Fundamental Physics Search with SRF Cavities

Yifan Chen, Niels Bohr Institute
yifan.chen@nbi.ku.dk
SHANHE Collaboration

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Electromagnetic Detection of Ultralight Bosons and HFGW

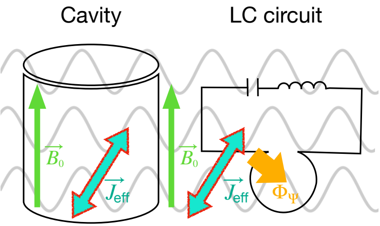
$\vec{J}_{\rm eff}$ in a cavity or shield room: $\Box\vec{A} = \vec{J}_{\rm eff}$.

Dark photon A':
$\vec{J}_{\rm eff} = \epsilon\, m_{A'}^2 \vec{A}'$;
No background field.

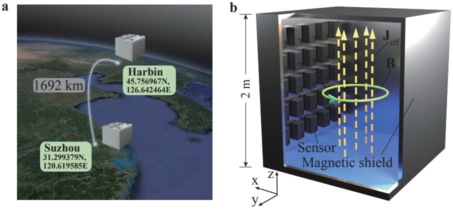
Axion a:
$\vec{J}_{\rm eff} = g_{a\gamma} \vec{B}_0 \partial_t a$;
Background $\vec{B}_0$.

GW h:
$\vec{J}_{\rm eff} \sim \partial(h\, F_0)$;
Background $\vec{E}_0$ or $\vec{B}_0$.

- **Resonant cavity**: $\omega_{\rm rf} \sim \omega_{J_{\rm eff}}$.
- **Circuit/magnetometer**: $B \sim |\vec{J}_{\rm eff} V^{1/3}|$ from $\vec{J}_{\rm eff}$ where $1/\omega_{J_{\rm eff}} \gg V^{1/3}$.



e.g. ADMX, HAYSTAC, CAPP, ORGAN, DM radio...



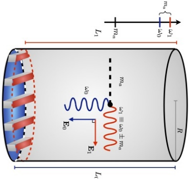
[Jiang et al, Nature Commun, 2305.00890]

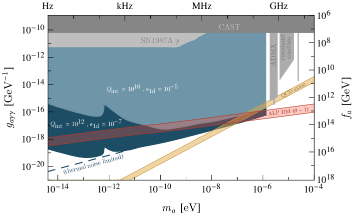
Superconducting Radio-Frequency (SRF) Cavity

- **SRF cavities** are widely used for accelerators.
- Significant $Q_0 > 10^{10}$ compared to copper cavity with $Q_0 \leq 10^6$.
- High Q_0 boosts **dark photon searches** [SERAPH 22', SHANHE 23']:

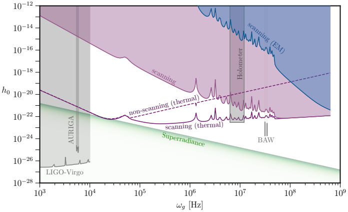
$$\epsilon \approx 10^{-16} \left(\frac{10^{10}}{Q_0}\right)^{\frac{1}{4}} \left(\frac{4\,\mathrm{L}}{V}\right)^{\frac{1}{2}} \left(\frac{100\,\mathrm{s}}{t_{\mathrm{int}}}\right)^{\frac{1}{4}} \left(\frac{\mathrm{GHz}}{f_0}\right)^{\frac{1}{4}} \left(\frac{T_{\mathrm{amp}}}{3\,\mathrm{K}}\right)^{\frac{1}{2}}.$$

- **Heterodyne upconversion** [Berlin et al 19']: $\omega_{\mathrm{rf}} - \omega_0 \approx \omega_a$ or ω_h.
- Both **EM** and **mechanical** coupling from GW [Berlin et al 21' 23'].





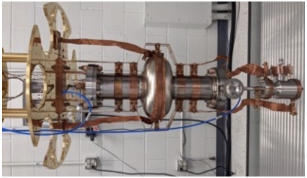
[Berlin et al 19']



[Berlin et al 23']

- Niobium superconductor requires $B_0^{\max} < 0.2\mathrm{T}$, still $g_{a\gamma}$ or $h \sim 1/Q_0^{1/4}$.

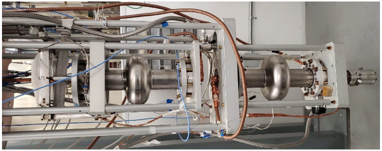
International SRF Campaigns

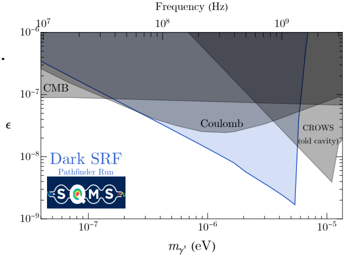


- **Fermilab SQMS**

  •**SERAPH**:
  Dark photon dark matter searches.

  •**Dark SRF**:
  Light-shining-wall search for dark photon.



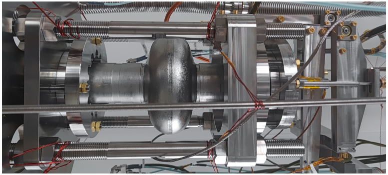


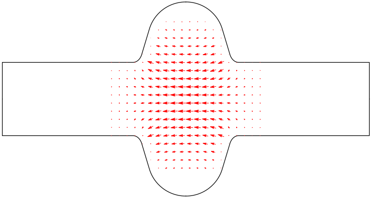
- **DESY/SQMS**:

  •**MAGO 2.0**
  HFGW searches with mechanical couplings.

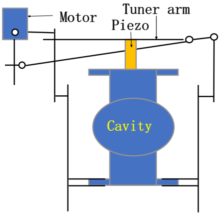
DPDM Scan Search [SHANHE Collaboration]

- TM_{010} of **1-cell elliptical cavity**: largest overlapping with DPDM.

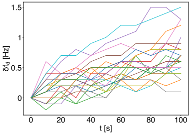


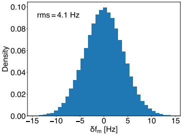


- Cavity and amplifier positioned in 2 K **liquid helium**.
- **Mechanical turner** scans resonant frequency f_0.
- Each scan is followed by calibration of f_0 and its stability range Δf_0.



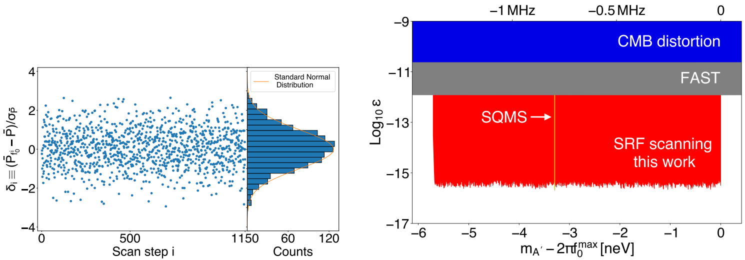
- Drift $\delta f_d \leq 1.5\,\mathrm{Hz}$ and microphonics $\sigma_{f_0} \approx 4\,\mathrm{Hz}$ $\rightarrow \Delta f_0 \approx 10\,\mathrm{Hz}$.





Data Analysis and Constraints

- Total 1150 scan steps with each 100 s integration time.
- **Group every 50 adjacent bins** and perform a constant fit to address small helium pressure fluctuation.
- Normal power excess shows **Gaussian distribution**:



[SHANHE, PRL 2305.09711]

- **Scan search with SRF and most stringent constraints in most exclusion space near** $f_0 \approx 1.3$ **GHz**.

Cavity as Radio Telescope for Dark Photon

Galactic boosted dark photon from dark matter decay:

- Perturbative cascade decay from **standard halo**.
  [ADMX Dror et al 23']

- Parametric resonant production from **scalar clump**?

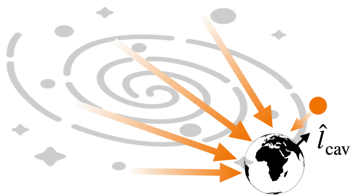
Polarization-dependent production:

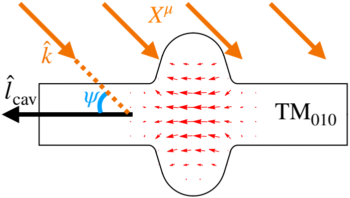
- Longitudinal mode
  from a dark higgs.

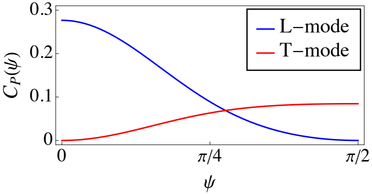
- Transverse mode
  from axion-photon-type coupling.

Cavity as **radio telescope** for dark photon.

- **Diurnal modulation** to distinguish direction and polarization.







Cavity as Radio Telescope for Dark Photon

Galactic boosted dark photon from dark matter decay:

- Perturbative cascade decay from **standard halo**.
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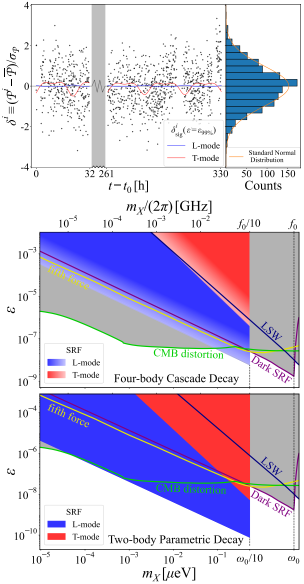
- Parametric resonant production from **scalar clump**?

Polarization-dependent production:

- Longitudinal mode from a dark higgs.
- Transverse mode from axion-photon-type coupling.

Diurnal modulation constraints:

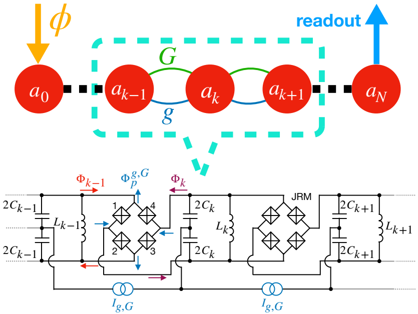
- L-mode enhanced from $|\vec{A}'| \sim \omega_{A'}/m_{A'} \gg 1$.

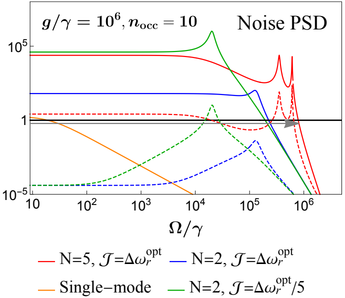


[SHANHE, 2402.03432]

Response Width for Multi-mode Resonators

Broadened response in multi-mode resonators [YC et al, PRR 2103.12085, 2309.12387]:





$$H_{\rm ch} = \sum_{k=0}^{N-1} \left(\mathrm{i} g \hat{a}_k \hat{a}_{k+1}^\dagger + \mathrm{i} G \hat{a}_k \hat{a}_{k+1} + h.c. \right) .$$

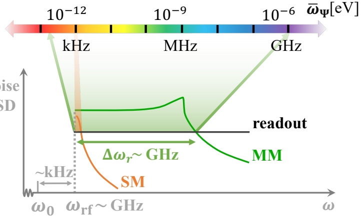
- **Realization** via Josephson junctions [Wurtz et al 21', Jiang et al 23', CEASEFIRE].
- New sensitivity limit for multi-mode resonators, optimal for SRF cavity:

$$\frac{\Delta\omega_r^{\rm MM}}{\Delta\omega_r^{\rm SM}} \propto \left(\frac{g}{\gamma n_{\rm occ}} \right)^{\frac{2N}{2N+1}} \rightarrow \frac{Q_0}{n_{\rm occ}} \text{ as } N \gg 1, \;\; g \rightarrow \omega_{\rm rf}, \;\; \Delta\omega_r^{\rm MM} \rightarrow \omega_{\rm rf} .$$

Simultaneous Resonant and Broadband Detection

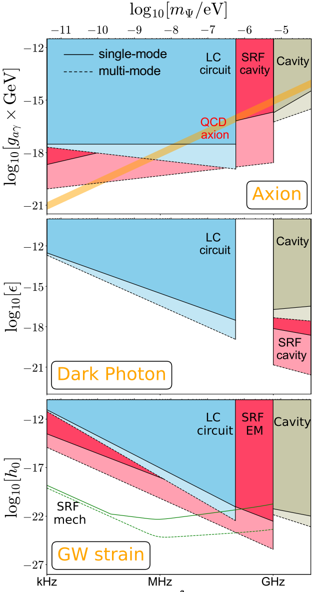
- *e*-fold time: 10^7 s.
- **DC cavity and LC circuits**: $\frac{\mathrm{SNR}^2_{\mathrm{MM}}}{\mathrm{SNR}^2_{\mathrm{SM}}} \simeq \frac{Q_0}{n_{\mathrm{occ}}}$.

  High n_{occ} of LC circuits at low frequency made enhancement ineffective.
- **SRF heterodyne upconversion**:



Simultaneous scan $N_e = 6$ orders of ω_Ψ with **significant response**:

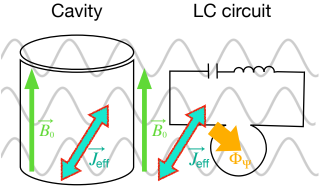
$$\frac{\mathrm{SNR}^2_{\mathrm{MM}}}{\mathrm{SNR}^2_{\mathrm{SM}}} \simeq N_e \frac{\bar{\omega}_\Psi \, Q_0}{\omega_{\mathrm{rf}} \, n_{\mathrm{occ}}}$$



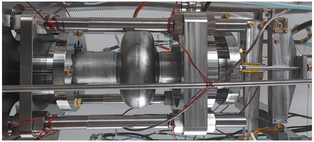
[YC et al, 2309.12387]

Summary

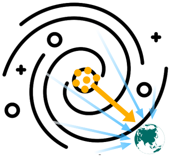
- **Resonant cavities or circuits** are powerful detectors for ultralight bosons and HFGW.



- **SRF cavity** with significant Q_0 has significant sensitivity: $\epsilon/g_{a\gamma}/h \propto 1/Q_0^{1/4}$.



- **SRF cavity** as **radio antenna** for dark photon: dissection of polarization and angular distribution.



- Multi-mode resonantors have broadened response.

  $\rightarrow$ **Simultaneous resonant and broadband** detection for **SRF upconversion**.

Thank you!

Appendix

Ultralight Bosons

$$-\frac{1}{2}\nabla^{\mu}a\nabla_{\mu}a - \frac{1}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - V(\Psi), \quad \Psi = a, \phi \text{ and } A'_{\mu}.$$

- Axion: hypothetical pseudoscalar motivated by strong CP problem.
- Prediction from fundamental theories with **extra dimensions**:

  e.g. $g_{MN}(5D) \to g_{\mu\nu}(4D) + A'_{\mu}(4D), \quad A'_{M}(5D) \to A'_{\mu}(4D) + a(4D).$

  String axiverse/photiverse: logarithmic mass window, $m_{\Psi} \propto e^{-\mathcal{V}_{6D}}$.
- **Coherent wave** dark matter candidates when $m_{\Psi} < 1$ eV:

$$\Psi(x^{\mu}) \simeq \Psi_0(\mathbf{x}) \cos \omega t; \qquad \Psi_0 \simeq \frac{\sqrt{\rho}}{m_{\Psi}}; \qquad \omega \simeq m_{\Psi}.$$

Axion Photon Coupling and Cavity Haloscope

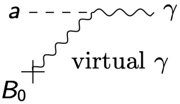
- **Axion photon coupling**: $\propto g_{a\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu}$,

  mixture with π_0 and anomaly generation.

$$\to \nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J} - g_{a\gamma}\left(\mathbf{E} \times \nabla a - \mathbf{B}\partial_t a\right).$$

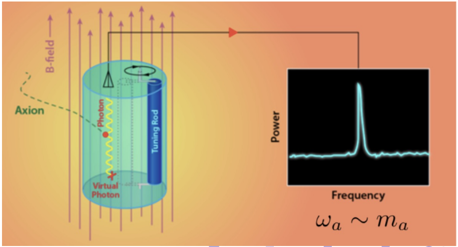
- Resonant cavity haloscope [Sikivie 83']:

$$\mathbf{J}_{\rm eff}(t) = g_{a\gamma}\mathbf{B}_0\partial_t a \qquad \left(\partial_t^2 + \gamma\partial_t + \omega_{\rm rf}^2\right)\mathbf{E}_{\rm rf} = \partial_t \mathbf{J}_{\rm eff}(t).$$



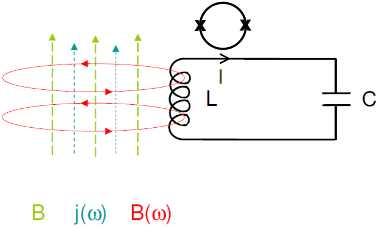
- Background static $\mathbf{B_0}$
  $\to$ resonant when
  $\omega_{\rm rf} = m_a \sim V^{-1/3} \sim \mathcal{O}(1)$ GHz.

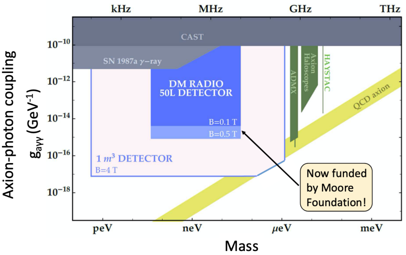
  e.g. ADMX, HAYSTAC, CAPP, ORGAN $\cdots$



Resonant LC circuit

- Resonant conversion happens when $m_a \simeq \omega_{\rm rf} = \frac{1}{\sqrt{LC}}$ [Sikivie et al 13'].
- Scanning the mass from 100 Hz to 100 MHz by **tuning the capacitor C**.



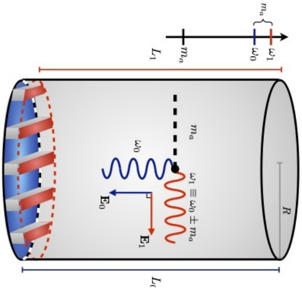


Assumptions: T=10 mK, Q=10^6, 3.5 year integration time, quantum-limited readout

e.g. DM radio, ADMX-SLIC

Heterodyne Upconversion with SRF Cavity

$$\left(\partial_t^2 + \gamma\partial_t + \omega_{\rm rf}^2\right)\mathbf{E}_{\rm rf} = g_{a\gamma}\partial_t\left(\mathbf{B_0}\partial_t a\right).$$



- **Heterodyne upconversion** [Berlin et al 19']:

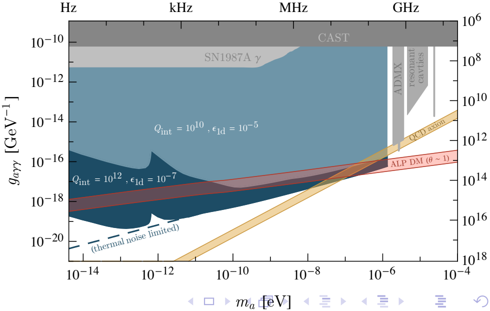
  injecting AC pump mode

  $\partial_t(\mathbf{B_0}) = i\omega_0\mathbf{B_0}, \quad \omega_{\rm rf} \simeq \omega_0 + m_a.$

  Large overlapping between $\mathbf{B_0}$ and $\mathbf{E}_{\rm rf}$ is required.

- Tune $\omega_{\rm rf} - \omega_0$ from Hz to GHz:

- Sensitivity benefits from superconducting nature: $Q_0 > 10^{10}$.



Dark Photon

- A new $U(1)$ vector couples in different portals with SM particles:

$$\epsilon F'_{\mu\nu} F^{\mu\nu} + A'_\mu \bar{\psi}\gamma^\mu (g_V + g_A\gamma_5)\psi + F'_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}(g_M + g_E\gamma_5)\psi.$$

- Cavity/circuits for **kinetic mixing**, optomechanics for **hidden U(1)**, spin sensors for **dipole couplings**...

- Similar to axion: extra dimensions, misalignment production (or during inflation), coherent wave.

- Novel aspects: **three polarization degrees of freedom**:

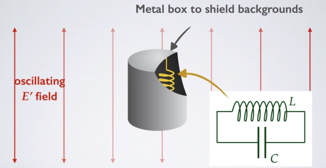
  Longitudinal mode: $\vec{\epsilon}_0(\vec{k}) \propto \vec{k}$.

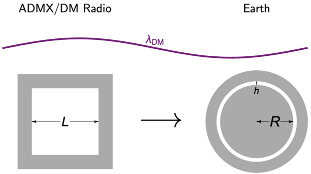
  Transverse modes: $\vec{\epsilon}_{R/L} \perp \vec{k}$.

- Signals projected to the **sensitive direction of a vector sensor**: $\sim \vec{\epsilon}\cdot\hat{l}$.

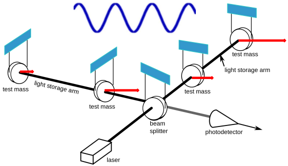
Kinetic Mixing and Hidden U(1) Dark Photon

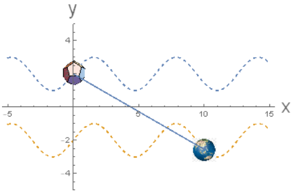
- Effective currents from $\epsilon F'_{\mu\nu} F^{\mu\nu}$: $A'_\mu \to \vec{J}_{\rm eff}$.
  Kinetic mixing U(1) dark photon shows up in circuit/cavity. [Chaudhuri et al 15'] or geomagnetic fields [Fedderke et al 21'];





- Force from $g_V A'_\mu \bar{\psi}\gamma^\mu\psi$: $A'_\mu \to \vec{F}$.
  U(1) B-L & B shows up in optomechanics [Graham et al 15', Pierce et al 18'] or astrometry [Graham et al 15', PTA, GAIA].



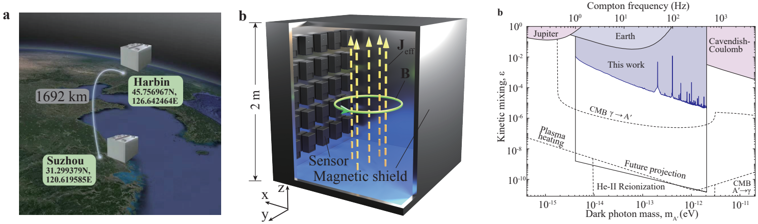


Large Shield Room for Dark Photon Dark Matter

Dark photon dark matter induces

$$B \approx |\vec{\mathbf{J}}_{\rm eff}|\, V^{1/3} \approx 10^{-12}\,\varepsilon \left(\frac{m_{A'}}{10\,\mathrm{Hz}}\right) \left(\frac{V^{1/3}}{1\,\mathrm{m}}\right) \mathrm{T},$$

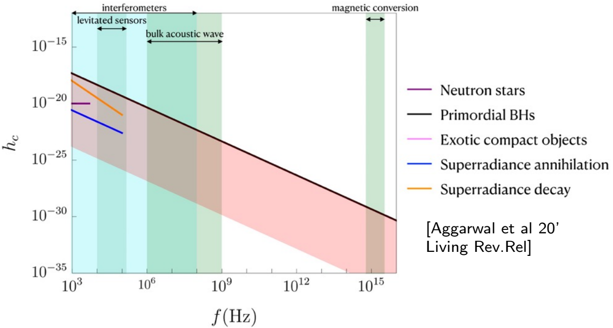
- **Two spatially separated large shield room** (8m^3) with **magnetometers** placed on the wall: [Jiang et al 23']



- Long-baseline correlation suppresses common-mode noise.

High-Frequency Gravitational Waves

- Gravitational waves (GW) above 10 kHz have no known astrophysical origins.



[Aggarwal et al 20' Living Rev.Rel]

- **Inverse Gertsenshtein effect**:

$$\frac{1}{2}h^{\mu\nu}T^{\mathrm{EM}}_{\mu\nu} \to J^{\mu}_{\mathrm{eff}} = \partial_\nu\left(\frac{1}{2}hF^{\mu\nu} + h^{\nu}{}_{\rho}F^{\rho\mu} - h^{\mu}{}_{\rho}F^{\rho\nu}\right).$$

- **Mechanical resonance**: cavity deformation and mode transition, [MAGO 2.0].

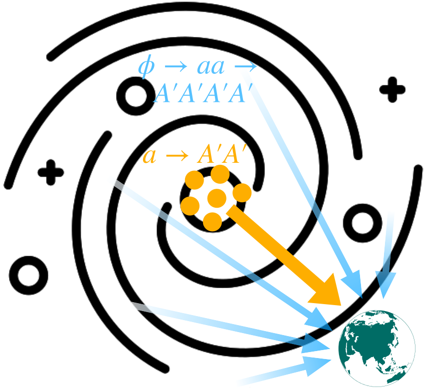
Galactic Boosted Dark Photon

Galactic boosted dark photon from dark matter decay:

- Perturbative cascade decay from **standard halo**.
  [ADMX Dror et al 23']
- Parametric resonant production from **scalar clump**?

Polarization-dependent production:

- Longitudinal mode from a dark higgs.
- Transverse mode from axion-photon-type coupling.



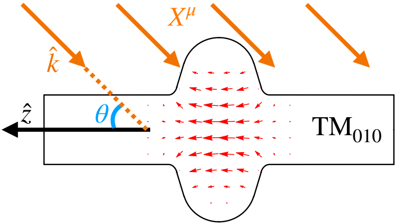
Diurnal Modulation from Earth Rotation

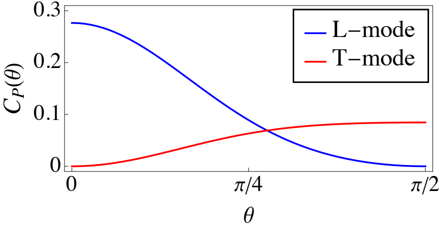
- **Angular-dependent sensitivity** to relativistic dark photon characterized by **overlapping factor** $C(\theta)$.
  [ADMX Dror et al 23' for galactic axion]

- Detector is rest at Earth frame while Earth is **rotating in galactic frame**.

- **Diurnal modulation** of the signals in cavity.

- Longitudinal and transverse modes show opposite variation.





SRF Constraints for Galactic Dark Photon

Same dataset as dark photon dark matter searches:

- Total scan range of $\sim 1\,\mathrm{MHz}$:
  within bandwidth of
  galactic dark photon.

- Total experimental time of
  $\sim 60\,\mathrm{hours}$:
  daily modulation tests.

- Cosmology requires
  $\rho_{A'} \leq 1000\,\rho_\gamma$ on Earth.

- Constraints for longitudinal modes are more stringent due to its spatial wavefunction is $\sim \omega_{A'}/m_{A'}$.

Simultaneous Resonant and Broadband Detection for Dark Sectors

based on

arxiv: 2103.12085, Phys. Rev. Res. **4** (2022) no.2, 023015

arxiv: 2309.12387

in collaboration with

Jiang, Li, Liu, Ma, Shu, Yang, Zeng

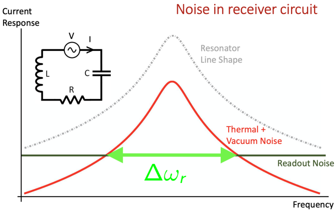
Quantum noise limit for resonant detection

- **Standard quantum limit for power law detection**: [Chaudhuri et al 18']

  Noise PSD: resonant intrinsic noise S_{int} + flat readout noise S_{r} .

- Sensitivity to S_{sig} and S_{int} is the same.

  $$\mathbf{SNR}^2 \propto \Delta\omega_r \; (S_{\mathrm{int}} \gg S_{\mathrm{r}}).$$



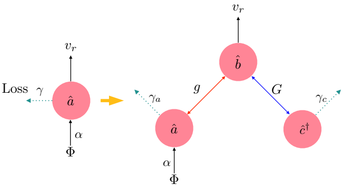
$S_{\mathrm{int}} \propto$ Cauchy distribution

- **Beyond standard quantum limit**:

  Squeezing S_{r}, e.g., HAYSTAC.

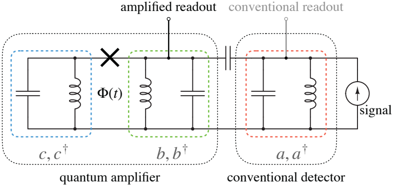
  Increasing the sensitivity to S_{sig}, e.g., white light cavity in optomechanics/GW detection [Miao et al 15'].

White Light Cavity for Axion [Li et al 20']



Single Cavity Design

Detector Design with $\mathcal{PT}$ symmetry



Probing mode: $\hbar\alpha(\hat{a} + \hat{a}^\dagger)\Psi$

- **Beam-splitting**: $\hbar g(\hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b})$.
- **Non-degenerate parametric interaction**: $\hbar G(\hat{b}\hat{c} + \hat{b}^\dagger\hat{c}^\dagger)$.
- **$\mathcal{PT}$-symmetry ($\hat{a} \leftrightarrow \hat{c}^\dagger$) emerges** when $g = G$.

$$(\dot{\hat{a}} + \dot{\hat{c}}^\dagger) = -i(g - G)\hat{b} - i\alpha\Psi + \cdots ;$$
$$\dot{\hat{b}} = -\gamma_r\hat{b} - ig(\hat{a} + \hat{c}^\dagger) + \cdots .$$

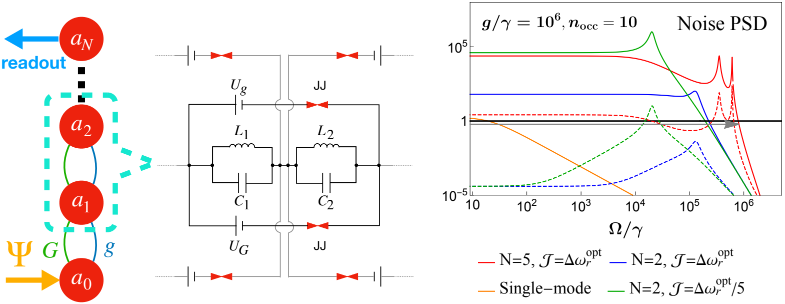
- Coherent cancellation leads to **double resonance**. $S_{\rm sig}$ is largely enhanced when $g \gg$ **intrinsic dissipation** γ:

$$S_{\rm sig}^{\rm WLC}(\Omega) = \frac{2\gamma_r\alpha^2 S_\Psi(\Omega)}{(\gamma + \gamma_r)^2 + \Omega^2}\left(\frac{g^2}{\gamma^2 + \Omega^2}\right).$$

Readout coupling γ_r

Response Width for Multi-mode Resonators

Signal response width can be significantly broadened in a multi-mode system compared to single-mode ones:



$$\frac{\Delta\omega_r^{\rm MM}}{\Delta\omega_r^{\rm SM}} \propto \left(\frac{g}{\gamma n_{\rm occ}}\right)^{\frac{2N}{2N+1}} \to \frac{Q_0}{n_{\rm occ}}.$$

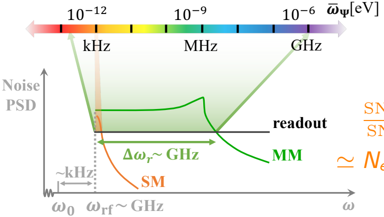
- New sensitivity limit for multi-mode resonators.

Simultaneous Resonant and Broadband Detection

- e-fold time: 10^7 s.
- **DC cavity and LC circuits**: $\frac{\mathrm{SNR}^2_{\mathrm{MM}}}{\mathrm{SNR}^2_{\mathrm{SM}}} \simeq \frac{Q_0}{n_{\mathrm{occ}}}$.

  High n_{occ} of LC circuits at low frequency made enhancement ineffective.

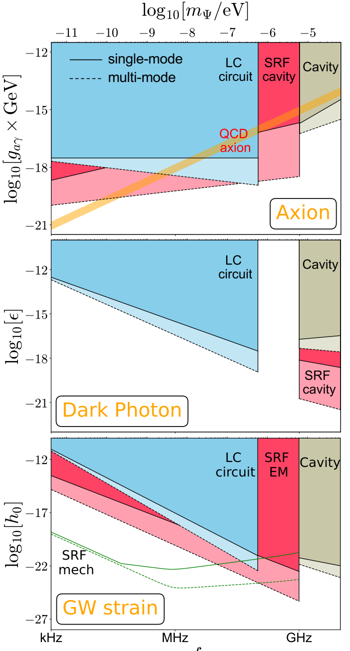
- **SRF heterodyne upconversion**:



$$\frac{\mathrm{SNR}^2_{\mathrm{MM}}}{\mathrm{SNR}^2_{\mathrm{SM}}} \simeq N_e \frac{\bar{\omega}_\Psi}{\omega_{\mathrm{rf}}} \frac{Q_0}{n_{\mathrm{occ}}}$$

Simultaneous scan $N_e = 6$ orders of ω_Ψ with significant response.

High Q_0 and constant n_{occ} enable reaching QCD axion with $m_a >$ kHz.



Property of Ultralight Dark Matter

Galaxy formation: virialization $\rightarrow \sim 10^{-3}c$ velocity fluctuation, thus kinetic energy $\sim 10^{-6} m_\Psi c^2$.
Effectively coherent waves:

$$\Psi(\vec{x}, t) = \frac{\sqrt{2\rho_\Psi}}{m_\Psi} \cos\left(\omega_\Psi t - \vec{k}_\Psi \cdot \vec{x} + \delta_0\right).$$

- Bandwidth: $\delta\omega_\Psi \simeq m_\Psi \left\langle v_{\rm DM}^2 \right\rangle \simeq 10^{-6} m_\Psi$, $Q_\Psi \simeq 10^6$.
- Correlation time: $\tau_\Psi \simeq \text{ms}\,\frac{10^{-6}\text{eV}}{m_\Psi}$.
  Power law detection is used to make integration time longer than τ_Ψ.
- Correlation length: $\lambda_d \simeq 200\ \text{m}\frac{10^{-6}\text{eV}}{m_\Psi} \gg \lambda_c = 1/m_\Psi$.
  Sensor array can be used within λ_d.

Quantization of Cavity Modes

- Quantized EM modes with wavefunctions $\vec{\epsilon}_n(\vec{r})$ In Coulomb gauge:

$$\vec{A} = \sum_n \frac{1}{\sqrt{2\omega_{\mathrm{rf}}^n}} \hat{a}_n^\dagger \vec{\epsilon}_n(\vec{r}) e^{-\mathrm{i}\omega_{\mathrm{rf}}^n t} + h.c..$$

- The Hamiltonian for each mode reduces to **harmonic oscillator**:

$$H_0 = \frac{1}{2}\int_V \left(\vec{E}^2 + \vec{B}^2\right) \mathrm{d}V = \sum_n \omega_{\mathrm{rf}}^n \left(\hat{a}_n^\dagger \hat{a}_n + \frac{1}{2}\right),$$

- Interaction with effective currents:

$$H_{\mathrm{int}} = \int_V \vec{A} \cdot \vec{J}_{\mathrm{eff}} \, \mathrm{d}V = \alpha\Psi \left(\hat{a}\, e^{\mathrm{i}\omega_{\mathrm{rf}} t} + \hat{a}^\dagger \, e^{-\mathrm{i}\omega_{\mathrm{rf}} t}\right)/\sqrt{2},$$

where α contains geometric overlapping factor $\eta_n \propto \int_V \vec{\epsilon}_n \cdot \vec{J}_{\mathrm{eff}} \, \mathrm{d}V$.

Quantization of Circuit Modes

- Energy stored in an inductor and a capacitor:

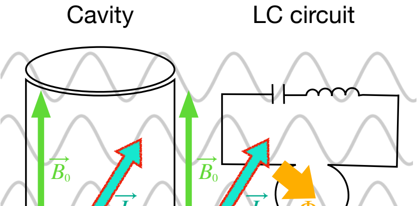
$$H_0 = \frac{\Phi^2}{2L} + \frac{Q^2}{2C} = \omega_{\rm rf}\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right).$$

- Interaction with external Φ_Ψ:

$$H_{\rm int} = \frac{\Phi\,\Phi_\Psi}{L} = \alpha\Psi\left(\hat{a}\,e^{{\rm i}\omega_{\rm rf}t} + \hat{a}^\dagger\,e^{-{\rm i}\omega_{\rm rf}t}\right)/\sqrt{2}.$$

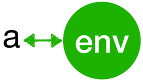
- **Circuit representation** of cavity modes with an antenna:

$$\Phi = \int_{\rm Ant} \vec{A}(\vec{r},t)\cdot {\rm d}\vec{l}.$$



Open quantum system

A system interacting with environment:



- System mode $\hat{a}$ couples to infinite degrees of freedom $\hat{w}_\omega$:

$$i\hbar\sqrt{2\gamma_r}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}[\hat{a}^\dagger\hat{w}_\omega - \hat{a}\hat{w}_\omega^\dagger] + \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\hbar\omega\hat{w}_\omega^\dagger\hat{w}_\omega .$$

- Fourier transformation: **0-dim localized mode** $\hat{a}$ couples to an 1-dim bulk w_ξ (transmission line):

$$i\hbar\sqrt{2\gamma_r}\hat{a}^\dagger\hat{w}_{\xi=0} + \mathrm{h.c.} + i\hbar\int_{-\infty}^{+\infty}d\xi\hat{w}_\xi^\dagger\partial_\xi\hat{w}_\xi .$$

- Equations of motion for $\hat{a}$ and outgoing mode $\hat{w}_{0_+}$:

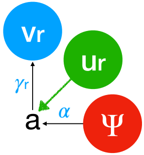
$$\dot{\hat{a}} = -\gamma_r\hat{a} + \sqrt{2\gamma_r}\hat{w}_{0_-}; \qquad \hat{w}_{0_+} = \hat{w}_{0_-} - \sqrt{2\gamma_r}\hat{a}$$

Single-mode Resonator as Quantum Sensor

- For a resonator $\hat{a}$ **probing weak signal** Ψ: $\alpha\left(\hat{a}+\hat{a}^{\dagger}\right)\Psi$

- Readout for outgoing mode $\hat{v}_r \equiv \hat{w}_{0_+}$:

$$\hat{v}_r = \frac{\Omega - i\gamma_r}{\Omega + i\gamma_r}\hat{u}_r + \frac{\sqrt{2\gamma_r}\alpha}{\Omega + i\gamma_r}\Psi.$$

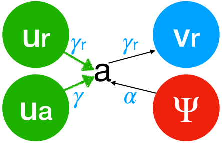


- Fluctuations in incoming mode $\hat{u}_r \equiv \hat{w}_{0_-}$ with quantum limited power spectral density $S_{\rm r} = 1$.

- Resonant signal spectrum $S_{\rm sig} = \frac{\gamma_r \alpha^2}{\gamma_r^2+\Omega^2}\, S_\Psi(\Omega)$.

- Trade-off between peak sensitivity and bandwidth by tuning γ_r.

Intrinsic loss and fluctuation



- **Intrinsic loss** $\propto \gamma$ exists, characterized by quality factor $Q_{\rm int} \equiv \omega/\gamma$.

- **Fluctuation-dissipation theorem** predicts intrinsic loss fluctuations

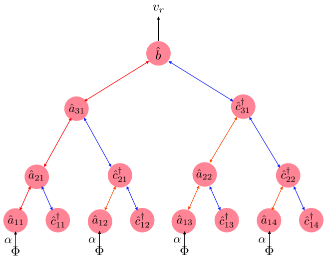
$$S_{\rm int}(\Omega) = \frac{4\gamma\gamma_r}{(\gamma+\gamma_r)^2+\Omega^2} n_{\rm occ}.$$

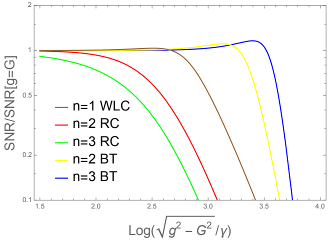
- Using scattering matrix elements:

$$S_{\rm sig} = |S_{0r}|^2 \frac{\alpha^2}{4\gamma} S_\Psi, \qquad S_{\rm noise} = |S_{0r}|^2 n_{\rm occ} + |S_{rr}|^2 \frac{1}{2} + \frac{1}{2}.$$

- **Standard quantum limit for power law detection: resonant $S_{\rm int}$+ flat $S_{\rm r}$.** [Chaudhuri et al 18']

Binary Tree Haloscope





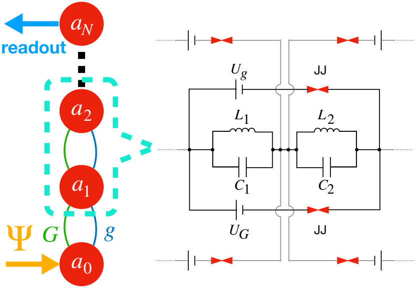
- **Fully $\mathcal{PT}$-symmetric setup** with $\hat{a}_{ij} \leftrightarrow \hat{c}^\dagger_{ij}$ brings strong **robustness**.

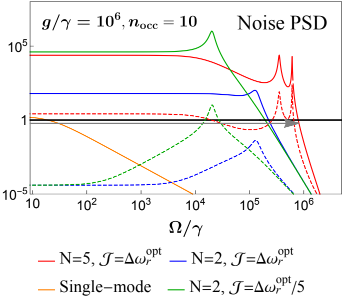
- Multi-probing sensors leads to **coherent enhancement**:

$$S^{\mathrm{BT}}_{\mathrm{sig}}(\Omega) = 2^{2n-2} S^{\mathrm{RC}}_{\mathrm{sig}}(\Omega).$$

Quantum Limit for Multi-mode resonators

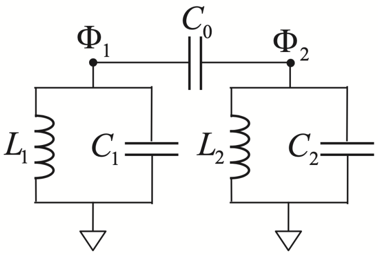
Scan bandwidth can be significantly increased in a multi-mode system.





- Far beyond the one of single-mode resonators.
- New quantum limit for multi-mode resonators.

Beam splitting coupling



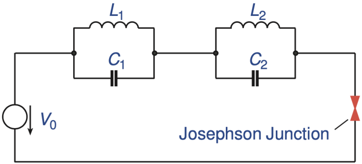
- Use an additional capacitor to couple two LC circuits:

$$H = \frac{1}{2}C_1\dot{\Phi}_1^2 + \frac{1}{2}C_2\dot{\Phi}_2^2 + \frac{1}{2L_1}\Phi_1^2 + \frac{1}{2L_2}\Phi_2^2 + \frac{1}{2}C_0(\dot{\Phi}_1 - \dot{\Phi}_2)^2.$$

- Conjugate momentum to Φ_i involves mixing. Interaction potential:

$$\beta\hbar\sqrt{\omega_1\omega_2}(\hat{a}_1 - \hat{a}_1^\dagger)(\hat{a}_2 - \hat{a}_2^\dagger) \sim \hat{a}_1\hat{a}_2^\dagger + h.c.,$$

Non-Degenerate Parametric amplifier coupling



- Use a DC voltage and a Josephson junction to couple two LC circuits:

$$
\begin{aligned}
V &= -\frac{\hbar I_J}{2e_0}\cos\left(\omega_0 t + \frac{2e_0}{\hbar}(\Phi_2 + \Phi_3)\right) \\
&= -\frac{\hbar I_J}{2e_0}\cos\left(\omega_0 t + \kappa_2(a_2 + a_2^\dagger) + \kappa_3(a_3 + a_3^\dagger)\right) \\
&\sim \frac{\hbar I_J}{4e_0}\kappa_2\kappa_3[a_2 a_3 + a_2^\dagger a_3^\dagger],
\end{aligned}
$$