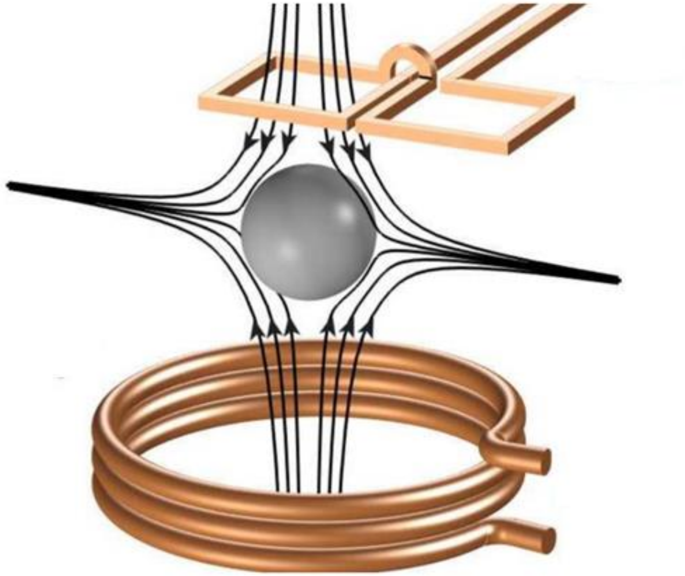
Maglev for Dark Matter



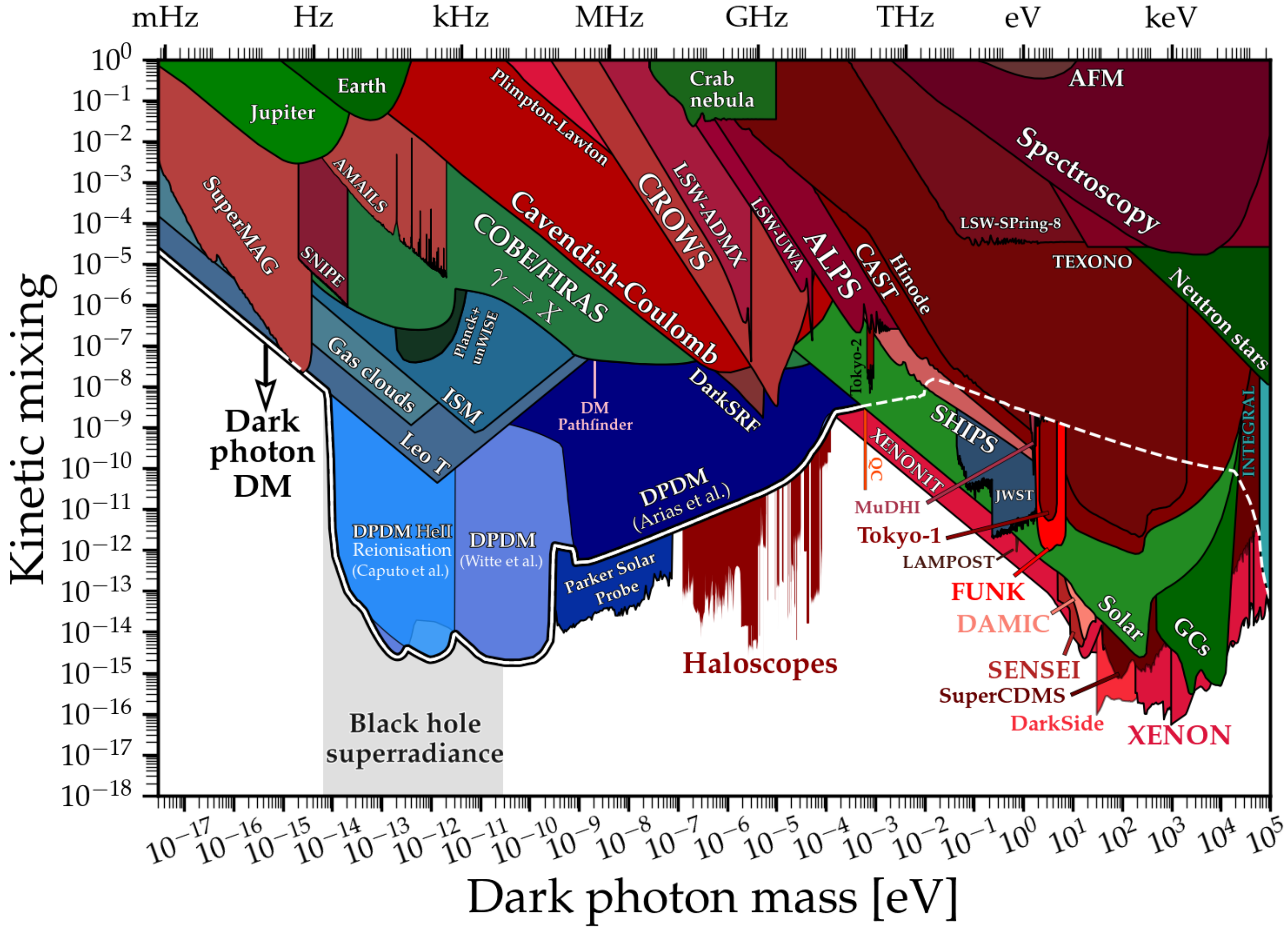
Saarik Kalia

19th Patras Workshop

September 17, 2024

Based on G. Higgins, S. K., and Z. Liu, Phys. Rev. D **109** (2024) 055024
and S. K., D. Budker, et al., arXiv:2408.15330

Introduction



[Caputo et al., Phys. Rev. D. **104**, 095029 (2021)]

Introduction

- Ultralight DM interactions can generate AC magnetic fields:
  - Dark photon kinetic mixing
  - Axion-photon coupling
  - Axion-electron coupling
- Many experiments utilize EM resonances → $f_{\mathrm{DM}} \gtrsim \mathrm{kHz}\ (m_{\mathrm{DM}} \gtrsim 10^{-12}\,\mathrm{eV})$
- Can use mechanical resonance for lower frequencies
- Mechanical system + sensitive to magnetic fields → magnetic levitation

Outline

- Dark matter candidates
- Magnetic levitation
  - Levitated superconductors
  - Levitated ferromagnets
- Noise sources
- Sensitivity

Kinetically mixed dark photon

- Massive vector A'^{μ}
- Non-relativistic → $\boldsymbol{A}'$ uniform in space, oscillates with frequency $m_{A'}$
- Coupled to EM via $\varepsilon m_{A'}^2 A'^{\mu} A_{\mu}$ → effective current $J_{\text{eff}}^{\mu} = -\varepsilon m_{A'}^2 A'^{\mu}$
- Can source EM fields via Ampère's Law

$$\nabla \times \boldsymbol{B} - \partial_t \boldsymbol{E} = \boldsymbol{J}_{\text{eff}}$$

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$$\nabla \times \boldsymbol{B} - \cancel{\partial_t \boldsymbol{E}} = \boldsymbol{J}_{\text{eff}}$$

- When λ_{DM} larger than apparatus, $\boldsymbol{E}$ negligible → only $\boldsymbol{B}$ signal

Axionlike particle

- Massive pseudoscalar a (oscillates with mass m_a)
- Axion-photon coupling $g_{a\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu}$
  - Effective current $\boldsymbol{J}_{\text{eff}} = i g_{a\gamma} m_a a \boldsymbol{B}_0$ → similar to dark photon
  - For levitated superconductor, trap can act as $\boldsymbol{B}_0$!
  - For levitated ferromagnet, magnet can act as $\boldsymbol{B}_0$!

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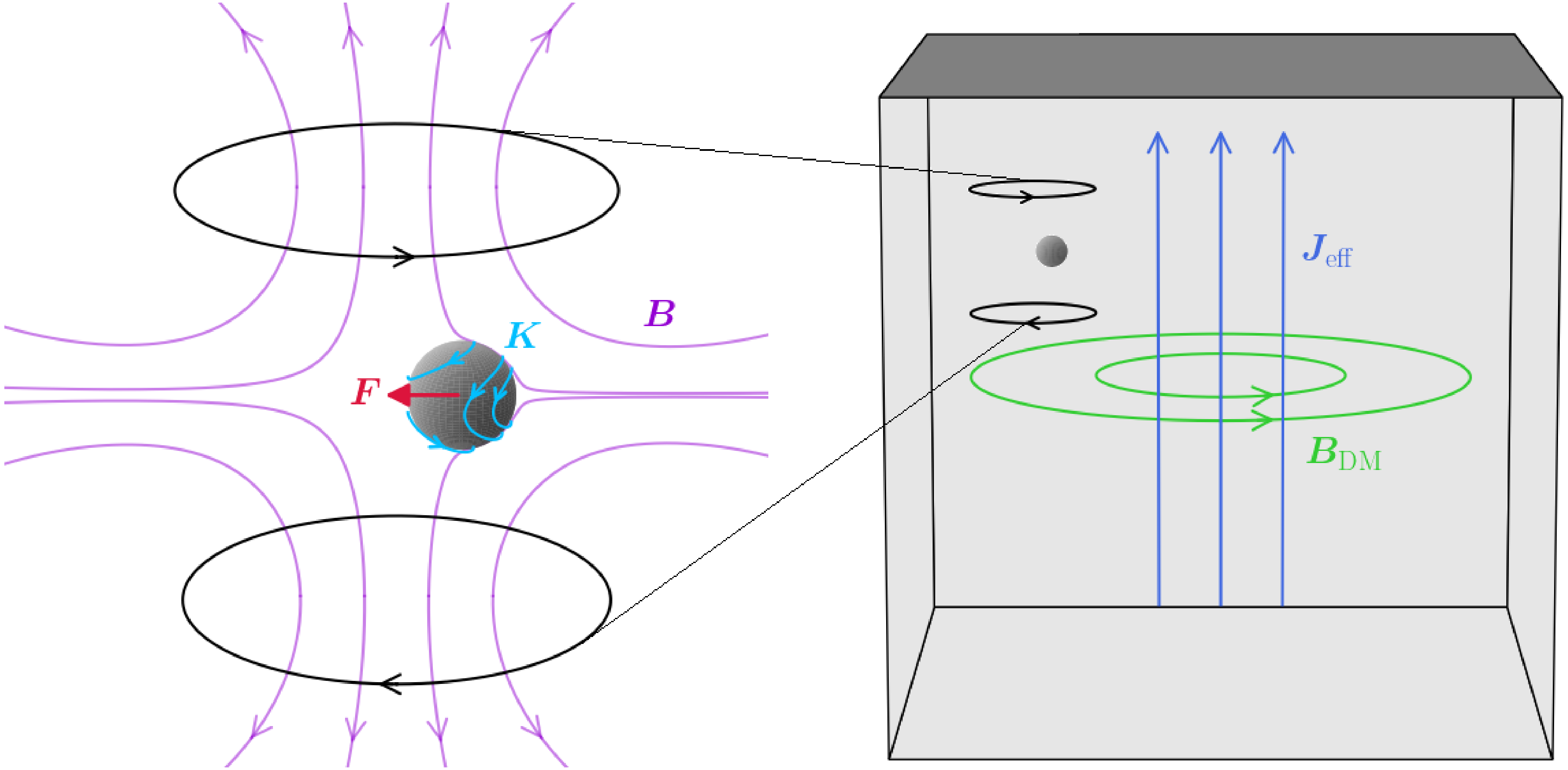
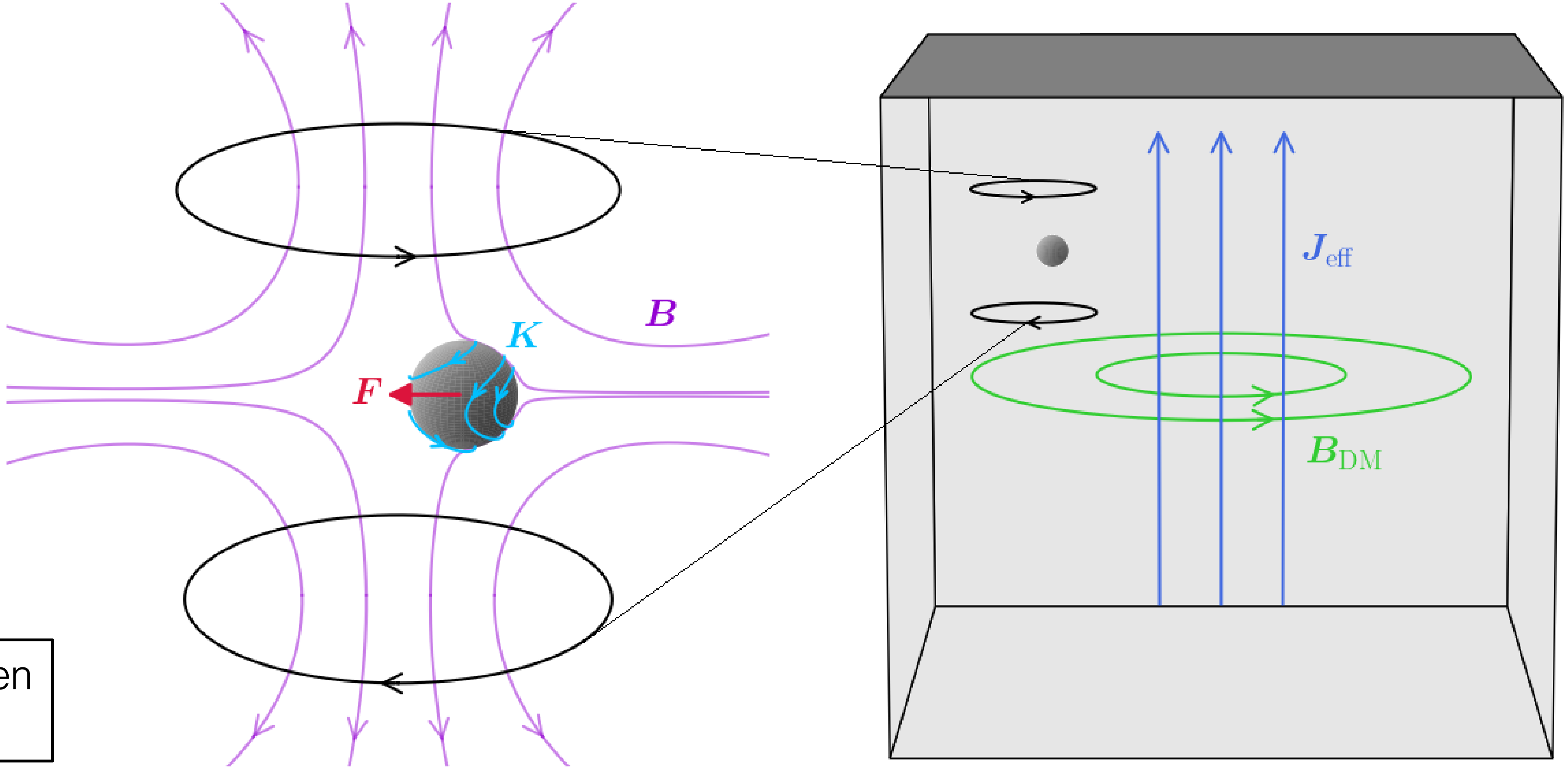
  - For levitated ferromagnet, magnet can act as $\boldsymbol{B}_0$!
- Axion-electron coupling $\frac{g_{ae}}{2m_e} \partial_\mu a \bar{\psi}_e \gamma^\mu \gamma_5 \psi_e$
  - Causes precession of electron spins
  - Effective magnetic field $\boldsymbol{B}_{ae} = -\frac{g_{ae}}{e} \nabla a$

Levitated superconductors

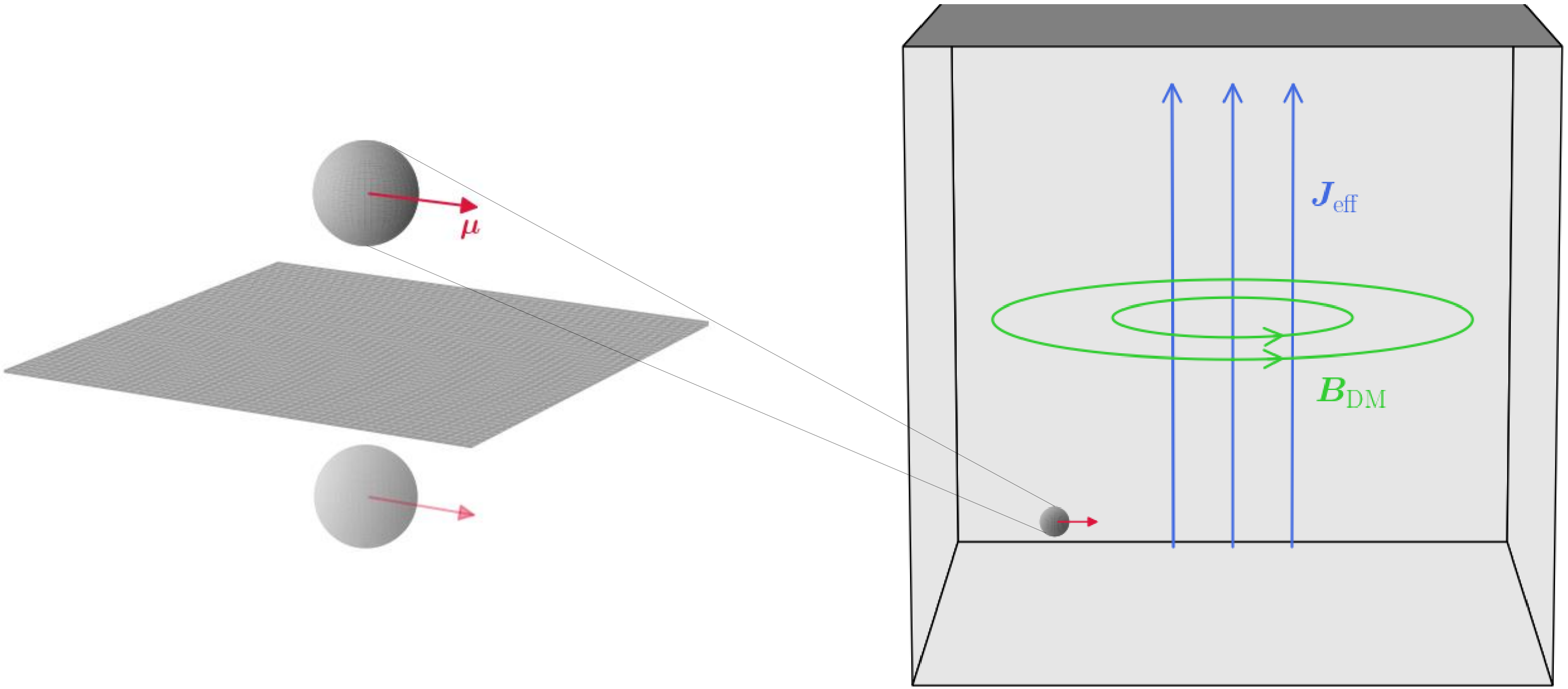


Levitated superconductors

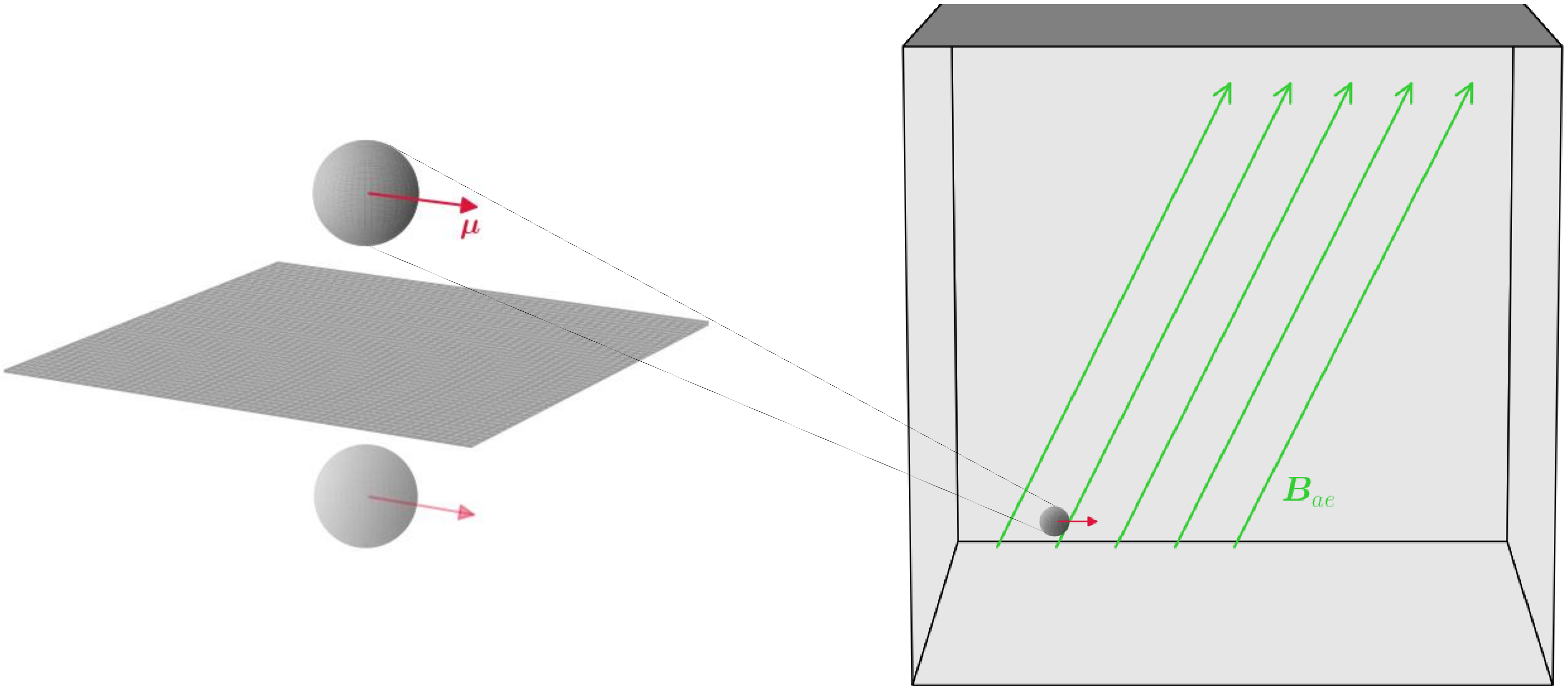


Resonant when
$m_{\rm DM} = f_0$

Levitated ferromagnets

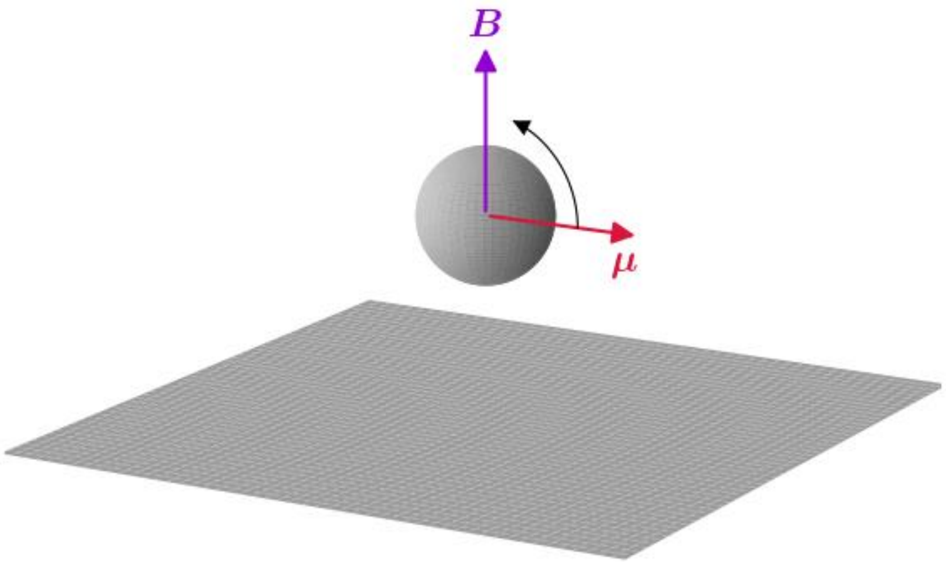


Levitated ferromagnets (axion wind)



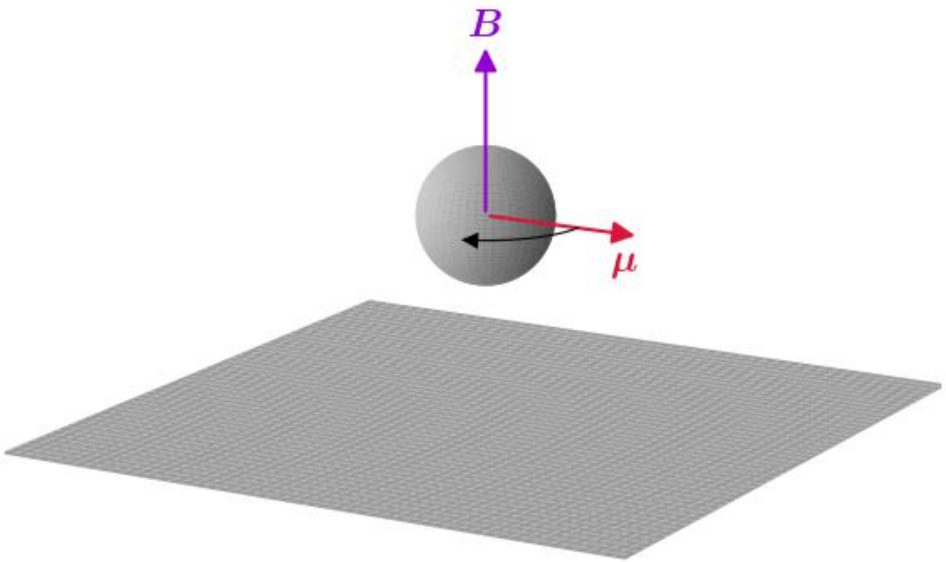
Ferromagnetic gyroscope

Libration (Compass)



$$I\Omega = L \gg S = N\hbar/2$$

Precession (Spin)



$$L \ll S$$

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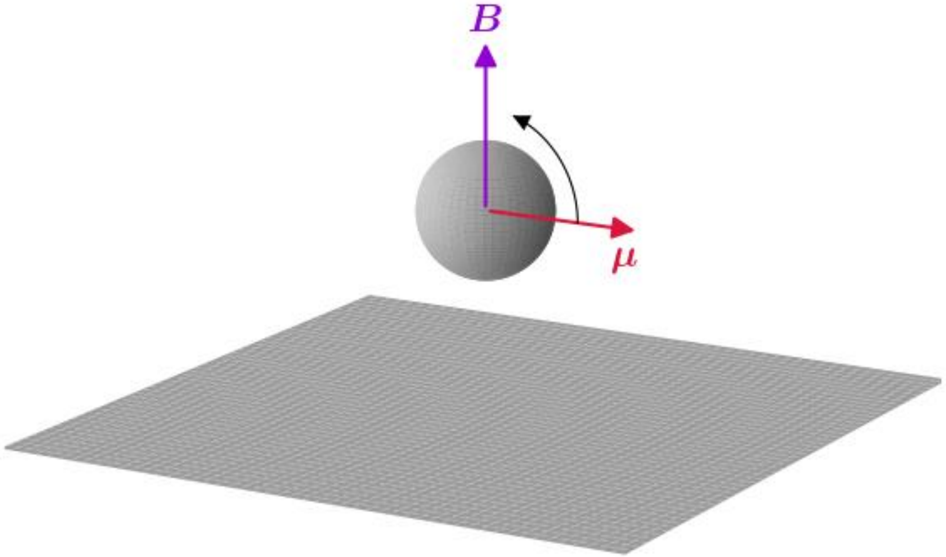

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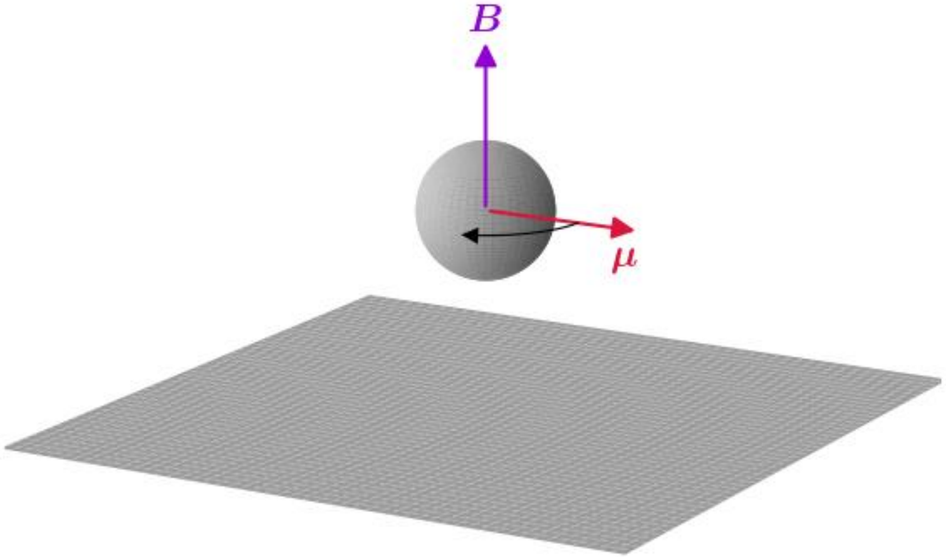

$$L \ll S$$

Depends on m_{DM}!

Effect of trapping potential

- Ferromagnet levitated in trapping potential $V(\boldsymbol{x}, \hat{\boldsymbol{n}})$, e.g.
    - Over superconductor $V \propto \frac{1+\cos^2\theta}{z^3}$
    - In freefall $V \approx 0$
- Resonances and behavior depend on angular trapping $v_{\alpha\alpha} \equiv \frac{2}{N\hbar}\partial_\alpha^2 V$:
    - Trapped ($v_{\alpha\alpha} \gg \omega_I$): only libration, resonances at $m_{\mathrm{DM}} = \sqrt{v_{\alpha\alpha}\omega_I}$
    - Gyroscope ($v_{\alpha\alpha} \ll \omega_I$): precession when $v_{\alpha\alpha} \ll m_{\mathrm{DM}} \ll \omega_I$, resonances at $m_{\mathrm{DM}} = \omega_I, \sqrt{v_{\theta\theta}v_{\phi\phi}}$

where $\omega_I = \frac{N\hbar}{2I}$

Noise sources

- Thermal: kicks from gas molecules

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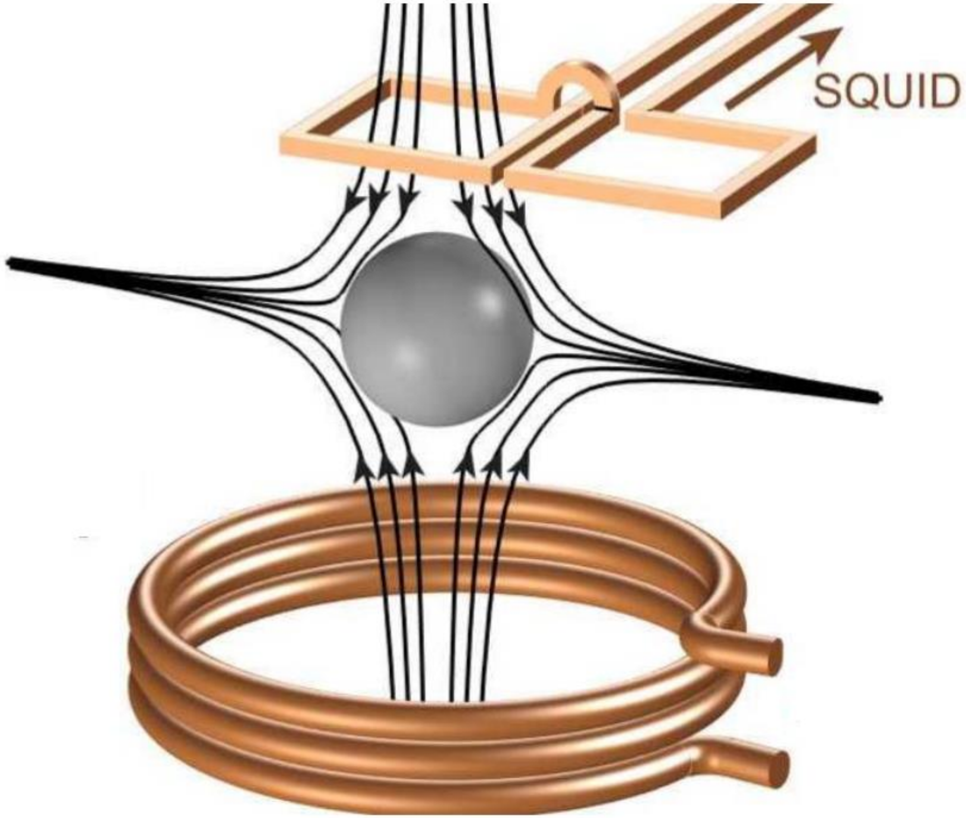
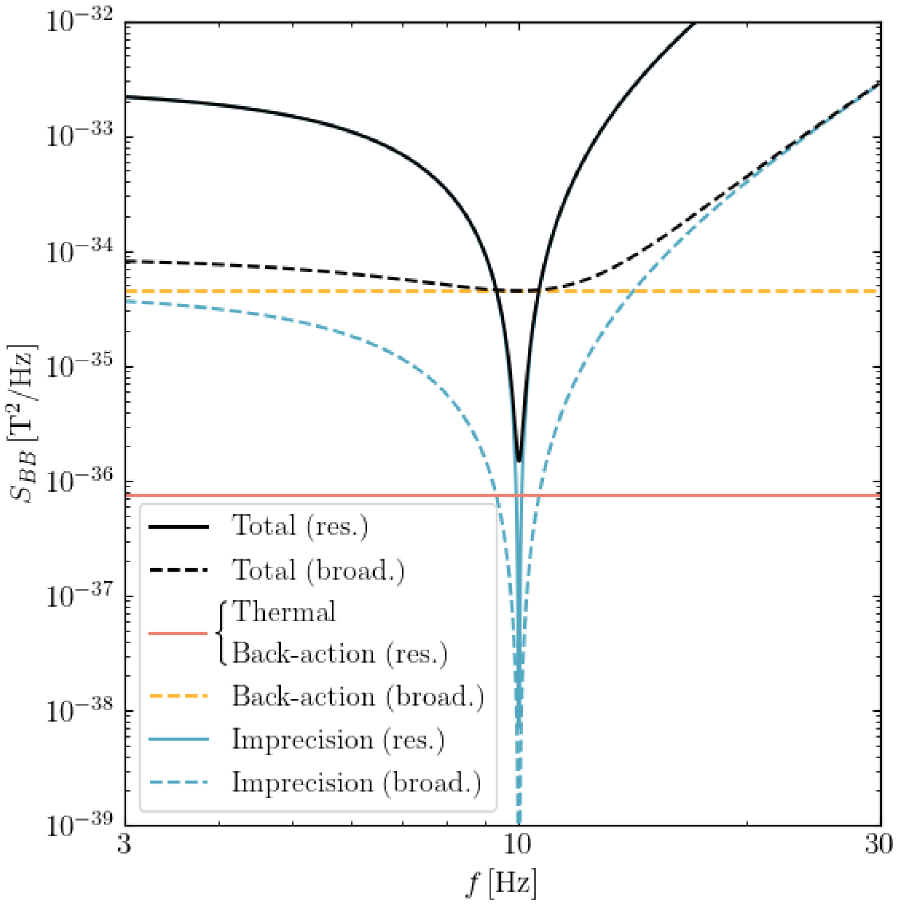
- Imprecision: flux noise → position/angle
- Back-action: current noise → force/torque



[Hofer et al., Phys. Rev. Lett. **131,** 043603 (2023)]

Noise sources

- Thermal: kicks from gas molecules
- Imprecision: flux noise → position/angle
- Back-action: current noise → force/torque
- If thermal subdominant, coupling trade-off
  - Small coupling → resonant detection
  - Large coupling → broadband detection

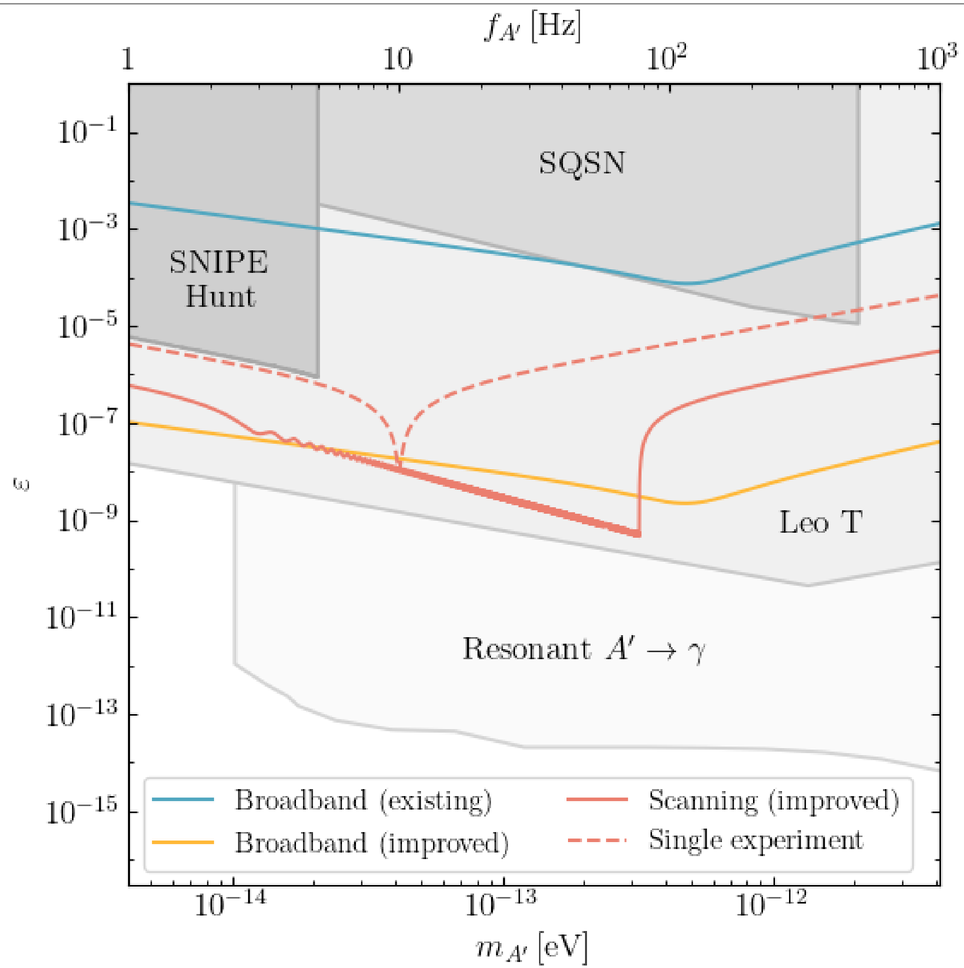


Dark photon sensitivity (SC)

Integration time: $1\,\mathrm{yr}$

Temperature: $10\,\mathrm{mK}$

	Existing	Improved
Mass	$10\,\mu\mathrm{g}$	$1\,\mathrm{g}$
Density	$10\,\mathrm{g/cm^3}$	$0.1\,\mathrm{g/cm^3}$
Shield size	$10\,\mathrm{cm}$	$1\,\mathrm{m}$
Quality factor	10^7	10^{10}



Axion-photon sensitivity (SC)

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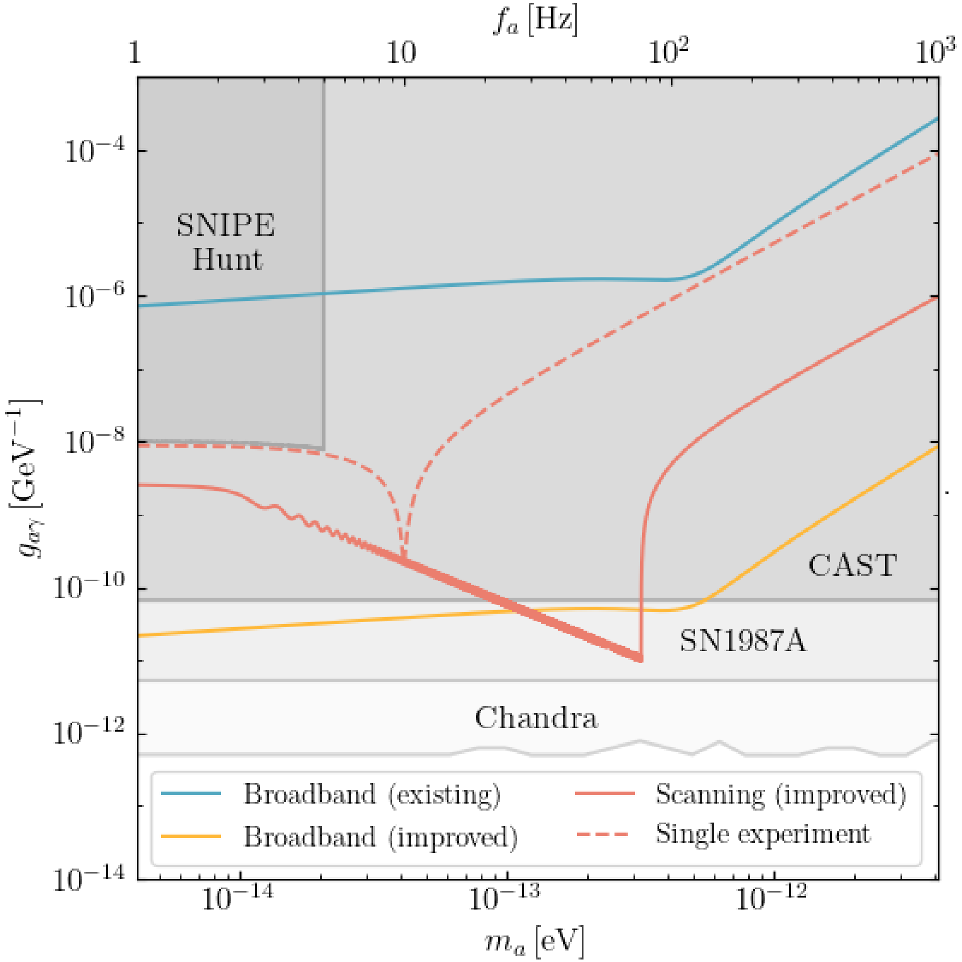
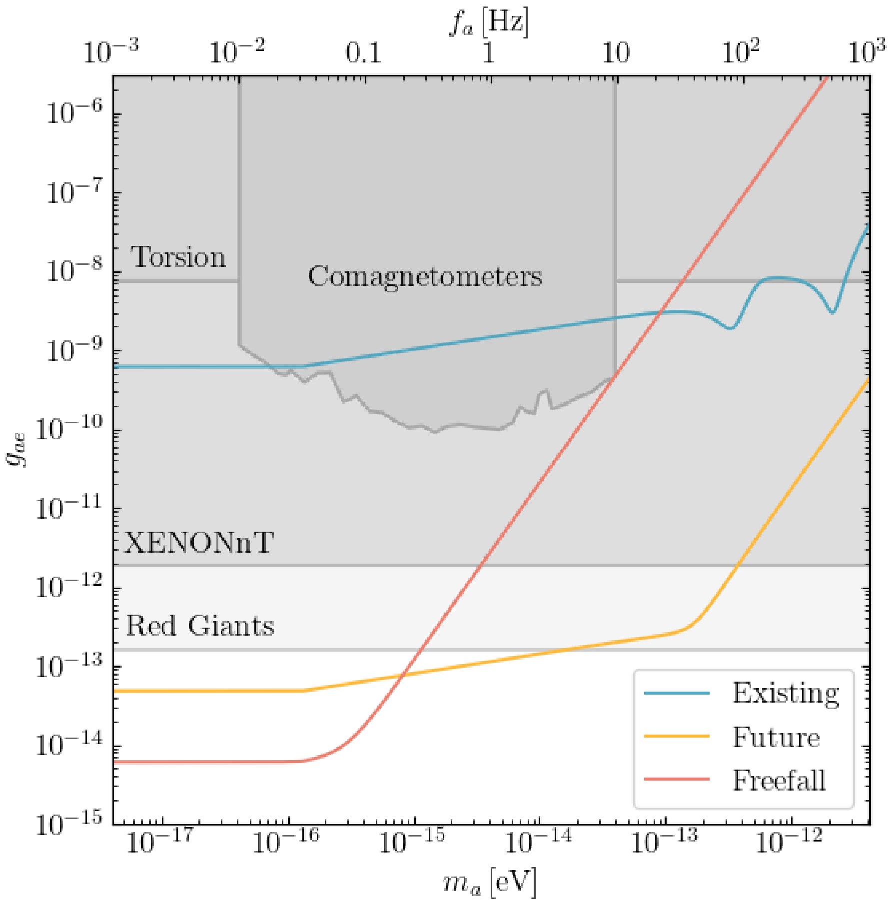


Axion-electron sensitivity (ferromagnet)

Integration time: $1\,\mathrm{yr}$

Magnetization: $7 \times 10^5\,\mathrm{A/m} \approx 0.9\,\mathrm{T}$

	Existing	Future	Freefall
Mass	$250\,\mathrm{ng}$	$250\,\mathrm{mg}$	$250\,\mathrm{g}$
Temperature	$4\,\mathrm{K}$	$50\,\mathrm{mK}$	$300\,\mathrm{K}$
Quality factors	3×10^8 7×10^6	5×10^{13} 5×10^{10}	5×10^{11} 5×10^{11}



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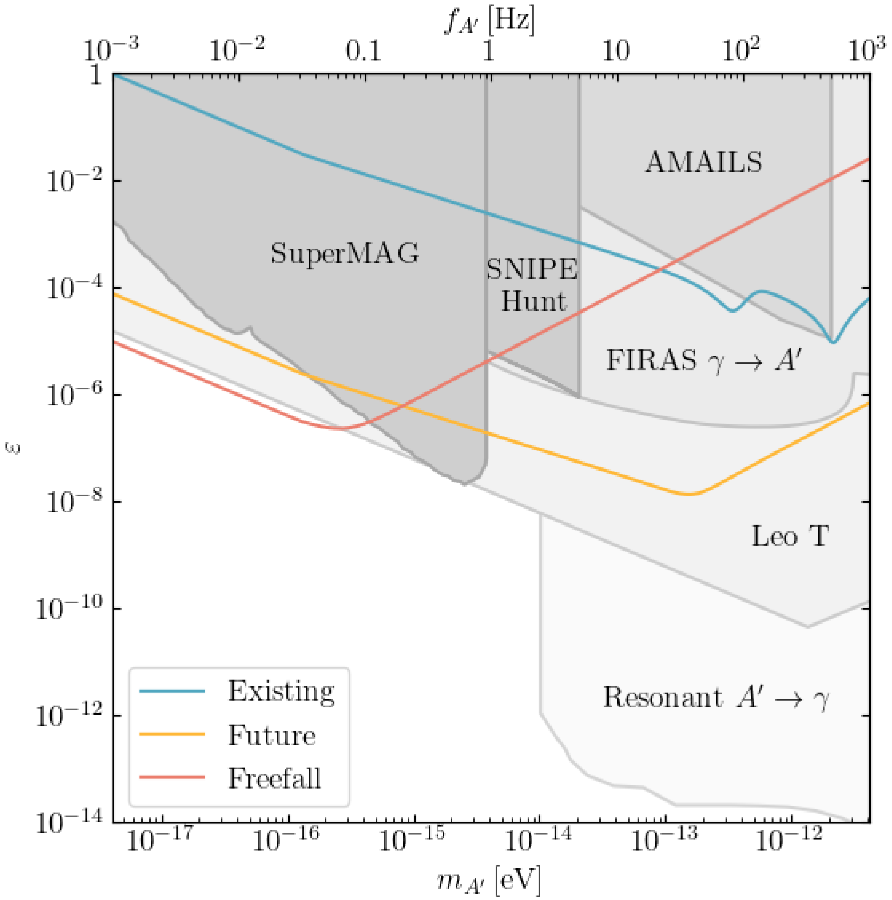

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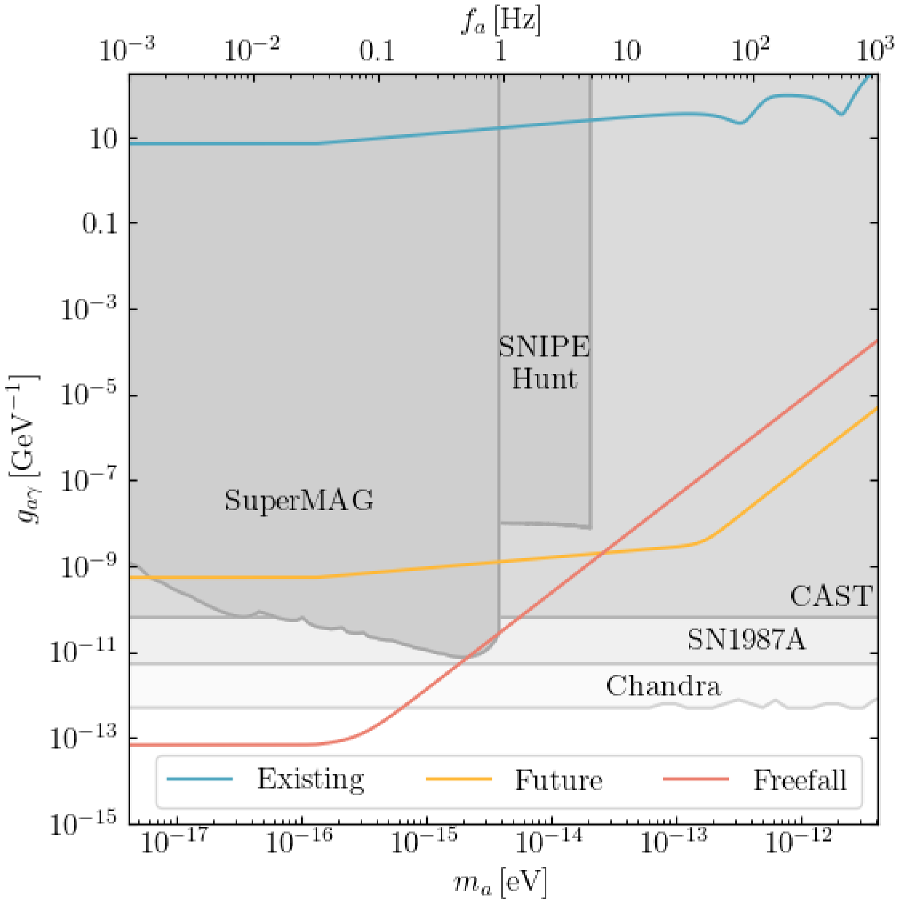


Conclusion

- Maglev systems can probe ultralight DM with $m_{\mathrm{DM}} \lesssim 10^{-12}\,\mathrm{eV}$
- Dark photon or axion-photon coupling source oscillating magnetic fields
  - Causes translational motion of levitated superconductor
  - Rotational motion of levitated ferromagnet (also sensitive to axion-electron coupling)
- Ferromagnet dynamics affected by trapping potential and m_{DM}
- Resonant and broadband schemes
- Dedicated setups can be leading laboratory probes of ultralight DM

Backup Slides

Physical considerations

- Vertical displacement:

$$\Delta z = \frac{g}{4\pi^2 f_z^2} \sim 3\,\mathrm{cm} \cdot \left(\frac{3\,\mathrm{Hz}}{f_z} \right)^2$$

- Maximum magnetic field:

$$B_{\mathrm{max}} \sim b_0 \mathcal{R} \sim 80\,\mathrm{mT} \cdot \left(\frac{m}{1\,\mathrm{g}} \right)^{1/3} \left(\frac{\rho}{0.1\,\mathrm{g/cm^3}} \right)^{1/6} \left(\frac{f_0}{100\,\mathrm{Hz}} \right)$$

- Pb and Ta have critical fields of $\sim 80\,\mathrm{mT}$

Sources of dissipation

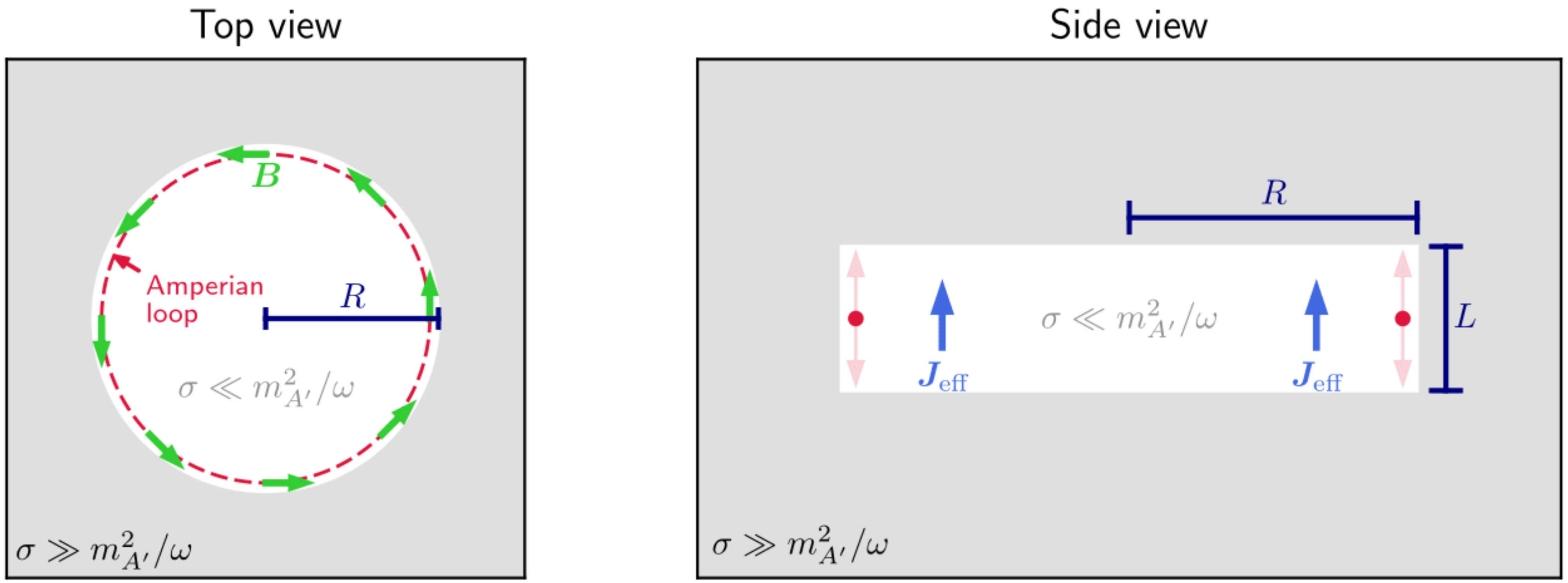
- Gas collisions:

$$\gamma \sim \frac{PA}{m\bar{v}_{\mathrm{gas}}} \sim 2\pi \cdot 10^{-8}\,\mathrm{Hz} \cdot \left(\frac{P}{10^{-7}\,\mathrm{Pa}}\right)\left(\frac{1\,\mathrm{g}}{m}\right)^{1/3} \cdot \left(\frac{0.1\,\mathrm{g/cm^3}}{\rho}\right)^{2/3} \sqrt{\left(\frac{m_{\mathrm{gas}}}{4\,\mathrm{Da}}\right)\left(\frac{10\,\mathrm{mK}}{T}\right)}$$

- Flux creep: movement of unpinned flux lines in type-II SC → use type-I SC

- Eddy current damping in nearby conductors → use only SCs and dielectrics

DPDM magnetic field signal



$$BR \sim \oint \mathbf{B} \cdot d\ell = \iint \mathbf{J}_{\rm eff} \cdot d\mathbf{A} \sim \varepsilon m_{A'}^2 R^2 A'$$

Noise trade-off

- Imprecision and back-action noise determined by coupling η:

$$S_{BB}^{\text{imp}} = \frac{2\rho S_{\phi\phi}}{3m^2\omega_0^2\eta^2|\chi(\omega)|^2} \qquad S_{BB}^{\text{back}} = \frac{2\rho\eta^2 S_{JJ}}{3m^2\omega_0^2}$$

- Flux and current noise satisfy uncertainty relation $\sqrt{S_{\phi\phi}S_{JJ}} = \kappa \geq 1$

- Can define $\tilde{\eta} = \eta\sqrt[4]{\frac{S_{JJ}}{S_{\phi\phi}}}$, so that

$$S_{BB}^{\text{imp}} = \frac{2\rho\kappa}{3m^2\omega_0^2} \cdot \tilde{\eta}^{-2}|\chi(\omega)|^{-2} \qquad S_{BB}^{\text{back}} = \frac{2\rho\kappa}{3m^2\omega_0^2} \cdot \tilde{\eta}^2$$

Noise trade-off (cont.)

- If $S^{\mathrm{th}}_{BB,\alpha\alpha} < \sqrt{S^{\mathrm{imp}}_{BB,\alpha\alpha}(\omega=0)\cdot S^{\mathrm{back}}_{BB,\alpha\alpha}}$ [or $\tilde{\eta}^{(\mathrm{res})} \leq \tilde{\eta}^{(\mathrm{broad})}$], then either:
  - Resonant detection: $\tilde{\eta}^{(\mathrm{res})} = \sqrt{\frac{4m\gamma T}{\kappa}}$
  - Broadband detection: $\tilde{\eta}^{(\mathrm{broad})} = \sqrt{m\omega_0}$
- Otherwise, can choose any $\tilde{\eta}^{(\mathrm{res})} \geq \tilde{\eta} \geq \left[\tilde{\eta}^{(\mathrm{broad})}\right]^2 / \tilde{\eta}^{(\mathrm{res})}$
  - Larger $\tilde{\eta}$ is better for higher frequencies

Signal-to-noise ratio

- Coherent:

$$\mathrm{SNR} = \frac{B_{\mathrm{DM}}^2}{S_{BB}^{\mathrm{tot}}/t_{\mathrm{int}}}$$

- Incoherent:

$$\mathrm{SNR} = \frac{B_{\mathrm{DM}}^2}{S_{BB}^{\mathrm{tot}}/t_{\mathrm{coh}}} \cdot \sqrt{\frac{t_{\mathrm{int}}}{t_{\mathrm{coh}}}}$$

- Multiple scans:

$$\mathrm{SNR}^2 = \sum_i \mathrm{SNR}_i^2$$

Bandwidth

- Bandwidth defined by:

$$S_{BB}^{\text{tot}}\left(\omega_0+\frac{\delta\omega}{2}\right)=2S_{BB}^{\text{tot}}(\omega_0)$$

- For resonant coupling,

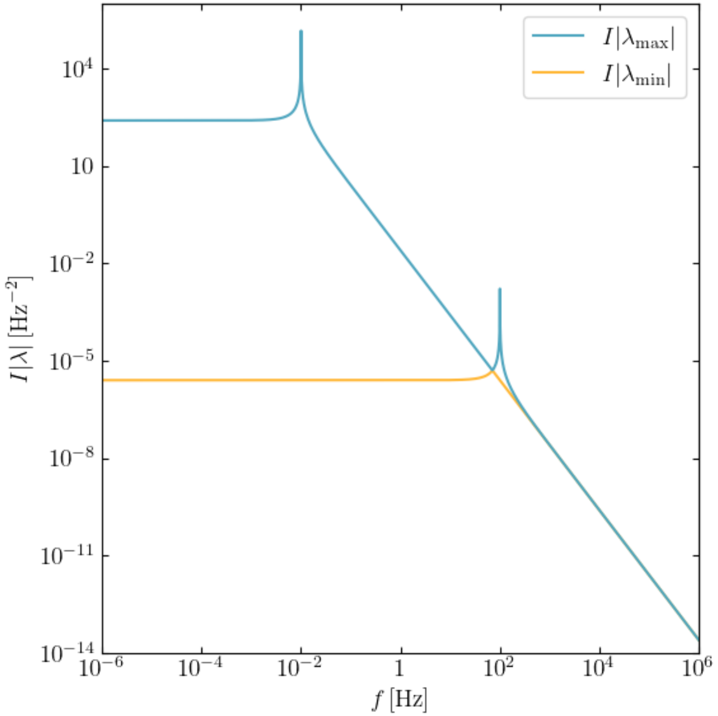
$$\delta\omega=\frac{4\sqrt{2}\gamma T}{\kappa\omega_0}\sim 2\pi\cdot 0.2\,\text{Hz}\left(\frac{\gamma}{2\pi\cdot 10^{-8}\,\text{Hz}}\right)\cdot\left(\frac{T}{10\,\text{mK}}\right)\left(\frac{5}{\kappa}\right)\left(\frac{10\,\text{Hz}}{f_0}\right)$$

- Total scan takes

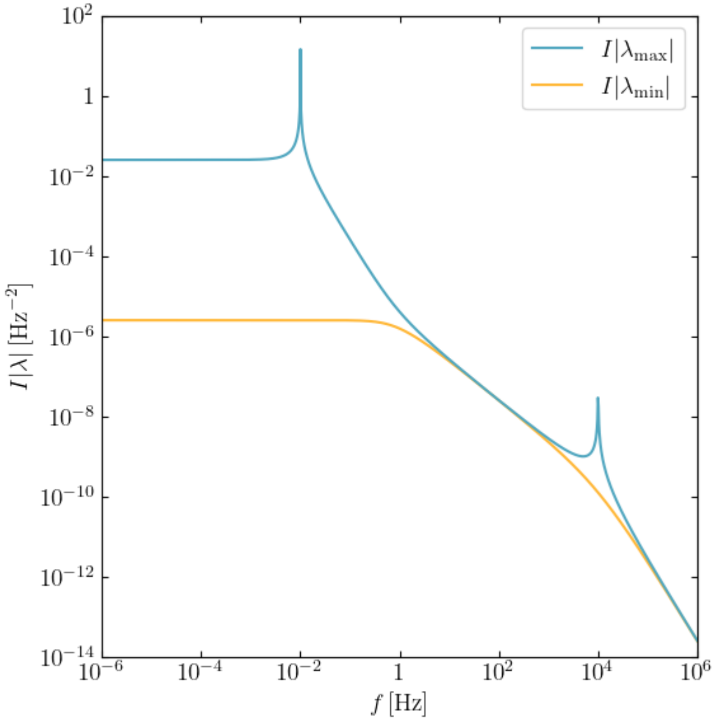
$$\sum_i t_{\text{int},i}=\frac{\kappa\pi}{2\sqrt{2}\gamma T v_{\text{DM}}^2}\Delta\omega\sim 1\,\text{yr}\left(\frac{\kappa}{5}\right)\left(\frac{2\pi\cdot 10^{-8}\,\text{Hz}}{\gamma}\right)\cdot\left(\frac{10\,\text{mK}}{T}\right)\left(\frac{\Delta f}{74\,\text{Hz}}\right)$$

Mechanical susceptibility

$$\chi(\omega)^{-1} = I\left[-\omega^2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i j_n \omega_I \omega \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} + \omega_I \begin{pmatrix}v_{\theta\theta} & 0\\ 0 & v_{\phi\phi}\end{pmatrix}\right]$$

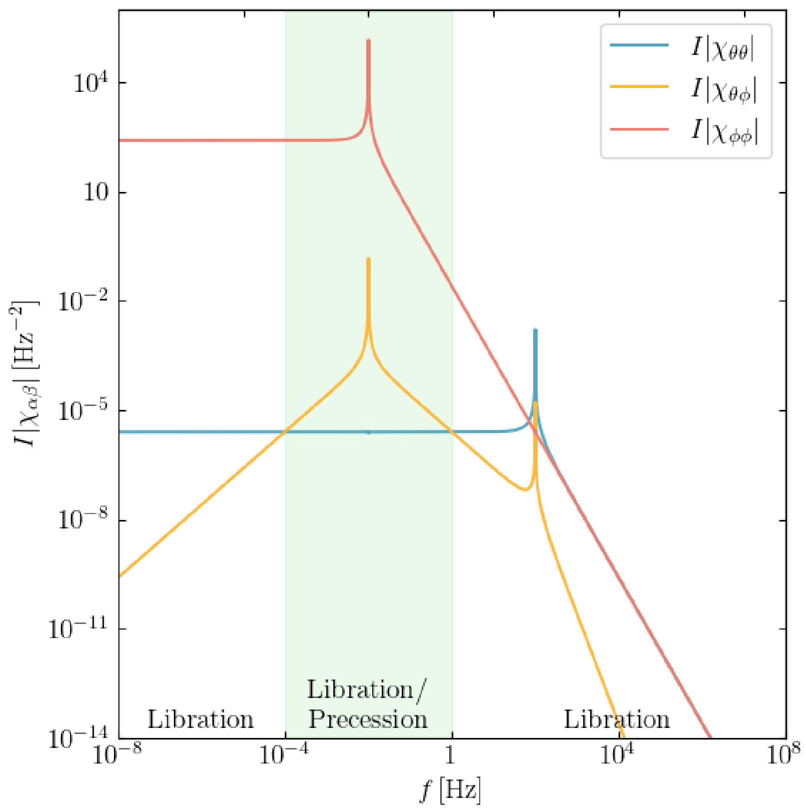


Partially Trapped ($v_{\theta\theta} \gg \omega_I \gg v_{\phi\phi}$)



Gyroscope ($\omega_I \gg v_{\theta\theta} \gg v_{\phi\phi}$)

Libration vs. Precession



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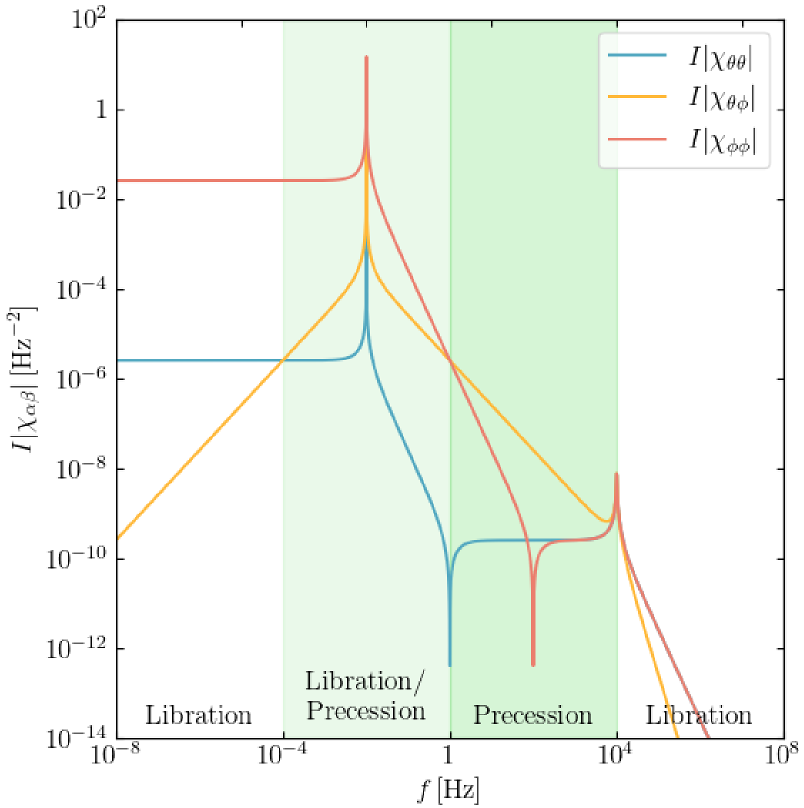

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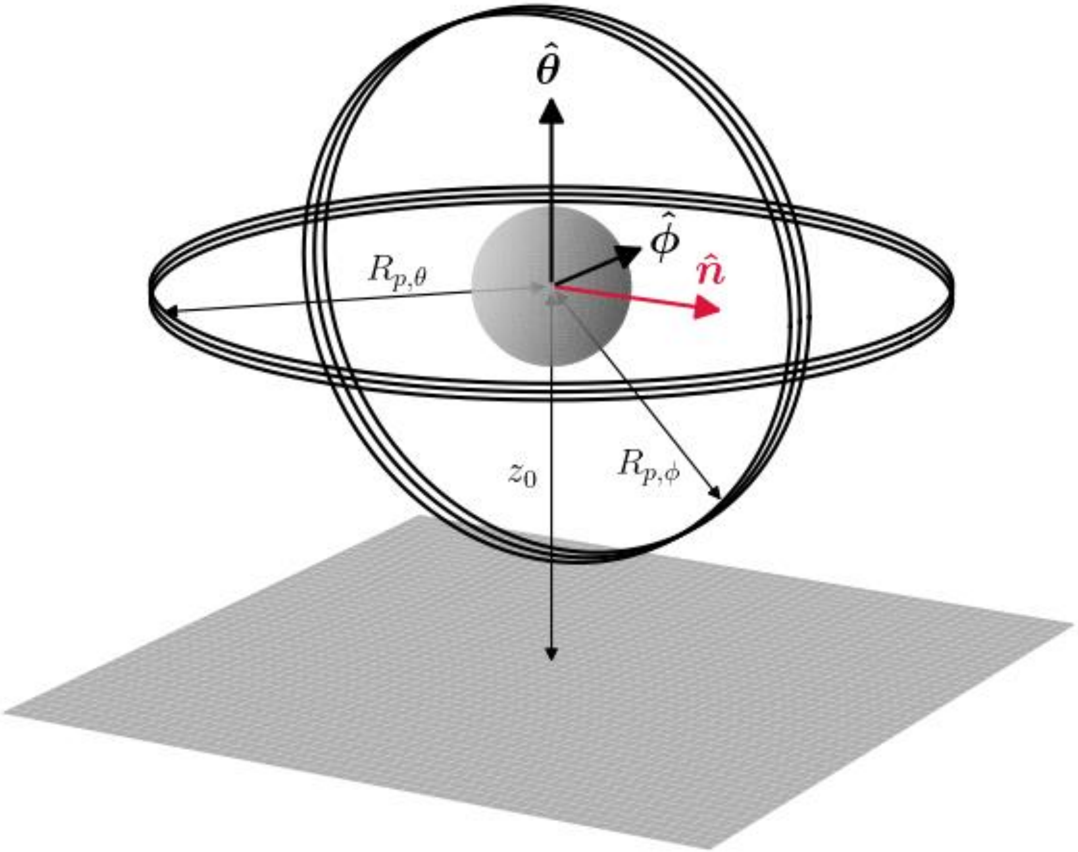
Ferromagnet readout



Ferromagnet noise curves