

# Bellell excess & Muon g-2 illuminating Light DM with Higgs portal



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Based on

arXiv: 2401.10112 with Shu-Yu Ho (KIAS), Pyungwon Ko (KIAS)

19th Patras Workshop on Axions, **WIMPs** and WISPs

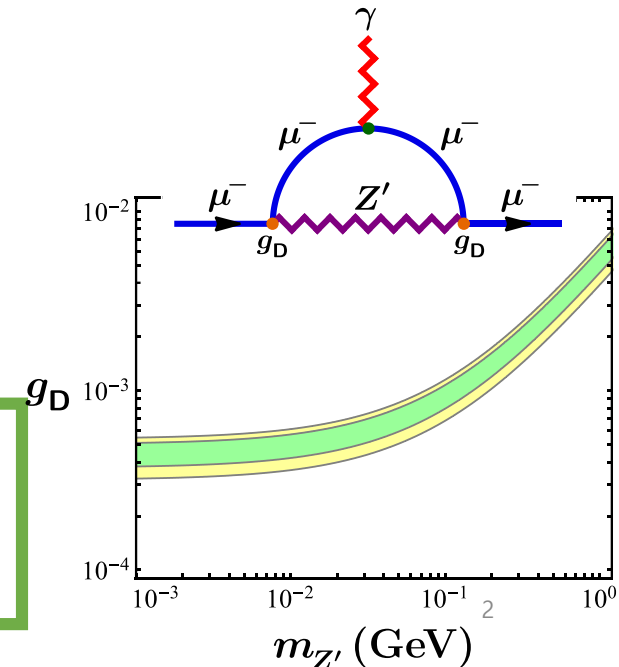
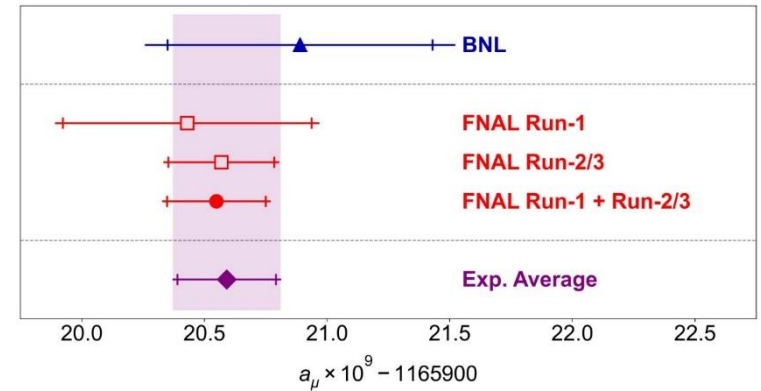
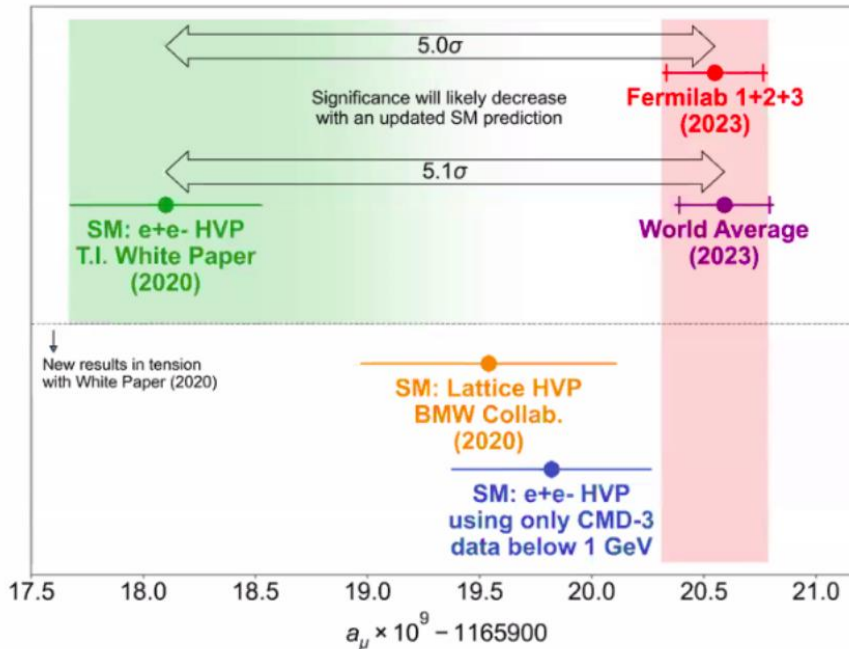
Sep 16 (Mon)



# Evidences – muon g-2

Muon g-2 collaboration, PRL 2023

- Muon g-2 experiment improves the precision of their previous result by a factor of 2



S. Baek, Deshpande, He, P. Ko, 2001

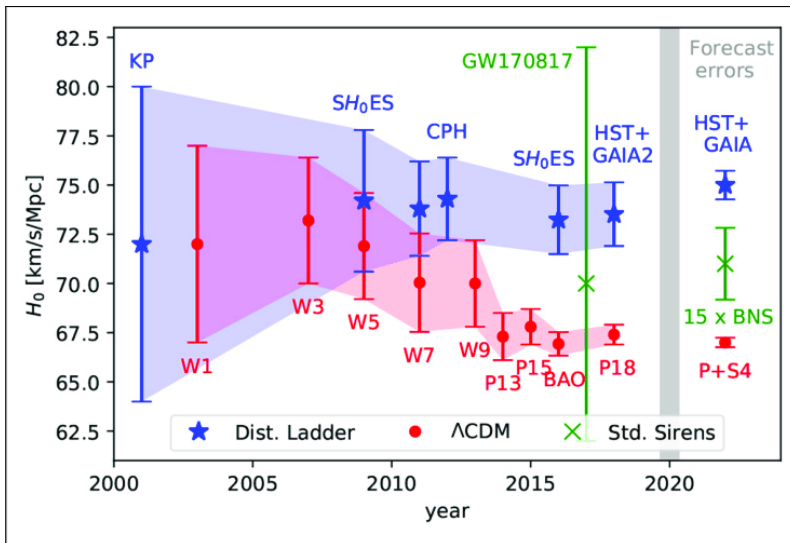
S. Baek, P. Ko, 2008

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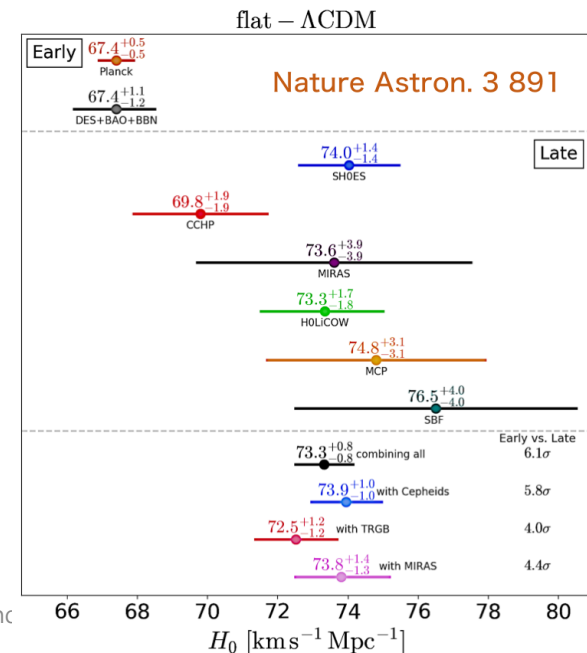
$$\Delta a_\mu = \frac{g_x^2}{4\pi^2} \int_0^1 dx \frac{m_\mu^2 x^2 (1-x)}{x^2 m_\mu^2 + (1-x)m_{Z'}^2}$$

# Evidences – Hubble tension

- Large difference between early and late  $H_0$  measurement
  - Late-time:  $H_0 = 73.2 \pm 1.3 \text{ kms}^{-1}\text{Mpc}^{-1}$
  - Early-time:  $H_0 = 67.4 \pm 0.5 \text{ kms}^{-1}\text{Mpc}^{-1}$
- The discrepancy either arises because
  - Our distance measurements are incorrect ( $\Delta G_N$ )
  - Cosmological model we use to fit all those distances is incorrect ( $\Delta N_{\text{eff}}$ )

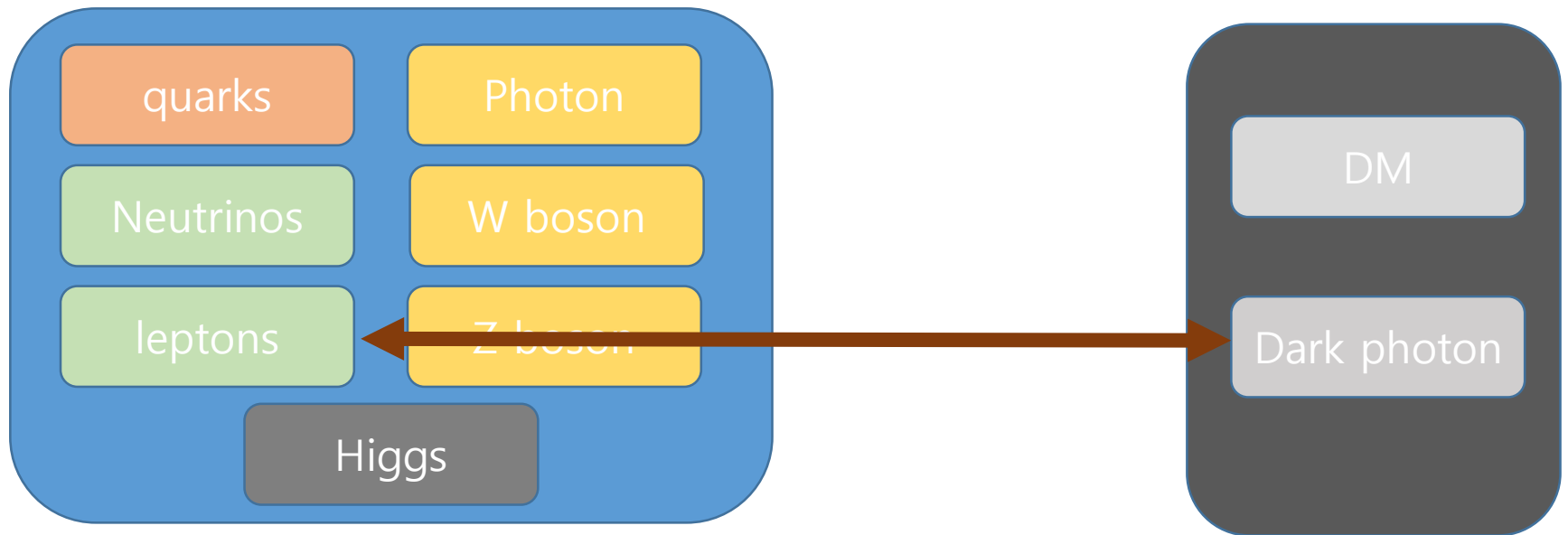


P. Shah et al, AAR 2021



# $U(1)_{L_\mu-L_\tau}$ -charged DM model

- $U(1)_{dark} \equiv U(1)_{L_\mu-L_\tau}$



- Let's call  $Z'$ ,  $U(1)_{L_\mu-L_\tau}$  gauge boson, dark photon since it couple to DM

# Gauged $U(1)_{L_\mu - L_\tau}$ $Z'$ model

- Gauge one of the differences of two lepton-flavor numbers

- $L_e - L_\mu, L_\mu - L_\tau, L_e - L_\tau$ : **anomaly free** without extension of fermion contents

X. G. He et al, PRD 1991

- Symmetry including  $L_e$  is strongly constrained

- Charge assignments:  $\widehat{Q}_{L_\mu - L_\tau}(\nu_\mu, \nu_\tau, \mu, \tau) = (1, -1, 1, -1)$

- No kinetic mixing between  $Z'$  and B @ high-energy

- Kinetic mixing is generated through



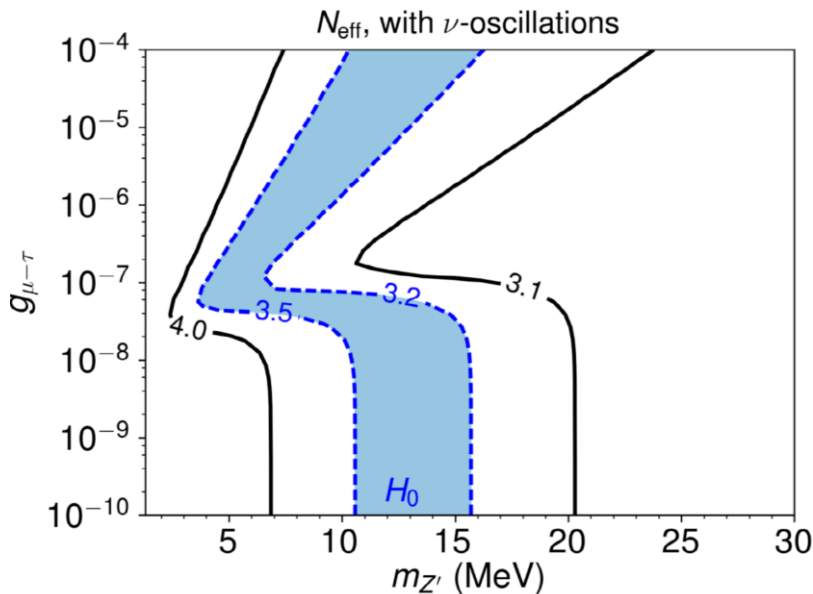
- $$\epsilon = -\frac{eg_{\mu-\tau}}{2\pi^2} \int_0^1 dx x(1-x) \log \left[ \frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2} \right] \xrightarrow{m_\mu \gg q} -\frac{eg_{\mu-\tau}}{12\pi^2} \log \frac{m_\tau^2}{m_\mu^2} \simeq -\frac{g_{\mu-\tau}}{70}$$

# Gauged $U(1)_{L_\mu - L_\tau}$ $Z'$ model

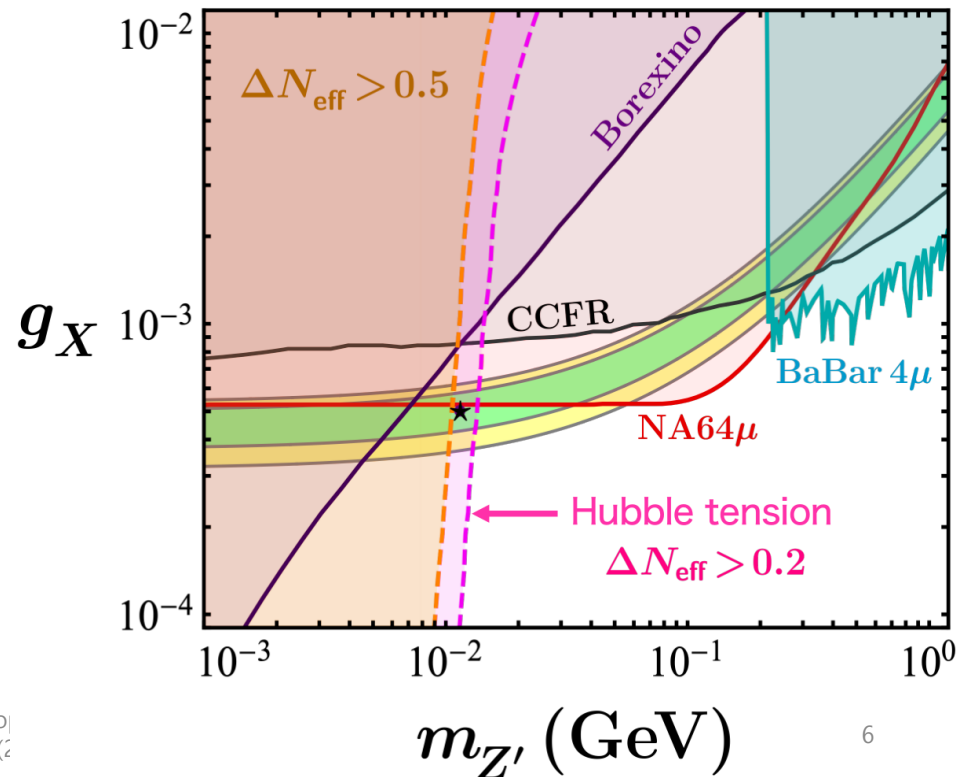
M. Escudero et al, JHEP 2019

## • Hubble tension

- $\sim 10\text{MeV}$   $Z'$  reached thermal equilibrium in the early Universe and decays, heating the neutrino population
- Delay the process of neutrino decoupling
- $0.2 < \Delta N_{\text{eff}}$ : substantially relaxes the tension



• BP :  $m_{Z'} = 11.5\text{MeV}$ ,  $g_X = 5 \times 10^{-4}$



# $U(1)_{L_\mu - L_\tau}$ -charged DM model

- $U(1)_{L_\mu - L_\tau}$ -charged scalar DM model

$$\mathcal{L}_{\text{int}} = ig_X Z'_\mu (X^* \partial^\mu X - X \partial^\mu X^*) + g_X Z'_\alpha \sum Q_\ell \bar{\ell} \gamma^\alpha \ell$$

- Free parameters:  $\{m_{Z'}, g_X, m_X, Q_X\}$
- Dark Photon  $Z'$  plays a role of messenger particle between DM and the SM leptons
- Dark Photon mass is generated Proca or Stueckelberg mechanism



Only when  $m_X > m_{Z'}$

- Consider  $Z'$  boson only &  $g_X \sim (3 - 5) \times 10^{-4}$  for the muon g-2
  - $\chi \bar{\chi} (X \bar{X}) \rightarrow f_{SM} \bar{f}_{SM}$  : dominant annihilation channels

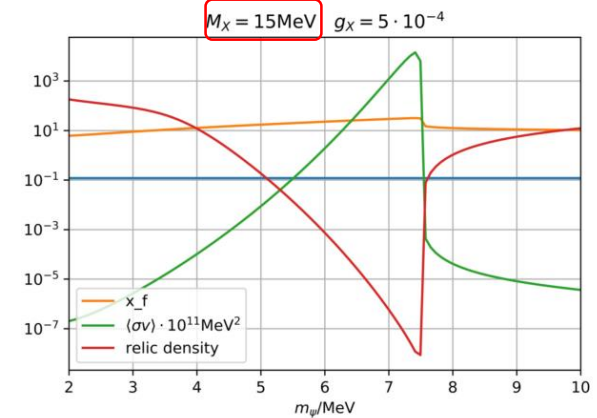
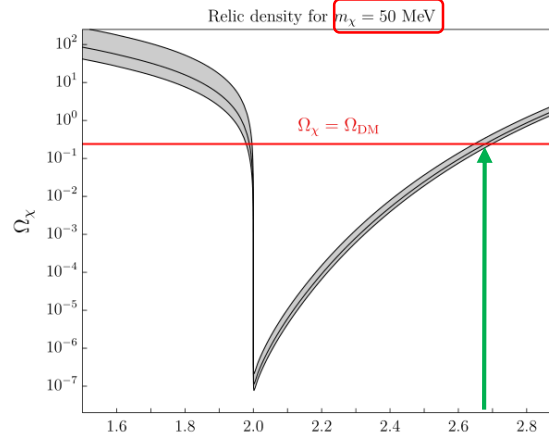
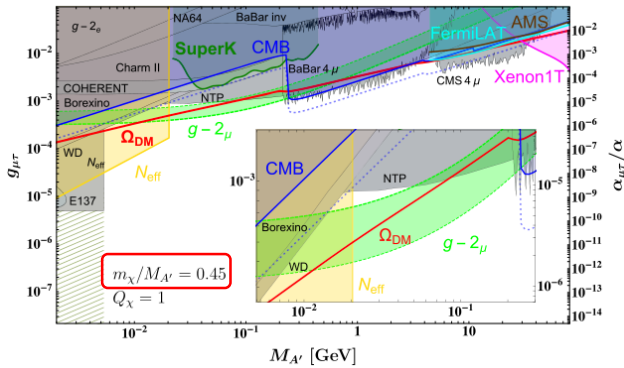
# $U(1)_{L_\mu - L_\tau}$ -charged DM model

- $XX^\dagger \rightarrow Z'^* \rightarrow \nu\bar{\nu}$  : dominant annihilation channels
  - $m_{Z'} \sim 2m_X$  with the **s-channel  $Z'$  resonance** only gives the correct relic density

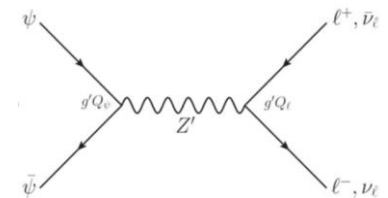
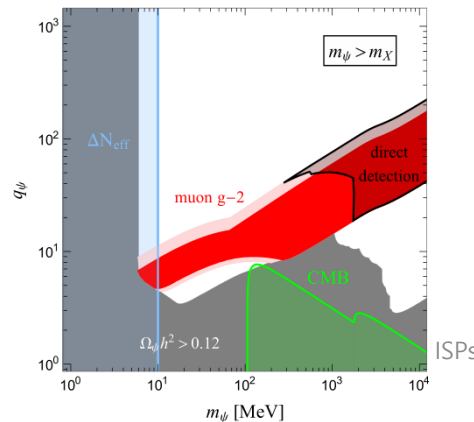
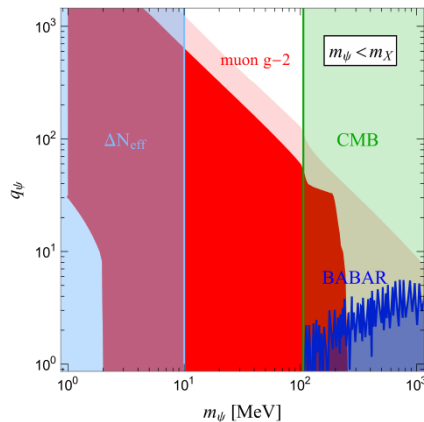
P. Foldenauer, PRD 2019

I. Holst, D. Hooper, G. Krnjaic, PRL 2022

M. Drees, W. Zhao, PLB 2022

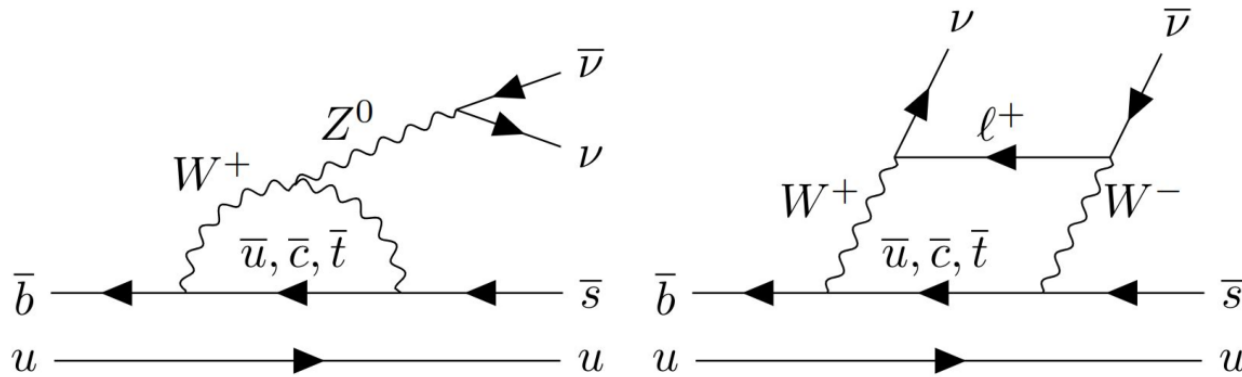


- Large DM charges Asai, Okawa, Tsumura, JHEP 2021



# Measurement of $B^+ \rightarrow K^+ \nu \bar{\nu}$

- The  $B^+ \rightarrow K^+ \nu \bar{\nu}$  process is known with high accuracy in the SM:
  - $Br(B^+ \rightarrow K^+ \nu \bar{\nu}) = (4.97 \pm 0.37) \times 10^{-6}$  HPQCD, PRD 2023

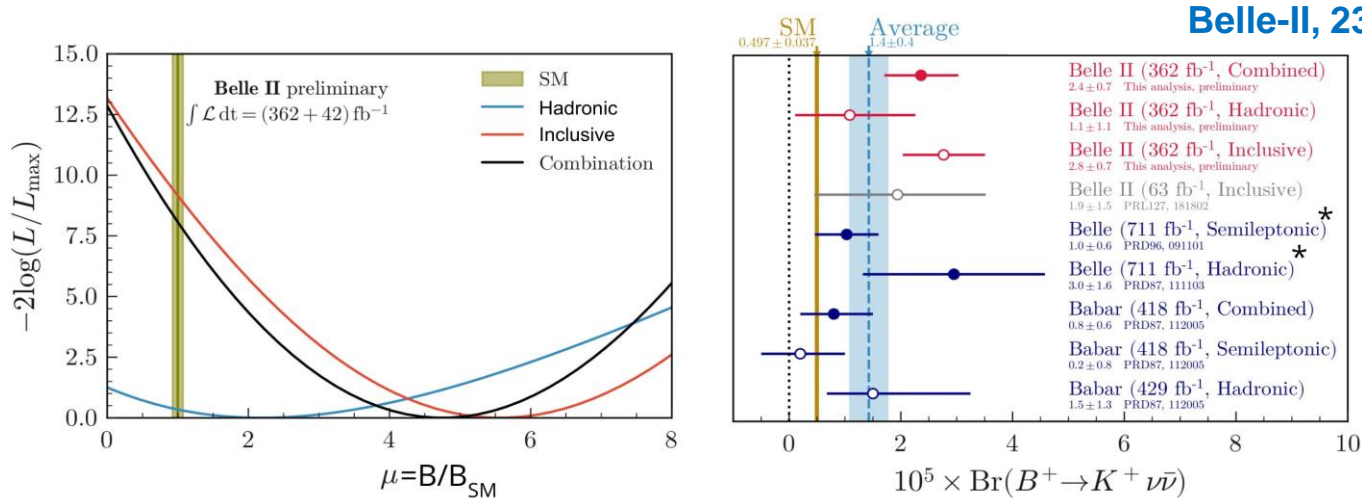


$$\mathcal{L}_{b \rightarrow s \nu \bar{\nu}} = -C_\nu \bar{s}_L \gamma^\mu b_L \bar{\nu} \gamma^\mu \nu$$

$$C_\nu = \frac{g_W^2}{M_W^2} \frac{g_W^2 V_{ts}^* V_{tb}}{16\pi^2} \left[ \frac{x_t^2 + 2x_t}{8(x_t - 1)} + \frac{3x_t^2 - 6x_t}{8(x_t - 1)^2} \ln x_t \right],$$

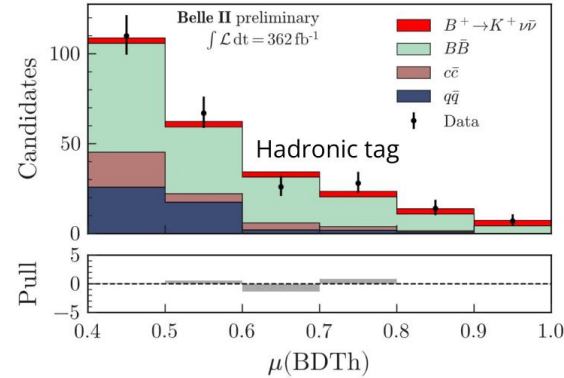
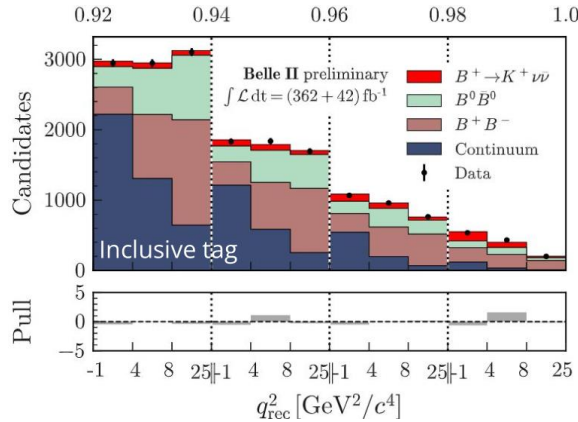
where  $x_t = m_t^2 / M_W^2$ .

# Measurement of $B^+ \rightarrow K^+ \nu \bar{\nu}$



- $Br(B^+ \rightarrow K^+ \nu \bar{\nu})_{Exp} = (2.3 \pm 0.7) \times 10^{-5}$ 
  - Prob(null signal from  $B^+ \rightarrow K^+ \nu \bar{\nu}$ ) = 0.012%
  - ➔ Significance of observation:  $3.5 \sigma$
  - Prob( $B^+ \rightarrow K^+ \nu \bar{\nu}$ )<sub>SM</sub> = 0.17% ( $2.8\sigma$  tension with the SM prediction)
- $Br(B^+ \rightarrow K^+ E_{\text{mis}})_{NP} = (1.8 \pm 0.7) \times 10^{-5}$ 
  - **Indirect NP effects:** The presence of heavy NP particles
  - **Direct NP effects:** the presence of new invisible particles

# Solutions: 2- or 3-body decay

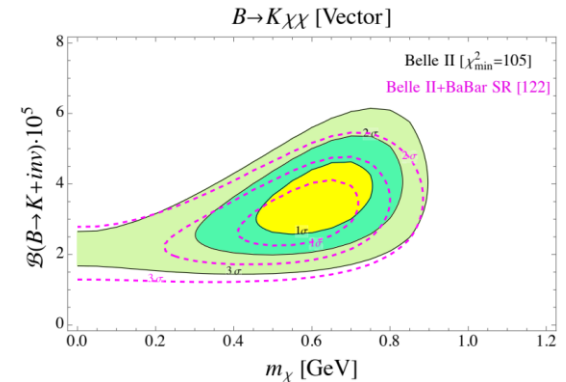
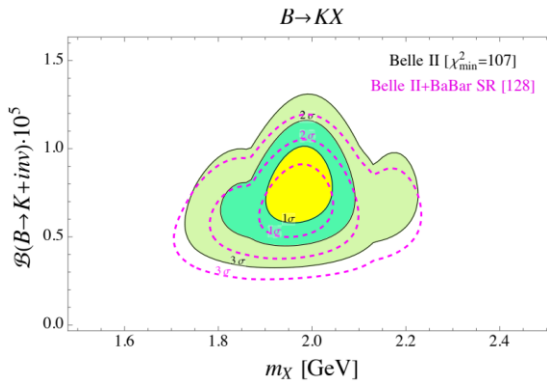
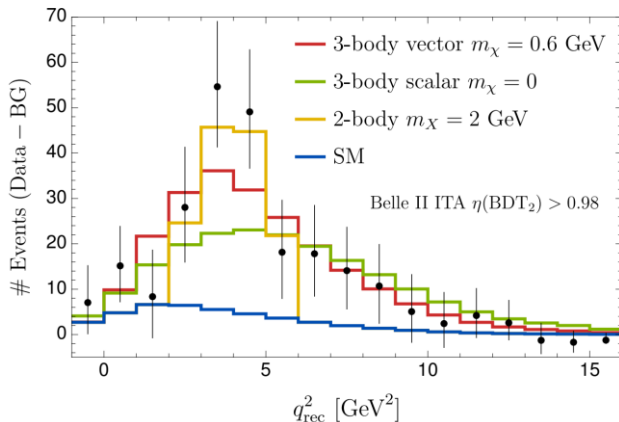


• Belle II provides information on the  $q_{rec}^2$  spectrum

- A **peak** localized around  $q_{rec}^2 = 4 \text{ GeV}^2$
- Two-body decay ( $B \rightarrow KX$ ),  $m_X = 2 \text{ GeV}$
- Three-body decay ( $B \rightarrow KXX$ ),  $m_X < 0.6 \text{ GeV}$

W. Altmannshofer et al, 2311.14629

K. Fridell et al, 2312.12507



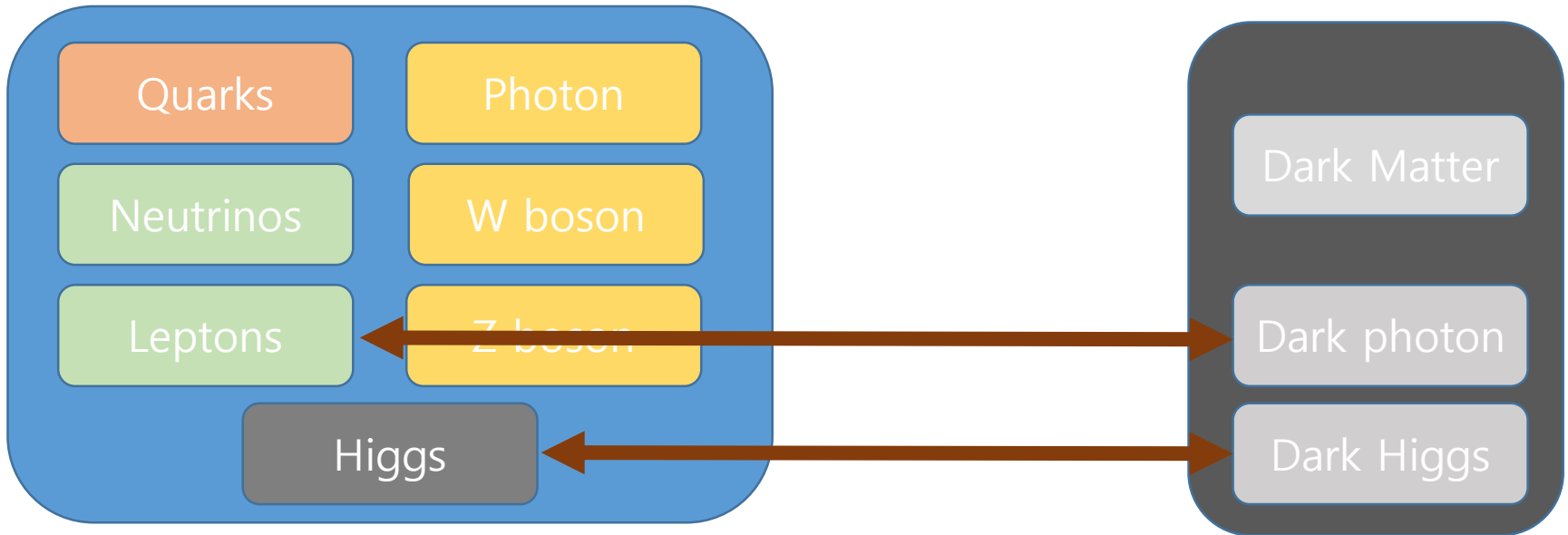
Can we find the integrated solution of  $\Delta a_\mu$ , DM relic density, Hubble tension and  $B^+ \rightarrow K^+ \nu \bar{\nu}$  at Belle II?

# $U(1)_{L_\mu - L_\tau}$ -charged DM model

- Muon g-2:  $g_X \sim 10^{-4}$  is **too small** to get  $\Omega h^2 = 0.12$ 
  - Only sub-GeV **DM** available
  - $m_{Z'} \sim 2m_X$  with the **s-channel  $Z'$  resonance**
  - Tight correlation between DM mass and  $Z'$  mass
- **No DM direct detection bound**
  - DM-nucleon scattering
  - DM-electron scattering
- **BelleII excess**
  - $B \rightarrow KZ'$  (2body decay)  
→ disfavored by  $q^2$  spectrum
  - $B \rightarrow KXX^\dagger$  (3body decay)  
→ suppressed by kinetic mixing and  $g_X \sim 10^{-4}$

# $U(1)_{L_\mu-L_\tau}$ -charged DM + Dark Higgs

- $U(1)_{dark} \equiv U(1)_{L_\mu-L_\tau}$ 
  - Let's call  $Z'$ ,  $U(1)_{L_\mu-L_\tau}$  gauge boson, **dark photon** since it couple to DM



- **UV complete**  $U(1)_{L_\mu-L_\tau}$ -charged **scalar** DM model
- Dark photon  $Z'$  gets massive through  $U(1)_{L_\mu-L_\tau}$  breaking
- A new singlet scalar (**Dark Higgs**), which mixes with the SM Higgs

# $U(1)_{L_\mu - L_\tau}$ -charged DM + Dark Higgs

- After electroweak and  $U(1)_{L_\mu - L_\tau}$  symmetry breaking

$$\mathcal{H} = \frac{1}{\sqrt{2}}(0 \ v_H + h)^\top, \quad \Phi = \frac{1}{\sqrt{2}}(v_\Phi + \phi)$$

- Dark photon  $Z'$  gets massive:  $m_{Z'} = g_X |Q_\Phi| v_\Phi$
- Two CP-even neutral scalar bosons mix each other

$$H_1 = \phi \cos \theta - h \sin \theta, \quad H_2 = \phi \sin \theta + h \cos \theta$$

dark Higgs boson                      SM-like Higgs boson                      mixing angle

$$m_{H_1} < m_{H_2} \simeq 125 \text{ GeV}$$

# $U(1)_{L_\mu - L_\tau}$ -charged DM + Dark Higgs

- Additional interactions with the dark Higgs

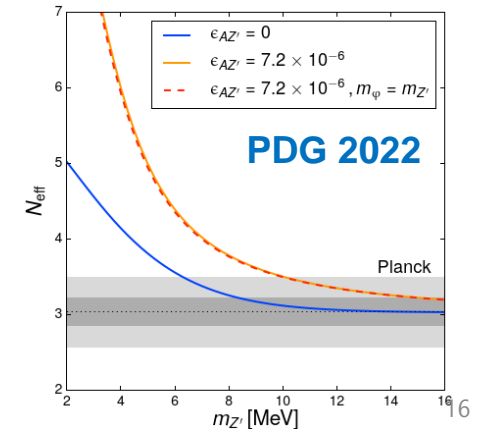
$$\mathcal{L}_\phi \supset \frac{1}{2} g_X^2 Q_\Phi^2 Z'^\mu Z'_\mu \phi^2 + g_X^2 Q_\Phi^2 v_\Phi Z'^\mu Z'_\mu \phi - \lambda_\Phi v_\Phi \phi^3 - \lambda_H v_H h^3 - \frac{\lambda_{\Phi H}}{2} v_\Phi \phi h^2 - \frac{\lambda_{\Phi H}}{2} v_H \phi^2 h$$

## • The SM-like Higgs invisible decay

- $H_2 \rightarrow H_1 H_1, Z' Z', X X^\dagger$
- SM Higgs mainly decays into dark photon and dark Higgs

$$\Gamma_{H_2 \rightarrow H_1 H_1} \simeq \Gamma_{H_2 \rightarrow Z' Z'} \propto \frac{\sin^2 \theta m_{H_2}^3}{v_\Phi^2} \gg \Gamma_{H_2 \rightarrow X X^\dagger} \propto \frac{\sin^2 \theta \lambda_{\Phi X}^2 v_\Phi^2}{m_{H_2}}$$

- $\text{Br}(H_2 \rightarrow \text{inv.}) = \frac{\Gamma_{H_2}^{ZZ^* \rightarrow 4\nu} + \Gamma_{H_2}^{H_1 H_1} + \Gamma_{H_2}^{Z' Z'} + \Gamma_{H_2}^{X X^\dagger}}{\Gamma_{H_2}^{\text{SM}} + \Gamma_{H_2}^{H_1 H_1} + \Gamma_{H_2}^{Z' Z'} + \Gamma_{H_2}^{X X^\dagger}} < 13\%$
- $\sin \theta \leq 0.01$  to satisfy the Higgs invisible decay



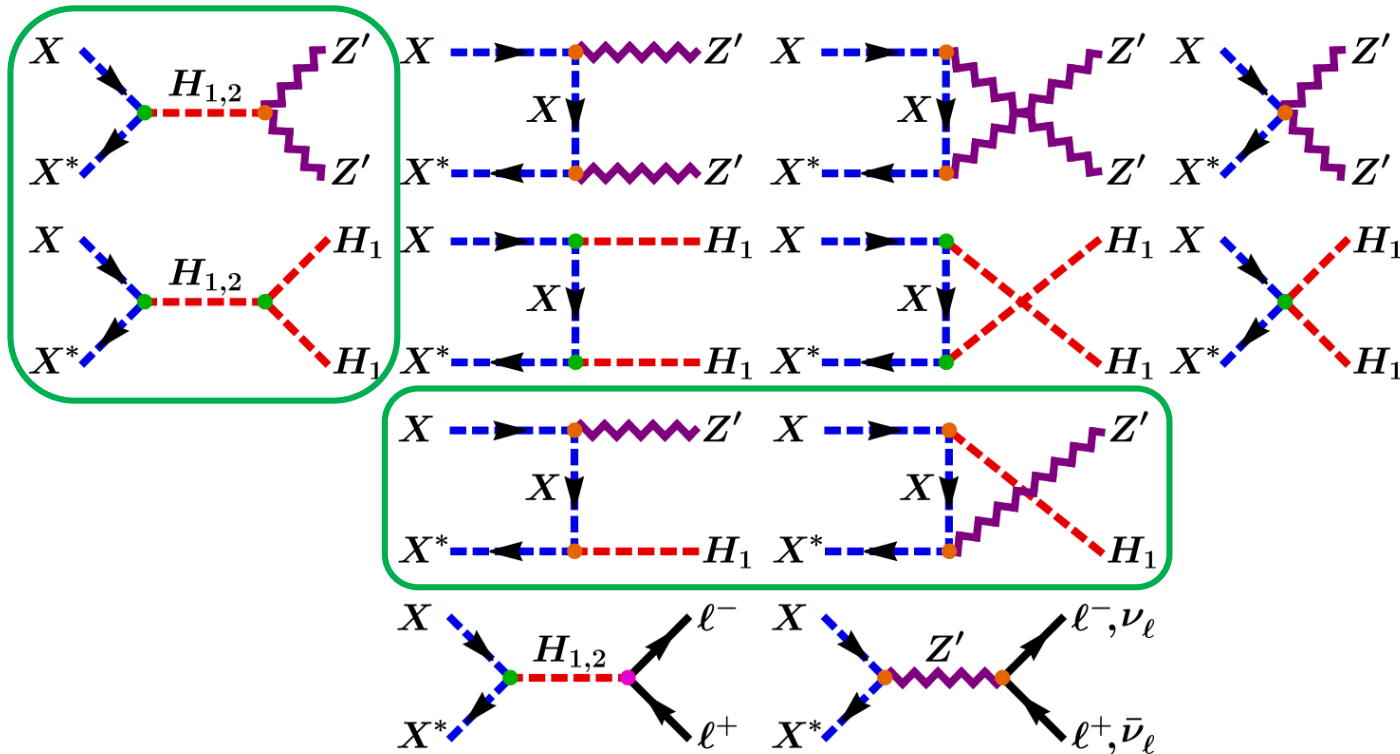
# $U(1)_{L_\mu-L_\tau}$ -charged DM + Dark Higgs

- UV-complete  $U(1)_{L_\mu-L_\tau}$ -charged scalar DM model

Baek, JK, Ko, 2204.04889

$$\mathcal{L}_{\text{DM}} = |D_\mu X|^2 - m_X^2 |X|^2 - \lambda_{\Phi X} |X|^2 \left( |\Phi|^2 - \frac{v_\Phi^2}{2} \right)$$

- Free parameters:  $\{m_{Z'}, g_X, \sin \theta, m_X, m_{H_1}, Q_\Phi, \lambda_{\Phi X}\}$



# BelleII excess: 2- or 3-body decay

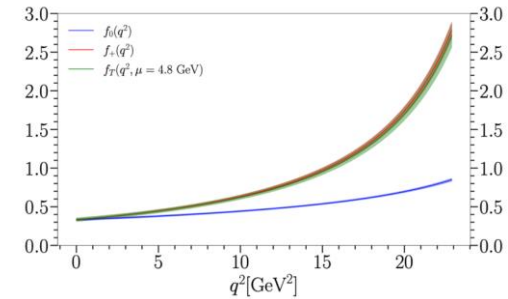
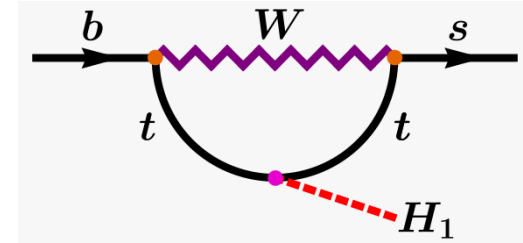
- When  $m_{H_1} < m_B - m_K$ ,  $H_1$  is on-shell

$$\Gamma_{B^+ \rightarrow K^+ H_1} \simeq \frac{|\kappa_{cb}|^2 \sin^2 \theta \left( \frac{m_{B^+}^2 - m_{K^+}^2}{m_b - m_s} \right)^2 [f_0(m_{H_1}^2)]^2}{64\pi m_{B^+}^3} \sin \theta \ll 1$$

$$\times \sqrt{\mathcal{K}(m_{B^+}^2, m_{K^+}^2, m_{H_1}^2)} \quad \text{form factor}$$

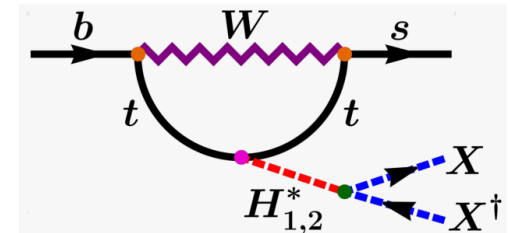
$$|\kappa_{cb}| \simeq 6.7 \times 10^{-6} \quad \mathcal{K}(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$$

W. G. Parrott, C. Bouchard & C. T. H. Davies, ORD 2023



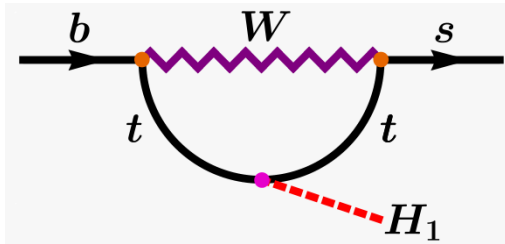
- When  $m_{H_1} > m_B - m_K$ ,  $H_1$  is off-shell  $\rightarrow$  three-body decay

$$\Gamma_{B^+ \rightarrow K^+ X X^\dagger} \simeq \frac{\lambda_{\Phi X}^2 v_\Phi^2 |\kappa_{cb}|^2 \sin^2 \theta \left( \frac{m_{B^+}^2 - m_{K^+}^2}{m_b - m_s} \right)^2 (m_{H_1}^2 - m_{H_2}^2)^2}{1024\pi^3 m_{B^+}^3} \times \int_{4m_X^2}^{(m_{B^+} - m_{K^+})^2} dq^2 \frac{\sqrt{1 - 4m_X^2/q^2} \sqrt{\mathcal{K}(m_{B^+}^2, m_{K^+}^2, q^2)} [f_0(q^2)]^2}{(q^2 - m_{H_1}^2)^2 (q^2 - m_{H_2}^2)^2}$$

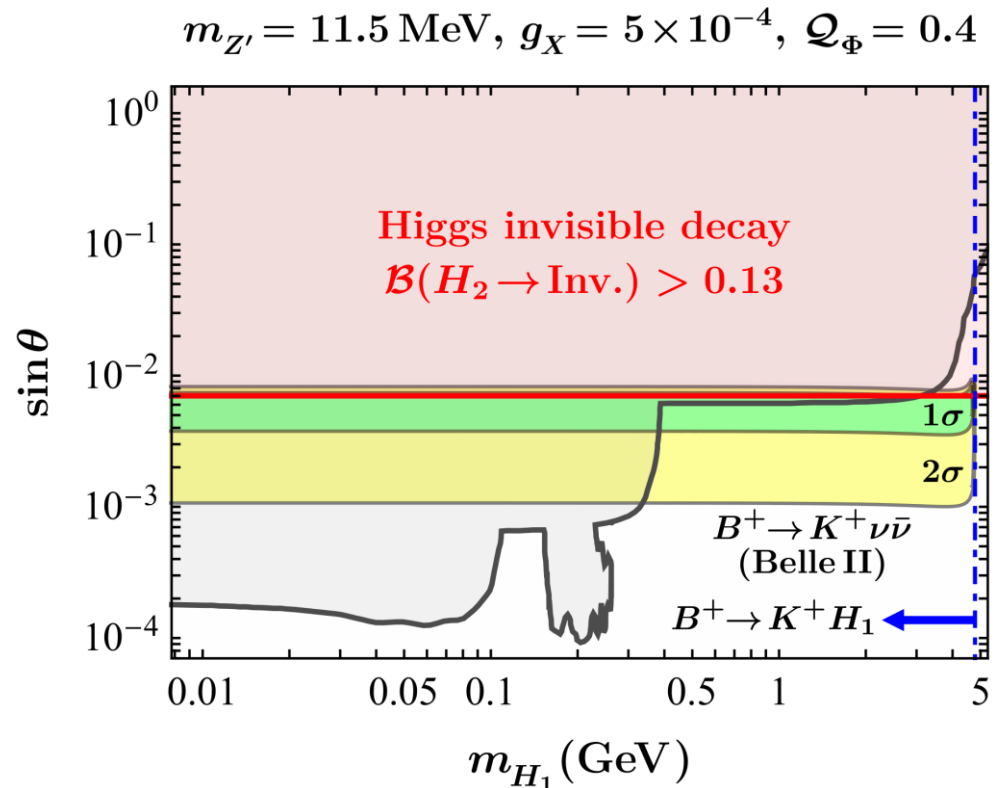


# BelleII excess: 2-body decay

- When  $m_{H_1} < m_B - m_K$ ,  $H_1$  is on-shell
- The gray shaded area is excluded by BelleII  $B^0 \rightarrow K^{*0} \nu \bar{\nu}$ , KOTO  $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$  & NA62  $K^+ \rightarrow \pi^+ + \text{inv.}$



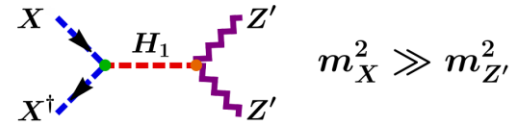
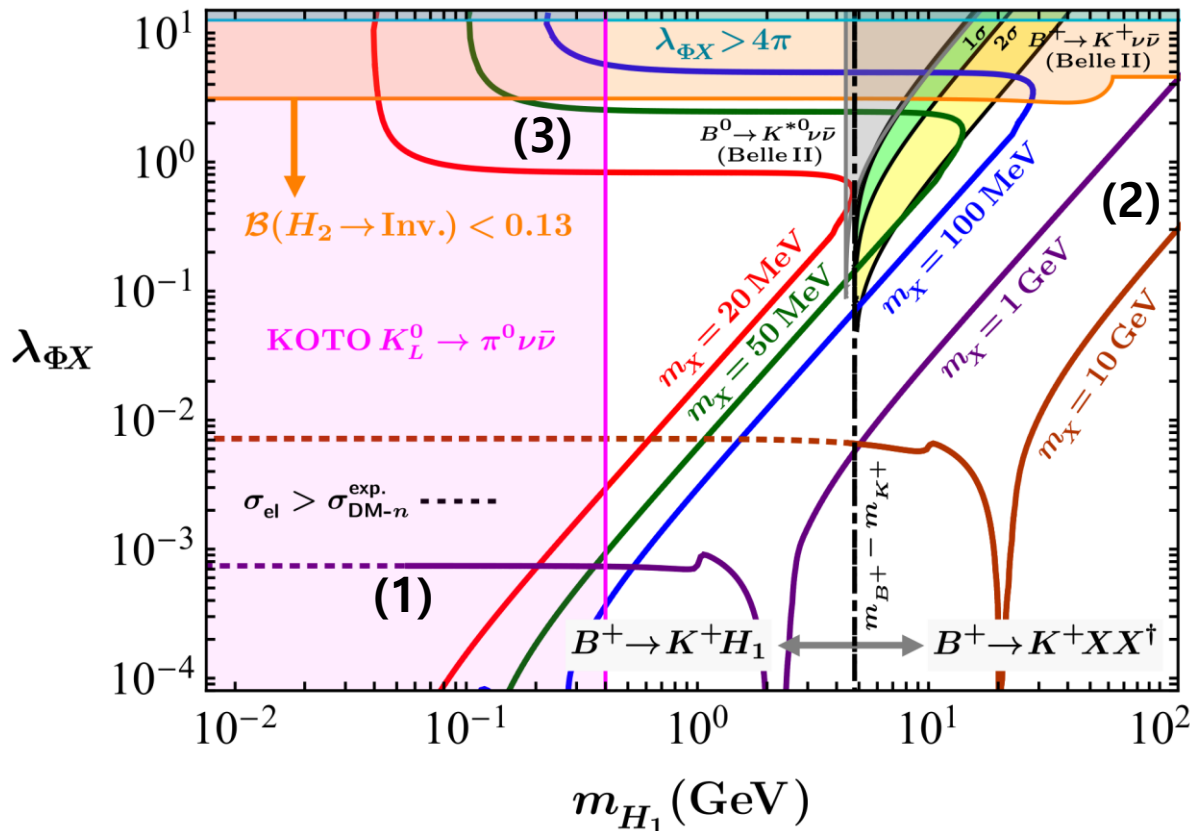
- $H_1$  decay process
  - $H_1 \rightarrow \mathbf{XX}^\dagger, \mathbf{Z}'\mathbf{Z}', ff^\dagger$
- Allowed value
  - $10^{-3} \leq \sin \theta \leq 7 \times 10^{-3}$
  - take  $\sin \theta = 6 \times 10^{-3}$



# BelleII excess : 2- or 3-body decay

- When  $m_{H_1} > (<) m_B - m_K$ ,  $H_1$  is off(on)-shell  $\rightarrow$  3(2)-body decay
  - Two-body decay:  $m_X \lesssim 10 \text{ GeV}$  ( $m_{H_1} < m_B - m_K$ )
  - Three-body decay:  $20\text{MeV} < m_X \lesssim 60\text{MeV}$  ( $m_{H_1} > m_B - m_K$ )

$$m_{Z'} = 11.5 \text{ MeV}, g_X = 5 \times 10^{-4}, \mathcal{Q}_\Phi = 0.4, s_\theta = 6 \times 10^{-3}$$



$$\langle \sigma v \rangle \propto \frac{\lambda_{\Phi X}^2 m_X^2}{(m_{H_1}^2 - 4m_X^2)^2 + \Gamma_{H_1}^2 m_{H_1}^2}$$

$$\Gamma_{H_1}^2 m_{H_1}^2 \propto \lambda_{\Phi X}^4 v_\Phi^4 \sqrt{1 - 4m_X^2/m_{H_1}^2}$$

(1)  $\lambda_{\Phi X} \ll 1$  &  $m_X^2 \gg m_{H_1}^2$

$$\Rightarrow \langle \sigma v \rangle \propto \frac{\lambda_{\Phi X}^2}{m_X^2} \Rightarrow \lambda_{\Phi X} \simeq \text{const.}$$

(2)  $\lambda_{\Phi X} \ll 1$  &  $m_{H_1}^2 \gg m_X^2$

$$\Rightarrow \langle \sigma v \rangle \propto \frac{\lambda_{\Phi X}^2 m_X^2}{m_{H_1}^4} \Rightarrow \lambda_{\Phi X} \propto m_{H_1}^2$$

(3)  $\lambda_{\Phi X} \gg 1$  &  $m_{H_1}^2 \gg m_X^2$

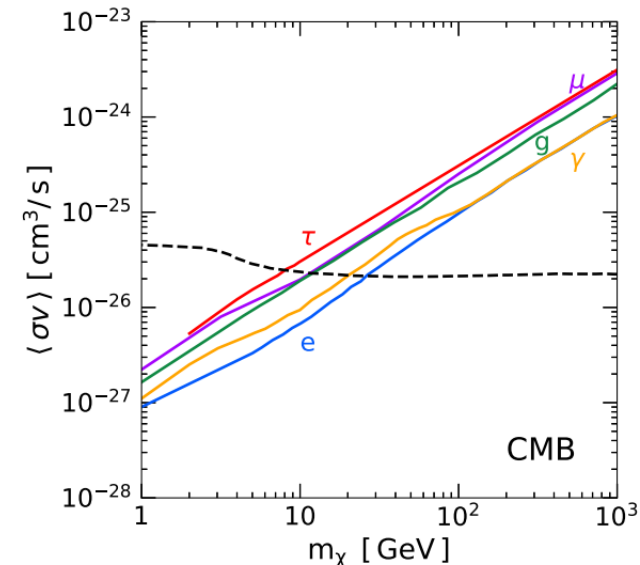
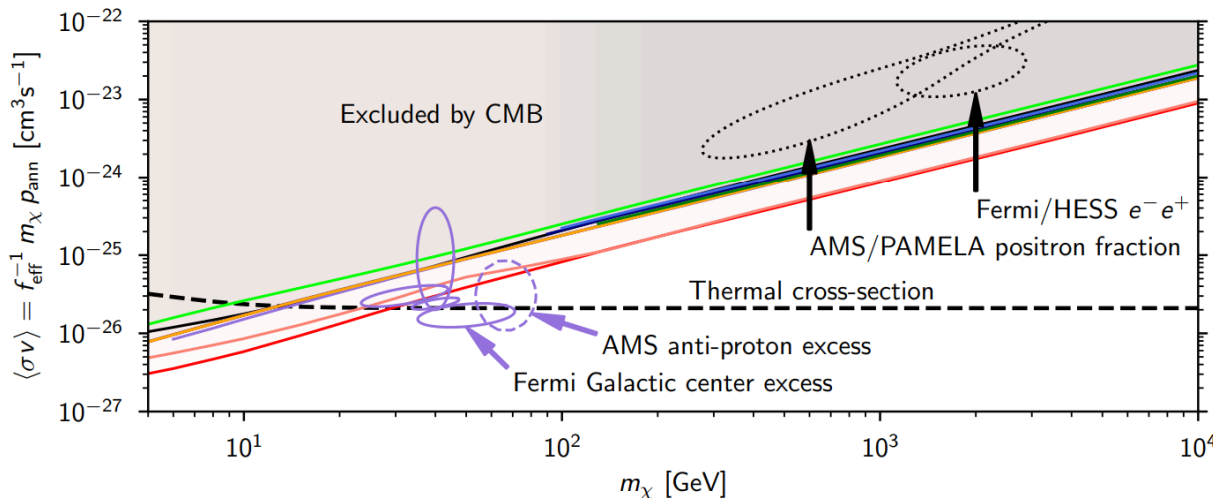
$$\Rightarrow \langle \sigma v \rangle \propto \frac{m_X^2}{\lambda_{\Phi X}^2 v_\Phi^4} \Rightarrow \lambda_{\Phi X} \simeq \text{const.}$$

# CMB constraints

- Any injection of ionizing particles modifies the ionization history of hydrogen and helium gas, perturbing CMB anisotropies
  - DM annihilations to the charged SM particles
- Measurements of these anisotropies provide robust constraints on production of ionizing particles from DM annihilation products.

$$\langle \sigma v \rangle \leq \frac{4.1 \times 10^{-28} \text{ cm}^3 \text{ sec}^{-1}}{f_{\text{eff}}} \left( \frac{m_{\text{DM}}}{\text{GeV}} \right)$$

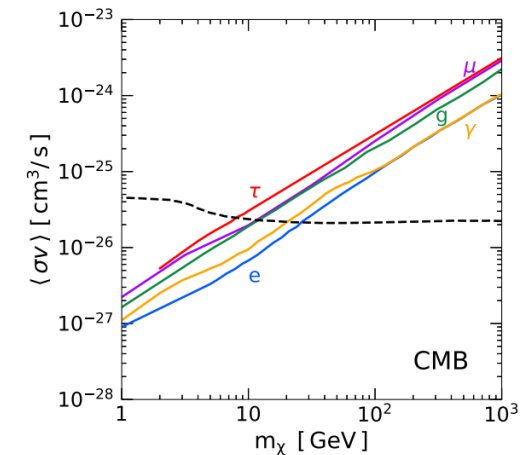
Planck 2018,  
R. K. Leane et al, PRD 2018



# CMB constraints

- For  $m_X \lesssim 20\text{GeV}$ , CMB bound (DM annihilation @  $T \sim \text{eV}$ ) excludes the thermal DM freeze-out determined by s-wave annihilation
  - DM annihilation should be mainly in **p-wave**
  - ...
- Dominant DM annihilation channel
  - $XX^\dagger \rightarrow Z'Z', H_1H_1$ : **s-wave** annihilation
  - $XX^\dagger \rightarrow Z'H_1$ : **p-wave** annihilation
- $Z'$  decay
- $H_1$  decay

$$\sigma v = \underset{\substack{\uparrow \\ \text{s-wave}}}{a} + \underset{\substack{\downarrow \\ \text{p-wave}}}{b}v^2 + O(v^4)$$



# CMB constraints

- For  $m_X \lesssim 20\text{GeV}$ , CMB bound (DM annihilation @  $T \sim \text{eV}$ ) excludes the thermal DM freeze-out determined by s-wave annihilation

- DM annihilation should be mainly in **p-wave**
- ...

- Dominant DM annihilation channel

- $XX^\dagger \rightarrow Z'Z', H_1H_1$ : **s-wave** annihilation
- $XX^\dagger \rightarrow Z'H_1$ : **p-wave** annihilation

- $Z'$  decay

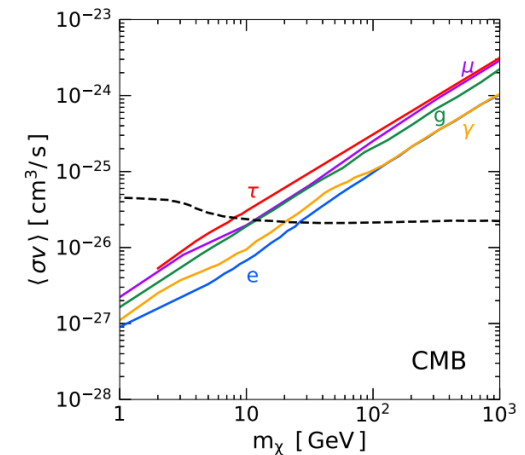
- A pair of  $\nu$  ( $m_{Z'} = 11.5\text{MeV}$ ,  $g_X = 5 \times 10^{-4}$ )

- $H_1$  decay

- A pair of DM (open when  $m_{H_1} > 2m_X$ )
- A pair of  $Z'$  ( $Z' \rightarrow \nu\nu$ )
- SM particles (suppressed due to small Yukawa coupling &  $\sin \theta$ )

$$\sigma v = a + b v^2 + O(v^4)$$

↑ s-wave  
↓ p-wave



# Conclusion

- BelleII data shows a mild excess of  $B^+ \rightarrow K^+ \nu \bar{\nu}$  over the SM prediction
- This mild excess can be interpreted as  $B^+ \rightarrow K^+ +$  dark sector particles through a dark Higgs portal
- CMB constraints can be evaded because DM pair annihilations into  $H_1 H_1, H_1 Z', Z' Z'$ , all of which are invisible in  $U(1)_{L_\mu - L_\tau}$  models with complex scalar DM in our benchmark point
- We can accommodate the muon g-2 and subsequently relax the tension in the Hubble constant with extra radiation

# Conclusion

- BelleII data shows a mild excess of  $B^+ \rightarrow K^+ \nu \bar{\nu}$  over the SM prediction

- This part

- CM into mod

Thank you  
very much

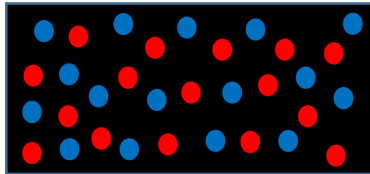
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- We can accommodate the muon g-2 and subsequently relax the tension in the Hubble constant with extra radiation

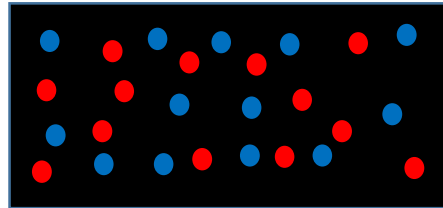
# Back-up Slides

# Evidences – Dark Matter

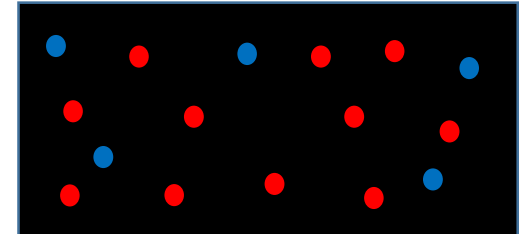


$$T \gg M_{DM}$$

- ● : Dark Matter
- ● : Standard Model



$$T \approx M_{DM}$$

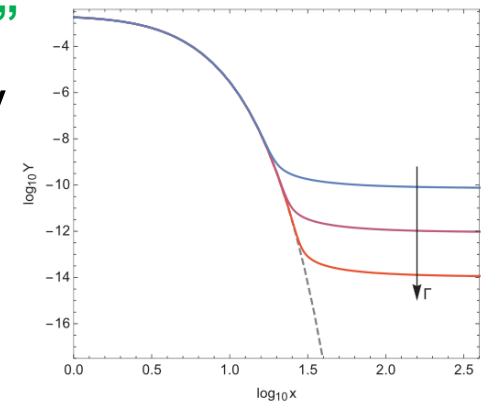
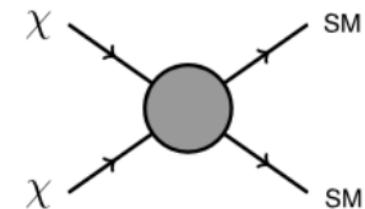


$$T \ll M_{DM}$$

- Dark matter population in an **expanding** Universe
  - **Dark matter particles can no longer annihilate**
  - **The number of dark matter particles “freeze-out”**
- Standard calculation for WIMP DM relic density
  - The Boltzmann equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle (n_\chi^2 - n_{eq}^2)$$

- **Relic density**:  $\Omega h^2 = 0.12 \rightarrow \langle\sigma v\rangle \sim 10^{-9} \text{GeV}^{-2}$



# Gauged $U(1)_{L_\mu - L_\tau}$ $Z'$ model

- **Neutrino trident production**

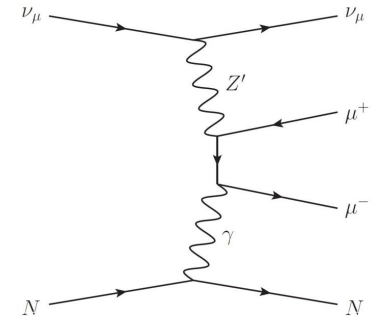
W. Altmannshofer et al, PRL 2014

- Production of a muon pair from the scattering of a muon neutrino with heavy nuclei

- $R_{\text{CCFR}} \equiv \frac{\sigma_{\text{CCFR}}}{\sigma_{\text{SM}}} = 0.82 \pm 0.28.$

- **NA64** Y. Andreev, 2401.01708

- $\mu^- N \rightarrow \mu^- N Z', (Z' \rightarrow \text{inv.})$
- Upper limit on  $g_X$  for  $1\text{MeV} \leq m_{Z'} \leq 1\text{GeV}$



- **$\Delta N_{\text{eff}}$**

M. Escudero et al, JHEP 2019

- $Z'$  will reheat the neutrino gas, resulting in a higher expansion rate
- Increase the effective number of neutrinos  $N_{\text{eff}}$
- $\Delta N_{\text{eff}} < 0.5$

- **BOREXINO**

R. Harnik et al, JCAP 2012

- $\nu - e$  scattering

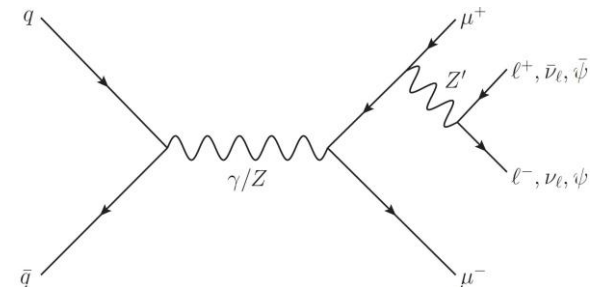
# BaBar, LHC $4\mu$ channels

- $e^+e^- \rightarrow \mu^+\mu^-Z'$ ,  $Z' \rightarrow \mu^+\mu^-$ 
  - Upper limit on  $g_X$  for  $200\text{MeV} \leq M_{Z'} \leq 10\text{GeV}$

BaBar Collaboration, PRD 2016

CMS Collaboration, PLB 2019

- The lowest order  $Z'$  production process at collider
  - Produce a charged lepton pair through Drell-Yan process
  - $Z'$  is radiated from one of leptons



- Final states
  - two pair of charged-leptons
  - A pair of charged-lepton plus missing energy

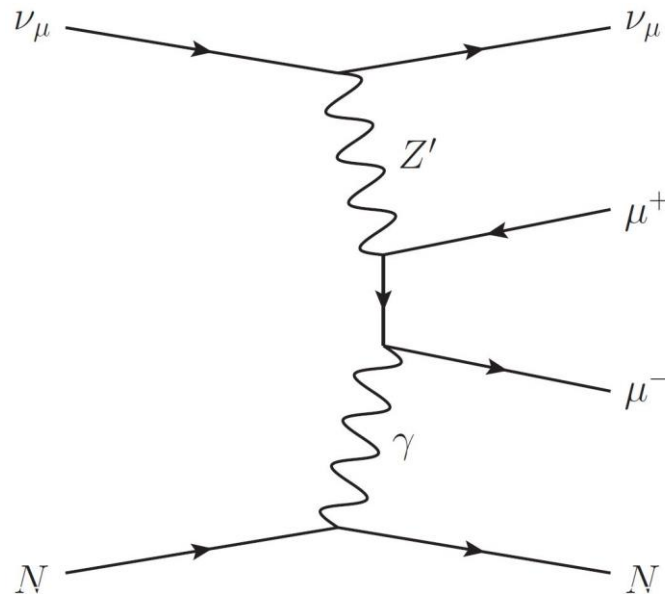
# Neutrino trident production

- Production of a muon pair from the scattering of a muon neutrino with heavy nuclei

- $R_{\text{CCFR}} \equiv \frac{\sigma_{\text{CCFR}}}{\sigma_{\text{SM}}} = 0.82 \pm 0.28.$

W. Altmannshofer et al, PRL 2014

- The leading order  $Z'$  contribution:



# Borexino: $\nu - e$ scattering

- Borexino is a liquid scintillator experiment measuring solar neutrino scattering off electron
  - Probe non-standard interactions between neutrinos and target
  - Limits from Borexino for the  $U(1)_{B-L}$  gauge boson have been derived.

R. Harnik et al, JCAP 2012

- Rescale the constraints on  $U(1)_{B-L}$  boson as

$$\alpha_{B-L}^2 \rightarrow \begin{cases} \left[ \sum_{i,j=1}^3 f_i |(U^\dagger Q_{\mu e} U)_{ij}|^2 \right]^{1/2} \alpha_{\mu e}^2, & \text{for } U(1)_{L_\mu - L_e}, \\ \left[ \sum_{i,j=1}^3 f_i |(U^\dagger Q_{e\tau} U)_{ij}|^2 \right]^{1/2} \alpha_{e\tau}^2, & \text{for } U(1)_{L_e - L_\tau}, \\ \left[ \sum_{i,j=1}^3 f_i |(U^\dagger Q_{\mu\tau} U)_{ij}|^2 \right]^{1/2} \alpha \alpha_{\mu\tau} \epsilon_{\mu\tau}(q^2), & \text{for } U(1)_{L_\mu - L_\tau}, \end{cases}$$

$$Q_{\mu\tau} = \text{diag}(0, 1, -1)$$



# $U(1)_{L_\mu - L_\tau}$ -charged DM model

- Conventional  $U(1)_{L_\mu - L_\tau}$ -charged fermion DM model

$$\mathcal{L} \supset \mathcal{L}_{\text{SM}} - \frac{1}{4} Z'_{\alpha\beta} Z'^{\alpha\beta} + \frac{1}{2} m_{Z'}^2 Z'_\alpha Z'^\alpha + i\bar{\chi}\gamma^\alpha \partial_\alpha \chi - m_\chi \bar{\chi}\chi$$

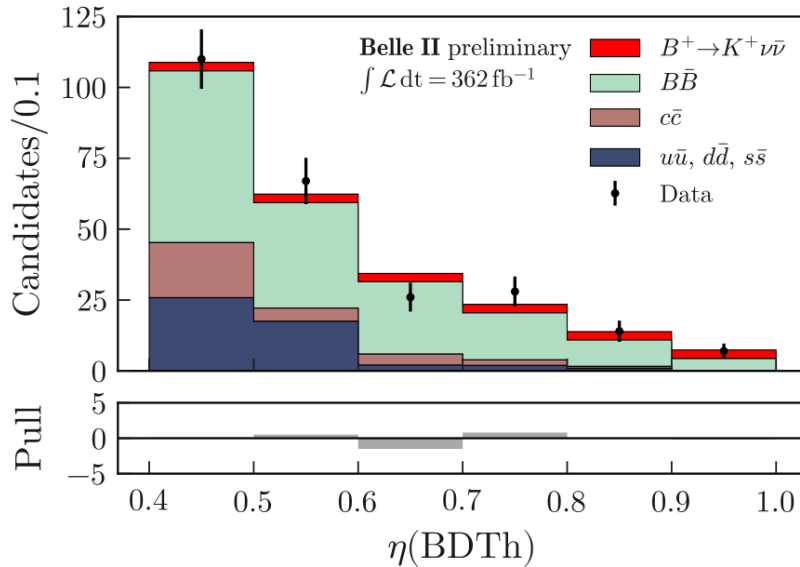
$$+ g_X Q_\chi Z'_\alpha \bar{\chi}\gamma^\alpha \chi + g_X Z'_\alpha \sum Q_{\ell} \bar{\ell}\gamma^\alpha \ell$$

- Dark Photon  $Z'$  plays a role of messenger particle between DM and the SM leptons
- Dark Photon mass is generated by hand or Stueckelberg mechanism
- New parameters:  $\{g_X, m_{Z'}, m_\chi, Q_\chi\}$
- Consider  $Z'$  boson only &  $g_X \sim (3 - 5) \times 10^{-4}$  for the muon  $g-2$ 
  - $\chi\bar{\chi}(X\bar{X}) \rightarrow f_{\text{SM}}\bar{f}_{\text{SM}}$  : dominant annihilation channels
  - $g_X \sim 10^{-4}$  is too small to get  $\Omega_\chi h^2 = 0.12$

# Measurement of $B^+ \rightarrow K^+ \nu \bar{\nu}$

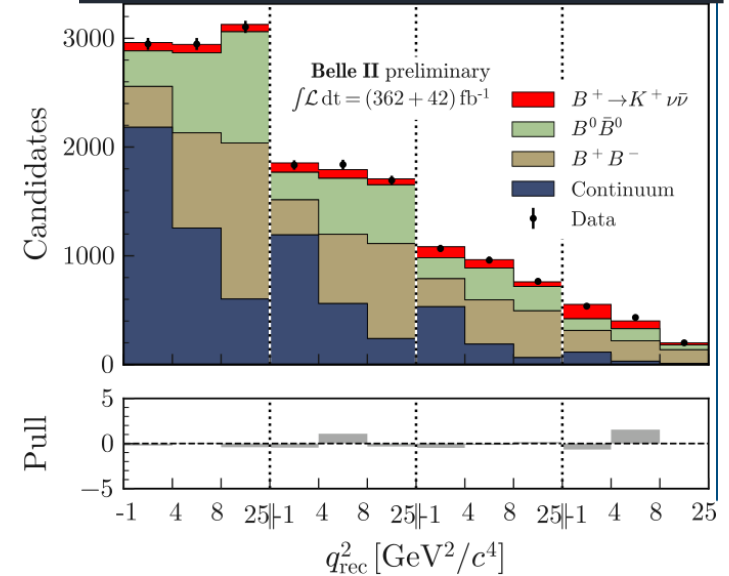
- Two ways of tagging

## Hadronic tagging (HTA)



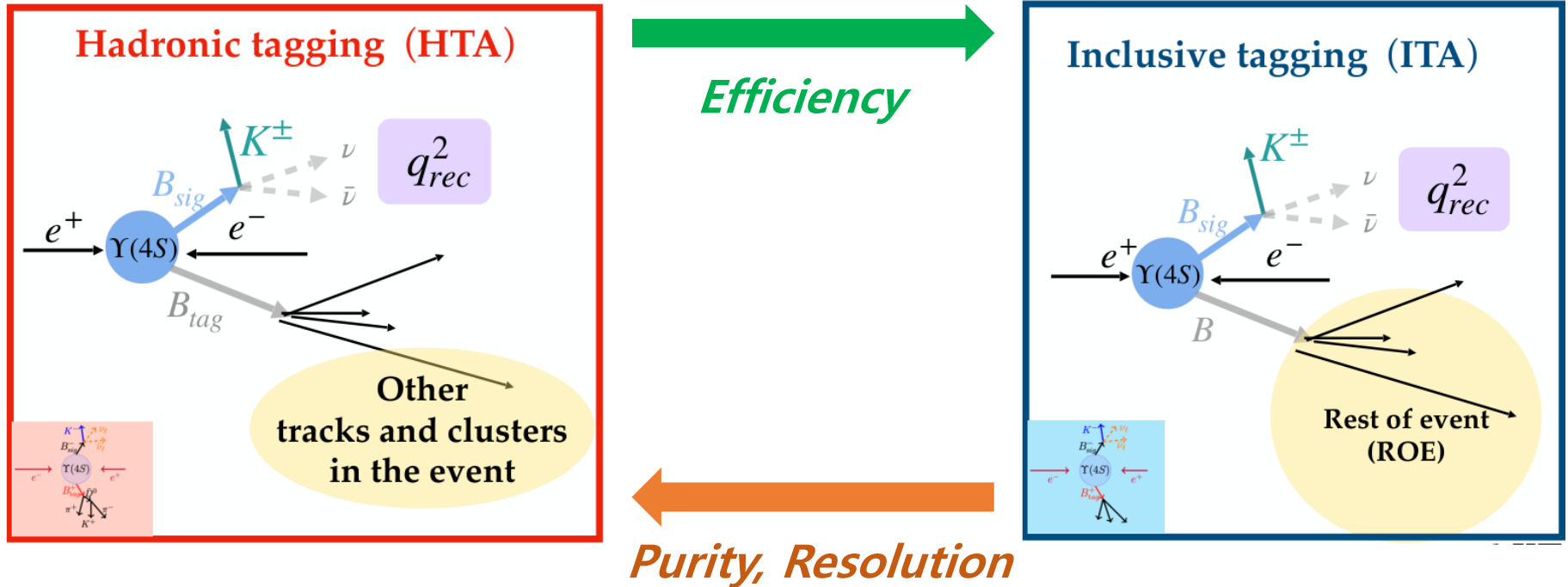
- $\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{HTA}} = (1.1^{+0.9+0.8}_{-0.8-0.5}) \times 10^{-5}$
- $\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{ITA}} = (2.7 \pm 0.5 \pm 0.5) \times 10^{-5}$

## Inclusive tagging (ITA)



# Measurement of $B^+ \rightarrow K^+ \nu \bar{\nu}$

- Two ways of tagging



- $q_{rec}^2$ : mass squared of the neutrino pair
- Inclusive tagging: It allows one to reconstruct inclusively the decay  $B^+ \rightarrow K^+ \nu \bar{\nu}$  from the charged kaon

# Measurement of $B^+ \rightarrow K^+ \nu \bar{\nu}$

- **Challenges** in reconstructing the events
  - Searches for  $B \rightarrow K^{(*)} \nu \bar{\nu}$  have only been performed at the B factories Belle and BaBar
- Using the same techniques in Belle, BaBar
  - Semileptonic tagged analyses
  - Hadronic-tagged analyses
- **Inclusive tag analysis** (Belle & Belle II )
  - Allow one to reconstruct inclusively the decay  $B^+ \rightarrow K^+ \nu \bar{\nu}$  from the charged kaon