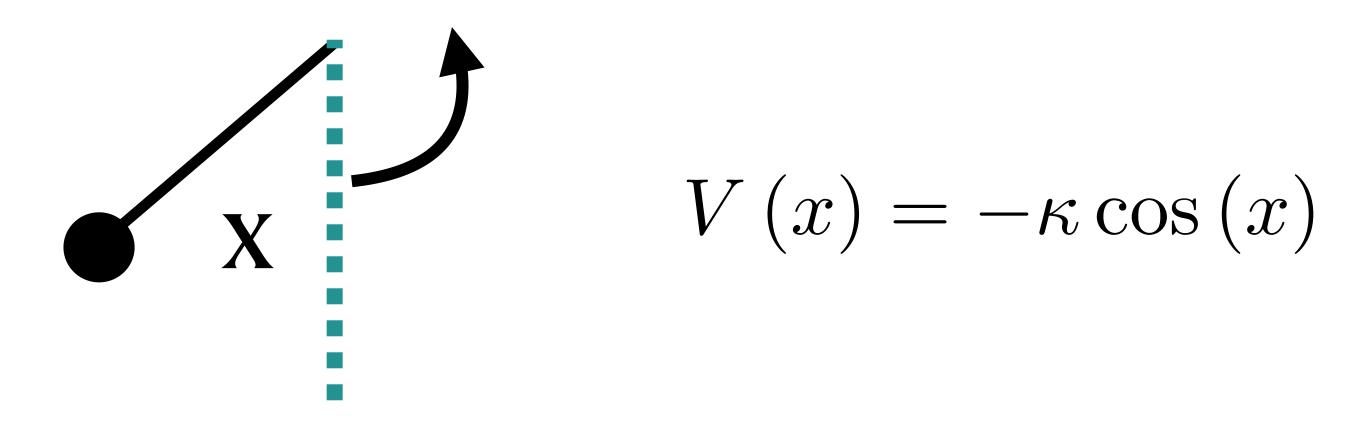


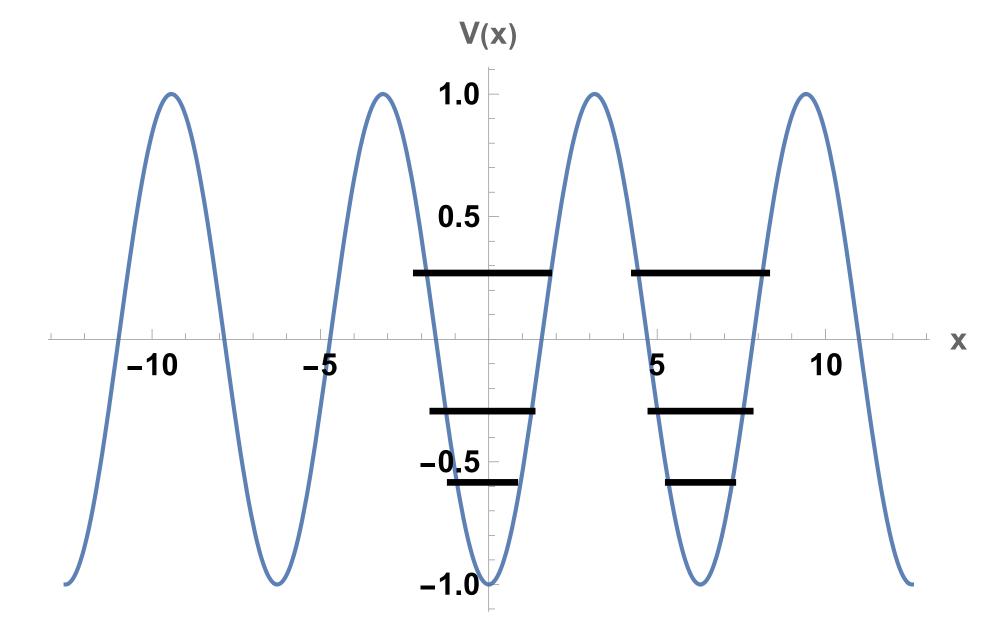


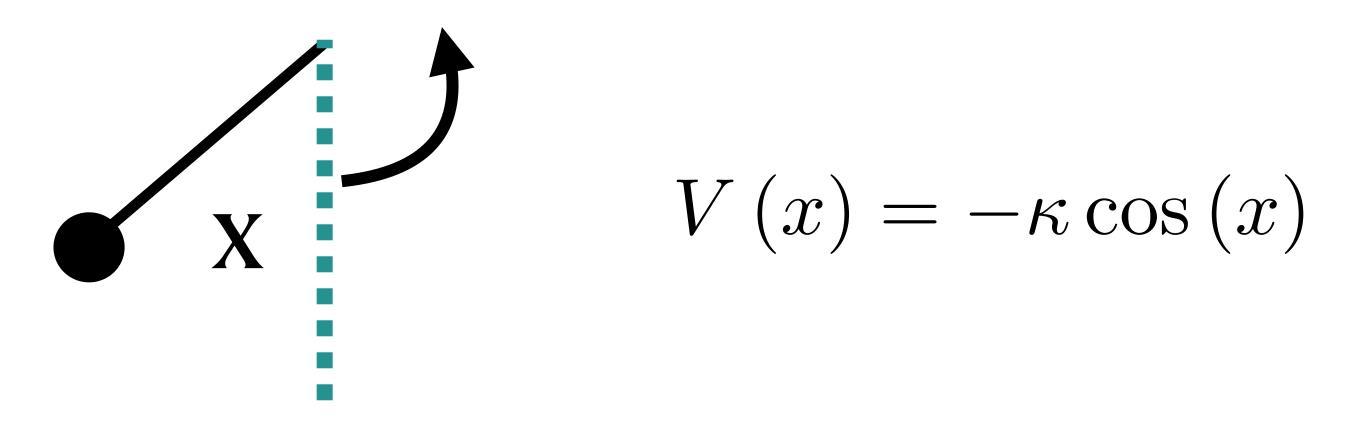




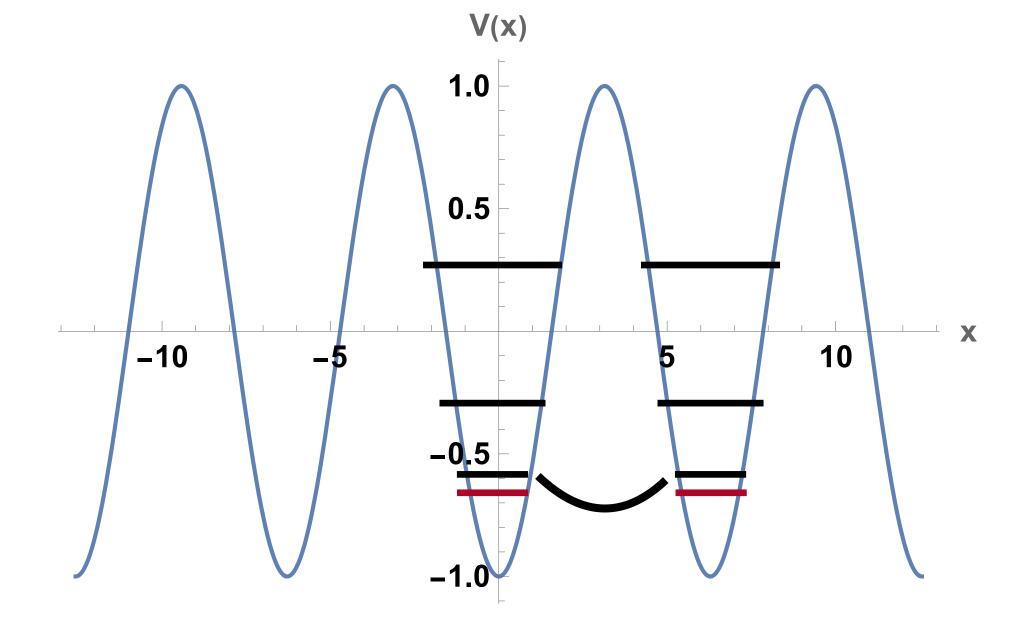




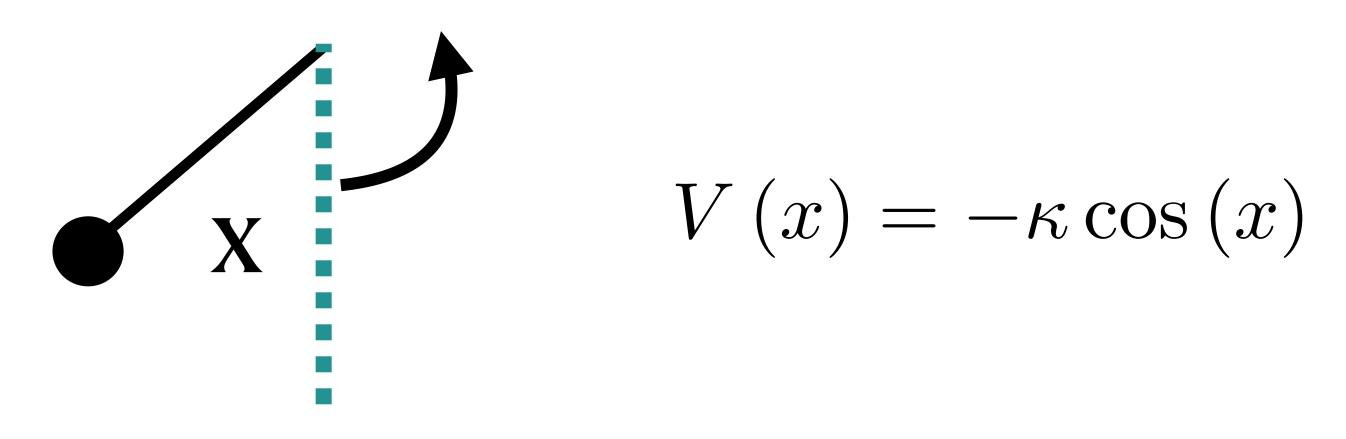




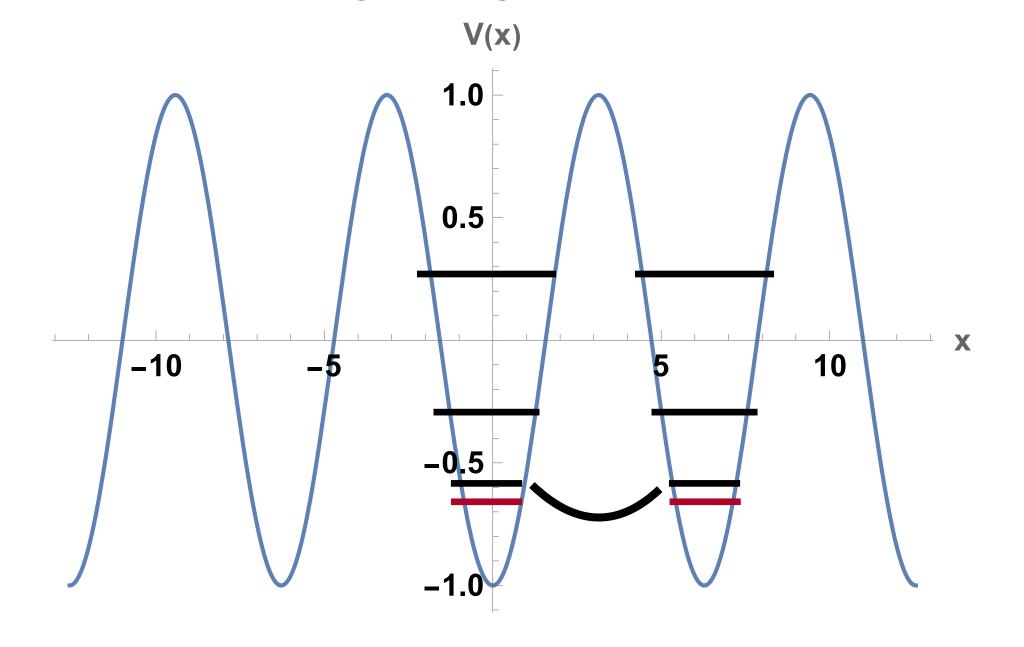
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Quantum Tunneling



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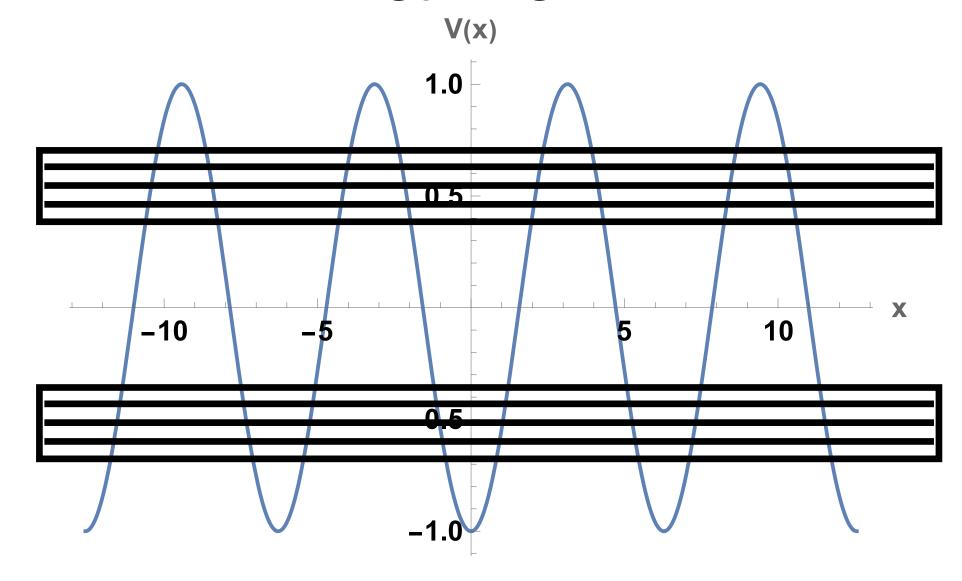
Nearly degenerate ground states

Eigenstates of Parity (x -> -x)

What do the energy eigenstates look like?

Band Structure (Bloch Waves)

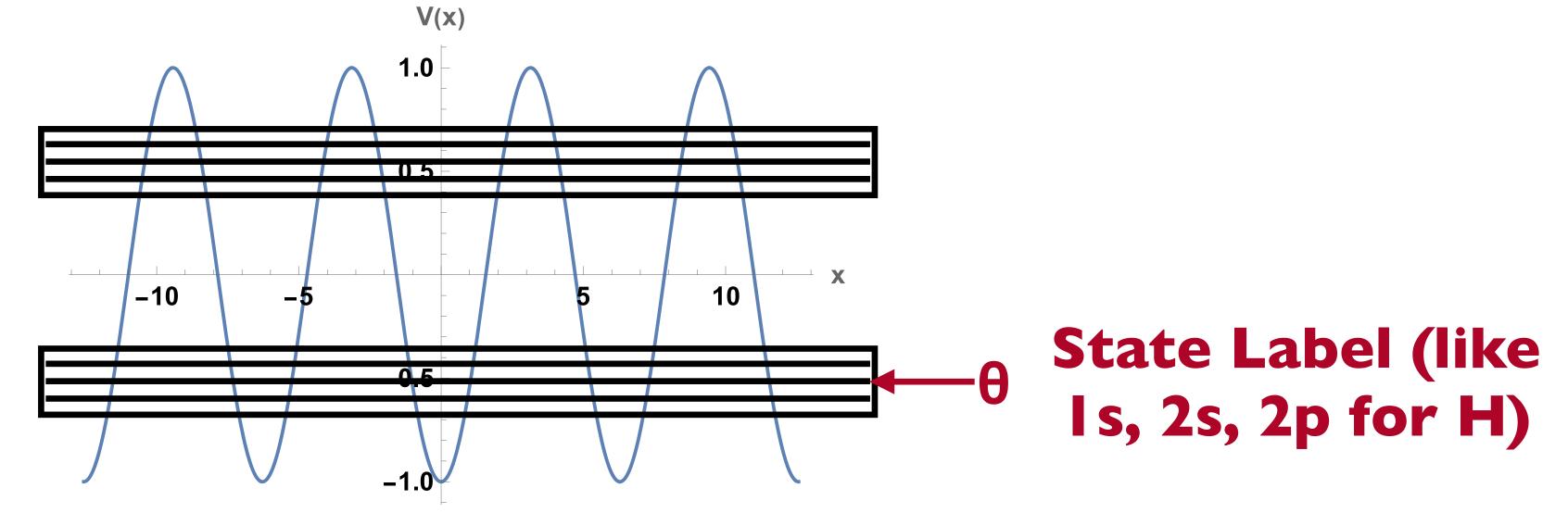
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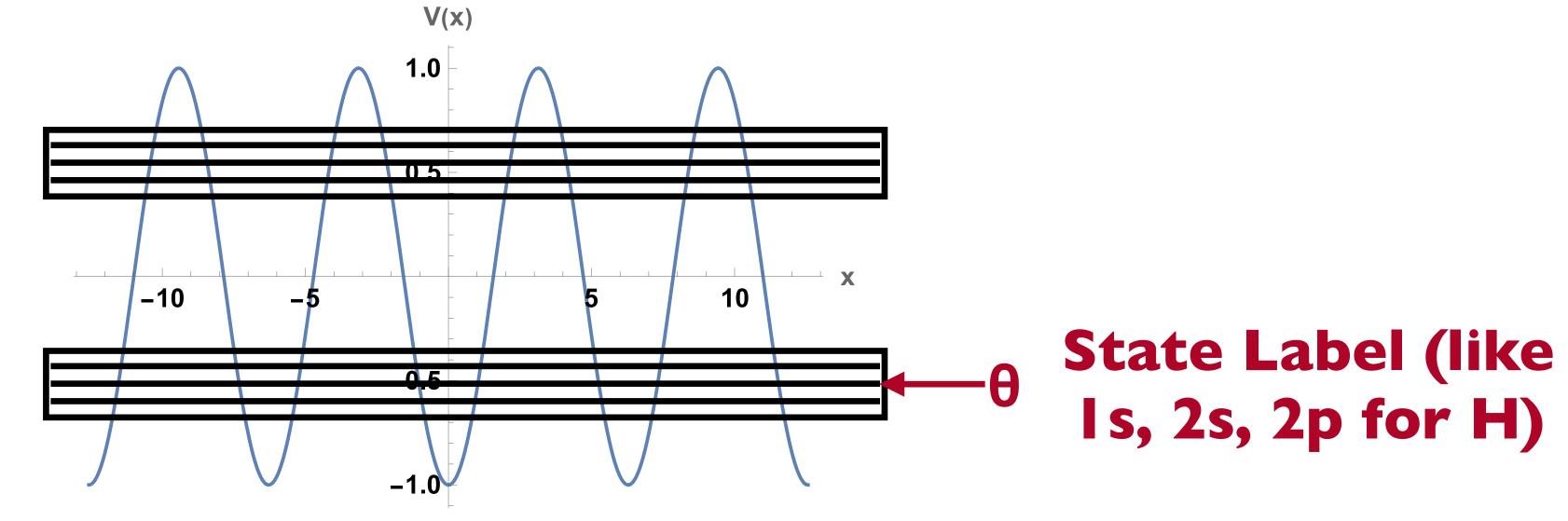


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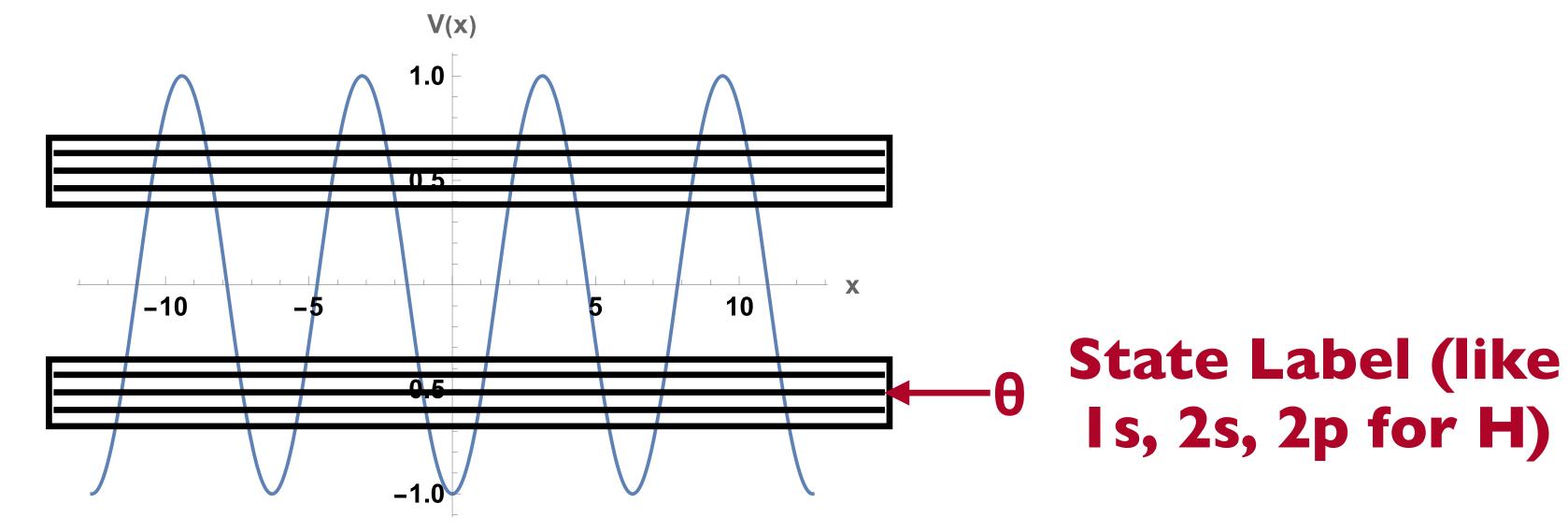
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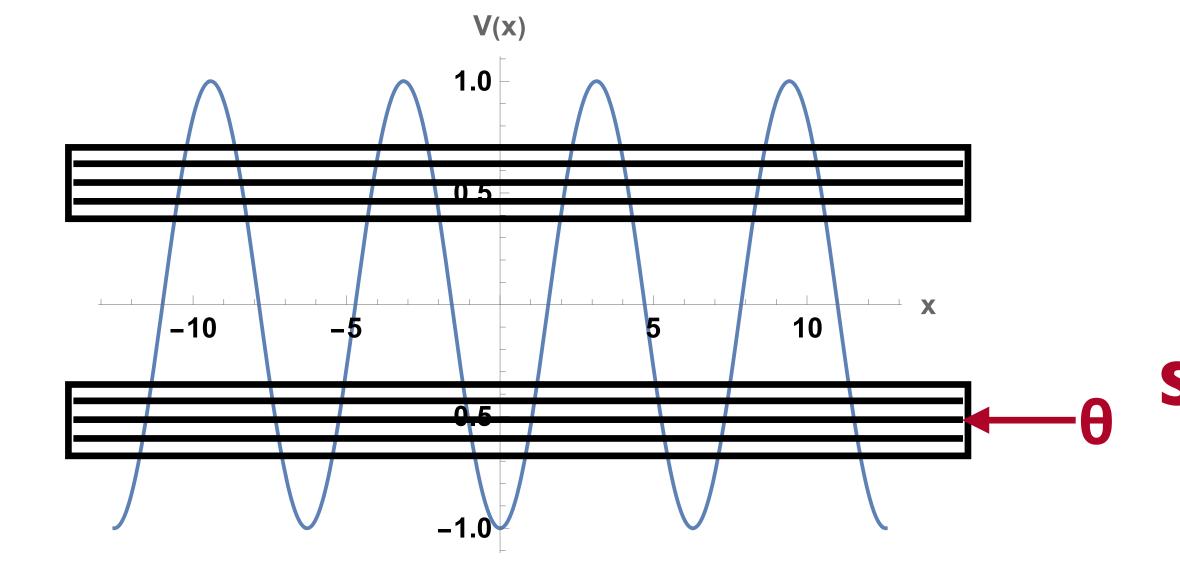
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NO: Hamiltonian already Parity symmetric. States need not be

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State Label (like Is, 2s, 2p for H)

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Only Dynamical Solutions Possible - e.g. couple to photon to allow decay

Hamiltonian To Lagrangian

Given $I\Psi(0)>$, what is $I\Psi(T)>$?

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$$K(y,T;x,0) = \langle y|e^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma e^{i\int_0^T dt L(x,\dot{x})}$$

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But - this is the same quantum problem and thus the same band structure

$$|\Psi(T)\rangle = \int dx \, dy \, f(x) \, K(y, T; x, 0) \, |y\rangle$$

Got to pick initial state - states labelled by θ

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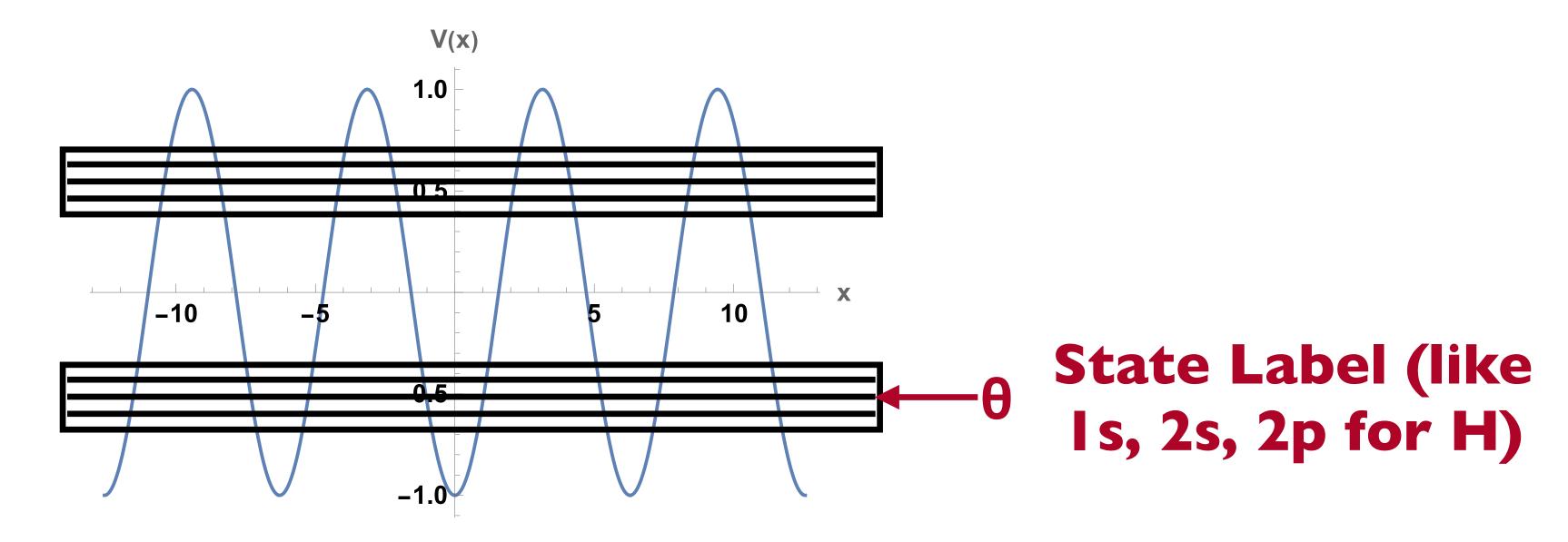
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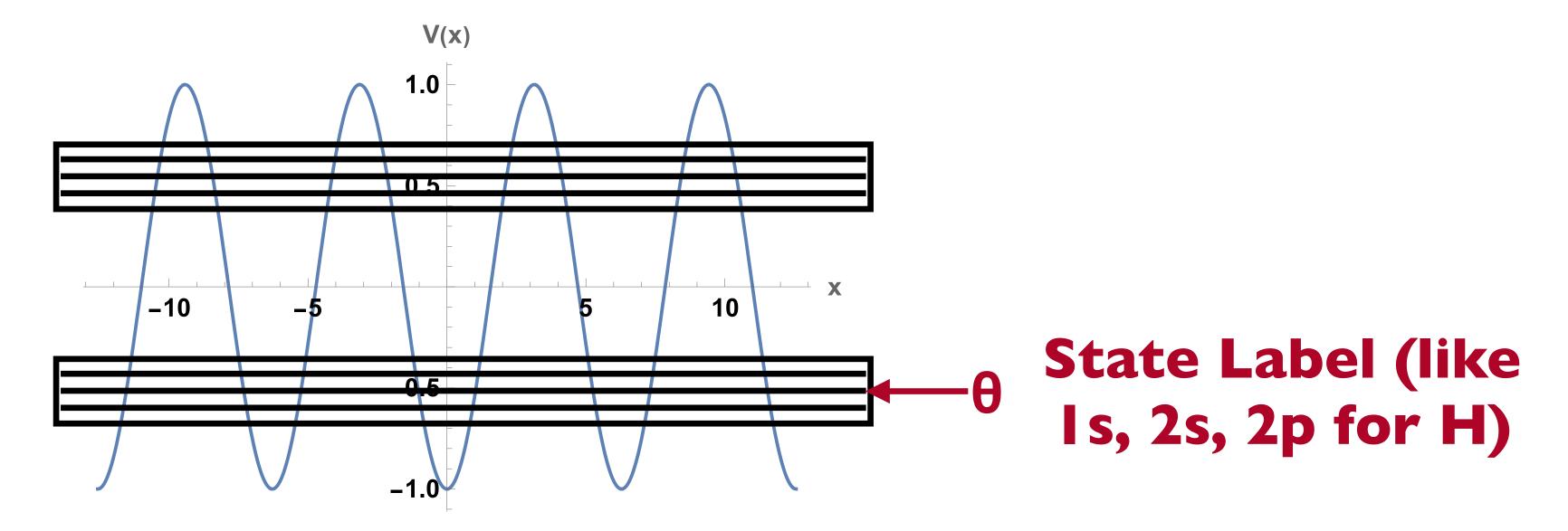
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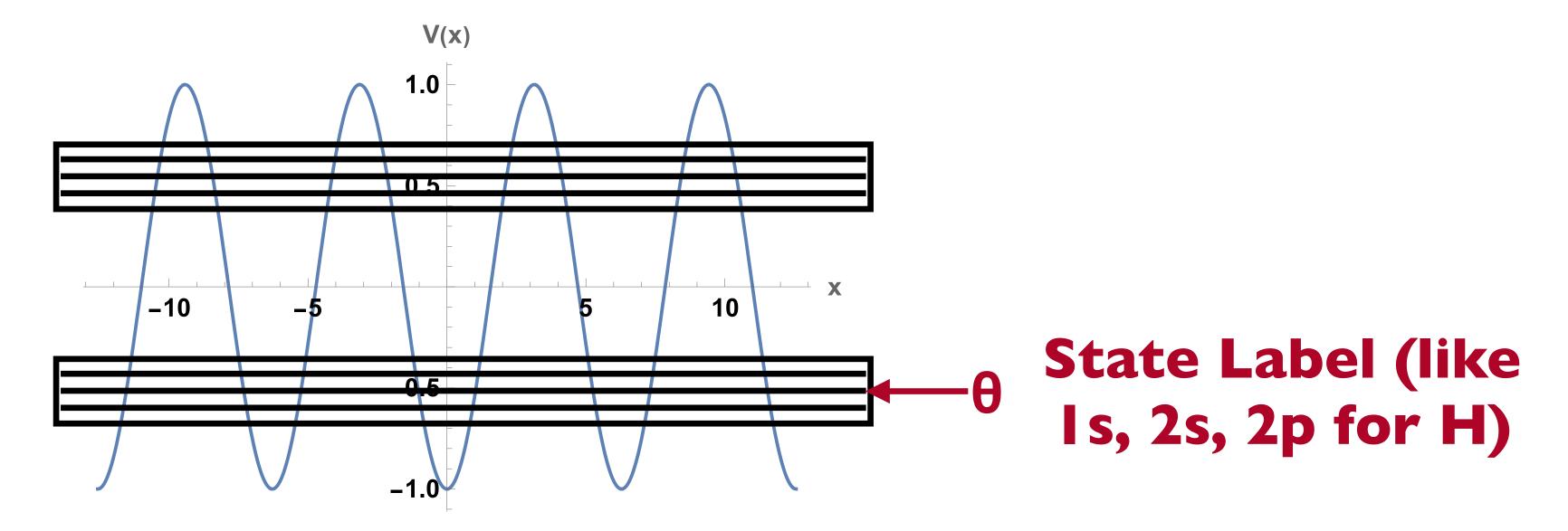


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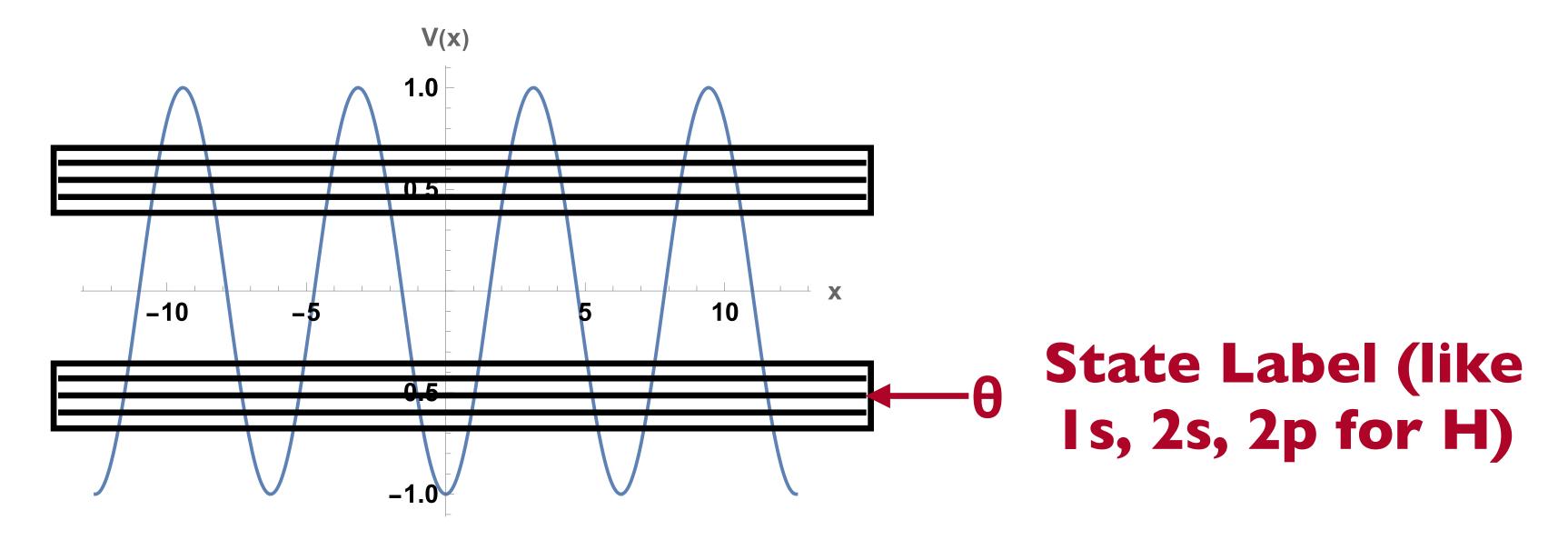
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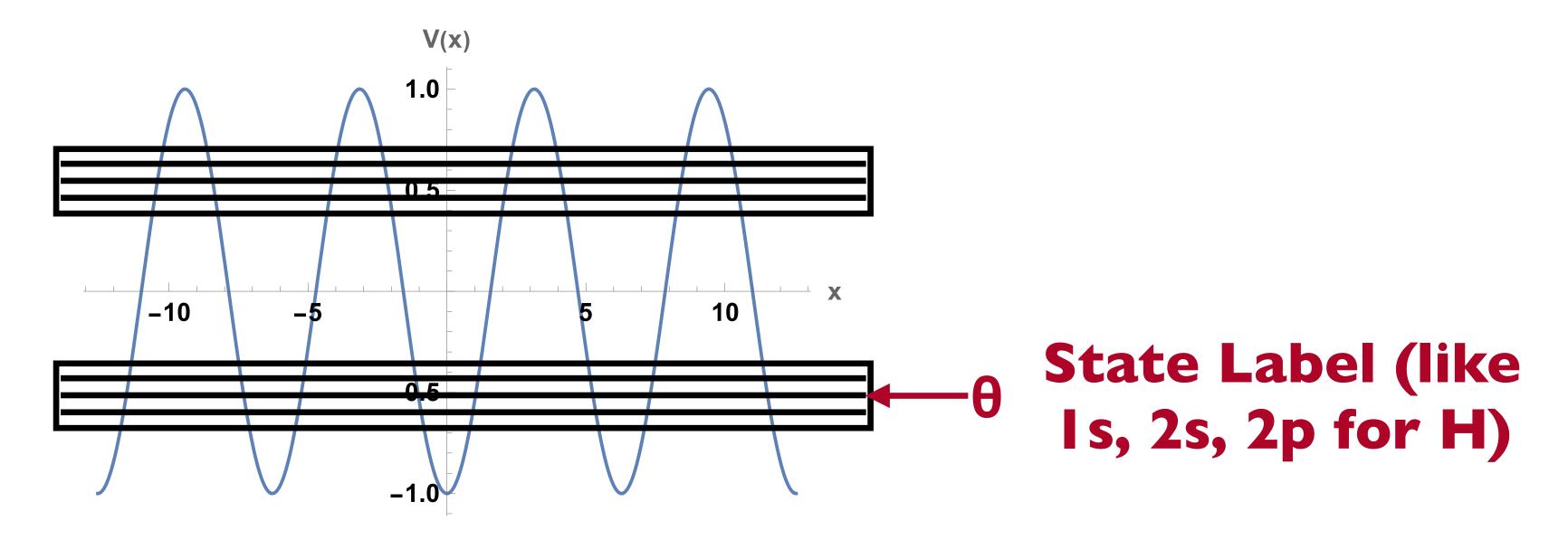


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V(x) still periodic - band structure persists. Choice of quantum state

Need dynamical solution

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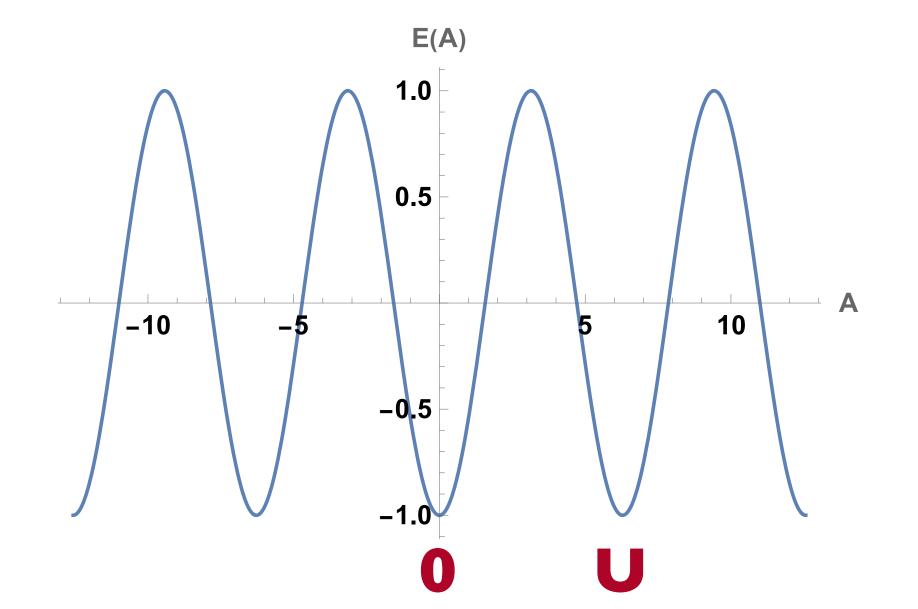
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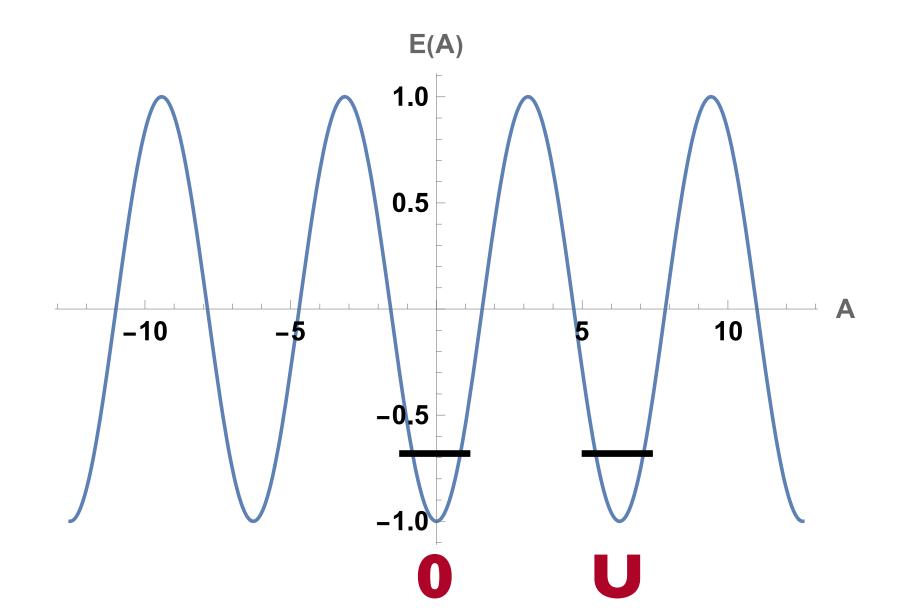
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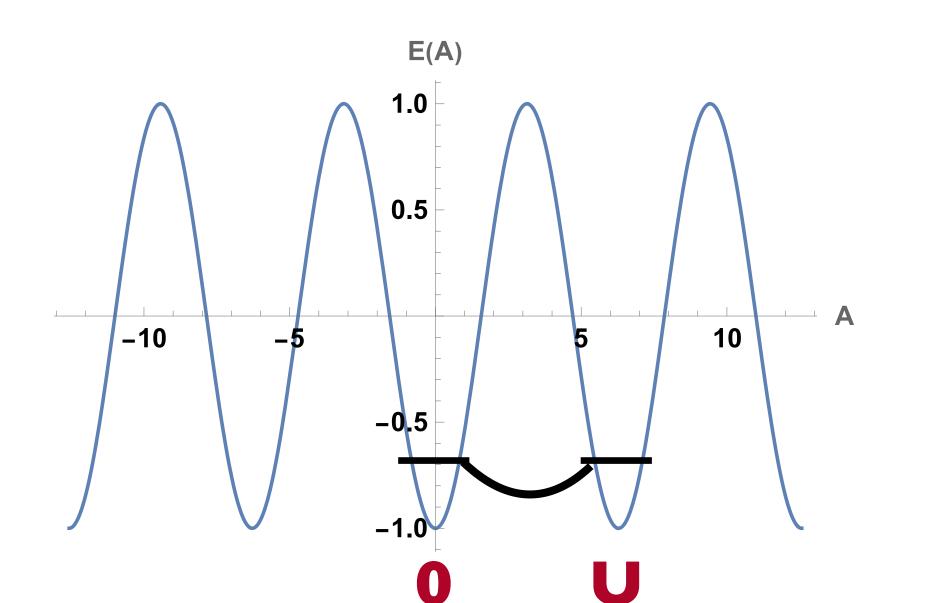
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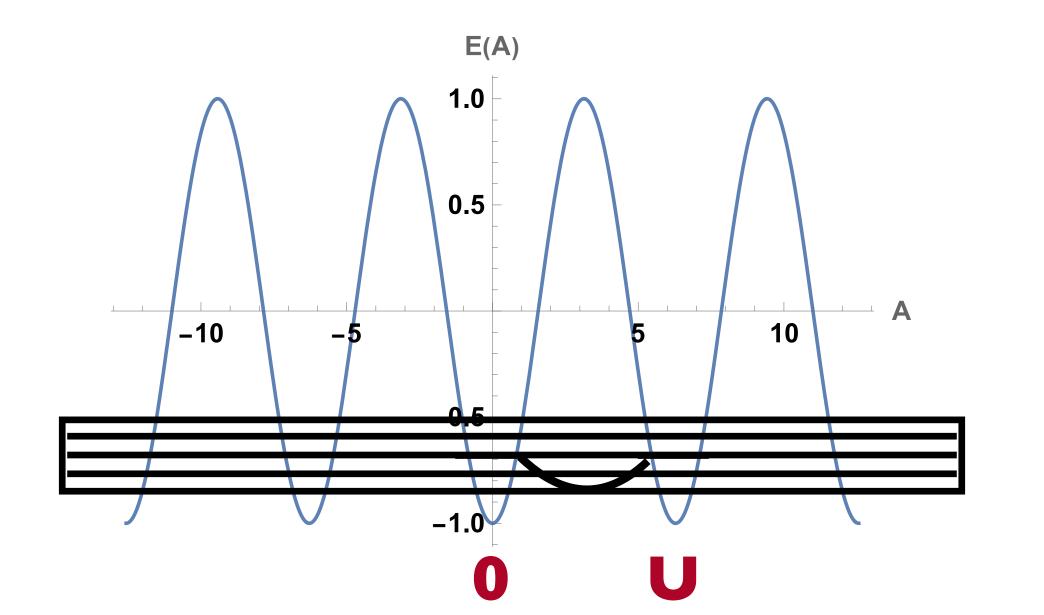
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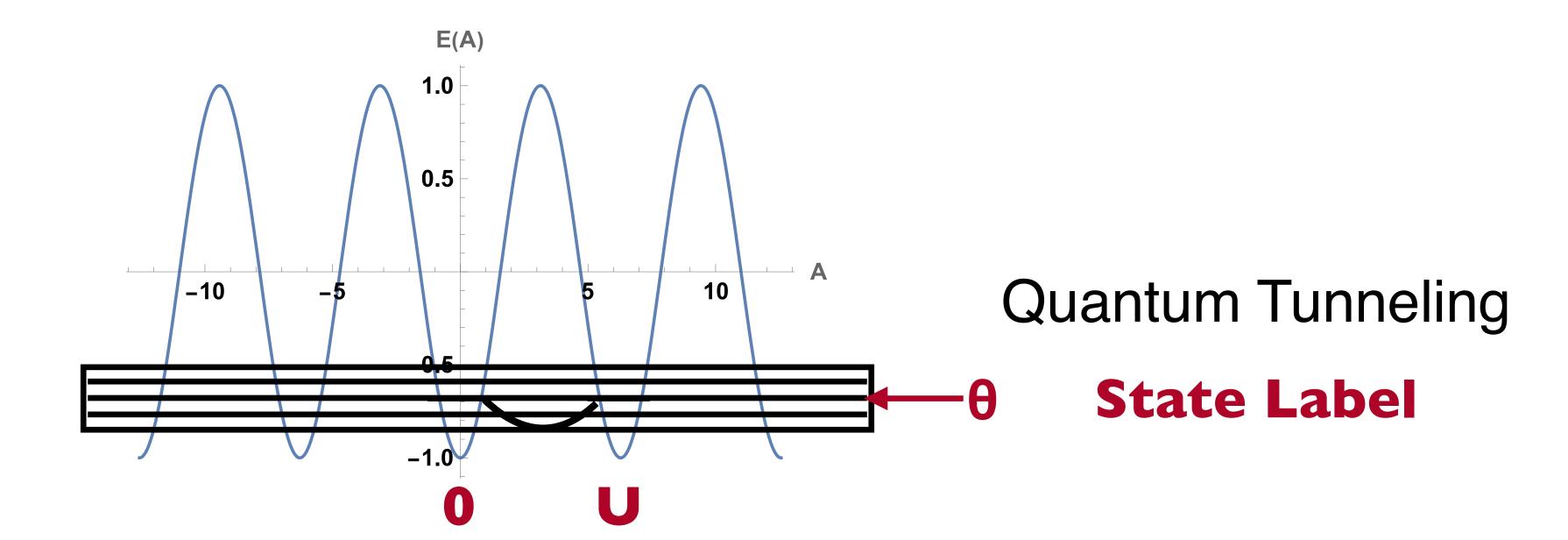
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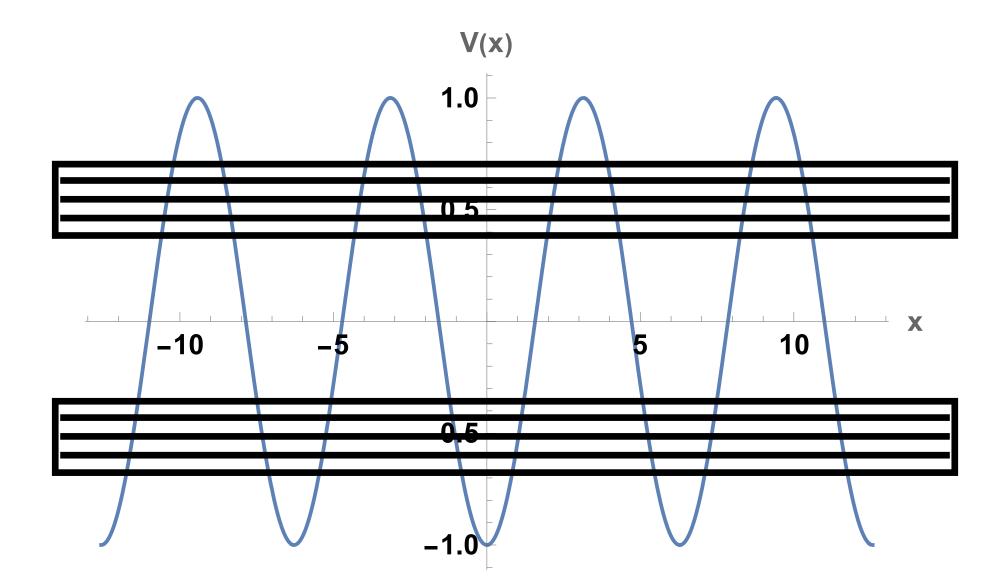
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From state perspective, pure gauge states separated by large gauge transformations still exist - band structure persists.

θ : choice of quantum state, not a parameter controlled by symmetry

Strong CP Problem

Pendulum in Gravity



Start Pendulum at High Energy

Why end up in $\theta = 0$?

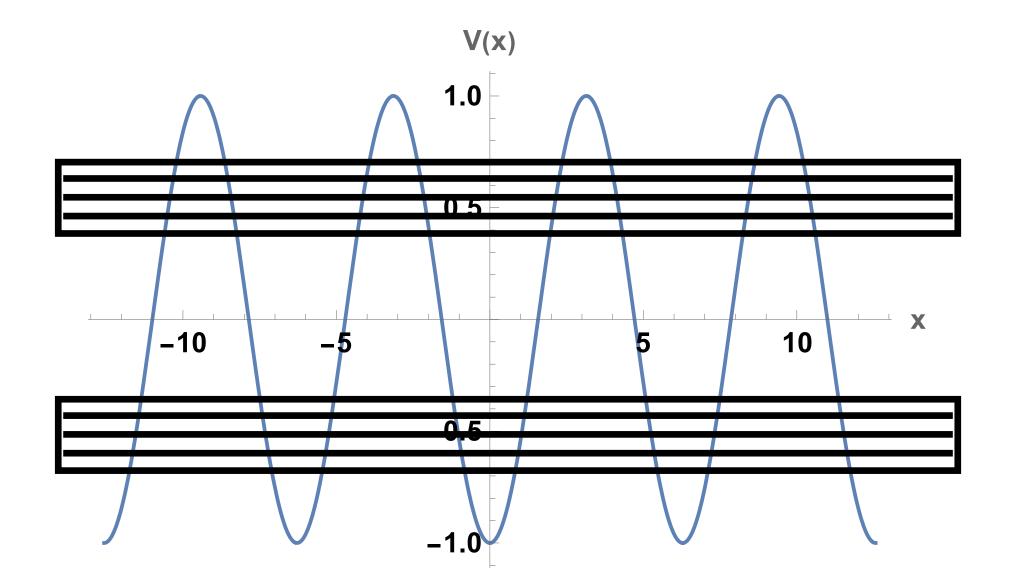
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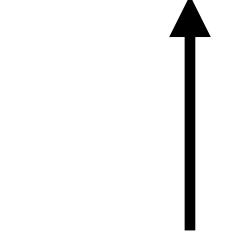
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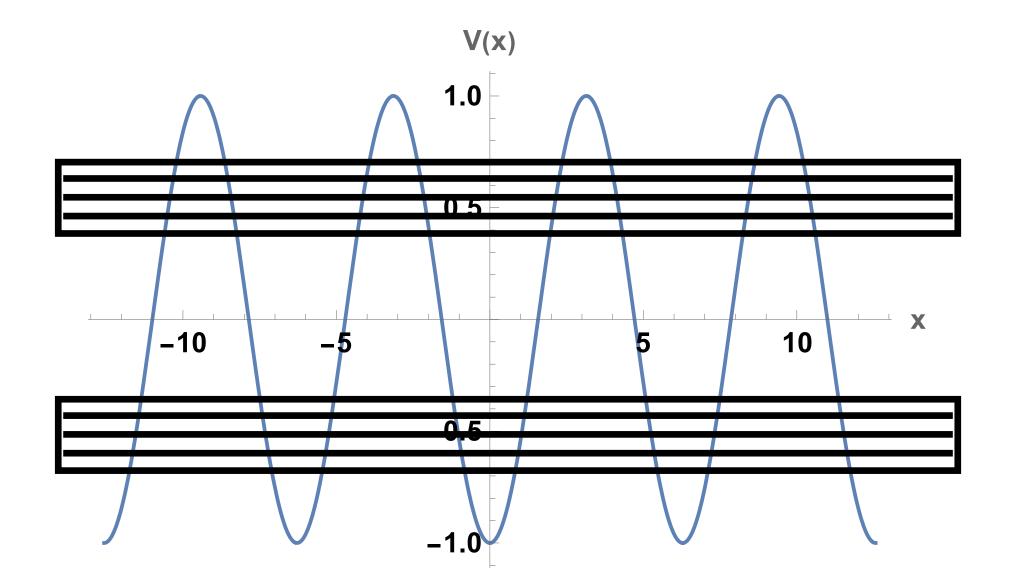


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QCD

Gauge Field A initially has some random value

Why end up in $\theta = 0$?

Need Dynamical Solution

QCD Axion

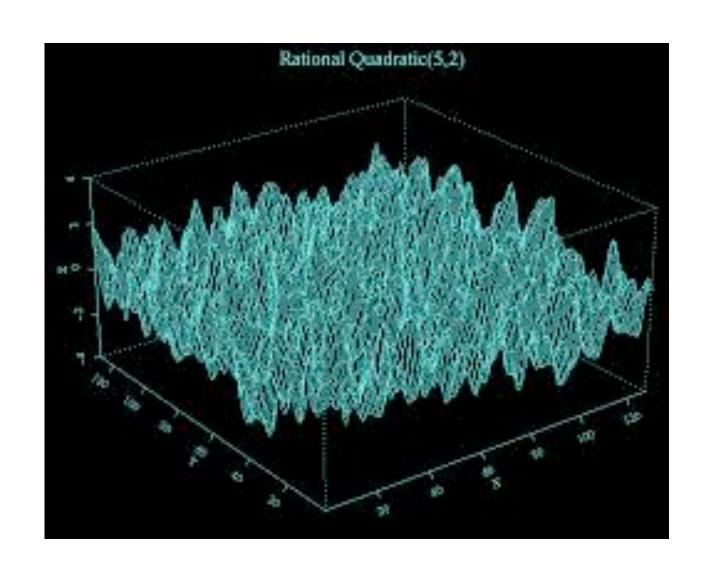
Non-zero cosmological abundance!

Experiment

(With Reza Ebadi, David E. Kaplan and Ron Walsworth)

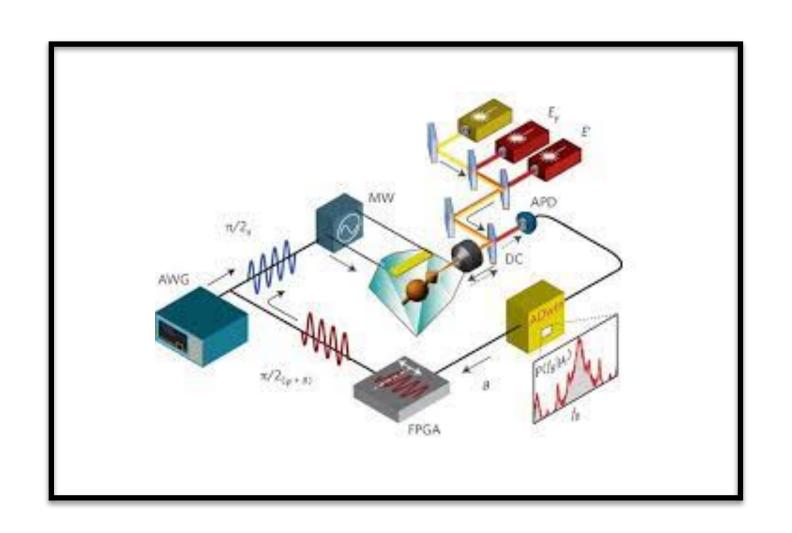
The Axion Landscape

Cosmological Source

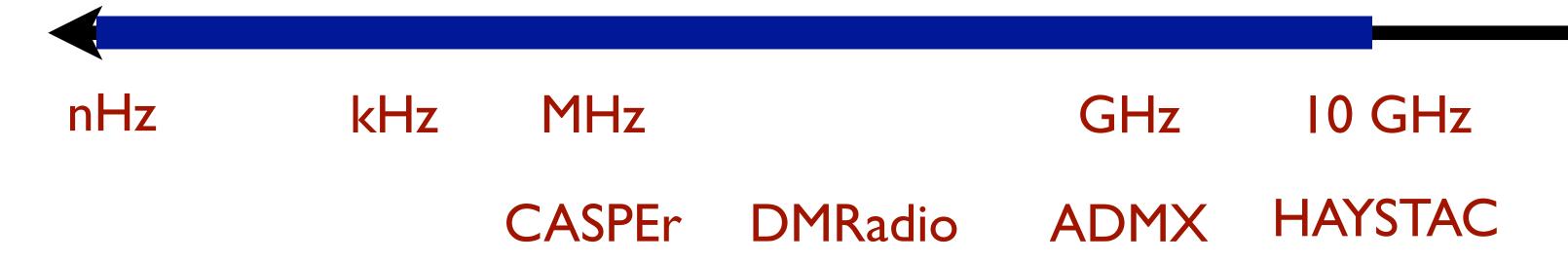


Axion Dark Matter

Oscillating signal, narrow band $(Q \sim 10^6)$

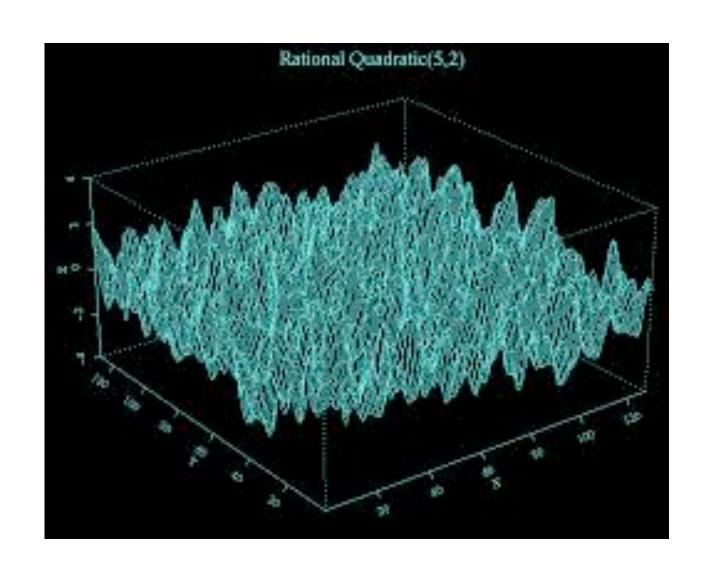


Detect Using suitable Resonator



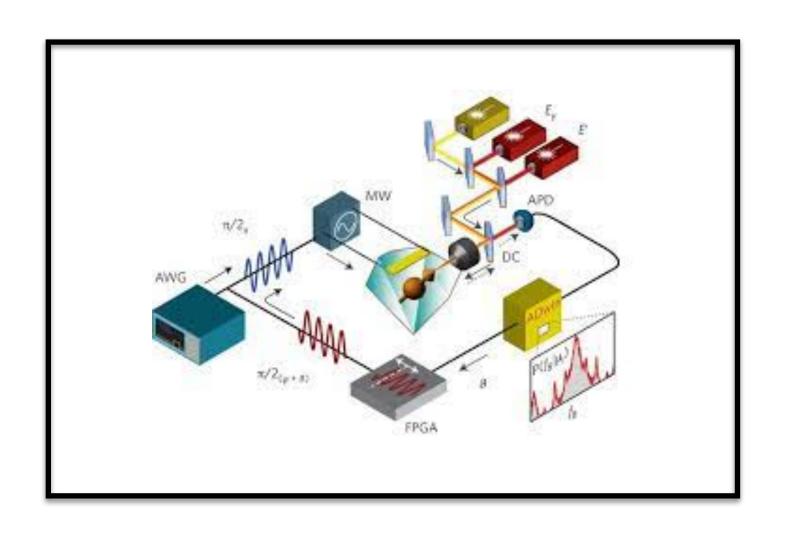
The Axion Landscape

Cosmological Source

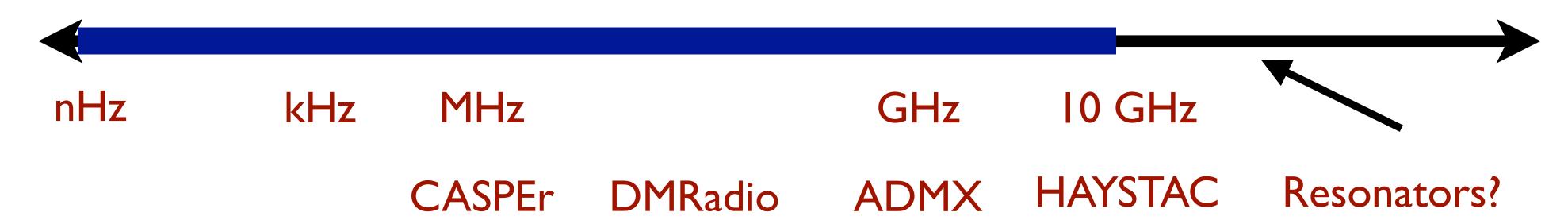


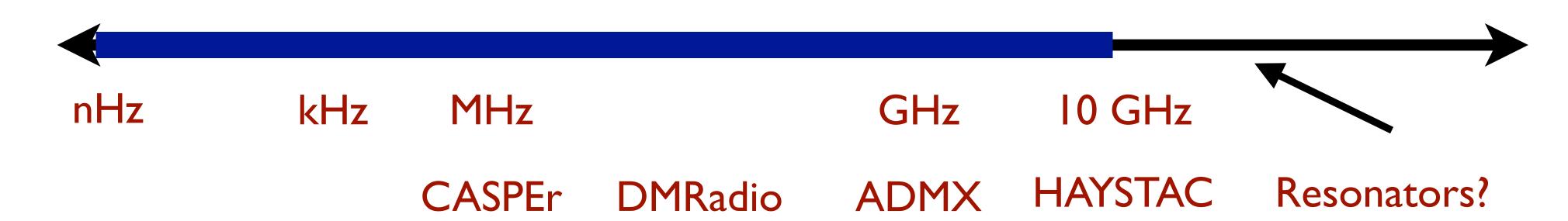
Axion Dark Matter

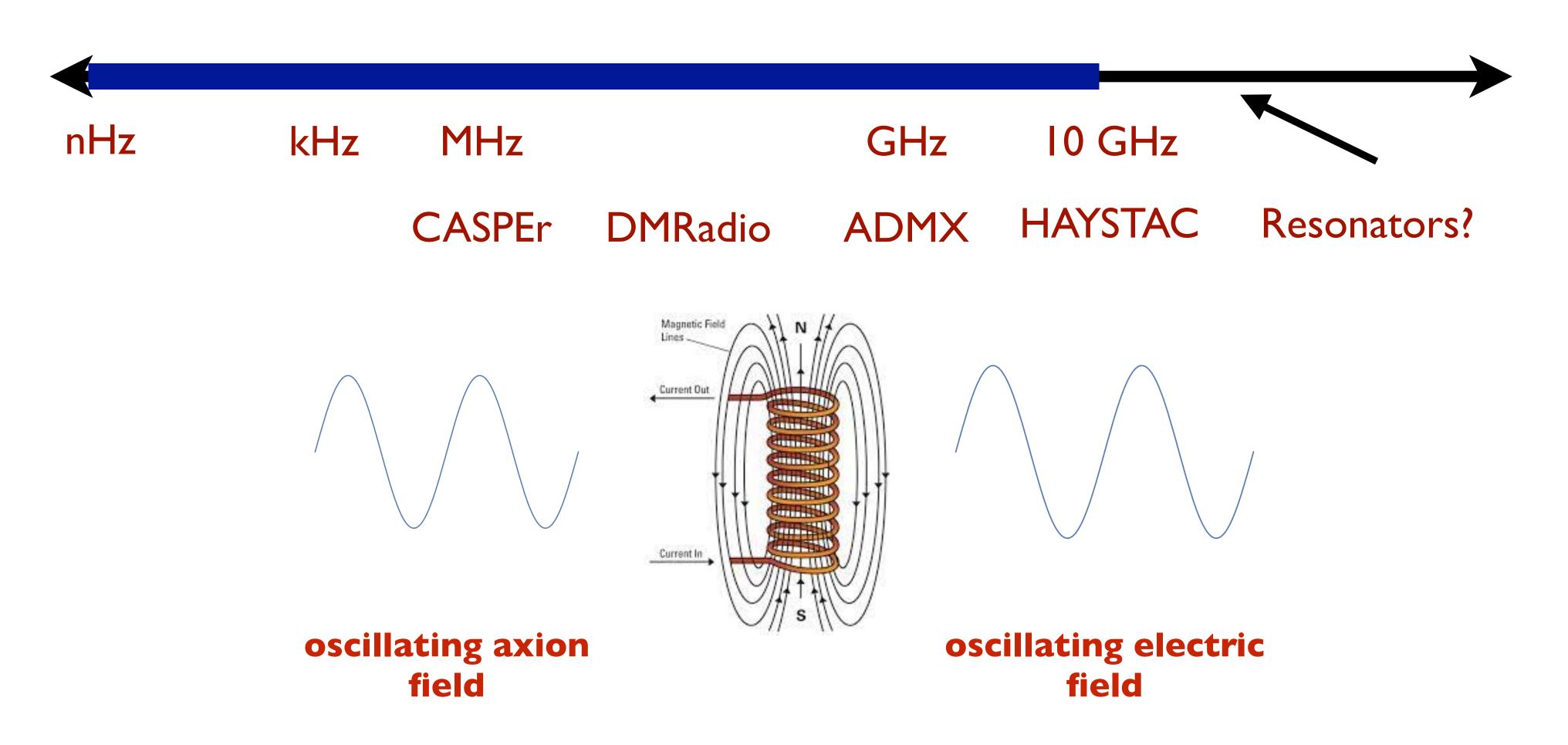
Oscillating signal, narrow band (Q ~ 106)

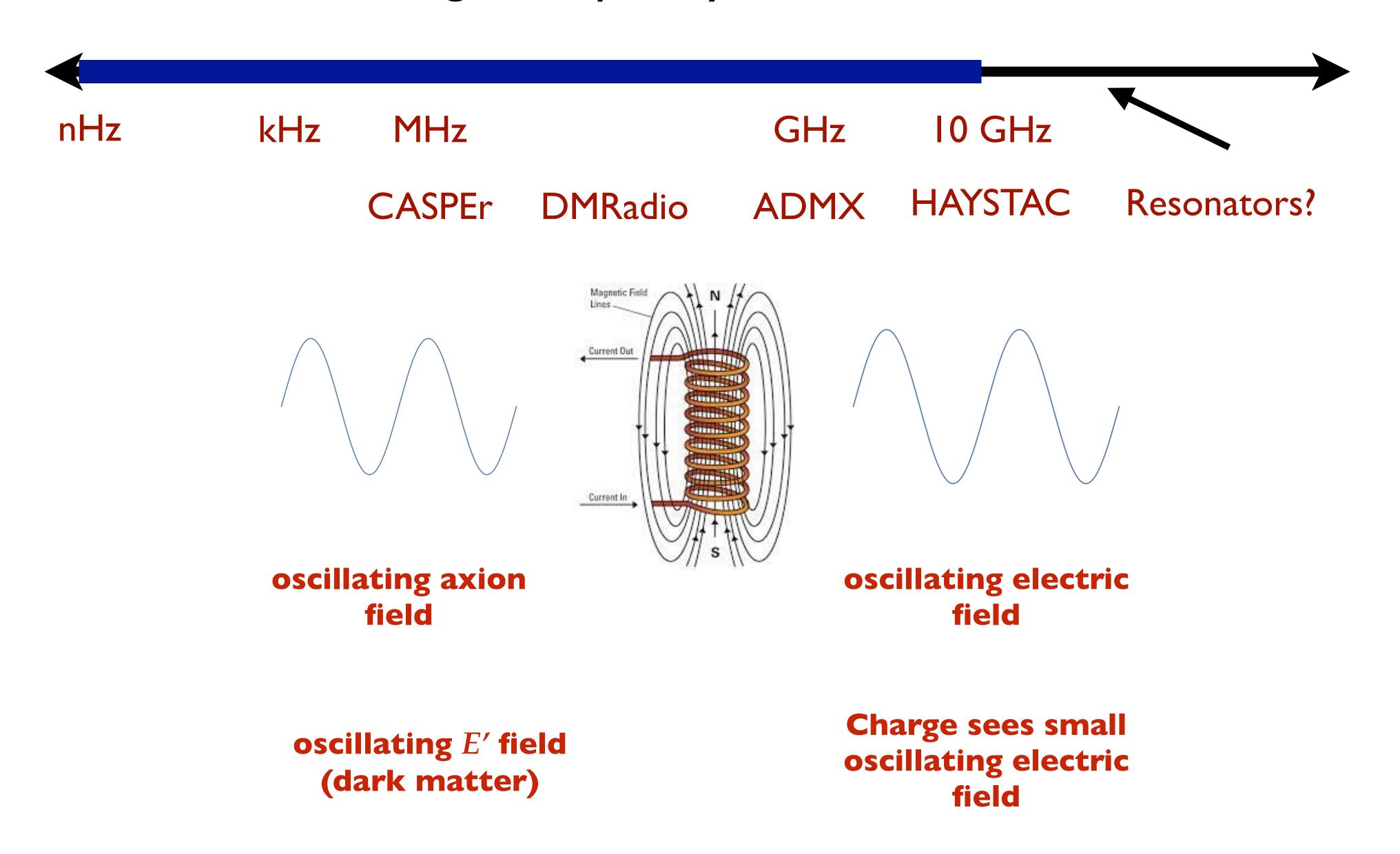


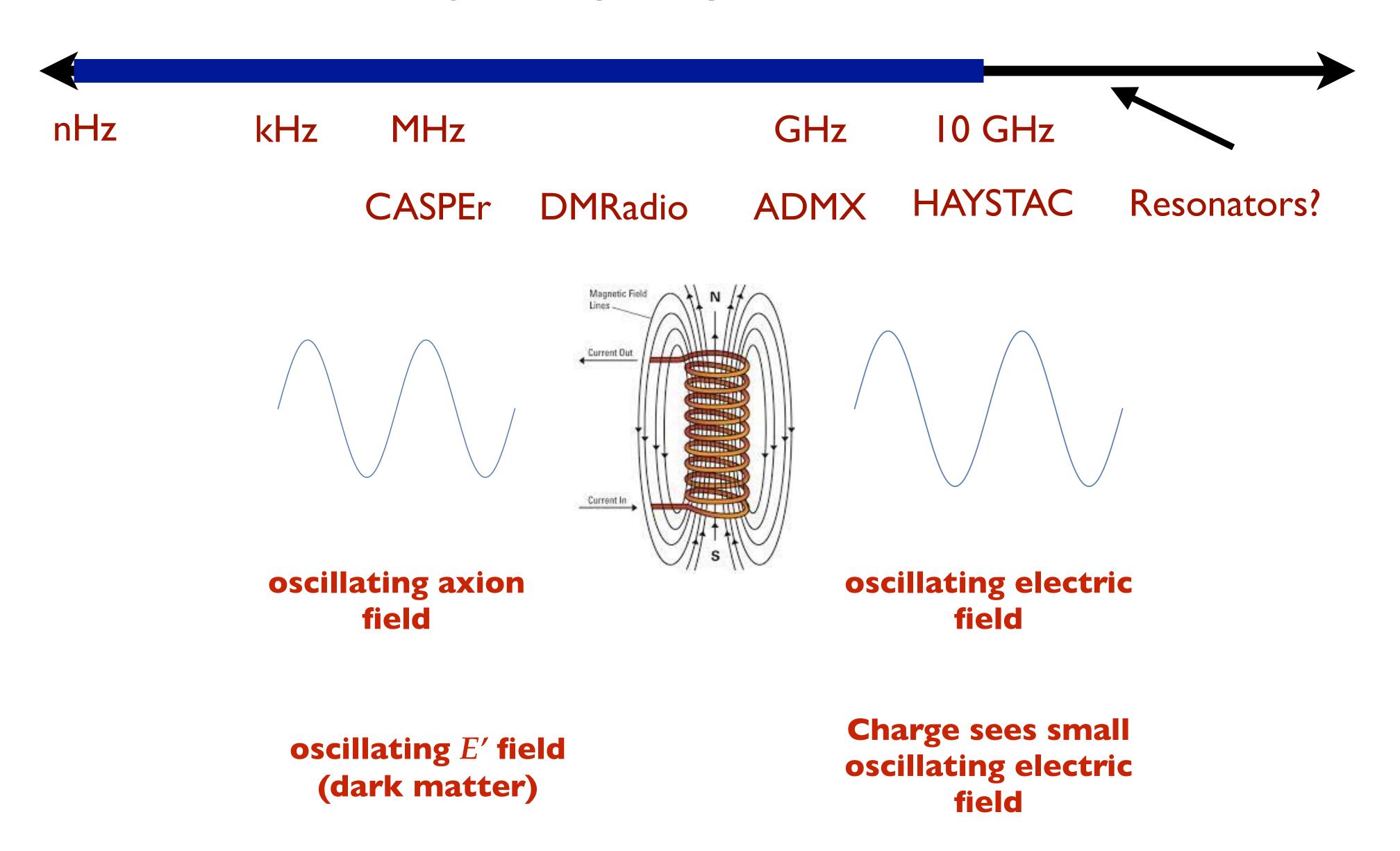
Detect Using suitable Resonator











Detect high frequency oscillating electric field

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Crystal with index of refraction with a linear dependence on electric field (e.g. Lithium Niobate)

Create Optical Cavity with Lithium Niobate - choose length to set resonance frequency

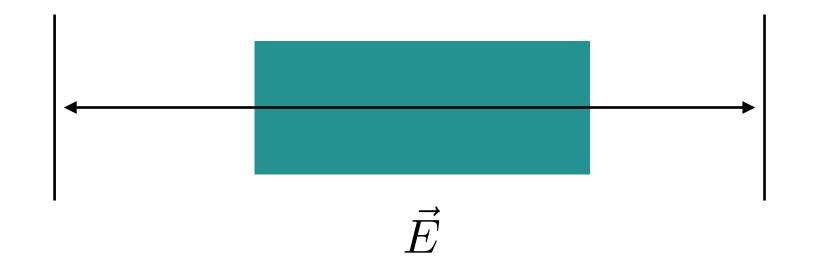
Send light through optical resonator - measure phase shift

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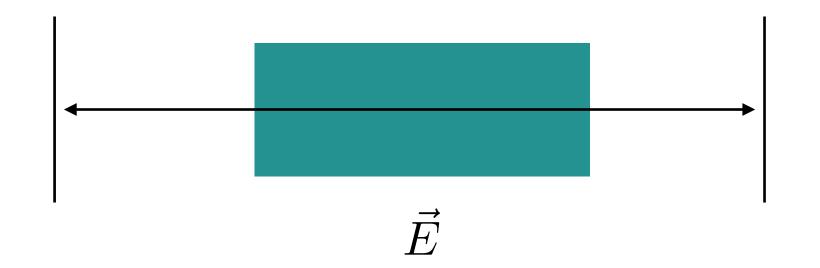
Phase shift depends on index of refraction - measures time varying electric field

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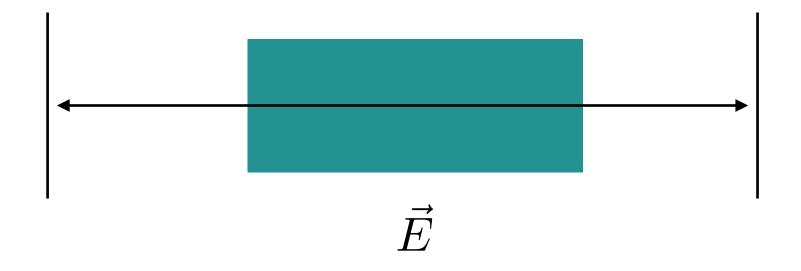
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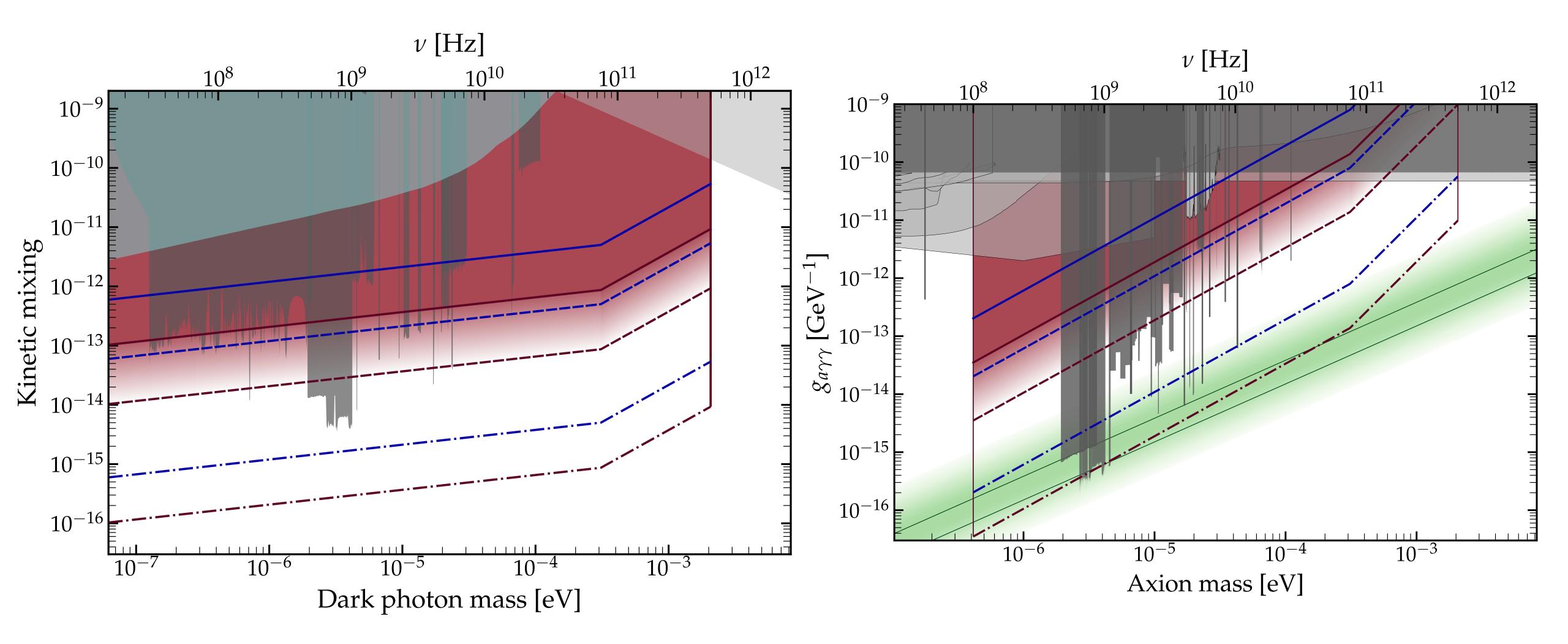


Phase shift depends on index of refraction - measures time varying electric field

Lowest frequency set by absorption length of light (~km)

High frequency cut-off: Nyquist limit, response time of crystal (> THz)

Projected Sensitivity



Conservative Solid Lines: 1 s averaging Super aggressive dot dashed: ~ 1 yr averaging, 10 db Squeezing, 10 W

Conclusions

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1. Strong CP Problem requires dynamical solution, strongly boosts searches for QCD Axion

2. Exciting opportunities with nonlinear optical elements